Transportation Research Part B 000 (2017) 1–21

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Integrated public transport timetable synchronization and vehicle scheduling with demand assignment: A bi-objective bi-level model using deficit function approach

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ARTICLE INFO

Article history:
Received 17 August 2017
Accepted 18 August 2017
Available online xxx

Keywords:
Public transport
Timetable synchronization
Vehicle scheduling
Demand assignment
Bi-objective bi-level model
Deficit function

ABSTRACT

In the operations planning process of public transport (PT), timetable synchronization is a useful strategy utilized to reduce transfer waiting time and improve service connectivity. However, most of the studies on PT timetable synchronization design have treated the problem independently of other operations planning activities, and have focused only on minimizing transfer waiting time. In addition, the impact of schedule changes on PT users' route/trip choice behavior has not been well investigated yet. This work develops a new bi-objective, bi-level integer programming model, taking into account the interests of PT users and operators in attaining optimization of PT timetable synchronization integrated with vehicle scheduling and considering user demand assignment. Based on the special structure characteristics of the model, a novel deficit function (DF)-based sequential search method combined with network flow and shifting vehicle departure time techniques is proposed to achieve a set of Pareto-efficient solutions. The graphical features of the DF can facilitate a decision-making process for PT schedulers for finding a desirable solution. Two numerical examples are illustrated to demonstrate the methodology developed.

1. Introduction

1.1. Background and motivation

One of the most challenging problems in transportation planning is shifting a significant number of private car users to public transport (PT) in a sustainable manner. The problem of improving PT patronage and its compatibility with user needs is multifaceted. However, one may rather intuitively assume that improving PT service reliability, from the user perspective, and reducing operations costs from the perspective of PT operators will lead to an increase in ridership and competiveness compared with private car use. For most PT systems, inter-route or intermodal transfer connections are important factors affecting service reliability. Timetable synchronization is a useful strategy used to reduce passenger transfer waiting time, provide a well-connected service and improve PT service reliability (Ceder, 2001,2016).

Most previous studies (e.g., Daganzo, 1990; Bookbinder and Desilets, 1992; and Ceder et al., 2001) on the PT timetable synchronization problem have treated the problem separately, rather than coupled with, or isolated from, other operation

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http://dx.doi.org/10.1016/j.trb.2017.08.024

planning activities, such as vehicle scheduling, transit assignment and network design. However, as pointed out by Ceder (2001, 2016), it is preferable that all operation planning activities can be planned simultaneously in order to exploit the system's capability to the greatest extent, and thereby, maximize its productivity and efficiency. Some researchers (e.g. Ceder and Stern, 1984; Van den Heuvel et al., 2008; Ibarra-Rojas et al., 2014; Liu and Ceder, 2016a; and Liu et al., 2017) have focused on integrating the PT timetable synchronization problem (TSP) with the vehicle scheduling problem (VSP) in order to analyze the tradeoff between level of service and operations costs. Various new mathematical programming models have been developed to serve as the basis for this tradeoff analysis for the integrated timetable synchronization and vehicle scheduling (ITSVS) problem.

To date, there are two main decision-making problems that have not yet been addressed regarding an integrated approach to simultaneously analyze timetable synchronization and vehicle scheduling tasks. First, almost all previous studies focused on optimizing operation parameters only, such as trip offset times (departure time of the first trip), route headways and vehicle trip chains. However, the effect of the changes of these operation parameters on PT users' trip choice behavior which have a heavy impact on other planning activities, is not taken into consideration in the timetable optimization process. Second, there is a lack of effective and efficient solution methods for finding a set of Pareto-efficient solutions so as to help for the purpose of assisting in multi-criteria decision analysis from the perspectives of both PT users and operators. To bridge these gaps, this study addresses the integrated PT timetable synchronization and vehicle scheduling problem with transit assignment (ITSVS-TA) for tactical and operational planning purposes.

1.2. Literature review

The problem of identifying the optimal synchronized PT timetable is essentially the problem of deciding on the best dispatching policy for transit vehicles on fixed routes. This has been dealt with quite extensively in the literature. Several approaches and computer-aided software packages have been developed to design synchronized timetables for PT networks with transfers. According to the different features, the approaches developed can be categorized into four groups: (i) interactive graphical optimization approach, (ii) analytical modelling approach, (iii) mathematical programming approach, and (iv) control theory approach.

In the first group, interactive graphical optimization techniques, which are recognized as the earliest approach actually applied in practice to determining synchronized timetables, have been proposed by a few researchers. Rapp and Gehner (1976) described a computer-aided, interactive graphic system for transfer optimization in the Basel Transit System. The system adopts a graphical person-computer interactive approach to reduce transfer delay and the number of vehicles required. Désilets and Rousseau (1992) and Fleurent et al. (2004) described the graphical interactive timetable planning tools used in the HASTUS system. Another graphical optimization method used in the timed transfer system (TTS) in a Philadelphia suburb was described by Vuchic (2005). This TTS used a clock-type diagram to provide graphical presenattions of synchronized schedules. Other earlier theoretical investigations of the PT timetable synchronization problem (PT-TSP) are mainly focused on how to set route headways and offset times. Salzborn (1980) studied a special inter-town route connected by a string of feeder routes. Some intuitive rules are provided to set the departure and arrival times of buses on the feeder routes. Daganzo (1990) examined the single transfer node case, and provided some intuitive rules for setting the headways of the inbound and outbound routes.

The second approach to solve the PT-TSP employs analytical formulations for idealized PT systems. Wirasinghe et al. (1977) and Wirasinghe (1980) developed approximate analytical models for investigating the optimal design parameters of a coordinated rail and bus transit system atop rectangular grid or ring-radial networks. A series of follow-up studies (e.g., Lee and Schonfeld, 1991; Chien and Schonfeld, 1998; Chowdhury and Chien, 2002; Ting and Schonfeld, 2005; Sivakumaran et al., 2012; and Kim and Schonfeld, 2014) have been conducted. Knoppers and Muller (1995) investigated the impact of fluctuations in passenger arrival times on the possibilities and limitations of synchronized PT transfers. They concluded that transfer synchronization is gainful when the arrival time of the feeder line is within a time window relative in length to the headway of the connecting line. As pointed out by Liu and Ceder (2017a), one limitation of the analytical modelling approach is that it fails to accurately calculate the measures of the cost components of the objective functions considered.

The third approach widely found in the literature adopts mathematical programming models. Klemt and Stemme (1988) and Domschke (1989) provided a quadratic programming model of the problem to minimize passenger transfer waiting time. A set of heuristics, such as regret methods, improvement algorithms and simulated annealing, are proposed to solve the model problem. Bookbinder and Desilets (1992) developed an integer programming model and an iterative improvement heuristic procedure was provided to minimize mean transfer disutility. Voß (1992) proposed a 0–1 integer programming and a tabu search algorithm to minimize transfer waiting time. Ceder et al. (2001) developed a mixed integer linear programming model and several heuristic algorithms to maximize the number of simultaneous bus arrivals at the transfer nodes of PT networks. Based on this seminal work, a series of follow-up studies have been conducted by other researchers (e.g., Shafahi and Khani, 2010; Ibarra-Rojas and Rios-Solis, 2012; Aksu and Akyol, 2014; Ibarra-Rojas et al., 2015; Fouilhoux et al., 2016; Wu et al., 2016). Wong et al., (2008) developed a mixed integer programming model and an optimization-based heuristic method to minimize the total passenger transfer waiting time for the MTR system in Hong Kong. Ibarra-Rojas et al., (2014) developed a bi-obejctive, integer programming model to maximize the number of passengers benefiting from well-timed transfers and minimize vehicle operating costs.

In the fourth and last group, control theory based models were uilized at the operations control level to control the movement of vehicles in order to improve the service reliability, and schedule adherence and headway regularity of the planned synchronized timetable. This control approach is based on feasible operational control strategies, such as holding (Dessouky et al., 2003; Hadas and Ceder, 2010; Liu et al., 2014; Daganzo and Anderson, 2016), skip-stop (Ceder et al., 2013), short-turn (Nesheli et al., 2015) and a combination of different selected control strategies (Liu et al., 2015; Nesheli et al., 2015; Liu and Ceder, 2016b). Generally speaking, a preplanned synchronized timetable created at the planning level serves as the basis of the control theory-based approach.

The objective function in most of the aforementioned work is either to minimize the total passenger transfer waiting time or maximize the number of direct transfers. One limitation of these works is that they failed to consider other system performance measures. For example, the minimization of passenger transfer waiting time may lead to impaired performance of other system measures, such as increased vehicle fleet size, increased passenger travel time and more empty-seat vehicle hours. In addition, PT user route/trip choice behavior responding to the changes of the system design parameters, such as route headways and offset times, has not been taken into consideration in previous studies. Therefore, a comprehensive, systematic, and multi-criteria decision-making framework that can take various system performance measures and users' route/trip choice behavior into account is needed.

1.3. Objectives and contributions

This study addresses the ITSVS problem with demand assignment using the mathematical programming approach. The purpose of this study is to provide a bi-objective, bi-level decision-making framework, together with a deficit function-based solution scheme, for the ITSVS problem considering passenger demand assignment. The theoretical contributions of this study to the current literature are threefold. First, a new bi-objective, bi-level integer programming model is developed for the ITSVS-TA problem. To the best of our knowledge, this is the first time that PT passenger route choice behavior is incorporated into the ITSVS problem. Second, in the upper level ITSVS problem, comprehensive objective function components that take both PT users and operators interests into account are formulated. In addition, the vehicle scheduling model can take various modelling features into consideration, such as multi-depots and heterogeneous fleet vehicles. Third, an innovative solution method that is based on the deficient function theory is developed.

This work is comprised of six sections including this introductory section. Section 2 provides background on the DF theory and the optimization framework for the problem. Section 3 presents the mathematical formulations. The DF-based solution method is presented in Section 4. Section 5 illustrates the proposed model and solution method with two detailed examples. Section 6 concludes our work and proposes future extensions of it.

2. Deficit function and optimization framework

The first part of this section provides a concise description of the deficit function theory. The optimization framework of the problem is provided in the second part.

2.1. Background on the deficit function

Following is a concise description of a step function approach proposed by Ceder and Stern (1981) and Ceder (2007, 2016) for assigning the minimum number of vehicles to a given timetable. Linis and Maksim (1967) and Gertsbach and Gurevich (1977) have called this step function a DF as its value represents the deficit number of vehicles required at a particular terminal in question in a multi-terminal PT system. The DF is a step function that increases by one at the time of each trip departure and decreases by one at the time of each trip arrival. To construct a set of DFs, the only information needed is a timetable of required trips. The DF graphical modelling method has been applied to various kinds of PT operations planning activities (Liu and Ceder, 2017b). Let $G = \{g\colon g = 1, \ldots, n\}$ denote a set of required trips. The trips are conducted between a set of terminals$U = \{u\colon u = 1, \ldots, q\}$, each trip is serviced by a single vehicle, and each vehicle is able to service any trip. Each trip g can be represented as a 4-tuple(p^g, t_s^g, q^g, t_e^g), in which the ordered elements denote departure terminal, departure (start) time, arrival terminal, and arrival (end) time. It is assumed that each trip g lies within a schedule horizon $[T_1,T_2]$, i.e., $T_1 \le t_s^g \le t_e^g \le T_2$. The set of all trips $S = \{(p^g, t_s^g, q^g, t_e^g) : p^g, q^g \in U, g \in G\}$ constitutes the timetable. Two trips, g, g' may be serviced sequentially (feasibly joined) by the same vehicle if and only if (a) $t_e^g \le t_s^{g'}$ and (b) $q^g = p^{g'}$. Let $d(u, t, S)$ denote the DF for terminal u at time t for schedule S. The value of $d(u, t, S)$ represents the total number of departures minus the total number of trip arrivals at terminal$u$, up to and including time$t$. The maximum value of $d(u, t, S)$ over the schedule horizon $[T_1,T_2]$, designated $D(u, S)$, depicts the deficit number of vehicles required at u.

Let t_s^g and t_e^g denote the start and end times of trip g, $g \in G$. It is possible to partition the schedule horizon of $d(u, t, S)$ into a sequence of alternating hollow and maximal intervals. The maximal intervals $[s_i^u, e_i^u], i = 1, \ldots, n(u)$ define the interval of time over which $d(u, t)$ takes on its maximum value. Note that the S will be deleted when it is clear which underlying schedule is being considered. Index i represents the ith maximal intervals from the left and $n(u)$ represents the total number of maximal intervals in $d(u, t)$. A hollow interval H_l^u, $l = 0, 1, 2, \ldots, n(u)$ is defined as the interval between two maximal intervals including the first hollow from T_1 to the first maximal interval, and the last hollow from the last interval to T_2. Hollows may consist of only one point, and if this case is not on the schedule horizon boundaries (T_1 or T_2),

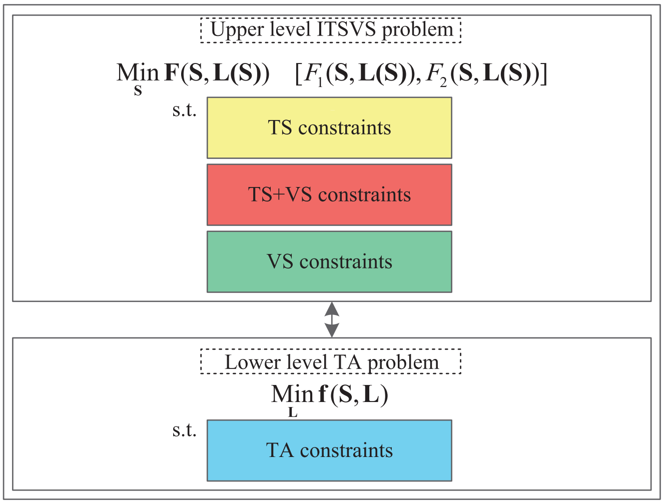


Fig. 1. Schematic representation of the bi-objective, bi-level optimization framework. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the graphical representation of $d(u, t)$ is emphasized by a clear dot. The sum of all DFs over u is defined as the overall DF, $g(t) = \sum_{u \in U} d(u, t)$. This function g(t) represents the number of trips that are simultaneously in operation; i.e., a count, from a bird's-eye view at time t, of the number of transit vehicles in actual service over the entire transit network of routes. The maximum value of g(t), G(S) is exploited for a determination of the initial lower bound on the fleet size.

Theorem 1. (The deficit function fleet size theorem). *If, for a set of terminals U and a fixed set of required trips G, all trips start and end within the schedule horizon [T_1, T_2] and no deadheading (DH) insertions are allowed, then the minimum number of vehicles required to service all trips in G is equal to the sum of all the deficits.*

$$MinN_{DF}(S) = \sum_{u \in U} D(u, S) = \sum_{u \in U} \max_{t \in [T_1, T_2]} d(u, t, S) \tag{1}$$

Proof. A formal proof of this theorem can be found in Ceder (2016).

When deadheading (DH), or empty, vehicle trip insertion and shifting departure times (SDT) are allowed, the fleet size may be further reduced below the level described in Eq. (1). The DF graphical modelling method has been applied to various kinds of PT operations planning activities, including vehicle scheduling, timetable design, network route design, deployment planning of bus rapid transit systems, operational parking space design, and crew scheduling.

2.2. Outline of the optimization framework

The ITSVS problem can be described as a Stackelberg or leader-follower game, where the leader is a PT operator who creates timetables and vehicle schedules (**S**) to optimize system-wide performance measures, and the followers are the PT users who choose their travel paths in a user optimal manner responding to the operator's decision. The operator's decision-making on **S** can influence, but cannot control PT users' travel choice behavior, which will result in passenger loads (**L**) on vehicles. The derivation of passenger loads **L** is actually a non-linear and non-continuous mapping of **S**. The passenger loads defined by the lower level TA serve as the input of the upper level ITSVS optimization problem. This kind of problem can be mathematically formulated as a bi-level optimization programming model (Yang and Bell, 1998; Yin, 2002).

The bi-objective, bi-level optimization framework of the problem is systematically outlined in Fig. 1. The upper level model from the perspective of PT operators aims to: (i) minimize total operation costs, which are related to fleet size, and (ii) minimize total passenger-hour cost, which include the total passenger in-vehicle travel time, total passenger initial waiting time, total passenger transfer waiting time, and total passenger load discrepancy or overcrowding hours. The lower level problem from the perspective of PT passengers is a standard schedule-based transit assignment problem with capacity constraints.

3. Model formulation

This section presents the notations and mathematical formulations of the model. Section 3.1 introduces the notations. Sections 3.2 and 3.3 present the upper level ITSVS model and the lower level schedule-based TA model. Then, Section 3.4 provides the integrated bi-objective bi-level model.

3.1. Nomenclature

Consider a connected network composed of a directed graph $G = \{N, A\}$ with a finite number of nodes $|N|$ connected by $|A|$ arcs. The notations used throughout the paper are listed as follows unless otherwise specified:

Route	progressive path initiated at a given PT terminal and terminated at a certain node while traversing given arcs in sequence
Transferpath	progressive path that uses more than one route
$R = \{r\}$	set of PT routes
$S = \{s\}$	set of PT route segments on which the expected load is more than the desired occupancy
$M = \{m\}$	set of number of departures
$TR = \{tr\}$	set of all transfer paths
N_r	set of nodes located on route r
N_{tr}	set of nodes located on transfer path tr
d_{ij}^r	passenger demand between i and j, $i, j, \in N$, riding on route r
d_{ij}^{tr}	passenger demand between $i$and j along the transfer path tr
d_{ij1}	initial aggregated passenger O-D demand between $i$and j
F_r	vehicle frequency associated with route r
$F_{\min}$	minimum frequency (reciprocal of policy headway) required
t_{ij}^r	average travel time between i and j on route r
t_{ij}^{tr}	average travel time between i and j on transfer path tr (do not include transfer penalties)
t_r^s	average travel time of route segment s of route r on which the expected load is more than the desired occupancy
t_r	overall travel time on route r between its start and end
p_r	maximum passenger load on route r
w_r	passenger initial waiting time on route r
L_{i1}	initial aggregated passenger load at i
C_v	vehicle capacity
d_o	desired occupancy on each vehicle (load standard)
λ	overload factor $\lambda > 1$
a_r^{tr}	$\begin{cases} 1, & \text{transfer tr moves throught route } r \\ 0, & \text{otherwise} \end{cases}$
x_r^m	$\begin{cases} 1, & \text{if m even – headway departrures are selected for route } r \\ 0, & \text{otherwise} \end{cases}$

3.2. The upper level integrated timetable synchronization and vehicle scheduling (ITSVS) problem

3.2.1. Two principal objective functions

The upper level ITSVS problem is based on two principal objective functions, minimum Z_1 and minimum Z_2, across the different sets of PT routes:

$$Z_1 = \alpha_1 \sum_{i,j \in N} PH(i,j) + \alpha_2 \sum_{i,j \in N} IWT(i,j) + \alpha_3 \sum_{i,j \in N} TWT(i,j) + \alpha_4 \sum_{r \in R} LD_r \tag{2}$$

$$Z_2 = FS \tag{3}$$

where

$PH(i, j)$	= passenger hours between nodes i and j, $i, j, \in N$ (defined as passenger riding time in a vehicle on an hourly basis; it measures the time spent by passengers in vehicles between the two nodes).
$IWT(i, j)$	= initial waiting time between nodes i and j, $i, j, \in N$ (defined as the amount of time passengers spend at the boarding stops between the two nodes).
$TWT(i, j)$	= transfer waiting time between nodes i and j, $i, j, \in N$ (defined as the amount of time passengers spend at the transfer stops between the two nodes).
LD_r	= passenger load discrepancy on route r (defined as the difference between the expected load and the desired occupancy; passenger load discrepancy measures the overcrowding seat use on vehicles).
FS	= fleet size (defined as the number of vehicles needed to provide all trips along a chosen set of routes).
α_k	= monetary or other weights, $k = 1, 2, 3, 4$.

For given weights of 1 or without units, Eq. (2) results in units of passenger hours (pass-h). Eq. (3) is simply the minimum fleet size required.

3.2.2. Objective function components

Eqs. (2) and (3) essentially combine five objective function components. The first objective component is to minimize the total passenger in-vehicle travel hours in the system. This is strictly from the perspective of PT users. The formulation of this objective component takes the following form:

$$Min\ \alpha_1 \sum_{i,j\in N} PH(i,j) \tag{4}$$

where α_1 is the monetary value of 1 h in-vehicle travel time. Specifically, its formulation is:

$$\sum_{i,j\in N} PH(i,j) = \sum_{r\in R}\sum_{i,j\in N_r} d_{ij}^r t_{ij}^r + \sum_{tr\in TR}\sum_{i,j\in N_{tr}} d_{ij}^{tr} t_{ij}^{tr} \tag{5}$$

The second objective component is to minimize total initial passenger waiting time at boarding stops. This is strictly from the perspective of the PT users. The following is the formulation of this objective component:

$$Min\ \alpha_2 \sum_{i,j\in N} IWT(i,j) \tag{6}$$

where α_2 is the monetary value of 1 h initial waiting time. Different formulations of the expected initial waiting time for PT passengers can be found in Marguier and Ceder (1984). For randomly arriving passengers, the expected initial waiting time on route r can be calculated as:

$$w_r = \frac{E(H)}{2}\left[1 + \frac{Var(H)}{E^2(H)}\right], \text{ for all } r \in R \tag{7}$$

where $E(H)$ and $Var(H)$ are, respectively, the mean and variance of headway time H between vehicles.

Assuming that PT passengers are arriving randomly at the stops and that the vehicle headways are relatively short and distributed in a deterministic and regular manner (with no variation of the headways, i.e., $Var(H) = 0$), this expected initial waiting time on route r is the half headway:

$$w_r = \frac{1}{2F_r}, \text{ for all } r \in R \tag{8}$$

where F_r can be calculated using the maximum load point method [See Ceder (2016) for details.]:

$$F_r = \max\left(\frac{L_r}{d_o}, F_{\min}\right) \tag{9}$$

If $F_r = F_{\min}$, then the load profile will have no influence on the frequency determination. Therefore, the total initial passenger waiting time can be calculated by:

$$\sum_{i,j\in N} IWT(i,j) = \sum_{r\in R}\frac{1}{2F_r}\left(\sum_{i,j\in N_r} d_{ij}^r + \sum_{i,j\in N_{tr}} d_{ij}^{tr} a_r^{tr}\right) \tag{10}$$

The third objective component is to minimize the total passenger transfer waiting time at transfer stops. This is strictly from the perspective of the PT users. The formulation of this objective component takes the following form:

$$Min\ \alpha_3 \sum_{i,j\in N} TWT(i,j) \tag{11}$$

where α_3 is the monetary value of 1 h transfer waiting time.

Consider a group of passengers $p_{k(i)q(j)}^n$ transferring from the $k(i)$th trip of the route that node i is in to the $q(j)$th trip of the route that node j is in at transfer stop n. Let $\Omega_{q(j)}$ represent the set of feasible transfer connecting trip patterns defined by:

$$\Omega_{q(j)} = \left\{q(j) \middle| D_{q(j)} - A_{k(i)} - W_{k(i)q(j)n} - \Delta \geq 0, k(i) \in K(i), q(j) \in Q(j), n \in N_{tr}(i,j)\right\} \tag{12}$$

where $A_{k(i)}$ is the arrival time of the $k(i)$th trip of the route that node i is in at transfer stop n; $D_{q(j)}$ is the departure time of the $q(j)$th trip of the route that node j is in at transfer stop n; $W_{k(i)q(j)n}$ is the transfer walking time needed from the arrival place of the$k(i)$th trip of the route that node i is in to the departure place of the $q(j)$th trip of the route that node j is in at transfer stop n; Δ is the constant time needed for alighting and boarding. It is assumed that the same group of passengers has the same transfer walking time and constant time needed for alighting and boarding, and passengers will board the first feasible transfer connecting trip (i.e., the vehicle capacity is sufficient), then the transfer waiting time for the passenger group $p_{k(i)q(j)}^n$ can be represented as follows:

$$T_{k(i)q(j)}^n = D_{q'(j)} - A_{k(i)} - W_{k(i)q'(j)n} - \Delta, k(i) \in K(i), n \in N_{tr}(i,j) \tag{13}$$

where $q'(j)$, the first feasible transfer connecting trip, is represented by:

$$q'(j) = \arg\min_{q(j)\in\Omega_{q(j)}} q(j) \tag{14}$$

Thus, the total passenger transfer waiting time can be represented by:

$$\sum_{i,j\in N} TWT(i,j) = \sum_{n\in N_{tr}(i,j)} \sum_{k(i)\in K(i),q(j)\in Q(j)} p^n_{k(i)q(j)}\left(D_{q'(j)} - A_{k(i)} - W_{k(i)q'(j)n} - \Delta\right) \tag{15}$$

The fourth objective component is to minimize the total passenger load discrepancy. It is from the perspectives of both the PT users and operators. The formulation of this objective component takes the following form:

$$Min\alpha_4 \sum_{r\in R} LD_r \tag{16}$$

where α_4 is the monetary value of 1 h in-vehicle crowding travel time.

Define p_r as the total number of passengers observed at the maximum load point of route r. p_r is a common service measure used by PT agencies worldwide, as described by Ceder (2016) and Liu and Ceder (2016a). The passenger load discrepancy cost of route r associated with x^m_r is defined as:

$$LD^m_r = \sum_{s\in S} \max\left[(p_r - m\cdot d_o), 0\right]\cdot t^s_r \tag{17}$$

Thus,

$$\sum_{r\in R} LD_r = \sum_{r\in R}\sum_{m\in M} x^m_r LD^m_r \tag{18}$$

The fifth objective component is to minimize the number of vehicles required. This is strictly from the perspective of the PT operators who wish to perform all PT trips using a minimum number of vehicles. This objective component takes the form:

$$MinFS \tag{19}$$

Remark 1. Estimation of the minimum fleet size can utilize the DF theory described in the previous section. Note that it may be sufficient to use the DF modelling technique to determine the stronger FS lower and upper bounds for this estimation. Practically speaking, the ITSVS problem usually involves a vast quantity of computations of route sets. Thus, the lower and upper bounds-based FS calculations can ease the computation effort for each route considered.

Objective function components (4), (6), (11), and (16) are all in terms of passenger-hour cost. Therefore, for the sake of simplicity, they can be summed up to Min Z_1 as shown in Eq. (2). The objective function component (18) stands alone to some extent and is termed Min Z_2 in Eq. (3).

3.2.3. Constraints

The mathematical mode includes four groups of constraints. The first group of constraints are the bundle departure constraints:

$$\sum_{m=L_r}^{U_r} x^m_r = 1, \forall r\in R \tag{20}$$

where m is an index of the number of departures running from L_r to U_r, i.e., $m = L_r, L_r+1, L_r+2, \cdots U_r-1, U_r$; L_r and U_r are respectively the lower and upper bounds of the number of vehicle departures for a given route r. This group of constraints ensures that only one bundle of departures is selected for a given route.

The second group of constraints is the DF bounds constraints:

$$d(u,t) \le D(u), t\in[T_1, T_2], u\in U \tag{21}$$

where $d(u, t)$ is the net number of departures less arrivals that occur before or at time t at terminal u as determined by the value of x^m_r. Thus, the left hand side of Eq. (21) can be represented as a linear function of the x^m_r variables. The value of this function for a given solution is that of an associated DF for terminal u at time t. This group of constraints ensure that the number of vehicles used at a given terminal u before and up to time t does not exceed the number of vehicles $D(u)$ assigned to terminal u.

The second group of constraint is the fleet size constraint:

$$\sum_{u\in U} D(u) \le N_0 = FS \tag{22}$$

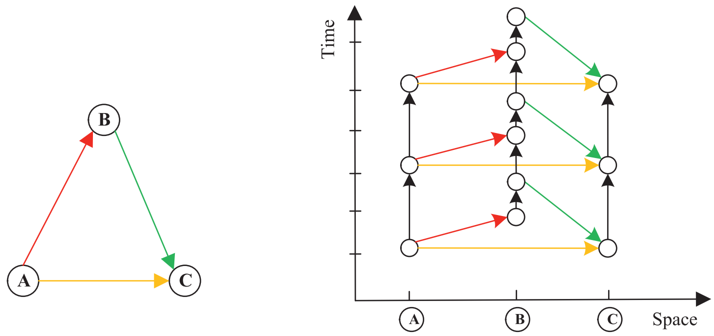
$$D(u)\in \mathbb{N}^0, \forall u\in U \tag{23}$$

where N_0 is the total fleet size. This constraint indicates that the sum of vehicles assigned to all termini should not be more than the required minimum fleet size FS=N_0.

The fourth group of constraints is the decision variable constraints:

$$x^m_r = \begin{cases} 1, & \text{if m even headway departrures are selected for route r} \\ 0, & \text{otherwise.} \end{cases}, \forall m\in M, r\in R \tag{24}$$

Please cite this article as: T. Liu, Avishai (Avi) Ceder, Integrated public transport timetable synchronization and vehicle scheduling with demand assignment: A bi-objective bi-level model using deficit function approach, Transportation Research Part B (2017), http://dx.doi.org/10.1016/j.trb.2017.08.024



(a) A physical network (b) The corresponding generic diachronic graph

Fig. 2. An illustration of transition from a physical network to a diachronic graph. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The optimal solution set $\{\bar{x}_r^m\}$ indicates the optimal number of departures. From $\{\bar{x}_r^m\}$, an optimal timetable can be constructed with proper route offset times.

Remark 2. Given the generated timetable, a vehicle schedule can be easily derived using the first-in-first-out (FIFO) rule [see Ceder and Stern, 1981 and Ceder, 2016 for details] or the chain extraction (CE) procedure [see Gertsbach and Gurevich, 1977 for details]. The vehicle schedule indicates the sequence of departures (trips) assigned to each vehicle in the fleet.

3.3. The lower level transit assignment (TA) problem

The lower level of the bi-level model represents the PT user route and vehicle run choice behavior responding to the upper level timetables and vehicle schedules. The accurate calculation of the number of passengers assigned to each vehicle run needs a sound transit assignment model. In the literature, there are various kinds of transit assignment models which have been developed. Generally speaking, they can be classified into frequency-based models and schedule-based models. The frequency-based models utilize aggregated representation of PT service and are usually used for strategic or long range planning problems. The schedule-based models are based on the actual vehicle run trajectories. They can determine the passenger loads on each vehicle run. Thus, they are more suitable for operation design problems. For a detailed description of the recent developments of the frequency-based and schedule-based models, the readers may refer to Gentile et al. (2016). Since we are doing a timetable design problem that aims at optimizing passenger transfers in a network, it is clear that schedule-based transit assignment models should be adopted. According to Gentile et al. (2016), there are three main approaches used for developing schedule-based models: (i) the diachronic graph-based approach, (ii) the dual graph-based approach, and (iii) the passenger choice set based approach. The diachronic graph-based approach is adopted in this study for its advantages in modelling run based assignment that is closely related to DF based modelling for vehicle scheduling.

In a diachronic graph, also called a space-time network, a node is modelled through a specific sub-graph whose nodes have space and time coordinates according to the timetable (see Nuzzolo et al., 2001; Nuzzolo, 2002 and Gentile et al., 2016 for details). Fig. 2 shows an example of how to transform a three node physical network to a generic diachronic graph in which black arcs represent waiting activities. The diachronic graph-based approach provides a more natural way to model the schedule-based transit assignment problem. In a diachronic graph, time related elements are built into its topological structure and thus it can clearly describe each vehicle run in a chronological order.

The lower level assignment model utilizes a utility function to describe users' perceived travel costs, i.e., generalized travel cost, travelling from origin i to destination j (Tong and Wong, 1999; Parbo et al., 2014). The formation of this generalized travel cost function, which is expressed in minutes, is given as follows:

$$t_{ij}^{\rho} = \beta_1 t_1^{\rho} + \beta_2 t_2^{\rho} + \beta_3 t_3^{\rho} + \beta_4 t_4^{\rho} + \beta_5 t_5^{\rho} + \beta_6 n^{\rho} \tag{25}$$

where t_1^{ρ} is the total walking time needed for getting from the origin place to the first stop and from the last stop to the destination place, t_2^{ρ} is the waiting time at the first stop, t_3^{ρ} is the in-vehicle travel time, t_4^{ρ} is the transfer walking time, t_5^{ρ} is the transfer waiting time, n^{ρ} is the time cost of the number of transfers involved in using path ρ, $\beta_i (i = 1, 2, ..., 6)$ are the corresponding weighting factors.

Remark 3. The generalized travel cost function can be extended by: (i) considering a time-dependent version of it, and (ii) incorporating the values of times (VoTs) to different groups of users.

Based on the calculation of generalized travel costs, a shortest (minimum cost) tree can be easily constructed on a diachronic graph by processing all nodes and using the Pallottino algorithm. [See Pallottino and Scutellà, 1998; and Gentile et al., 2016 for details.] With the help of this shortest tree, a network loading procedure is performed to conduct the demand assignment. The network loading procedure is done by performing a capacity-restrained incremental assignment, which is outlined as follows, by taking vehicle capacity constraints into account.

Remark 4. It should be pointed out that the proposed capacity-restrained incremental assignment procedure assumes that PT passengers have complete knowledge of the entire network and timetables, and that vehicles always keep to their schedules, i.e., without schedule delays. It will be interesting to extend it to a stochastic version to capture the perception and measure errors and heterogeneity of preferences of PT passengers.

3.4. Model integration

By combining the upper level ITSVS model and the lower level TA model, the resulting bi-objective, bi-level integrated timetable synchronization and vehicle scheduling problem with passenger assignment (ITSVS-TA) can be represented as follows:

$$\underset{\mathbf{S}}{Min}\, Z_1(\mathbf{S}, \mathbf{L}(\mathbf{S})) = \alpha_1 \sum_{i,j \in N} PH(i, j) + \alpha_2 \sum_{i,j \in N} IWT(i, j) + \alpha_3 \sum_{i,j \in N} TWT(i, j) + \alpha_4 \sum_{r \in R} LD_r \tag{26}$$

$$\underset{\mathbf{S}}{Min}\, Z_2(\mathbf{S}, \mathbf{L}(\mathbf{S})) = FS \tag{27}$$

$$\text{s.t.} \quad \text{Timetable synchronization and vehicle scheduling constraints: } (20) - (24) \tag{28}$$

where the passenger load on vehicles $\mathbf{L}(\mathbf{S})$ is obtained by solving the following TA problem:

$$\underset{\mathbf{L}}{\text{Min}}\, \mathbf{f}(\mathbf{S}, \mathbf{L}) \tag{29}$$

$$\text{s.t.} \quad \text{vehicle capacity constraint: } L_i \le \lambda C_v, \quad \forall i \in N \tag{30}$$

$$\text{passenger vehicle run choice and flow propagation constraints defined in Algorithm 1} \tag{31}$$

The nature of the upper level ITSVS model is non-linear, bi-objective integer programming with linear constraints, which is a special case of the integer quadratic programming problem that is known to be a NP-hard problem. In addition, solving the upper level ITSVS problem requires solving the lower level TA problem that is in effect a non-linear constraint of the upper level model, which makes the whole problem non-convex. Due to its intrinsic, non-linear and non-convex complexity, the bi-objective, bi-level ITSVS-TA problem is extremely difficult, to solve mathematically, especially for large scale networks, for an optimum global solution. In the next section, we propose a DF-based heuristic solution method for this problem by exploring its special structure properties.

4. Solution method

The successful implementation of this methodology largely depends on developing efficient solution algorithms. A new DF-based solution method has been developed to handle this problem for practical implementations. Before describing the DF-based solution method, a max flow technique for fixed schedule vehicle scheduling is introduced. A possible shifting trip departure time procedure is also introduced to further optimize the results obtained by the DF-based solution method.

4.1. Network flow technique for vehicle scheduling with fixed schedule

A network flow technique is employed to estimate the minimum fleet size of a given schedule**S**. A trip joining array for **S** may be constructed by associating the gth row with the arrival event of the gth trip, and the g'th column with the departure event of the g'th trip. Cell (g, g') will be admissible if g and g' can be joined feasibly. Otherwise, (g, g') will be an inadmissible cell. Let $x_{gg'}$be a 0–1 variable associated with cell (g, g') and G be the set of required trips. Then, consider the following problem:

$$MaxN_1 = \sum_{g \in G} \sum_{g' \in G} x_{gg'} \tag{32}$$

$$\text{s.t.} \quad \sum_{g' \in G} x_{gg'} \le 1, \quad g \in G \tag{33}$$

$$\sum_{g \in G} x_{gg'} \le 1, g' \in G \tag{34}$$

$$\left.\begin{array}{ll} x_{gg'} \in \{0, 1\}, & \text{all } (g, g') \text{ admissible} \\ x_{gg'} = 0, & \text{all } (g, g') \text{ inadmissible} \end{array}\right\} \tag{35}$$

Table 1
Trip schedule **S** for the example problem.

Trip number g	Departure terminal p^g	Departure time t_s^g	Arrival terminal q^g	Arrival time t_e^g
1	b	6:00	b	6:30
2	a	7:05	c	8:05
3	c	7:10	a	8:00
4	b	8:30	a	9:20
5	a	9:00	b	9:45

Table 2
Average DH travel time (min) matrix for the example in Table 1.

		Arrival terminal		
		a	b	c
Departure terminal	a	0	30	50
	b	35	0	45
	c	45	40	0

A solution with $x_{gg'} = 1$ indicates that trips g and g' are joined. The objective function maximizes the number of such joinings. Constraint (33) insures that each trip may be joined with no more than one successor trip. Similarly, constraint (34) indicates that each trip may be joined with no more than one predecessor trip. This problem is equivalent to a special arrangement of the maximum flow (max-flow) problem. The max-flow algorithm that solves the vehicle scheduling problem with DH trips is called an augmenting path algorithm. It is addressed at length in the classic book by Ford and Fulkerson (1962). A complete description of the augmenting path algorithm can be found in Ceder (2016). The vehicle scheduling problem can be transformed to a unit capacity bipartite network in which the solution time has the complexity of $O(n^{1/2}m)$ with n nodes (departure times) and m arcs. The following theorem states that maximizing N_1 is tantamount to minimizing the number of chains for a trip schedule of size n.

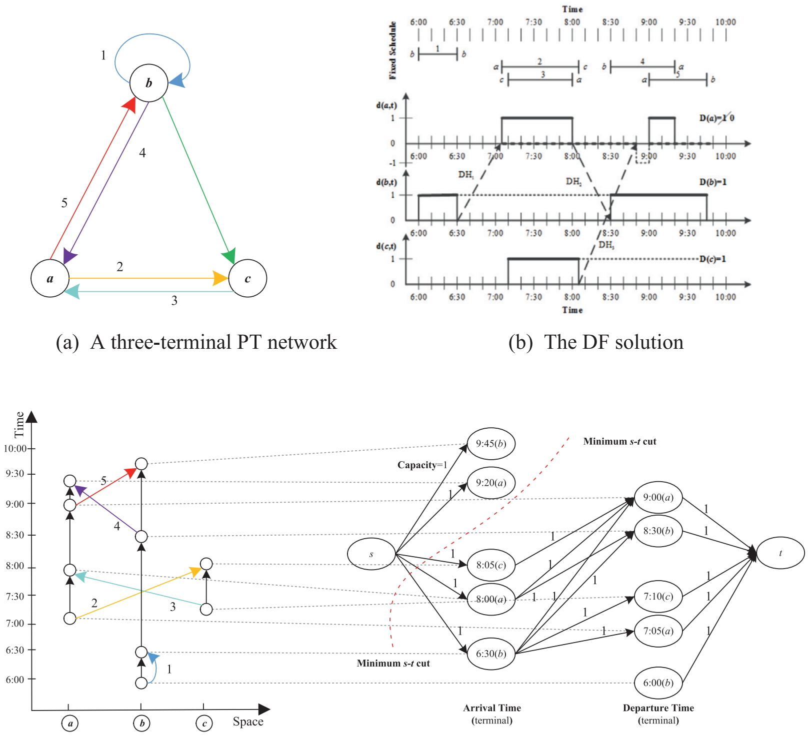
Theorem 2. (The max-flow fleet size theorem). *Let $N_{MF}(S)$ and n denote the number of chains and trips of schedule S, respectively. Then,*

$$MinN_{MF}(S) = n - MaxN_1 \tag{36}$$

Proof. Given a set of $G = \{g\text{: } g = 1, \ldots, n\}$ required trips. Assigning each trip separately to an individual vehicle results in a fleet size of n vehicles. If $x_{gg'} = 1$, then trip g' can be performed after trip g by the same vehicle v_g. Thus, the vehicle $v_{g'}$ assigned to trip g' can be saved. The required fleet size thus can be reduced from n to $n - 1$. Similarly, the value of max-flow $MaxN_1$ means $MaxN_1$ vehicles can be saved by linking trips together. Thus, the minimum number of vehicles required to perform all trips in G is $n - MaxN_1$. This completes the proof. □

Example: Consider the three terminal problem defined by the data in Tables 1 and 2 and Fig. 3(a). The data in Table 1 and Fig. 3(a) are transformed into the generic diachronic graph in Fig. 3(c) and network-flow representation in Fig. 3(d), which has two dummy nodes: a source node s and a sink node t. The nodes, being connected from s, are the arrival times of the example, with an indication, in parentheses, of the arrival terminal. The nodes connected to t are the departure times, with an indication of the departure terminal. Feasible connections between the arrival and departure times, utilizing the DH data in Table 2, establish the arcs between the left and right nodes, based on Eq. (32). Each arc capacity represents the number of connections that can flow through the arc. In our case, there is only a unit capacity assigned to each arc, because only one connection (if any) between a given arrival time and terminal and a given departure time and terminal is possible. The more flow created, the fewer chains will be required as stated by Theorem 2. The objective function N_1 equals the flow to be created at s and absorbed at t. Since max-flow = minimum s-t cut in the original network flow. This minimum s-t cut is shown in Fig. 3(d). The result of the example is max-flow $= MaxN_1 = 3$, and, following Theorem 2, $MinN_{MF}(S) = n - MaxN_1 = 5 - 3 = 2$ chains. Restated, the timetable in Table 1 can be carried out by a minimum of two vehicles having the connections shown in Fig. 3(d).

The equivalent DF solution, shown in Fig. 3(b), also results in two vehicles:$MinN_{DF}(S) = D(a) + D(b) + D(c) = 0 + 1 + 1 = 2$. Explicitly the two blocks, by their trip number in 1, are **[**1-DH$_1$-2-DH$_3$-5**]** and **[**3-DH$_2$-4**]**. These two blocks have three DH trips for connecting arrival and departure terminals: DH$_1$(b-a) and DH$_3$(c-a) (in the first block) and DH$_2$(a-b) (in the second block), with the total of $35 + 45 + 30 = 110$ min DH time respectively. The $MinN_{MF}(S)$ can be used as an initial lower bound of the required fleet size when using a DF-based sequential search method for solving the bi-objective, bi-level ITSVS-TA problem, which is described in the following section.



(a) A three-terminal PT network (b) The DF solution

(c) The corresponding generic diachronic graph (d) The max-flow solution

Fig. 3. An example vehicle scheduling problem using max-flow technique and its equivalent DF solution. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

4.2. Deficit function based sequential search method

Based on the special structure of the model, we developed a novel DF-based sequential search (DF-SS) method to generate Pareto-efficient solutions. The key point behind the DF-SS method is to decompose the original bi-objective programming model to a series of one-objective programming models. The decomposition is achieved by estimating the lower bound and upper bound of objective function Z_2, i.e., the fleet size, which can be accomplished by using the network flow and DF techniques. The proposed DF-SS method is outlined below in a step by step manner.

In Step 2, some available integer programming solvers, such as CPLEX, GAMS and MATLAB, can be utilized to efficiently solve the resulting single objective integer programming model. In the DF-SS method, the firstly identified lower and upper bounds of the required vehicle fleet size can significantly reduce the complexity of the problem.

4.3. A possible shifting departure time (SDT) procedure

It is certain that the results may be improved by allowing the timetable obtained by the DF-SS method to be shifted within given shifting tolerances. The implications are that for each transfer stop and for each two time points: t_1, t_2 ($t_1 < t_2$) at which there are vehicle arrivals at the transfer stop, there is an attempt to shift departure times (offset times) of all vehicles arriving at the node at t_1 so that they will arrive at time t_2. If this succeeds, the timetable is changed accordingly, and the passenger-hour cost may be reduced. In addition, after performing SDT, the vehicle fleet size may be also further reduced (Ceder 2002). However, it should be noted that due to the impact of the changed schedule on passengers' route/trip choice behavior, the vehicle passenger loads **L** may also be changed after the SDT procedure, which may lead to unbalanced passenger loads, i.e., overcrowding or empty seat hours, and bus bunching problems. Therefore, the SDT procedure must be performed very carefully by experienced schedulers with the help of DF graphical displays of the optimized timetables. See Gertsbakh and Stern (1978), Ceder (2001) and Ceder (2002) for details.

4.4. Overall solution procedure

An overview of the solution procedure is outlined as follows:

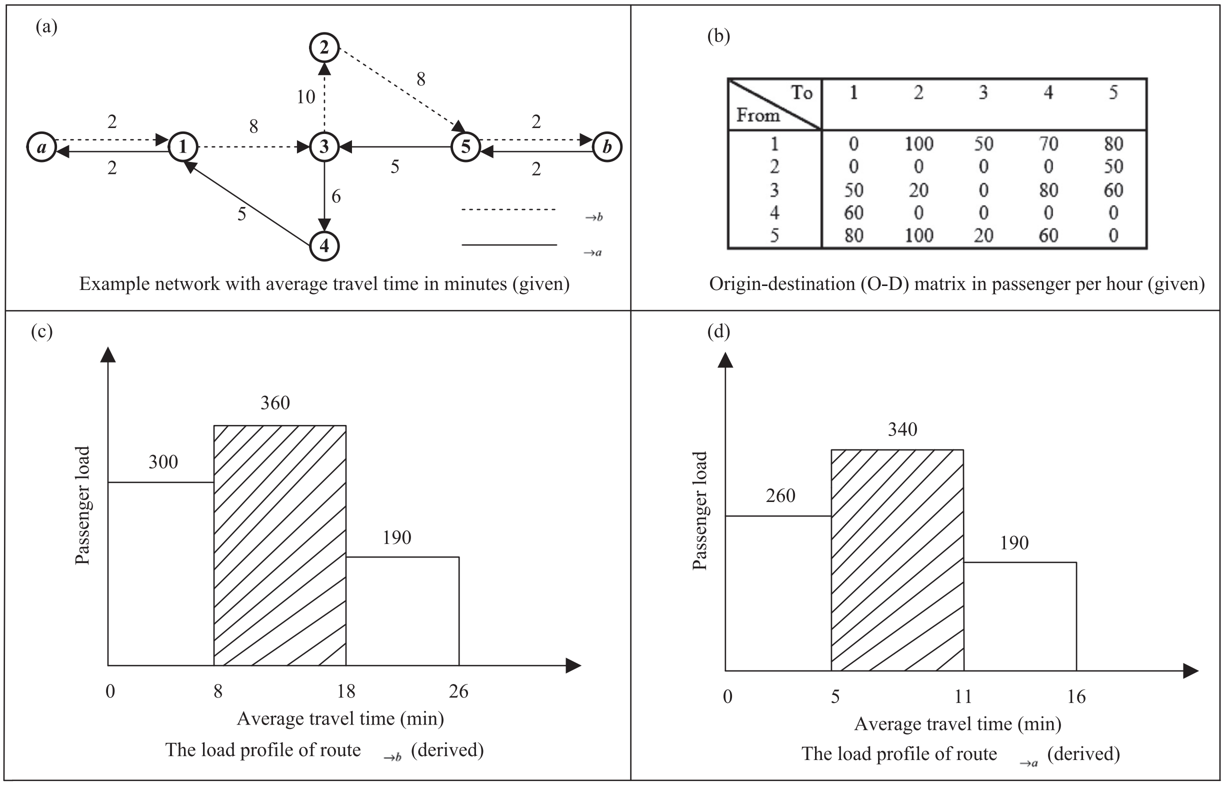


Fig. 4. Example network, its demand data, and the construction of load profiles.

Step 0	Estimate possible route frequencies based on O-D demand, vehicle capacity, policy frequency requirement and other parameter values.
Step 1	Generate Pareto-efficient solutions using Algorithm 1 and Algorithm 2.
Step 2	Modify route offset times using the SDT procedure, based on schedulers' DF observations, personal preferences or other practical considerations, so as to generate more possible Pareto-efficient solutions.
Step 3	Display the combined Pareto-efficient solutions obtained in Step 1 and Step 2 in a fleet cost two-dimensional (2D) space.

5. Numerical studies

In this section, two numerical examples are presented for comprehending the application of the proposed model and DF-based solution method for the bi-objective, bi-level ITSVS-TA problem. The first is a detailed small-sized example network. The second is a medium-sized network adapted from Spiess and Florian (1989).

5.1. A small illustrative example

A small example is provided to illustrate the proposed model and solution method. The example PT network, which is adapted from Liu and Ceder (2017a) and shown in Fig. 4(a) has two termini ($a$and$b$), two routes ($r_{a\rightarrow b}$ and $r_{b\rightarrow a}$) and one transfer stop (node 3). The input data during the given time period (7:00–8:00) consist of average travel times (in minutes) and an estimated O-D demand matrix in part b of Fig. 4. Based on the input data, the passenger load profiles presented in Fig. 4(c) and Fig. 4(d) are constructed for routes $r_{a\rightarrow b}$ and $r_{b\rightarrow a}$, respectively. For route $r_{a\rightarrow b}$, the load on route segment 3–2 is the maximum load (360 passengers) of the O-D demand: 1–2, 1–5, 3–2, 3–5, 5–2 (see Fig. 1(b)). Similarly to route $r_{b\rightarrow a}$, the load on route segment 3–4 is the maximum load (340 passengers) of the O-D demand: 5–4, 5–1, 3–4, 3–1, 1–4. Other input data for this example network are presented in Tables 3–5. The average route travel times for route $r_{a\rightarrow b}$ and route $r_{b\rightarrow a}$ are set as 30 min and 20 min, respectively. The desired occupancy for both of the two routes is set up to 70 passengers. In addition, vehicle dwell times at transfer stop 3 of vehicles of the two routes are 1 min. The transfer walking time $W_{k(i)q(j)n}$ needed from the arrival place to the departure place is set up to 0.5 min. The constant time Δ needed for alighting and boarding is set as 0.5 min. The four weights $\alpha_k, k = 1, 2, 3, 4$ take the value of 1.

Note that in this example network, there is only one feasible path between each O-D pair. Thus, the stop/link passenger flows resulted from the lower level transit assignment will remain fixed. Indeed, the generalized travel costs of paths obtained by Eq. (25) will not impact passengers' travel path choice behavior.

Table 3
The basic input data for the numerical example.

Route	Average route travel time (min)	Time span	Number of passengers P	Desired occupancy d_o	Number of departures (Frequencies)							
					$q=3$		$q=4$		$q=5$		$q=6$	
					LD cost (pass.h)	Variable	LD cost (pass.h)	Variable	LD cost (pass.h)	Variable	LD cost (pass.h)	Variable
$r_{a\to b}$	30	7:00–8:00	360	70	–	–	13.33	x_1	1.67	x_2	0	x_3
$r_{b\to a}$	20	7:00–8:00	340	70	–	–	6	x_4	0	x_5	–	–

Algorithm 1 Capacity restrained incremental assignment.

Step 0 (*Preliminaries*): Calculate initial passenger O-D demand d_{ij1} and number of passengers L_{i1} on vehicle at stop i. Set n: =1.
Step 1 (*Incremental loading*): If $d_{ijn} \le \lambda C_v - L_{in}$, then the packet of d_{ijn} passenger demand is loaded onto the n-shortest path, i.e., $L_{in} := L_{in} + d_{ijn}$, and stop; otherwise, the packet of $(\lambda C_v - L_{in})$ demand is loaded onto the n-shortest path, i.e., $L_{in} := L_{in} + (\lambda C_v - L_{in})$.
Step 2 (*Update*): Set n: $=n+1$, $d_{ijn} = d_{ij(n-1)} - (\lambda C_v - L_{i(n-1)})$, and go to Step 1.

Algorithm 2 Deficit function-based sequential search (DF-SS) method.

Step 0 (*Lower bound and upper bound calculation*): Apply the network flow technique to obtain an initial lower bound of the required fleet size N'_L. Construct the DF for each route r with $m=L_r$, $r \in R$, and calculate the lower bound of the fleet size as $N_L = min\{N'_L, \sum_{u\in U} D(u)\}$; construct the DF for each route r with $m=U_r$, $r \in R$, and calculate the upper bound of the fleet size as $N_U = min\{n, \sum_{u\in U} D(u)\}$.
Step 1 (*Initialization*): Set $v=1$, $Z_2^1 = 1$, $\mathbf{Z} = [0]_{|N_U - N_L + 1|\times 2}$, $\mathbf{X} = [0]_{|N_U-N_L+1|\times|R|}$.
Step 2 (*Z_1^v Calculation*): Decompose the original bi-objective model to a single objective model with Z_2^v known. Solve the resulting single objective integer programming model. This yields the value of Z_1^v and a solution set of $[x_{r_1}^{m,v}, x_{r_2}^{m,v}, \cdots, x_{r_{|R|}}^{m,v}]$. Replace the v-th row of $\mathbf{Z}$ with $[Z_1^v, Z_2^v]$; replace the v-th row of $\mathbf{X}$ with $[x_{r_1}^{m,v}, x_{r_2}^{m,v}, \cdots, x_{r_{|R|}}^{m,v}]$.
Step 3 (*Move*): Set $Z_2^{v+1} = Z_2^v + 1$.
Step 4 (*Stopping rule*): If $Z_2^{v+1} > N_U$, go to Step 5; otherwise, set v: $=v+1$ and go to Step 3.
Step 5 (*Solution output*): Generate Pareto-efficient solutions from matrix $\mathbf{X}$ with associated objective function values (Z_1,Z_2) generated by matrix $\mathbf{Z}$.

The input data result in three sets of number of departures ($q=4$,$q=5$ and $q=6$) for route $r_{a\to b}$, and two sets of number of departures ($q=4$ and $q=5$) for route $r_{b\to a}$. Thus, there are five associated variables, namelyx_1, x_2, x_3, x_4 and x_5, as shown in Table 3. The LD cost is calculated by using Eq. (17) based on the maximum load data, route segment travel time, desired occupancy and number of departures. For example, for decision variable x_1, its associated LD cost is $\{\max[(360-4\times 70),\ 0]\times 10\}/60 = 13.33$ pass-h. The LD costs for other decision variables, as displayed in Table 3, are calculated in this way. The initial passenger waiting time is calculated by using Eq. (10). For this example problem, for passengers taking route $r_{a\to b}$ with $q=4$ departures, the initial waiting time is $IWT(r_{a\to b}, x_1) = [1/(2\times 4)] \times [(50+100+80+70)+(20+60)+(50)+(100)] = 66.25$ pass-h. For other decision variables, the initial passenger waiting time can also be calculated in this way. Thus, we have: $IWT(r_{a\to b}, x_2) = 53.00$ pass-h, $IWT(r_{a\to b}, x_3) = 44.17$ pass-h, $IWT(r_{b\to a}, x_4) = 65.00$ pass-h and $IWT(r_{b\to a}, x_5) = 52.00$ pass-h. The total passenger transfer waiting time cost is calculated by using Eq. (15), which is based on the number of transferring passengers and the transfer waiting time. In this example problem, we assume a uniform distribution of transferring passengers during the time period. Thus, the total passenger transfer waiting time cost can be calculated by using the average transfer waiting time of different groups of transferring passengers. That is $\sum_{i,j\in N} TWT(i,j) = \begin{pmatrix} (1/60)\times 70\times(12x_1x_4 + 4.5x_1x_5 + 6x_2x_4 + 9x_2x_5 + 7x_3x_4 + 6x_3x_5) \\ +(1/60)\times 100\times(3x_1x_4 + 4.5x_2x_4 + 5.5x_3x_4 + 6x_1x_5 + 3x_2x_5 + 5x_3x_5) \end{pmatrix} = 19x_1x_4 + 29.25x_1x_5 + 14.5x_2x_4 + 15.5x_2x_5 + 17.33x_3x_4 + 15.33x_3x_5$. The in-vehicle passenger riding time is calculated using Eq. (5). For this example network, Eq. (4) yields $\sum_{i,j\in N} PH(i,j) = (1/60)\times \begin{bmatrix} (300\times 8 + 360\times 10 + 140\times 8)+ \\ (260\times 5 + 340\times 6 + 190\times 5) \end{bmatrix} = 190.17$ pass-h. Adding the four passenger hour cost components yields the objective function:

$$Z_1 = \begin{pmatrix} \underbrace{13.33x_1 + 1.67x_2 + 6x_4}_{\sum_{r\in R} LD_r} + \underbrace{66.25x_1 + 53.00x_2 + 44.17x_3 + 65.00x_4 + 52.00x_5}_{\sum_{i,j\in N} IWT(i,j)} \\ + \underbrace{19x_1x_4 + 29.25x_1x_5 + 14.5x_2x_4 + 15.5x_2x_5 + 17.33x_3x_4 + 15.33x_3x_5}_{\sum_{i,j\in N} TWT(i,j)} + \underbrace{190.17}_{\sum_{i,j\in N} PH(i,j)} \end{pmatrix}.$$

The even headway departure and arrival times of each of the five x_i,$i=1, 2, 3, 4, 5$ are listed in Table 4, which are used to construct the DF bounds constraints. The example problem can now be formulated as the following bi-objective nonlinear

Table 4
Departure and arrival times at terminal nodes and node 3, and passenger ready for departure times at node 3 (route $r_{a\to b}$).

Departure terminal	A														
Arrival terminal	B														
Variable	x_1				x_2					x_3					
Departure time	7:15	7:30	7:45	8:00	7:12	7:24	7:36	7:48	8:00	7:10	7:20	7:30	7:40	7:50	8:00
Arrival time	7:45	8:00	8:15	8:30	7:42	7:54	8:06	8:18	8:30	7:40	7:50	8:00	8:10	8:20	8:30
Arrive time at node 3	7:25	7:40	7:55	8:10	7:22	7:34	7:46	7:58	8:10	7:20	7:30	7:40	7:50	8:00	8:10
Departure time From node 3	7:26	7:41	7:56	8:11	7:23	7:35	7:47	7:59	8:11	7:21	7:31	7:41	7:51	8:01	8:11
Passenger ready for departure time at node 3	7:26	7:41	7:56	8:11	7:23	7:35	7:47	7:59	8:11	7:21	7:31	7:41	7:51	8:01	8:11

integer programming model with linear constraints.

$$Min\ Z_1 = \begin{pmatrix} 19x_1x_4 + 29.25x_1x_5 + 14.5x_2x_4 + 15.5x_2x_5 + 17.33x_3x_4 + 15.33x_3x_5 \\ +79.58x_1 + 54.67x_2 + 44.17x_3 + 71.00x_4 + 52.00x_5 + 190.17 \end{pmatrix} \tag{37}$$

$$Min\ Z_2 = FS = N_0 \tag{38}$$

$$\text{s.t.} \quad x_1 + x_2 + x_3 = 1 \tag{39}$$

$$x_4 + x_5 = 1 \tag{40}$$

$$2x_1 + 2x_2 + 3x_3 \le D(a) \tag{41}$$

$$2x_1 + 3x_2 + 4x_3 - x_4 - 2x_5 \le D(a) \tag{42}$$

$$3x_1 + 4x_2 + 5x_3 - 2x_4 - 2x_5 \le D(a) \tag{43}$$

$$4x_1 + 5x_2 + 6x_3 - 2x_4 - 3x_5 \le D(a) \tag{44}$$

$$2x_4 + 3x_5 \le D(b) \tag{45}$$

$$-x_1 - x_2 - x_3 + 3x_4 + 3x_5 \le D(b) \tag{46}$$

$$-x_1 - x_2 - 2x_3 + 3x_4 + 4x_5 \le D(b) \tag{47}$$

$$-2x_1 - 2x_2 - 3x_3 + 4x_4 + 5x_5 \le D(b) \tag{48}$$

$$D(a) + D(b) \le N_0 \tag{49}$$

$$D(a), D(b) \in \mathbb{N}^0 \tag{50}$$

$$x_i = 0 \text{ or } 1;\ i = 1, 2, 3, 4, 5 \tag{51}$$

where Eqs. (37) and (38) are the two principal objective functions; Eqs. (39) and (40) are the bundle departure constraints; Eqs. (41)–(48) are the DF bound constraints, which are constructed by using the timetable information given in Tables 4 and 5. Accordingly, each possible combination of the net number of departures for a given terminal is restricted so as not to exceed the number of vehicles (maximal value of the DF) assigned to that terminal. For example, the constraint in Eq. (41) refers to $7{:}00 \le t < 7{:}32$ regarding the net number of departures in terminal a. Eqs. (42)–(48) are constructed in the same way. For a detailed description of the procedure for constructing the DF bounds constraints, readers are referred to Ceder (2016). Eqs. (49) and (50) are the fleet size constraints. Eq. (51) represents the decision variable constraints.

Table 5
Departure and arrival times at terminal nodes and node 3, and passenger ready for departure times at node 3 (route $r_{b\to a}$).

Departure terminal	*b*								
Arrival terminal	*A*								
Variable	x_4				x_5				
Departure time	7:15	7:30	7:45	8:00	7:12	7:24	7:36	7:48	8:00
Arrival time	7:35	7:50	8:05	8:20	7:32	7:44	7:56	8:08	8:20
Arrive time at node 3	7:22	7:37	7:52	8:07	7:19	7:31	7:43	7:55	8:07
Departure time from node 3	7:23	7:38	7:53	8:08	7:20	7:32	7:44	7:56	8:08
Passenger ready for departure time at node 3	7:23	7:38	7:53	8:08	7:20	7:32	7:44	7:56	8:08

Table 6
Route data.

Route	Commercial speed (km/h)	Vehicle capacity (pax)
Route 1-Red	24	80
Route 2-Green	30	80
Route 3-Blue	45	80
Route 4-Black	18	80

By applying this fleet size estimation method in Algorithm 2, the fleet size lower bound and upper bound are obtained by the DFs shown in Fig. 5(a) and 5(b). Fig. 5(a) is based on the minimum number of departures for trip bundles, i.e., (x_1,x_4), which yields the fleet size lower bound $N_L = D(a) + D(b) = 2 + 2 = 4$. Fig. 5(b) is based on the maximum number of departures for trip bundles, i.e.,(x_3,x_5), which yields the fleet size upper bound $N_U = D(a) + D(b) = 3 + 3 = 6$. Based on the calculated fleet size lower bound and upper bound, the original bi-objective nonlinear integer programming problem is decomposed to three one-objective integer programming problems with known objective function$Z_2 = FS = N_0 = 4$, 5, 6. The resulting three one-objective integer programming problems can be easily solved, yielding the following three sets of Pareto-efficient solutions:

$$\text{Set } 1 = (x_1, x_4) \rightarrow \begin{pmatrix} Z_1 = 359.75 \\ Z_2 = 4 \end{pmatrix}; \ \text{Set } 2 = (x_2, x_5) \rightarrow \begin{pmatrix} Z_1 = 312.34 \\ Z_2 = 5 \end{pmatrix}; \ \text{Set } 3 = (x_3, x_5) \rightarrow \begin{pmatrix} Z_1 = 301.67 \\ Z_2 = 6 \end{pmatrix}.$$

In addition, by applying the SDT procedure to modify the offset times of the timetable generated by solution set 1, as shown in Fig. 6, the fleet size can be reduced from 4 to 3, but which results in increased passenger-hour costs. This produces another Pareto-efficient solution: Set 4 $= \begin{pmatrix} Z_1 = 375.58 \\ Z_2 = 3 \end{pmatrix}$.

To facilitate the multi-criteria decision making process of PT schedulers, a graphical display of alternative solution sets and the associated objective functions in a fleet-cost 2D space are provided. For the example problem, the four Pareto-efficient solutions are depicted in Fig. 7. PT schedulers need to make a tradeoff between passenger-hour cost Z_1 and vehicle fleet size Z_2 displayed in a fleet-cost 2D space. With this graphical information in hand, the PT schedulers are able to choose a desired solution or a desired set of solutions based on their preferences and practical considerations, by taking account of both PT user and operator interests.

5.2. Application to the Spiess–Florian network

The model and solution method described in previous sections were applied to the network used by Spiess and Florian (1989), which is a classic network used in transit assignment problems. The network, depicted in Fig. 8, comprises four transit routes and four transfer stops. There are two transfer stops, stop *b* and *c*, in this network. The red route 1 operates in a separated corridor and the other three routes, routes 2, 3 and 4, operate along the same corridor. It is assumed that stop *d* is within a central business district (CBD), and another three stops are directed to stop *d*. In this network, travels have the choice between a number of different paths to reach their final destination. For example, for trips departing from stop *a* and arriving at stop *d*, there are five different paths for reaching stop *d*, namely 1-*a*-*d*, 2-*a*-*c*-3-*d*, 2-*a*-*c*-4-*d*, 2-*a*-*b*-3-*d*, 2-*a*-*b*-3-*c*-4-*d*. Thus, the impact of timetable and vehicle schedule on travelers' path/trip choice behavior should be taken into consideration while doing the integrated timetable synchronization and vehicle scheduling optimization.

The input data, which includes route data shown in Table 6, route segment data shown in Table 7 and O-D demand data shown in Table 8, are taken from Noekel et al. (2016). The schedule horizon is [7:00, 8:00] representing the morning peak hour. The weighting factors in objective functionZ_1 are set as $\alpha_1 = 1$,$\alpha_2 = 1.5$, $\alpha_3 = 2$ and $\alpha_4 = 1.2$. The weighting factors in the generalized travel cost function, Eq. (25) are set as $\beta_1 = 0$, $\beta_2 = 1.5$, $\beta_3 = 1$, $\beta_4 = 1.5$, $\beta_5 = 2$ and $\beta_6 = 5$ min per transfer stop. The policy frequencies (the minimum required frequencies) for all four routes are set as 1 veh/h. Note, that to simplify the analysis, the walking time needed for getting from the origin to the first stop and from the last stop to the destination are not considered. In addition, the transfer walking time for all the transfers made by travelers in the network are set as

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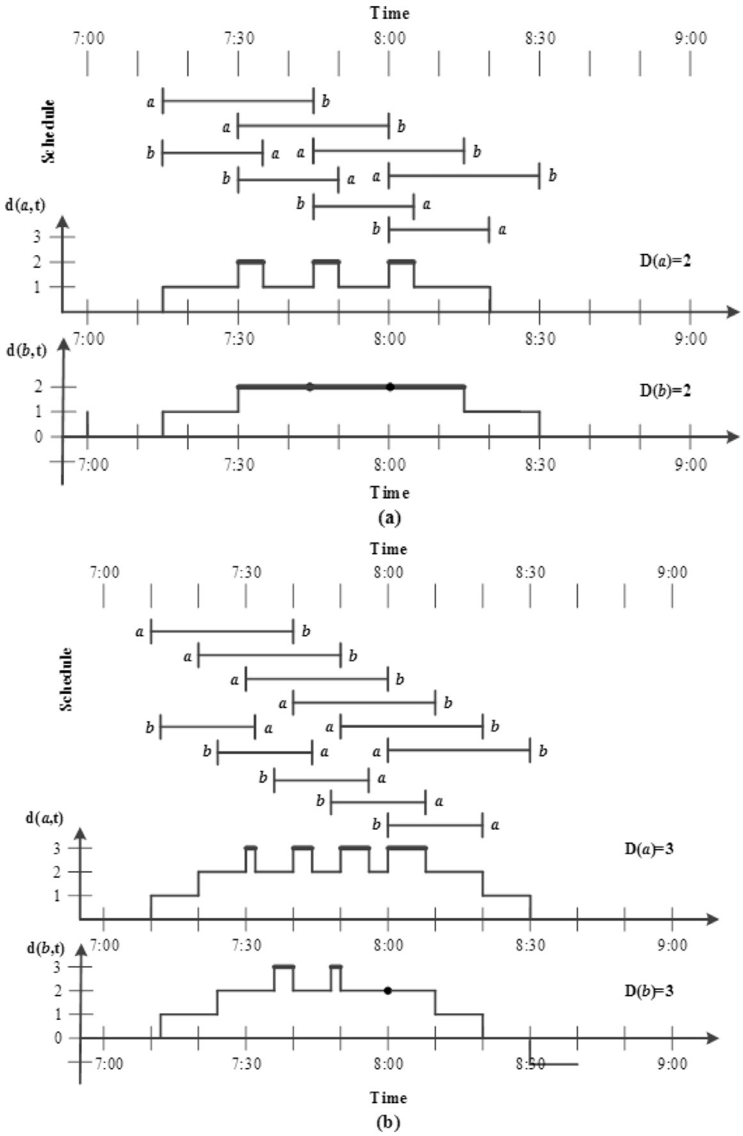

Fig. 5. Deficit function for estimating fleet size lower bound and upper bound of the example network timetable.

Table 7
Route segment data.

Route	Route segment	Length (km)	Running time (min)
Route 1-Red	(*a*,*d*)	10	25
Route 2-Green	(*a*,*b*)	3.5	7
Route 2-Green	(*b*,*c*)	3	6
Route 3-Blue	(*b*,*c*)	3	4
Route 3-Blue	(*c*,*d*)	3	4
Route 4-Black	(*c*,*d*)	3	10

Table 8
O-D pairs and travel demand.

Origin	Destination	Flow (pax/h)
Stop *a*	Stop *d*	300 = 5pax/min
Stop *b*	Stop *d*	360 = 6pax/min
Stop *c*	Stop *d*	240 = 4pax/min

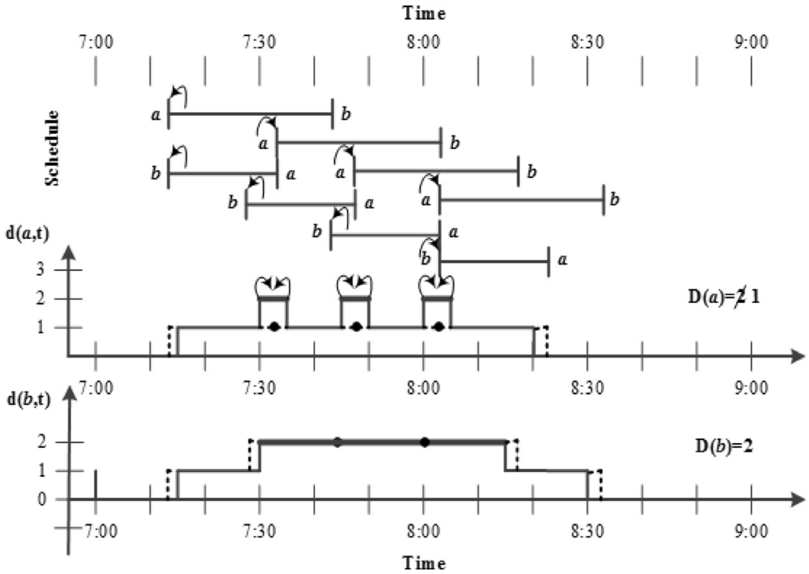

Fig. 6. Using SDT procedure to reduce the fleet size from 4 to 3. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

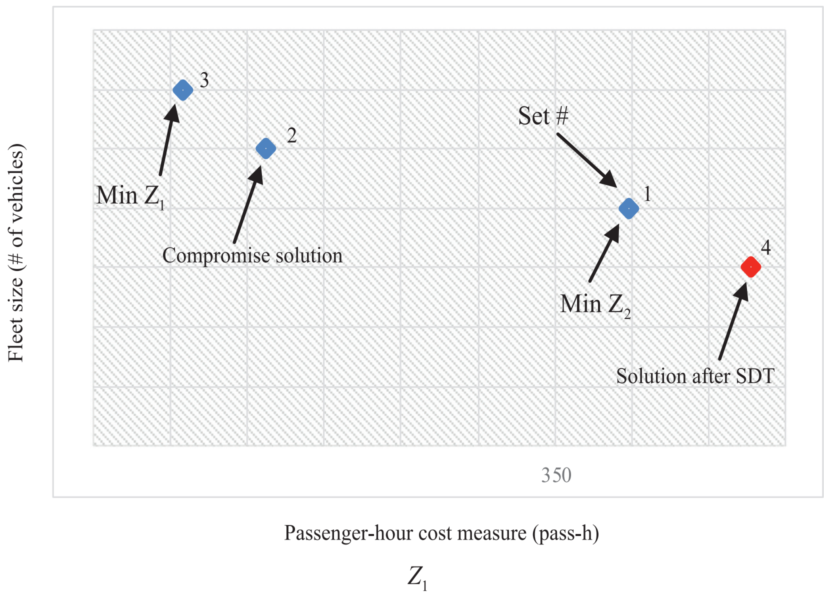

Fig. 7. Pareto-efficient solutions: tradeoff between passenger-hour cost Z_1 and vehicle fleet size Z_2 of the example problem in a fleet-cost 2D space. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

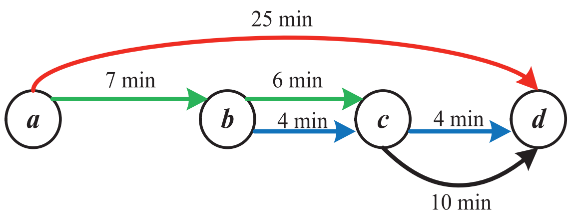

Fig. 8. The Spiess–Florian network with 4 routes and 4 stops. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

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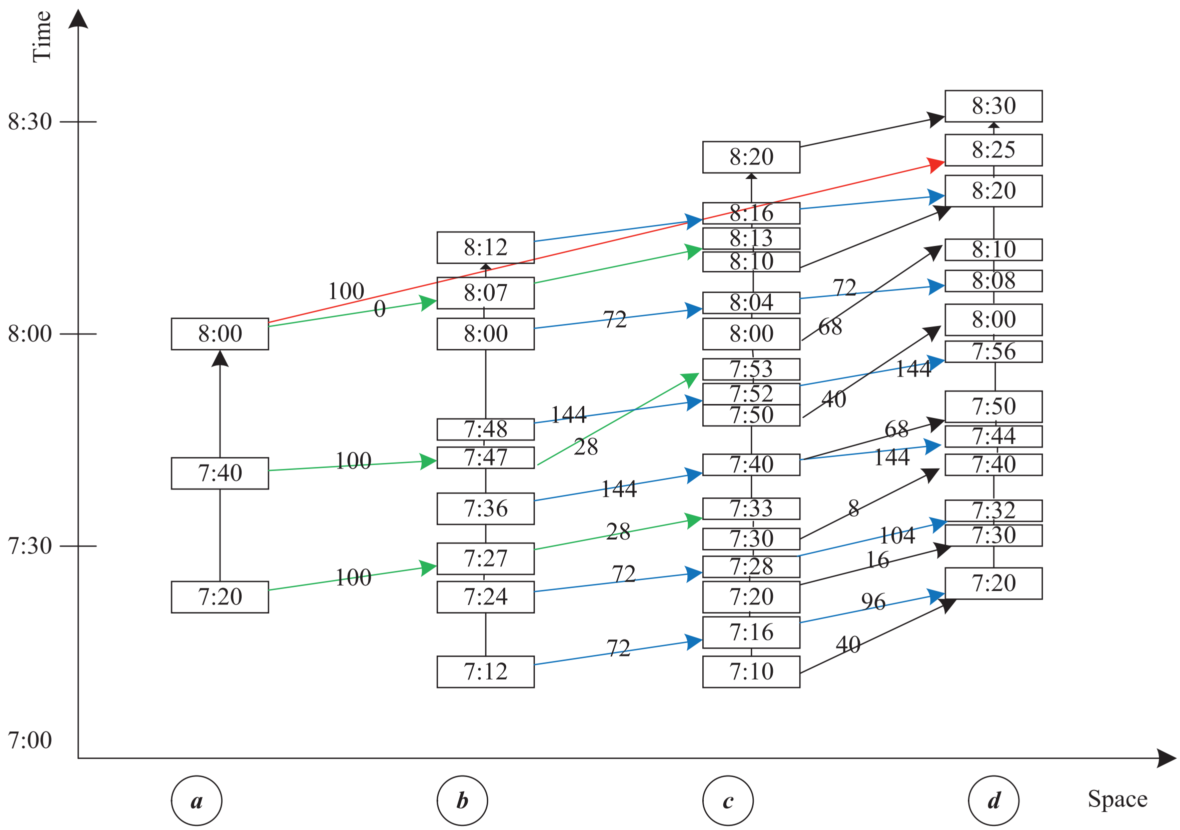

Fig. 9. An assignment result on the diachronic graph applied to the Spiess–Florian network. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

1 min. The desired occupancy for all four routes are set as the capacity of the vehicles, i.e., $d_o = C_v = 80$. The vehicle overload factor is set as $\lambda = 1.8$.

It is assumed that passenger O-D travel demands are evenly distributed in the schedule horizon. That is the O-D demands are not time dependent and without fluctuation. It is also assumed that all passengers relate to the VoT to the same degree. Passengers' path/trip choice behavior is assumed to follow the principle defined in Section 3.3.

The analysis begins with setting a set of possible number of departures (frequencies) for each route. To this end, a network synthesis process (see Ceder 2016 for details) is utilized to generate a set of initial frequencies based on passenger O-D demand. It results in:

$$f_1 + f_2 \geq \frac{300}{80} = 3.75, \quad f_1 + f_2 + f_3 \geq \frac{300 + 360}{80} = 8.25, \quad f_1 + f_3 + f_4 \geq \frac{300 + 360 + 240}{80} = 11.25.$$

To simplify the analysis, we set $f_1 + f_2 = 4$, $f_1 + f_2 + f_3 = 9$, $f_1 + f_3 + f_4 = 12$. Solving this system of linear equations following policy frequency constraints of $f_i \geq 1$, $i = 1, 2, 3, 4$ results in the following set of frequencies:

$$\text{Set 1} = (f_1, f_2, f_3, f_4) = (1, 3, 5, 6), \quad \text{Set2} = (f_1, f_2, f_3, f_4) = (2, 2, 5, 5), \quad \text{Set3} = (f_1, f_2, f_3, f_4) = (3, 1, 5, 4).$$

Based on the initial frequency sets, the lower level transit assignment process is conducted based on the resulting diachronic graph by using the capacity restrained incremental assignment algorithm. The weighted generalized shortest travel paths are constructed from a shortest tree, which is generated by using the Pallottino algorithm. Fig. 9 shows the demand assignment results for a timetable on its corresponding generic diachronic graph. Note that the numbers on the vehicle run lines indicate the resulting passenger flows. Based on the assignment results, the upper level objective function components can be easily calculated.

In this case study, vehicle DH trip may be inserted between any pair of terminals following DH travel time constraints. In addition, the allowed left and right shifting tolerances are both taken as 8 min. After performing the methodology, three Pareto-efficient solutions are obtained. The tradeoff situation between the passenger-hour cost Z_1 and the vehicle fleet size Z_2 with regard to this case study is depicted in a fleet-cost 2D space, as shown in Fig. 10. These solutions provide a Pareto-efficient frontier based on which the transit schedulers can easily choose their desired solutions.

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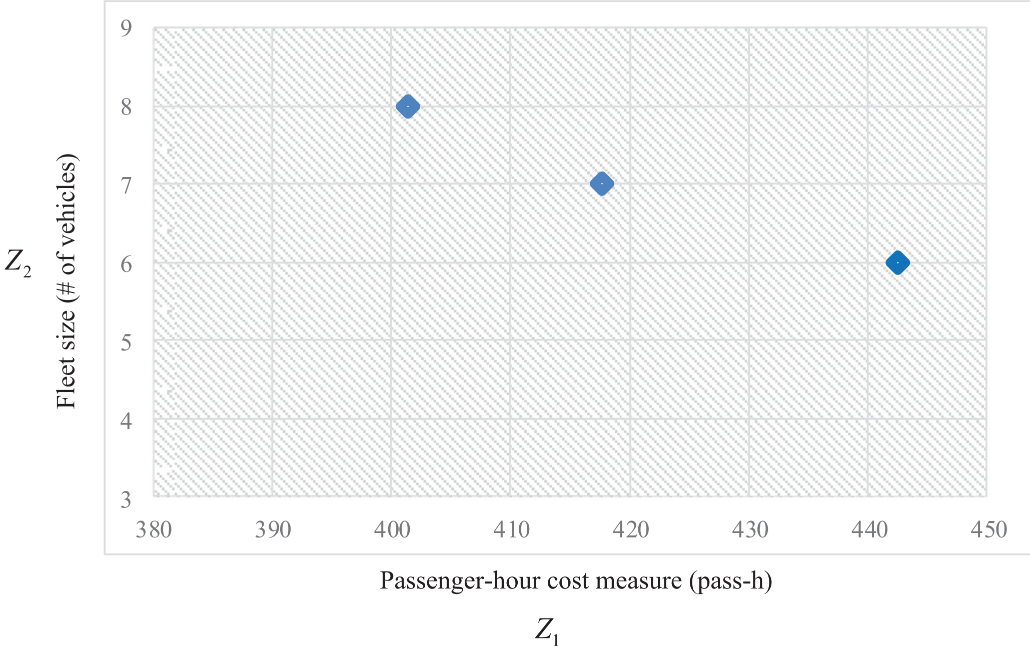

Fig. 10. Pareto-efficient solutions of the Spiess–Florian network case study.

6. Conclusions and further studies

The development of synchronized timetables for providing more user-oriented, system-optimal and well-connected public transport (PT) service is attracting ever-increasing attention. However, most previous studies on the PT timetable synchronization design problem are focused on maximizing the number of simultaneous arrivals (or arrivals within a time window) of vehicles at transfer stops, or on minimizing the total passenger transfer waiting time. Other operations activities and system performance measures are not explicitly taken into consideration, and nor has the impact of schedule changes on PT users' route/trip choice behavior been well investigated. To bridge these gaps, this study provides a new multi-criteria optimization modelling framework using a systems approach for the integrated PT timetable synchronization and vehicle scheduling problem with passenger demand assignment. A new bi-objective, bi-level mathematical programming model that takes into account both PT user and operator interests is proposed. The nature of the overall mathematical formulations of the new model is bi-objective, bi-level integer programming, which is non-linear and non-convex. Based on the special structural characteristics of the model, a novel deficit function (DF)-based sequential search method, which is combined with a network-flow technique and a shifting departure time (route offset time) procedure, is proposed to solve the problem to obtain a set of Pareto-efficient solutions. The graphical features of the DF and the two-dimensional fleet-cost space can facilitate the decision-making process of PT schedulers in finding a desirable solution. Numerical results from a small PT network and a case study of the Spiess–Florian network demonstrate that the proposed model and solution method are effective and have potential for being applied to large scale and realistic networks.

A limitation in the results of this study is the need for accurate calculations of the time-dependent passenger demand and the number of transferring passengers at transfer stops. More accurate and realistic demand assignment models are needed to better describe PT users' route/trip choice behavior, which has an important impact on the upper level timetable design and vehicle schedule optimization. With the increased use of information, communication, and computation technologies, especially the widespread use of smartphones, smartcards and automatic passenger counting systems, the expected passenger load demand data and transferring passenger data will be easily obtainable. In addition, the fluctuations of passenger demand may result in imbalanced passenger loads on successive vehicles, which may cause vehicle bunching problems.

Subsequent research, which the authors have begun, includes: (i) incorporation of sensitivity analysis of the weighting factors $\alpha_k(k=1, 2, 3, 4)$ and $\beta_i(i=1, 2, \ldots, 6)$ since PT users do not value different travel components of their trip equally, (ii) incorporation of useful heuristics developed by previous researchers (e.g., Daganzo, 1990; and Ceder et al., 2001) for intelligently and automatically setting route headways and offset times, (iii) extension of the lower level model to other more realistic, schedule-based transit assignment models, considering mode choice and demand elasticities, (iv) introducing time windows, reliability margins or slack times into the current deterministic model so as to improve the reliability of synchronized transfers, and (v) testing the model and solution methods in real-life cases with more routes and transfer stops with increased complexity, and taking heterogeneous vehicle fleet into consideration.

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