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Convective heat transfer flow of nanofluid in a porous medium over wavy surface

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ABSTRACT

In this letter, water base nanofluid flow over wavy surface in a porous medium of spherical packing beds is investigated. The copper oxides particles are taken into account. These properties are rehabilitated when fluid interacts with porous walls. For porous medium, Dupuit–Forchheimer model; an extension of Darcy's law model is utilized. The natures of velocity and temperature profiles of nanofluid are discussed graphically whereas the values of convection heat transfer coefficient in the presence of different nanoparticles concentrations in porous medium is presented in tabular form. The obtained results illustrate that convection heat transfer is improved by nanoparticles concentration but reduces when fluid attract to pores structured medium. On the other hand, when particles are added in fluid, convection heat transfer rate is improved but flow velocity is declined.

1. Introduction

Porous medium is characterized as a matrix containing pores. The structure of matrix is usually a solid and pores occupied by fluid. Network of pore and the solid matrix are continuous. Mostly, the properties of the medium like permeability, heat capacitance and thermal conductivity are sometime depended on solid matrix, media porosity and pores structure [1–3].

In industry, some processes are qualitatively influenced with deed of heat transfer enhancement. There are numerous ways to advance the efficiency in heat transfer. To improve convection heat transfer, one method is to use a medium which contains of solid matrix with an interconnected void called porous [4]. Transport and permeable media are becoming charming in design and analysis of heat exchanger and heat transfer devices. When the fluid moves through porous media, it contacts its wide area and heat transfer rate is enhanced in fluid due to tortuous shape of media. In addition, porous media is used to cooled or heated the fluids and to improve the thermal conductivity of fluids. Alkam and Al-Nimr [5] introduced a technique to improved convection heat transfer between fluid and tube walls. They used permeable substrates at both inner sides of wall of tubes and noted that coefficient of convective heat transfer is increase 50% in permeable medium as compare to without permeable medium. In another study [6], the flow and heat transfer characteristics of air in permeable media are investigated numerically and experimentally. Both experimental and numerical results illustrate that the convection heat transfer is improved significantly due high thermal conduction of solid metric in porous media. Ozgumus and Mobedi [7] investigated the Newtonian fluid flow in porous media which built by inline array of rectangular rods and calculated the influence of pore to throat size ratio on heat transfer coefficient. Their results show the enhancement in values of Nusselt number due the low pore to throat size ratios because the flow is entered the gaps deeply and walls of solid rectangular rods bands contribute on the heat transfer. Lin et al. [8] studied the effects of different pore size in bidisperse wick on heat transfer performance. Their results showed that the porosity and the permeability is not only factors in performance of heat transfer, but pore size distribution has great influence. Sumirat et al. [9] are discussed the properties of porosity on the thermal conductivity for nanoporous materials and give a way to design with higher or lower thermal conductivity to further used in heat exchanger or heat transfer applications.

Nanofluid is an important chemical fluid that is used in industries due to high heat transfer rates. Nanofluid show amazing thermo-physical properties that are not existed in the convective

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heat transfer fluids like water, oil etc. After discovery of nanofluids, many researches associated with various nanofluid's applications have been investigated. Some of the applications contain; electronic cooling, car radiators, heat pipes, nuclear plant, coolant in welding and machining, heat exchanger, etc. Since for the last one decade, studies on the nanofluid have been augmented rapidly [10–13]. Choi et al. [14] prepared oil base Al_2O_3/AIN-nanofluid. By this nanofluid, they found 8% improvement of thermal conduction and overall the value of heat transfer coefficient is improvement 20% at 0.5% nanoparticles volume fraction. The performance in heat transfer in water base hybrid nanofluid in tubular heat exchanger is investigated experimentally by Madhesh and Kalaiselvam [15] through titania-copper nanomaterial. Their results illustrate up to 30.4% improvement in heat transfer coefficient at 0.7% concentrations of hybrid nano composite. A fully developed flow of water base (Cu–Al_2O_3)-nanofluid along convective heat transfer behavior in heated circular tube were examined by Suresh et al. [16]. The experimental results of this showed a 6.09% enhanced in Nusselt number via Al_2O_3 nanoparticles at 0.1% concentration when related to water. In another study, Suresh et al. [17] calculated the heat transfer behavior in turbulent flow of water base (Al_2O_3–Cu) nanofluid and found 8.02% enhancement in heat transfer rate as compared to water.

Recently, porous media with nanofluid is suited highly due great potential for heat transfer enhancement. Many studies in literature [18–21] are exposed that the convective heat transfer is improved through porous medium as well as nanoparticles due to having high thermal conductivity. An opportunity is offered for engineers to develop highly effective and compact heat transfer equipment by porous media and nanofluid. Further investigations covering the aspect of said topic can be consulted through the studies [22–40].

Motivated to all above facts, nanofluid flow through porous medium over wavy surface is investigated in the present study. For porous medium, Dupuit–Forchheimer model is used which is extension of Darcy's law model. For nanofluid, consider water as base fluid and modify its physical properties by copper oxides particles. The velocity and temperature profiles as well as convection heat transfer coefficient is exam under various particle concentrations and different percentage of porosity medium.

2. Mathematical formulation

2.1. Flow modeling

Consider the non-Darcy flow of nanofluid over vertical wavy surface. The dimensional coordinates along and normal to surface are denoted by x and y respectively. The porous structure and fill fluid are in thermodynamic equilibrium and temperature of surface is considered greater than ambient fluid temperature. Under boundary layer and Boussinesq approximations, the governing equations are

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0, \tag{1}$$

$$\frac{\mu_{nf}}{K}u+\rho_{nf}cu^2=\frac{\mu_{nf}}{\varepsilon}\frac{\partial^2 u}{\partial y^2}+g(\rho\beta)_{nf}(T-T_\infty), \tag{2}$$

$$(\rho C_p)_{nf_{eff}}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_{nf_{eff}}\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

Subject to

$$\left.\begin{array}{l} u=v=0,\ T=T_w \quad \text{at } y=\sigma(x)=Sin(x) \\ u=0,\ T=T_\infty \quad \text{as } y\to\infty \end{array}\right\}. \tag{4}$$

In Eq. (2), K is the permeability and c is Forchheimer coefficient which determined as

$$K=\frac{d^2\varepsilon^3}{150(1-\varepsilon)^2}, \tag{5}$$

$$c=\frac{1.75(1-\varepsilon)}{d\varepsilon^3}, \tag{6}$$

where d is the particle diameter. The parameter ε shows the porosity of a packed-sphere bed.

Let us introduce the following non-dimensional variables

$$\begin{array}{lll} \bar{x}=\dfrac{x}{L}, & \bar{y}=\dfrac{y}{L}Gr^{1/4}, & \bar{u}=\dfrac{u}{u_o}, \\ \bar{v}=\dfrac{Gr^{1/4}v}{U_o}, & \bar{T}=\dfrac{T-T_\infty}{T_w-T_\infty}, & Gr=\left(\dfrac{u_oL}{\nu_f}\right)^2, \end{array} \tag{7}$$

where $u_o=[g\beta_f L(T_w-T_\infty)]^{1/2}$.

Substituting Eq. (7) in the Eqs. (1) to (4) and get the dimensionless forms of governing equations as follows

$$\frac{\partial\bar{u}}{\partial\bar{x}}+\frac{\partial\bar{v}}{\partial\bar{y}}=0 \tag{8}$$

$$\frac{\mu_{nf}}{\mu_f}\frac{1}{\Lambda}\bar{u}+\frac{\rho_{nf}}{\rho_f}C\bar{u}^2=\frac{1}{\varepsilon}\frac{\mu_{nf}}{\mu_f}\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+\frac{(\rho\beta)_{nf}}{(\rho\beta)_f}\bar{T}, \tag{9}$$

$$\frac{(\rho C_p)_{nf}}{(\rho C_p)_f}\Pr\left(\bar{u}\frac{\partial\bar{T}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{T}}{\partial\bar{y}}\right)=\frac{k_{nf}}{k_f}\frac{\partial^2\bar{T}}{\partial\bar{y}^2}, \tag{10}$$

subject to boundary conditions

$$\left.\begin{array}{l} \bar{u}=\bar{v}=0,\ \bar{T}=1 \quad \text{at } \bar{y}=\sigma(L\bar{x}) \\ \bar{u}=0,\ \bar{T}=0 \quad \text{at } \bar{y}\to\infty \end{array}\right\}. \tag{11}$$

In above $\wedge=KGr^{\frac{1}{2}}/L^2$ is dimensionless permeability parameter whereas $C=cL$ is dimensionless Forchheimer coefficient.

Further, using the stream function formulation

$$\left.\begin{array}{l} \psi=\bar{x}f(Y),\quad \overline{T}=\bar{x}\theta(Y),\quad Y=\bar{y}-\sigma(L\bar{x}) \\ \bar{u}=\dfrac{\partial\psi}{\partial\bar{y}},\quad \bar{v}=-\dfrac{\partial\psi}{\partial\bar{x}} \end{array}\right\}. \tag{12}$$

The resulting Eqs. (9)–(11) become

$$\frac{\mu_{nf}}{\mu_f}\frac{1}{\Lambda}f'+\frac{\rho_{nf}}{\rho_f}C^*f'^2=\frac{1}{\varepsilon}\frac{\mu_{nf}}{\mu_f}f'''+\frac{(\rho\beta)_{nf}}{(\rho\beta)_f}\theta, \tag{13}$$

$$\frac{k_{nf_{eff}}}{k_f}\theta''+\frac{(\rho C_p)_{nf_{eff}}}{(\rho C_p)_f}\Pr\left(f'\theta-f\theta'\right)=0, \tag{14}$$

with boundaries conditions

$$\left.\begin{array}{l} f=0,\ f'=0,\ \theta=1 \quad \text{at } \eta\to 0 \\ f'=0,\ \theta=0 \quad \text{at } \eta\to\infty \end{array}\right\}. \tag{15}$$

2.2. Nanofluid modeling

When nanoparticles are dispersed in base fluid, thermo-physical properties of fluids are improved which play important role in heat transfer, to minimize irreversible processes and other applications. So far, most studies on thermal properties of nanofluid have focused on viscosity and thermal conductivity. However, heat transfer characteristics also depend on other properties, such as specific heat, thermal expansion and density etc. In Eqs. (9) and (10), physical properties such as thermal expansion coefficient β_{nf}, density ρ_{nf} and viscosity μ_{nf} for the nanofluid are given as,

$$(\beta)_{nf} = \frac{\phi(\rho\beta)_s + (1-\phi)(\rho\beta)_f}{\rho_{eff}}, \tag{16}$$

$$\rho_{nf} = \phi\rho_s + (1-\phi)\rho_f, \tag{17}$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}. \tag{18}$$

Since the heat transfer is through the nanofluid in porous media, the effective thermal conductivity $k_{nf_{eff}}$ and heat capacitance $(C_p)_{nf_{eff}}$ models are specified as follows

$$k_{nf_{eff}} = \frac{k_m k_{nf}}{\varepsilon k_m + (1-\varepsilon)k_{nf}}, \tag{19}$$

$$(C_p)_{nf_{eff}} = \frac{\varepsilon(\rho C_p)_{nf} + (1-\varepsilon)(\rho C_p)_m}{\rho_{nf}}. \tag{20}$$

In above, the subscripts m and nf are used for medium and nanofluid. In the present of nanoparticles, heat capacitance and thermal conductivity models are given as

$$(C_p)_{nf} = \frac{\phi(\rho C_p)_s + (1-\phi)(\rho C_p)_f}{\rho_{nf}}, \tag{21}$$

$$k_{nf} = \frac{k_{pe} + 2k_f + 2(k_{pe} - k_f)(1+\beta^*)^3\phi}{k_{pe} + 2k_f - (k_{pe} - k_f)(1+\beta^*)^3\phi} k_f, \tag{22}$$

where thermal conductive of layer around the particles is defined by

$$k_{pe} = \frac{[2(1-\gamma) + (1+\beta^*)^3(1+2\gamma^*)]\gamma^*}{-(1-\gamma^*) + (1+\beta^*)^3(1+2\gamma^*)} k_s, \tag{23}$$

here $\gamma^* = k_{layer}/k_s$ is the ratio of nanolayer and particle thermal conductivities.

2.3. Heat transfer coefficient

The most imperative result to be resolved is the heat transfer between a surface and a fluid flowing past it is usually presented in terms of the Nusselt number is given as

$$Nu = \frac{hL}{k_f}, \tag{24}$$

where

$$h = \frac{-k_{nf}\frac{\partial(T-T_\infty)}{\partial y}|_{z=0}}{(T_w - T_\infty)}. \tag{25}$$

The local Nusselt number in terms of the new variable is

$$\frac{Nu}{Gr^{1/4}} = -\frac{k_{nf}}{k_f}\theta(0). \tag{26}$$

3. Solution of the problem

In this section, BVPh 2.0 package is used to get the analytic solution of governing equations. The BVPh 2.0 necessitates placing the ordinary differential equations subject to boundary conditions, auxiliary linear operators and initial guess. Thus, choose the auxiliary linear operators according to Eqs. (13) and (14) are

$$\left.\pounds_f = \frac{d^3}{d\eta^3} - \frac{d}{d\eta}, \quad \pounds_\theta = \frac{d^2}{d\eta^2} - 1\right\}. \tag{27}$$

The selected initial guess according to Eq. (27) are

$$f_0(\eta) = -1 + \eta + e^{-\eta}, \qquad \theta_0(\eta) = e^{-2\eta}. \tag{28}$$

After placing the desire and suitable linear auxiliary operators and initial guess, the nonlinear coupled Eqs. (13) and (14) subject to the Eq. (15) are directly solved by using package BVPh 2.0. The results for the stream function and the temperature distribution are calculated up to 20th iterations of package.

Further, the mathematical expression of results for the stream function and temperature up to first iteration are as follows

$$\begin{aligned} f = &\begin{pmatrix} \frac{1}{2} + \frac{71}{600}\frac{(\rho\beta)_{nf}}{(\rho\beta)_f} - \frac{71C}{6000}\frac{\rho_{nf}}{\rho_f} \\ -\frac{71\Lambda}{1200}\frac{\mu_{nf}}{\mu_f} - \frac{71}{600\varepsilon}\frac{\mu_{nf}}{\mu_f} \end{pmatrix} \\ &+ \begin{pmatrix} -1 - \frac{71}{300}\frac{(\rho\beta)_{nf}}{(\rho\beta)_f} + \frac{71C}{3000}\frac{\rho_{nf}}{\rho_f} \\ +\frac{71\Lambda}{480}\frac{\mu_{nf}}{\mu_f} + \frac{71}{600\varepsilon}\frac{\mu_{nf}}{\mu_f} \end{pmatrix} e^{-\eta} \\ &\times \begin{pmatrix} \frac{1}{2} + \frac{71}{600}\frac{(\rho\beta)_{nf}}{(\rho\beta)_f} \\ -\frac{71\Lambda}{600}\frac{\mu_{nf}}{\mu_f} - \frac{37}{600\varepsilon}\frac{\mu_{nf}}{\mu_f} \end{pmatrix} e^{-2\eta} \\ &+ \begin{pmatrix} -\frac{71C}{2400}\frac{\rho_{nf}}{\rho_f} + \frac{71\Lambda}{2400}\frac{\mu_{nf}}{\mu_f} \\ -\frac{71}{600\varepsilon}\frac{\mu_{nf}}{\mu_f} \end{pmatrix} e^{-3\eta} \\ &+ \left(\frac{71C}{3000}\frac{\rho_{nf}}{\rho_f}\right)e^{-4\eta} - \left(\frac{71C}{12000}\frac{\rho_{nf}}{\rho_f}\right)e^{-5\eta}, \end{aligned} \tag{29}$$

$$\begin{aligned} \theta = &\begin{pmatrix} \frac{4}{15}\frac{k_{nf}}{k_f} + \frac{3}{50}\frac{(\rho C_p)_{nf}}{(\rho C_p)_f} \\ 1 - \frac{7\,\mathrm{Pr}}{150}\frac{(\rho C_p)_{nf}}{(\rho C_p)_f} \end{pmatrix} e^{-\eta} + \begin{pmatrix} -\frac{4}{15}\frac{k_{nf}}{k_f} \\ +\frac{2\,\mathrm{Pr}}{15}\frac{(\rho C_p)_{nf}}{(\rho C_p)_f} \end{pmatrix} e^{-2\eta} \\ &- \begin{pmatrix} \frac{\mathrm{Pr}}{10}\frac{(\rho C_p)_{nf}}{(\rho C_p)_f} \\ +\frac{1}{10}\frac{(\rho C_p)_{nf}}{(\rho C_p)_f} \end{pmatrix} e^{-3\eta} + \begin{pmatrix} \frac{2}{75}\frac{(\rho C_p)_{nf}}{(\rho C_p)_f} \\ -\frac{4\,\mathrm{Pr}}{75}\frac{(\rho C_p)_{nf}}{(\rho C_p)_f} \end{pmatrix} e^{-4\eta}. \end{aligned} \tag{30}$$

4. Results and discussion

In this section, to understand the behavior of particle concentration and porosity on flow field and heat distribution are potted graphically and numerical values are computed for convection heat transfer coefficient. To observe the influences of parameters, some assumptions are taken to account. For nanofluid, consider water as base fluid and copper oxides is used as nanomaterial. In addition, consider thermal conductivity around nanoparticles of nanolayer is two times greater than base fluid and its thickness is taken 1 nm.

The explorations of nanoparticle concentration on velocities and temperature profiles are representing in Figs. 1 and 2. The different velocity curves are shown in resulting of diverse concentrations of nanoparticles. By different nanoparticles concentration, different collisions among adjacent particles of fluid are happened which shaped diverse velocity curves. It is noticed that velocity profile is decreased by increasing nanoparticles concentration. This behavior is occurred due to increasing the resistance between neighboring moving fluid's layers by increasing nanoparticles concentration which leads to fall the velocity profiles. In the Fig. 2, it is perceived that the temperature profile of nanofluid is improved by increasing nanoparticles volume fraction. In the enhancement of temperature profile, several factors are included like viscosity, thermal conductivity, density and heat capacity. The effects of porosity on velocity and temperature fields are illustrated in Figs. 3 and 4. It is known that permeability of a medium is related to the porosity. The permeability shows ability of a porous material to allow fluids to pass through it. In consequence of porosity enhancement,

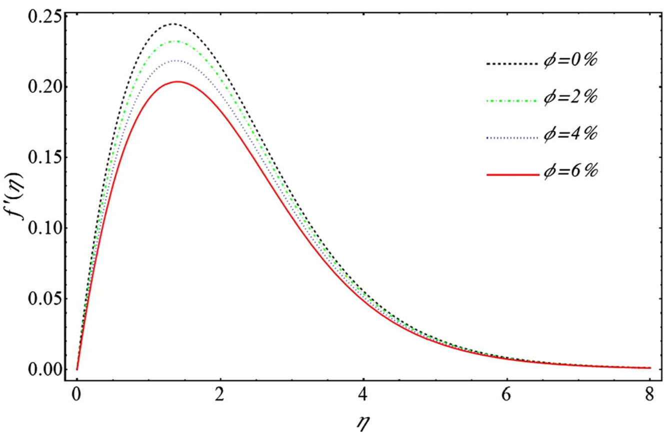

Fig. 1. Velocity profile corresponding to various nanoparticle concentrations.

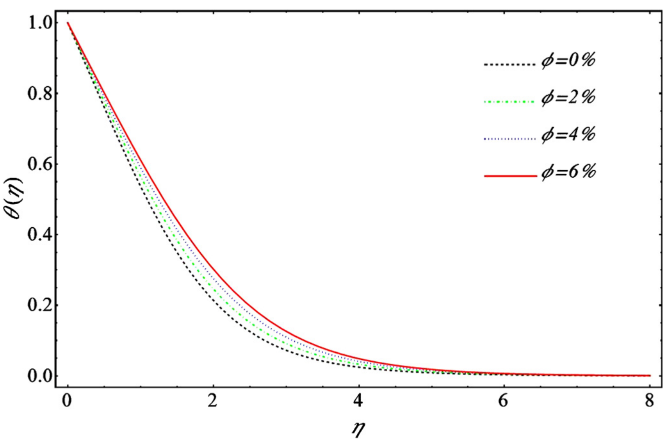

Fig. 2. Temperature profile corresponding to various nanoparticle concentration.

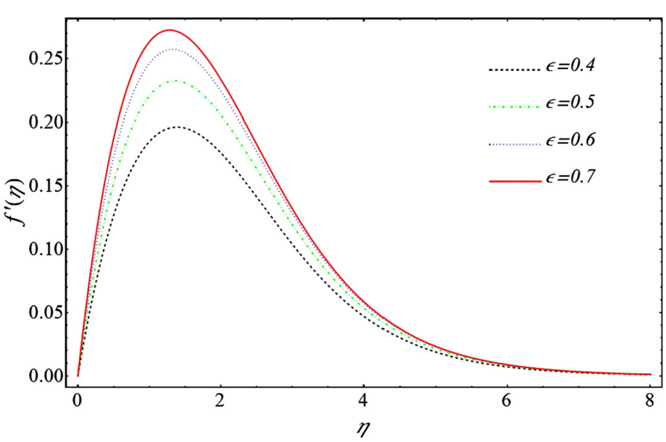

Fig. 3. Velocity profile corresponding to various values of porosity parameter.

permeability is amplified which leads to augmentation of fluid flow in supposed porous medium. Therefore, due to porosity enhancement, velocity field is increased which is displayed in Fig. 3. Fig. 4 demonstrates the influence of porosity parameter on temperature distribution and observed that temperature is declined by increasing of porosity parameter. The main reason in reduction of temperature is thermal conductivity and specific heat which decays in consequence of porosity influence.

The numerical sets of values show the results for parameters on local Nusselt number. The impacts of nanoparticle volume fraction and porosity on local Nusselt number are shown in Table 1. In this table, it is observed that when nanoparticle concentration is

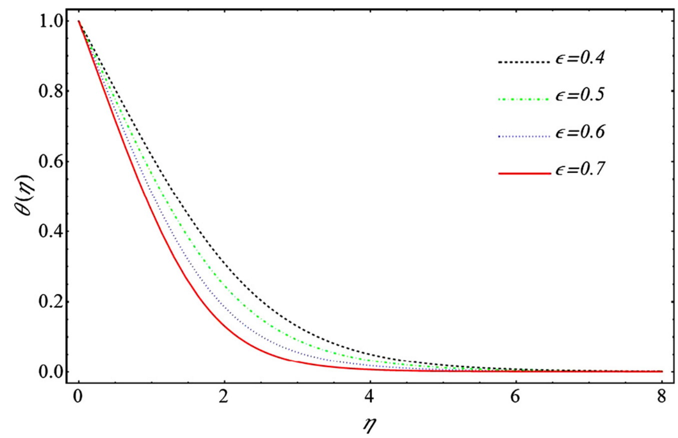

Fig. 4. Temperature profile corresponding to various values of porosity parameter.

Table 1
The effect of particle concentration and porosity on heat transfer coefficient.

ϕ	$Gr^{-1/4}Nu$	ε	$Gr^{-1/4}Nu$
0%	1.17331	0.4	1.30248
2%	1.20346	0.5	1.27361
4%	1.22491	0.6	1.24443
6%	1.24443	0.7	1.20401

enhanced, the value of local Nusselt number is increased which indicates the improvement in convective heat transfer from surface to fluid. In the improvement of convective heat transfer, thermal conductivity plays a main role which is increased by nanoparticles dispersion. On the other hand, the thermal conductivity is decreased by increasing of porosity parameter which became cause of decrement in convective heat transfer.

5. Conclusion

The present work examines fluid flow and heat transfer in Non-Darcian flow phenomena over wavy surface. Two main consequence are observed as follows

- It is observed that flow speed is improvement when porosity is increased while convection heat transfer rate is reduced due to declination in thermal conduction.
- On the other hand, when particles are added in fluid, convection heat transfer rate is improved but flow velocity is declined.

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