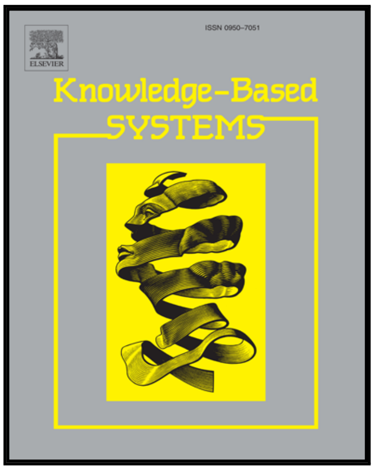

The stratified multi-criteria decision-making method

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Highlights

- This paper applies a recent concept, namely the concept of stratification.
- The dynamicity of decision environment when a decision is made is considered.
- This method mimics the brain decision making process and has the potential of future applications in artificial intelligence.
- This method will be useful to researchers who are applying different MCDM methods.

The stratified multi-criteria decision-making method

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Abstract:

Multiple Criteria Decision Making (MCDM) methods generally require the decision maker to evaluate alternatives with respect to decision criteria and also to assign importance weightings to the criteria. Then, based on the assigned weightings, the best alternative can be selected. However, after a decision is made it often happens that the decision maker becomes doubtful whether the right weightings have been assigned to the criteria given that a variety of eventualities may occur in the near future. The main aim of this paper is to address this concern and improve the application of MCDM methods by addressing possible fluctuations in the criteria weightings. The recently proposed concept of stratification (CST) is used in conjunction with MCDM methods to stratify the decision environment. The method is then applied to a supplier selection problem. The stratified MCDM (SMCDM) approach is in its early stages only and requires further research to reach its maturity.

Keywords: Multi Criteria Decision Making (MCDM); Concept of Stratification (CST); SMCDM; Uncertainty

1. Introduction

The concept of stratification (CST) is an innovative approach to problem solving which has recently been proposed by Zadeh (2016). CST is a system that receives inputs which are the basis for transitioning through different states (Asadabadi, Saberi, & Chang, 2017; Asadabadi, Saberi, & Chang, 2018a, 2018b). In each state the inputs are coupled with outputs and this enables the structuring of dynamic situations such as environments in which multiple criteria decisions are made.

Various Multiple Criteria Decision Making (MCDM) methods have previously been developed (Govindan, Rajendran, Sarkis, & Murugesan, 2015) and used where a number of alternatives need to be ranked based on selection criteria. However, it often occurs that the decision maker is doubtful about the final decision when an MCDM method is employed (Diaz-Balteiro, González-Pachón, & Romero, 2017). Such doubt is due to the fact that the future is always accompanied by uncertainty and the uncertainty makes the decision maker doubtful about the weights assigned to the criteria (Asadabadi, 2017). There are many cases, such as the case discussed in Section 4 of this paper, in which the decision maker is not quite certain when static weightings are provided for the criteria to be used in MCDM methods. So far, MCDM methods have not been empowered to consider fluctuations, in the weightings of decision criteria, in the way changes occur in the human brain. Reviewing the literature (in Section 2) we notice that there is a research gap and this problem has not been sufficiently investigated. Developing a method to resolve doubt in the decision-making process has motivated this study.

Such a method can be developed by utilising supportive concepts such as CST for considering changes that are likely to happen in the decision environment. While making a decision, observing the decision environment and anticipating and considering possible situations can increase the robustness of the final decision. Such an approach to decision-making mimics the decision processes of the human brain while eliminating the associated confusion. When the human brain takes multiple criteria into account in order to make the best decision, the brain considers many positive and negative situations that might happen (Steyvers, Lee, & Wagenmakers, 2009; Weng, Huang, & Li, 2010). This means frequent changes in the weightings of the criteria. For example, when deciding to rent a unit several ‘what ifs’ come to mind: the possibility of having guests, having kids, buying a car or getting a new job. Such uncertainties can change the importance weightings of the criteria such as price, distance and size of the unit. As the importance weightings start changing, the relative value of different units/alternatives may change, and this may impact the decision. Since the human brain is not capable of considering all of the relevant situations simultaneously (Tzeng & Huang, 2011), the decision maker might be confused about whether the right decision is being made.

The contribution of this paper is in providing an application of a recent concept, namely CST, and in showing how this concept can be utilised in combination with an MCDM method to structure the decision-making process in a way that is similar to what takes place in the human brain. In doing so, different eventualities are taken into account while making a multi-criteria decision. In the proposed method, the current state of a decision is identified and potential states that may occur, and are adjacent to the current state, are also engaged in making the decision. This consideration strengthens and empowers MCDM methods by enabling them to handle the dynamicity of the decision environment. A process that stratifies the environment and the associated precomputations benefits the decision maker by ensuring them that their concerns are taken into account in the decision process. As a result, there is less likelihood of regret in the future. The proposed integration should stimulate work on the future application of CST in conjunction with various MCDM methods, in particular, in artificial intelligence.

The remainder of this paper is structured as follows. The next section features a brief review of the current literature on MCDM methods and CST. Then, a combined method applying CST to a general version of MCDM methods, namely the stratified MCDM (SMCDM) method, is explained. The method is applied on a supplier selection problem experienced by a company. At the end, a discussion section proposes the limitations and future studies.

2. Literature review

In this section, papers dealing with applications of MCDM methods in different areas of decision-making , in particular in supplier selection, are reviewed. We focus on the uncertainty issue and how the researchers have tackled the problem so far. This is followed by a review of the literature on CST and its applications.

2.1 A review of MCDM methods

MCDM methods generally have been developed in order to facilitate the selection of an alternative with respect to multiple criteria. There are currently several MCDM methods in use. The more frequently employed are listed as follows.

- Analytical Hierarchy Process (AHP) (Ahmadi, Petrudi, & Wang, 2017; Liu, Yu, Pedrycz, & Zhang, 2018; Xu & Liao, 2014)
- Analytical Network Process (ANP) (Liao, Mi, Xu, Xu, & Herrera, 2018; Öztayşi & Kahraman, 2013b)
- Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE) (Boujelben, 2017; Liao & Xu, 2014)
- VlseKriterijumska Optimizacija I Kompromisno Resenje (Multi-criteria optimization and compromise solution or VIKOR) (Liao, Xu, & Zeng, 2015; Ren, Xu, & Wang, 2017)
- ELimination Et Choix Traduisant la REalité (ELimination and Choice Expressing Reality or ELECTRE) (Wan, Xu, & Dong, 2017)
- Best Worst Method (BWM) (Rezaei, 2015)
- Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) (Wang & Chen, 2017).
- Decision Making Trial and Evaluation Laboratory (DEMATEL) (Yazdani, Chatterjee, Zavadskas, & Zolfani, 2017)

MCDM methods have been integrated with many other tools and techniques in the last few decades. The integrations are mainly with the aim of strengthening and empowering MCDM methods to address various decision problems more effectively. While a review of all is not possible in this paper, a brief review of the recently used integrations that deal with the uncertainty and supplier selection issues is submitted here.

2.1.1 Combining MCDM methods with other tools and techniques

Integrations of MCDM methods with other tools and techniques are mainly to provide more accurate and therefore more effective rankings of alternatives. There are many recent examples of integrations of various tools and techniques and MCDM methods, such as the Markov Chain (Asadabadi, 2016; Nawaz et al., 2018), Quality Function Deployment (QFD) (Wu & Liao, 2018), fuzzy sets (Liao, Jiang, Xu, Xu, & Herrera, 2017; Liao, Wu, et al., 2018; Tang, 2017), Z numbers (Aboutorab, Saberi, Asadabadi, Hussain, & Chang, 2018) and others (Chai, Liu, & Ngai, 2013; Liu & Liao, 2017). A complete list of integrations would be too extensive to be included in this paper. However, a number of studies applying MCDM methods are reviewed in this section because of their recent contribution to knowledge.

Ahmadi et al. (2017) propose a combination of a version of grey relational analysis approaches and AHP to compute sustainable weights of criteria and to find the best ranking of potential suppliers. Xu et al. (2014) submit an integration of fuzzy sets and AHP to deal with uncertainty in the decision-making process. Using a similar approach, Liu et al. (2018) utilise this integration to address uncertainty which decision makers may confront while performing pairwise comparisons. In a more advanced approach, Liao et al. (2018) combine fuzzy sets with ANP to take into account the interrelationships of the elements, which are involved in the decision-making process. Öztayşi et al. (2013a) use ANP accompanied with fuzzy sets to deal with uncertainty in the performance measurement of a company's characteristics when numerically expressed.

Liao et al. (2018) employ fuzzy sets and aggregate individual decision matrices in a collective matrix in a linguistic environment. In contrast to the previous methods dealing with similar problems in linguistic environments, their method addresses a drawback of previous methods: namely the inability to provide an effective aggregation of large number of opinions. Boujelben (2017) utilises the principles of the PROMETHEE method to deal with a suppliers' segmentation problem. Liao et al. (2015) employ the VIKOR method in the process of group decision-making and additionally use a hesitant fuzzy linguistic technique to deal with hesitation in expressing the preferences for alternatives. Ren et al. (2017), in a similar approach, combine VIKOR with dual hesitant fuzzy sets. Wan et al. (2017) propose an integration of ELECTRE and ANP to deal with a supplier selection problem by considering a hierarchical structure among criteria.

Rezaei (2015) proposes a new MCDM method, namely BWM. The proposed method is compared with AHP and the merits of the method, for example the reduction in the required number of comparisons and the consistency ratio of the method, are highlighted. Aboutorab et al. (2018) improve the method to be able to take into account the uncertainty of real word decision and show that their method results in lower inconsistency ratios when compared with the original proposal of BWM. Wang et al. (2017) submit a combination of linear programming and TOPSIS. Linear programming is used to compute the optimal weights of criteria and is combinrf with TOPSIS in order to be able to provide the ranking of the alternatives. Asadabadi (2016) deals with the uncertainty that may exist while computing the weights of importance of criteria in ANP and proposes a Markov chain to calculate the final weights. Wu and Liao (2018) combine the QFD method (presenting matrices that show the strengths of relations between two or more sets of elements), and a ranking method, ORESTE, to find a prioritised list of design requirements based on customer requirements. Tang (2017) investigates a green supplier selection and uses fuzzy sets to deal with uncertainty of the information. Liao et al. (2017) propose an integration of hesitant fuzzy linguistic term sets, PROMETHEE, and linear programming to deal with the uncertainty of information in multi-criteria decision-making processes.

As we can see, quite a large percentage of papers proposing combinations of MCDM methods, and other tools and techniques, focus on addressing uncertainty in decision-making which is similar to the aim of the current study. In the next subsection, we concentrate on the uncertainty issue.

2.1.2 Uncertainty in decision-making

There are many studies that develop decision-making models to deal with the uncertainty issue. Some of the recent ones are worth reviewing. A linear programming approach is structured by Liu, Dong, Chiclana, Cabrerizo, and Herrera-Viedma (2016) to minimize the information deviation of the relations between decision makers' preferences when they have a different level of uncertainty. Moral, Chiclana, Tapia, and Herrera-Viedma (2017) study a group decision-making problem with uncertainty involved. The problem is investigated using a fuzzy approach to obtain experts' preferences while focusing on the convergence speed of the consensus of the preferences. They show that setting a number of rules can control the speed in the decision-making process. Wu, Chiclana, Fujita, and Herrera (2017) propose a visual interaction framework to facilitate reaching a consensus: based on different preferences obtained from a range of experts. A trust-based recommendation mechanism is then submitted to deal with the inconsistency in expressed preferences by different experts. The mechanism finds out whether an unknown expert can be trusted and so whether the associated preferences should be taken into account. Capuano, Chiclana, Fujita, Herrera-Viedma, and Loia (2017) propose a model that considers the real preference of an expert who might be influenced by the opinion of other experts. They assume that the expert is unable to express preferences on some alternatives and employ a user friendly fuzzy ranking model to obtain the preferences and to prevent the expert's uncertainty to affect the process. Zhanga, Dong, & Herrera-Viedma (2017) deal with significant conflicts in experts' preferences that may cause serious uncertainty in the decision process. They employ a selection process to divide decision makers into different clusters. Individual preference vectors are obtained, and a feedback adjustment process is utilised to help decision makers in adjusting their preferences.

The main difference between the previous study and this study is the fact that this study stratifies the decision environment and allows the ranking of alternatives to be transitioning through the strata and affected by uncertainty. Then, considering all the influences, the optimal ranking is found.

2.1.3 Applications of MCDM methods in ranking suppliers

In the case study later presented, we utilise SMCDM to address uncertainty in a supplier selection problem. Supplier selection is a multi-criteria decision-making problem that has a considerable effect on effectiveness and efficiency of a company's performance (Aouadni, Rebai, & Turskis, 2017; Asadabadi, 2017; Jamali, Asl, Zolfani, & Šaparauskas, 2017; Keshavarz Ghorabaee, Amiri, Zavadskas, Turskis, & Antucheviciene, 2017; Keshavarz Ghorabaee, Zavadskas, Amiri, & Esmaeili, 2016; Keshavarz Ghorabaee, Zavadskas, Amiri, & Turskis, 2016; Liao, Fu, & Wu, 2016; Qin, Liu, & Pedrycz, 2017; Stević, Pamučar, Vasiljević, Stojić, & Korica, 2017; Yazdani et al., 2017). It often happens that suppliers have different strengths and weaknesses so that it becomes difficult to objectively select the best supplier without using an MCDM method (Ho, Xu, & Dey, 2010). There are many examples of the application of MCDM methods to address such supplier selection problems. Aouadni et al. (2017) have worked on reference points of TOPSIS to improve its meaningfulness. Qin et al. (2017) utilise the interactive and multi-criteria decision-making method, developed by Gomes and Lima (1992), in a fuzzy and uncertain environment with several unknown factors to address the problem. Yazdani et al. (2017) employ a combination of QFD, DEMATEL and Complex Proportional Assessment (COPRAS) to provide a ranking of a number of potential suppliers. Liao et al. (2016) propose an integration of fuzzy analytic hierarchy process, fuzzy additive ratio assessment and multi-segment goal programming to deal with the uncertainty and vagueness involved in the supplier selection problem. Keshavarz Ghorabaee et al. (2016) propose a modified version of the weighted aggregated sum product assessment method in combination with interval type-2 fuzzy sets to handle the uncertainty of information. Keshavarz Ghorabaee et al. (2016) utilise a fuzzy version of evaluation based on distance from average solution method to deal with the uncertainty of the decision environment in order to provide a stable ranking of suppliers. Later, Keshavarz et al. (2017) employ an improved version of the method to deal with uncertainty of performance values of alternatives. Jamali et al. (2017) utilise the step-wise weight assessment ratio analysis method in combination with the Strengths, Weaknesses, Opportunities and Threats (SWOT) tool for evaluation purpose in supply chain analysis. Asadabadi (2017) deals with the uncertainty of weightings of criteria in supplier selection and utilises an application of a Markov chain in combination with ANP-QFD method to address that.

While instability of the decision and uncertainty about the weightings of decision criteria in MCDM methods and more particularly in their applications in supplier selection had been previously studied, there is a need for further investigations that consider the instability of the environment in which multi-criteria decisions are made. This study employs a recently proposed concept, namely CST as a means of taking into account the instability of the decision environment.

2.2 A review of the concept of stratification (CST)

There are various versions of stratification, including stratified logic (Zaniolo, 2015), approach (Herencia, 1996), analysis (Brenes & Gayo-Avello, 2009), and programming (Dascalu, Pasculescu, Woolever, Fritzinger, & Sharan, 2003), among others (Balmin, Ercegovac, Haas, Peng, & Sismanis, 2017; Qu, Shang, Shen, Mac Parthaláin, & Wu, 2015; Zadeh, 2016; Zhang, Wu, Yuan, Wang, & Dai, 2016) that have been previously developed and applied. Recently, an innovative version of stratification, namely CST, has been proposed by Zadeh (Zadeh, 2016). This method describes a system that transitions through different states to reach the target (Asadabadi et al., 2017). While CST seems to have a great potential of applicability to address complex problems, Zadeh does not provide any applications of the concept and encourages future research to apply it in different areas. Although approach is similar to dynamic programming, it has characteristics that differentiate it from the existing methods (Zadeh, 2016). One of the unique characteristics of Zadeh's method is its approach to reach the target that is called stratification (Asadabadi et al., 2018a). In this approach the environment, in which a system transitions to reach the target, is stratified.

Since the concept has been recently proposed (Zadeh, 2016), only a limited number of studies have been undertaken (Asadabadi et al., 2017; Asadabadi et al., 2018a, 2018b). Asadabadi et al. (2017) implement the concept for logistics informatics modelling. Their study is focused on providing examples that show how CST can be utilised to structure issues in two areas: information dominance and requirement specification in contracting. In particular, they show how the requirement elicitation and specification process can be modelled easily using this concept. This work was followed by two more studies by Asadabadi et al. (2018 a, 2018 b).

In Asadabadi et al. (2018b), the authors address one of the main shortcoming of the concept which is its inability to simultaneously consider more than one target of unequal weights of importance. They utilised CST to select a restaurant in their neighbourhood which is in some ways similar to a supplier selection problem. The case has two targets: one is to select the one with highest review rate, the other one is to select the one located at the closeest distance. They assign different weights to these criteria and solve the problem using CST. While they proposed an approach by following which two targets of different weights of importance can simultaneously be considered, the approach is unable to consider more than two targets. In Asadabadi et al. (2018a), they review the concept and propose potential applications to CST. The authors provide three extensions to CST, namely fuzzy CST, three dimensional CST, and multiple systems multiple CST, with the aim of drawing attention to this new and powerful concept.

With CST in its early stages, studies of the concept are quite limited, but the characteristics of CST are very promising for a wide variety of future applications. The characteristics make it suitable for application in many areas such as robotics, artificial intelligence, and planning and monitoring (Asadabadi et al., 2017). Although MCDM methods have been used in combination with different tools and techniques, a combination that empowers MCDM methods to stratify the decision-making environment has not yet been proposed and this is the contribution of this paper.

3. The integration of CST and MCDM

In this section we propose a model which takes into consideration events that are likely to happen and may influence the weightings of criteria when employing an MCDM method. The method integrates the concept of stratification (3.1) with a general version of MCDM methods (3.2).

3.1 Illustrating the concept of stratification (CST)

Stratification categorizes a number of states which belong to different strata where one or more states are considered as the members of the target set. CST transitions through the states in order to reach a state in the target set. To find paths to reach the target, the original target set is gradually degraded. This helps identification of the ways of reaching the target. The target degradation can be of interest when the original target is not reachable, being too costly in terms of time or other resources, or not clearly observable at the time that the problem is addressed (Asadabadi et al., 2017). In Figure 1, the target set, strata, and states are presented.

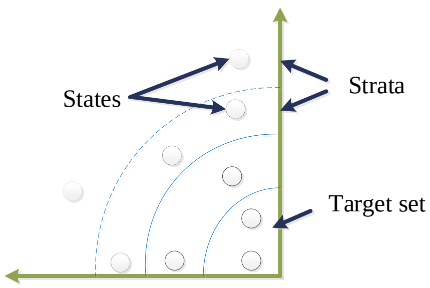

Fig. 1. Target set, states and strata in CST

The following concepts are required to define CST:

- **System:** A system is a collection of objects. It transitions through states toward the target set.
- **State:** A state is associated with values of the system variables. i^{th} state is labelled state w_i. The system transitions from one state to another as the values of variables change.
- **State-transition function:** The state-transition function presents the transitions of the system from i^{th} state to $(i+1)^{th}$ state where u_t is the input of the system at state S_t.

$$S_{(t+1)}=f(S_t, u_t) \quad (1)$$

- **Target state:** State w_i is in the target set if reaching w_i is an objective of the system.

- **Reachability:** w_j is reachable from w_i if there is a path to transition from w_i to w_j.
- **Input (u_t) and output (v_t):** In each state the system may receive an input. The input is associated with an output and determines the next state that the system transitions to.

To elaborate further, the concept is illustrated through the following example. Assume that the system is a patient who is described measuring two state variables: *body temperature* and *blood pressure*. This measurement determines the state that the patient is currently in. The input (u_t) can be considered as the medicine that the patient receives and can be either *medicine for body temperature* (α) or *medicine for blood pressure* (β). These two inputs are associated with two outputs: *reduced body temperature* (γ) and *reduced blood pressure* (δ). Assuming that the *body temperature* can be 'high (H)' or 'low (L)' and the *blood pressure* can be 'dangerously high (D)', 'high but not dangerous (ND)', or 'normal (N)', there can be 6 possible states that as shown in Table 1.

Table 1
Tabular CST for the patient example

S_t	Body temperature	blood pressure	u_t	S_{t+1}
w_1	H	D	α	w_2
			β	w_3
w_2	L	D	β	w_4
w_3	H	ND	α	w_4
			β	w_5
w_4	L	ND	β	w_6
w_5	H	N	α	w_6
w_6	N	N	Null	Null

In this example, the target set has only one state, namely w_6. Figure 2 graphs the system.

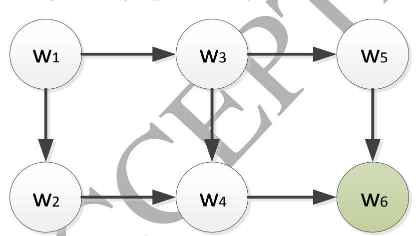

Fig. 2. Graphical CST for the patient example

In the above example, the system is the patient who is transitioning through different states toward the target set. The target set is a single state target and includes state w_6, which is reachable from all of the other five states. This state is considered as the target state because the following proposition is true: *'p = patient is cured'*.

We can see the potential of this concept can be applied in combination with MCDM methods. This is because a multiple criteria decision may confront various situations that change the weights of the criteria. While we admit that this paper does not utilise all of the abilities and components of CST, it uses the main concept to stratify the decision environment.

3.2 A general version of MCDM methods

In this paper we do not limit the proposed model to a particular MCDM method, rather a general version of MCDM methods is used. In MCDM methods the alternatives are scored with respect to the criteria and then multiplied by the weights of the criteria (Fu, Xu, & Xue, 2018; Zhang, Kou, Yu, & Guo, 2018). The sum of the numbers for each alternative, namely the value of the alternative, is the basis for ranking the alternative (Rezaei, 2015). Assume that 'Matrix A' shows the scores, $q_{11}\ldots,q_{nm}$, that the alternatives, $a_1 \ldots, a_n$, receive with respect to the criteria, $c_1 \ldots, c_m$.

$$A = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \ldots & c_m \end{array} \\ \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1m} \\ q_{21} & \vdots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ q_{n1} & \cdots & \cdots & q_{nm} \end{bmatrix} \end{array} \quad (2)$$

In order to select the best alternative, the value of each is computed. Assuming that normalised weightings, Wt:$\{wt_1\ldots, wt_m\}$, have been assigned to $c_1 \ldots, c_m$, the value of alternative *'i'* is as follows.

$$V_{ai} = \sum_{j=1}^{m} q_{ij} wt_j \quad (3)$$

MCDM methods provide a ranking of the alternatives taking into account the importance weightings of the criteria. However, the situation in which the decision is made may change, so that the weightings of the criteria used to make the decision may change. If the weights of the criteria change, the decision may change.

In this study we simulate the environment in which the decision is made in order to make a smarter decision. The decision takes into account other situations or states that might happen as well as the current state of the system.

3.3 The Stratified MCDM method (SMCDM)

In subsection 3.2, the general process of MCDM methods was presented. Based on the discussed model, Equation (3) is capable of computing the weight of alternative 'i' if the current situation, in which the decision is made, persists.

Again, we assume that there are 'm' criteria and 'n' alternatives, as presented in Matrix (2). The alternatives, a_1 to a_n, are compared with respect to the criteria, c_1 to c_m. However, the weights of the criteria, Wt:$\{wt_1\ldots, wt_m\}$, depend on whether the current situation continues.

Consider that the decision is a system that is currently at state w_i. We assume that there are 'h' different states, including the current state, that decision can be in or transition to. The states are the result of the incidents that may happen and take the system to different states. Given this, the system can move to ‘h-1’ states other than the current state.

$$W: \{w_1 \dots w_h\} \quad (5)$$

A visualised view of the states and transitions is represented in Figure 3. The weightings of criteria in each state may change. We assume that the decision maker in an organisation is able to:

- identify which states are likely to happen and
- either calculate or estimate the likeliness of their occurrences.
- estimate the weightings of criteria in each situation.

These are the prerequisites for running the model. After providing the information, the method can find the optimal ranking of the alternatives considering all possible eventualities.

Considering each situation as a state in CST, the weightings for criteria in k^{th} state are as follows.

$$Wt_k: \{wt_{k1}, \dots wt_{km}\} \quad (6)$$

The estates, W: $\{w_1 \dots w_h\}$, and their associated weights, Wt_k: $\{wt_{k1}, \dots wt_{km}\}$, are represented in Figure 3.

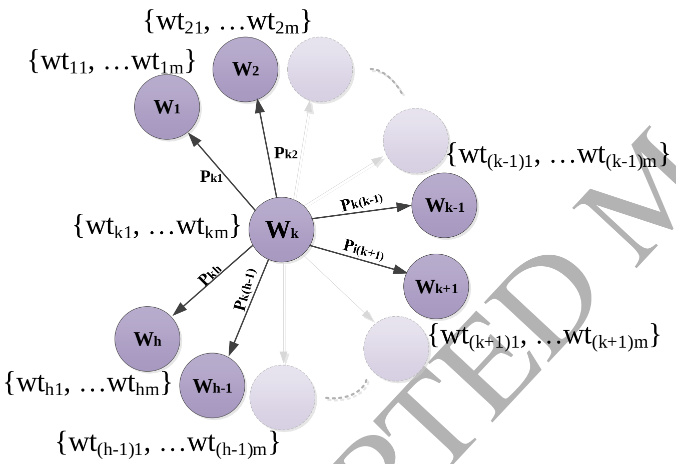

Fig. 3. Stratified weightings for criteria

Considering Figure 3, the weighting of criterion ‘j’ in state ‘f’ is presented as wt_{fj}.

The general form of the transition matrix, which includes all of the transition probabilities, is as follows.

$$P = \begin{array}{c c} & \begin{array}{cccc} w_1 & w_2 & \dots. & w_h \end{array} \\ \begin{array}{c} w_1 \\ w_2 \\ \vdots \\ w_h \end{array} & \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1h} \\ p_{21} & \vdots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ p_{h1} & \cdots & \cdots & p_{hh} \end{bmatrix} \end{array} \quad (7)$$

Now, assuming again that the system is at state ‘k’, the probabilities of transitioning to other states are as presented in the k^{th} row of Matrix ‘P’, which has to be provided to the model.

Being provided with the k^{th} row, the value of alternative ‘i’ is computed as follows.

$$V_{ai} = \sum_{t=1}^{m} q_{it} \sum_{j=1}^{h} wt_{jt} p_{kj} \quad (8)$$

Note that P_{kk} denotes the probability of transitioning from state ‘k’ to state ‘k’, which is the likelihood that the current situation persists. The method is represented in the Algorithm 1.

Algorithm 1. SMCDM algorithm

Input:

Wt : the weightings of criteria in all states

P : probabilities of transitioning from the current state to the other states

Q : the weightings of alternatives with respect to criteria

Output:

V_a : ranking of the alternatives

h ← the number of states
m ← the number of criteria
n ← the number of alternatives
wt_{jt} ← wightings for criteria t at state j
p_{kj} ←probability of moving from state k (current state) to state j
for (i=1; i <=n; i++)
  set $qwtp$ (0) to zero
  for (t=1; t $<= m$; t++)
    set $wtp(0)$ to zero
    for (int j=1; j $<= h$; j++)
      $wtp(j) = wt_{jt} p_{kj}$
      $wtp\ (j) \leftarrow wtp(j-1) + wtp(j)$
    end for
   $qwtp\ (t)$= q_{it} $wtp\ (j)$
   $qwtp\ (t) \leftarrow qwtp(t-1) + qwtp(t)$
  end for
  $V_{ai} \leftarrow qwtp\ (t)$
end for
return V_a

In the next section the method is illustrated by presenting its application to a real-world supplier selection problem.

4. A real world example

Doris Pars Company is a bathroom equipment and accessories wholesaler and manufacturer in Iran. The company has three potential suppliers for their high- and low-density polyethylene, labelled supplier ‘A’, ‘B’ and ‘C’ (the Doris company prefers their trading partners to be anonymous). The choice of the supplier should be made considering three criteria: *price*, *quality*, and *delivery*.

In subsection 4.1, the best supplier is selected by applying the general form of MCDM methods. The method fails to consider some managerial concerns that are explained in subsection 4.2. The concerns are considered when using the SMCDM method to address the problem in subsection 4.3.

4.1 Addressing the problem using the MCDM method

The three criteria mentioned above are compared and weighted by the managers. The weights are presented in Table 2.

Table 2
Weightings of the criteria

	Weights
Price	0.40
Delivery	0.40
Quality	0.20

Alternatives are compared with respect to each criterion separately, and then presented in Table 3 (equivalent to Matrix A in the previous section). For instance, supplier B is considerably better than the other two with respect to Delivery.

Table 3
Comparing suppliers with respect to criteria

	Quality	Price	Delivery
Supplier A	0.23	0.49	0.30
Supplier B	0.36	0.14	0.45
Supplier C	0.41	0.37	0.25

In order to consider the weights of the criteria for the values in Table 3, the weights of criteria (in Table 2) are multiplied by the relevant columns (in Table 3). Then, the sum of each row presents the value of the associated alternative/supplier. The results are shown in Table 4.

Table 4
The ranking using the MCDM method

	Quality	Price	Delivery	Weights	Rank
Supplier A	**0.05**	**0.19**	**0.12**	**0.36**	**1**
Supplier B	0.07	0.06	0.18	0.31	3
Supplier C	0.08	0.15	0.10	0.33	2

Supplier ‘A’ is selected as the best option for the Doris Pars Company. However, the managers are concerned with upcoming possible incidents that are likely to influence the weightings of the criteria (in Table 2). The upcoming incidents are explained in the next subsection.

4.2 Managerial concerns

The method has found the best supplier, but only if the current situation persists. In order to sign a contract with a supplier, the managers are concerned with other situations that are likely to happen in the near future. Supplier ‘A’ has been selected for the Doris Pars Company considering the current weightings of the criteria: quality, price, and delivery (0.20, 0.40, 40). However, the company currently is negotiating to sign contracts with two main consumers, namely company ‘X’ and ‘Y’. Company ‘X’ intends to sign a long-term contract while company Y prefers to have a conditional contract that allows them to terminate the contract within a year in case of conflict. Signing a contract with each of them changes the initial weights of the criteria. In addition, there is a potential investor that has an interest in becoming a partner. Therefore, the managers are concerned that the assigned weights of the criteria will not be valid if each of these three incidents occur in the near future, namely ‘signing a new contract with company ‘X’ or ‘Y’’ and ‘having a new investor’. Any of these eventualities changes the weightings of the criteria and the current application of MCDM method is not able to consider these concerns. In the next section, we see how these concerns can be resolved using SMCDM.

4.3 Addressing the concerns applying the SMCDM method

Three possible situations that can affect the weightings of the criteria in the near future are considered using CST. Using vectors facilitates the explanation of how inputs and outputs are bundled and simplifies the presentation of information in the Tabular form of CST. The three arrays vector of u_t =(a, b, c) is considered as the input vector at state s_t which is associated with an output vector, v_t =(x, y, z). The arrays of u_t may possess the following values.

$$a = \begin{cases} 0 & , No\ updates\ from\ company\ 'X' \\ 1 & , Company\ \ 'X' signs\ the\ contract \end{cases} \tag{9}$$

$$b = \begin{cases} 0 & , No\ updates\ from\ company\ 'Y' \\ 1 & , Company\ \ 'Y'\ signs\ the\ contract \\ -1 & , Company\ \ 'Y'\ terminates\ the\ contract \end{cases} \tag{10}$$

$$c = \begin{cases} 0 & , No\ updates\ from\ the\ investor \\ 1 & , The\ investor\ joins\ the\ company \end{cases} \tag{11}$$

The arrays of the outputs (x: the weight of quality, y: the weight of price, z: the weight of delivery) can be considered as the modified weights of the criteria.

The Doris Pars Company determines the weights of the criteria with respect to each of the circumstances. For example, the managers state that if the input is (0, 1, 0) which means that company ‘Y’ signs a contract with the company, the importance of the weighting of price decreases considerably. They suggest that the following vector be used as the weightings of the criteria, namely quality, price, and delivery: (0.52,0.11,0.37). In contrast, for example, if company ‘Y’ breaks and leaves the contract for any reason, the input is labelled (0, -1, 0) and the weights are modified respectively. Taking into account other likely situations, Table 5 is built.

Table 5
Tabular CST for Doris Pars Company

S_t	u_t	S_{t+1}	v_t
w_1	(0,0,1)	w_2	(0.21,0.21,0.58)
	(0,1,0)	w_3	(0.52,0.11,0.37)
	(1,0,0)	w_4	(0.23,0.38,0.39)
w_2	(0,1,0)	w_5	(0.32,0.05,0.63)
	(1,0,0)	w_6	(0.10,0.22,0.68)
w_3	(0,0,1)	w_5	(0.32,0.05,0.63)
	(1,0,0)	w_7	(0.29,0.03,0.68)
	(0,-1,0)	w_1	(0.20,0.40,0.40)
w_4	(0,0,1)	w_6	(0.10,0.22,0.68)
	(0,1,0)	w_7	(0.29,0.03,0.68)
w_5	(1,0,0)	w_8	(0.12,0.02,0.86)
	(0,-1,0)	w_2	(0.21,0.21,0.58)
w_6	(0,1,0)	w_8	(0.12,0.02,0.86)
w_7	(0,0,1)	w_8	(0.12,0.02,0.86)
	(0,-1,0)	w_4	(0.23,0.38,0.39)
w_8	(0,-1,0)	w_6	(0.10,0.22,0.68)

The above table is the tabular form of CST. The transitions can also be shown using a graph version of the concept, as presented in Figure 4.

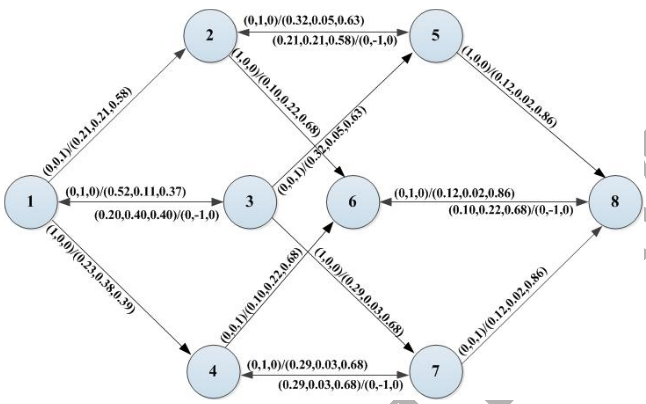

Fig. 4. Graphical CST for Doris Pars Company

Based on the information in Table 5, the weights of the criteria in each state, presented previously in Table 5, are presented in Table 6.

Table 6
Weightings of the criteria at different states

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
Quality	0.20	0.21	0.52	0.23	0.32	0.10	0.29	0.12
Price	0.40	0.21	0.11	0.38	0.05	0.22	0.03	0.02
Delivery	0.40	0.58	0.37	0.39	0.63	0.68	0.68	0.86

Considering the information in Table 3 and 6, the value of each supplier in each state can be computed. For example, the value of supplier B at w_2 is computed as follows.

$$V_{S_B w_2} = (0.21 \times 0.36) + (0.21 \times 0.14) + (0.58 \times 0.45) = 0.37 \qquad (12)$$

Following similar computations, the importance weightings of suppliers in different states can be computed as presented in Table 7.

Table 7
Weights of alternatives with respect to criteria

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
Supplier A	0.36	0.32	0.28	0.35	0.29	0.33	0.28	0.29
Supplier B	0.31	0.37	0.37	0.31	0.41	0.38	0.42	0.44
Supplier C	0.33	0.31	0.34	0.33	0.31	0.29	0.30	0.27
Best Supplier	A	B	B	A	B	B	B	B

Considering the information in Table 7, supplier A outperforms other suppliers when the current situation of the company is considered. If the outperformance continued, there would be no need to continue the method for obvious reasons. However, given Table 7, we need to move forward to the next step, which considers the likelihood of occurrence of other states as well as the continuation of the current situation. Since the current state is state one, the first row of the matrix presented below needs to be computed.

$$P = \begin{array}{c} \\ w_1 \\ w_2 \\ \vdots \\ w_8 \end{array} \begin{array}{c} \begin{array}{cccc} w_1 & w_2 & \dots. & w_8 \end{array} \\ \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{18} \\ p_{21} & \vdots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ p_{81} & \cdots & \cdots & p_{88} \end{bmatrix} \end{array} \qquad (13)$$

Although such probabilities can be too difficult to accurately compute, managers are expected to provide intuitive estimations of the probabilities. Note that the consideration of the other states, even though their probabilities may have a degree of error, should generally be preferred to entirely ignoring their occurrence likelihood.

In Matrix (13), for instance, P_{17} denotes the probability that both company 'X' and 'Y' sign contracts with the company, while the investor has not yet joined. P_{11} represents the probability that the current situation continues, which in this case was stated to be about 17%. The probabilities of moving from state one (the current state) to any other states are shown in Figure 5.

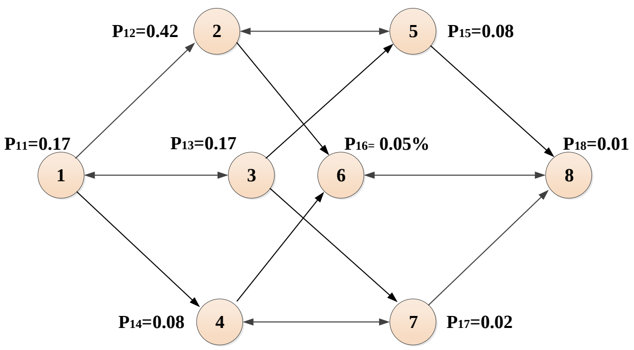

Fig. 5. Graphical CST for Doris Pars Company

In order to find the final weightings of each alternative/supplier, the weightings of each of them at different states need to be multiplied by the probability of that state occurring. In other words, the first row of Matrix (13) is multiplied by the columns of Table 7. The sum of each resulting row is the final value of each supplier. The results are presented in Table 8.

Table 8
Weightings of the alternatives

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	Final Weights	Ranking
Supplier A	0.061	0.135	0.048	0.028	0.023	0.017	0.006	0.003	0.321	2
Supplier B	0.053	0.155	0.063	0.025	0.033	0.019	0.008	0.004	0.360	1
Supplier C	0.056	0.130	0.059	0.027	0.025	0.015	0.006	0.003	0.319	3

The final weights of suppliers computed in Table 8, can also be obtained using the formula presented in Equation (8). For instance, the value for supplier B is computed as follows.

$$V_{Sup_B} = \sum_{t=1}^{3} \mathrm{q}_{\mathrm{Bt}} \sum_{\mathrm{j}=1}^{8} wt_{jt} p_{kj} \quad (14)$$

$$\begin{aligned} V_{Sup_B} = 0.36(&0.2 * 0.17 + 0.21 * 0.42 + 0.52 * 0.17 \\ &+ 0.23 * 0.08 + 0.32 * 0.08 + 0.10 \\ &* 0.05 + 0.29 * 0.02 + 0.12 * 0.01) \\ &+ 0.14(0.4 * 0.17 + 0.21 * 0.42 \\ &+ 0.11 * 0.17 + 0.38 * 0.08 + 0.05 \\ &* 0.08 + 0.22 * 0.05 + 0.03 * 0.02 \\ &+ 0.02 * 0.01) + 0.45(0.4 * 0.17 \\ &+ 0.58 * 0.42 + 0.37 * 0.17 + 0.39 \\ &* 0.08 + 0.63 * 0.08 + 0.68 * 0.05 \\ &+ 0.68 * 0.02 + 0.86 * 0.01) \end{aligned}$$

$$V_{Sup_B} = 0.360$$

Without considering other possible eventualities supplier ‘A’ would be the best supplier. However, applying the SMCDM method, other possible situations are considered, and hence supplier B is selected to sign the contract.

5. Discussion

There have been numerous studies that propose combinations of MCDM methods with other tools and techniques to address a variety of multi-criteria decision problems (Ho et al., 2010). More particularly, considerable effort has been made in applying MCDM methods in combination with fuzzy sets (Liao, Mi, et al., 2018). The application of fuzzy sets is commonly used to address the uncertainty associated with the information that MCDM methods receive either regarding the preference values for alternatives or the weightings of criteria. Considering the significant consequences of uncertainty in the decision-making process, more studies need to be undertaken.

One area that has the potential of further research is the uncertain environment in which a decision is made. It often happens that the decision maker assigns weights to criteria relevant to a certain situation. However, the situation on which the weights are computed might be unstable. The case discussed in Section 4 illustrates such instability. We observed in subsection 4.1 that the MCDM method could find a ranking of supplier, but the ranking was not able to cover the managerial concerns discussed in subsection 4.2. By changing the situation of Doris Company, the weights of importance of the criteria change. If company Y signs the contract, the importance of quality significantly increases. This is probably because the management knows that company Y would be sensitive about the quality of the product. However, the event of *‘company Y signs the contract’* is associated with a probability, and so its probability should be taken into account in order to consider its effect on the weights of criteria.

While the current situation of the company encourages the selection of supplier A, considering the fact that there is only 17 percent probability of experiencing no changes in a short time (remaining in the current situation), selecting supplier A would not be an optimal selection. This was calculated using the principles of CST in subsection 4.3. Without performing the stratification and considering the eight possible situations that are the consequences of the three incidents that were likely to happen, it was difficult to find the best supplier. In this case, the number of incidents were only three, each with one effective event; an effective event is an event which is capable of introducing the system to a new state (e.g. the event of ‘$company\ Y'\ terminates\ the\ contract$’ is not considered an effective event as it returns the system to a state which previously existed). Now, if we increase the number of incidents or their effective events, the situations that can be confronted significantly increase. Mathematically speaking, if there are *‘n’* incidents with n_i effective events, the number of possible situations, or the number of states (NOS) for CST, can be computed as follows.

$$NOS_{CST}=\prod_{i=1}^{n}(n_i+1) \quad (15)$$

SMCDM is an effective method for addressing uncertainty by giving the consideration of all the eventualities that are likely to happen in the near future. However, the method can be criticized for increasing the number of calculations. We admit that the proposed method increases the effort that the decision maker must make in order to achieve the final ranking. More particularly, when the number of effective events increases the number of states and consequently, the calculations considerably increase, and this could be time consuming. A similar situation occurs if the number of incidents increases. But, since the calculations are quite straightforward, they can be performed using software such as MS Excel or more appropriately through coding in computing environments such as MATLAB. Furthermore, we should take into account the fact that the consideration of possible events in SMCDM is with the aim of providing more accurate results and to resolving the decision maker's concerns (e.g. the case discussed in 4.2). Therefore, we believe that the increase in calculations is unlikely to discourage future applications of the method when given the benefits of the proposed method for decision makers. Besides, the decision maker does not need to be concerned about trivial events which have little likelihood of happening. This means that they can ignore those situations that have relatively low probabilities of occurrence. In deciding what to ignore, a predetermined threshold can be calculated. The threshold, for obvious reasons, will decrease as the number of states increases. For example, in Section 4 the threshold of 5% for the existing eight states seems suitable. Removing states which have probabilities of less than 5% does not have any major effect on the results, but significantly decreases the amount of computation.

This study contributes to the existing literature by submitting a utilisation of the recently proposed CST method. This utilisation enables MCDM methods to deal with uncertainties in multi-criteria decision environments. The method was examined with reference to a supplier selection problem. Applying CST in combination with MCDM methods empowers them to become more reliable when addressing a multi-criteria problem. However, the study has two limitations that require further investigation. First, although the method empowers MCDM methods to consider the dynamicity of the decision environment, it significantly increases the amount of computation involved. While this problem does not arise when dealing with situations with a few incidents, as is the case with supplier selection problem discussed in the paper, in other cases a considerable amount of computation may be required. This motivates designing the method to be software based. Such software would require a decision maker to answer a limited number of questions, then based on these it would perform the computations and provide the decision maker with the best ranking of the available alternatives. The second limitation is the complexity of computing the probabilities. This study assumed that the decision maker is able to provide us with estimations of probabilities for situations that are likely to happen in the near future. However, the likelihood of being in the current situation, e.g. situation {1}, is the sum of being in the current state plus transitioning to other states, e.g. {2, 3,…n} and returning to the current state. These possible transitions motivate the application of methods such as a Bayesian network (Abolbashari, Chang, Hussain, & Saberi, 2018), which can make the model more interesting. The SMCDM method will reach its maturity with the result of future studies that consider wider applications and integrations with other tolls and techniques.

6. Concluding remarks

CST is a recent and an innovative approach in problem solving which considers a number of states, each with its own inputs and outputs. The main purpose of this study was to take into account the dynamicity of an environment in which multiple criteria decisions are made. MCDM methods are capable of considering several criteria when making a decision. A typical challenge while applying MCDM methods is the possibility that the conditions under which a decision is being made may change. Such a change affects the weightings of criteria and hence may result in a different selection of alternatives. This paper applied the SMCDM approach and showed how the impact of such incidents can be taken into account so that smarter decisions can be made. The combined method reduces the doubt about making the best decision. This is because the method is able to cover various situations that have an impact on a decision. In other words, the environment in which a decision is made is considered under different conditions with respective probabilities. Then, the weightings of the criteria are computed with respect to each likely condition. These computed weightings of the criteria contribute to the final weightings, taking into account the likeliness of each condition happening. This approach was developed following an examination of the process of decision-making in the human brain and it is expected that it would be employed to in future studies improve decision-making in artificial intelligence. Future studies could also apply CST in combination with a variety of well-known MCDM methods and develop SMCDM methods, such as SAHP, SANP, SBWM and similar, to address various multi-criteria decision-making problems.

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