Neural Comput & Applic
DOI 10.1007/s00521-017-2916-9

ORIGINAL ARTICLE

The potential of nonparametric model in foundation bearing capacity prediction

Saadya Fahad Jabbar[1] · **Raed Ibraheem Hamed**[2] · **Asmaa Hussein Alwan**[1]

Received: 17 November 2016 / Accepted: 13 February 2017

Abstract Nonparametric mathematical models have gained a very massive attention in the last two decades in solving regression problem. The application of soft computing methodologies produced a very remarkable assistance to human abilities especially in solving nonlinear and non-stationary engineering problems. The current article investigates the utility of k-nearest neighbor (k-nn) approach in predicting ultimate bearing capacity of shallow foundation. The inspected application involves an experimental data set of foundation dimension and soil properties that suggested and calculated via manual computational methods. The predictive model is established using dimensional shallow foundation, and soil properties are an inputs variable, whereas the bearing capacity is the output variable. For the purpose of comparison and evaluating the modeling accuracy, multiple linear regression (MLR) model is chosen to diagnose the result accuracies. Couple of statistical indicators are utilized to exhibit the performance criteria of the predictive model including coefficient of determination (r^2), degree of agreement (d), root-mean-square error (RMSE) and mean absolute percentage error (MAPE). The results exhibited a very representable and high accuracies of the investigated k-nn model vis-à-vis MLR. For instance, the RMSE and MAPE were enhanced by 24 and 17%, respectively. In addition, the findings indicated that k-nn provides an accurate and reliable alternative predictive model to the manual computational methods.

Keywords K-nearest neighbor · Soft computing · Multiple linear regression · Bearing capacity · Predictive model

✉ Saadya Fahad Jabbar
saadya.fahad@ircoedu.uobaghdad.edu.iq

Raed Ibraheem Hamed
raed.alfalahy@uhd.edu.iq

Asmaa Hussein Alwan
asmaa.hussein@ircoedu.uobaghdad.edu.iq

[1] College of Education for Human Science/Ibn Rushed, University of Baghdad, Baghdad, Iraq

[2] Department of Information and Technology, College of Science and Technology, University of Human Development, Sulaymaniyah, Iraq

1 Introduction

Soil ultimate bearing capacity (*qu*) is indicated as the least pressure that would cause shear stress failure of the supporting soil immediately below and adjacent to the foundation [1]. The ultimate bearing capacity is also known as the limited settlement of foundation. The importance of this geotechnical term is its major requirement within the foundation design satisfaction. For any geotechnical engineering work (e.g., construction), the allowable *qu* is controlled by the amount of settlements and their criteria [2]. *qu* is usually computed via either experimentally or manual calculations through analytical procedure (i.e., empirical formulation). Based on the exist state-of-the-art literature, it has been observed that BC for soil which was proposed by [3] is based on homogenies criteria and on manual calculation method that can be not practically well organized. Several formulas have been proposed by several scholars to compute the BC, such as Terzaghi [3], Meyerhof [4], Hansen [5], and Kumbhojkar [6]. The conceptual theory of all those researches is based on the footing geometry and the shearing resistance angle. The current paper focuses on the utility of the k-nn model as an intelligent approach to predict bearing capacity of soil under shallow strip foundation loading.

Published online: 01 March 2017

Soft computing techniques are valuable intelligence models that have been remarkably in various engineering applications in comparison with the classical methods that are difficult to pursue or capture the high complicated relationship between parameters [7–11]. Based on the historical researches conducted in predicting bearing capacity, several studies accomplished the modeling utilization the soft computing techniques [12–16]. In 2005, the most primitive study was conducted in predicting the ultimate bearing capacity using artificial neural network via two main algorithms: backpropagation and radial basis function [17]. The used experimental data belong to shallow foundation on reinforced cohesion-less soil. In summary, the findings approved that the application of ANN outperformed the traditional methods. Another study was conducted for the same application using multiple layer perceptron (MLP) algorithm [18]. The investigated MLP algorithm was verified with the theoretical computation and showed a very comprehended modeling. An adaptive neuro-fuzzy inference system is inspected to capture the correlation between the basic status of soil foundation system and *qu* [19]. The discovery of the study showed a high potential of the inspected soft computing model to model the bearing capacity. Ornek et al. [20] estimated bearing capacity of circular footing placed on soft clay layer soil. The modeling constructed using artificial neural network (ANN) model. The data set was obtained from an extensive series of field test (i.e., seven different sizing dimensions used as input parameters). The results of ANN model are evaluated with MLR model. In conclusion, ANN model offers a simple and reliable tool for estimating bearing capacity of circular footings foundation in stabilized clay soil. In 2013, multiple linear regression model was used to predict bearing capacity of geosynthetic reinforced sand bed [21]. The investigated model was verified against laboratory experimental results. In conclusion, the statistical analysis showed that the aperture size of the reinforcement has high impact on the bearing capacity and also the regression model displayed a reasonable predictive model.

k-nn model has been proved to outperform the traditional method in solving the regression problems (i.e., as a predictive model) [22–24]. This nonparametric supervised machine learning model is motivated due to its flexibility and simplicity to incorporate different data types. The main concept of k-nn model is to predict a particular number of observations in which the closest to the desired output through the *K* value (the theoretical concept is described in Sect. 3.1). k-nn is efficiently used for discrete and continuous of classification and regression, respectively. Worth mentioning advantages of k-nn are as follows: (1) an instance machine learning model, (2) robust with small error ratio, (3) the basic algorithm based on local learning and (4) the modeled prediction based on limited data set [25]. Based on the mentioned advantages, the current research aim is to inspect the efficiency of nonparametric soft computing approach in determining the bearing capacity in shallow foundation. The merit of applying this method is to avoid the manual calculation and time consumption, and to produce an accurate intelligent model.

2 Experimental work and data set

The applied data sets present varying conditions, and the obtained results are modeled using the proposed nonparametric soft computing approach. The main parameters used in performing the experimental data sets include soil properties and foundation dimensions. Formula (1) indicates Terzaghi's equation [3]. Terzaghi's formulation for calculating ultimate bearing capacity of soil below shallow strip footing foundation is expressed graphically in Fig. 1 and theoretically as follows:

$$q_u = c \cdot N_c + \gamma D N_q + 0.5 B \gamma N_\gamma \tag{1}$$

where c is the soil cohesion, γ is the unit weight of soil, D is the foundation depth, and B is the footing width, whereas N_c, N_q and N_γ are the bearing capacity factors that rely on the value of the (φ), which is the angle of internal friction of soil [26–28].

$$N_c = \left(N_q - 1\right) \cot \varphi \tag{2}$$

$$N_q = \mathrm{e}^{(\pi \tan \varphi)} \tan^2\left(45 + \frac{\varphi}{2}\right) \tag{3}$$

$$N_\gamma = 2\left(N_q - 1\right) \tan \varphi \tag{4}$$

In the light of the previous studies, the geometry and the physical parameters were used to formulate the manual procedure to compute the ultimate bearing capacity. Those parameters are c, γ, D and B, as reported by Foye et al. [29]. Hence, we used those parameters as inputs to construct the proposed intelligent model, and the target parameter is the *qu*.

3 Models overview

3.1 K-nearest neighbors (k-nn)

The k-nn is a nonparametric predictive model similar to artificial neural network, adaptive neuro-fuzzy inference system and genetic programing. k-nn model characterized by the advantage of exploits the closeness ("neighborhood") between the most recent observations and K similar sets of observations chosen for an adequately large amount

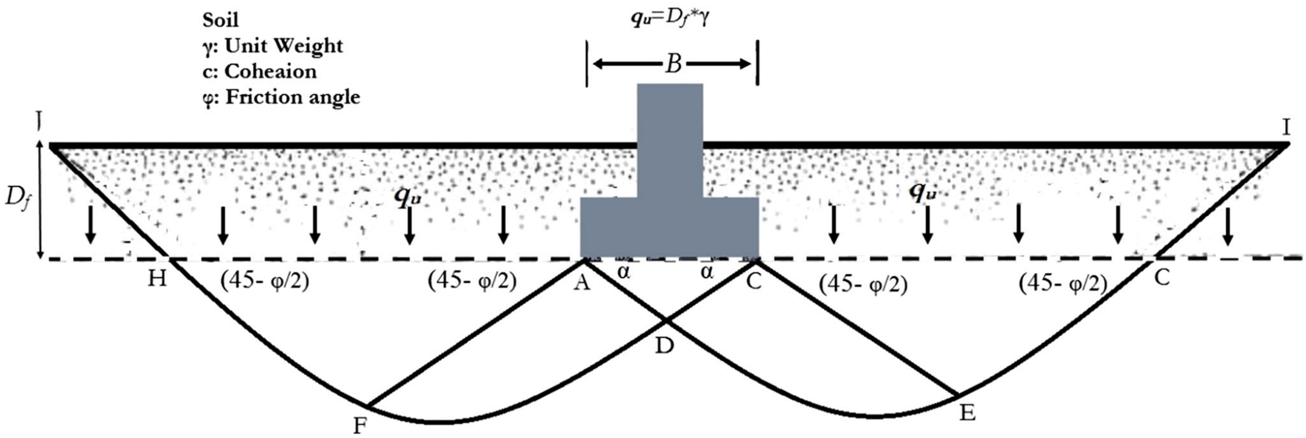

Fig. 1 Graphical presentation of the shear failure of shallow strip foundation footing

of data [30]. The applications of k-nn model have been explored and developed for several engineering and science problems for the purpose to solve the nonlinear, nonstationary, and dynamic features [31–36]. The implementation of this nonparametric method exhibited a robust predictive model that can mimic the human brain system and perform at high level of accuracies. In the current research, we investigate the capability of the classic k-nn model in order to predict a regression problem (i.e., ultimate bearing capacity).

Mathematically, the regression problem can be solved via formula (5) that can be expressed as follows:

$$y_i = \frac{1}{k}\sum_{j}^{k} y_j \tag{5}$$

k-nn model selects the k training data that are close to the testing phase data to predict the desired target variable. k value indicates the number of the nearest neighbors of y_j. The general formulation used to predict the testing phase data set is represented as follows:

$$y_i = \sum_{j=1}^{k} w_j y_j \tag{6}$$

where w_j is the weight of jth neighbor. This weight is adjustable in accordance to the number of the actual data set; it can be expressed by formula (7):

$$w_j = \frac{j}{n}. \tag{7}$$

The main concept behind the prediction processes using k-nn algorithm is to mimic the behavior of the bearing capacity that is highly nonlinear and dynamic due to the influence of several parameters such as the dimension of the footing foundation and the soil properties. The selection of the k number was defined from the simulated observations.

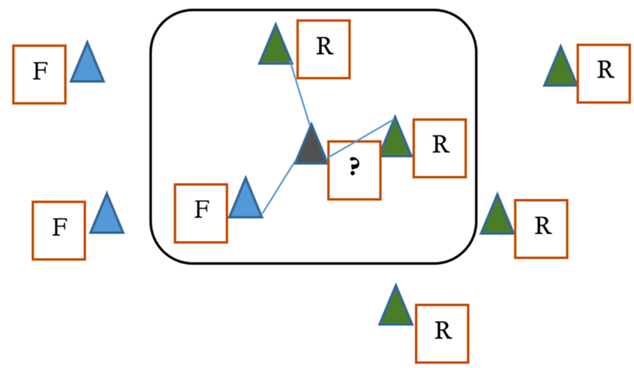

Fig. 2 Decision rule of the k-nn model, the unknown point ? is assigned to the nearest class based on the K search

3.2 Multiple linear regression (MLR)

In this research, MLR model was selected for the purpose of verification of the investigated nonparametric approach. In constructing a MLR model, the number of observation denoted by N (the number of data set) is used. In addition, the relationship between the inputs candidates (x_1, x_2, x_3,…, x_n) and the output variables (y_1, y_2, y_3,…, y_n) is examined and described as follows [37]:

$$y_{\mathrm{p}} = w_0 + w_1x_1 + w_2x_2 + \cdots + w_nx_n + e \tag{8}$$

where y_{p} is the predicted variable (qu in the current application), w_0 is the regression constant, $x_{1,2,\ldots n}$ are the input variables (the geometry dimensions and soil properties of the applied problem), and e is the error noise term.

3.3 Model development

The motivation of using k-nn model is owing to its features such as low computational cost and the simplicity of its implementation [38]. The mechanism of applying the k-nn algorithm model was based on the finding the closest candidates or the most similar that are needed for the prediction instead of using functional model to interpret the data set. The data set was divided into training and testing phases 75 and 25%, respectively. The testing phase data set consists of the records that are being predicted using k-nn model. Figure 2 displays the weighted euclidean distance that represents the closest distance between two components, where the nearest neighbors are selected using the euclidean distance.

3.4 Modeling evaluators

Several quantitative statistical indicators were used to evaluate and assess the prediction accuracies between the actual records and the prediction models output. The application and analysis (Sect. 4) discussed the best fit goodness and absolute error measurements indicators. Here, the best fit goodness indicators are non-dimensional metrics including coefficient of determination (r^2) and degree of agreement (d) that provide relative comparison between the models performances [39, 40], whereas the absolute error measurements including root-mean-square error (RMSE) and mean absolute percentage error (MAPE) are dimensional metrics and denote the unit of the ultimate bearing capacity [41, 42]. The mathematical definition of those indicators is expressed as follow:

$$r^2 = \frac{\sum_{t=1}^{n}\left[\left((y_a) - \overline{(y_a)}\right)\left((y_p) - \overline{(y_p)}\right)\right]}{\sqrt{\sum_{t=1}^{n}\left((y_a) - \overline{(y_a)}\right)^2 \sum_{t=1}^{n}\left((y_p) - \overline{(y_p)}\right)^2}} \quad (9)$$

$$d = 1 - \frac{\sum_{t=1}^{n}\left((y_a) - (y_p)\right)^2}{\sum_{t=1}^{n}\left(\left|(y_a) - (y_p)\right| + \left|(y_a) - (y_p)\right|\right)^2} \infty \le d \le 1 \quad (10)$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{t=1}^{n}\left((y_a) - (y_p)\right)^2} \quad (11)$$

$$\mathrm{MAPE} = \frac{1}{N}\sum_{t=1}^{n}\left|(y_a) - (y_p)\right|. \quad (12)$$

4 Application and analysis

In this section, a detailed analysis of the nonparametric (k-nn) model in comparison with multiple linear regression (MLR) model is discussed. The prediction of the ultimate bearing capacity of shallow foundation is shown in Fig. 3 for both models k-nn and MLR over the testing phase data set. Figure 3 shows that k-nn model performed the prediction for each single observation of *qu* more accurate and closer to the actual records than the MLR model.

The performance evaluation criteria measurements (r^2, d, RMSE and MAPE) for the testing phase are given in Table 1. The k-nn model showed a significant improvement in terms of all the skills indicators. As can be seen, there is remarkable enhancement in the utilization of k-nn model over MLR model. The RMSE and MAPE were improved by 24 and 17%, respectively. This improvement was noticed also in the non-dimensional indicators r^2 and d. The r^2 and d values obtained using MLR model were 0.93 and 0.96, whereas using k-nn model, 0.97 and 0.98 were obtained. It is necessary to state that the degree of agreement indicator was examined here due to the sensitivity to the outliers and the insensitivity to the proportional difference between actual and predicted values, respectively.

Figure 4 presents the scatter plots of k-nn and MLR models. The slope and the intercepts of regression equations can be considered as the relationship between the actual and predicted *qu* records. The slope designates the relative relationship between (x) and (y), while the intercept specifies the lag between prediction and observations. The slope of one and an intercept of zero define the best model. Here, the coefficient of determination indicates the proportion of the variance in observation data explained by the model [43]. Standard regression or correlation coefficient (r) is another indicator used frequently to measure the correlation between the measured and simulated values. The value of r can be determined by the square root of (r^2). Also, the correlation coefficient was used to estimate the degree of linear relationship between the two variables. The major observation can be discussed between Fig. 4a, b is that the diversion from the ideal perfect line using k-nn model is closer than MLR model. This clearly verified the applicability of k-nn model in capturing the high complexity relationship between the predictor candidates and the predictand.

Another significant diagnosis has been undertaken which is the distribution of the relative error (RE) that can be best expressed in the following formula:

$$\mathrm{RE} = \frac{(y_a) - (y_p)}{(y_a)} * 100 \quad (13)$$

where (y_a) and (y_p) indicated the actual and predicted records of *qu*.

Testing phase was inspected using this robust evaluator which is (RE). Figure 5 shows the RE for each single record of *qu* using MLR and k-nn models, respectively. The figure specified the limitation of the RE based on the over- and under-predicted values. In other words, the graphical presentation ranged between positive scale and negative scale. In general, there was a variance between both model performances. MLR model was specified the performing of the RE between (+15 and −17%); while k-nn model obtained maximum positive error 8% and minimum negative 16%. In more analytical manner, the prediction error of the k-nn model is limited between +5 and −5% for more than 85% of

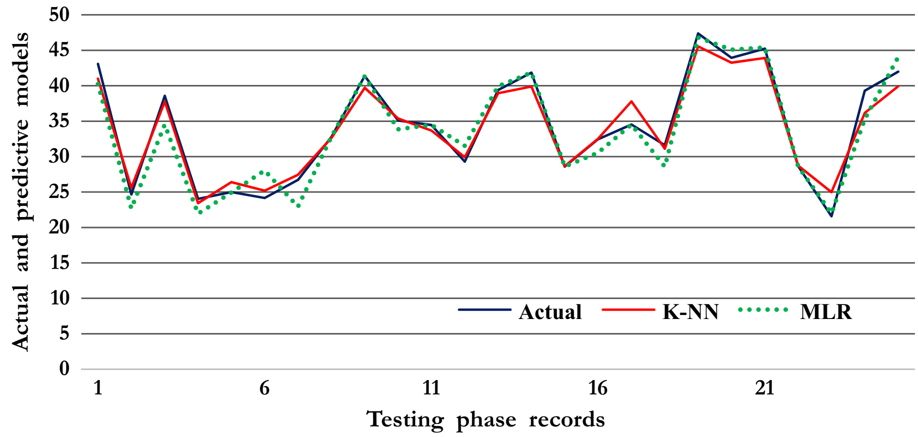

Fig. 3 Actual and predictive models (i.e., k-nn and MLR) for the ultimate bearing capacity over the testing phase (25%) of the whole data set

Table 1 Performance criteria for both models (MLR and k-nn) including coefficient of determination (r^2), degree of agreement (d), root-mean-square error (RMSE) and mean absolute percentage error (MAPE) over the testing phase data set

Performance indicators	r^2	d	RMSE	MAPE
MLR	0.93	0.96	2.03	1.44
k-nn	0.97	0.97	1.54	1.19

the whole data set, except a couple of error events exceeding this limitation. On the contrary, MLR model error distribution is more fluctuated between −10 and +10%.

From our prospective, it is an excellent idea to compare the current research results with the literature researches that have been conducted on the same application with different soft computing approaches. For instance, Padmini et al. [19] predicted bearing capacity of shallow foundation on cohesion-less soil using artificial neural network (ANN), adaptive neuro-fuzzy inference system (ANFIS) and pure fuzzy inference system (FIS) models. The obtained RE of the applied all three models was indicated +24 and −49% for ANN, +16 and −16% for ANFIS and +53 and −41% for FIS. Another research was conducted by [44]; the authors estimated the *qu* of non-cohesive soil using the application of ANN. The best architecture of neural network yielded highest score of coefficient of determination (r^2) 0.93. However, our current state of the art stated that the prediction model based on nonparametric approach outperformed the state-of-the-art researches in terms of the accuracies.

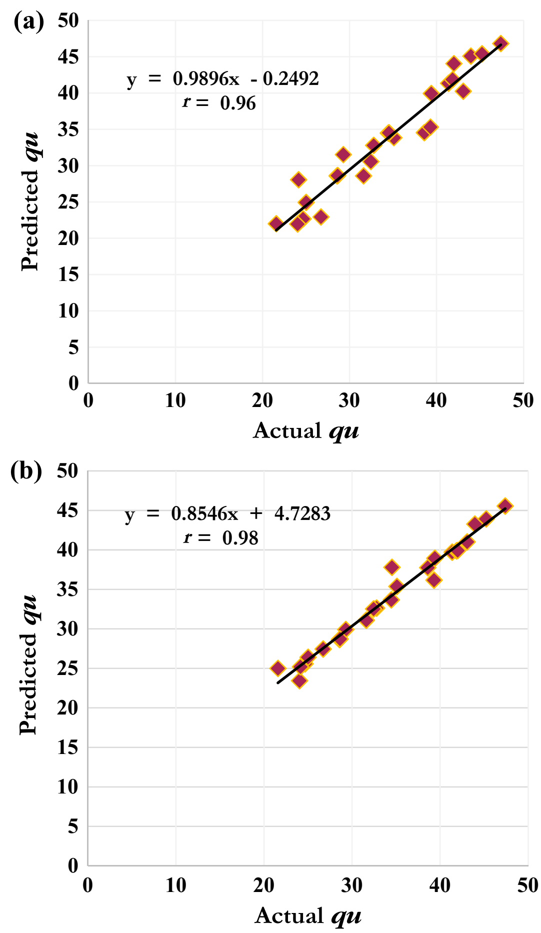

Fig. 4 Scatter plots for the predictive models belonging to the testing phase **a** MLR model and **b** k-nn model

5 Conclusions

In this research, the potential capability of nonparametric mathematical model called k-nearest neighbor was inspected to predict the ultimate bearing capacity (*qu*) of shallow foundation. The modeling was developed based on geometry parameters (i.e., foundation dimensions) and soil properties; on the other hand, the target parameter was *qu*. The motivation of this study is to explore an alternative intelligent model to predict *qu* with less complication, save execution time and produce high level of accuracy. k-nn was verified against common regression model that has been applied for the application by numerous scholars which is MLR. In general, the results of this study

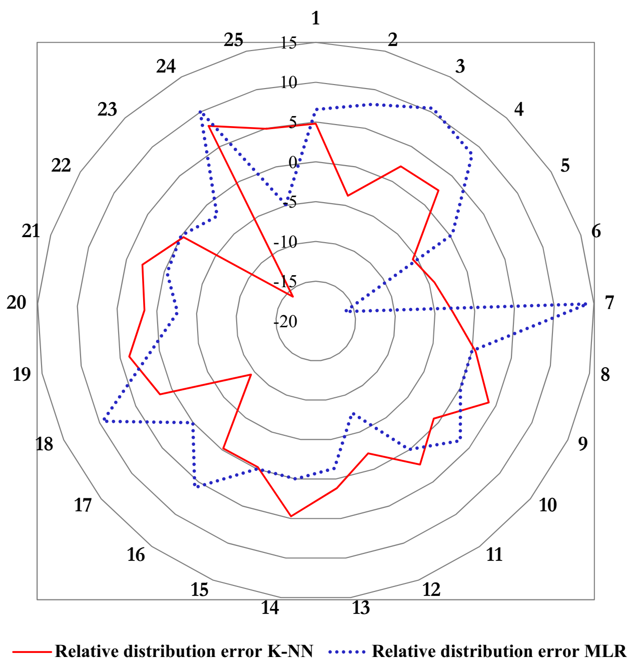

Fig. 5 Relative error distribution for both models over the testing phase

indicated that k-nn model provided reliable predictive model for the geotechnical engineering, particularly to predict ultimate bearing capacity of shallow foundation. In more numerical details, the augmentation of the absolute error measurements between k-nn and MLR models was 24 and 17% for RMSE and MAPE, respectively.

Compliance with ethical standards

Conflict of interest We declare that there is no conflict of interests regarding submission of this research.

References

1. Popescu ME (1984) Foundation analysis and design. Eng Geol 20:269. doi:10.1016/0013-7952(84)90010-3
2. Shahnazari H, Tutunchian MA (2012) Prediction of ultimate bearing capacity of shallow foundations on cohesionless soils: an evolutionary approach. KSCE J Civ Eng 16:950–957. doi:10.1007/s12205-012-1651-0
3. Terzaghi K (1943) Earth pressure and shearing resistance of plastic clay: a symposium: liner-plate tunnels on the Chicago (IL) subway. Trans Am Soc Civ Eng 108:100–970
4. Meyerhof GG (1963) Some recent research on the bearing capacity of foundations. Can Geotech J 1:16–26. doi:10.1139/t63-003
5. Hansen JB (1970) A revised and extended formula for bearing capacity. Danish Geotechnical Institute, Copenhagen, Denmark, Bulletin No. 28, pp 5–11
6. Kumbhojkar AS (1993) Numerical evaluation of Terzaghi's Nγ. J Geotech Eng 119:598–607. doi:10.1061/(ASCE)0733-9410(1993)119:3(598)
7. Yaseen ZM, El-Shafie A, Afan HA et al (2015) RBFNN versus FFNN for daily river flow forecasting at Johor River. Neural Comput Appl, Malaysia. doi:10.1007/s00521-015-1952-6
8. Elzwayie A, El-shafie A, Yaseen ZM et al (2016) RBFNN-based model for heavy metal prediction for different climatic and pollution conditions. Neural Comput Appl. doi:10.1007/s00521-015-2174-7
9. Goel N, Singh S, Aseri TC (2013) A comparative analysis of soft computing techniques for gene prediction. Anal Biochem 438:14–21. doi:10.1016/j.ab.2013.03.015
10. Tsakonas A, Dounias G, Shtovba S, Vivdyuk V (2002) Soft computing-based result prediction of football games. In: Proceedings of the First International Conference on Inductive Modelling (ICIM'2002). Lviv, Ukraine
11. Akhlaghinia MJ, Lotfi A, Langensiepen C (2007) Soft computing prediction techniques in ambient intelligence environments. IEEE Int Conf Fuzzy Syst. doi:10.1109/FUZZY.2007.4295608
12. Tsai HC, Tyan YY, Wu YW, Lin YH (2013) Determining ultimate bearing capacity of shallow foundations using a genetic programming system. Neural Comput Appl 23:2073–2084. doi:10.1007/s00521-012-1150-8
13. Jahed Armaghani D, Shoib RSNSBR, Faizi K, Rashid ASA (2015) Developing a hybrid PSO–ANN model for estimating the ultimate bearing capacity of rock-socketed piles. Neural Comput Appl. doi:10.1007/s00521-015-2072-z
14. Ornek M (2014) Estimation of ultimate loads of eccentric-inclined loaded strip footings rested on sandy soils. Neural Comput Appl 25:39–54
15. Cure E, Turker E, Uzuner BA (2014) Analytical and experimental study for ultimate loads of eccentrically loaded model strip footings near a sand slope. Ocean Eng 89:113–118. doi:10.1016/j.oceaneng.2014.07.018
16. Chen Q, Abu-Farsakh M (2015) Ultimate bearing capacity analysis of strip footings on reinforced soil foundation. Soils Found 55:74–85. doi:10.1016/j.sandf.2014.12.006
17. Soleimanbeigi A, Hataf N (2005) Predicting ultimate bearing capacity of shallow foundations on reinforced cohesionless soils using artificial neural networks. Geosynth Int 12:321–332. doi:10.1680/gein.2005.12.6.321
18. Kuo YL, Jaksa MB, Lyamin AV, Kaggwa WS (2009) ANN-based model for predicting the bearing capacity of strip footing on multi-layered cohesive soil. Comput Geotech 36:503–516. doi:10.1016/j.compgeo.2008.07.002
19. Padmini D, Ilamparuthi K, Sudheer K (2008) Ultimate bearing capacity prediction of shallow foundations on cohesionless soils using neurofuzzy models. Comput Geotech 35:33–46
20. Ornek M, Laman M, Demir A, Yildiz A (2012) Prediction of bearing capacity of circular footings on soft clay stabilized with granular soil. Soils Found 52:69–80. doi:10.1016/j.sandf.2012.01.002
21. Latha GM, Somwanshi A, Reddy KH (2013) A multiple regression equation for prediction of bearing capacity of geosynthetic reinforced sand beds. Indian Geotech J 43:331–343. doi:10.1007/s40098-013-0053-7
22. Eronen AJ, Klapuri AP (2010) Music tempo estimation with k-NN regression. IEEE Trans Audio Speech Lang Process 18:50–57. doi:10.1109/TASL.2009.2023165
23. Kpotufe S (2011) *k*-NN regression adapts to local intrinsic dimension. In: Neural information processing systems 2011, Granada, Spain
24. Cheng CB, Lee ES (1999) Nonparametric fuzzy regression—k-NN and kernel smoothing techniques. Comput Math Appl 38:239–251. doi:10.1016/S0898-1221(99)00198-4
25. Cover T, Hart P (1967) Nearest neighbor pattern classification. IEEE Trans Inf Theory 13:21–27. doi:10.1109/TIT.1967.1053964
26. Lee J, Salgado R (2005) Estimation of bearing capacity of circular footings on sands based on cone penetration test. J Geotech Geoenviron Eng 131(4):442–452
27. Caquot A, Kerisel J (1953) Sur le terme de surface dans le calcul des fondations en milieu pulverulent. In: Proceedings of the third

international conference on soil mechanics and foundation engineering, pp 336–337

28. Chummar AV (1972) Bearing capacity theory from experimental results. J Geotech Geoenviron Eng Div, ASCE 98(12):1257–1276
29. Foye KC, Salgado R, Scott B (2006) Assessment of variable uncertainties for reliability-based design of foundations. J Geotech Geoenviron Eng 132:1197–1207. doi:10.1061/(ASCE)1090-0241(2006)132:9(1197)
30. Fukunage K, Narendra PM (1975) A branch and bound algorithm for computing k-nearest neighbors. IEEE Trans Comput 100(7):750–753
31. Horton P, Nakai K (1997) Better prediction of protein cellular localization sites with the k nearest neighbors classifier. Proc Int Conf Intell Syst Mol Biol 5:147–152
32. Li H, Sun J, Sun BL (2009) Financial distress prediction based on OR-CBR in the principle of k-nearest neighbors. Expert Syst Appl 36:643–659. doi:10.1016/j.eswa.2007.09.038
33. Lee BH, Scholz M (2006) A comparative study: prediction of constructed treatment wetland performance with K-nearest neighbors and neural networks. Water Air Soil Pollut 174:279–301. doi:10.1007/s11270-006-9113-2
34. Gangopadhyay S, Clark M, Rajagopalan B (2005) Statistical downscaling using K-nearest neighbors. Water Resour Res 41:1–23. doi:10.1029/2004WR003444
35. Franco-Lopez H, Ek AR, Bauer ME (2001) Estimation and mapping of forest stand density, volume, and cover type using the k-nearest neighbors method. Remote Sens Environ 77:251–274. doi:10.1016/S0034-4257(01)00209-7
36. Maltamo M, Kangas A (1998) Methods based on k-nearest neighbor regression in the prediction of basal area diameter distribution. Can J For Res 28:1107–1115. doi:10.1139/x98-085
37. Preacher KJ, Curran PJ, Bauer DJ (2006) Computational tools for probing interactions in multiple linear regression, multilevel modeling, and latent curve analysis. J Educ Behav Stat 31:437–448. doi:10.3102/10769986031004437
38. Alkhatib K, Najadat H, Hmeidi I, Shatnawi MKA (2013) Stock price prediction using K-nearest neighbor (k NN) algorithm. Int J Bus Humanit Technol 3(3):32–44
39. Legates DR, McCabe GJ (1999) Evaluating the use of "goodness-of-fit" measures in hydrologic and hydroclimatic model validation. Water Resour Res 35(1):233–241
40. Willmott CJ, Robeson SM, Matsuura K (2012) A refined index of model performance. Int J Climatol 32:2088–2094. doi:10.1002/joc.2419
41. Willmott CJ, Matsuura K (2005) Advantages of the mean absolute error (MAE) over the root mean square error (RMSE) in assessing average model performance. Clim Res 30:79–82. doi:10.3354/cr030079
42. Chai T, Draxler RR (2014) Root mean square error (RMSE) or mean absolute error (MAE)? Arguments against avoiding RMSE in the literature. Geosci Model Dev 7:1247–1250. doi:10.5194/gmd-7-1247-2014
43. Moriasi DN, Arnold JG, Van Liew MW et al (2007) Model evaluation guidelines for systematic quantification of accuracy in watershed simulations. Trans ASABE 50:885–900. doi:10.13031/2013.23153
44. Gupta R, Goyal K, Yadav N (2016) Prediction of safe bearing capacity of noncohesive soil in arid zone using artificial neural networks. Int J Geomech 16:4015044. doi:10.1061/(ASCE)GM.1943-5622.0000514