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Market entry waves and volatility outbursts in stock markets☆

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ABSTRACT

We develop a simple agent-based financial market model in which speculators' market entry decisions are subject to herding behavior and market risk. In addition, speculators' orders depend on price trends, market misalignments and fundamental news. Using a mix of analytical and numerical tools, we show that a herding-induced market entry wave may amplify excess demand, triggering lasting volatility outbursts. Eventually, however, higher stock market risk reduces stock market participation and volatility decreases again. Simulations furthermore reveal that our approach is also able to produce bubbles and crashes, excess volatility, fat-tailed return distributions and serially uncorrelated price changes. Moreover, trading volume is persistent and correlated with volatility.

1. Introduction

The goal of our paper is to develop a simple agent-based financial market model to explain a number of important stylized facts of stock markets. In particular, we analytically and numerically demonstrate that speculators' market entry and exit behavior may give rise to volatility clustering. Our model's key features and its main implications may be summarized as follows. We assume that there is a market maker who adjusts stock prices with respect to speculators' orders, which, in turn, use technical and fundamental trading rules to determine their trading behavior. Speculators' market entry decisions depend on two socio-economic principles. First, speculators are subject to herding behavior and increasingly enter the stock market as the number of active speculators increases. Second, speculators react to stock market risk. The higher the past volatility of the stock market, the lower the probability that a speculator will enter the stock market. As it turns out, the stock market is relatively stable if the number of active speculators is low. Since stock market risk is then perceived as negligible, more and more speculators become active. Consequently, excess demand increases, the market maker adjusts stock prices more strongly and volatility picks up. Due to the increase in stock market risk, stock market participation eventually decreases again. Confronted with a lower excess demand, the market maker needs to adjust stock prices less strongly.

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We also show that the repeated inflow and outflow of speculators along with their heterogeneous trading behavior may produce bubbles and crashes, excess volatility, serially uncorrelated returns and a fat-tailed return distribution. Moreover, trading volume displays significant memory effects and is strongly correlated with volatility. Keeping track of the individual speculators' wealth dynamics reveals that heterogeneity among speculators may be a persistent phenomenon of financial markets, i.e. neither do a few speculators accumulate all the wealth and dominate the market nor does a substantial fraction of speculators go bankrupt and vanish from the market.

Our paper adds to the burgeoning stream of literature on agent-based financial market models (see Chiarella et al., 2009a; Hommes and Wagener, 2009; Lux, 2009 for surveys). Within these models, speculators apply technical and fundamental trading rules to determine their orders. Technical trading rules (Murphy, 1999) are usually based on trend extrapolation and tend to destabilize the dynamics of financial markets. In contrast, fundamental trading rules (Graham and Dodd, 1951) bet on mean reversion, exercising a stabilizing impact on the dynamics of financial markets. Models by Day and Huang (1990), De Grauwe et al. (1993), Brock and Hommes (1998), LeBaron et al. (1999), Farmer and Joshi (2002), Chiarella et al. (2007), Franke and Westerhoff (2012) and Jacob Leal and Napoletano (2017), for example, show that (non-linear) interactions between speculators relying on technical and fundamental trading rules can produce dynamics which resembles the dynamics of actual financial markets quite closely. Without question, this line of research helps us to improve our understanding of the functioning of financial markets. For instance, agent-based financial market models reveal that a bubble may emerge if speculators forcefully rely on technical analysis while a crash can be set in motion if speculators put more weight on fundamental analysis. Such a time-varying impact of technical and fundamental trading rules can also produce volatility clustering. Financial markets tend to be relatively stable when speculators prefer fundamental analysis but turn wilder when speculators opt for technical analysis.

Herding behavior plays a prominent role in a number of agent-based financial market models. In Alfarano and Lux (2007); Kirman (1993); Lux and Marchesi (1999), speculators' herding behavior influences whether they choose technical or fundamental trading rules to determine their orders. Cont and Bouchaud (2000) and Stauffer et al. (1999) assume that speculators' herding behavior influences whether they are optimistic or pessimistic. Bischi et al. (2006) show that complex asset price dynamics may emerge if speculators mimic the buying and selling behavior of other speculators. LeBaron and Yamamoto (2008) study imitation behavior which results from social learning and show that it may be responsible for long memory effects in trading volume and volatility. Tedeschi et al. (2012) develop a model in which speculators imitate the behavior of more successful speculators. In Schmitt and Westerhoff (2017), speculators' herding behavior may lead to changes in the heterogeneity of trading rules applied. Compared to these models, we assume in our paper that speculators' herding behavior affects their stock market participation.

In fact, empirical evidence suggests that stock market participation changes over time and is influenced by social interactions. Most importantly for our approach, Hong et al. (2004, 2005), Brown et al. (2008) and Shiller (2015) report that households and professional investors regard a stock market as increasingly attractive the more of their peers participate in it. Surprisingly, there are only a few agent-based models which explicitly study speculators' market entry and exit behavior. Alfi et al. (2009b, 2009c, 2009a) show that agent-based models with a fixed number of speculators may lose their ability to produce realistic dynamics if the number of speculators is set either too high or too low. Against this background, they endogenize the number of speculators and explore under which conditions the model dynamics may self-organize such that the number of active speculators approaches a level which generates realistic dynamics. Iori (1999, 2000) develops a more involved agent-based simulation framework with heterogeneous interacting agents. Due to trade frictions, such as trading costs or information processing constraints, speculators may become inactive. However, communication and imitation among speculators may lead to a spontaneous spark in stock market participation and elevate price fluctuations. To study the effects of transaction taxes, Westerhoff and Dieci (2006) develop a model in which speculators have the choice between technical trading, fundamental trading and being inactive. Speculators' choices depend on the past profitability of these alternatives. Schmitt and Westerhoff (2016) show that although speculators' inflow and outflow may create bubbles and crashes, their market entry and exit behavior is not subject to herding effects.

Our approach differs to these contributions in several dimensions. One advantage of our model is that its deterministic skeleton allows us to derive a number of analytical insights which make the model's functioning and the origin of volatility clustering rather transparent. For instance, our model possesses a steady state in which prices reflect their fundamental values and in which all speculators are active. We analytically show that this steady state becomes unstable (via a Neimark–Sacker bifurcation) if speculators strongly extrapolate past price trends. Simulations reveal that the dynamics we then observe are characterized by alternating periods of high volatility, pushing destabilizing speculators out of the stock market, and periods of low volatility, attracting destabilizing speculators to the stock market. The same forces are at work in a stochastic version of our model which is able to mimic a number of important time series properties of stock markets. It is important to note that our results are not driven by speculators who constantly lose money or by speculators who become very rich.

The rest of our paper is organized as follows. In Section 2, we present our simple agent-based financial market model. In Section 3, we study the properties of the model's deterministic skeleton. In Section 4, we illustrate that the model's stochastic version is able to replicate a number of important stylized facts of stock markets. In Section 5, we conclude our paper and point out some avenues for future research.

2. A simple agent-based financial market model

The key elements of our agent-based financial market model may be summarized as follows. We consider a stock market which is populated by a single market maker and a time-varying number of heterogeneous speculators. A market maker adjusts the stock price with respect to speculators' orders which, in turn, depend on the stock market's price trend, its misalignment and current fundamental news. The probabilistic market entry decision of a speculator is repeated at the beginning of each trading period. We assume that the probability that a given speculator will enter the market increases with current stock market participation and decreases with current stock market risk. Since the total number of speculators is fixed, the number of active speculators follows a binomial distribution. As we will see, a gradual inflow and outflow of speculators may lead to alternating periods of high and low volatility.

Let us turn to the details of our model. We assume that the stock market's log fundamental value follows a random walk. To be precise, the stock market's log fundamental value in period $t+1$ is given by

$$F_{t+1} = F_t + n_{t+1}. \tag{1}$$

Fundamental shocks n_t are normally distributed with mean zero and constant standard deviation σ^n. Note that fundamental shocks represent the only extrinsic force in our model.

Following Day and Huang (1990), a market maker adjusts the stock price using a log-linear price-adjustment rule, i.e.

$$P_{t+1} = P_t + a\sum_{i=1}^{N_t} D_{t,i}, \tag{2}$$

where P_t stands for the log of the stock price at time t, a is a positive price adjustment parameter, N_t represents the number of active speculators, and $D_{t,\,i}$ denotes the order placed by an active speculator.[1] Accordingly, the market maker increases the stock price if buying exceeds selling, and vice versa.

As in Chiarella and Iori (2002); Chiarella et al. (2009b); Pellizzari and Westerhoff (2009), the order placed by an active speculator i depends on a linear blend of technical and fundamental trading signals. In addition, speculator i's trading behavior is influenced by the arrival of new information. The order placed by speculator i is formalized as

$$D_{t,i} = b_{t,i}(P_t - P_{t-1}) + c_{t,i}(F_t - P_t) + d_{t,i}(F_t - F_{t-1}). \tag{3}$$

The first component of (3) reflects speculator i's technical trading (Murphy, 1999). Speculator i receives a buying (selling) signal if prices increase (decrease). Parameter $b_{t,\,i} > 0$ defines how strongly speculator i reacts to the price signal. The second component of (3) formalizes speculator i's fundamental trading (Graham and Dodd, 1951). Since $c_{t,\,i}$ is a positive reaction parameter, speculator i obtains a buying signal when the market is undervalued and a selling signal when it is overvalued. The third component of (3) indicates that speculator i also reacts to the arrival of new information (Pearce and Roley, 1985). Positive news stimulates buying orders while negative news triggers selling orders. Of course, reaction parameter $d_{t,\,i}$ is also positive. In the following, we assume that reaction parameters $b_{t,\,i}$, $c_{t,\,i}$ and $d_{t,\,i}$ are uniformly distributed, i.e. $b_{t,i} \sim U(b-\beta, b+\beta)$, $c_{t,i} \sim U(c-\gamma, c+\gamma)$ and $d_{t,i} \sim U(d-\delta, d+\delta)$, with $b > \beta \geq 0$, $c > \gamma \geq 0$ and $d > \delta \geq 0$. Hence, all speculators follow their own time-varying trading strategy.

Before we continue with the description of our approach, let us derive a convenient model property. First, inserting (3) in (2) reveals that

$$\begin{aligned} P_{t+1} &= P_t + a\sum_{i=1}^{N_t}\left(b_{t,i}(P_t - P_{t-1}) + c_{t,i}(F_t - P_t) + d_{t,i}(F_t - F_{t-1})\right) \\ &= P_t + a(P_t - P_{t-1})\sum_{i=1}^{N_t} b_{t,i} + a(F_t - P_t)\sum_{i=1}^{N_t} c_{t,i} + a(F_t - F_{t-1})\sum_{i=1}^{N_t} d_{t,i}. \end{aligned} \tag{4}$$

Recall next that the sum of independently uniformly distributed random variables follows a uniform sum distribution.[2] Defining $\sum_{i=1}^{N_t} b_{t,i} = B_t$, $\sum_{i=1}^{N_t} c_{t,i} = C_t$ and $\sum_{i=1}^{N_t} d_{t,i} = D_t$ yields $B_t \sim USD(N_t, \{b-\beta, b+\beta\})$, $C_t \sim USD(N_t, \{c-\gamma, c+\gamma\})$ and $D_t \sim USD(N_t, \{d-\delta, d+\delta\})$, respectively, and we can rewrite (4) as

$$P_{t+1} = P_t + a(B_t(P_t - P_{t-1}) + C_t(F_t - P_t) + D_t(F_t - F_{t-1})). \tag{5}$$

Apparently, our setup has the convenient property that it is not necessary to evaluate the trading rules of all N_t active speculators, each consisting of three different components, to simulate its dynamics. We simply need to generate three uniform sum distributed random variables. Moreover, the means and variances of the three uniform sum distributed random variables are given by $\mu_B = bN_t$, $\mu_C = cN_t$, $\mu_D = dN_t$, $\sigma_B^2 = \frac{N_t}{3}\beta$, $\sigma_C^2 = \frac{N_t}{3}\gamma$ and $\sigma_D^2 = \frac{N_t}{3}\delta$, respectively. In particular, note that the means of the uniform sum distributed random variables increase with the number of active speculators, i.e. if there is an inflow of speculators, there is, on average, stronger trend extrapolation trading, a stronger mean reversion behavior and

[1] For notational convenience, we use the index $i = 1, 2, \ldots, N_t$ to refer to active speculators in trading period t. Clearly, index i does not stand for a specific speculator.

[2] A uniform sum distribution, also called an Irwin–Hall distribution, approaches a normal distribution as the number of added random variables increases.

a stronger reaction to new information. It is easily imaginable that this will have a destabilizing impact on the model dynamics, at least for some parameter constellations. We also remark that the variances of the three uniform sum distributed random variables vanish if β, γ and δ approach zero.

Let us now return to our model. At the beginning of each trading period, speculators decide whether to enter the stock market. We assume that speculators' probabilistic market entry decisions are influenced by two socio-economic principles. In line with empirical evidence reported by Hong et al. (2004, 2005), Brown et al. (2008) and Shiller (2015), speculators regard a stock market as increasingly attractive when more speculators are already active. A similar herding perspective is taken in Iori (1999, 2000). Moreover, speculators' market entry decisions also depend on market circumstances: the higher the stock market risk, the less attractive the stock market appears to be. As in Alfi et al. (2009b, 2009c, 2009a), stock market risk is represented by the stock market's volatility

$$V_t = mV_{t-1} + (1-m)(P_t - P_{t-1})^2, \tag{6}$$

where $0 \le m < 1$ is a memory parameter controlling how strongly current and past price changes affect volatility. We summarize both socio-economic principles by the following relative fitness function

$$A_t = hN_{t-1} - \nu V_t, \tag{7}$$

where h and ν are positive parameters. Accordingly, market participation is regarded as increasingly attractive the more speculators are active in the market and less attractive the higher the stock market's past volatility.

We use exponential replicator dynamics (Hofbauer and Sigmund, 1988; Hofbauer and Weibull, 1996) to model speculators' probabilities of entering the market. The probability that a speculator will enter the stock market can thus be written as

$$W_t = \frac{W_{t-1}}{W_{t-1} + (1 - W_{t-1})\exp[-\lambda A_t]}, \tag{8}$$

where parameter $\lambda > 0$ denotes speculators' intensity of choice. Note that the exponential replicator dynamics term has three important properties. First, speculators' probabilities of entering the market depend positively on the stock market's relative fitness. The higher the stock market's relative fitness, the more probable it is that speculators will enter the market. Second, an increase in λ implies that speculators react more sensitively to the stock market's relative fitness. If speculators' intensity of choice approaches zero, they have a 50% probability of entering the market. If speculators' intensity of choice goes to plus infinity, the probability that they will enter the market is either 100% if the herding component dominates the risk component or zero percent otherwise. Third, market entry probabilities display a mild form of inertia. If W_{t-1} is either close to zero or close to one, market entry probabilities depend less strongly on the stock market's relative fitness.[3]

Obviously, the number of active speculators is binomially distributed, i.e.

$$N_t \sim B(N, W_t), \tag{9}$$

where $N > 0$ denotes the total number of speculators. As is well known, the mean and variance of the number of active speculators are given by NW_t and $NW_t(1 - W_t)$, respectively.

3. Analysis of the model's deterministic skeleton

In this section, we explore the model's deterministic skeleton. In Section 3.1, we derive the model's dynamical system and analyze under which conditions the model's steady states are locally asymptotically stable. In Section 3.2, we introduce a base parameter setting to explain the functioning of our deterministic model. In Section 3.3, we study how the model's parameters affect its global dynamics. In Section 3.4, we show that the model may also give rise to coexisting attractors and produce bubbles and crashes. In Section 3.5, we briefly discuss the dynamics of our model for an alternative parameter setting.

3.1. Dynamical system, steady states and local stability

By setting $\beta = \gamma = \delta = \sigma_n = 0$ and introducing the auxiliary variable $\tilde{P}_t = P_{t-1}$, we can summarize our model by the four-dimensional nonlinear map

$$X : \begin{cases} P_{t+1} = P_t + aN_t\{b(P_t - \tilde{P}_t) + c(F - P_t)\} \\ \tilde{P}_{t+1} = P_t \\ V_{t+1} = mV_t + (1-m)(P_{t+1} - P_t)^2 \\ N_{t+1} = N\frac{N_t}{N_t + (N - N_t)\exp[-\lambda(hN_t - \nu V_{t+1})]} \end{cases}. \tag{10}$$

[3] Dindo and Tuinstra (2011) provide a deeper discussion of exponential replicator dynamics. Further economic examples in this direction include Bischi et al. (2015), Kopel et al. (2014) and Schmitt et al. (2017).

Since we set the scaling parameters a and λ to 1, the dynamics depends solely on seven parameters: b, c, N, h, v, F and m.[4]

Straightforward computations reveal that our dynamical system may give rise to two steady states, namely

$$X_1^* = (P^*, \tilde{P}^*, V^*, N^*) = (F, F, 0, N) \tag{11}$$

and

$$X_2^* = (P^*, \tilde{P}^*, V^*, N^*) = (P^*, \tilde{P}^*, 0, 0). \tag{12}$$

As can be seen, the steady-state price is given by the fundamental value, the stock market's volatility is zero and all speculators are active at X_1^*, while X_2^* has zero active speculators, an indeterminate price and also a volatility of zero. Since the second steady state is economically uninteresting, we will focus now on X_1^*, which we also call the fundamental steady state of our model.

To determine the stability of X_1^*, we derive the characteristic polynomial from the Jacobian matrix of (10), i.e.

$$J(X_1^*) = \begin{pmatrix} 1 + bN - cN & -bN & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & e^{-hN} \end{pmatrix}, \tag{13}$$

and obtain

$$(e^{-hN} - z)(m - z)(z^2 + z(cN - bN - 1) + bN) = 0. \tag{14}$$

The steady state is locally asymptotically stable if the eigenvalues of the polynomial are less than one in modulus (see, e.g. Gandolfo, 2009 or Medio and Lines, 2001). It is easy to see from (14) that the first two eigenvalues are given by $z_1 = e^{-hN}$ and $z_2 = m$. Since we assume that $h > 0$, $N > 0$ and $0 \leq m < 1$, we have $|z_{1,\,2}| < 1$. However, the eigenvalues of $z^2 + z(cN - bN - 1) + bN$ are less than one in modulus if and only if

$$c > 0, \tag{15}$$

$$c < c_c = \frac{2}{N} + 2b \tag{16}$$

and

$$b < b_c = \frac{1}{N} \tag{17}$$

simultaneously apply. Recall that c is a positive reaction parameter, which is why condition (15) is always fulfilled. According to (16), the fundamental steady state becomes unstable if c crosses c_c, a situation which leads to a flip bifurcation and the onset of a period-two cycle. If b exceeds its critical value b_c, condition (17) is violated, which is associated with a Neimark–Sacker bifurcation, i.e. the emergence of a cyclical motion. In economic terms, these two conditions imply that the steady state becomes unstable if speculators react to market misalignments or to price trends too strongly. Note that (16) and (17) also depend on the total number of speculators. Hence, the stock market also becomes unstable if N increases.

To visualize our analytical results, we depict in Fig. 1 combinations of b and c for which the model's fundamental steady state is locally asymptotically stable. Since the two black lines represent stability conditions (16) and (17), the model's fundamental steady state is always locally asymptotically stable for parameter combinations within these two lines. As indicated by the arrows, an increase in parameter b may cause a loss of stability via a Neimark–Sacker bifurcation while an increase in parameter c may cause a loss of stability via a flip bifurcation. Moreover, the gray shaded area indicates the parameter space for which the steady state becomes unstable if the number of speculators increases from N to N'.

For the sake of completeness, note that the Jacobian matrix of (10) at the second steady state is given by

$$J(X_2^*) = \begin{pmatrix} 1 & 0 & 0 & c(F - P^*) \\ 1 & 0 & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{18}$$

from which the characteristic polynomial

$$-z(m - z)(1 - z)^2 = 0 \tag{19}$$

can be derived. Therefore, we obtain $z_1 = 0$, $z_2 = m$ and $z_{3,4} = 1$, which implies that the second steady state is always unstable.

[4] Note that N_t is given by NW_t, i.e. we focus in this section on the mean dynamics of the active number of speculators. Such a procedure is common in this line of research, see, for instance, Sandholm (2015). In fact, numerical experiments confirm that our analytical results predict the properties of the non-mean dynamics quite well. The same is true for the simulation results presented in Section 3. For simplicity, we call N_t the active number of speculators.

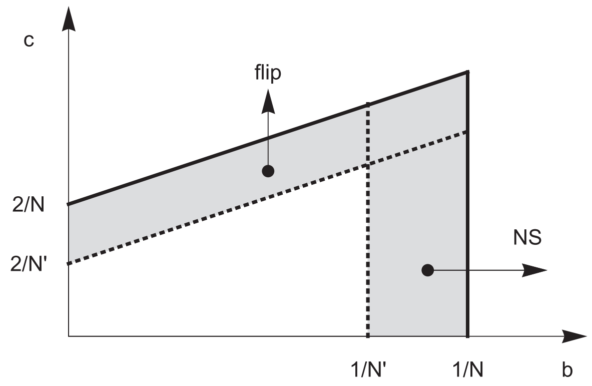

Fig. 1. Combinations of b and c for which the fundamental steady state is locally asymtotically stable. The two black lines represent stability conditions (16) and (17), i.e. $c = \frac{2}{N} + 2b$ and $b = \frac{1}{N}$, respectively.

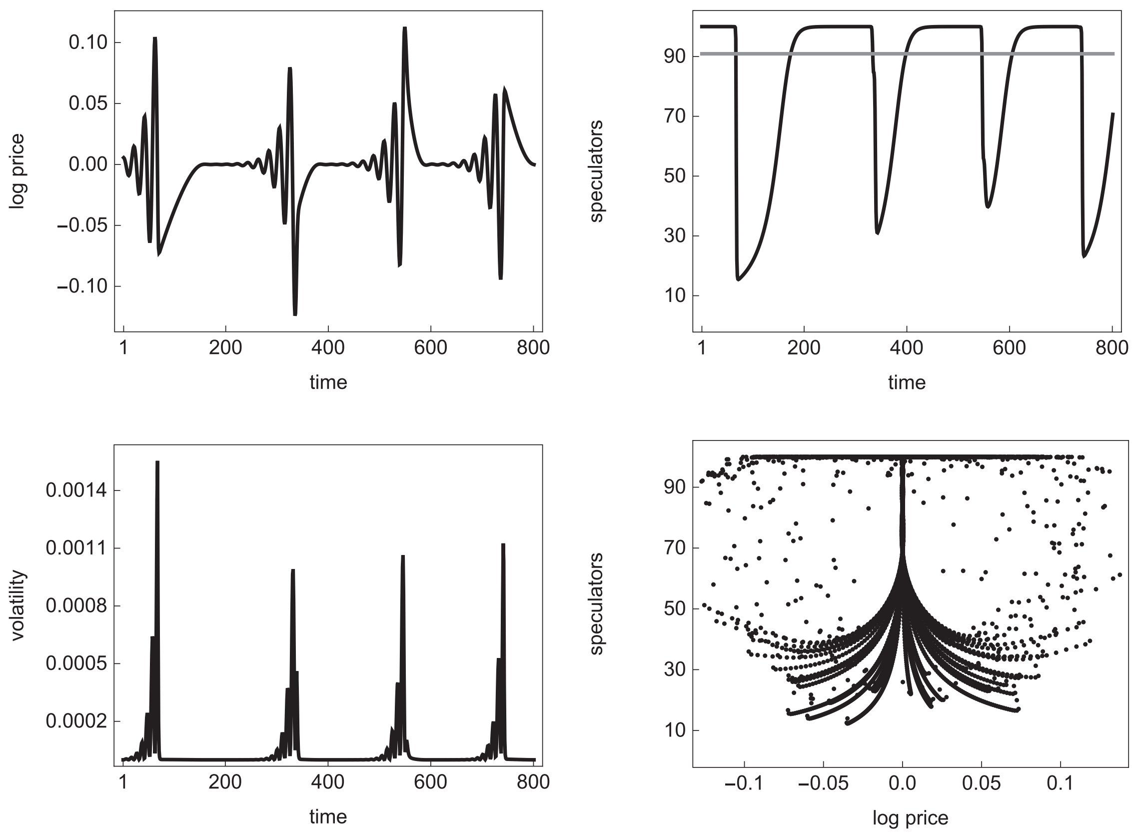

Fig. 2. Dynamics of the deterministic model for our base parameter setting. The panels show the evolution of the log price, the number of active speculators, the stock market's volatility and the number of active speculators versus the log price, respectively. The underlying parameter setting is given in Section 3.2.

3.2. Base parameter setting and functioning of the model

To be able to explain the functioning of our deterministic model, represented by the four-dimensional nonlinear map (10), we make use of the following parameter setting: $b = 0.011$, $c = 0.001$, $N = 100$, $h = 0.001$, $v = 2000$, $m = 0.25$ and $F = 0$. Recall that the scaling parameters a and λ are set equal to 1. Accordingly, we have $b > 1/N$, which implies that our fundamental steady state, i.e. $X_1^* = (F, F, 0, N)$, is unstable and that its instability is due to a Neimark–Sacker bifurcation. Fig. 2 shows a representative simulation run for 800 periods (after omitting a longer transient period). The first three panels depict the evolution of the log price, the number of active speculators and the stock market's volatility, respectively, while the fourth panel presents the number of active speculators versus the log price.

Obviously, our model is able to produce intricate dynamics, in particular, alternating periods with low and high volatility. In a nutshell, the working of our deterministic model may be summarized as follows. Note first that the gray line in the top right panel of Fig. 2 indicates the threshold for the number of active speculators for which the dynamics of our deterministic model becomes unstable, i.e. $N_c = \frac{1}{b} \approx 90.91$. If the number of active speculators is below N_c, the market is stable and prices converge slowly towards their fundamental value. Since volatility is relatively low during these periods, speculators' herding behavior dominates their risk aversion and more and more speculators enter the stock market. However, the picture changes if the number of active speculators exceeds N_c. Once $N_t > N_c$, the model dynamics becomes unstable, i.e. price fluctuations are characterized by oscillations with an increasing amplitude. As a result, volatility increases up to the point where speculators' risk aversion offsets their herding behavior. Speculators then start to exit the market and initiate a new period of relative stability. During the 800 depicted time steps, we witness four marked volatility outbursts. The strange attractor, visible in the bottom right panel, illustrates the complexity of the model dynamics.

3.3. The impact of the model's parameters on its global dynamics

To demonstrate how the global dynamics of our deterministic model depends on its parameters, we present a number of simulations in this section. Fig. 3 contains examples of how parameters *b, c* and *N* may influence the model dynamics. The first, second and third rows depict bifurcation diagrams for $0 < b < 0.020$, $0 < c < 0.004$ and $80 < N < 120$, respectively. While the left side presents their effect on log prices, the right side shows how they affect the number of active speculators. As predicted by our analytical results, we have $P^* = F = 0$ and $N^* = N = 100$ for $b < b_c = \frac{1}{N} = 0.01$. As soon as *b* exceeds this critical value, the fundamental steady state loses its stability and endogenous dynamics emerges. While the two top panels reveal that our model dynamics becomes unstable if technical trading is too aggressive, the second row shows that a stronger fundamental trading reduces the amplitude of price fluctuations. The two panels at the bottom also confirm our previous analytical results. The fundamental steady state loses its stability at $N = N_c = \frac{1}{b} \approx 90.91$, after which the amplitude of price dynamics and speculators' market entry and exit behavior increases with *N*.

In Fig. 4, we show bifurcation diagrams for $0 < h < 0.004$, $1000 < \nu < 3000$ and $0 < m < 1$. The left panels reveal again how log prices react to an increase in parameters *h*, ν and *m* and the right panels illustrate how this affects the number of active speculators. It can be seen from the two top panels that a stronger herding behavior increases the amplitude of price fluctuations as well as fluctuations in the number of active speculators. In contrast, price dynamics is less pronounced if speculators show a stronger risk-sensitive behavior. However, the amplitude of price fluctuations also increase with *m*. Of course, this also causes higher fluctuations in speculators' market entry and exit behavior.

3.4. Special features: coexisting attractors and bubbles and crashes

As is well known, nonlinear dynamical systems may give rise to a number of complicated dynamic phenomena. As indicated by the (asymmetric) bifurcation diagram in the top left panel of Fig. 3, our model may also produce coexisting attractors. The left panels of Fig. 5 provide an example of this outcome. The panels show from top to bottom the evolution of the log price, the number of active speculators and the number of active speculators versus the log price, respectively. We use the base parameter setting, except that we set $b = 0.0155$ (instead of $b = 0.011$). Moreover, the dynamics is depicted for two different initial values, represented in black and red. The top left panel of Fig. 5 shows that one set of initial conditions produces a sequence of bull markets while the other set of initial conditions produces a sequence of bear markets. As it turns out, the evolution of the number of active speculators is identical for both price trajectories. The bottom left panel of Fig. 5 reveals that the bull market dynamics is intricately intertwined with the bear market dynamics. In the absence of exogenous shocks, the model generates either persistent bull or persistent bear market dynamics. However, it is clear that the addition of some exogenous noise may easily push the dynamics from one attractor to the other. The overall dynamics is then characterized by erratic switches between bull and bear market dynamics.

Interestingly, the right panels of Fig. 5 demonstrate that our model is able to generate endogenous boom-bust dynamics. Once more we use the base parameter setting but assume that $c = 0.00004$ (instead of $c = 0.001$). The top right panel of Fig. 5 demonstrates that a boom period can be followed by another boom period but also by a severe crash (and since the model is symmetric, the same is true the other way around). The right panel in the center of Fig. 5 indicates that the dynamics is again driven by speculators' market entry and exit behavior. Since speculators' mean reversion trading is now relatively weak, we do not observe repeated volatility outbursts but the emergence of pronounced and lasting bull and bear market dynamics. It is easy to check that the instability of the model's fundamental steady state is again caused by a Neimark–Sacker bifurcation. However, the strange attractor visible in the bottom right panel of Fig. 5 reveals that the model dynamics is quite complicated for the underlying parameter setting.

3.5. Alternative parameter setting

So far, we focused mainly on the Neimark–Sacker bifurcation scenario. We now turn briefly to the flip bifurcation scenario. Fig. 6 is based on an alternative parameter setting: $a = 1$, $b = 0.005$, $c = 0.0301$, $N = 100$, $h = 0.005$, $\nu = 10$, $m = 0.1$, $\lambda = 1$ and $F = 0$. Hence, the model's fundamental steady state is unstable due to a flip bifurcation. Since the flip bifurcation occurs at $c = c_c = 0.03$, the onset of endogenous dynamics, initially in the form of a period-two cycle and then in the form

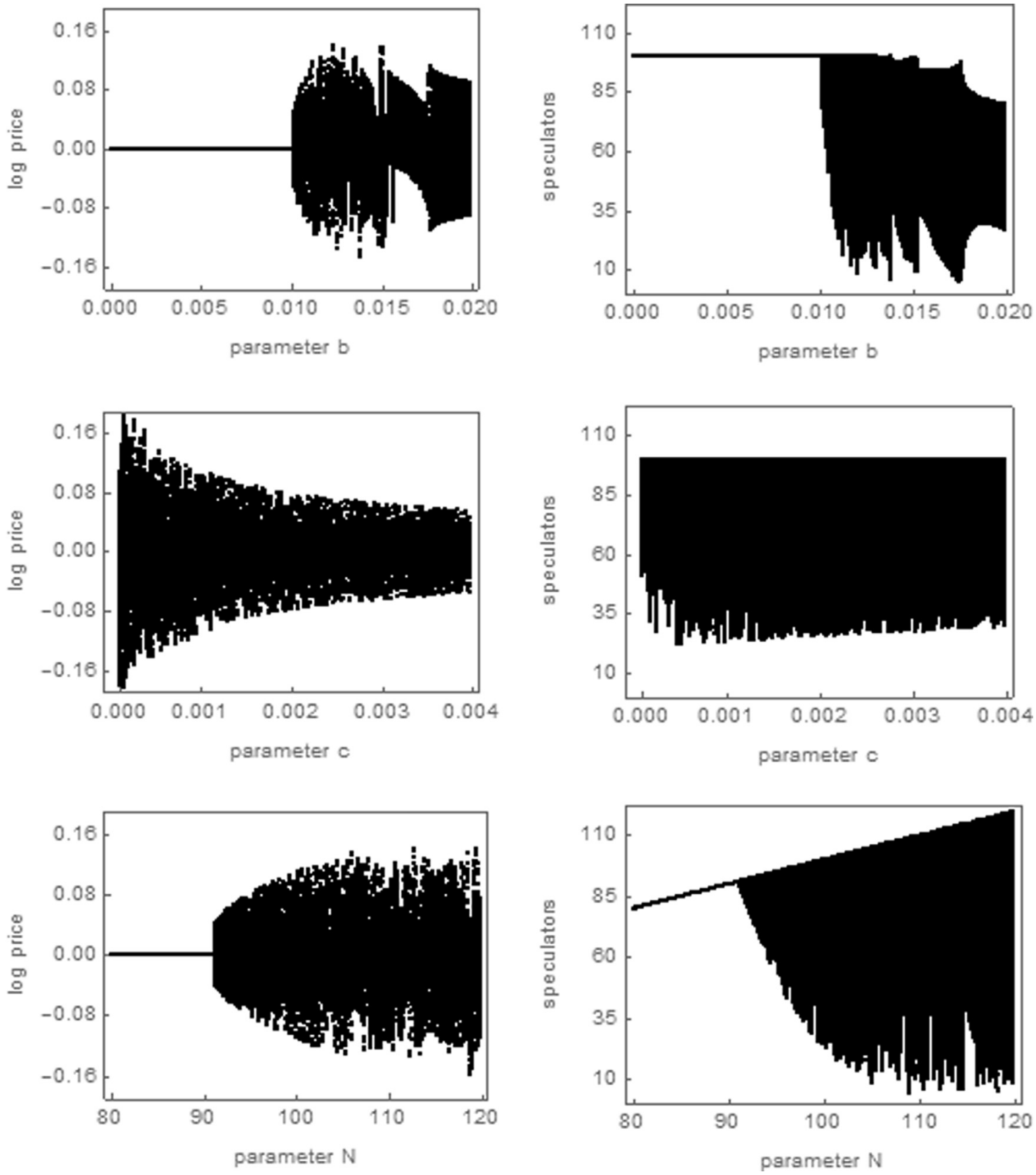

Fig. 3. The impact of *b, c* and *N*. The first, second and third rows show bifurcation diagrams for parameters *b, c* and *N*, respectively. While the left side presents their effect on log prices, the right side depicts how they affect the number of active speculators. Parameters are as in our base parameter setting.

of more complicated dynamics, is caused by speculators' excessively aggressive fundamental trading. The first three panels of Fig. 6 show the evolution of the log price, the number of active speculators and the stock market's volatility for 800 periods, respectively. As can be seen, the model is also able to produce volatility clustering for the alternative parameter setting. The reason for this is similar to before. If the number of active speculators is low, the market is stable. Since speculators' herding behavior outweighs their risk aversion, they quickly enter the stock market. This process renders the dynamics unstable and we observe increasing (improper) oscillations. Eventually, the associated increase in stock market risk makes the stock market become unattractive. Speculators exit the stock market and there is a brief period of market stability in which the price approaches its fundamental value. Then, the process repeats itself, albeit in an intricate manner. This is also confirmed by the right panel in the center of Fig. 6, which presents the corresponding dynamics in (N_t, P_t) space.

The bottom two panels of Fig. 6 show bifurcation diagrams for parameter *c*. The left panel reveals how the log price reacts to increasingly aggressive fundamental trading, while the right panel shows how this affects the number of active

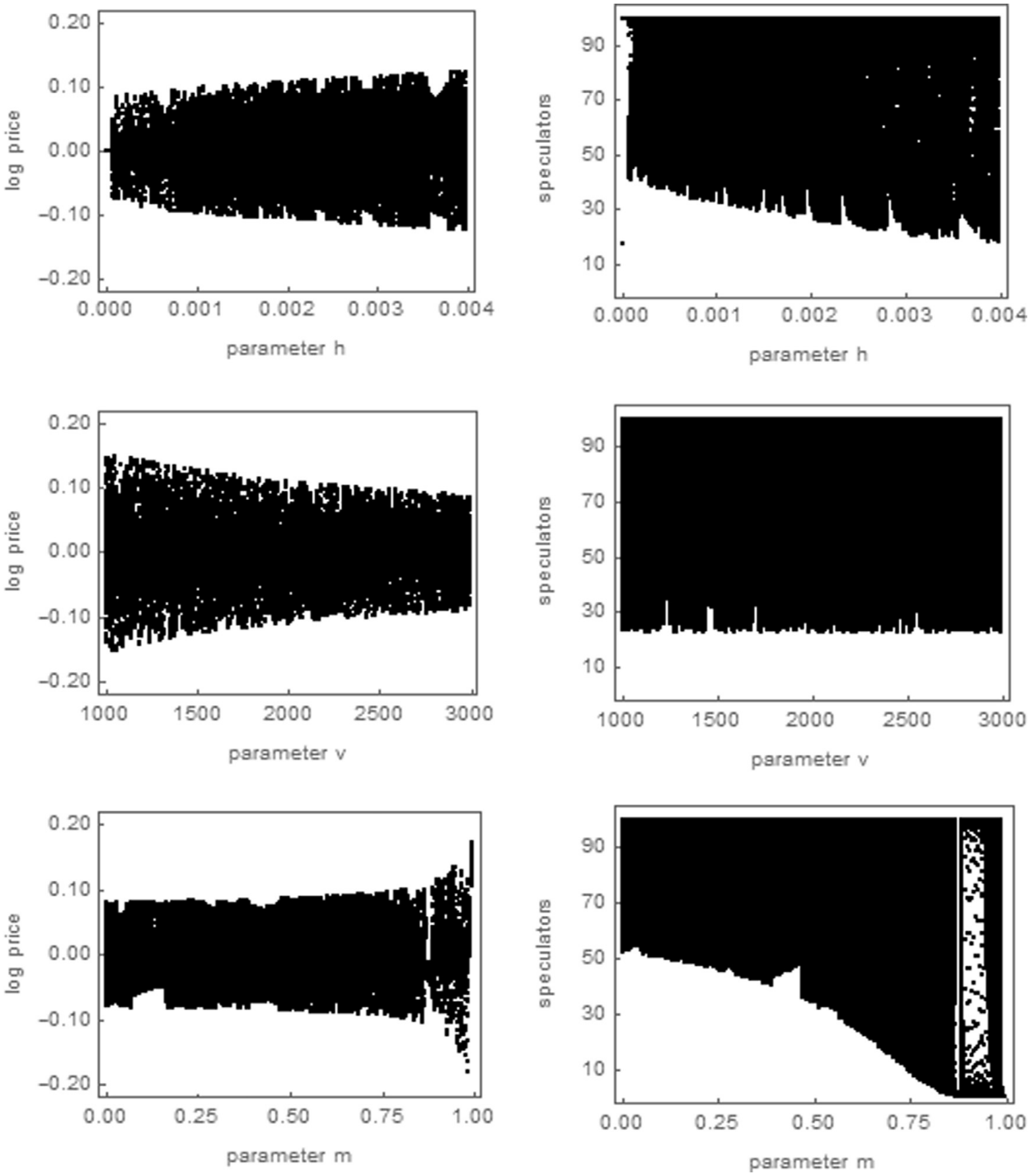

Fig. 4. The impact of *h*, *v* and *m*. The first, second and third rows show bifurcation diagrams for parameters *h*, *v* and *m*, respectively. While the left side presents their effect on log prices, the right side depicts how they affect the number of active speculators. Parameters are as in our base parameter setting.

speculators. As predicted by our analytical results, the model dynamics approaches the fundamental steady state for $c < 0.03$. However, a flip bifurcation occurs at $c = c_c = 0.03$. Within a small parameter range, the dynamics is then characterized by a period-two cycle. Afterwards, we observe the start of more complex dynamics, as already depicted in the first four panels of Fig. 6. Note furthermore that the amplitude of the price dynamics increases with parameter c. Here we have an example where excessively aggressive mean reversion trading by speculators leads to a destabilization of the stock market. As price fluctuations increase, the number of active speculators also displays more pronounced fluctuations.

4. Stochastic dynamics

In Section 3, we show that the deterministic version of our simple agent-based financial market model is – at least in a qualitative sense – able to produce bubbles and crashes, excess volatility, extreme price changes, complex price dynamics

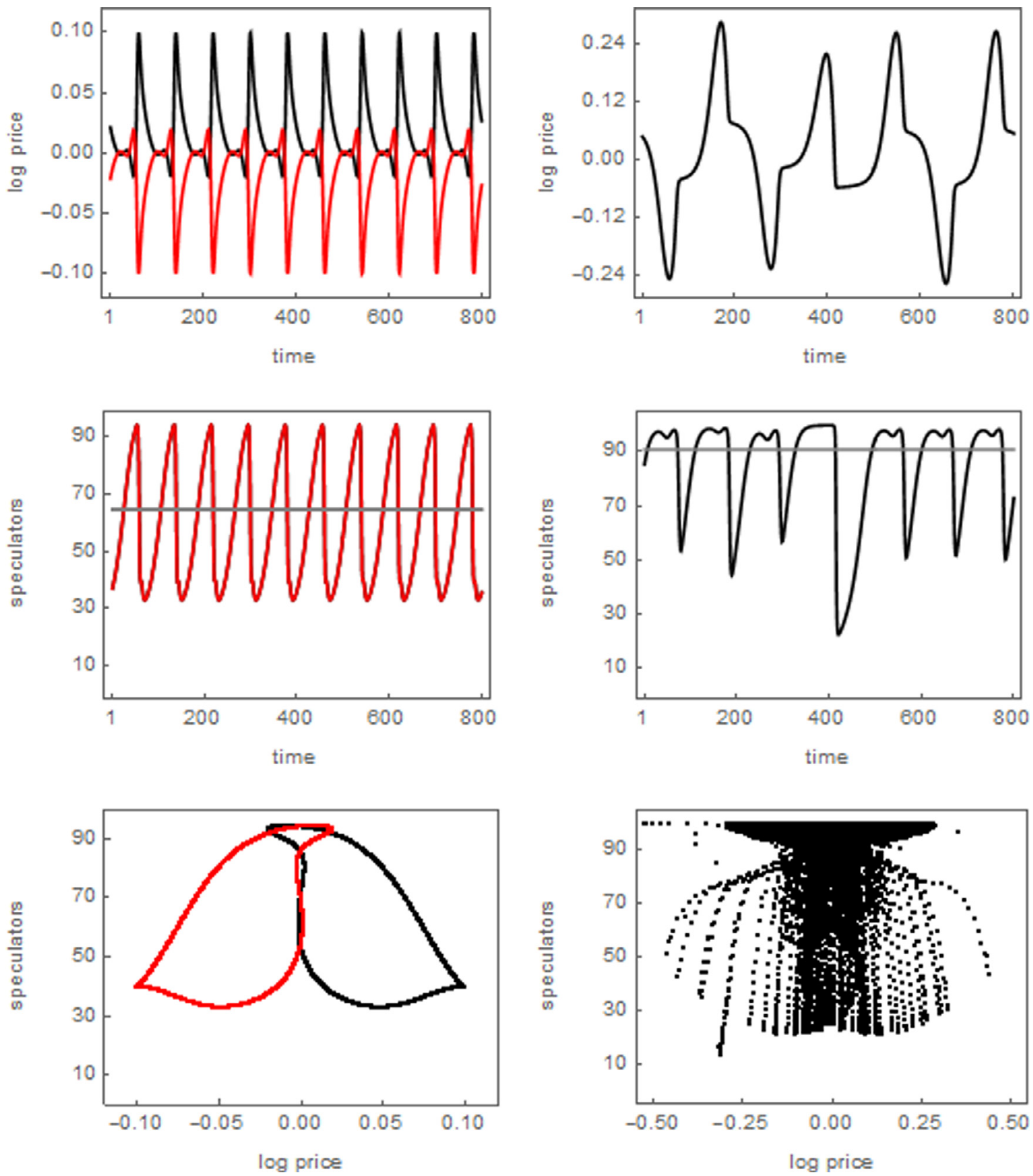

Fig. 5. Emergence of bubbles and crashes. The left panels show for two different sets of initial conditions (black and red) the evolution of log prices, the number of active speculators and the number of active speculators versus the log price for our base parameter setting, except for $b = 0.0155$. The right panels show the same for a single initial condition and our base parameter setting, except for $c = 0.00004$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

and volatility clustering. In this section, we go one step further and demonstrate that the stochastic version of our model may also replicate a number of key statistical properties of actual stock markets in finer detail. In Section 4.1, we first review the stylized facts of stock markets. Then, we discuss the dynamics of our stochastic model and explain its functioning in Section 4.2. In Section 4.3, we explore the wealth dynamics of individual speculators.

4.1. Stylized facts of stock markets

As is well known, the dynamics of stock markets is characterized by bubbles and crashes, excess volatility, fat-tailed return distributions, serially uncorrelated returns and volatility outbursts. Moreover, trading volume is persistent and cor-

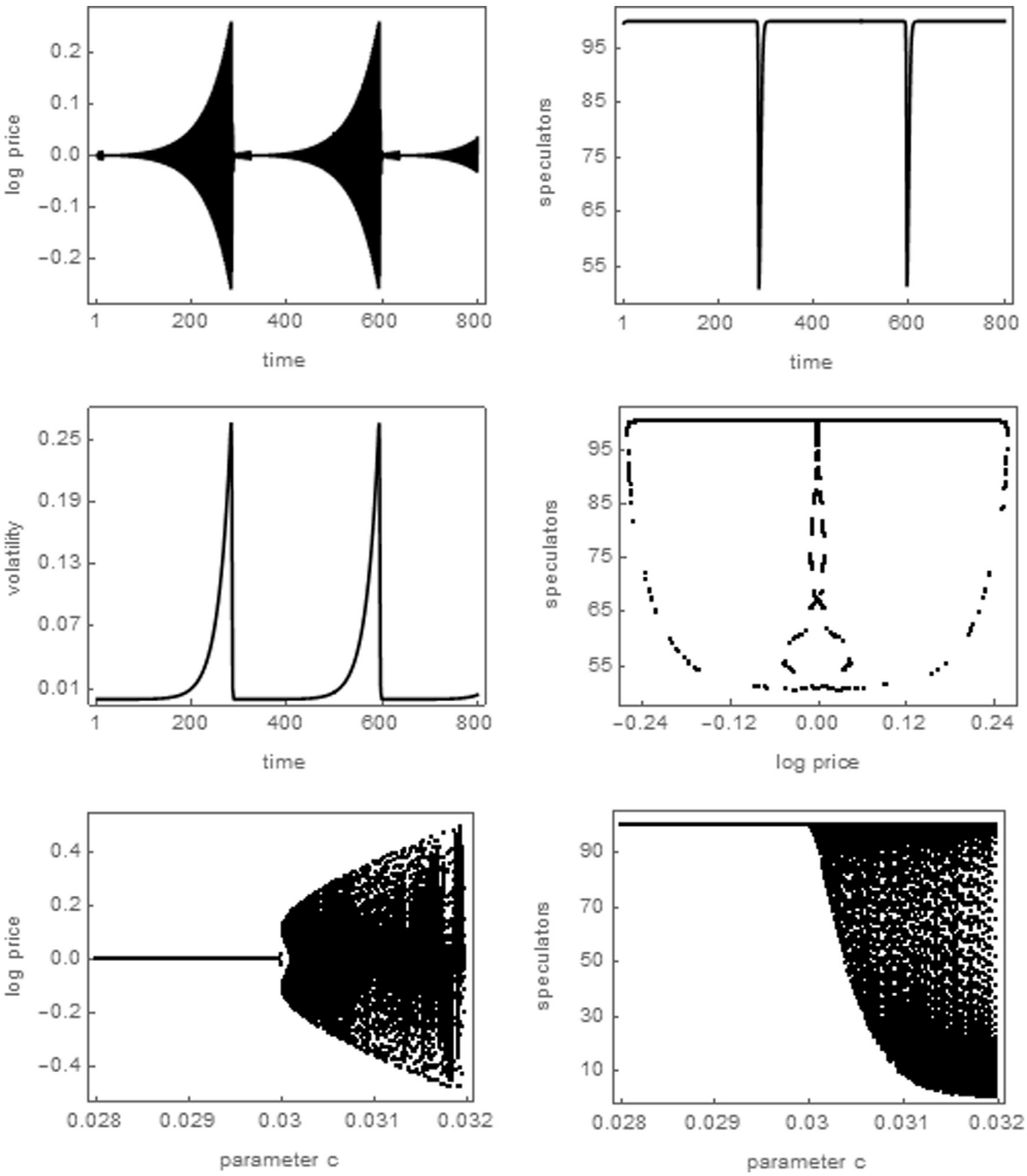

Fig. 6. Dynamics of the deterministic model for an alternative parameter setting. The panels show the evolution of the log price; the number of active speculators; the stock market's volatility; and the number of active speculators versus the log price. The last two panels show bifurcation diagrams for the log price and the number of active speculators with respect to parameter *c*. The underlying parameter setting is given in Section 3.5.

related with volatility. The boom-and-bust behavior of stock markets and their volatile nature is discussed thoroughly in Shiller (2015). Moreover, Mantegna and Stanley (2000), Cont (2001) and Lux and Ausloos (2002) provide excellent surveys about the statistical properties of financial markets. For illustrative reasons, we focus below on the behavior of the Dow Jones Industrial Average between 1981 and 2016, as depicted in Fig. 7. The underlying data set comes from Thomson Reuters Datastream and contains about 9000 daily observations. The top left panel of Fig. 7 shows the development of the Dow Jones Index. Despite its long-run upwards movement, a number of severe crashes can be spotted. For instance, the Dow Jones Index witnessed dramatic depreciations around 2001 and 2007. The top right panel of Fig. 7 presents the corresponding return time series. Overall, the Dow Jones Index may be regarded as quite volatile. Just to give one example, the standard deviation of the return time series is about 0.011. Furthermore, there are several larger returns visible and calm periods obviously alternate with turbulent periods.

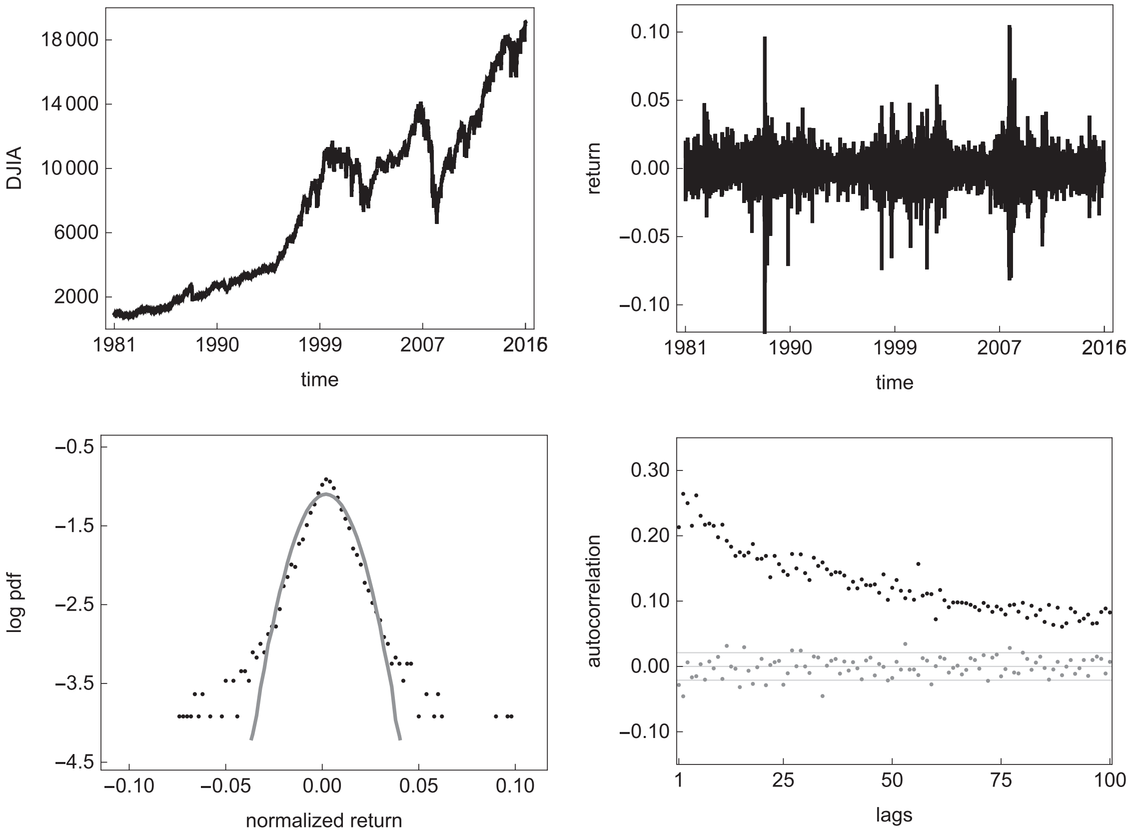

Fig. 7. Properties of the Dow Jones Industrial Average. The panels show the evolution of the Dow Jones Index between 1981 and 2016, the corresponding returns, the log probability density function of normalized empirical returns (black) and standard normally distributed returns (gray) and the autocorrelation function of raw returns (gray) together with the autocorrelation function of absolute returns (black).

The bottom right panel of Fig. 7 compares the log probability density function of normalized Dow Jones Index returns (black dots) with standard normally distributed returns (gray line). Apparently, the distribution of the returns of the Dow Jones Index contains more probability mass in the center, less probability mass in the shoulders, and again more probability mass in the tails than warranted by a normal distribution with identical mean and standard deviation. Since the kurtosis of empirical returns is 42.66 and thus much larger than the kurtosis of a normal distribution, namely 3, there is clear evidence of excess kurtosis. However, the tail index provides a more reliable measure to quantify the fat-tail property of the distribution of returns (Gopikrishnan et al., 1999; Lux, 1996; Lux and Alfarano, 2016). Using the largest 5% of the observations, the Hill tail index estimator indicates a typical tail index of about 3.02 for this time series, suggesting that the fourth moment of the distribution of the returns does not exist. The bottom right panel of Fig. 7 shows the autocorrelation coefficients of absolute returns (black dots) and raw returns (gray dots) for the first 100 lags, together with their 95% confidence bands (thin gray lines). The absence of autocorrelation of raw returns demonstrates that the evolution of the Dow Jones Index is hardly possible to predict, i.e. that its path is close to a random walk. In contrast, the autocorrelation coefficients of absolute returns are highly significant, implying a temporal persistence of volatility for more than 100 days. Finally, we remark that trading volume shows clear signs of long-memory effects and is highly correlated with volatility. Due to missing data availability, these properties are not depicted here. However, see Brock and LeBaron (1996), Cont (2001) and Schmitt and Westerhoff (2014) for a deeper empirical account.

4.2. Properties and functioning of the stochastic model

In the last couple of years, considerable progress has been made in estimating agent-based financial market models, see, e.g. Alfarano et al. (2005); Amilon (2008); Boswijk et al. (2007); Chiarella et al. (2014); Hommes and in 't Veld (2017). One powerful method to estimate such models is given by the method of simulated moments, which seeks to align a selection of empirical moments, i.e. certain summary statistics which quantify the stylized facts of financial markets, with model generated moments. Contributions in this direction include Gilli and Winker (2003), Winker et al. (2007), Franke (2009) and Franke and Westerhoff (2012). Unfortunately, the large number of parameters of our model prevents us from using this method (which requires a multi-dimensional grid search in parameter space). Instead, we rely on a more informal calibration approach. After a tedious and time-consuming trial-and-error exercise, we can at least show that our model has some ability to match the stylized facts of stock markets.

To be precise, we use the following parameter setting to discuss the dynamics of our stochastic model: $a = 1$, $b = \beta = 0.0001$, $c = \gamma = 0.000005$, $d = \delta = 0.01$, $h = 0.00008$, $\nu = 130$, $m = 0.99$, $\lambda = 1$, $\sigma^n = 0.005$ and $N = 500$. Fig. 8 depicts the outcome of a typical simulation run with 9,000 observations. The top left panel of Fig. 8 displays the evolution of the stock price. While the stock price fluctuates quite erratically, there are a number of stronger price appreciations and depreciations, resembling the boom-and-bust behavior of the Dow Jones Index (since the fundamental value follows a random walk in our model, there is no long-run upwards trend in the simulated stock price dynamics). The top right panel of Fig. 8 depicts the corresponding return time series. The standard deviation of simulated returns is given by 0.011, i.e. our model matches the average volatility of the Dow Jones Index quite well. Moreover, the standard deviation of the fundamental value only amounts to 0.005. Hence, returns are roughly twice as volatile as justified by changes in the fundamental value. This panel also reveals that there are a number of larger returns as well as occasional volatility outbursts.

The bottom left panel of Fig. 8 relates the log probability density function of normalized returns (black dots) with the log probability density function of standard normally distributed returns (gray line). As can be seen, the distribution of simulated returns possesses more probability mass in the center, less probability mass in the shoulders, and again more probability mass in the tails than warranted by a normal distribution with identical mean and standard deviation. The fat-tail property is also confirmed by estimates of the kurtosis for which we obtain a value of 4.62. While this value indicates excess kurtosis, it should be noted that it is much lower than the value we observe for the Dow Jones Index. Estimates of the tail index point in the same direction. For the simulated time series, the Hill tail index estimator produces a value of 4.56, implying that the fourth moment of the distribution of returns exists. Clearly, simulated returns have more probability mass in the tails of their distribution than normally distributed returns but less than actual returns.[5] The bottom right panel of Fig. 8 presents the autocorrelation coefficients of absolute returns (black dots) and raw returns (gray dots) for the first 100 lags. Raw returns are serially uncorrelated, i.e. also simulated prices are hardly possible to predict. The autocorrelation coefficients of absolute returns are highly significant, revealing strong evidence of volatility clustering.

Fig. 9 presents some properties of trading volume and how it relates to volatility. The panels show the evolution of trading volume, the autocorrelation function of trading volume, the return dynamics and the cross-correlation function of trading volume and absolute returns, respectively. Following Schmitt and Westerhoff (2014), we assume that all speculators trade directly with the market maker, i.e. speculators do not trade with other speculators. Trading volume can then be defined by $TV_t = \sum_{i=1}^{N_t} |D_{t,i}|$. As can be seen, trading volume is highly persistent, i.e. autocorrelation coefficients of trading volume are positive and decay rather slowly. Moreover, trading volume is positively correlated with volatility. While there is a strong contemporaneous correlation between trading volume and volatility, lagged correlations are rather low, as is the case for real markets (Brock and LeBaron, 1996).

Overall, we may thus conclude that the stochastic version of our simple agent-based financial market model is able to replicate key empirical regularities of actual stock markets. To explain its functioning in more detail, we continue with the simulation run depicted in Fig. 8 but focus our attention on a shorter time window. The four panels of Fig. 10 show from top left to bottom right the evolution of stock prices (black line) and fundamental values (gray line), the corresponding returns, the stock market's volatility and the number of active speculators between periods 6150 and 7650. During this time period, there are three pronounced volatility outbursts. Note also that volatility tends to increase with the number of active speculators. Accordingly, the functioning of our stochastic model may be understood as follows. Suppose that stock market volatility is low. In such a situation, speculators' herding behavior dominates their risk aversion. Consequently, more and more speculators enter the stock market and volatility picks up. Eventually, speculators' risk aversion offsets their herding behavior. As the number of speculators declines, the stock market becomes more stable. However, this leads directly to the next market entry wave and to another high volatility episode. Of course, higher stock market participation also drives trading volume up, i.e. stock market participation, trading volume and volatility are correlated.

It is interesting to note that the functioning of the stochastic version of our agent-based model is very similar to the functioning of its deterministic counterpart. In the deterministic setup, endogenous dynamics and volatility outbursts emerge when a model parameter crosses the Neimark–Sacker bifurcation boundary, either because speculators react too strongly to price trends or because there are too many speculators. While the calibrated parameter setting of our stochastic model implies that the fundamental steady state of the corresponding deterministic model is locally stable, the model's cyclical nature prevails. We remark that this phenomenon, i.e. realistic model dynamics for parameter settings in which the fundamental steady state of the model's deterministic skeleton is locally stable, is quite common in this line of research. As it turns out, it is the interplay of nonlinear forces and random elements that causes realistic dynamics. Nevertheless, the analytical and numerical insights we gain from studying the deterministic framework prove to be instrumental in our understanding of the much more complicated stochastic framework.

[5] Although our model does a fairly good job of matching the stylized facts of stock markets, it produces too few extreme returns. Further experiments (available upon request) reveal that simple model extensions can alleviate this issue. In particular, our model may produce quite realistic tail indices if certain model parameters, such as speculators' reaction to fundamental shocks, are allowed to vary over time – without destroying its ability to match the other stylized facts. Since our main focus is on explaining volatility outbursts in stock markets, we abstain, for simplicity, from such model extensions.

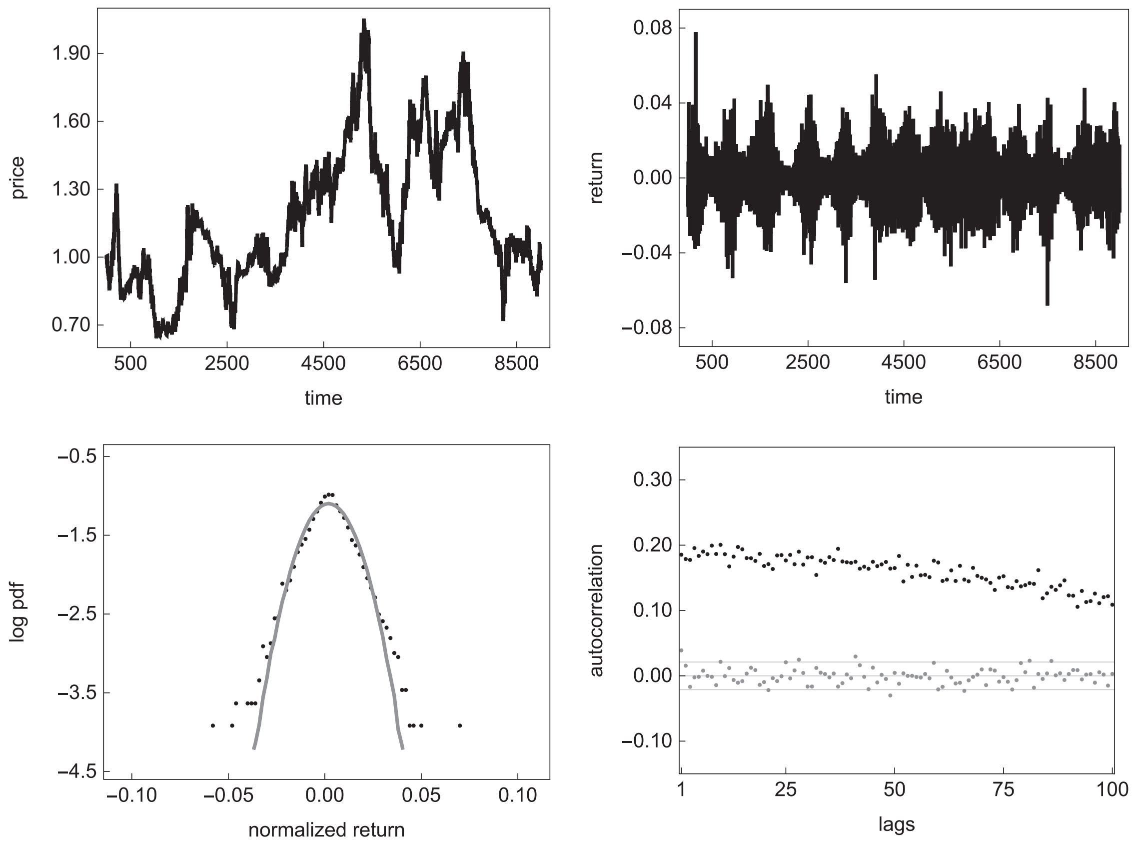

Fig. 8. Properties of the stochastic model. The panels show the evolution of the stock price for 9000 observations, the corresponding returns, the log probability density function of normalized model returns (black) and standard normally distributed returns (gray) and the autocorrelation function of raw returns (gray) together with the autocorrelation function of absolute returns (black). The underlying parameter setting is given in Section 4.2.

Random elements in our model stem from speculators' probabilistic market entry decisions, from their time-varying trading rules and from changes in the fundamental value. As can be seen in the top left panel of Fig. 10, stock prices and fundamental values tend to move in the same direction. However, stock prices may substantially disconnect from fundamental values. This is particularly true if the number of active speculators is rather high or rather low – the stock market then reacts too strongly or too weakly to incoming fundamental shocks. Since positive and negative fundamental shocks are equally likely, stock price changes are basically random. Moreover, in an environment in which stock prices already fluctuate quite erratically, speculators' trend extrapolation behavior does not add predictable structure to the return time series. Whether the technical part of speculators' trading rules produces a buy or sell signal is essentially equally likely. Large price changes occur if a large number of active speculators receive a strong trading signal, either because of significant price trends, pronounced misalignments or distinct fundamental shocks, or if the time-varying reaction parameters of their trading rules suggest aggressive trading. Of course, extreme returns may emerge if these forces act together, i.e. if many speculators act aggressively on heavy trading signals.

4.3. Wealth dynamics of individual speculators

Stock price changes induce diverging wealth dynamics among speculators relying on heterogeneous trading rules. In the long run, the evolutionary pressure originating from such wealth dynamics may act as a natural selection device among speculator types. While some speculator types may turn out to be successful and survive evolutionary competition, other speculator types may fail and become extinct. The so-called market selection hypothesis (Blume and Easley, 1992) therefore predicts that heterogeneity among speculators can only be a short-run phenomenon. Note that the implications of this hypothesis may be far-reaching. In particular, naive speculator types who persistently lose wealth may eventually vanish from the market, implying that stock prices would then only be subject to smart speculator types who manage to make a profit. However, Anufriev and Dindo (2010), Bottazzi and Dindo (2014) and Bottazzi et al. (2017) demonstrate that heterogeneity among speculators may prevail in stock markets, especially if speculators differ in their risk aversion, beliefs or sentiments. Overall, this important line of research points out that the relation between rationality and survival is rather weak and that stock prices are also influenced by boundedly rational speculators.

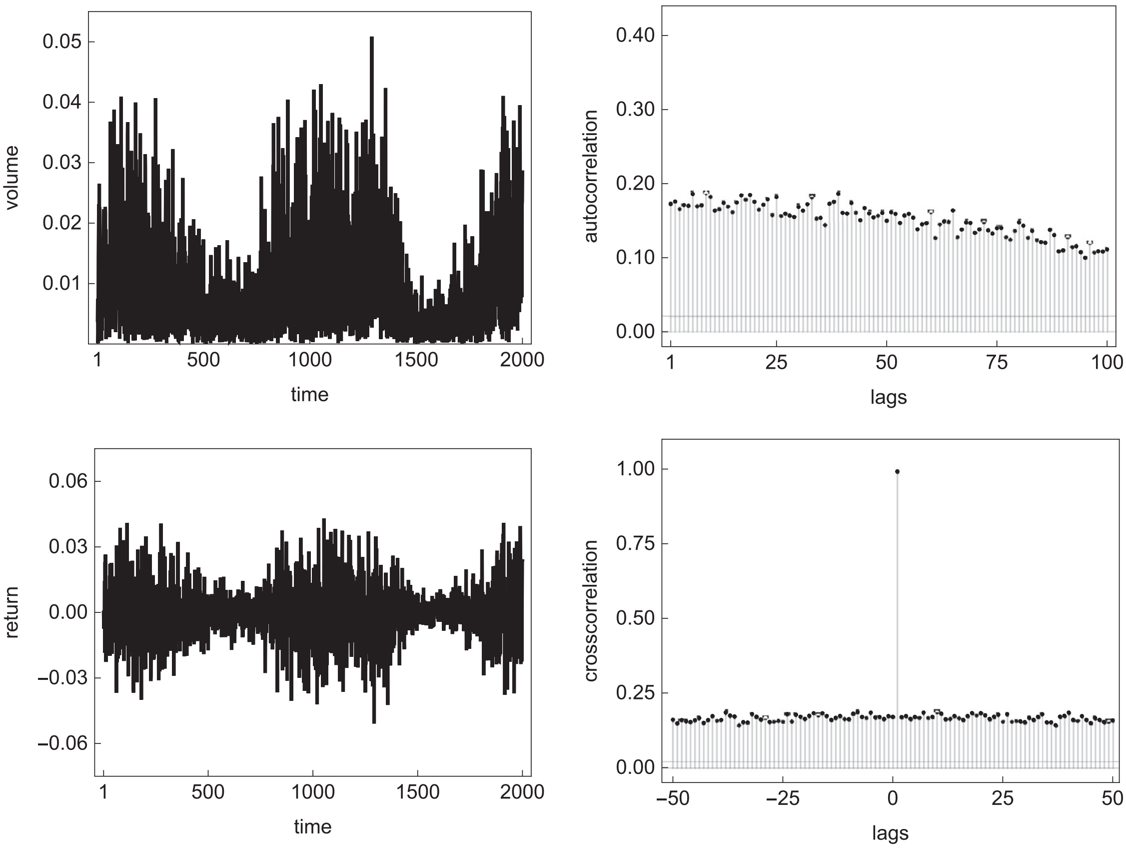

Fig. 9. The relation between trading volume and volatility. The panels show the evolution of trading volume, the autocorrelation function of trading volume, the return dynamics and the cross-correlation function of trading volume and absolute returns, respectively. The parameter setting is as in Fig. 8.

In the following, we explore how speculators' wealth evolves within our model. Fig. 11 illustrates our main results. Its top left panel shows the development of log prices for 250 trading periods, i.e. for a time span of about one year. Two things deserve our attention. First, the stock price in period 1 roughly corresponds to the stock price in period 250. Second, speculators have triggered a bubble in between. The center left panel of Fig. 11 presents the orders of two randomly selected speculators, say speculator 1 (red line) and speculator 2 (black line). Apparently, speculators' order flows differ substantially. The bottom left panel of Fig. 11 depicts (myopic) profits of the two speculators. Here we use the same time structure and definition of profits as Westerhoff and Dieci (2006) do. Accordingly, an order submitted in period $t-2$ is filled at the price in period $t-1$. This transaction then appears as a profit or as a loss depending on the price in period t. Formally, profits of speculator i in period t are given with $\pi_{t,i} = (exp[P_t] - exp[P_{t-1}])D_{t-2,i}$ (recall that P_t refers to log stock prices). As can be seen, it is difficult for speculators to beat the market. More precisely, the probability that speculator i will make a profit in period t is – due to the stock market's random walk nature – equal to the probability that he will make a loss, namely 50%.

The top right panel of Fig. 11 shows the evolution of the positions of speculators 1 and 2 while the center right panel of Fig. 11 displays the corresponding wealth dynamics. For simplicity, we abstract from trading costs, interest rates and dividend payments. Moreover, the wealth level is initially set to zero and then updated according to $\Pi_{T,i} = exp[P_T] \sum_{t=3}^{T} D_{t-2,i} - \sum_{t=3}^{T} exp[P_{t-1}]D_{t-2,i}$. Hence, the wealth of speculator i after T trading periods depends on the value of his position, given by the stock price in period T multiplied by the sum of his orders, and his actual expenditures and revenues from accumulating his position, given by his executed transactions. Note that speculator 1 (red line) builds up a positive position and initially benefits from the bubble. When the bubble bursts, however, much of his wealth vanishes again. In contrast, speculator 2 enters a short position around period 75 and loses wealth. Between periods 75 and 125, speculator 2 then increases his stock position and finally depletes it again. In the end, the wealth gains and losses of both speculators are roughly equal and close to zero.

The bottom right panel of Fig. 11 extends this experiment by visualizing the wealth dynamics of 50 randomly selected speculators (alternating red and black lines). While there is a somewhat larger wealth dispersion during the height of the bubble, speculators' wealth differentials diminish after the market's mispricing reduces. Hence, our model's stock price dy-

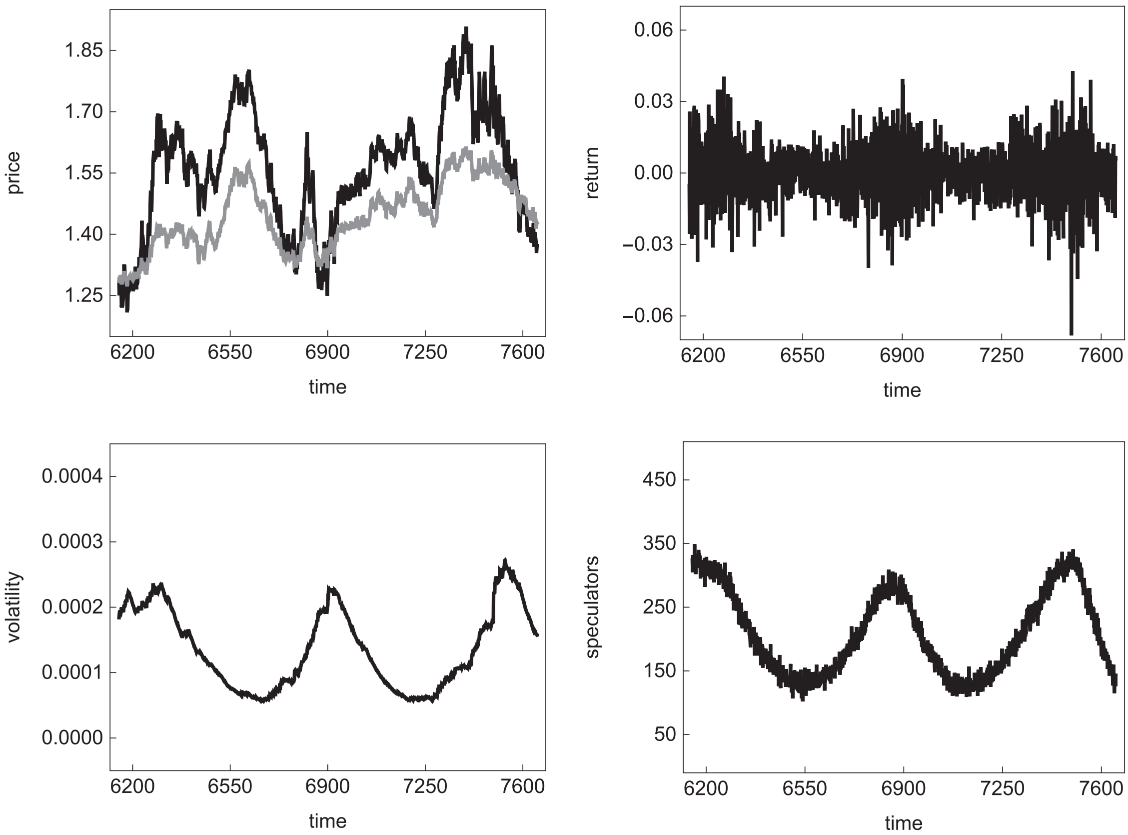

Fig. 10. Functioning of the stochastic model. The panels highlight the evolution of stock prices (black) and fundamental values (gray), the returns, the stock market's volatility and the number of active speculators between periods 6150 and 7650 of the simulation run depicted in Fig. 8.

namics does not imply that some speculators consistently lose money (and should thus eventually vanish from the market) nor that some speculators become richer and richer (and should thus dominate the market in the long run). Put differently, our analysis reveals that stock markets are difficult to beat. Some speculators may be lucky and make money; other speculators may be unlucky and lose money. However, no speculator encounters systematic profits or losses. Of course, actual speculators are not infinitely lived. In reality, speculators will eventually leave the stock market permanently, either with a higher or a lower wealth level, and newborn speculators will emerge. Taking such a perspective, the outcome depicted in Fig. 11 may be regarded as a representative snapshot of a stock market's long-run wealth dynamics.

5. Conclusions

We develop an agent-based financial market model with heterogeneous interacting speculators to explain a number of important statistical regularities of stock markets. Speculators base their orders on current price trends, the market's mispricing and new information. Speculators are heterogeneous in the sense that each of them follows his own time-varying trading rule. However, not all speculators are always active in the stock market. Two socio-economic principles govern speculators' probabilistic market entry decisions. First, speculators' market entry decisions are subject to herding behavior. The more speculators are active in the stock market, the more attractive the stock market appears to them. Second, speculators' market entry decisions depend on stock market risk. The higher the stock market risk, measured by the past volatility of the stock market, the less attractive the stock market appears to them. All orders placed by speculators are matched by a market maker who adjusts stock prices with respect to excess demand. The only extrinsic forces in our model are exogenous shocks which drive the random evolution of the fundamental value.

We use a mix of analytical, numerical and empirical tools to investigate our model. Our main result is that sporadic market entry waves may cause volatility outbursts in stock markets. To be precise, we show that a herding-induced inflow of speculators leads to rather unstable market dynamics with high volatility while a consecutive risk-driven outflow of speculators leads to more stable market dynamics with low volatility. This kind of volatility clustering is observed in the deterministic skeleton of our model, for which we provide a full steady-state and stability analysis, and in the stochastic version of our model, which we calibrate to the stylized facts of stock markets. The latter exercise demonstrates that our model is able to generate bubbles and crashes, excess volatility, fat-tailed return distributions, serially uncorrelated returns

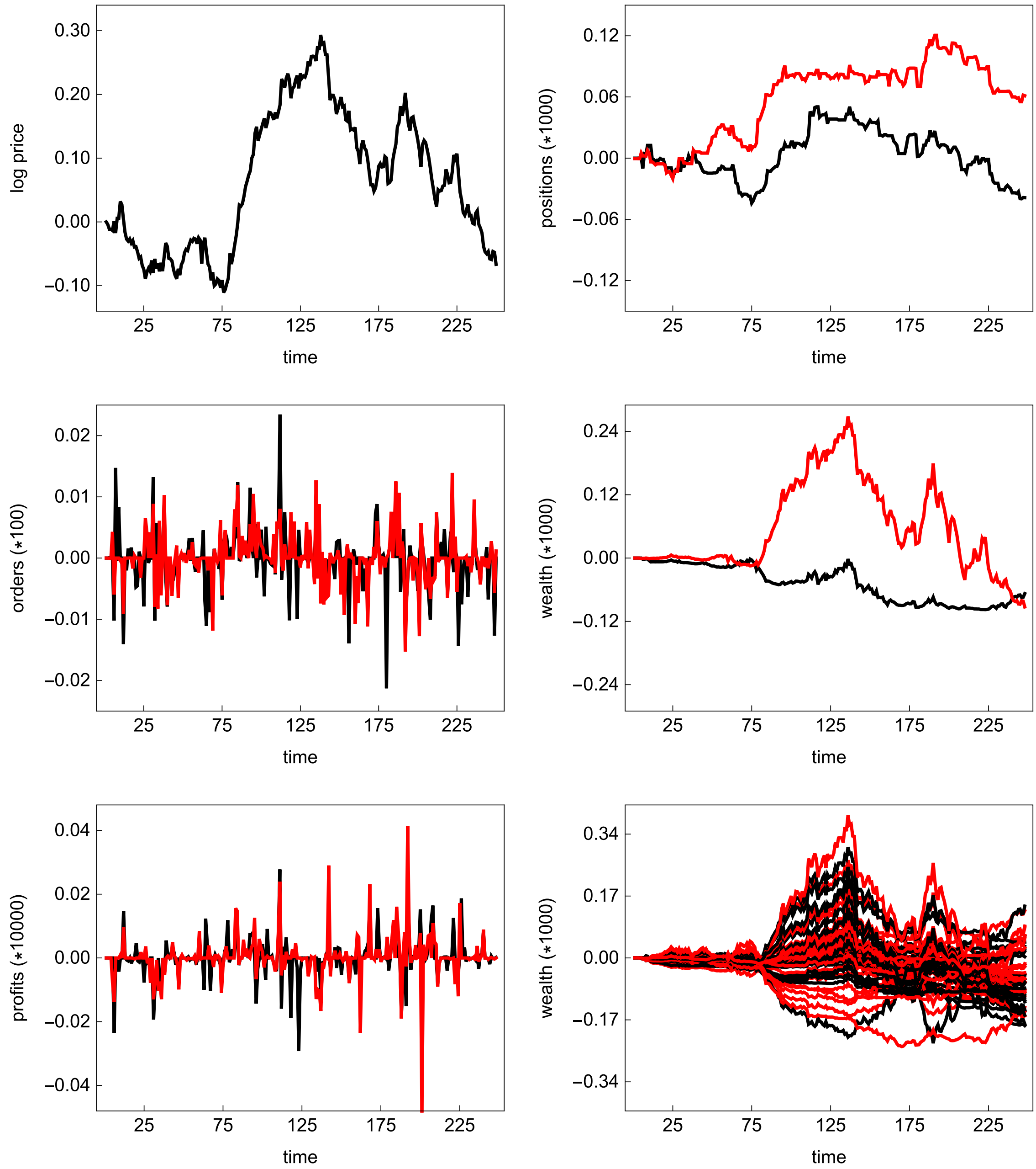

Fig. 11. Wealth dynamics of individual speculators. The panels show the evolution of log prices, the positions of two individual speculators, their orders, wealth and profits and the wealth of 50 individual speculators, respectively. The parameter setting is as in Fig. 8. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

and, as already mentioned, volatility clustering. In addition, trading volume is persistent and correlated with volatility. In this sense, our model may be regarded as validated.

Our analytical results prove instrumental in our understanding of the functioning of our model. In particular, we show that the model's fundamental steady state becomes unstable once too many speculators enter the stock market. Since the instability of the fundamental steady state is due to a Neimark–Sacker bifurcation, we observe the onset of (quasiperiodic) endogenous dynamics. In this respect, it is worth mentioning that our stochastic agent-based model starts from the description of the trading behavior of a large number of individual speculators but can, after some straightforward transformations, be expressed as a four-dimensional deterministic nonlinear map. In this way, it is possible to obtain valuable analytical insights for a rather complex agent-based model. Moreover, the reduced model version greatly reduces computational efforts when it comes to a simulation-based model calibration. Of course, once the model is calibrated, one may simulate the original agent-based framework and monitor various aspects of the behavior of individual speculators, e.g. their wealth dynamics.

We conclude our paper by pointing out a few avenues for future research. First, speculators follow a linear blend of technical and fundamental trading rules in our model. One interesting extension of our model could be to let active investors make a behavioral choice for a specific trading rule. We could then have situations with a large number of active speculators who prefer fundamental analysis or situations with a small number of active speculators who favor technical analysis – just to give two examples. Such a rule selection behavior could be modeled along the lines of Brock and Hommes (1997); Lux and Marchesi (2000) or Franke and Westerhoff (2012). Second, speculators who do not enter the stock market in our model are simply inactive. Another interesting extension of our model could be to model speculators' outside option. For instance, Dieci et al. (2018) develop a model in which speculators can invest their wealth in stock, bond and housing markets. Alternatively, one may assume that speculators switch between different stock markets. Research in that direction is surprisingly scant so far. Third, one may also use our model to conduct policy experiments (see, e.g. Jacob Leal et al., 2016 and Jacob Leal and Napoletano, 2017). Our model implies that (destabilizing) speculators increasingly enter the stock market if stock market volatility is low. This model feature may prove a real challenge for regulatory measures which seek to tame stock market fluctuations. To sum up, we hope that our paper stimulates more research in this direction. The financial crisis at the end of the noughties has not only made clear that our understanding of the dynamics of financial markets is still incomplete – it revealed, once again, how important it is to make scientific and real progress in this area.

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