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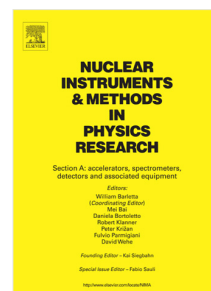
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Modeling charge transport in photon-counting detectors

Yuan Fang^a, Cheng Xu^b, Yuan Yao^c, Norbert Pelc^d, Mats Danielsson^b, Aldo Badano^e

^a*Division of Radiological Health, Office of In Vitro Diagnostics and Radiological Health, Center for Devices and Radiological Health, Food and Drug Administration, 10903 New Hampshire Ave, Silver Spring, MD 20993 USA.*

^b*Department of Physics, Royal Institute of Technology, Stockholm, Sweden.*

^c*Department of Bioengineering, Stanford University, Stanford, California; Department of Radiology, Stanford University, Stanford, California 94305.*

^d*Department of Bioengineering, Stanford University, Stanford, California; Department of Radiology, Stanford University, Stanford, California; Department of Electrical Engineering, Stanford University, Stanford, California.*

^e*Division of Imaging, Diagnostics, and Software Reliability, Office of Science and Engineering Laboratories, Center for Devices and Radiological Health, Food and Drug Administration, 10903 New Hampshire Ave, Silver Spring, MD 20993 USA.*

Abstract

1 The purpose of this study is to review and compare simulation methods for
2 describing the transport of charge clouds in silicon based semiconductor detec-
3 tors and investigate the effects on energy spectrum for silicon based photon-
4 counting strip detectors. Charge clouds and detailed carrier transport are sim-
5 ulated and compared using two different approaches including analytical and
6 Monte Carlo schema. The results of the simulations are evaluated using pulse-
7 height spectra (PHS) for a silicon strip detector with edge on geometry at two
8 energies (25 and 75 keV) at various x-ray absorption locations relative to the
9 pixel boundary and detector depth. The findings confirm carrier diffusion plays
10 a large role in the charge sharing effect in photon counting detectors, in par-
11 ticular when the photon is absorbed near the pixel boundary far away from
12 the pixel electrode. The results are further compared in terms of the double-
13 counting probability for x-ray photons absorbed near the pixel boundary as a
14 function of the threshold energy. Monte Carlo and analytical models show rea-
15 sonable agreement (2% relative error in swank factor) for charge sharing effects
16 for a silicon strip detector with edge-on geometry. For 25 keV mono-energetic
17 photons absorbed at 5 μm from the pixel boundary, the theoretical threshold

18 energy at 10% double-counting probability based on charge sharing is 5.5, 8.5
19 and 9.2 keV for absorption depths of 50, 250 and 450 μm from the electrode,
20 respectively. The transport of charge clouds affects the spectral characteristics
21 of photon counting detectors and the double-counting probability results show
22 the theoretical threshold energy to avoid double-counting as a function of x-
23 ray energy and x-ray interaction locations for silicon and can be considered for
24 future studies of charge sharing effects.

Keywords:

photon-counting, silicon detector, charge-sharing, double-counting probability,
Monte Carlo

25 1. Introduction

26 Photon-counting detectors with energy discrimination capabilities have been
27 recently developed for many medical x-ray imaging applications[1, 2] promising
28 several advances including the ability to estimate the energy of transmitted
29 photons at each pixel location. This technological development could enable
30 improved material decomposition, higher spatial resolution, and implementa-
31 tion of beam hardening corrections. In most cases, photon counting detectors
32 with energy discrimination can achieve higher signal-to-noise ratio[3] leading to
33 improvements in existing modalities or allowing novel applications.[1, 4–8] In
34 addition, photon-counting detectors can be used in spectral CT applications.[9–
35 15]

36 One major challenge for photon-counting detectors is a phenomenon gener-
37 ally known as charge sharing. Under an externally applied bias, a cloud of
38 charge carriers created by the energy imparted by an absorbed x-ray photon
39 travels within the semiconductor and reaches the detector electrode. Near a
40 pixel boundary, the cloud may be divided and detected simultaneously by mul-
41 tiple pixels recording energies lower than the energy carried by the x-ray quan-
42 tum. The distribution of energy causes distortions in the spectral response. The
43 significance of this effect depend on the detection material, charge carrier mobil-

44 ity, pixel size, absorption location with respect to the pixel boundary, depth of
45 interaction within the active detector layer, temperature, applied bias (including
46 non-uniform electric field effects due to Frisch grid structures).

47 Different models have been proposed to simulate charge-sharing effects. An-
48 analytical models vary in complexity due to compromises made towards com-
49 putational simplicity and/or practical solution of the theoretical electrostatic
50 equations. Barrett *et al.* [16] derived a one-dimensional solution to the trans-
51 port of charge clouds in semiconductor detectors with 2D anode arrays and
52 described how to compute the pulse-height spectra. Rossi[17] adopted a one-
53 dimensional Gaussian model and provided a simple solution in determining the
54 charge split in a microstrip detector. In this model, thermal diffusion was the
55 sole deterministic factor to the spread of the Gaussian profile and the charge
56 loss was ignored. Kozorezov *et al.*[18] developed a drift-diffusion model for com-
57 puting the collected electron charges as the product of two error functions for
58 the limited trapping case. The solution of the general drift-diffusion model was
59 obtained by factorizing the carrier motion into two separate components, the
60 first describing a purely diffusive process in the lateral direction and the second
61 a mixture of drift and random walk along the normal z direction. This model
62 was generally applicable to the pixelated photon-counting detectors and is ma-
63 terial independent. Later on, an extension by Engel *et al.*[19] described the
64 additional contribution from detector polarization in a pixelated x-ray semi-
65 conductor detector containing an inhomogeneous electric field parallel to the
66 depth axis caused by different concentration of ionized dopants. X-ray energy
67 deposition and charge movements within the detector were modeled in Monte
68 Carlo simulations giving access to statistical analysis of electron drift time and
69 current pulse widths for various degrees of static polarization.

70 While analytical models can describe initial trends and be applicable to sim-
71 plified geometries, Monte Carlo techniques provide an accurate and more flexible
72 method to simulate the response of photon-counting semiconductor detectors for
73 a variety of geometries and system configurations. The interaction and transport
74 mechanisms for x-ray photons in the detector and the generation of secondary

75 particles can be tracked in detail until energy is either transformed into secon-
76 daries (electron-hole pairs) or lost out of the detector volume through a radiative
77 process. The mechanisms of photon and electron interaction and transport in
78 semiconductors are complex and can be modeled with existing available codes,
79 e.g. PENELOPE,[20, 21] GEANT4,[22] MCNP,[23] or EGSnrc,[24]. Recently, a
80 comprehensive (spatial and temporal) MC simulation tool, ARTEMIS,[25, 26]
81 for the simulation of photon and secondary electron interactions with the ad-
82 dition of electron-hole pair charged-carrier transport in the presence of a bias
83 electric field became publicly available.

84 In addition, other numerical techniques including finite element methods
85 (FEM) can be utilized for solving the transport problem in a wide range of
86 semiconductor devices.[27, 28] Based on classical electrodynamics, the electric
87 potential in the detector is computed numerically by solving the Poisson equa-
88 tion with properly set boundary conditions. The existence of surface charge
89 below the oxide layer in the region between electrodes affects the distribution of
90 electric potential. Advanced finite element analysis (FEA) programs can in this
91 case be used to model the charged carrier behavior. FEM and MC methods can
92 also be combined to solve carrier transport problems.[29, 30] In particular, the
93 PENELOPE package has been successfully combined with COMSOL to model
94 charge induction by carriers in photon-counting detectors.[31] In general, numer-
95 ical techniques using analytical (or semi-analytical) models less calculation time
96 compared to MC techniques. However, they suffer from numerically instabil-
97 ity, uncertain appropriate boundary conditions for obtaining meaningful results,
98 and lack of stochastic models necessary for modeling radiation absorption and
99 transport in semiconductor materials.

100 With many MC and analytical approaches available for modeling radiation
101 transport in semiconductor materials, this work provides a comparison of two
102 models for transport of charged carriers. We study the effect of charge sharing
103 due to the transport of charged carriers in photon-counting silicon detectors as a
104 function of x-ray energy, absorption depth, and absorption location with respect
105 to the pixel boundary with two different models for simulating x-ray and charge

106 carriers in semiconductor materials.[25, 32] Specifically, both analytical[32] and
107 MC[25] models are studied for the charged carrier transport.

108 **2. Methods**

109 In this section, we present the details of the models used in this study for
110 modeling charge sharing effects in photon-counting imaging detectors and de-
111 scribe the computational experiments used to compare the results obtained with
112 the different approaches. The bubble-line and ARTEMIS models utilize MC
113 methods for simulation of photon and electron interactions. The difference be-
114 tween the two models arise from the simulation of transport of charged carriers
115 which is analytical in the bubble-line model.

116 *2.1. Analytical*

117 The production and distribution of initially-released electron-hole pairs, i.e.
118 initial charge cloud, is the first step of the simulation of charge cloud transport.
119 Although a detailed Monte Carlo simulation is preferable, simulations can be
120 time-consuming especially for higher energy photons. Instead, some simplified
121 models are commonly used assuming the initial charge cloud as a sphere with
122 charge inside following either a uniform or Gaussian distribution.[33, 34]

123 The spherical approximation of initial charge clouds, which is also the ba-
124 sis of analytical approaches, is not capable of modeling the detector response
125 accurately for high-energy photons in medical x-ray imaging energy range. Re-
126 cently, a statistical model has been proposed to simulate the shape of initial
127 charge clouds by using pre-computed statistical distributions based on Monte
128 Carlo simulation data with promising results.[32]

129 For the so-called bubble-line model, the initial charge cloud produced by a
130 photon interaction is represented through three pre-extracted parameters: the
131 magnitude and polar angle of the center-of-gravity (COG) vector of the initial
132 charge distribution, and the track size. For a certain photon-deposited energy,
133 the above parameters can be sampled from the corresponding pre-computed

134 probability distributions (two Gaussian distributions and one Weibull distribu-
135 tion), and the parameters of each distribution are simply the results of quadratic
136 functions with the deposition energy as variable and the coefficients known. It is
137 named as bubble-line model because a certain amount of the deposited energy
138 is distributed into a bubble located at the COG and the left energy along a
139 line through the COG. By sampling the COG position and track size statisti-
140 cally based on the pre-simulated MC tracks, the random characteristics of the
141 initial charge distributions are reproduced more accurately than the spherical
142 approximation, and comparable to pure Monte Carlo simulations, with much
143 less simulation time.

144 After being released, the charge cloud moves towards the corresponding elec-
145 trodes along the electric field lines and enlarged during the charge-collection
146 process. The charge transport include drift and diffusion and generation of
147 signals by Shockley-Ramo theorem.[35, 36]

148 Field simulation is performed by solving Poisson equations with, for exam-
149 ple, Successive Over-Relaxation method (electric field and weighting field) or by
150 more advanced software which can capture the field distribution at Si-SiO₂ in-
151 terface. For some types of semiconductors with low mobility of charge carriers as
152 a result of deep impurities and structural defects, trapping and recombination of
153 charge carriers should be taken into account and modeled accurately to produce
154 the correct detector response. Depending on the initially-released positions, the
155 charge produced by a photon interaction might be collected by more than one
156 pixel (i.e. charge sharing), leading to double counting in photon-counting semi-
157 conductor detectors if the shared charge is larger than the minimum threshold
158 in the neighboring pixel.

159 *2.2. Monte Carlo*

160 ARTEMIS (pArticle transport, Recombination, and Trapping in sEMicon-
161 ductor Imaging Simulations) is an open-source Monte Carlo simulation package
162 for modeling the charge transport process in radiation imaging detectors.[25,
163 26] ARTEMIS relies on PENELOPE for the simulation of photon and high-

164 energy electron transport coupled with charge transport routines for the spatio-
165 temporal simulation of electron-hole pair transport under an applied bias. The
166 charge transport routines include three-dimensional spatial and temporal models
167 of electron-hole pair transport taking into account recombination and trapping.
168 Many electron-hole pairs are created simultaneously in bursts from energy de-
169 position events. Carrier transport processes include drift due to external field
170 and Coulombic interactions, and diffusion due to Brownian motion.

171 When a photon interacts within the photoconductor, high-energy electrons
172 are created and moved in a random walk while they deposit energy at random
173 locations. Electron-hole pairs are generated in bursts from the deposition ener-
174 gies, based on the interaction coordinates, the energy of the interacting particle,
175 and the amount of energy deposited. The number of electron-hole pairs created
176 is sampled based on a Poisson distribution, with the mean calculated based on
177 the ionization energy equation developed by Que and Rowlands.[37] The ioniza-
178 tion energy also contributes to the initial electron-hole pair separation described
179 by the Knight-Davis Equation,[38] as a function of the amount of energy de-
180 posited and applied electric field. Once the electron-hole pairs are initialized
181 in bursts, the electric field pulls the particles toward opposed electrodes and
182 the electrons and holes may get trapped or recombine.[39] Recombination can
183 occur when an electron and a hole travel toward each other, and trapping can
184 occur when an electron or a hole reaches a lower energy state due to material
185 impurities. During transport, carriers are subject to drift from both the applied
186 electric field and Coulomb field due to other charge carriers. Each carrier is also
187 subject to random Brownian diffusion as a function of temperatures and carrier
188 mobility.

189 The ARTEMIS material properties including carrier mobility reflect those
190 of the simulated detectors. Recombination and trapping are disabled while
191 Coulombic interactions are not considered because, compared to Selenium car-
192 rier interactions in silicon such as recombination and trapping from initial charge
193 cloud creation do not lead to significant loss in the detector signal. The elec-
194 tronic noise during readout is not modeled and only electrons are detected.

Material property	Si	a-Se	CdZnTe
Bandgap (eV)	1.1	2.3	1.7
Applied E-field ($V/\mu\text{m}$)	0.30	10	0.25
Ionization energy (eV)	3.62	~ 5	4.6
Measured Fano factor	0.11	0.059	0.089
Hole mobility, μ_h (cm^2/Vs)	480	0.14	50
Electron mobility, μ_e (cm^2/Vs)	1350	5×10^{-3}	1000
Hole lifetime, τ_h (s)	2×10^{-3}	10^{-6}	10^{-6}
Electron lifetime, τ_e (s)	10^{-3}	10^{-6}	3×10^{-6}

Table 1. Material and transport properties of semiconductor materials.[37, 40–44]

195 These modifications have led to significant speed-ups in the simulation by at
 196 least ten times depending on the simulation conditions. Table 1 lists the mate-
 197 rial properties and transport parameters for different semiconductor materials
 198 including Si, a-Se and CdZnTe. All models in this study use the same set of
 199 simulation parameters and edge on geometry described in Figure 1.

200 A comparison of the bubble-line and ARTEMIS models for x-ray and electron
 201 transport, charge cloud creation and carrier transport models can be found in
 202 Table 2.

203 2.3. Detector Simulation Geometry

204 Figure 1 illustrates the silicon strip detector geometry and set up used in
 205 this study. Specifically, the pixel size is $50 \mu\text{m}$ wide, with a strip length of 1
 206 cm and a thickness of $500 \mu\text{m}$. We simulated 9 x-ray absorption locations to
 207 cover a range relative to the inter-pixel boundary and electrodes. This includes
 208 3 incident x-ray locations 5 , 15 and $25 \mu\text{m}$ away from the pixel boundary and
 209 3 x-ray absorption depths, 50 , 250 and $450 \mu\text{m}$ from the pixel electrode. This
 210 configuration is applicable to all the models in order to provide a basis for fair
 211 comparison of the simulation results.

212 2.4. Double-counting

213 The double-counting probability ($DC_{probability}$) of a carrier being absorbed
 214 in a secondary pixel is defined as the product of probability of absorption in the

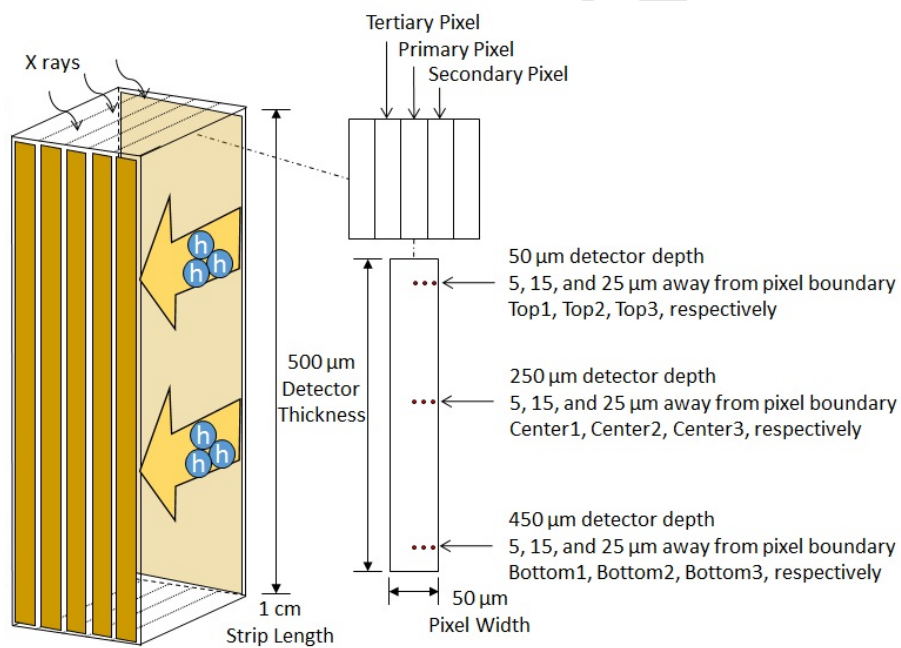


Figure 1. Silicon strip detector used for simulation purposes. The edge-on geometry has x rays incident from the top and electrodes on the side. Large arrows within the strip detector indicate the direction of carrier motion for holes. The pixel size is 50 μm , and the incident x ray location are 5, 15 and 25 μm from the pixel boundary. The detector thickness is 500 μm , and the absorption depth are 50, 250 and 450 μm from the pixel electrode.

Model	X-ray and electron transport	Cloud creation model	Charge transport model
Bubble-line[32]	PENELOPE[20]	All energy deposited in the semiconductor material is uniformly distributed in a sphere at center-of-gravity of the electron track and along a line through COB	Carrier drift, Brownian diffusion, holes only (electrons are not considered).
ARTEMIS[25]	PENELOPE[20]	The electrons are simulated and tracked individually with energy deposited at site of each electron interaction	Carrier drift, Brownian diffusion, electrons only (holes are not considered). Disabled features include Coulombic interactions, recombination of electron-hole pairs, carrier trapping and electronic noise.)

Table 2. Models used in this paper to compare charge-sharing effects.

215 primary (p_p) and secondary (p_s) pixels shown below as a function of threshold
 216 energy (E_t):

$$DC_{probability}(E_t) = p_p(E_t)p_s(E_t) , \quad (1)$$

217 where the probability of count in the pixel i is defined as the pulse-height spec-
 218 trum (\mathcal{P}) integrated and then divided by the threshold energy and photon energy
 219 (E_p) over the entire count of \mathcal{P} .

$$p_i(E_t) = \frac{\int_{E_t}^{E_p} \mathcal{P}_i(E)dE}{\int_0^{E_p} \mathcal{P}_i(E)dE} . \quad (2)$$

220 The secondary and tertiary pixels are the two neighboring pixels next the
 221 the primary pixel where the x-ray photon is absorbed shown in Fig. 1. When the
 222 x-ray photon is absorbed near the pixel boundary, not in the pixel center, the
 223 secondary pixel is defined as the pixel closer the x-ray interactions site, and the
 224 tertiary is the pixel on the opposite side further away from the x-ray interaction
 225 location. In this definition, the double-counting probability is a value between
 226 0 and 1. In addition, the double-counting probability is only for the charge
 227 sharing effects and not take into account timing information such as pile-up in
 228 the detector.

229 An alternative metric can be defined as the double counting percentage
 230 ($DC_{percentage}$) as a ratio of collected charges[45]:

$$DC_{percentage}(E_t) = \frac{(C_s + C_t)}{C_p}, \quad (3)$$

231 where the amount of charge collected in the primary (C_p), secondary (C_s) and
 232 tertiary (C_t) pixels are considered.

233 3. Results

234 3.1. Pulse-height spectra

235 Pulse-height simulation results for 25 keV mono-energetic x-ray photons are
 236 presented in Figure 2. The x-axis is the number of holes collected from each
 237 interacted x-ray photon and the y-axis is the number of primary x-ray absorption
 238 events normalized to peak of unity. For the 25 keV case where the incident x-
 239 ray is absorbed very close ($5 \mu\text{m}$) to the pixel boundary and $50 \mu\text{m}$ from the
 240 pixel electrode, \mathcal{P} contains a spectral tail due the diffusion of charge carriers.
 241 \mathcal{P} further degrades in terms of total count of carriers detected and the spectral
 242 width increases as the absorption depth increases from the pixel electrode (250
 243 and $450 \mu\text{m}$). This is due to the more significant diffusion effects as a result
 244 of the increase in the distance the charge carrier must travel to reach the pixel
 245 electrode and due to the random walk by the high-energy electron leading to
 246 a charge cloud of electron-hole pairs created. As the incident x-ray photon
 247 interaction location moves further away (15 and $25 \mu\text{m}$) from the pixel boundary,
 248 \mathcal{P} recovers and the effects of charge carrier transport become less pronounced.

249 The bubble-line and ARTEMIS models show good agreement in the simula-
 250 tion results. This is because semiconductor detectors such as silicon have much
 251 higher carrier mobility and lifetime compared to Se-based detectors as shown
 252 in Table 1, resulting in negligible trapping and recombination rates. In these
 253 types of materials, charge sharing is mainly due to carrier diffusion affected by
 254 the x-ray photon absorption location near the pixel boundary and distance to
 255 the electrode. That said, there are some differences in \mathcal{P} , especially for cases

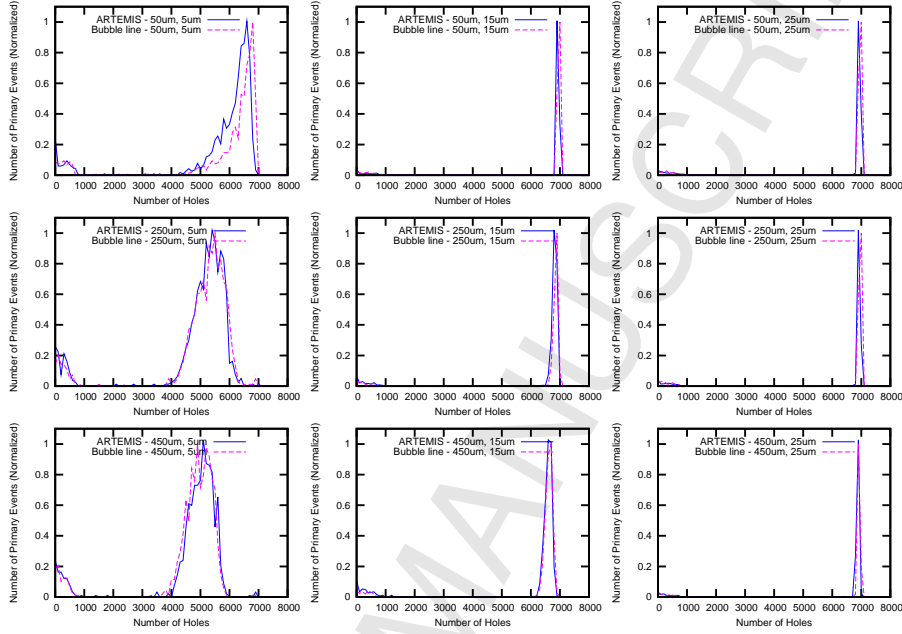


Figure 2. Pulse-height simulation results for 25 keV mono-energetic x-ray photon with bubble-line and ARTEMIS models including a wide range of x-ray absorption locations and pixel boundaries.

256 where the photon is absorbed near the pixel boundary which can be attributed
 257 to statistical MC noise.

258 For the 75 keV case shown in Figure 3, similar trends are observed in \mathcal{P}
 259 as a function of x-ray interaction location from pixel boundary and absorption
 260 depth in the detector. The main difference between the 25 and 75 keV cases is
 261 the increase of counts at lower energies due to Compton scattering.

262 3.2. Double-counting

263 Figure 4 (a) and (b) show the probability of a signal is counted in the primary,
 264 secondary and tertiary pixels as a function of threshold energy. For both 25 and
 265 75 keV cases, the probability of count in the tertiary pixel is very low and is
 266 not considered in the double-counting probability calculations in this work. In
 267 addition, the double-counting probability is only for the charge sharing effects

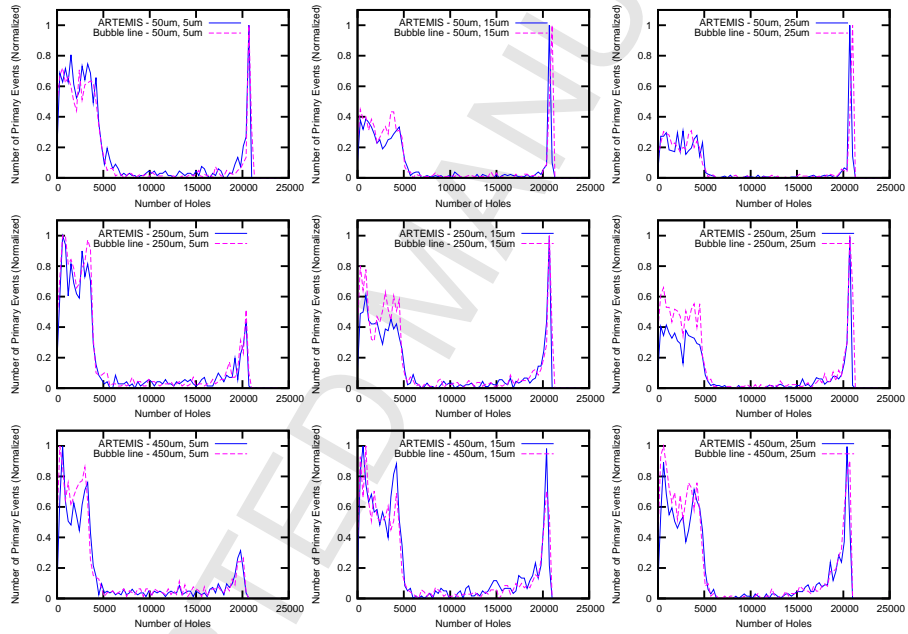


Figure 3. Pulse-height simulation results for 75 keV mono-energetic x-ray photons with bubble-line and ARTEMIS models including a wide range of x-ray absorption locations and pixel boundaries.

268 and not take into account timing information such as pile-up in the detector.

269 The double-counting probability is calculated based on the ARTEMIS \mathcal{P}
270 simulation results are shown in Figure 4 (c) and (d). The double-counting
271 probability is highest for the case where the x-ray interaction is closest to the
272 pixel boundary ($5 \mu\text{m}$) and farthest from the pixel electrode ($450 \mu\text{m}$) and slowly
273 decreases when the absorption depth is closer to the pixel electrode. This is
274 consistent with the results for \mathcal{P} , where \mathcal{P} improves as the absorption depth from
275 pixel electrode is reduced. For the 25keV case with x-ray interaction $5 \mu\text{m}$ from
276 the pixel boundary, the threshold energy at 10% double-counting probability is
277 5.5, 8.5 and 9.2 keV for absorption depth 50, 250 and $450 \mu\text{m}$ from electrode,
278 respectively. This calculated is for mono-energetic x-ray photons, and considers
279 primary and secondary pixels. However, based on a $50 \mu\text{m}$ pixel width, the effect
280 of tertiary and higher order pixels is very small, as shown by the probability of
281 interaction based on threshold energy plots in Figure 4. Therefore, depending on
282 the depth in the detector where the photons are absorbed, the double-counting
283 probability varies.

284 In addition, double counting is also evaluated using Equation 3. The double-
285 counting percentage for 25 keV mono-energetic photons is shown in Figure 5
286 with the same conditions as in Figure 4 (c). For various photon absorption
287 depths (50 , 250 and $450 \mu\text{m}$) the double-counting percentage and probability
288 results exhibit similar trends as a function of threshold energy.

289 4. Discussion

290 We compared the bubble-line and ARTEMIS models for the characterization
291 of charge-sharing effects in semiconductor x-ray detectors. The simulated PHS
292 results shows good agreement when no trapping and recombination are used.
293 This comparison is based on the relative error in Swank factor[46, 47] calculated
294 based on the PHS. The models attribute charge sharing mainly to carrier diffu-
295 sion affected by the x-ray photon absorption location near the pixel boundary
296 and distance to the electrode. This result is applicable to semiconductor mate-

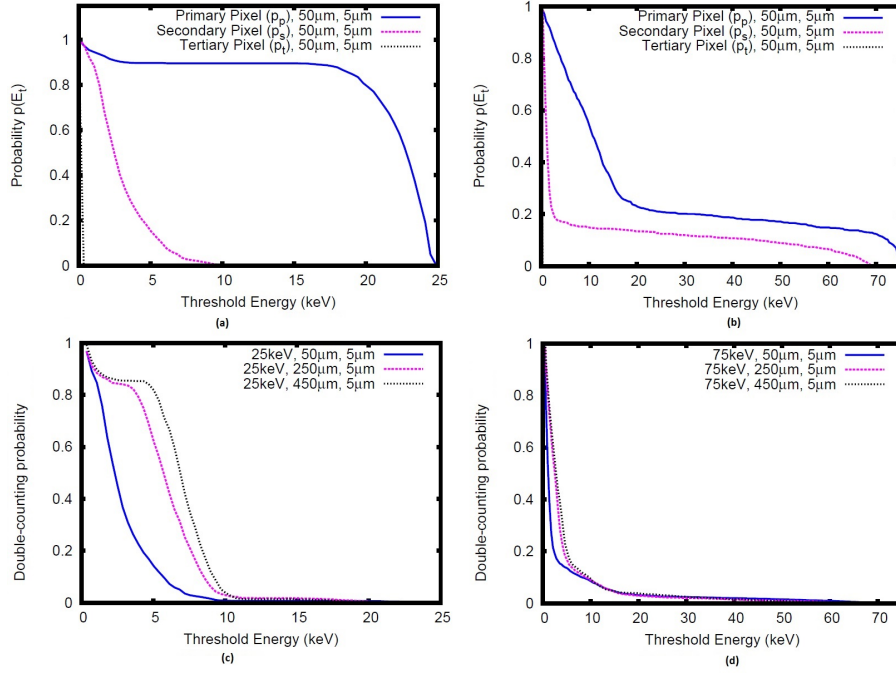


Figure 4. Simulated double-counting probability results: (a) Probability of a signal being counted in the primary, secondary and tertiary pixels as a function of threshold energy for 25 keV mono-energetic photons. (b) The probability a signal is counted in the primary, secondary and tertiary pixels as a function of threshold energy for 75 keV mono-energetic photons. Double-counting probability results for (c) 25 keV mono-energetic photons, and (d) for 75 keV mono-energetic photons.

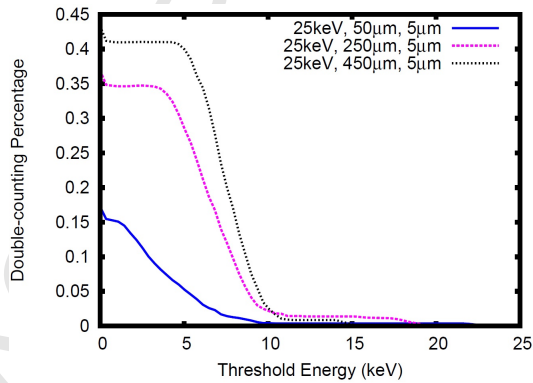


Figure 5. Double-counting percentage results for 25 keV mono-energetic photons.

297 rials such as silicon with high carrier mobility and lifetime but might not apply
298 to relatively higher density materials such as Se and CdZnTe. A speed up of
299 more than 10 times was observed for the ARTEMIS model simulation times by
300 not taking into account recombination and trapping.

301 The effect of charge sharing depends strongly on the location of x-ray ab-
302 sorption near the pixel boundary. In this study, three absorption locations were
303 used. The case at $25\ \mu\text{m}$ from the pixel boundary is at the pixel center with 50
304 μm pixel width. The closest photon absorption location to the pixel boundary
305 is $5\ \mu\text{m}$. In future work, more absorption locations could be studied to fully
306 describe the charge-sharing effect. In addition, we used mono-energetic x-ray
307 photons to minimize Swank noise from a polychromatic spectrum. In order
308 to study charge-sharing effects in clinical x-ray systems, clinical spectral x-ray
309 distributions should be used. At any rate, the study of double-counting proba-
310 bility can be used to understand the fundamental energy threshold limitations
311 for system design in photon counting applications including the use of multiple
312 energy thresholds for novel imaging applications.

313 For double-counting, we used two different metrics that are calculated using
314 the charge transport simulation results described in this paper. The probability
315 definition of double-counting depends on the probability of count in specific
316 pixels (primary, secondary, tertiary, etc) and is a value between 0 and 1. This
317 definition can be used to isolate the double-counting probability in individual
318 pixels and normalized to 1 independently of pixel size. The percentage metric
319 is based on the amount of charge collected and is a ratio between counts in the
320 primary and neighbouring pixels, and can be greater than 1 for small pixels.

321 Overall, the comparison of models described in this paper has the following
322 limitations. First, in ARTEMIS, recombination and trapping effects were dis-
323 abled to provide a direct comparison with the bubble-line method. Since carrier
324 mobility is high in silicon compared to a-Se, recombination and trapping were
325 assumed to be negligible. However, for other types of materials, these effects
326 may be significant and can be included in future work. Table 1 lists the material
327 properties for Si, a-Se and CdZnTe. Finally, the detection geometry can play

328 a significant role in charge sharing. In particular, the results obtained for the
329 edge on geometry used in this study for silicon detectors might be different from
330 results obtained for other detection materials with vertical geometry as seen in
331 spectral CT systems being developed.[6, 7]

332 5. Conclusion

333 Charge clouds and their transport affect the spectral characteristics of pho-
334 ton counting detectors. These effects are most pronounced when interactions
335 occur near pixel boundaries and can be simulated with Monte Carlo and ana-
336 lytical tools. Results of the model comparison show reasonable agreement for
337 the pulse-height spectra simulations (2% relative error in Swank factor) between
338 the bubble-line and ARTEMIS models when considering a silicon strip detec-
339 tor with two mono-energetic beams (25 and 75 keV). The comparison results
340 indicate that carrier diffusion plays a large role in photon-counting detectors,
341 particularly when the photon is absorbed near the pixel boundary far away from
342 the pixel electrode. In addition, the double-counting probability and percentage
343 for x-ray photons absorbed near the pixel boundary as a function of the thresh-
344 old energy has been simulated. This work contributes to our understanding
345 of modeling efforts designed to guide future studies of charge-sharing effects in
346 different detection materials, detector arrangements, absorption locations and
347 at different levels of x-ray energy thresholds.

348 Acknowledgments

349 The mention of commercial products herein is not to be construed as either
350 an actual or implied endorsement of such products by the Department of Health
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352 copyright.

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Fig. 1 Silicon strip detector used for simulation purposes. The edge-on geometry has x rays incident from the top and electrodes on the side. Large arrows within the strip detector indicate the direction of carrier motion for holes. The pixel size is $50\ \mu\text{m}$, and the incident x ray location are 5, 15 and $25\ \mu\text{m}$ from the pixel boundary. The detector thickness is $500\ \mu\text{m}$, and the absorption depth are 50, 250 and $450\ \mu\text{m}$ from the pixel electrode.

Fig. 2 Pulse-height simulation results for 25 keV mono-energetic x-ray photon with bubble-line and ARTEMIS models including a wide range of x-ray absorption locations and pixel boundaries.

Fig. 3 Pulse-height simulation results for 75 keV mono-energetic x-ray photons with bubble-line and ARTEMIS models including a wide range of x-ray absorption locations and pixel boundaries.

Fig. 4 Simulated double-counting probability results: (a) Probability of a signal being counted in the primary, secondary and tertiary pixels as a function of threshold energy for 25 keV mono-energetic photons. (b) The probability a signal is counted in the primary, secondary and tertiary pixels as a function of threshold energy for 75 keV mono-energetic photons. Double-counting probability results for (c) 25 keV mono-energetic photons, and (d) for 75 keV mono-energetic photons.

Fig. 5 Double-counting percentage results for 25 keV mono-energetic photons.