Tunnelling and Underground Space Technology 78 (2018) 146–158

Contents lists available at ScienceDirect

Tunnelling and Underground Space Technology

journal homepage: www.elsevier.com/locate/tust

Simplified solution for tunnel-soil-pile interaction in Pasternak's foundation model

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ARTICLE INFO

Keywords:
Tunnel-soil-pile interaction
Ground shearing displacements
Effects of lateral soil displacements
Influencing factors
Simplified solution

ABSTRACT

The current design practice for predicting the interaction mechanics for tunnel-soil-pile is generally based on Winkler's foundation, which is subject to some important limitation, such as ignoring the continuity of the soil foundation. Furthermore, the current analytical studies are mostly employed the plane strain analyses and do not consider the influences of lateral soil displacements on pile behaviour. To improve the accuracy for the pile behaviour prediction induced by tunnelling, the analytical method should account for the effects of a number of parameters, such as the ground shearing displacements, and the influence of lateral soil displacements next to the pile. This paper focuses on a simplified solution based on Pasternak's foundation model to predict the lateral displacements and internal forces of a single-pile and group-piles induced by tunnelling considering the effects of lateral soil displacements. First, the simplified solution of tunnel-soil-pile interaction, which reflects the influence of shearing displacements of foundation, is established on Pasternak's foundation model. Second, the equivalent concentrated forces are supplied to the pile through the shear layer to consider the influence of lateral soils beside the pile. The validity of the solutions is verified by the boundary element program results, centrifuge test data, and field measurements. The calculated results are also compared with and without considering the effects of tunnel-soil-pile interaction. When the influences of lateral soil displacements are considered, the results are shown to be closer to the monitored in-situ data and the centrifuge test data. In addition, the influencing factors of a single-pile and group-piles displacements are also investigated, including the shear layer modulus, pile diameter, ground-loss ratio, pile-tunnel distance, and pile spacing. The influence of soil shear displacements on pile response cannot be ignored, and an error may occur when Winkler's foundation model is used to solve this problem.

1. Introduction

The rapidly increasing demand for public transport construction in congested urban areas will promote tunnel excavation adjacent to existing buildings and services due to the lack of surface space. Adverse effects on nearby pile foundations may be appeared due to changes in ground stress and, hence, building movements. Accurate predicting of effects of tunnelling on pile foundations poses a major challenge during civil engineering design and construction.

Increasing attention has been paid to evaluating the effects of shield tunnelling on adjacent piles. The methods used for analyzing this problem may be broadly classified into three categories: numerical analyses, simplified analytical methods and laboratory tests. A variety of research has been conducted on this subject based on the numerical approaches (Surjadinata et al., 2006; Jongpradist et al., 2013; Hong et al., 2015; Fu et al., 2016). The most common method is finite element (FE) method and the simulation results are obtained with the condition on the tunnel, pile and soil as a whole. Several researchers investigated the effects of tunnelling on the bearing capacity and deflection of the piles by the centrifuge model tests (Loganathan et al., 2000; Jacobsz et al., 2004; Lee and Chiang, 2007; Marshall and Mair, 2011; Ng and Lu, 2013; Ng et al., 2013, 2014; Franza and Marshall, 2018). In addition, some researchers performed experimental tests in laboratory to study the effects of tunnelling on pile foundations (Lee and Bassett, 2007; Meguid and Mattar, 2009; Bel et al., 2016).

Researchers have been studying different analytical approaches to predict the pile responses during tunnel excavation (Chen et al., 1999; Huang et al., 2009; Mu et al., 2012; Xiang and Feng, 2013; Basile, 2014;

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https://doi.org/10.1016/j.tust.2018.04.025
Received 18 January 2017; Received in revised form 10 April 2018; Accepted 11 April 2018

Marshall, 2012; Marshall and Haji, 2015; Franza et al., 2017). In order to obtain a better mechanical understanding of the effects of tunnelling on adjacent piles and provide a rapid predication of the response characteristics of existing structures, a simplified two-stage approach is presented in their study. The simplified method to analyze such a problem is carried out in two steps: first, the estimation of green-field ground movements induced by tunnelling, which would occur if the existing piles were not present; second, the calculation of the response of the existing piles to green-field ground movements.

Recent studies (Huang et al., 2009; Mu et al., 2012) have investigated the effects of tunnelling on existing piles and evaluated the complex pile-soil interaction, which usually relies on Winkler's foundation model. According to Winkler's foundation model, the soil is modelled as a series of closely spaced, mutually independent, linear elastic lateral springs, which provide resistance in direct proportion to the deflection of the pile. In Winkler's foundation model, the soil properties are described only by the sub-ground parameters, which represent the stiffness of the lateral springs. However, due to its inability to take into account the soil continuity or cohesion, Winkler's foundation model is considered a rather crude approximation of the mechanical behaviour of soil material. Therefore, it cannot always give accurate predictions. The model assumes that an applied load is transmitted to each individual spring without any interaction between other springs, also resulting in the influence of the soil on either side of the pile being overlooked. For a uniformly distributed load applied to a homogenous pile, the behaviour of the elastic pile corresponds to that of a rigid body without the existence of the bending moment and shear force inside it. In reality, however, piles may not be perfectly rigid and can show curvilinear displacement profiles, as seen in Fig. 1. To overcome this weakness, the two-parameter elastic foundation models have been suggested, such as Pasternak's foundation model (Pasternak, 1954; Tanahashi, 2008). In this model, a shear layer is added to the Winkler's model to provide mechanical interaction among spring elements. The first parameter represents the stiffness of the springs, as in Winkler's foundation model. The second parameter is introduced to account for the coupling effect of the linear elastic springs. Lee et al. (2004) analyzed the retaining wall movements caused by ground excavations based on Pasternak's two-parameter beam-column model. Zhang and Zhang (2013) estimated the longitudinal deflection and internal forces of existing pipelines due to tunnelling using Kerr's three-parameter elastic foundation model (Kerr, 1965). However, the tunnel-pile interaction analyses are generally based on Winkler's foundation model and further studies must be conducted based on Pasternak's foundation model.

The simplified analytical methods available in the literature have mostly focused on the analyses of passive piles subjected to tunnelling without considering the effects of lateral soil displacements. These methods may not be accurate in evaluating the behaviour of piles subjected to concurrently tunnel excavation and lateral soil

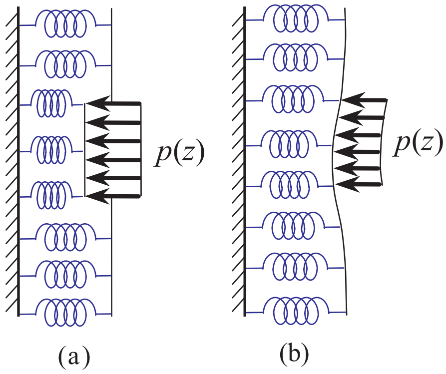

Fig. 1. Foundations under uniformly distributed loads: (a) Winkler's foundation model; (b) actual displacement of foundations.

displacements. In fact, the interaction between tunnel and pile is a three-dimensional effect issue. The lateral displacements of soil will also have an impact on the pile deflection. Current analytical studies are mostly based on the plane strain analysis and ignore the effects of lateral soil displacements on pile behaviour. Fig. 2 shows three tunnelling conditions of short-range excavated (Fig. 2(a)), medium-range excavated (Fig. 2(b)) and fully excavated (Fig. 2(c)), which mean the whole excavation process. The condition selected in this paper is the worst operating condition, that is, the third condition (Fig. 2(c)). As shown in Fig. 2(c), surface A in this figure shows the ground displacements without piles. Due to the limitations of the pile, the soil displacement in a piled foundation is slightly smaller than it is without the pile. Surface C shows the soil displacements in a piled foundation without considering the effect of lateral soil displacements, which is a plane-strain case. In fact, the existence of the pile constrains the lateral soil displacements within a certain range, whereas the displacements value of the lateral soil far from the pile is approximately equal to free soil displacements. The actual soil displacements in the piled foundation is shown as surface B in Fig. 2(c), and the calculation error may occur in the current plane strain analyses.

In this paper, a simplified method based on Pasternak's two-parameter foundation model is presented for estimating the lateral deflection and internal forces of existing piles. It is interesting that the proposed method can consider the shearing displacements of foundation and the influence of lateral soil displacements beside the pile. A parametric analysis is performed to discuss the influence of various factors on the deflection and bending moment of single and group-piles, and the reliability of the proposed method is demonstrated by comparing predictions with the boundary element program results, centrifuge test data, and field measurements.

2. Pile response without considering the influences of lateral soils

2.1. Analysis of single-pile response

The two-parameter foundation model for tunnel-soil-pile interaction is deduced using finite difference method. As shown in Fig. 3, in Pasternak's foundation model, a shear layer is added to Winkler's foundation model to consider interaction among spring elements. The basic assumptions of the model are as follows: (a) in the longitudinal direction, the pile is considered as a rectangular beam, with width of D and stiffness of EI; (b) the shear force can be transferred between springs, and the shear layer produces only shear displacement (x-direction); (c) the pile connects closely with surrounding soil and pile displacements is equal to displacements at the pile-soil contacting surface; and (d) there exists the friction in x-direction only, and the lateral friction between foundation and pile is not considered.

When the effects of lateral soil displacements beside the pile are not considered, the equilibrium equation of a single-pile is established as follows:

$$EI\frac{d^4w}{dz^4}-GD\frac{d^2w}{dz^2}+kDw=pD \tag{1}$$

where w is the lateral displacement of pile; p is the additional load on the pile; D and EI are equivalent width and bending stiffness of the pile, respectively; and k is the stiffness of the springs (Vesic,1961) and G is the stiffness the shear layer, and are calculated as follows (Tanahashi, 2008):

$$k=\frac{0.65}{D}\left(\frac{E_sD^4}{EI}\right)^{\frac{1}{12}}\frac{E_s}{1-\nu_s{}^2}\text{ and} \tag{2}$$

$$G=\frac{E_st}{6(1+\nu_s)} \tag{3}$$

where E_s and ν_s are the elastic modulus and Poisson ratio of soils, respectively, and t is the thickness of the shear layer and is related to the

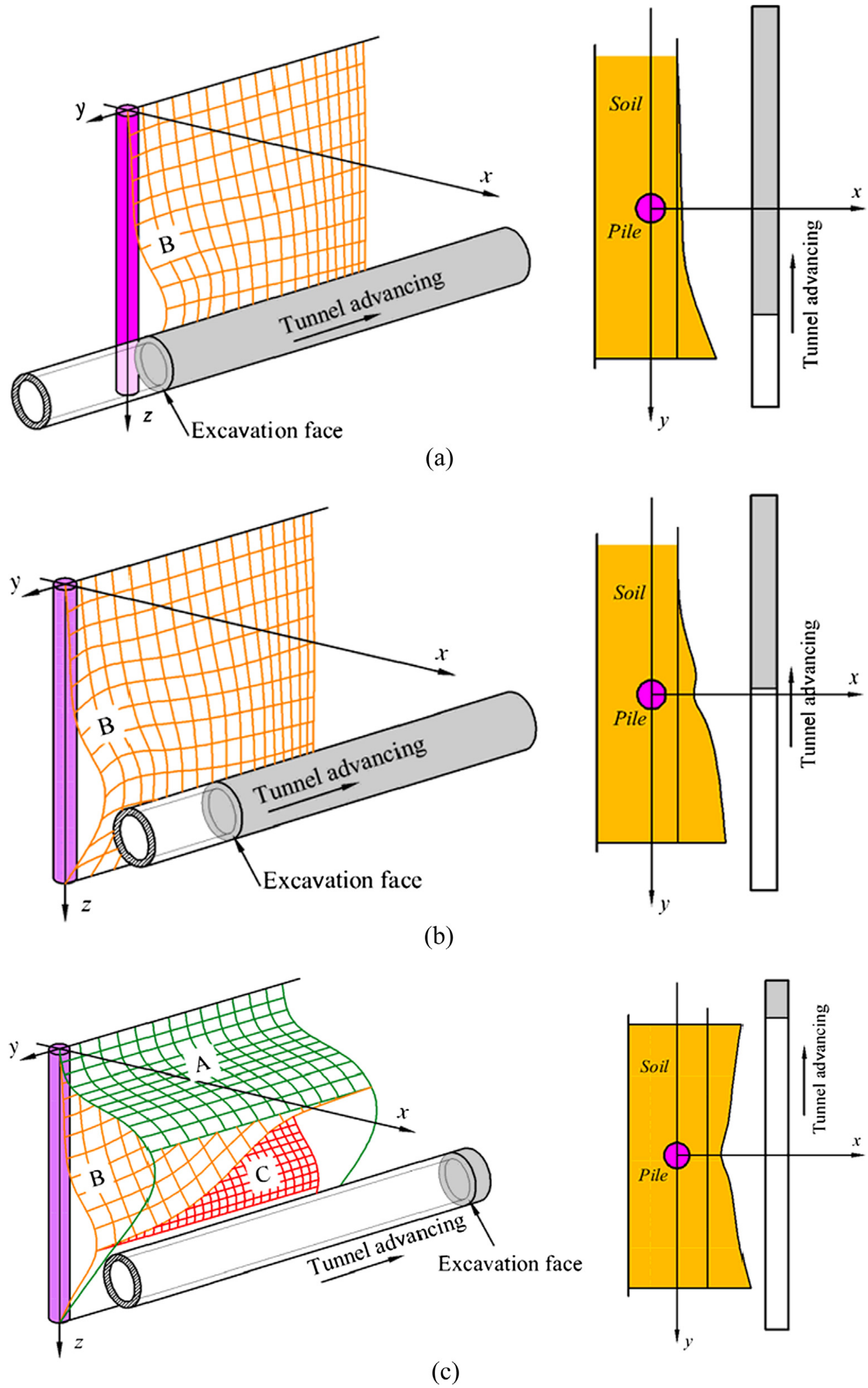

Fig. 2. Schematic view for effects of tunnelling on nearby piles: (a) short-range excavated; (b) medium-range excavated; (c) fully excavated.

soil characteristics. The variable p is related to the space variables z, and the variable w is obtained according to p, which leads to the dependence of variable w and p on the space variables z.

Eq. (1) can be solved in general by numerical methods, e.g., the finite difference method or the finite element method. Here, a simple finite difference procedure is followed. Assuming that the pile is divided into n segments, the ordinary differential equation can be converted into a difference equation:

$$\alpha w_{i-2} + \beta w_{i-1} + \gamma w_i + \beta w_{i+1} + \alpha w_{i+2} = p_i \tag{4}$$

in which $i = 0, 1, 2, 3, \cdots, n$ and α, β and γ are expressed as follows:

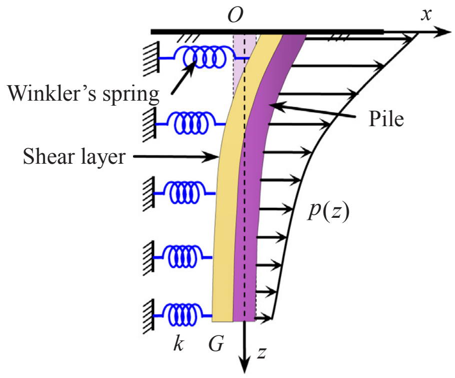

Fig. 3. Pile-soil interaction model in Pasternak's foundation.

$$\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ -4 & -1 & 0\\ 6 & 2 & 1\end{bmatrix}\cdot\left\{\frac{EI}{Dh^4}\quad \frac{G}{h^2}\quad k\right\}^T \tag{5}$$

in which h is the length of each segment, i.e., $h = L/n$ with L being the length of the whole pile, and in Eq. (4), p_i is the additional load on the pile induced by tunnelling, a load that can be expressed as follows:

$$p(z)=kS_x(z)-GS_x''(z) \tag{6a}$$

$$p_i=k(S_x)_i-G\frac{(S_x)_{i+1}-2(S_x)_i+(S_x)_{i-1}}{h^2} \tag{6b}$$

in which S_x is the free-field soil movement at the pile location due to tunnelling. According to Loganathan and Poulos (1998), $(S_x)_i$ is given by the following equation:

$$(S_x)_i=\varepsilon R^2 x_0\left[\frac{1}{x_0{}^2+(H-ih)^2}+\frac{3-4\nu_s}{x_0{}^2+(H+ih)^2}-\frac{4ih(ih+H)}{(x_0{}^2+(H+ih)^2)^2}\right]\cdot e^{-\frac{1.38x_0{}^2}{(H+R)^2}-\frac{0.69(ih)^2}{H^2}} \tag{7}$$

where R is the radius of the tunnel, H is the buried depth of the tunnel axis, x_0 is the offset from the tunnel axis and ε is the equivalent ground-loss ratio.

The rotation angle θ, bending moment M and shear force Q of the pile can be expressed as follows:

$$\theta_i=\left(\frac{\mathrm{d}w}{\mathrm{d}z}\right)_i=\frac{1}{2h}(w_{i-1}-w_{i+1}) \tag{8a}$$

$$M_i=EI\left(\frac{\mathrm{d}^2w}{\mathrm{d}z^2}\right)_i=\frac{EI}{h^2}(w_{i-1}-2w_i+w_{i+1})\ \text{and} \tag{8b}$$

$$Q_i=EI\left(\frac{\mathrm{d}^3w}{\mathrm{d}z^3}\right)_i=\frac{EI}{2h^3}(w_{i-2}-2w_{i-1}+2w_{i+1}-w_{i+2}) \tag{8c}$$

When the pile is unconstrained at the top and tip, the boundary conditions are as follows:

$$w_{-1}-2w_0+w_1=0 \tag{9a}$$

$$w_{-2}-2w_{-1}+2w_1-w_2=0 \tag{9b}$$

$$w_{n-3}-2w_{n-2}+2w_n-w_{n+1}=0\ \text{and} \tag{9c}$$

$$w_{n-1}-2w_n+w_{n+1}=0 \tag{9d}$$

Combining Eqs. Eqs. (4) and (9) and assembling discretized equations for each node leads to a finite difference formulation for the whole pile in matrix-vector form:

$$\{W\}_{(n+1)\times 1}=\{K\}^{-1}_{(n+1)\times(n+1)}\cdot\{P\}_{(n+1)\times 1} \tag{10}$$

in which $\{W\}$ and $\{P\}$ are the vector of pile lateral displacement and the additional loading vector in the horizontal direction, respectively, (i.e. $\{W\}=\{w_0, w_1, w_2 \cdots w_{n-1}, w_n\}^T$, $\{P\}=\{p_0, p_1, p_2 \cdots p_{n-1}, p_n\}^T$) and $\{K\}$ is the vertical stiffness matrix of the pile:

$$\{K\}=\begin{bmatrix}\gamma+2\beta+4\alpha & -4\alpha & 2\alpha & & & & \\ \beta+2\alpha & \gamma-\alpha & \beta & \alpha & & & \\ \alpha & \beta & \gamma & \beta & \alpha & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & \alpha & \beta & \gamma & \beta & \alpha \\ & & & \alpha & \beta & \gamma-\alpha & \beta+2\alpha \\ & & & & 2\alpha & -4\alpha & \gamma+2\beta+4\alpha\end{bmatrix} \tag{11}$$

Similarly, when the pile is unconstrained at the top and fixed at the tip, the lateral stiffness matrix can be expressed as follows:

$$\{K\}=\begin{bmatrix}\gamma+2\beta+4\alpha & -4\alpha & 2\alpha & & & & \\ \beta+2\alpha & \gamma-\alpha & \beta & \alpha & & & \\ \alpha & \beta & \gamma & \beta & \alpha & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & \alpha & \beta & \gamma & \beta & \alpha \\ & & & \alpha & \beta & \gamma & \beta \\ & & & & \alpha & \beta & \gamma+\alpha\end{bmatrix} \tag{12}$$

When the pile is fixed at the top and tip, the lateral stiffness matrix can be expressed as follows:

$$\{K\}=\begin{bmatrix}\gamma+\alpha & \beta & \alpha & & & & \\ \beta & \gamma & \beta & \alpha & & & \\ \alpha & \beta & \gamma & \beta & \alpha & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & \alpha & \beta & \gamma & \beta & \alpha \\ & & & \alpha & \beta & \gamma & \beta \\ & & & & \alpha & \beta & \gamma+\alpha\end{bmatrix} \tag{13}$$

Once the lateral displacement of the pile is obtained, the rotation angle, bending moment and shear force of the pile can also be obtained according to Eqs. (8a)–(8c).

2.2. Analysis of group-piles response

In general, the piles do not exactly follow the free-field movements generated by tunnelling at the pile location and the soil movement surrounding the pile may be altered due to the presence of piles. Mroueh and Shahrour (2002) found that the lateral soil movement induced by tunnelling decreased due to the existence of piles, especially at the level of the tunnelling centre axis. Such a phenomenon is often considered as shielding effect; i.e., the real displacement of the pile is equal to the free-field soil movement with the addition of a shielding movement, in which the shielding movement is opposite to the free-field soil movement.

The problem of the interaction between two piles subjected to soil movement due to tunnelling is depicted in Fig. 4, where two piles with pile spacing s are adjacent to a tunnel. The horizontal distances between two piles and the tunnel axis are x_1 and x_2, respectively.

Without the presence of pile 1, it is assumed that the free-field lateral soil movement caused by tunnelling at the position of pile 1 is $S_{x1}(z)$ and the lateral deflection of pile 1 due to tunnelling is δ_{11} (this also considers the soil movement around pile 1 owing to the displacement compatibility). The shielding displacement due to the shielding effect of pile 1 can then be expressed as follows:

$$\Delta\delta_1(z)=\delta_{11}(z)-S_{x1}(z) \tag{14}$$

At the location of pile 2, if the lateral stiffness of pile 2 is neglected, the lateral deflection of pile 2 will follow the soil movement exactly due to the shielding effect of pile 1, which is as follows:

$$S_{x21}(z)=\lambda(s,z)\cdot\Delta\delta_1(z)=\lambda(s,z)[\delta_{11}(z)-S_{x1}(z)] \tag{15}$$

where s represents the spacing between pile 1 and pile 2; $\lambda(s,z)$ represents the lateral displacement influence factor between two piles

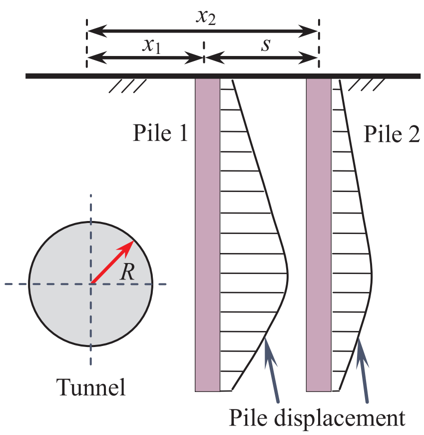

Fig. 4. Computing model for group-piles adjacent to tunnelling.

based on Mindlin's solution as follows (Huang et al., 2009):

$$\lambda(s,z) = \frac{S_{x2}(z)}{S_{x1}(z)} \tag{16}$$

where $S_{x1}(z)$ and $S_{x2}(z)$ are the free-field lateral soil movements caused by tunnelling at the positions of piles 1 and 2, respectively. Thus, the lateral controlling equation of pile 2 considering the shielding effect of pile 1 can be expressed as follows:

$$\frac{EI}{D}\frac{d^4\delta_{21}}{dz^4} - \frac{G}{D}\frac{d^2\delta_{21}}{dz^2} + k\delta_{21} = kS_{x21} - G\frac{d^2S_{x21}}{dz^2} \tag{17}$$

where δ_{21} is the lateral displacement of pile 2 caused by tunnelling considering the shielding effect of pile 1.

Similarly, for a particular pile i, the lateral pile deflection caused by tunnelling can be obtained according to Eq. (10). The lateral shielding displacement due to the shielding effect of pile j is obtained by the following:

$$\frac{EI}{D}\frac{d^4\delta_{ij}}{dz^4} - \frac{G}{D}\frac{d^2\delta_{ij}}{dz^2} + k\delta_{ij} = kS_{xij} - G\frac{d^2S_{xij}}{dz^2} \tag{18}$$

where δ_{ij} represents the lateral shielding displacement of pile i due to the shielding effect of pile j, and S_{xij} represents the lateral soil shielding movement at the position of pile i due to the shielding effect of pile j.

The lateral shielding displacement of pile i at any depth z can be expressed as follows:

$$\delta_i(z) = \sum_{j=1}^{n} \delta_{ij} \tag{19}$$

The rotation angle, bending moment and shear force of the pile can also be obtained according to Eqs. (8a)–(8c).

3. Pile response considering the influences of lateral soils

3.1. Analyses of single-pile response

Undoutly, the effects of tunnelling on a pile are a three-dimensional problem. The additional stresses induced by tunnelling are imposed not only on a pile, but also on the lateral soils around the pile. For the interaction mechanics, the lateral soils will affect the pile displacements inconsistently. Therefore, the pile response should be investigated by considering the effects of lateral soil displacements.

Considering the effects of lateral soil displacements in Pasternak's foundation (shown as Fig. 5), the basic assumptions are as follows: (a) the parameters of the soil around the pile will not vary with the tunnel excavation; (b) the lateral forces on the pile are delivered by the shear layer through the forces T_1 and T_2 on both sides of the pile, as shown in Fig. 5(a) and (b); (c) the pile deforms consistently with the shear layer,

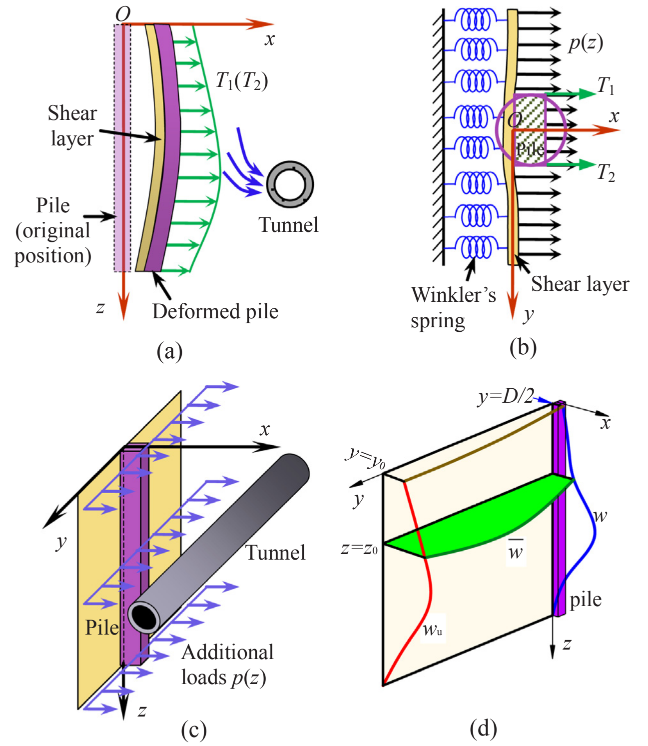

Fig. 5. Effects of lateral soils on piles: (a) deformed piles induced by lateral forces; (b) forces on piles delivered by shear layer; (c) additional loads applied on piles and lateral soils; (d) deformation of shear layer beside piles.

and no slip occurs at the interface of the pile and shear layer, i.e., the displacements at the interface of pile and shear layer is equal to the pile displacements, as shown in Fig. 5(a); and (d) the additional load $p(z)$ is imposed on the pile and lateral soils at the same time in y direction, assuming that the influence range is sufficiently wide, as shown in Fig. 5(b) and (c).

In this study, the problem is simplified to two plane analyses in different directions, as shown in Fig. 5(d). For a certain plane $z = z_0$, the balance equation is as follows:

$$p(z)|_{z=z_0} = -G\frac{d^2\bar{w}}{dy^2} + k\bar{w} \tag{20}$$

where $\bar{w}$ represents the lateral displacements of the shear layer in this plane ($z = z_0$).

For $y \geq D/2$, the general solution of Eq. (20) is as follows:

$$\bar{w} = C_1 e^{-\sqrt{k/G}(y-D/2)} \tag{21}$$

where C_1 is a coefficient that can be determined by the boundary condition.

As shown in Fig. 5(d), the pile axis corresponds to $y = 0$. Assume that the plane $y = y_0$ is sufficiently far from the pile, the displacements of the shear layer at this plane is w_u, and w_u is a particular solution of Eq. (20). The solution of Eq. (20) can be expressed as follows:

$$\bar{w} = w_u + C_1 e^{-\sqrt{k/G}(y-D/2)} \tag{22}$$

According to the boundary condition, the lateral displacement of shear layer $\bar{w}$ equals the pile displacement w when $y = D/2$, that is, $C_1 = w - w_u$. Then, Eq. (22) can be written as follows:

$$\bar{w} = w_u + (w - w_u)e^{-\sqrt{k/G}(y-D/2)} \tag{24}$$

For the plane $z = z_0$, the force of lateral shear layer imposed on pile is as follows:

$$T_1 = T_2 = G\frac{d\bar{w}}{dy}\bigg|_{y=D/2} = (w_u - w)\sqrt{Gk} \tag{25}$$

The distribution of the load $T_1(T_2)$ on the pile is shown in Fig. 5(a).

As shown in Fig. 5(c) and (d), in the plane $y = y_0$, the load $p(z)$ is imposed on the soil directly. The balance equation of lateral shear layer is as follows:

$$p(z) = -G\frac{d^2 w_u}{dz^2} + kw_u \tag{26}$$

The balance equation of pile can be expressed as follows:

$$EI\frac{d^4 w}{dz^4} + q(z)D = 2T_1 \tag{27}$$

where $q(z)$ is the stress of pile, which can be expressed as follows:

$$q(z) = -G\frac{d^2 w}{dz^2} + kw \tag{28}$$

and by introducing this and Eq. (25) into Eq. (27), the following relation is obtained:

$$\frac{EI}{2\sqrt{Gk}}\frac{d^4 w}{dz^4} - \frac{GD}{2\sqrt{Gk}}\frac{d^2 w}{dz^2} + \left(\frac{Dk}{2\sqrt{Gk}} + 1\right)w = w_u \tag{29}$$

where w_u can be solved through Eq. (26).

The finite difference formulation of Eq. (26) can be written as follows:

$$p_i = -\frac{G}{h^2}[(w_u)_{i+1} - 2(w_u)_i + (w_u)_{i-1}] + k(w_u)_i \tag{30}$$

When the pile is unconstrained at the top and tip, the boundary conditions are as follows:

$$-\frac{G}{h^2}(w_u)_{i+1} + \left(\frac{2G}{h^2} + k\right)(w_u)_i - \frac{G}{h^2}(w_u)_{i-1} = p_i \tag{31a}$$

$$(w_u)_{n-1} - 2(w_u)_n + (w_u)_{n+1} = 0 \text{ and} \tag{31b}$$

$$(w_u)_{-1} - 2(w_u)_0 + (w_u)_1 = 0 \tag{31c}$$

Again, assembling discretized equation for each node and combining with boundary conditions lead to a finite difference formulation for the shear layer in matrix-vector form as follows:

$$\begin{bmatrix} Q+2J & 0 & & & \\ J & Q & J & & \\ & \ddots & \ddots & \ddots & \\ & & J & Q & J \\ & & & 0 & Q+2J \end{bmatrix} \cdot \begin{bmatrix} (w_u)_0 \\ \vdots \\ (w_u)_n \end{bmatrix} = \begin{bmatrix} p_0 \\ \vdots \\ p_n \end{bmatrix} \tag{32}$$

where $J = -G/h^2$, $Q = 2G/h^2 + k$.

Similarly, when the pile is unconstrained at the top and fixed at the tip, the lateral stiffness matrix can be expressed as follows:

$$\begin{bmatrix} Q+2J & 0 & & & & \\ J & Q & J & & & \\ & \ddots & \ddots & \ddots & & \\ & & J & Q & J & \\ & & & J & Q & J \\ & & & & J & Q \end{bmatrix} \cdot \begin{bmatrix} (w_u)_0 \\ \vdots \\ (w_u)_n \end{bmatrix} = \begin{bmatrix} p_0 \\ \vdots \\ p_n \end{bmatrix} \tag{33}$$

When the pile is fixed at both ends, the lateral stiffness matrix can be expressed as follows:

$$\begin{bmatrix} Q & J & & & & \\ J & Q & J & & & \\ & \ddots & \ddots & \ddots & & \\ & & J & Q & J & \\ & & & J & Q & J \\ & & & & J & Q \end{bmatrix} \cdot \begin{bmatrix} (w_u)_0 \\ \vdots \\ (w_u)_n \end{bmatrix} = \begin{bmatrix} p_0 \\ \vdots \\ p_n \end{bmatrix} \tag{34}$$

Thus, the lateral displacement w_u of the shear layer can be obtained from Eq. (32), and the pile lateral displacement w can be obtained according to Eq. (29). The finite difference formulation of Eq. (29) can be written as follows:

$$\lambda_1 w_{i-2} + \lambda_2 w_{i-1} + \lambda_3 w_i + \lambda_2 w_{i+1} + \lambda_1 w_{i+2} = (w_u)_i \tag{35}$$

where λ_1, λ_2 and λ_3 are calculated as follows:

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & -1 & 0 \\ 6 & 2 & 1 \end{bmatrix} \cdot \left[\frac{EI}{2h^4\sqrt{Gk}} \quad \frac{GD}{2h^2\sqrt{Gk}} \quad \frac{Dk}{2\sqrt{Gk}} + 1 \right]^T \tag{36}$$

To simplify calculation, four virtual nodes (nodes -2, -1, $n+1$ and $n+2$) are added at pile-top and pile-tip. The rotation angle, bending moment and shear force of the pile are as follows:

$$\theta_i = \left(\frac{dw}{dz}\right)_i, \tag{37a}$$

$$M_i = EI\left(\frac{d^2 w}{dz^2}\right)_i \text{ and} \tag{37b}$$

$$Q_i = EI\left(\frac{d^3 w}{dz^3}\right)_i, \tag{37c}$$

assuming that the pile is unconstrained at the top and tip, i.e., $Q = 0$ and $M = 0$ when $z = H$ and $z = 0$.

Combining with boundary conditions, the virtual nodes are eliminated. Assembling a discretized equation for each node leads to a finite difference formulation for the whole pile in matrix-vector form as follows:

$$\{w\}_{(n+1)\times 1} = \{K\}^{-1}_{(n+1)\times(n+1)} \cdot \{w_u\}_{(n+1)\times 1} \tag{38}$$

where $\{w\}$ is the lateral displacement vector of pile; $\{w_u\}$ is the nodal lateral displacement vector of shear layer at the plane $y = y_0$, which can be obtained from Eq. (32); and $\{K\}$ is the lateral stiffness matrix of the pile in x direction, a stiffness that can be expressed as follows:

$$\{K\} = \begin{bmatrix} 4\lambda_1+2\lambda_2+\lambda_3 & -4\lambda_1 & 2\lambda_1 & & & & \\ 2\lambda_1+\lambda_2 & \lambda_3-\lambda_1 & \lambda_2 & \lambda_1 & & & \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_2 & \lambda_1 & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_2 & \lambda_1 \\ & & & \lambda_1 & \lambda_2 & \lambda_3-\lambda_1 & 2\lambda_1+\lambda_2 \\ & & & & 2\lambda_1 & -4\lambda_1 & 4\lambda_1+2\lambda_2+\lambda_3 \end{bmatrix} \tag{39}$$

Similarly, when the pile is unconstrained at the top and fixed at the tip, the lateral stiffness matrix can be expressed as follows:

$$\{K\} = \begin{bmatrix} 4\lambda_1+2\lambda_2+\lambda_3 & -4\lambda_1 & 2\lambda_1 & & & & \\ 2\lambda_1+\lambda_2 & \lambda_3-\lambda_1 & \lambda_2 & \lambda_1 & & & \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_2 & \lambda_1 & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_2 & \lambda_1 \\ & & & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_2 \\ & & & & \lambda_1 & \lambda_2 & \lambda_3+\lambda_1 \end{bmatrix} \tag{40}$$

When the pile is fixed at the top and tip, the lateral stiffness matrix can be expressed as follows:

$$\{K\} = \begin{bmatrix} \lambda_1+\lambda_3 & \lambda_2 & \lambda_1 & & & & \\ \lambda_2 & \lambda_3 & \lambda_2 & \lambda_1 & & & \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_2 & \lambda_1 & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_2 & \lambda_1 \\ & & & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_2 \\ & & & & \lambda_1 & \lambda_2 & \lambda_1+\lambda_3 \end{bmatrix} \tag{41}$$

3.2. Analyses of group-piles response

When the effects of lateral soil displacements near the pile are considered, the solution of group-piles response is similar to the solution without considering these effects. The solution can be expressed as follows:

$$\delta_i(z) = \sum_{j=1}^{n} \delta_{ij} \tag{42}$$

where δ_{ij} represents the lateral shielding displacement of pile i due to the shielding effect of pile j, a condition that can be solved by Eq. (18). δ_{ii} represents the lateral displacement of a single-pile i due to tunnelling, a condition that can be solved by Eq. (38). Then, the rotation angle, bending moment and shear force of the pile can be obtained according to Eqs. (8a)–(8c).

4. Verifications and parametric analysis for single-pile

4.1. Verifications for single-pile

4.1.1. Comparison with boundary element program

Xu and Poulos (2001) analyzed the response of a single-pile due to tunnelling with different volume losses using the boundary element program GEPAN. Soil and the pile are assumed to be elastic materials. The elastic modulus and Poisson ratio of soils are 24 MPa and 0.5, respectively. The tunnel diameter and the depth of the tunnel axis are 6 m and 20 m, respectively. A ground loss ratio ε of 2.5% is considered. The single-pile with 0.5 m diameter and 25 m length is located horizontally 4.5 m away from the tunnel axis; the elastic modulus of the pile is 30 GPa. Assuming that the top and tip of the pile are free, the calculating diagram is shown in Fig. 6.

Fig. 7 shows the lateral displacement and bending moment of the single-pile using the proposed method based on the Pasternak model. It is also compared with those from GEPAN by Xu and Poulos (2001). The solutions for the Pasternak model are obtained with and without considering the effect of lateral soil displacements (ELSD). In addition, the analytical solutions based on the Winkler model are also compared with the above-mentioned results. It can be seen that the response profiles estimated by the Pasternak model considering the effect of lateral soil displacements (ELSD) are more similar compared with those obtained by using GEPAN. According to the Pasternak model, the lateral displacement and bending moment profiles with depth for a single-pile are almost identical, while the maximum lateral deflection and bending moment occur slightly above tunnel axis. The maximum lateral displacements and bending moment are underestimated compared with those calculated from GEPAN. However, the solution based on the Winkler model is shown as poor agreement with the results from GEPAN and the proposed method based on the Pasternak model, compared with the solutions from the Winkler model, is more suitable in predicting the pile response caused by tunnelling.

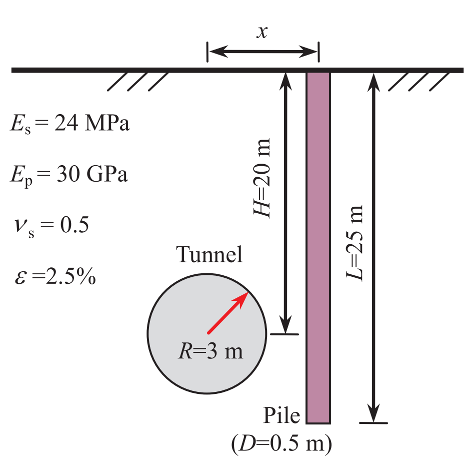

Fig. 6. Calculating diagram for effects of single-pile due to tunnelling (Xu and Poulos, 2001).

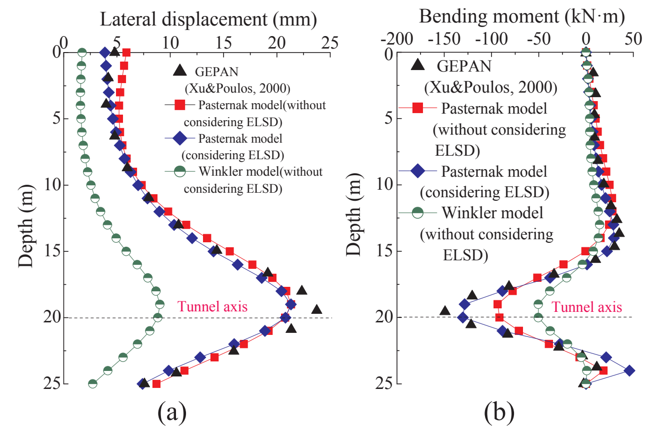

Fig. 7. Comparison of pile lateral displacement and bending moment in case 1: (a) lateral displacement and (b) bending moment.

4.1.2. Comparison with field measurements

Lee et al. (1994) reported a case study to analyze pile behaviour as a result of the construction of a nearby tunnel. The pile is 28 m in length and 1.2 m in diameter. The modulus of the pile is 30 GPa. The tunnel axis line is approximately 5.7 m from the centreline of the pile, and the depth of the tunnel axis level is 15 m. The tunnel is excavated using hand tools in two stages: the first a pilot tunnel of 4.5 m diameter and the second an enlargement of 8.25 m diameter. Measured ground loss ratios were approximately 1.5% for the pilot tunnel and 0.5% for the tunnel enlargement (Loganathan et al., 2001). An average soil elastic modulus E_s of 54 MPa is considered. The pile-top and pile-tip are assumed to be free.

Pile lateral displacements based on the Pasternak and Winkler model are compared with measured data, as shown in Fig. 8. According to the solutions from the Pasternak model, the location of the computed maximum pile deflection is slightly above the measured deflection. When the effect of lateral soil displacements is not considered, the computed maximum pile deflection is 11.94 mm, where a difference of 19% compared to measured data. When the effect of lateral soil displacements is considered, the maximum lateral displacement of the pile is 9.86 mm, where a difference of 2% compared to measured data.

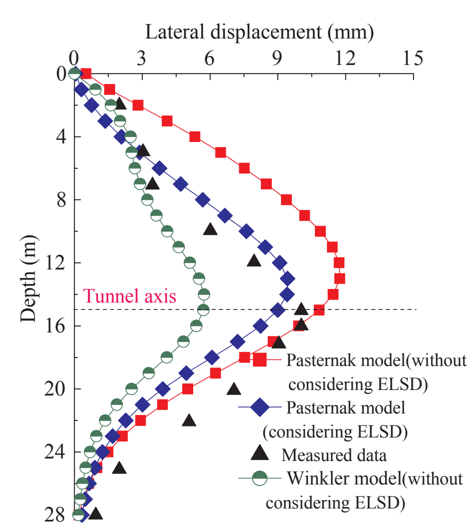

Fig. 8. Comparison of pile lateral displacement in case 2.

Overall, good agreement is observed between the computed and measured pile deflection profiles considering the effect of lateral soil displacements, especially for those above the tunnel axis. However, the solutions based on the Winkler model obviously under-predict the horizontal movements.

4.1.3. Comparison with centrifuge test data

Loganathan et al. (2000) conducted three centrifuge tests based on uniform radial displacements of soil. Heavily over-consolidated (OCR at tunnel axis level is approximately 5.2) Kaolin clay is used. The test is performed at a centrifuge acceleration of $n = 100g$. The prototype consists of a 6-m diameter tunnel excavated in a stiff clay stratum. Depth to the tunnel axis is varies (15 m, 18 m and 21 m) between tests. The three tests are identical in all aspects except the depth of the model tunnel. The diameter and length of the prototype foundation piles are 0.8 m and 18 m, respectively. A single-pile is installed on the side of the tunnel. The distance from the tunnel axis to the single-pile is 5.5 m. A pile elastic modulus of 20.5 GPa is considered in calculation. The average ground loss is taken as 1%; the elastic modulus of the soil is taken as 30 MPa.

The lateral displacements and bending moments of the single-pile calculated by the proposed method based on the Pasternak model in three tests are compared, both when the effect of lateral-soil displacements (ELSD) is considered and when it is not, as shown in Fig. 9. Furthermore, the solutions based on the Winkler model are also compared with the above-mentioned results. Regarding the shapes of distribution profiles, the calculated solution based on the Pasternak model when the effect of lateral soil displacements is considered is much closer to the observed results than when the effect is not considered. Because the pile is assumed to be unconstrained at both ends, but the tests cannot actually guarantee complete freedom of pile end, the lateral displacement in the simplified calculation method herein has some

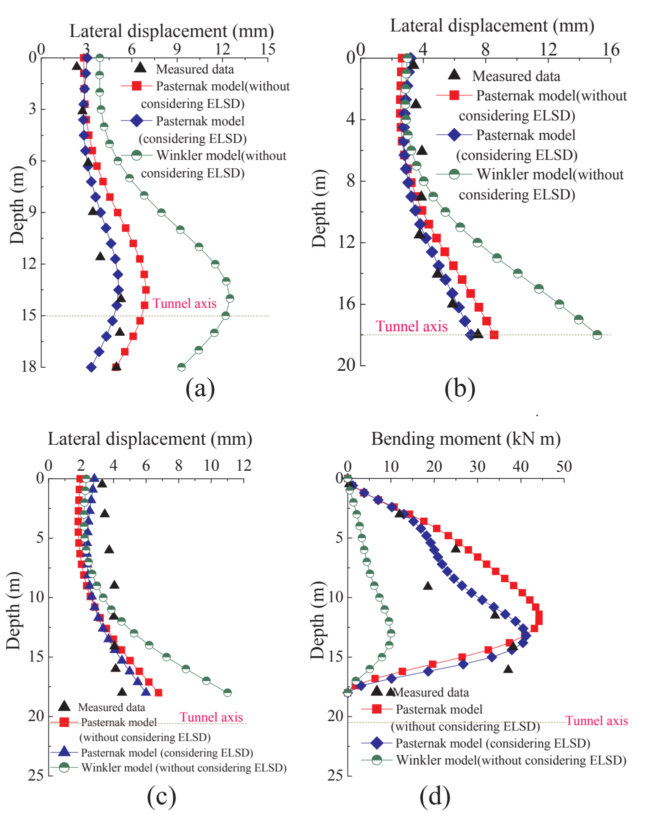

Fig. 9. Comparison of pile lateral displacement and bending moment in case 3: (a) lateral displacement in test 1; (b) lateral displacement in test 2; (c) lateral displacement in test 3; (d) bending moment in test 3.

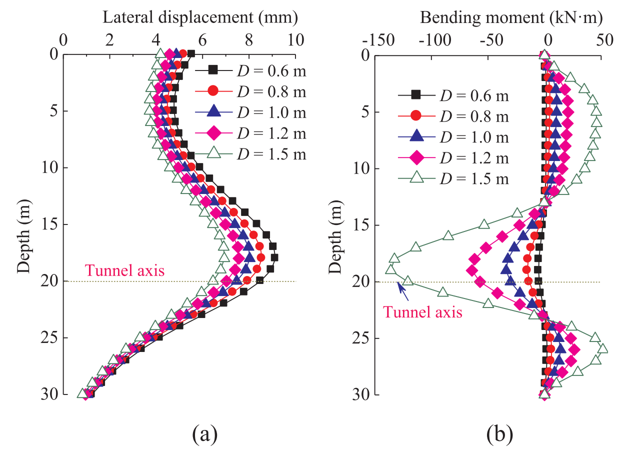

Fig. 10. Lateral displacement and bending moment of pile with different pile diameters: (a) lateral displacement; (b) bending moment.

discrepancy with the experimental data near the end of the pile.

4.2. Parametric analysis for single-pile

4.2.1. Influence of pile diameter

The tunnel's diameter and depth are 6.4 m and 20 m, respectively. The ground loss ratio ε is set as 2%. The pile axis is located horizontally 8 m away from the tunnel axis. The length and elastic modulus of the pile are 30 m and 30 GPa, respectively. The elastic modulus and Poisson ratio of soils are 30 MPa and 0.4, respectively. The top and tip of the pile are assumed to be unconstrained. The proposed simplified method is used considering effect of lateral soil displacements. The comparison for single-pile displacements and bending moments is shown as Fig. 10 according to cases of pile diameters of 0.6 m, 0.8 m, 1.0 m, 1.2 m and 1.5 m. The smaller the lateral displacements of the single-pile are, the larger the diameter of the pile is. However, the bending moments of the single-pile increase as the diameter of the pile grows.

4.2.2. Influence of ground loss ratio

The tunnel's diameter and depth are 6.4 m and 20 m, respectively. The diameter, length, and elastic modulus of the pile are 1 m, 30 m, and 30 GPa, respectively. The pile axis is located horizontally 8 m away from the tunnel axis. The elastic modulus and Poisson ratio of soils are 30 MPa and 0.4, respectively. The ground loss ratio ε for five cases is set as 0.5%, 1%, 2.0%, 3% and 5%. The top and tip of the pile are assumed to be unconstrained. The effect of lateral soil displacements is taken into account for this study. Fig. 11 shows the influences of tunnelling on the displacements and bending moments of the single-pile considering the different ground loss ratio. The response of pile displacements and bending moments is increasingly significant as the ground loss ratio increases.

4.2.3. Influence of pile-tunnel distance

The ground loss ratio ε is set as 2%. The tunnel's diameter and depth are 6.4 m and 20 m, respectively. The diameter, length, and elastic modulus of the pile are 1 m, 30 m, and 30 GPa, respectively. The elastic modulus and Poisson ratio of soils are 30 MPa and 0.4, respectively. Horizontal distances between the pile axis and tunnel axis for five cases are set as 4 m, 6 m, 8 m, 10 m, and 12 m. The top and tip of the pile are assumed to be free. Fig. 12 shows the displacements and bending moments of the single-pile induced by tunnelling under the condition of different cases. The calculation results are also conducted considering the influences of lateral soil displacements. The pile displacement and bending moments decrease as the tunnel-pile distance and the maximum value increase, generally beside by the tunnel centreline.

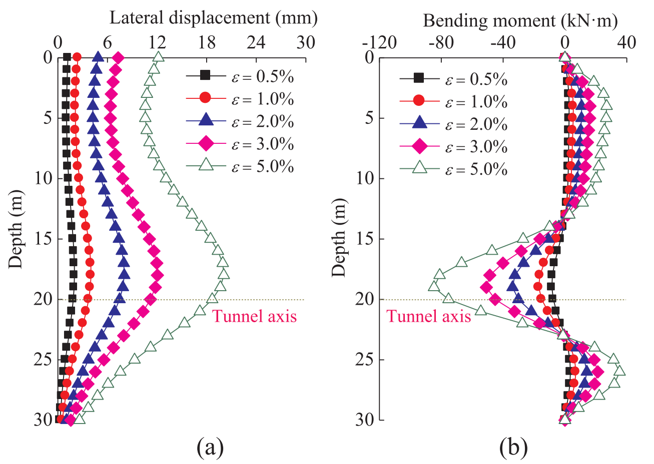

Fig. 11. Lateral displacement and bending moment of pile with different ground loss: (a) lateral displacement; (b) bending moment.

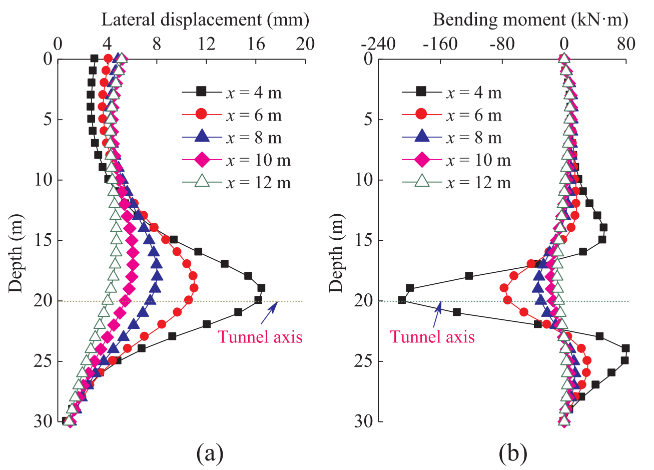

Fig. 12. Lateral displacement and bending moment of pile with different tunnel-pile distance: (a) lateral displacement; (b) bending moment.

5. Verifications and parametric analysis for group-piles

5.1. Verifications for group-piles

Loganathan et al. (2001) calculated the lateral displacement and bending moment of a 2 × 2 capped-pile group due to tunnelling using the boundary element program GEPAN. The modulus and Poisson ratio of the soil are 24 MPa and 0.5, respectively. A ground loss ratio ε of 1% is considered. The diameter of the tunnel is 6 m, buried with its axis at a depth of 20 m. Piles in the group are spaced at 2.4 m with 0.8 m diameter and 25 m length. The elastic modulus of each pile is 30 GPa. The horizontal distance between the front row pile and the tunnel axis is 4.5 m. The calculating diagram for the group-piles is shown in Fig. 13.

Figs. 14 and 15 show comparisons of the response of the front and rear piles calculated by the analytical solutions based on the Pasternak and Winkler model with those computed by GEPAN in Loganathan et al. (2001). As shown in the figures, in the displacement analyses of group-piles, the deviations of the results are obvious in the two situations of considering and or not considering the effect of lateral soil displacements (ELSD). According to the solutions using the Pasternak model, pile deflection and bending moment when considering effect of lateral soil displacements are closer to the GEPAN-computed results, further validating the reliability of the method. Thus, the effect of lateral soil displacements cannot be ignored in the analyses of pile-soil interaction. According to the solutions using the Winkler model, the presence of ground continuity or cohesion of the foundation may have contributed

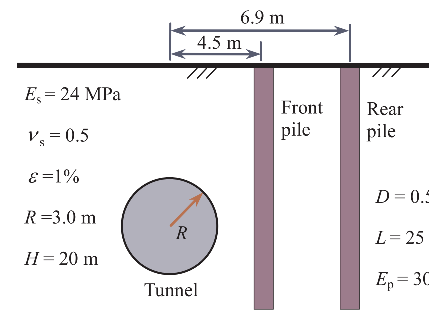

Fig. 13. Calculating diagram for the effect of 2 × 1 pile group due to tunnelling (Loganathan et al., 2001).

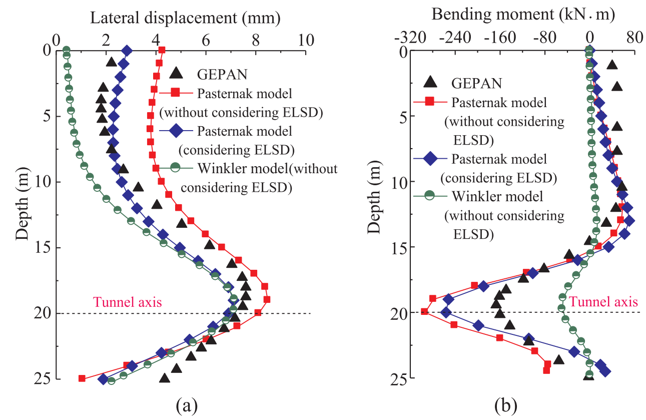

Fig. 14. Comparison of lateral displacement and bending moment of front pile in case 4: (a) lateral displacement; (b) bending moment.

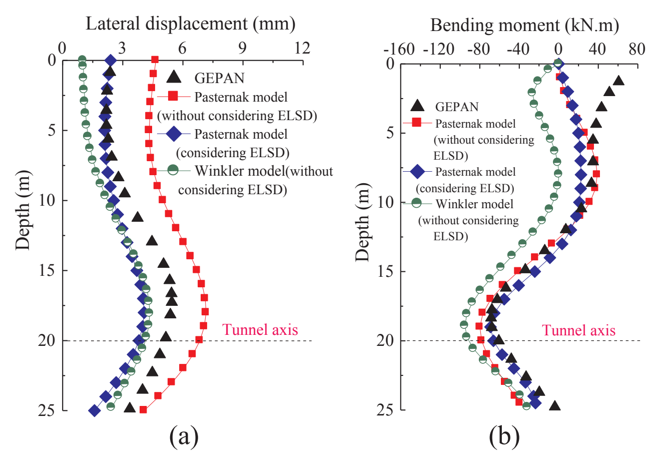

Fig. 15. Comparison of lateral displacement and bending moment of rear pile in case 4: (a) lateral displacement; (b) bending moment.

to the less satisfactory agreement between the predicted results and GEPAN-computed.

5.2. Parametric analyses for group-piles

The pile displacement and bending moment due to tunnelling can be predicted by the presented method for examining the influence of different factors, including the modulus of shear layer G, pile diameter d, ground loss ratio ε, pile-tunnel distance x and pile spacing s. The following parameters are adopted in this section: 2 × 1 group-piles with depths of 30 m and elastic modulus of 30 GPa located close to a tunnel. The soil nature unit weight is 20 kN/m^3 and the modulus and Poisson ratio of soil are 30 MPa and 0.4, respectively. The depth of the shield

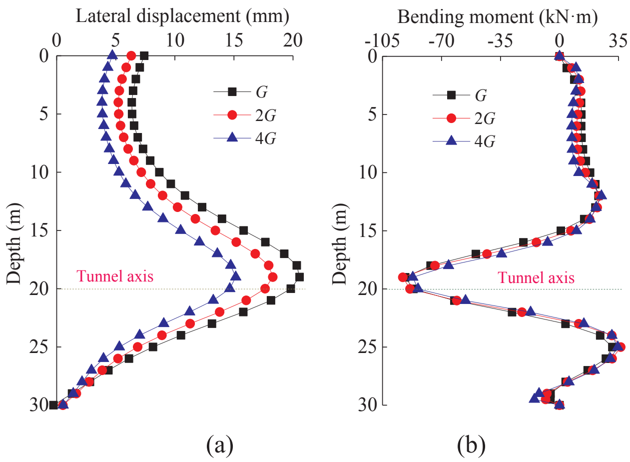

Fig. 16. Lateral displacement and bending moment of front pile with different foundation shear modulus: (a) lateral displacement; (b) bending moment.

tunnel's center is 20 m and the tunnel's diameter is 6 m. To achieve sufficient precision, the pile is divided into 30 segments. The proposed method of considering effect of lateral soil displacements is used in calculation. The top and tip of the pile are assumed to be unconstrained.

5.2.1. Influence of shear layer modulus

A 2 × 1 pile group is located horizontally 6 m away from the tunnel axis with the pile diameter of 0.5 m and pile spacing of 1.5 m. A ground loss ratio ε of 4.0% is considered. In analysis, three cases are considered, in which the modulus of the shear layer is G, $2G$, and $4G$. Figs. 16 and 17 show the comparison for front pile and rear pile displacements and bending moments in these three cases. The pile lateral displacement decreases as the shear layer modulus increases. It indicates that Pasternak's foundation model can fully reflect the effect of shear displacements of foundation soil. Therefore, the influence of soil shear displacements on pile response cannot be ignored, and an error may occur when Winkler's foundation model is used.

5.2.2. Influence of pile diameter

The front pile is located horizontally 6 m away from the tunnel axis with the spacing of 1.5 m. A ground loss ratio ε of 3.0% is considered. In analysis, three cases are considered, in which the diameters of the group-piles are 0.3, 0.5 and 0.8 m. Figs. 18 and 19 show the comparison for front pile and rear pile displacements and bending moments in different pile diameters. As the pile diameter increases, the pile lateral displacement decreases, but the bending moment grows. The main reason for this phenomenon is that the flexural rigidity EI increases with the pile diameter d, and its resistance to displacement increases, but bending moments also increase.

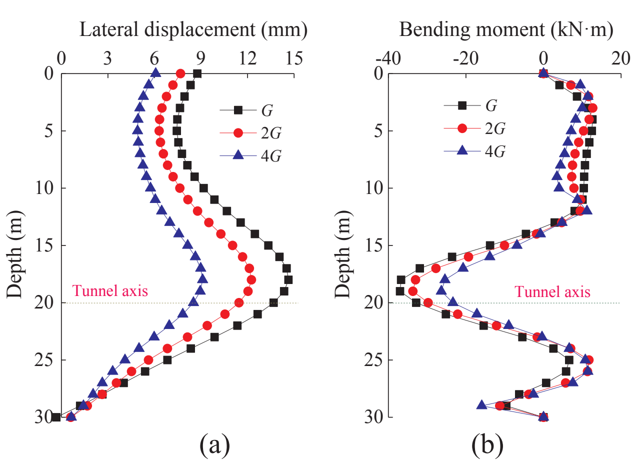

Fig. 17. Lateral displacement and bending moment of rear pile with different foundation shear modulus: (a) lateral displacement; (b) bending moment.

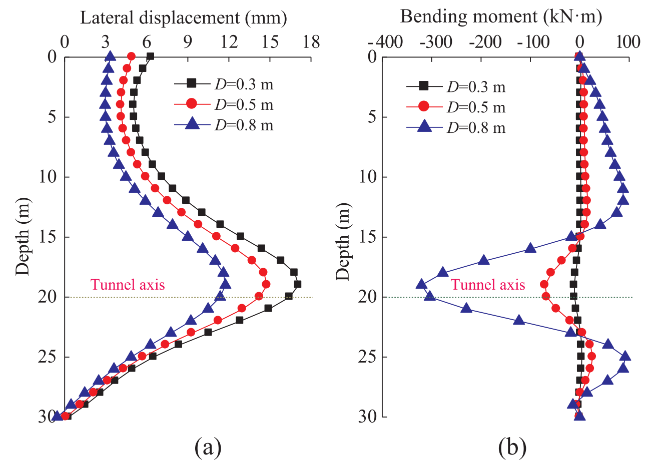

Fig. 18. Lateral displacement and bending moment of front pile with different pile diameters: (a) lateral displacement; (b) bending moment.

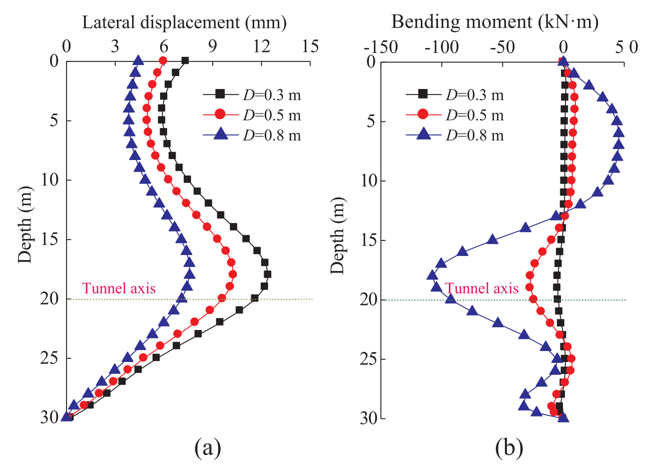

Fig. 19. Lateral displacement and bending moment of rear pile with different pile diameters: (a) lateral displacement; (b) bending moment.

5.2.3. Influence of ground loss ratio

The front pile is located horizontally 6 m away from the tunnel axis with a pile diameter of 0.5 m and pile spacing of 1.5 m. The ground loss ratios ε are 1.0%, 2.0% and 3.0% in three cases. Figs. 20 and 21 give the variations of pile displacement and bending moment as the ground loss ratio increases. The effect of the ground loss ratio is significant on pile displacements and bending moment. The influences of tunnelling on pile lateral displacement and bending moment are developed as ground loss ratios ε increase.

5.2.4. Influence of pile-tunnel distance

A 2 × 1 pile group is located close to a tunnel and has a diameter of 0.5 m and a spacing of 1.5 m. A ground loss ratio ε of 3.0% is considered. The horizontal distances between the tunnel axis line and the centreline of the front piles are 6 m, 8 m and 10 m. Figs. 22 and 23 show the comparison for lateral displacements and bending moments of the front pile and rear pile in different pile-tunnel distances. The shapes of the distribution profiles are similar, and the maximum lateral displacement and bending moment occur slightly above tunnel axis. The pile displacement and bending moment decrease nonlinearly as the pile-tunnel distance x increases.

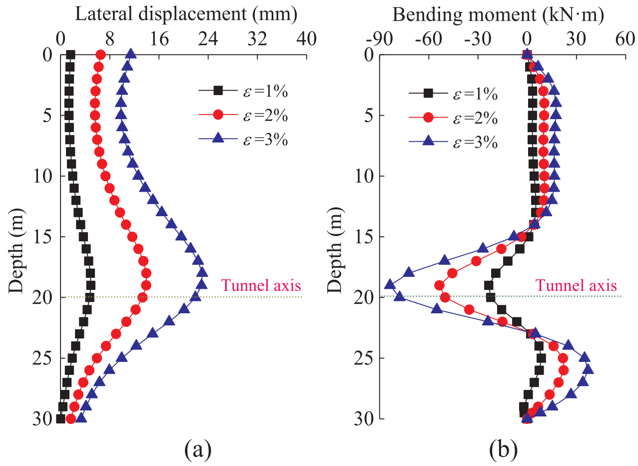

Fig. 20. Lateral displacement and bending moment of front pile with different ground loss: (a) lateral displacement; (b) bending moment.

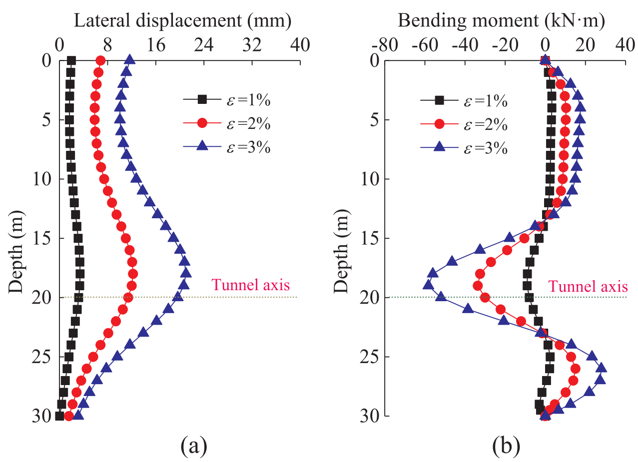

Fig. 21. Lateral displacement and bending moment of rear pile with different ground loss: (a) lateral displacement; (b) bending moment.

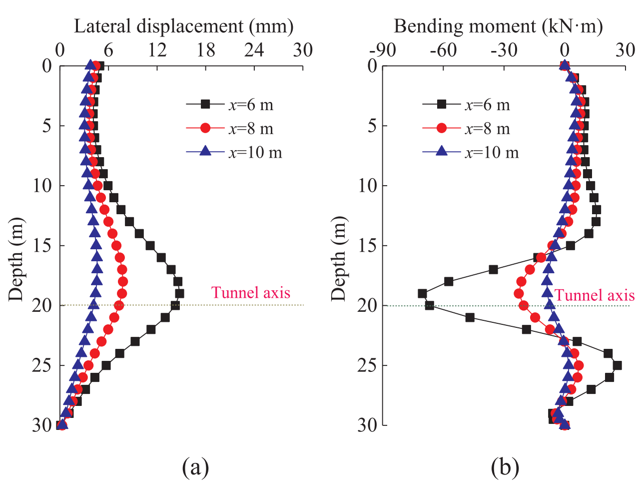

Fig. 22. Lateral displacement and bending moment of front pile with different pile-tunnel distance: (a) lateral displacement; (b) bending moment.

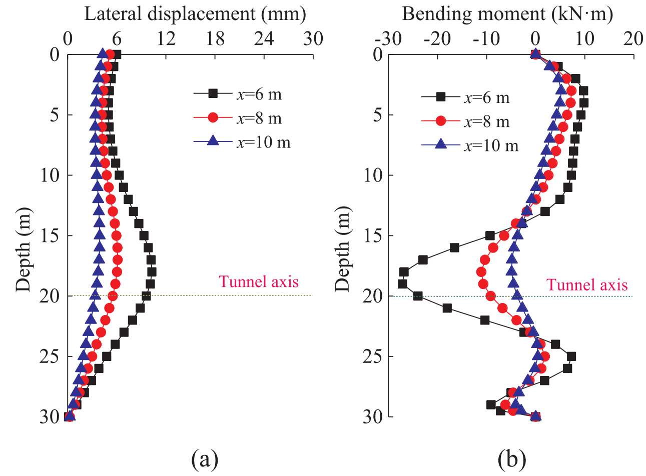

Fig. 23. Lateral displacement and bending moment of rear pile with different pile-tunnel distance: (a) lateral displacement; (b) bending moment.

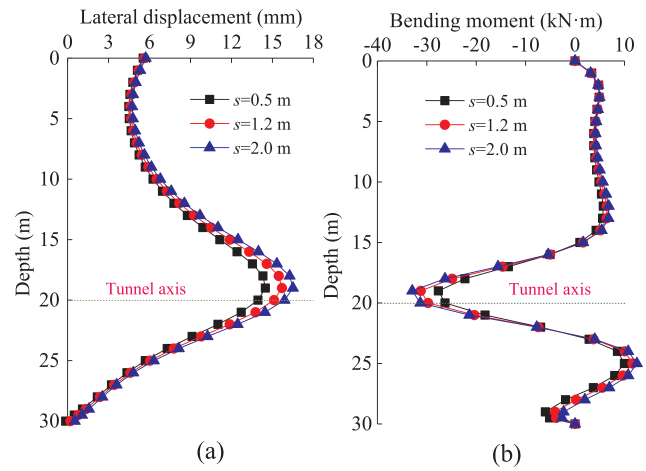

Fig. 24. Lateral displacement and bending moment of front pile with different pile spacing: (a) lateral displacement; (b) bending moment.

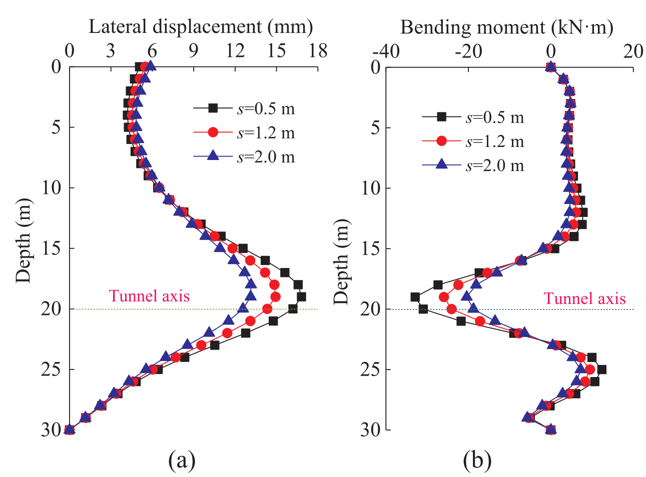

Fig. 25. Lateral displacement and bending moment of rear pile with different pile spacing: (a) lateral displacement; (b) bending moment.

5.2.5. *Influence of pile spacing*

A 2 × 1 pile group with a pile diameter of 0.4 m is located close to a tunnel. A ground loss ratio ε of 3.0% is considered. The horizontal distance between the centreline of the front pile and the tunnel axis line is 6 m. Pile spacings of 0.5 m, 1.2 m and 2.0 m are considered. Figs. 24 and 25 show the comparison for lateral displacements and bending moments of the front pile and rear pile in different pile spacing. The maximum lateral displacement of the front pile increases as the pile spacing increases, whereas the maximum bending moment increases slightly. However, for the rear pile, the maximum lateral displacement and bending moment decrease as the pile spacing increases. This occurs because the distance between the rear pile and tunnel is increased as the pile spacing increases.

6. Conclusions

To avoid the drawbacks of the plane strain analyses and Winkler's foundation, this paper addresses the influence problem of tunnel-soil-pile interaction, and a simplified method is proposed based on Pasternak's foundation model. The analyses aim to provide an efficient means to assess the effects of tunnelling on the lateral displacements and bending moments of an adjacent single-pile and group-piles. The pile response induced by tunnelling is presented firstly based on Pasternak's foundation model considering the shearing displacements of foundation. To obtain more accurate results, the simplified solution is derived secondly considering the effect of lateral soil displacements. The response of a single-pile is determined by imposing the free-field soil movement profile estimated by Loganathan-Poulos' analytical expression to the passive pile based on Pasternak's foundation model. The shielding effect of passive group-piles due to pile-soil-pile interaction is then considered, and Mindlin's solution for the lateral response is adopted to simulate the pile-pile interaction. The responses of group-piles due to tunnelling are finally obtained by the superposition principle. The proposed method is verified through comparisons with published solutions by the boundary element program GEPAN, centrifuge test data and field measurements. The displacements and bending moments of a single-pile and group-piles induced by tunnelling are calculated through the presented method considering the effects of lateral soil displacements.

The presented method can consider the impacts of lateral soil displacements on pile response and reflect the effects of tunnel-soil-pile interaction. The proposed method considering the effects of lateral soil displacements provides reliable estimates for the response of a passive single-pile and group-piles induced by tunnelling. The method is based on Pasternak's foundation model and two parameters, k and G, to account for the shearing interaction between surrounding soils. The results show that parameter G has a certain impact on the pile response. Therefore, the influence of soil shear displacements on pile response cannot be ignored for higher precision.

In the analyses of responses of group-piles, the simplified solution considering the effects of lateral soil displacements better predicts pile displacements induced by tunnelling. The lateral displacement and bending moment of group-piles are studied by the proposed method considering the effects of lateral soil displacements to discuss the influence of different factors, including the modulus of shear layer, pile diameter, ground loss ratio, pile-tunnel distance and pile spacing. The influence of soil shear displacements on pile response cannot be ignored. When Winkler's foundation model is used, an error may occur.

However, the major limitation of the proposed method stems from the simplifying assumptions of linearity and perfect bonding between pile and soil. These assumptions may generate differences between the calculated results and the centrifuge test data or field measurements. It is noted that the soil is not perfectly elastic but generally non-linear. In addition, the pile side friction effects are existed in actual projects. In this study, we cannot consider the characteristics of soil non-linearity and the pile side friction. The core object of this study is to investigate the ground shearing displacements, and the influence of lateral soil displacements beside the pile. In addition, the simplified analytical method in this paper is only valid for uniform clayey soils. In regard to the stratified soils, the soil parameters for an equivalent homogeneous foundation are calculated using the weighted average. Although the presented analytical solution is limited in scope, it can be used for a preliminary design of tunnels in clays. Therefore, further research on the analyses considering the nonlinear soil behaviour, the layered soils, and the gap between the existing piles and soils is still required to decisively assess the responses of piles due to tunnelling-induced ground movements.

Acknowledgments

The authors acknowledge the financial support provided by the National Key R&D Program of China (No. 2016YFC0800200), and by Natural Science Foundation of China (No. 41772331), and by Project Program of Key Laboratory of Urban Underground Engineering of Ministry of Education (No. TUE2017-04).

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