



# Compensation and profit distribution for cooperative green pickup and delivery problem



Junwei Wang<sup>a</sup>, Yang Yu<sup>b,\*</sup>, Jiafu Tang<sup>c</sup>

<sup>a</sup> Department of Industrial and Manufacturing Systems Engineering, the University of Hong Kong, Pokfulam Road, Hong Kong

<sup>b</sup> State Key Laboratory of Synthetic Automation for Process Industries, Department of Intelligent Systems, Northeastern University, Shenyang 110819, PR China

<sup>c</sup> College of Management Science and Engineering, Dongbei University of Finance and Economics, Dalian 116023, PR China

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## ABSTRACT

Cooperation is a powerful strategy to achieve the objective of the green pickup and delivery problem (GPDP) that minimizes carbon emissions of pickup and delivery service. However, the cooperative GPDP may not be accepted by all the partners, as the cost of cooperative GPDP may be higher than that of the non-cooperative minimum cost PDP. Therefore, a reasonable compensation mechanism is desired to form an acceptable cooperative GPDP, and a fair method of profit distribution, based on the mechanism, is needed to stabilize the cooperation. In this paper, we analyze the situations in which a compensation is needed and develop the lower bound of the compensation. Further, we propose an exact method to calculate the actual compensation and the profit distribution based on cooperative game theory. The proposed exact method can efficiently solve the largest scale instance in Li & Lim benchmarks, i.e., pdptw1000-LR1\_10\_1 with 1,054 customers and 19,306 products. The proposed compensation and profit distribution mechanism based on cooperative game theory is also applied to a real-world GPDP and achieve satisfactory performance. Some interesting and important managerial insights are found and discussed.

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## 1. Introduction

Vehicle emissions contribute to various greenhouse gas. About more than 80% of urban air pollution results from the transportation sector, according to a report of [United Nations Economic and Social Council \(2009\)](#). The transportation sector is the largest consumer of petroleum products and the chief culprit for air pollution worldwide ([Fan and Lei, 2016](#); [Zhou et al., 2017](#)). The growth of energy consumption in transportation is faster than any other sectors due to the strong transport demands arising from rapid economic development.

Therefore, green logistics in urban areas has been an important issue. This paper focuses on the green pickup and delivery problem (GPDP), an extension of the green vehicle routing problem (GVRP), which provides logistics service between pickup and delivery points. Pickup and delivery problem (PDP) is a famous extension of the vehicle routing problem (VRP). Practical applications of PDP include the dial-a-ride ([Cordeau, 2006](#)), airline scheduling, bus routing, and share-ride-problem ([Caulfield 2009](#)). We consider cooperation as a potentially powerful strategy to reduce carbon emissions due to its effectiveness in improving the utilization of vehicles. In a cooperative GPDP, the objective is to minimize carbon emissions; while

\* Corresponding author.

E-mail address: [yuyang@ise.neu.edu.cn](mailto:yuyang@ise.neu.edu.cn) (Y. Yu).

each partner company involved in the cooperation has its own objective that is to minimize its cost. The cost of a company with a cooperation strategy may be more than that without a cooperation strategy. As such, the cooperation targeting on the lowest carbon emissions may not be accepted by all partners. The contradiction between the environmental target and the economic benefit calls for a reasonable compensation mechanism to form a feasible cooperative GPDP. Unfortunately, such a compensation mechanism for the cooperative GPDP has not been examined in the current literature. Compensation means that the cooperation can ensure a non-negative profit. Further, a method that can fairly distribute the profit obtained by the cooperation and compensation needs to be developed.

### 1.1. Related work

The strategies and measures for the vehicle routing problem (VRP) aimed at reducing carbon emissions have been extensively studied. [Bektas and Laporte \(2011\)](#) investigated GVRP, which they called the Pollution-Routing Problem (PRP). Bi-objective PRP ([Demir et al., 2014b](#)) and time-dependent PRP ([Franceschetti et al., 2013](#)) were investigated. [Dabia et al. \(2017\)](#) developed an exact method for a variant of PRP. [Demir et al. \(2014a\)](#) surveyed the existing heuristic algorithms for the GVRP. [Fukasawa et al. \(2015, 2016\)](#) developed two exact algorithms for the GVRP. [Koç et al. \(2014\)](#) and [Xiao and Konak \(2016\)](#) studied how to reduce carbon emissions through replacing homogeneous vehicles with heterogeneous vehicles in the GVRP. [Yu et al. \(2014\)](#) stated that approximately, 16% carbon emissions were reduced by replacing a single-trip mode with a multi-trip mode in the GVRP. [Yu et al. \(2016\)](#) demonstrated that approximately, 10.8% carbon emissions were reduced by integrated scheduling instead of non-integrated scheduling in the GVRP.

Cooperation is a potentially powerful strategy for the GVRP due to its effectiveness in improving the utilization of vehicles. However, it has never been considered in the current literature. It is noted that cooperation has been employed as an important strategy to reduce operation cost in the transportation sector ([Lin, 2008](#); [Lozano et al., 2013](#)), such as the cooperation in grocery distribution ([Caputo and Mininno, 1996](#)), freight carriers ([Krajewska et al., 2008](#)), forest ([Frisk et al., 2010](#)), and railway transportation ([Sherali and Lunday, 2011](#)). It is noted that the focus in the studies on cooperation is to calculate and distribute the cost saving or profit. Most studies used heuristic or meta-heuristic algorithms to estimate the cost saving of cooperation due to the larger-scale instances caused by cooperation. [Krajewska et al. \(2008\)](#) used the heuristic proposed by [Ropke and Pisinger \(2006\)](#) to solve horizontal cooperation among freight carriers. To minimize execution costs for a coalition of freight forwarders, [Ergun et al. \(2007\)](#) used a greedy heuristic to solve the instance. As a result, the profit brought by cooperation is not exact. The exact Shapley value for cost allocation or profit distribution has been studied in some special cases. [Littlechild and Owen \(1973\)](#) gave a famous simple expression for the Shapley value for airport runway cost games. [Kuipers et al. \(2013\)](#) studied the exact expression of Shapley value for the cost sharing in highways.

However, the aforementioned studies on cooperation have not considered the problem of reduction of carbon emissions. The reason for this omission is that there is a conflict between the cooperative GPDP and regular non-cooperative minimum cost PDP; in particular, a cooperation that reduces carbon emissions may lead to higher cost. To resolve this conflict, a compensation mechanism is desired.

In short, to implement a cooperation strategy in the cooperative GPDP, we need to handle two key issues, compensation calculation, and profit distribution. The objective of this paper is to develop a systematic methodology to address the two issues and achieve a better cooperation for the GPDP.

### 1.2. Our contributions

This paper is originally motivated by carbon emissions reduction brought by cooperative GPDP. However, the carbon emissions reduction may make cost increase. We thus investigate how to provide the least compensation to make cooperation accepted. Our main contributions are summarized as follows.

First, we analyze all situations of the cooperative GPDP and identify the situations in which a compensation is needed to form a feasible cooperation that reduces carbon emissions. Second, we develop a lower bound of the compensation. Third, we develop an exact method to calculate the actual compensation and the profit distribution based on cooperative game theory by finding a Pareto-optimal solution with the minimum cost ( $POS_C$ ) and a Pareto-optimal solution with the lowest carbon emissions ( $POS_{CE}$ ). In particular, the exact method is expected to reduce the solution space by cutting non-Pareto-optimal trips.

### 1.3. Overview of the paper

The paper is structured as follows. [Section 2](#) describes the non-cooperative minimum cost PDP and cooperative GPDP, respectively, and defines two special Pareto-optimal solutions, i.e.,  $POS_C$  and  $POS_{CE}$ . [Section 3](#) enumerates all situations of the cooperative GPDP and identifies the situations in which a compensation is needed to form the cooperation. [Section 4](#) presents the methodology for making a better cooperation for GPDP, including the definition of compensation along with its lower bound, and the definition of the profit along with its distribution. [Section 5](#) analyzes the key characteristics of the cooperative GPDP and develops an exact method to calculate the compensation and to distribute the profit based on cooperative game theory for solving  $POS_C$  and  $POS_{CE}$ ; further, the proposed method is validated by solving

the largest scale instance in Li & Lim benchmarks, i.e., pdptw1000-LR1\_10\_1 with 1,054 customers and 19,306 products. Section 6 validates the proposed methodology by a real-world case of the pickup and delivery problem.

## 2. Non-cooperative minimum cost PDP and cooperative GPDP

### 2.1. Notations

The notations are defined as follows.

#### Indices

$c$	Company $c$ , $c \in C$ ;
$w_c$	Customer $w$ of company $c$ , $w_c \in W_c$ ;
$ci$	Trip $i$ of company $c$ , $i \in R_c$ ;
$w$	Customer $w$ in the cooperative GPDP, $w \in W$ ;
$i$	Trip $i$ of cooperation, $i \in R$ .

#### Parameters

$C$	The set of the companies operating PDP;
$ C $	The number of companies;
$W_c$	The set of the customers of company $c$ ;
$R_c$	The set of all the feasible trips of company $c$ ;
$W$	The set of customers in the cooperative GPDP, $W = \cup_{c \in C} W_c$ . The different companies' customers with the same pickup and delivery time windows and coordinates are incorporated into one customer;
$R$	The set of all the feasible trips of cooperation;
$q_{wc}$	Product of customer $w_c$ ;
$c_{ci}$	Cost of trip $i$ of company $c$ ;
$ce_{ci}$	Carbon emissions of trip $i$ of company $c$ ;
$q_w$	Product of customer $w$ . $q_w$ may be more than $Q$ because of customer incorporation. If $q_w > Q$ , $\lfloor \frac{q_w}{Q} \rfloor$ vehicles are used to serve $\lfloor \frac{q_w}{Q} \rfloor * Q$ customers in advance and $q_w - \lfloor \frac{q_w}{Q} \rfloor * Q$ customers are left;
$ce_i$	Carbon emissions of trip $i$ ;
$ct_i$	Cost of trip $i$ ;
$d_i$	Distance of trip $i$ ;
$0$	Depot;
$d$	Delivery point;
$Q$	The capacity of the vehicle;
$v$	Speed of the vehicle;
$c_d$	Cost per unit distance.

#### Decision variables

$x_{ci}$	$\begin{cases} 1, & \text{if trip } i \text{ of company } c \text{ is selected;} \\ 0, & \text{otherwise} \end{cases}$ ;
$a_{w_c, ci}$	$\begin{cases} 1, & \text{if customer } w_c \text{ is in trip } ci, \\ 0, & \text{otherwise} \end{cases}$ ;
$x_i$	$\begin{cases} 1, & \text{if trip } i \text{ is selected;} \\ 0, & \text{otherwise} \end{cases}$ ;
$a_{w, i}$	$\begin{cases} 1, & \text{if customer } w \text{ is in trip } i \\ 0, & \text{otherwise} \end{cases}$ .

### 2.2. Non-cooperative minimum cost PDP

A complete undirected graph  $G = \{V, E\}$  is given, where  $V$  is the set of vertices and  $E$  is the set of edges. For company  $c$ , we have  $V = \{0\} \cup W_c \cup \{d\}$ , where vertex  $0$  represents the depot and  $W_c$  represents the set of customers, each one having  $q_{wc}$  units of product to be picked up and delivered to the delivery point  $d$  and having the preferred time windows of pickup and delivery. A fleet of identical vehicles is located at the depot. Each vehicle has a capacity of  $Q$  and runs at a constant speed of  $v$ . Every  $q_{wc}$  is not more than  $Q$ .  $|C|$  companies operate the PDP solely. The objective is to minimize operation cost of company  $c$ .

### 2.3. Cooperative GPDP

A complete undirected graph  $G = \{V, E\}$  is given, where  $V$  is the set of vertices and  $E$  is the set of edges. We have  $V = \{0\} \cup W \cup \{d\}$ , where vertex  $0$  represents the depot and  $W$  represents the set of customers, each one having  $q_w$  units of product to be picked up and delivered to the delivery point  $d$  and having the preferred time windows of pickup and

delivery. A fleet of identical vehicles is located at the depot. Each vehicle has a capacity of  $Q$  and runs at a constant speed of  $v$ .  $|C|$  companies cooperate to operate the GPDP. The objective is to minimize carbon emissions through the cooperation. As the entire society is the beneficiary of reducing carbon emissions, we assume that the compensation is provided by the society or the carbon emissions management department. For simplicity, we use the society as the financial supporter who provides the compensation in the remainder of this paper.

2.4. Method to calculate the cost of a trip

The cost of a trip is expressed as follows.

$$\text{Cost of trip } i = c_f + c_d * d_i, \tag{1}$$

where  $c_f$  and  $c_d$  denote the fixed cost per vehicle and the cost per unit distance.

2.5. Method to calculate carbon emissions of a trip

So far, many approaches have been developed to model vehicle fuel consumption to estimate carbon emissions. This research adopts the fuel consumption proposed by Bektaş and Laporte (2011). The comprehensive emission model of vehicle can be found in Barth et al. (2005), Barth and Boriboonsomsin (2008, 2009). The carbon emissions ( $CE$ ) of a trip directly relates to the energy consumed ( $P$ ) through the relation  $CE = rfP$ , where  $f$  is a fuel consumption index parameter and  $r$  is a carbon emissions index parameter. The total amount of energy consumed on the arc ( $ij$ ) is approximately expressed with the speed and load as follows (Bektaş and Laporte, 2011):

$$P_{ij} \approx a_{ij}(w_{ij} + f_{ij})d_{ij} + \beta v_{ij}^2 d_{ij}, \tag{2}$$

where  $w_{ij}$  is the empty vehicle weight;  $f_{ij}$  is the load carried by the vehicle on this arc, and  $a_{ij}$  and  $\beta_{ij}$  are expressed by Eqs. (3) and (4), respectively.

$$a_{ij} = a + g \sin \theta_{ij} + g C_r \cos \theta_{ij}, \tag{3}$$

where  $a$  is the acceleration ( $m/s^2$ );  $g$  is the gravitational constant ( $9.8 m/s^2$ );  $\theta_{ij}$  is the arc angle;  $C_r$  is the coefficients of rolling resistance.  $a_{ij}$  is an arc specific constant, and  $a_{ij}(w_{ij} + f_{ij})d_{ij}$  represents load-induced energy requirements.

$$\beta = 0.5 C_d A \rho, \tag{4}$$

where  $C_d$  is the drag coefficient,  $A$  is the frontal surface area of the vehicle ( $m^2$ ), and  $\rho$  is the air density ( $kg/m^3$ ).  $\beta$  is a vehicle specific constant.  $\beta = 0.5 C_d A \rho$  represents speed-induced energy requirements.

Given the environment and the type of vehicle,  $a_{ij}$ ,  $w_{ij}$ ,  $\beta$ ,  $f$  and  $r$  are all constants.

2.6. Mathematical model for non-cooperative minimum cost PDP

A set partitioning model, i.e., model 1, is established for company  $c$  in the non-cooperative minimum cost PDP.

**Model 1:**

$$\text{Minimize } \sum_{ci \in R_c} c_{ci} x_{ci}, \tag{5}$$

$$\text{subject to } \sum_{ci \in R_c} a_{w_c, ci} x_{ci} = 1, \quad w_c \in W_c, \tag{6}$$

$$x_{ci} = 0 \text{ or } 1, \quad ci \in R_c. \tag{7}$$

Eq. (5) is to minimize the cost for company  $c$ . Eq. (6) requires that each  $w_c$  is severed once exactly. Eq. (7) is the decision variable constraint.

It is noted that the optimal objective value, i.e., the minimum cost, of model 1, is unique; however, the optimal solutions may be multiple. To facilitate the analysis in Section 3, we borrow the concept of Pareto-optimal solution to represent the one with the lowest carbon emissions within these multiple optimal solutions. A detailed definition is given below.

**Definition 2.1. (Pareto-optimal solution with the minimum cost, POS\_C):** Pareto-optimal solution with the minimum cost is the solution that achieves the lowest carbon emissions within all the optimal solutions that achieve the minimum cost.

## 2.7. Mathematical model of cooperative GPDP

A set partitioning model, i.e., model 2, is established for cooperative GPDP.

### Model 2:

$$\text{Minimize } \sum_{i \in R} ce_i x_i, \quad (8)$$

$$\text{subject to } \sum_{i \in R} a_{w,i} x_i = 1, \quad w \in W, \quad (9)$$

$$x_i = 0 \text{ or } 1, \quad i \in R. \quad (10)$$

Eq. (8) is to minimize carbon emissions of GPDP. Eq. (9) requires that each  $w$  is severed once exactly. Eq. (10) is the decision variable constraint.

It is noted that the optimal objective value of model 2, i.e., the lowest carbon emissions, of the model is unique; however, the optimal solutions may be multiple. To facilitate the analysis in Section 3, we borrow the concept of Pareto-optimal solution to represent the one with the minimum cost within these multiple optimal solutions. A detailed definition is given below.

**Definition 2.2. (Pareto-optimal solution with the lowest carbon emissions, POS\_CE):** Pareto-optimal solution with the lowest carbon emissions is the solution that achieves minimum cost within all the optimal solutions that achieve the lowest carbon emissions.

## 3. Cooperative GPDP and all its feasible situations

### 3.1. Notations

The notations are defined as follows.

$mc_c$ : Minimum cost of company  $c$  in the non-cooperative PDP, i.e., cost of POS\_C of company  $c$ ;

$ce\_mc_c$ : Carbon emissions of company  $c$  in the non-cooperative PDP with the minimum cost;

$C$ : Cooperation with all companies for the lowest carbon emissions;

$S$ : Cooperation  $S$ , i.e., cooperation with  $|S|$  companies for the lowest carbon emissions,  $S \subseteq C$ ;

$\sum_{c \in S} mc_c$ : Minimum total cost of the non-cooperative PDP of the companies in  $S$ ;

$\sum_{c \in S} ce\_mc_c$ : Total carbon emissions of the companies in  $S$  in the non-cooperative PDP;

$lce(S)$ : Lowest carbon emissions of cooperation  $S$ , i.e., carbon emissions of POS\_CE of  $S$ ;

$c\_lce(S)$ : Cost of POS\_CE of cooperation  $S$ ;

$RCE(S)$ : Reduced carbon emissions by cooperation  $S$ ,  $RCE(S) = \sum_{c \in S} ce\_mc_c - lce(S)$ ;

$RC(S)$ : Saved cost by cooperation  $S$ ,  $RC(S) = \sum_{c \in S} mc_c - c\_lce(S)$ ;

$C(S)$ : Compensation to form the acceptable cooperation  $S$ ,  $C(S) \geq 0$ ;

$LB\_C(S)$ : Lower bound of  $C(S)$ , i.e., the minimum compensation to form cooperation  $S$ .

### 3.2. Acceptable cooperative GPDP

**Definition 3.1. (Acceptable cooperation S):** An acceptable cooperation  $S$  is defined as a cooperation that satisfies two conditions:

- (1)  $RCE(S) \geq 0$ ; and
- (2)  $RC(S) \geq 0$ , where at least one inequality is strict.

Condition (1) is the society acceptance constraint to guarantee the carbon emissions of a cooperation is not higher than those of non-cooperation, so that the cooperation is accepted by the society. It is noted that we have assumed the society is the financial supporter who provides compensation. Condition (2) is the participant acceptance constraint to guarantee the cost of the cooperation is not higher than that of non-cooperation, so that the cooperation is accepted by all the participants.

**Theorem 3.1.** For any cooperation  $S (S \subseteq C)$ , we have  $RCE(S) \geq 0$ .

**Proof.** Let  $S$  be an arbitrary cooperation and  $lce_c$  be the lowest carbon emissions of company  $c$  in non-cooperation. Assume  $ts\_lce_c$  is the solution (i.e., trip set) of  $lce_c$ , then  $\cup_{c \in S} ts\_lce_c$  is a feasible solution of cooperation  $S$ . Carbon emissions of  $\cup_{c \in S} ts\_lce_c$  (i.e.,  $\sum_{c \in S} lce_c$ ) is not more than  $lce(S)$  because  $lce(S)$  is the lowest carbon emissions of cooperation  $S$ . Thus,  $lce(S) \leq \sum_{c \in S} lce_c, \forall S \subseteq C$ .

Moreover, for arbitrary company  $c$ , we have  $lce_c \leq ce\_mc_c$  because  $lce_c$  and  $ce\_mc_c$  are the lowest carbon emissions of company  $c$  and carbon emissions of POS\_C of company  $c$ , respectively. Thus, we have  $\sum_{c \in S} lce_c \leq \sum_{c \in S} ce\_mc_c, \forall S \subseteq C$ .

Therefore,  $lce(S) \leq \sum_{c \in S} lce_c \leq \sum_{c \in S} ce\_mc_c, \forall S \subseteq C$ , i.e.,  $RCE(S) \geq 0$ .  $\square$

**Theorem 3.1** indicates that the cooperation is able to reduce carbon emissions.

**Theorem 3.2.** *In a cooperation S, if  $RC(S) < 0$  then at least one company's cost in the cooperative GPDP is higher than that in the non-cooperative PDP.*

**Proof.** For a cooperation S, assume that  $RC(S) < 0$ , i.e.,  $c\_lce(S) > \sum_{c \in S} mc_c$  and no company's cost in the cooperative GPDP is higher than that in the non-cooperative PDP. Let  $c\_lce(c)$  be the cost of company c in cooperation S. No company's cost in the cooperative GPDP is higher than that in the non-cooperative PDP, i.e.,  $c\_lce(c) \leq mc_c$ , and so  $\sum_{c \in S} c\_lce(c) \leq \sum_{c \in S} mc_c$ . Therefore,  $c\_lce(S) = \sum_{c \in S} c\_lce(c) \leq \sum_{c \in S} mc_c$ .

Because of the assumption that  $RC(S) < 0$  and no company's cost in the cooperative GPDP is higher than that in the non-cooperative PDP led to a contradiction,  $RC(S) < 0$  means at least one company's cost in the cooperative GPDP is higher than that in the non-cooperative PDP.  $\square$

**Theorem 3.2** means when  $RC(S) < 0$ , at least one company does not accept cooperation S for the lowest carbon emissions. However, for a cooperation S, it is possible that  $RC(S) = \sum_{c \in S} mc_c - c\_lce(S) < 0$ . Some examples of  $RC(S) < 0$  are shown in Table 7.

We enumerate all the four situations of cooperative GPDP by combining different situations of  $RCE(S)$  and  $RC(S)$ .

### 3.3. All situations of the cooperative PDP for the lowest carbon emission

**Definition 3.2. (All situations of the cooperative GPDP):** All situations of the cooperative GPDP can be enumerated as follows.

- Situation (1):  $RCE(S) = 0$  and  $RC(S) \leq 0$ ;
- Situation (2):  $RCE(S) = 0$  and  $RC(S) > 0$ ;
- Situation (3):  $RCE(S) > 0$  and  $RC(S) < 0$ ; and
- Situation (4):  $RCE(S) > 0$  and  $RC(S) \geq 0$ .

In Situation (1), neither carbon emissions nor cost is reduced by the cooperation. As a result, the cooperation S cannot be accepted.

In Situation (2), carbon emissions are not reduced but cost is reduced by the cooperation. As a result, the cooperation S is accepted.

In Situation (3), carbon emissions are reduced but cost is increased by the cooperation. As a result, the cooperation S cannot be accepted by at least one participant unless a compensation is provided.

In Situation (4), carbon emissions are reduced and cost is not increased by the cooperation. As a result, the cooperation S is accepted.

## 4. Compensation, profit, and profit distribution based on cooperative game theory

Since the cost of the cooperative GPDP may be higher than that in the non-cooperative PDP, compensation should be provided to form an acceptable cooperation to reduce carbon emissions.

### 4.1. Compensation of cooperation s

**Definition 4.1. (Compensation of cooperation S, C(S)):** C(S) is the compensation to make cooperation S acceptable when  $RC(S) < 0$ . Obviously,  $C(S) \geq 0$ .

According to **Theorem 3.1**, the carbon emissions of the cooperative GPDP are always not higher than those in the non-cooperative PDP. Therefore, the key to form an acceptable cooperation S is to make every participant obtain a non-negative profit so as to make all participants accept the cooperation. Next, we define the profit of the cooperative GPDP.

### 4.2. Profit of cooperation s and the lower bound of C(S)

**Definition 4.2. (Profit of cooperation S, v(S)):** v(S) is the profit of cooperation S, every v(S) must satisfy the following three conditions:

- (1)  $v(\emptyset) = 0$ ;
- (2)  $v(S) \geq 0, S \subseteq C$ ; and
- (3)  $v(S \cup T) \geq v(S) + v(T), S \cap T = \emptyset, S, T \subseteq C$ .

Condition (1) is a convention in which a void cooperation has a zero value. Condition (2) makes cooperation  $S$  accepted by all participants. Condition (3) guarantees that two acceptable cooperation can form a new acceptable cooperation.

**Lemma 4.1.**  $v(\emptyset) = 0$ .

**Proof.** Obviously,  $c\_lce(\emptyset) = \sum_{c \in \emptyset} mc_c = 0$ , then we have  $v(\emptyset) = \sum_{c \in \emptyset} mc_c - c\_lce(\emptyset) = 0$ .  $\square$

**Lemma 4.2.** The lower bound of  $C(S)$  is  $LB\_C(S)$ ,  $LB\_C(S) = \begin{cases} 0, & RC(S) \geq 0 \\ -RC(S), & RC(S) < 0 \end{cases}$

**Proof.** There are two cases.

Case 1:  $RC(S) \geq 0$ . Since  $RC(S) \geq 0$  and  $C(S) \geq 0$ ,  $v(S) = RC(S) + C(S) \geq 0$ . Thus, the lower bound of  $C(S)$  is 0.

Case 2:  $RC(S) < 0$ .  $C(S) = v(S) - RC(S)$  and  $v(S) \geq 0$ , so  $C(S) \geq 0 - RC(S)$ . Thus, the lower bound of  $C(S)$  is  $-RC(S)$ .  $\square$

Lemma 4.2 indicates that at least the lower bound of  $C(S)$  should be provided to make the cooperation  $S$  acceptable, i.e., to satisfy the condition (2) in Definition 4.2.

**Lemma 4.3.**  $v(S \cup T)$  may be less than  $v(S) + v(T)$ .

**Proof.** Several cases of  $v(S \cup T) \geq v(S) + v(T)$  are given in Table 8. A case of  $v(S \cup T) < v(S) + v(T)$  is given in Table 9.  $\square$

Lemma 4.3 indicates that when  $v(S \cup T) < v(S) + v(T)$ , the compensation of  $C(S \cup T)$  should be provided to satisfy  $v(S \cup T) \geq v(S) + v(T)$ ,  $S \cap T = \emptyset$ ,  $S, T \subset C$ , i.e., condition (3) in Definition 4.2.

**Definition 4.3. (Compensation of cooperation  $S \cup T$ ,  $C(S \cup T)$ ):**  $C(S \cup T)$  is the compensation to make the cooperation  $S \cup T$  acceptable when  $v(S \cup T) < v(S) + v(T)$ ,  $S \cap T = \emptyset$ ,  $S, T \subset C$ . Obviously,  $C(S \cup T) \geq 0$ .

**Lemma 4.4.** The lower bound of  $C(S \cup T)$  is  $LB\_C(S \cup T)$ ,  $LB\_C(S \cup T) = \begin{cases} 0, & v(S \cup T) \geq v(S) + v(T) \\ v(S) + v(T) - v(S \cup T), & v(S \cup T) < v(S) + v(T) \end{cases}$

**Proof.** When  $v(S \cup T) \geq v(S) + v(T)$ , obviously,  $C(S \cup T)$  does not need to be provided. When  $v(S \cup T) < v(S) + v(T)$ ,  $C(S \cup T)$  needs to be provided to form the cooperation  $S \cup T$ , satisfying  $v(S \cup T) + C(S \cup T) \geq v(S) + v(T)$ . Thus,  $LB\_C(S \cup T)$  is  $v(S) + v(T) - v(S \cup T)$ .  $\square$

**Lemma 4.5.**  $v(S) = RC(S) + LB\_C(S) + \max\{LB\_C(P \cup T), P \cup T = S, P \cap T = \emptyset\}$ .

**Proof.** If  $RC(S) < 0$ , at least  $LB\_C(S)$  needs to be provided to guarantee the participants to be formed. If  $v(S) < v(P) + v(T)$ ,  $\forall P, \forall T | P \cup T = S, P \cap T = \emptyset$ , at least  $LB\_C(P \cup T)$  is provided to guarantee cooperation  $P$  and  $T$  to form the cooperation  $S$ . Therefore,  $v(S) = RC(S) + LB\_C(S) + \max\{LB\_C(P \cup T), P \cup T = S, P \cap T = \emptyset\}$ .  $\square$

Lemma 4.5 means that  $v(S)$  satisfies the conditions (2) and (3) in Definition 4.2.

#### 4.3. Profit distribution solution

Let  $x_c$  be the distributed profit of company  $c$ . A vector  $x = (x_1, \dots, x_{|S|})$  is a profit distribution solution of cooperation  $S$  if it satisfies the two following conditions:

- (1)  $x_c \geq 0$ ,  $c \in S$ ; and
- (2)  $\sum_{c \in S} x_c = v(S)$ .

Condition (1) is the individual rationality that no company can accept a negative allocation. Condition (2) is the group rationality that the total profit is fully shared.

Obviously,  $v(S) \geq 0$  is the premise that vector  $x$  is a profit distribution solution. Compensation of  $C(S)$  and  $C(S \cup T)$  guarantees  $v(S) \geq 0$  and  $v(S \cup T) \geq v(S) + v(T)$ .

#### 4.4. Profit distribution based on cooperative game theory

Most profit distribution cases have been studied based on cooperative game theory. The set of solutions includes the core, the nucleolus (Kuipers et al., 2013), and the Shapley value (Shapley, 1953).

A profit distribution solution of  $x = (x_1, \dots, x_{|C|})$  is in the core if it satisfies the two following conditions: (1)  $\sum_{c \in S} x_c \geq v(S)$ ; and (2)  $\sum_{c \in C} x_c = v(C)$ . However, the core usually is not unique. Thus, the core center (González Díaz and Sánchez Rodríguez, 2007) is used here.

The definition of the nucleolus is conceptual. The concept of the nucleolus can be found in Maschler et al. (1979). If the core is non-empty, the nucleolus is in the core and is always unique.

The Shapley value is computed as  $\phi(v) = (\phi_1(v), \phi_2(v), \dots, \phi_{|C|}(v))$ , where  $\phi_c(v) = \sum_{c \in S} \frac{(|C|-|S|)! (|S|-1)!}{|C|!} [v(S) - v(S - \{c\})]$ ,  $c = 1, 2, \dots, |C|$ ;  $S$  is any cooperation including company  $c$ ; and  $v(S) - v(S - \{c\})$  is the marginal contribution of company  $c$  to cooperation  $S$ . Therefore, the Shapley value allocates to each company the weighted sum of his contributions.

All of the core center, the nucleolus and the Shapley value of cooperative GPDP can be obtained based on the exactly solved  $v(S)$ . Thus, we develop an exact method to obtain  $v(S)$  of cooperative GPDP.

**5. Exact algorithms to obtain the POS\_C and POS\_CE**

To exactly obtain  $v(S)$ , we develop two exact algorithms to solve POS\_C and POS\_CE respectively. Subsequently, the exact profit distribution of cooperative PDP for the lowest carbon emissions can be calculated.

*5.1. An exact algorithm to obtain the POS\_C of company c*

It is difficult to obtain all the Pareto-optimal solutions of PDP or VRP. Fortunately, POS\_C is the special Pareto-optimal solution with the minimum cost, and so it is unnecessary to solve all Pareto-optimal solutions. Consequently, we propose an exact algorithm including two set-partition models to solve POS\_C. The first set-partition model obtains the minimum cost. The obtained minimum cost is input in the second set-partition model as a constraint, then POS\_C can be exactly solved.

The first set-partition model to obtain the minimum cost ( $mc_c$ ) is expressed as model 1. The second set-partition model to obtain POS\_C of company  $c$  is expressed as follows, i.e., model 3.

**Model 3:**

$$\text{Minimize } \sum_{i \in R_c} ce_{ci}x_{ci}, \tag{11}$$

$$\text{subject to } \sum_{i \in R_c} c_{ci}x_{ci} = \text{obtained } mc_c, \tag{12}$$

$$\sum_{i \in R_c} a_{w_c, ci}x_{ci} = 1, \quad w_c \in W_c, \tag{13}$$

$$x_{ci} = 0 \text{ or } 1, \quad ci \in R_c. \tag{14}$$

In constraint (12),  $mc_c$  is exactly obtained by solving model 1 using CPLEX. Also, if model 3 can be exactly solved by CPLEX, then  $ce\_mc_c$  can be obtained. Thus, POS\_C of company  $c$  can be exactly solved.

*5.2. An exact algorithm to obtain the POS\_CE of cooperation*

In the exact algorithm to obtain the POS\_CE of cooperation, the first set-partition model is expressed as model 2, and the second set-partition model is expressed as follows, i.e., model 4.

**Model 4:**

$$\text{Minimize } \sum_{i \in R} ct_i x_i + |S| * ccpc \tag{15}$$

$$\text{subject to } \sum_{i \in R} ce_i x_i = \text{obtained } lce(C), \tag{16}$$

$$\sum_{i \in R} a_{w, i} x_i = 1, \quad w \in W, \tag{17}$$

$$x_i = 0 \text{ or } 1, \quad i \in R. \tag{18}$$

In objective (15),  $ct_i$  is the cost of trip  $i$ , as shown in Eq. (1). The  $ccpc$  is the cost of cooperation per company, and  $|S|*ccpc$  is the total cooperative cost, e.g., the cost of information integration.

In constraint (16),  $lce(C)$  is exactly obtained by solving model 2 using CPLEX. Also, model 4 can be exactly solved by CPLEX, and  $c\_lce(C)$  can be obtained. Thus, POS\_CE of cooperation  $C$  can be exactly solved.

*5.3. Pareto-optimal trip*

Although models 1–4 are linear, the computing time by CPLEX depends on the numbers of the used trips. Models 1–4 use all the feasible trips. To reduce the solution space to exactly solve POS\_C and POS\_CE, we define Pareto-optimal trip, as given in Definition 5.1. All the feasible trips and Pareto-optimal trips for a case with customers 1, 2 and 3 are given in Table 1.

Table 1 shows the expression, cost, and carbon emission for the 6 feasible trips of the case with 3 customers. Only 2 trips are Pareto-optimal in the case, and others are non-Pareto-optimal trips. We prove that both POS\_C and POS\_CE are composed of Pareto-optimal trips, and so the non-Pareto-optimal trips can be cut. Thus, the solution space of the used trips are reduced significantly.



**Table 1**  
All the feasible trips and Pareto-optimal trips for a case with 3 customers.

Trips	Expression	Cost	Carbon emission	Pareto-optimal trip
1	0->1->2->3->d	22	4.5	✓
2	0->3->2->1->d	22	4.7	-
3	0->2->1->3->d	23	4.6	-
4	0->3->1->2->d	23	4.8	-
5	0->1->3->2->d	21	4.6	✓
6	0->2->3->1->d	21	4.7	-

**Definition 5.1. (Dominance relation >):** Let  $\vec{t}'$  and  $\vec{t} \in R$ .  $\vec{t}' > \vec{t}$  if and only if  $t'_c \leq t_c$  and  $t'_{ce} \leq t_{ce}$ , where at least one inequality is strict;  $t'_c$  and  $t'_{ce}$  are the cost and carbon emissions of  $\vec{t}'$  respectively; and  $t_c$  and  $t_{ce}$  are the cost and carbon emissions of  $\vec{t}$ , respectively.

**Definition 5.2. (Pareto-optimal trip):** Let  $SFT$  be the set of all the feasible trips of a customer set. A trip  $\vec{x} \in SFT$  is a Pareto-optimal trip of the customer set if and only if  $\nexists \vec{x}' \in SFT$  such that  $\vec{x}' > \vec{x}$ .

**Theorem 5.1.** In  $POS_C$  and  $POS_{CE}$ , all trips are Pareto-optimal.

**Proof.** Let a set of trips ( $ST$ ) be the solution of  $POS_C$  or  $POS_{CE}$  with customer set of  $W$ . Assume at least one trip is non-Pareto-optimal in  $ST$ .  $ST = \{t_1, t_2, \dots, t_{|ST|}\}$ , where  $t_i$  expresses the trip  $i$ .

Case 1:  $|ST| = 1$ ,  $ST$  contains only one trip, i.e.,  $ST = \{t_1\}$ . According to the assumption,  $t_1$  is non-Pareto-optimal. Let  $pot_1$  be one of the Pareto-optimal trips of the customer set of  $W$ . Let  $PST = \{pot_1\}$ , then  $PST$  is a feasible solution. Since  $|ST| = 1$ , the amount of carbon emissions of  $ST$  equals  $t_{1ce}$  and cost of  $ST$  equals  $t_{1c}$ . Also, the amount of carbon emissions of  $PST$  equals  $pot_{1ce}$  and cost of  $PST$  equals  $pot_{1c}$ . Since  $pot_1 > t_1$ ,  $PST > ST$ . But this contradicts the assumption that  $ST$  is one Pareto-optimal solution, i.e.,  $POS_C$  or  $POS_{CE}$ .

Case 2:  $|ST| \geq 2$ ,  $ST$  contains at least two trips. Assume  $t_i$  is non-Pareto-optimal. The amount of carbon emissions of  $ST$  ( $ST_{ce}$ ) is  $\sum_{j=1}^{i-1} t_{jce} + t_{ice} + \sum_{j=i+1}^{|ST|} t_{jce}$  and cost of  $ST$  ( $ST_c$ ) is  $\sum_{j=1}^{i-1} t_{jc} + t_{ic} + \sum_{j=i+1}^{|ST|} t_{jc}$ , where  $t_{ice}$  and  $t_{ic}$  are carbon emissions and cost of  $t_i$ , respectively. Let  $pot_i$  be one of Pareto-optimal trips of the customer set of  $t_i$ . Let  $PST = \{t_1, \dots, t_{i-1}, pot_i, t_{i+1}, \dots, t_{|ST|}\}$ . Customer set of  $pot_i$  is the same as that of  $t_i$ , therefore,  $PST$  is a feasible solution. The amount of carbon emissions of  $PST$  ( $PST_{ce}$ ) is  $\sum_{j=1}^{i-1} t_{jce} + pot_{ice} + \sum_{j=i+1}^{|ST|} t_{jce}$  and cost of  $PST$  ( $PST_c$ ) is  $\sum_{j=1}^{i-1} t_{jc} + pot_{ic} + \sum_{j=i+1}^{|ST|} t_{jc}$ . Therefore,  $PST_{ce} - ST_{ce} = pot_{ice} - t_{ice}$  and  $PST_c - ST_c = pot_{ic} - t_{ic}$ . Since  $pot_i > t_i$ ,  $pot_{ice} - t_{ice} \leq 0$  and  $pot_{ic} - t_{ic} \leq 0$ , where at least one inequality is strict. Therefore,  $PST > ST$ . But this contradicts the assumption that  $ST$  is one Pareto-optimal solution, i.e.,  $POS_C$  or  $POS_{CE}$ .

These cases cover all the possibilities. Since the assumption that both  $POS_C$  and  $POS_{CE}$  are composed of non-Pareto-optimal trips leads to a contradiction, in both  $POS_C$  and  $POS_{CE}$ , all trips are Pareto-optimal. □

Thus, the solution space is reduced by cutting the non-Pareto-optimal trips.

5.4. Computing time of the exact algorithm for the largest-scale instance

We use the largest benchmark instance of PDP from Li & Lim benchmarks, i.e., pdptw1000-LR1\_10\_1 with 1,054 customers and 19,306 products, to evaluate the performance of the proposed exact algorithm. Delivery point ( $d$ ) has the coordinates of (50, 30). The coordinates of depot (0) are (20, 30). For the instance of pdptw1000-LR1\_10\_1 with  $Q=70$ , the computing time of producing all 109,868 Pareto-optimal trips is 3 min, and the computing time of obtaining  $POS_C$  or  $POS_{CE}$  is 14 min.

5.5. The algorithm to exactly obtain compensation and profit distribution

To form and stabilize the cooperative GPDP, we develop an exact method with three steps to exactly calculate the actual compensation and obtain the exact profit distribution based on cooperative game theory. The first step produces all the Pareto-optimal trips. The second step exactly obtains  $POS_C$  of non-cooperation. The third step exactly obtains  $POS_{CE}$  of cooperation, compensation, and exact profit distribution. The procedure of the exact algorithm is described in brief in Table 2.

In sub-step 1.1, a feasible subset means that it is possible to use a vehicle to deliver the products of all the customers in the subset to delivery point.

In sub-step 1.2, a feasible trip means that all constraints of the customers are satisfied by the trip.

In sub-steps 2.1 and 2.2, Model 1 and Model 3 are solved by CPLEX, respectively.

In sub-steps 3.1 and 3.2, Model 2 and Model 4 are solved by CPLEX, respectively.

Algorithm 1 exactly calculates the lower bound of compensation, as shown in the following description.

**Table 2**

The exact method to solve POS\_C, POS\_CE, compensation and profit distribution.

Step	Sub-step	Description
1	1.1	Produce all the feasible subsets of each $W_c$ and $W$ respectively.
	1.2	Produce all the feasible trips of each feasible subset of each $W_c$ and $W$ respectively.
	1.3	Obtain all the Pareto-optimal trips of each feasible subset of each $W_c$ and $W$ respectively, using all the feasible trips obtained in sub-step 1.2.
	1.4	Obtain all the Pareto-optimal trips of each $W_c$ and $W$ respectively based on sub-step 1.3.
2	2.1	For every company $c$ , calculate $mc_c$ by Model 1 using all the Pareto-optimal trips of company $c$ .
	2.2	For every company $c$ , calculate $ce\_mc_c$ by Model 3 using all the Pareto-optimal trips of company $c$ .
	2.3	Calculate $\sum_{c \in C} mc_c$ and $\sum_{c \in C} ce\_mc_c$ of non-cooperation.
3	3.1	For every $S, S \subseteq C$ , calculate $lce(S)$ by Model 2 using all the Pareto-optimal trips of $W$ .
	3.2	For every $S, S \subseteq C$ , calculate $c\_lce(S)$ by Model 4 using all the Pareto-optimal trips of $W$ .
	3.3	Calculate lower bound of compensation and $v(S)$ of every $S$ . See <a href="#">Algorithm 1</a> .
	3.4	Obtain profit distribution of every $c$ based on $v(S)$ using TUGlab ( <a href="#">Mirás Calvo and Sánchez Rodríguez, 2006</a> ).

**Table 3**  
Experiment parameters.

Parameter name	Parameter value
(v) speed of the vehicles	40 km/h
(Q) capacity of the vehicles	4
(f) fuel consumption index parameter	1/44,000 J/g
(r) carbon emissions index parameter	3.125 g/g
(w) weight of empty vehicle	1400 kg
(f) weight of a customer and his/her baggage	100 kg
(C <sub>d</sub> ) drag coefficient	0.45
(C <sub>r</sub> ) coefficients of rolling resistance	0.015
(A) frontal surface area of the vehicle	1 m <sup>2</sup>
(c <sub>f</sub> ) fixed cost per vehicle	10 RMB
(c <sub>d</sub> ) cost per unit distance	2 RMB
(ccpc) cost of cooperation per company	300 RMB
(ρ) air density	1.225 kg/m <sup>3</sup>
(0) depot coordinate	(20, 30)
(d) delivery point coordinate	(50, 30)

**Table 4**  
The used benchmark instances.

Companies	Benchmark	Name	NC	TP	POS_C		
					mc <sub>c</sub>	ce_mc <sub>c</sub>	Used vehicles
1	LR1_2_1-100	C1-100	100	1832	21,371	657	59
	LR1_2_1	C1-210	210	3662	37,372	1205	99
2	LR1_4_1-300	C2-300	300	5466	79,733	2700	144
	LR1_4_1	C2-416	416	7474	10,2769	3479	191
3	LR1_6_1-500	C3-500	500	8907	20,8694	6869	233
	LR1_6_1	C3-634	634	11402	25,9972	8647	290

Step (2) eliminates Situation (1), where cooperation C cannot be formed even by compensation. Therefore, the situation of cooperation C is one of Situations (2), (3), and (4).

Step (2-1) calculates the lower bound of C(S); adds it into *compensationSet*, and calculates  $v(S)$ .

Step (3) gives lower bound of compensation of every S.

## 6. Computational results

### 6.1. Experiment instances

To demonstrate the carbon emissions reduction by cooperation, compensation, and exact profit distribution, we perform extensive experiments using benchmark instances and a real-world PDP, i.e., picking up and delivering customers to airport service (PDAS). The experiment parameters are listed in Table 3. The used benchmark instances of PDP from Li & Lim benchmarks are shown in Table 4.

*Name*: the used name of benchmark in Table 6; *NC*: number of customers in an instance; *TP*: total product in an instance; *mc<sub>c</sub>*: the minimum cost of company *c* in the non-cooperation; *ce\_mc<sub>c</sub>*: the carbon emissions of company *c* in the non-cooperative PDP with the minimum cost.

In Table 4, instances of “LR1\_2\_1”, “LR1\_4\_1”, and “LR1\_6\_1” are the instances in Li & Lim benchmarks. “LR1\_2\_1-100” is the “LR1\_2\_1” with the first 100 customers. The results of POS\_C of 9 non-cooperation are solved by the proposed exact algorithm. Using the six benchmark instances, we performed 12 cooperation experiments with two companies and 8 cooperation experiments with three companies to reduce carbon emissions. The 20 cooperation results are given in Table 6.

The used practical instances are shown in Table 5. The practical data were obtained through one-month investigation from Zhongshan company, which is the famous agency performing the service of picking up and delivering customers to the airport in Shenyang in China (Yu et al., 2016). Table 5 shows nine practical instances from three companies labeled as 1, 2, and 3. The name of an instance includes the number of customer points and the total number of customers. For example, the instance of “40\_40” means 40 customer points and 40 served customers. The results of POS\_C of 9 non-cooperation are solved by the proposed exact algorithm. Using the nine instances from three logistics companies, we performed 27 cooperation experiments with two companies and 27 cooperation experiments with three companies to reduce carbon emissions. The 54 cooperation results are given in Table 7.

**Table 5**  
The Used Practical Instances.

Companies	Instance name	POS_C		
		mc <sub>c</sub>	ce_mc <sub>c</sub>	Used vehicles
1	40_40	1945	45.8	20
	40_88	2544	60.6	29
	40_117	3103	74.0	37
2	45_100	6413	153.7	32
	50_100	7062	168.5	34
	55_100	3368	80.6	28
3	80_160	11,217	269.3	47
	85_170	11,820	283.8	52
	90_180	12,303	295.6	53

**Table 6**  
Results of 20 cooperative GPDPs based on the benchmark instances.

No.	Companies			R_CE%	R_C%	LB_C(S)	v(S)
	1	2	3				
1	C1-100	C2-300	-	8.03	2.62	0	2647
2	C1-100	C2-416	-	6.28	1.47	0	1828
3	C1-210	C2-300	-	8.50	3.29	0	3854
4	C1-210	C2-416	-	6.87	2.40	0	3357
5	C1-100	-	C3-500	4.68	1.35	0	3111
6	C1-100	-	C3-634	4.55	1.25	0	3516
7	C1-210	-	C3-500	4.99	1.62	0	3977
9	C1-210	-	C3-634	4.78	1.14	0	3403
9	-	C2-300	C3-500	7.67	3.83	0	11,038
10	-	C2-300	C3-634	7.14	3.20	0	10,886
11	-	C2-416	C3-500	7.32	3.65	0	11,357
12	-	C2-416	C3-634	7.00	3.18	0	11,522
13	C1-100	C2-300	C3-500	8.80	18.60	0	57,624
14	C1-100	C2-300	C3-634	8.13	4.80	0	17,344
15	C1-100	C2-416	C3-500	8.38	5.30	0	17,625
16	C1-100	C2-416	C3-634	7.95	4.74	0	18,189
17	C1-210	C2-300	C3-500	8.98	5.67	0	18,475
18	C1-210	C2-300	C3-634	8.33	5.03	0	18,972
19	C1-210	C2-416	C3-500	8.54	5.41	0	18,875
20	C1-210	C2-416	C3-634	8.12	4.82	0	19,286

No. is the index of cooperative GPDP;  $R_{CE} = \frac{RC(S)}{\sum_{c \in S} ce\_mc_c} * 100\%$ ;  $R_C = \frac{RC(S)}{\sum_{c \in S} mc_c} * 100\%$ .

6.2. Carbon emissions reduction and compensation of cooperation

The results of 20 cooperative GPDPs based on the benchmark instances are solved by the proposed exact algorithm, as listed in Table 6. The 20 cooperative GPDPs reduce carbon emissions and save costs simultaneously. No compensation is provided, i.e., all LB\_C(S) equal to 0.

The compensation and profit of 54 cooperative GPDPs based on the practical instances are solved by the proposed exact algorithm. The results are listed in Table 7. Fig. 1 shows R\_CE and R\_C in the 54 cooperative GPDPs. The horizontal axis represents the number of cooperative GPDPs and the vertical axis shows R\_CE and R\_C in each GPDP.

Fig. 1 shows the results of all the 54 cooperative GPDPs. However, 11 cooperative GPDPs spend higher costs than non-cooperative PDPs; therefore, the compensation (see the column of LB\_C(S)) should be provided to form the cooperation.

Based on the results of v(S) in Table 7, we can easily obtain the profit distribution for any cooperative GPDP using TUGlab. The cooperative cases 37–45 are used to show the compensation of cooperation and the detailed profit distribution of each company in the cooperation, as shown in Table 8.

No. is the index of cooperative GPDP in Table 7; the data in the columns of Core center, Shapley value, and Nucleolus expresses the obtained profit of the company by the cooperation using the profit distribution method. For example, in column of Shapley value of cooperation 37, “1–54.4” means that the obtained profit of company 1 is 54.4 by using the profit distribution method of Shapley value.

Cooperation 45 shows that the compensation of 319 = 164 + 155 (see the column of LB\_C(S)) RMB is provided but the amount of carbon emissions is reduced by 7.2% through the cooperation.

In Table 8, when RC(S) ≥ 0 in the cooperation with 3 companies, LB\_C(P ∪ T) always equal 0. However, according to Lemma 4.4, it is possible that even RC(S) ≥ 0 LB\_C(P ∪ T) may be more than 0. Thus, the compensation needs to be provided

**Table 7**  
 Compensation and profit of 54 cooperative GPDPs based on the real-life instances.

No.	Companies			$R_{CE}\%$	$RC(S)$	$LB_C(S)$	$v(S)$	$R_C\%$	Time (s)
	1	2	3						
1	40_40	45_100	–	7.3	127	0	127	1.4	5.2
2	40_40	50_100	–	9.5	412	0	412	4.3	4.1
3	40_40	55_100	–	6.2	–28	28	0	–0.3	6.3
4	40_88	45_100	–	4.2	–157	157	0	–1.6	4.2
5	40_88	50_100	–	5.3	–14	14	0	–0.1	5.2
6	40_88	55_100	–	4.6	–164	164	0	–1.7	5.2
7	40_117	45_100	–	3.3	–248	248	0	–2.4	5.2
8	40_117	50_100	–	5.4	32	0	32	0.3	4.2
9	40_117	55_100	–	4.4	–144	144	0	–1.4	4.2
10	40_40	–	80_160	5.1	217	0	217	1.6	6.4
11	40_40	–	85_170	5.8	384	0	384	2.6	5.3
12	40_40	–	90_180	4.4	175	0	175	1.2	6.4
13	40_88	–	80_160	3.0	–105	105	0	–0.7	6.4
14	40_88	–	85_170	3.5	29	0	29	0.2	4.2
15	40_88	–	90_180	2.5	–155	155	0	–1.0	6.4
16	40_117	–	80_160	3.0	–94	94	0	–0.6	5.4
17	40_117	–	85_170	3.1	–38	38	0	–0.2	6.3
18	40_117	–	90_180	1.8	–266	266	0	–1.6	5.3
19	–	45_100	80_160	3.9	148	0	148	0.8	5.2
20	–	45_100	85_170	5.0	420	0	420	2.2	4.0
21	–	45_100	90_180	3.7	160	0	160	0.8	5.1
22	–	50_100	80_160	6.6	720	0	720	3.8	5.4
23	–	50_100	85_170	6.3	671	0	671	3.4	5.3
24	–	50_100	90_180	7.0	876	0	876	4.3	5.0
25	–	55_100	80_160	6.5	602	0	602	3.3	5.3
26	–	55_100	85_170	6.1	549	0	549	2.9	8.4
27	–	55_100	90_180	5.8	549	0	549	2.8	6.4
28	40_40	45_100	80_160	8.1	942	0	942	4.6	5.3
29	40_40	45_100	85_170	8.8	1,139	0	1,139	5.4	6.1
30	40_40	45_100	90_180	6.7	696	0	696	3.2	6.3
31	40_40	50_100	80_160	10.4	1,513	0	1,513	7.1	9.8
32	40_40	50_100	85_170	10.3	1,537	0	1,537	7.0	6.3
33	40_40	50_100	90_180	10.5	1,617	0	1,617	7.2	6.2
34	40_40	55_100	80_160	9.7	1,240	0	1,240	6.0	6.3
35	40_40	55_100	85_170	10.3	1,431	0	1,431	6.7	8.5
36	40_40	55_100	90_180	9.0	1,168	0	1,168	5.4	8.5
37	40_88	45_100	80_160	5.3	311	0	311	0.0	6.3
38	40_88	45_100	85_170	6.4	616	0	616	2.8	5.3
39	40_88	45_100	90_180	5.1	360	0	360	0.0	7.4
40	40_88	50_100	80_160	7.6	863	0	863	3.9	6.4
41	40_88	50_100	85_170	7.9	997	0	997	4.4	7.5
42	40_88	50_100	90_180	8.2	1,105	0	1,105	4.8	7.3
43	40_88	55_100	80_160	7.6	766	0	766	3.6	8.6
44	40_88	55_100	85_170	8.4	1,010	0	1,010	4.6	5.3
45	40_88	55_100	90_180	7.2	778	0	778	3.5	6.2
46	40_117	45_100	80_160	4.5	141	0	141	0.6	5.3
47	40_117	45_100	85_170	5.9	546	0	546	2.4	5.3
48	40_117	45_100	90_180	4.3	167	0	167	0.7	6.4
49	40_117	50_100	80_160	7.4	873	0	873	3.9	7.5
50	40_117	50_100	85_170	7.1	812	0	812	3.5	7.5
51	40_117	50_100	90_180	7.8	1,040	0	1,040	4.4	6.4
52	40_117	55_100	80_160	7.3	738	0	738	3.4	6.4
53	40_117	55_100	85_170	7.9	911	0	911	4.0	6.3
54	40_117	55_100	90_180	6.7	696	0	696	3.0	6.6

even  $RC(S) \geq 0$ . Table 9 shows compensation and profit distribution of a cooperation with 3 companies under  $RC(S) \geq 0$ . The instances of company 1, 2 and 3 are “40\_117”, “45\_100” and “80\_160”, respectively.

In Table 9, for the cooperation of  $C = \{1,2,3\}$ , both  $R_{CE}(C)$  and  $RC(C)$  are greater than 0. Therefore, the cooperation of  $\{1,2,3\}$  is accepted. However,  $RC(\{1,2\})$  and  $RC(\{1,3\})$  are smaller than 0, and thus the compensation needs to be provided, i.e.,  $LB_C(\{1,2\})$  is 248 and  $LB_C(\{1,3\})$  is 94. In addition, according to Lemma 4.5, if  $v(S) < v(P) + v(T)$ ,  $\forall P, \forall T | P \cup T = S, P \cap T = \emptyset$ , then the compensation needs to be provided to form cooperation  $S$ . To form the cooperation of  $\{1,2,3\}$ , the maximum  $LB_C(\{1,2,3\})$  is  $7 = 148 - 141$ . Table 9 shows that the compensation of  $349 = 248 + 94 + 7$  (see the columns of  $LB_C(S)$  and  $LB_C(P \cup T)$ ) RMB is provided but the amount of carbon emissions is reduced by 4.5% through cooperation.

**Table 8**  
Profit distribution of 9 cooperative GPDs with 3 companies based on Table 7.

Cooperation	No.	R_CE%	RC(S)	LB_C(S)	LB_C (P ∪ T)	v(S)	Corecenter	Nucleolus	Shapley value
37	4	4.2	-157	157	0	0	1-71.86	1-81.5	1-54.4
	13	3.0	-105	105	0	0	2-119.57	2-114.75	2-128.3
	19	3.9	148	0	0	148	3-119.57	3-114.75	3-128.3
	37	5.3	311	0	0	311	-	-	-
38	4	4.2	-157	157	0	0	1-92.16	1-98	1-70.2
	14	3.5	29	0	0	29	2-260.70	2-259	2-265.7
	20	5.0	420	0	0	420	3-263.14	3-259	3-280.1
	38	6.4	616	0	0	616	-	-	-
39	4	4.2	-157	157	0	0	1-87.18	1-100	1-66.8
	15	2.5	-155	155	0	0	2-136.41	2-130	2-146.6
	21	3.7	160	0	0	160	3-136.41	3-130	3-146.6
	39	5.1	360	0	0	360	-	-	-
40	5	5.3	-14	14	0	0	1-69.34	1-71.5	1-47.8
	13	3.0	-105	105	0	0	2-396.83	2-395.75	2-407.6
	22	6.6	720	0	0	720	3-396.83	3-395.75	3-407.6
	40	7.6	863	0	0	863	-	-	-
41	5	5.3	-14	14	0	0	1-152.60	1-163	1-113.5
	14	3.5	29	0	0	29	2-421.45	2-417	2-434.5
	23	6.3	671	0	0	671	3-422.95	3-417	3-449
	41	7.9	997	0	0	997	-	-	-
42	5	5.3	-14	14	0	0	1-110.08	1-114.5	1-76.4
	15	2.5	-155	155	0	0	2-497.46	2-495.25	2-514.3
	24	7.0	876	0	0	876	3-497.46	3-495.25	3-514.3
	42	8.2	1105	0	0	1,105	-	-	-
43	6	4.6	-164	164	0	0	1-78.72	1-82	1-54.8
	13	3.0	-105	105	0	0	2-343.64	2-342	2-355.6
	25	6.5	602	0	0	602	3-343.64	3-342	3-355.6
	43	7.6	766	0	0	766	-	-	-
44	6	4.6	-164	164	0	0	1-208.01	1-230.5	1-158.5
	14	3.5	29	0	0	29	2-400.42	2-389.75	2-418.5
	26	6.1	549	0	0	549	3-401.57	3-389.75	3-433
	44	8.4	1010	0	0	1,010	-	-	-
45	6	4.6	-164	164	0	0	1-107.92	1-114.5	1-76.4
	15	2.5	-155	155	0	0	2-335.04	2-331.75	2-350.8
	27	5.8	549	0	0	549	3-335.04	3-331.75	3-350.8
	45	7.2	778	0	0	778	-	-	-

**Table 9**  
Compensation and profit distribution of a cooperation with 3 companies under  $RC(S) \geq 0$ .

Cooperation	R_CE%	RC(S)	LB_C(S)	LB_C (P ∪ T)	v(S)	Corecenter	Nucleolus	Shapley value
{1}	0	0	-	-	0	0	0	0
{2}	0	0	-	-	0	74	74	74
{3}	0	0	-	-	0	74	74	74
{1,2}	3.3	-248	248	0	0	-	-	-
{1,3}	3.0	-94	94	0	0	-	-	-
{2,3}	3.9	148	0	0	148	-	-	-
{1,2,3}	4.5	141	0	7	148	-	-	-

**Algorithm 1**  
CalComp().

**Input:**  $mc_c$  and  $ce\_mc_c$  of every company  $c, c \in C$ ,  $lce(S)$  and  $c\_lce(S)$  of every  $S, S \subseteq C$ .

**Output:** Lower bound of compensation of every  $S$ .

- (1) Initialize.  
 $compensationSet$  (Set of lower bound of the compensations of all companies)  $\leftarrow \emptyset$
- (2) **if** ( $!(RCE(C) = 0$  and  $RC(C) \leq 0)$ ) **then**
  - (2-1) **for** (each  $S \subseteq C$ ) **do**
    - Calculate the lower bound of  $C(S)$ , i.e.,  $LB_C(S)$ .
    - $compensationSet \leftarrow compensationSet \cup \{LB_C(S)\}$
    - Calculate  $v(S)$
  - end for**
- end if**
- (3) Output  $compensationSet$ .

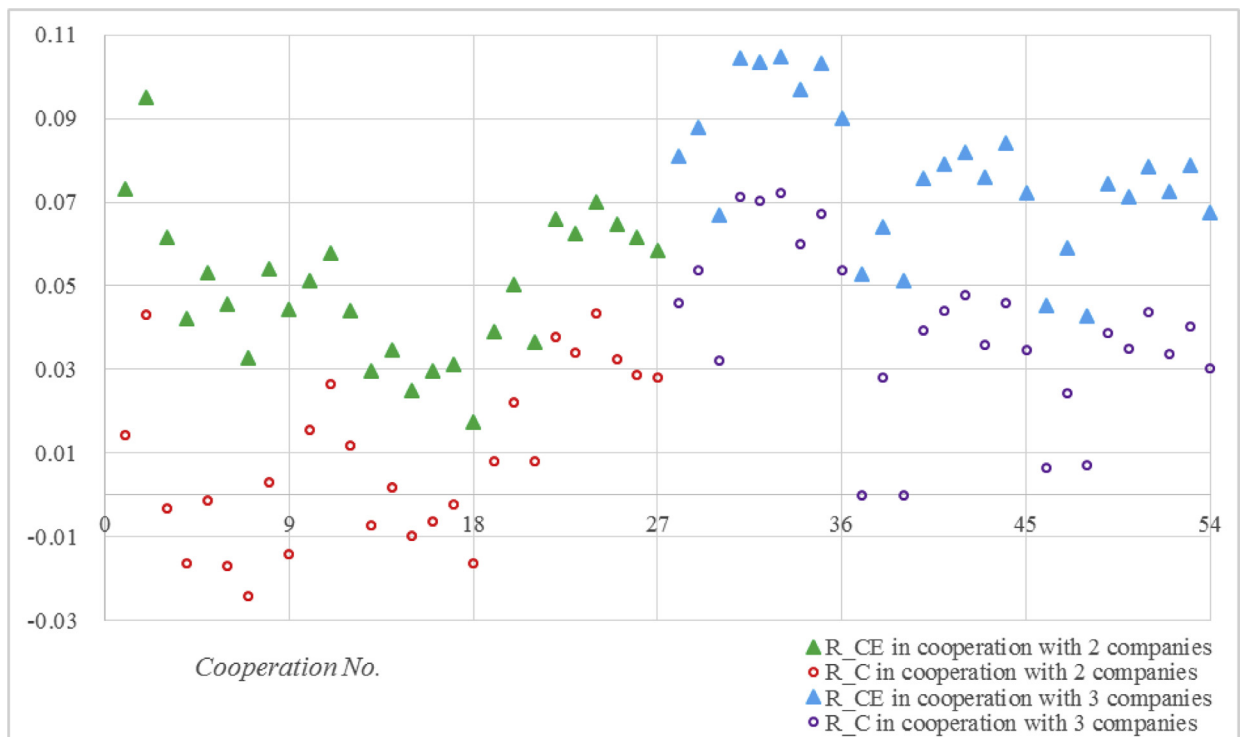


Fig. 1. Reduced carbon emissions and cost in 54 cooperative GPDPs.

After providing the compensation, the exact profit distribution of each company can be obtained using  $v(S)$ , see the columns of *Core center*, *Shapley value*, and *Nucleolus*.

### 6.3. Managerial insights

Based on the results of the above experiments, we obtain the following insights.

1. The cooperation strategy leads to a significant performance improvement for a GPDP that has fewer products in the customers and more partner companies due to the higher utilization of vehicles.
2. For many GPDP cases, the cooperation strategy can significantly reduce carbon emissions and costs simultaneously.
3. Compensation is required in only a few cooperative GPDP cases; furthermore, in these cases, limited compensation can significantly reduce carbon emissions.

## 7. Conclusions

In this paper, we conducted a comprehensive study on the cooperation strategy for GPDP. We first investigated the situations in which the compensation needs to be provided to form an acceptable cooperation. Then, we defined the compensation to form a cooperation and developed the lower bound of compensation. An exact method was proposed to calculate the compensation and to distribute the profit to a particular GPDP. Finally, we employed the largest benchmark PDP instance and a real-world PDP with the real-life data to validate the proposed method. Some interesting and important managerial insights were concluded. The cooperation strategy has been proved to be effective, especially for a GPDP with fewer products in the customers and more partner companies; further, compensation is required in only a few cooperative GPDP cases and limited compensation will achieve a significant carbon emissions reduction.

We have not considered the potential effects of the introduction of road charging schemes that may lead to strong economic incentives for cooperation. This issue will be investigated in our future work.

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