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Intuitionistic Fuzzy Analytic Network Process

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***Abstract*—Since the Analytic network process (ANP) is much more flexible than the analytic hierarchy process (AHP) in handling the multiple criteria decision making (MCDM) problems in which the criteria or sub-criteria are interdependent, it has attracted many scholars' attention and has been applied into many different areas. Given the powerfulness of intuitionistic fuzzy set in representing positive, negative and indeterminate information, this paper investigates the ANP framework for the MCDM problems in which all the pairwise comparison judgment information over the objects are represented by intuitionistic fuzzy numbers. We first justify the way to decompose the MCDM problem into a holarchy and network structure, based on which, the intuitionistic fuzzy preference relations (IFPRs) can be constructed through pairwise comparisons over the goals, criteria, clusters as well as the elements. Considering that not all the IFPRs are consistent, we then propose a new method to derive the priorities from the IFPRs no matter the IFPRs are consistent or not. After that, we address the way to construct the supermatrix for those interdependent elements. The complete algorithm of intuitionistic fuzzy ANP (IFANP) is given and illustrated by a flow chart. To show the applicability and efficiency of the IFANP, we implement the method to a case study concerning the brand management of the six golden flowers of Sichuan liquor. Some comparative analyses are given to clarify the advantages and invalidation of the IFANP.**

***Index Terms*—Intuitionistic fuzzy analytic network process, intuitionistic fuzzy preference relation, analytic hierarchy process, multiple criteria decision making, Sichuan liquor.**

I. Introduction

ANALYTIC network process (ANP), firstly brought out by Saaty in 1996 [1], allows one to include all the factors and criteria that have impacts on making a best decision. It is of great use in assisting the mind to organize its thoughts and experiences and to elicit judgments recorded in memory and qualify them in the form of priorities. It allows the decision makers (DMs) to represent diverse opinions after discussion and debate. The ANP, as a generalized model of the analytic hierarchy process (AHP) [2], not only can solve the AHP problems, but also can tackle interdependent relationships within multiple criteria decision making (MCDM) problems by replacing hierarchies with networks. It is based on deriving ratio scale measurements and can be used to allocate resources according to their ratio-scale priorities. It provides a way to the input judgments and measurements to derive ratio scale priorities for the distribution of influences among the factors and groups of factors in the decision making problem. The steps of ANP involve the following steps: ① constructing control hierarchy and network and identifying feedbacks or dependences among the elements and clusters, ② constructing the pairwise comparison matrices regarding to the elements and clusters, ③ deriving local priorities and constructing unweighted supermatrix with local priorities, ④ adjusting the unweighted supermatrix to the weighted supermatrix (also called as column stochastic matrix), ⑤ limiting the weighted supermatrix by raising it to an arbitrarily large power and calculating the limit priorities from it , and ⑥ deriving the final priorities of the alternatives.

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Although the traditional ANP is widely used in management science and operations research, we shall not ignore its drawback: the DMs cannot guarantee that the judgements valued by them are exact and crisp. During the pairwise comparison procedure, the DMs usually are required to give the value of the preference relation by crisp numbers based on the knowledge and experience they owned. However, only by the crisp numbers cannot express the DMs' uncertainty on the preference relation. If the experts cannot clearly comprehend the problem, they are unwilling to give their judgements by crisp values, and then we cannot successfully solve the problem. In order to overcome this shortcoming of ANP, Mikhailov and Singh [3] made enormous strides in the direction of fuzzy ANP (FANP) and its applications in MCDM. So far, the FANP have been applied in many areas, such as evaluating region agricultural drought risks [4], selecting container ports [5], selecting social media platform [6], choosing supplier [7], evaluating ship maneuverability [8], and determining the importance of hospital information system adoption factors [9].

In 1986, Atanassov [10] developed the intuitionistic fuzzy set (IFS), which includes three-dimensional degrees, i.e., the membership degree, the non-membership degree and the hesitancy degree, to represent the DMs' positive, negative and indeterminate cognitions. Given that the intuitionistic fuzzy preference exhibits the characteristics of affirmation, negation and hesitation, it can be used to depict the uncertain and hesitant preference information flexibly in the case that the DMs are unwilling/unable to discriminate explicitly the degree to which an alternative is better than others [11]. Decision making with intuitionistic fuzzy preference relations (IFPRs)

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[12,13] has been investigated by many scholars over the past two decades. Xu and Liao [11] firstly proposed the comprehensive framework of intuitionistic fuzzy AHP (IFAHP) and implemented it in global supplier development. Later, Liao et al. [14] introduced the general framework of group decision making with IFPRs and then implemented the group decision making algorithm to the outstanding PhD student selection for the China Scholarship Council. Other applications related to intuitionistic fuzzy decision making can be found in the project manager selection [15], the alternative energy exploitation schemes evaluation problem [16], the flexible manufacturing system selection problem [17], the electronic learning management problem [18], and the motorcycle performance evaluation problem [19]. The surveys related to intuitionistic fuzzy decision making approaches can be referred in Ref. [20-22].

As we all know, the key points of decision making with preference relations are checking the consistency of the preference relations and deriving the priorities from the preference relations. Many scholars have paid their attention to investigating the consistency of the IFPR, which mainly involves two categories, i.e., the additive consistency [23-25] and the multiplicative consistency [17,26,28,29]. Liao and Xu [21] made a comprehensive survey on these different consistency conditions. In this paper, we do not want to pay much attention to this issue any more. Readers who have interests can refer to Refs. [17,21,23-26,28,29] for details. Considering that not all the IFPRs are consistent, we try to propose a new method to derive the priorities from the IFPRs no matter the IFPRs are consistent or not.

Based on the above analyses, we can observe that both the ANP/FANP and the IFPRs are very useful in handling the complicated decision making problems. However, till now, as far as we know, there is no research combining these two well-known techniques together. This is the research gap for both of the two theories and the motivation of this paper to propose the intuitionistic fuzzy ANP (IFANP) paradigm. In case that we use the ANP under intuitionistic fuzzy environment, the judgements we acquired from experts do not only express the agreement or the disagreement, but also express the indeterminacy or hesitation. Thus, the IFANP paradigm should be much more reasonable and comprehensive than the traditional ANP as well as the FANP.

The main contributions of this paper are summarized as follows:

- ✓ We decompose the complicated problem into the ANP holarchy and how to find out the problem's clusters and elements. We also address the way to construct the IFPRs regarding to different objects.
- ✓ We propose a new linear programming method to derive the crisp priorities from the IFPRs, which can be used in some software. This model can also be used to measure the consistency of each IFPR.
- ✓ We introduce the way to establish the supermatrix, which is a large difference between the AHP and the ANP. The supermatrix consists of all local priorities derived from each IFPR.
- ✓ The step by step algorithm of the IFANP is developed for the simplicity of application. Compared to ANP, we add the steps to check the IFPRs' consistency and repaired it automatically. It is time-saving and remains the initial, useful and consistent information.
- ✓ To show the potential application, we apply the IFANP to evaluate six golden flowers of Sichuan liquor and compare the results with those derived by some other methods (the IFAHP, the FANP and the value function of IFV-based method).

The advantage of the IFANP over these methods has been shown by comparative analysis, especially the IFANP is more actual and retains more information among the models of decision making problems.

The reminder of this paper is organized as follows: Section II introduces the decomposition of the problem and the forms of IFPRs. Section III develops a new priority method to check the consistency of IFPR and derive the corresponding priorities, simultaneously. Section IV concentrates on supermatrix. Section V proposes the algorithm of IFANP and Section VI gives an example of its application. Section VII focuses on comparative analysis with some similar methods and Section VIII gives some concluding remarks.

II. Decompose The Problem And Comparative Judgment Under Intuitionistic Fuzzy Environment

A. *Decompose the MCDM problem*

As the departure of the ANP, we should decompose the problem to construct the holarchy and the network structure. The holarchy includes the control hierarchy which is the same as that in AHP (readers can refer to Ref. [2] for details). The network structure expresses the interdependency and feedback of the elements. For simplicity, we assume that the problem can be decomposed to ① goals (at least one goal), ② criteria, which are denoted as $CRR=\{CRR_1,CRR_2,\cdots,CRR_m\}$, ③ clusters, which are denoted as $CLS=\{CLS_1,CLS_2,\cdots,CLS_Q\}$, and ④ elements, which are represented as c_{in_i} $(i=1,2,\cdots,Q)$, showing the n_i th element that belongs to the i th cluster. The former two parts consist of the control hierarchy, and the network structure is made up with the clusters and the elements. The number of the four parts depends on the decision makers' personal factors and the complexity and significance of the MCDM problem. Generally speaking, the more complex the problem is and the stronger the decision makers' will is, the more intricate the ANP holarchy would be.

To tackle a MCDM problem with ANP, we should decompose the problem and find out the problem's clusters and elements. The AHP has the linear structure whose hierarchy is from top to bottom, while the ANP structure includes the outer and inner dependences among the elements. The comparison between AHP and ANP can be shown in Fig. 1.

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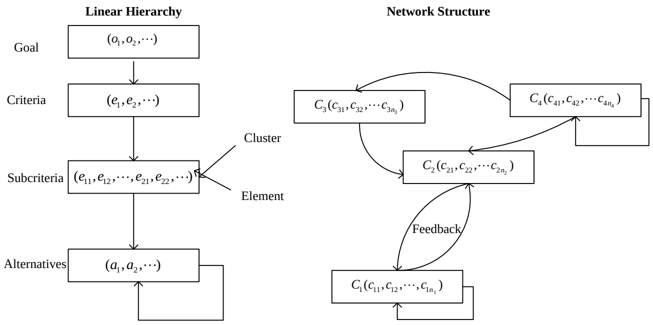

Fig. 1. Comparison between the AHP and ANP structures

Note: The loop in a cluster indicates the inner dependence among the elements of the cluster in regard to a common property. The arc, for example, from cluster C3 to C2, indicates the outer dependence among the elements in C2 on the elements in C3 with respect to a common property.

B. Comparative judgments under intuitionistic fuzzy environment

After constructing the structure of the ANP, we need to identify the pairwise judgments between clusters and elements under intuitionistic fuzzy environment. The pairwise comparisons under the intuitionistic fuzzy environment can be denoted by the 2-dimensional sequential pairs which are called the intuitionistic fuzzy value (IFV) [26].

The concept of IFS was firstly introduced by Atanassov [10]. The definition of IFS is the series of ordering triples, which can be denoted as $\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x) \right) | \, x \in X \right\}$. In the definition of IFS, $\mu_{\tilde{A}}$ and $v_{\tilde{A}}$ are the membership and non-membership functions which are mapped from X to $[0,1]$, respectively, with the condition $0 \le \mu_{\tilde{A}} + v_{\tilde{A}} \le 1$. $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - v_{\tilde{A}}(x)$ is called the intuitionistic index of x in $\tilde{A}$.

Generally, we should cultivate the decision makers or the experts to represent their comparative judgments by IFVs. On condition that all the pairwise judgments are denoted by IFVs, an intuitionistic fuzzy preference relation (IFPR) can be acquired naturally. Xu [13] defined the IFPR as a matrix $\tilde{R} = (\tilde{r}_{pq})_{n \times n}$ over the alternative set $A = \{a_1, a_2, ..., a_n\}$. For each $\tilde{r}_{pq} = < \mu(a_p, a_q), v(a_p, a_q) >$, $\mu(a_p, a_q)$ denotes the degree that a_p is more advantageous than a_q; $v(a_p, a_q)$ denotes the degree that a_p is more disadvantageous than a_q; $\pi(a_p, a_q) = 1 - \mu(a_p, a_q) - v(a_p, a_q)$ denotes the degree of hesitancy or uncertainty, with the condition:

$$\mu(a_p, a_q), v(a_p, a_q) \in [0,1], \; 0 \le \mu(a_p, a_q) + v(a_p, a_q) \le 1,$$
$$\mu(a_p, a_q) = v(a_q, a_p), \; \mu(a_p, a_p) = v(a_p, a_p) = 0.5,$$
$$\text{for } p, q = 1, 2, \cdots, n \tag{1}$$

IF we want to compare any each IFVs $\tilde{r}_{hl} = (\mu(a_h, a_l), v(a_h, a_l))$ and $\tilde{r}_{sw} = (\mu(a_s, a_w), v(a_s, a_w))$, the value function of IFV can be shown as follows [27]:

$$\psi(\tilde{r}_{hl}) = \frac{1 - v_{hl}}{1 + \pi_{hl}} \tag{2}$$

There is a positive correlation between the value of $\psi(\tilde{r}_{hl})$ and the IFV. If $\psi(\tilde{r}_{hl}) > \psi(\tilde{r}_{sw})$, then $\tilde{r}_{hl}$ is larger than $\tilde{r}_{sw}$.

According to the above knowledge, the pairwise comparison judgments indicated by IFVs could form a comparison matrix. However, before we use the matrix to rank the alternatives, it is essential to check its consistency as the inconsistent IFPR would yield unreasonable results.

III. A New Priority Determining Method From The IFPR

A. Priority vector and consistency of IFPR

To highlight our presentation, here we focus our attention on the priority derivation method from the IFPR.

Suppose that the underlying priority vector of the elements is $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$. Generally, for the fuzzy preference relation $B = (b_{ij})_{n \times n}$ whose elements are represented by the 0.1-0.9 scale, if B is multiplicative consistent, then

$$b_{ij} = \frac{\omega_i}{\omega_i + \omega_j}, \; i, j = 1, 2, \cdots, n \tag{3}$$

It should be noted that the IFPR expresses the preferences of the DMs by 0.1-0.9 scale. Also note that the IFV (μ_{ij}, v_{ij}) can be transformed into its equivalent interval value $[\mu_{ij}, 1 - v_{ij}]$. Motivated by these two points, Xu [23] gave the definition of multiplicative consistent IFPR:

Definition 1. [23] Let $\tilde{R} = (\tilde{r}_{ij})_{n \times n}$ with $\tilde{r}_{ij} = (\mu_{ij}, v_{ij})$ ($i, j = 1, 2, ..., n$) be an IFPR, if there exists a vector $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$, such that

$$\mu_{ij} \le \frac{\omega_i}{\omega_i + \omega_j} \le 1 - v_{ij}, \; \text{for all } i, j = 1, 2, ..., n \tag{4}$$

where $\omega_i \ge 0$, ($i = 1, 2, ..., n$), $\sum_{i=1}^{n} \omega_i = 1$. Then, we call $\tilde{R}$ a multiplicative consistent IFPR.

Based on the above definition, Liao and Xu [19] introduced a model to derive the underlying crisp weight vector from a multiplicative consistent IFPR. However, this method can only be used to derive the priorities under the condition that the IFPR is consistent. Once the IFPR is inconsistent, their method is invalid.

Thus, in the next subsection we propose a new method to yield the priorities from IFPR, which is motivated by the idea in Ref. [3].

B. A new priority determining method: the optimal priority optimization model

Given that the IFPR is symmetric as indicated by Eq. (1), for the simplicity of presentation, in the following, we only consider the upper triangular part of the IFPR. As Eq. (4) holds only when the IFPR is consistent, if the IFPR is inconsistent, then the priority vector $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ does not exist. However, it is reasonable for us to find a vector which fits Eq. (4) "as much as possible". In other words, the approximate solution of $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ should satisfy

$$\mu_{ij} \,\tilde{\le}\, \frac{\omega_i}{\omega_i + \omega_j} \,\tilde{\le}\, 1 - v_{ij}, \; i = 1, 2, ..., n, \; j = 2, ..., n-1 \tag{5}$$

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where the symbol "$\tilde{\leq}$" implies the statement "fuzzy less than or equal to". Eq. (5) is equivalent to

$$\begin{cases} \mu_{ij} - \dfrac{\omega_i}{\omega_i + \omega_j} \tilde{\leq} 0 \\ \dfrac{\omega_i}{\omega_i + \omega_j} - 1 + v_{ij} \tilde{\leq} 0 \end{cases}, \ i = 1,2,...,n, \ j = 2,...,n-1$$

That is

$$\begin{cases} (\mu_{ij} - 1)\omega_i + \mu_{ij}\omega_j \tilde{\leq} 0 \\ v_{ij}\omega_i + (v_{ij} - 1)\omega_j \tilde{\leq} 0 \end{cases}, \ i = 1,2,...,n, \ j = 2,...,n-1 \quad (6)$$

As there are $n(n-1)$ inequalities as given in Eq. (6), we can rewrite them in a matrix form as:

$$P\omega \tilde{\leq} 0 \quad (7)$$

where P is a $m \times n$ matrix indicating the coefficients of the priorities and $m = n(n-1)$.

Eq. (7) defines the fuzzy linear constraint of the underlying priorities. To continue our derivation process, we should further represent the fuzzy symbol "$\tilde{\leq}$". Generally, the l-th row of Eq. (7), represented as $P_l\omega \tilde{\leq} 0$, can be transformed into a membership function as:

$$m(P_l\omega) = \begin{cases} 1 - \dfrac{P_l\omega}{t_l}, & P_l\omega \leq t_l \\ 0, & P_l\omega > t_l \end{cases}, \ l = 1,2,\cdots,m \quad (8)$$

where t_l is a parameter given by the DM, denoting the approximate satisfaction of the inequality $P_l\omega \tilde{\leq} 0$, $l = 1,2,\cdots,m$. The membership degree of $m(P_l\omega)$ represents the intensity of satisfaction of the priorities to the inequalities as given in Eq. (4). If $m(P_l\omega) = 0$, it means that the priorities violate the l-th constraint completely. Generally, the value of $m(P_l\omega)$ lies between zero and one.

For a given minimum ratio τ of $P_l\omega/t_l$ with respect to any tolerance parameter t_l, it is reasonable to make an assumption with respect to the selection rule of the optimal priorities that the optimal priority vector should make the membership degree of $m(P_l\omega)$ attain the highest degree. In other words, the optimal priorities should satisfy

$$\tau = \max_{l=1,2,\cdots,m} \min_{\substack{i=1,2,\cdots,n; \\ j=2,\cdots,n-1}} \{m(P_1\omega),\cdots,m(P_l\omega),\cdots,m(P_m\omega)\} \quad (9)$$

Eq. (9) can be transformed equivalently into the following linear programming model, called the optimal priority optimization (OPO) model.

$$\begin{aligned} &\max \quad \tau \\ &s.t. \begin{cases} 1 - \dfrac{P_l\omega}{t_l} \geq \tau, l = 1,2,\cdots,m \\ m = n(n-1) \\ 0 \leq \omega_i \leq 1, i = 1,2,\cdots,n \\ \sum_{i=1}^{n} \omega_i = 1 \end{cases} \end{aligned} \quad (10)$$

The optimal solution ω^* of the OPO Model is the underlying priority vector which maximizes the membership degrees of those fuzzy linear constraints denoted in Eq. (6). The optimal objective function value τ^* measures the maximum satisfaction degree of those fuzzy constrains. It can be taken as the indicator to measure the inconsistency of the DM's assessments. In other words, it does somehow like the consistency index in the classical AHP framework. When the IFPR is consistent, τ^* should be greater than or equal to one; while if the IFPR is inconsistent, τ^* should be varied between zero and one, depending on the degree of inconsistency and the values of t_l given by the DM.

Generally, for a given programming model, we cannot always guarantee that the optimal or feasible solution exists. The following theorem shows that the optimal solution of the OPO model always exists for any IFPR, either consistent or inconsistent.

Theorem 1. (Existence) The optimal solution of the OPO model always exists for any IFPR.

The proof of Theorem 1 can be seen in Appendix.

The OPO model can be solved easily by some optimization package such as Lingo or Matlab. Without loss of generality, we can set the parameter t_l to be equal if the DMs have no preferences over the pairwise assessments [3].

Example 1. Suppose that we have an IFPR in terms of the following form:

$$R_1 = \begin{pmatrix} (0.50, 0.50) & (0.55, 0.45) & (0.69, 0.31) \\ (0.45, 0.55) & (0.50, 0.50) & (0.65, 0.35) \\ (0.31, 0.69) & (0.35, 0.65) & (0.50, 0.50) \end{pmatrix}$$

Based on the OPO model, if we set the tolerance parameter $t_l = 1$ for $l = 1,2,\cdots,6$, then we can construct the following optimization problem:

$$\begin{aligned} &\max \quad \tau \\ &s.t. \begin{cases} 1\text{-}[(0.55\text{-}1)\omega_1 + 0.55\omega_2] \geq \tau \\ 1\text{-}[(0.69\text{-}1)\omega_1 + 0.69\omega_3] \geq \tau \\ 1\text{-}[(0.65\text{-}1)\omega_2 + 0.65\omega_3] \geq \tau \\ 1\text{-}[0.45\omega_1 + (0.45\text{-}1)\omega_2] \geq \tau \\ 1\text{-}[0.31\omega_1 + (0.31\text{-}1)\omega_3] \geq \tau \\ 1\text{-}[0.35\omega_2 + (0.35\text{-}1)\omega_3] \geq \tau \\ 0 \leq \omega_1 \leq 1, 0 \leq \omega_2 \leq 1, 0 \leq \omega_3 \leq 1 \\ \omega_1 + \omega_2 + \omega_3 = 1 \end{cases} \end{aligned}$$

Solving this optimization problem, we can get that the optimal solution is $\omega_1^* = (0.44, 0.36, 0.20)^T$, and the maximal value of the objective function is $\tau_1^* = 0.999 \approx 1$, which implies that the IFPR $\tilde{R}_1$ attains the complete consistency. Note that in the IFPR $\tilde{R}_1$, each membership degree plus its associated non-membership degree is equal to one. That is to say, this IFPR can be reduced to the fuzzy preference relation. By the priority determining formula of fuzzy preference relation as given below [17,29]:

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$$\omega_i = \begin{cases} 1\Big/\sum_{j=1}^{n} \dfrac{r_{ji}}{r_{ij}}, & r_{ij} \neq 0 \\ 0, & r_{ij} = 0 \end{cases} \quad \text{for all } i = 1, 2, \cdots, n \tag{11}$$

We can obtain that the underlying weight vector is also $\omega_1^* = (0.44, 0.36, 0.20)^T$. This shows that the priority derivation method for IFPR given by the OPO model is convincing. The result is also coincident with those derived by the methods in Ref. [17] and Ref. [19].

Example 2. Given an IFPR shown as:

$$R_2 = \begin{pmatrix} (0.5,0.5) & (0.8,0.1) & (0.6,0.4) \\ (0.1,0.8) & (0.5,0.5) & (0.7,0.1) \\ (0.4,0.6) & (0.1,0.7) & (0.5,0.5) \end{pmatrix}$$

Based on the OPO model, for each $l = 1, 2, \cdots, 6$, we give the tolerance parameter's value $t_l = 1$. Then we can construct the following optimization problem:

$$\max \quad \tau$$
$$s.t. \quad \begin{cases} 1\text{-}[(0.8\text{-}1)\omega_1 + 0.8\omega_2] \geq \tau \\ 1\text{-}[(0.6\text{-}1)\omega_1 + 0.6\omega_3] \geq \tau \\ 1\text{-}[(0.7\text{-}1)\omega_2 + 0.7\omega_3] \geq \tau \\ 1\text{-}[0.1\omega_1 + (0.1\text{-}1)\omega_2] \geq \tau \\ 1\text{-}[0.4\omega_1 + (0.4\text{-}1)\omega_3] \geq \tau \\ 1\text{-}[0.1\omega_2 + (0.1\text{-}1)\omega_3] \geq \tau \\ 0 \leq \omega_1 \leq 1, 0 \leq \omega_2 \leq 1, 0 \leq \omega_3 \leq 1 \\ \omega_1 + \omega_2 + \omega_3 = 1 \end{cases}$$

Solving this optimization problem, we can get the optimal solution $\omega_2^* = (0.54, 0.24, 0.22)^T$, and the maximal of the objective function τ_2^*=0.92 , which means the IFPR $\tilde{R}_2$ almost possesses the consistency. Meanwhile we can find that each IFV's membership degree plus non-membership degree is approximately equal to one.

C. *On the inconsistency reparation*

From the above two examples, we can conclude that τ actually reflects that the IFPR is consistent or not and there is a positive relation between the value of τ and the consistency of IFPR when the value of t_l given by the DM is certain. Note that t_l is random regarding to different opinions, and in this paper, we always suppose t_l=1 .

It is of great importance that consistency should be checked in nature. Even if three values are available to represent the preference relations, the experts who lack professional knowledge and comprehend understanding may not give the certain value. Thus, if the consistency value τ is less than the requirement value, we'd better ask the original experts to repair the consistency of the preference relation until it is acceptable. Because of time-consuming for the experts' revaluations, the experts may not have the will to reevaluate. Thus, it is essential to repair the consistency automatically. As to the IFPR, Xu and Liao [11] proposed the following algorithm to repair the inconsistent IFPR:

Algorithm I. Repairing the inconsistent IFPR

Step 1: For $k > i+1$, let $\bar{r}_{ik} = (\bar{\mu}_{ik}, \bar{v}_{ik})$, where

$$\bar{\mu}_{ik} = \frac{\sqrt[k-i-1]{\prod_{t=i+1}^{k-1} \mu_{it}\mu_{tk}}}{\sqrt[k-i-1]{\prod_{t=i+1}^{k-1} \mu_{it}\mu_{tk}} + \sqrt[k-i-1]{\prod_{t=i+1}^{k-1} (1-\mu_{it})(1-\mu_{tk})}}, \ k > i+1 \tag{12}$$

$$\bar{v}_{ik} = \frac{\sqrt[k-i-1]{\prod_{t=i+1}^{k-1} v_{it}v_{tk}}}{\sqrt[k-i-1]{\prod_{t=i+1}^{k-1} v_{it}v_{tk}} + \sqrt[k-i-1]{\prod_{t=i+1}^{k-1} (1-v_{it})(1-v_{tk})}}, k > i+1 \tag{13}$$

Step 2: For $k = i+1$, let $\bar{r}_{ik} = r_{ik}$.

Step 3: For $k < i$, let $\bar{r}_{ik} = (\bar{v}_{ki}, \bar{\mu}_{ki})$.

Step 4: The repaired IFPR is obtained as $\tilde{R} = (\tilde{r}_{ik})_{m\times m}$, where

$$\tilde{\mu}_{ik} = \frac{(\mu_{ik})^{1-\sigma}(\bar{\mu}_{ik})^{\sigma}}{(\mu_{ik})^{1-\sigma}(\bar{\mu}_{ik})^{\sigma} + (1-\mu_{ik})^{1-\sigma}(1-\bar{\mu}_{ik})^{\sigma}}, i,k = 1,2,...,n \tag{14}$$

$$\tilde{v}_{ik} = \frac{(v_{ik})^{1-\sigma}(\bar{v}_{ik})^{\sigma}}{(v_{ik})^{1-\sigma}(\bar{v}_{ik})^{\sigma} + (1-v_{ik})^{1-\sigma}(1-\bar{v}_{ik})^{\sigma}}, i,k = 1,2,...,n \tag{15}$$

For the sake of consistency, we set $\sigma = 1$ in this text.

IV. CONSTRUCT THE SUPERMATRIX

In this section, we follow the procedure of ANP to construct the supermatrix from the IFPRs.

Suppose that the criteria of the control level and the clusters of the network are the set $CRR = \{CRR_1, CRR_2, \cdots, CRR_m\}$ and $CLS = \{CLS_1, CLS_2, \cdots, CLS_Q\}$, respectively. In addition, the elements of the clusters are denoted as $c_{in_i}\ (i = 1, 2, \cdots, Q)$. For example, we take the criterion CRR_s of the control levels and the elements $c_{i1}, c_{i2}, \cdots, c_{in_i}$ in the cluster CLS_i as criterion and subcriteria, respectively. For the elements in the different clusters CLS_i and CLS_j , there may be some dependent relationships among them. Under this condition, we should take comparative judgements between all the elements $c_{i1}, c_{i2}, \cdots, c_{in_i}\ (i = 1, 2, \cdots, Q)$ in the cluster CLS_i and all the elements $c_{j1}, c_{j2}, \cdots, c_{jn_j}\ \ (j = 1, 2, \cdots, Q)$ in the cluster CLS_j according to its dominance or importance. That is to say, we construct the matrices under the criterion CRR_s . Usually, we could obtain the values given by the experts.

Here we suppose that all the matrices are IFPRs. This is different from the traditional ANP in which the values of the matrices are given in Saaty's scale [2]. In the traditional ANP method, the ordering vector $(\omega_{i1}^{jk}, \omega_{i2}^{jk}, \cdots, \omega_{in_i}^{jk})^T$ is inferred by the eigenvector method. But in IFANP, we use the OPO model proposed in Section III to obtain the weight vectors from different IFPRs.

The pairwise comparison matrix of the elements $c_{i1}, c_{i2}, \cdots, c_{in_i}\ (i = 1, 2, \cdots, Q)$ in the cluster CLS_i over the element c_{jn_j} in the cluster CLS_j is shown in Table I.

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TABLE I
WEIGHT VECTOR OF THE ELEMENTS IN CLUSTER OVER THE ELEMENT IN ANOTHER CLUSTER

c_{jn_j}	c_{1n_1}	c_{2n_2}	$\cdots$	c_{in_i}	Weight vector ω^{jn_j}
c_{i1}	$(\mu_{i1}^{1n_1},\nu_{i1}^{1n_1})$	$(\mu_{i1}^{2n_2},\nu_{i1}^{2n_2})$	$\cdots$	$(\mu_{i1}^{in_i},\nu_{i1}^{in_i})$	$\omega_{i1}^{jn_j}$
c_{2n_2}	$(\mu_{i2}^{1n_1},\nu_{i2}^{1n_1})$	$(\mu_{i2}^{2n_2},\nu_{i2}^{2n_2})$	$\cdots$	$(\mu_{i2}^{in_i},\nu_{i2}^{in_i})$	$\omega_{i2}^{jn_j}$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
c_{in_i}	$(\mu_{in_i}^{1n_1},\nu_{in_i}^{1n_1})$	$(\mu_{in_i}^{2n_2},\nu_{in_i}^{2n_2})$	$\cdots$	$(\mu_{in_i}^{in_i},\nu_{in_i}^{in_i})$	$\omega_{in_i}^{jn_j}$

If all the weight vectors ω^{jn_j} ($j=1,2,\cdots,Q$) pass the consistency check, we can get the local weight vector matrix W_{ij}, which is shown as:

$$W_{ij}=\begin{bmatrix}\omega_{i1}^{j1} & \omega_{i1}^{j2} & \cdots & \omega_{i1}^{jn_j}\\ \omega_{i2}^{j1} & \omega_{i2}^{j2} & \cdots & \omega_{i2}^{jn_j}\\ \vdots & \vdots & \ddots & \vdots\\ \omega_{in_i}^{j1} & \omega_{i2}^{j2} & \cdots & \omega_{in_i}^{jn_j}\end{bmatrix}$$

Each column vector of W_{ij} is the priority ordering vector of the elements $c_{i1},c_{i2},\cdots,c_{in_i}\,(i=1,2,\cdots,Q)$ in the cluster CLS_i with respect to the elements c_{jn_j} in the cluster CLS_j $(j=1,2,\cdots,Q)$. Note that if the elements of CLS_i and CLS_j are independent with respect to each other, then $W_{ij}=0$.

For each cluster CLS_i and CLS_j ($i,j=1,2,\cdots,Q$), the pairwise comparisons between the element c_{in_i} and the element c_{jn_j} should be performed. Then under the control element CRR_s, the supermatrix W could be formed as:

$$W=\begin{bmatrix}W_{11} & W_{12} & \cdots & W_{1Q}\\ W_{21} & W_{22} & \cdots & W_{2Q}\\ \vdots & \vdots & \ddots & \vdots\\ W_{Q1} & W_{Q2} & \cdots & W_{QQ}\end{bmatrix}$$

As for W, each element is a submatrix and the sum of its any column vector should be equal to 1. That is to say, each column of W_{ij} ($i,j=1,2,...,Q$) is normalized. But the whole matrix W is not normalized. To obtain the convergent supermatrix, W should multiply a weighted matrix A to get a normalized matrix $\hat{W}$. Furthermore, under the condition of the criterion CRR_s, the pairwise comparisons between CLS_j $(j=1,2,\cdots,Q)$ should be executed and the results are shown in Table II.

TABLE II
NORMALIZIED WEIGHTED EIGENVECTOR AMONG CLUSTERS

CRR_s	CLS_1	CLS_2	$\cdots$	CLS_Q	Normalized weight vector
CLS_1	(μ_1,ν_1)	(μ_1,ν_2)	$\cdots$	(μ_1,ν_Q)	a_{1j}
CLS_2	(μ_2,ν_1)	(μ_2,ν_2)	$\cdots$	(μ_2,ν_Q)	a_{2j}
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
CLS_Q	(μ_Q,ν_1)	(μ_Q,ν_2)	$\cdots$	(μ_Q,ν_Q)	a_{Qj}

Note that if the cluster has nothing to do with CLS_j, the corresponding ordering vector should be zero. Then we can acquire the weighted matrix A:

$$A=\begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1Q}\\ a_{21} & a_{22} & \cdots & a_{2Q}\\ \vdots & \vdots & \ddots & \vdots\\ a_{Q1} & a_{Q2} & \cdots & a_{QQ}\end{bmatrix}$$

The normalized supermatrix $\hat{W}=(\hat{W}_{ij})$ should be inferred from the supermatrix W multiplying the weighted matrix A, where $\hat{W}_{ij}=a_{ij}W_{ij}$, $i=1,2,\cdots,Q$, $j=1,2,\cdots,Q$.

To reflect the interdependency and feedback between elements, we should make the normalized supermatrix $\hat{W}$ stable. In other words, we need to calculate each supermatrix limit ordering vector:

$$W^{\infty}=\lim_{k\to\infty}\frac{1}{N}\sum_{k=1}^{N}\hat{W}^{k} \tag{16}$$

If the limit is convergent and exclusive, the elements of the final ordering value should be calculated by the limit vectors.

V. THE ALGORITHM OF INTUITIONISTIC FUZZY ANALYTIC NETWORK PROCESS

Based on the above analyses, to conduce the IFANP method, we should first construct the control hierarchy and the network, and identify the feedbacks or dependences among the elements and the clusters. Then, we need to construct the IFPRs based on the pairwise comparisons between the elements in different clusters regarding to different criteria. Afterwards, we need to derive the local priority vectors from those IFPRs by the fuzzy programming model and then construct the unweighted supermatrix with these local priority vectors. We need to further adjust the unweighted supermatrix to the weighted supermatrix (also called the column stochastic matrix). Based on this supermatrix, we can calculate the limit priority from the stochastic matrix, and then limit the weighted supermatrix by raising it to an arbitrarily large power. Finally, we can calculate the final priorities and then rank the alternatives.

For the convenience of application, in the following, we develop the step by step procedure of IFANP:

Algorithm II. IFANP

Step 1: Collapse the decision making problem and find the goal, control criteria, clusters as well as the elements. After that, we construct the control hierarchy with control criteria, which is the same as that in AHP, and the network which consists of clusters and elements. Go to the next step.

Step 2: Identify the feedbacks and dependences among the clusters and the elements. Go to the next step.

Step 3: Determine the comparative judgments under intuitionistic fuzzy environment between the elements in different clusters regarding to each criterion. Then the IFPRs can be acquired by the pairwise comparisons. Go to the next step.

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Step 4: Calculate the local priority vector $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ and the optimal objective function value from each IFPR through the OPO Model. Go to the next step.

Step 5: Check the consistency of each IFPR according to the objective function value τ of the OPO model. If τ is less than or equal to the consistency threshold ξ, then the consistency level is unacceptable, and thus we go to Step 6; otherwise, go to Step 7.

Step 6: Repair the inconsistent IFPRs according to Algorithm I. Go to Step 4.

Step 7: Construct the unweighted supermatrix with local priority vector and adjust it to the weighted supermatrix. Go to the next step.

Step 8: Raise the weighted supermatrix to infinite powers until it is convergent. Go to the next step.

Step 9: Calculate the global priority vector and rank the alternatives according it. Go to the next step.

Step 10: End.

Step 1 and Step 2 are the data initializations. From Step 3 to Step 6, it is specially designed for IFANP yet the other steps are similar to those in IFANP. In this algorithm, we only need the DM to give their intuitionistic fuzzy preferences on pairwise judgements and some parameters that we can utilize to check each IFPR's consistency. If the IFPR is not consistent, this algorithm can repair it by fusing the origin one and the repaired IFPR to a fused IFPR automatically. After that, the local priorities, the weighted supermatrix, the limit priorities could be executed. Finally, the ranking of alternatives could be acquired by the limit priorities. Fig. 2 illustrates the flow chart of the IFANP.

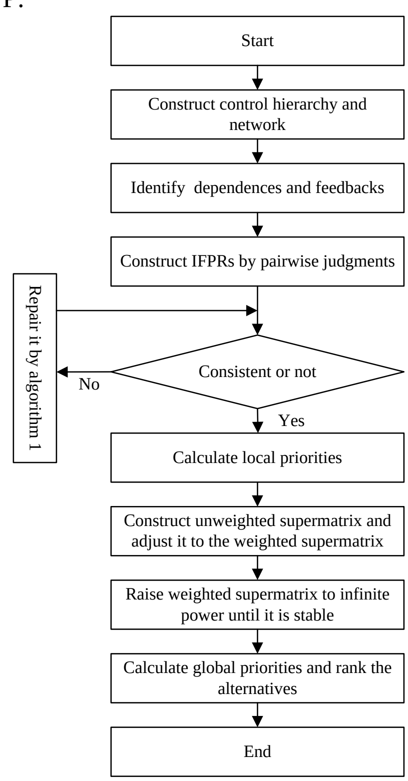

Fig. 2. The flow chart of the IFANP

VI. Apply The IFANP To Brand Management Of Six Golden Flowers Of Sichuan Liquor

An increasing climb experienced in the development of Sichuan liquor among Chinese liquor between the 4-year period from 2012 to 2015. To be specific, the portion of Sichuan liquor in Chinese liquor increased to 2.65% (from 25.60% to 28.25%), which is always the largest part of Chinese liquor. With the fantastic spur liquor industry in Sichuan, people are more likely to choose good products from different kinds of Sichuan liquor. However, there emerge many problems in the development of Sichuan liquor, especially in brand management. For example, some firms give an exaggerated account of some properties of a wine product, which results in bad influence on their brands. People may have a bad impression on a brand because of purchasing incorrect win products. To the better development of Sichuan liquor, we'd better figure out the feasible reasons for its blossom and recession. It is commonly believed that the six golden flowers of Sichuan liquor, which are Wu Liangye, Luzhou Liquor, Jian Nanchun, Lang Liquor, Quanxing Daqu, Tuopai Shede [31], play a key role in Sichuan liquor.

In this passage, we take the six golden flowers of Sichuan liquor into consideration and use the IFANP to analyze the problem which concentrates on the brand management in six golden flowers of Sichuan liquor. In the following we describe the IFANP algorithm II steps applied to this case of study.

Step 1: Decompose this problem and find that the goal is to pick out best brand of Sichuan liquor. The control criteria are the actors of product, the origin and firm, and the times of brand. The times of brand consist of introduction, growth and mature [32]. The control hierarchy with the above control criteria and the networks with clusters and elements (in this text, the clusters and elements are extracted from Ref. [33]) can be seen in Fig. 3.

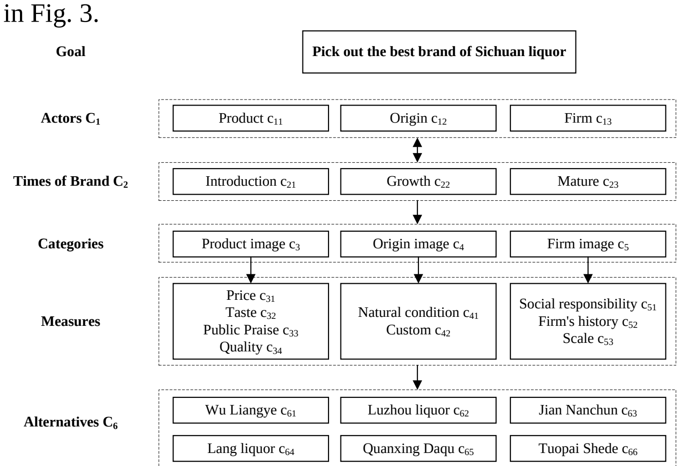

Fig. 3. The model of brand management of six golden flowers of Sichuan liquor

Step 2: In this example, the only dependence and feedback exist among the control criteria. The arrow between the actors and the times of brand is two-way, and the other arrows are one-way. Considering the complexity of Sichuan liquor brand management, we construct the structure shown in Fig. 3 to analyze the problem.

Step 3: Do pairwise judgments among the clusters and the elements under intuitionistic fuzzy environment. Then the IFPRs can be acquired.

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Step 4: Calculate the local priority vector $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ and the optimal objective function value from each IFPR through the OPO Model. For example, the IFPRs R_1,R_2,R_3 among product (c_{11}), origin (c_{12}) and firm (c_{13}) and the corresponding local priority vectors are furnished as:

$$R_1=\begin{pmatrix}(0.50,0.50) & (0.55,0.45) & (0.69,0.31)\\ (0.45,0.55) & (0.50,0.50) & (0.65,0.35)\\ (0.31,0.69) & (0.35,0.65) & (0.50,0.50)\end{pmatrix},\ \omega_1=\begin{pmatrix}0.44\\0.36\\0.20\end{pmatrix}$$

$$R_2=\begin{pmatrix}(0.5,0.5) & (0.8,0.1) & (0.6,0.4)\\ (0.1,0.8) & (0.5,0.5) & (0.7,0.1)\\ (0.4,0.6) & (0.1,0.7) & (0.5,0.5)\end{pmatrix},\ \omega_2=\begin{pmatrix}0.54\\0.24\\0.22\end{pmatrix}$$

$$R_3=\begin{pmatrix}(0.5,0.5) & (0.85,0.1) & (0.1,0.8)\\ (0.1,0.85) & (0.5,0.5) & (0.85,0.1)\\ (0.8,0.1) & (0.1,0.85) & (0.5,0.5)\end{pmatrix},\ \omega_3=\begin{pmatrix}0.3572\\0.3243\\0.3185\end{pmatrix}$$

Other IFPRs among the six elements product (c_{11}), origin (c_{12}), firm (c_{13}), introduction (c_{21}), Growth(c_{22}), Mature(c_{23}) are listed in Appendix.

Step 5: Check the consistency of each IFPR according to the objective function value τ of the OPO model. Solved by the OPO model, we can acquire only $\tau_{R_3}=0.7779<0.8$, which shows the consistency of the IFPR R_3 is not acceptable and other IFPRs' consistencies are acceptable. It is necessary to repair the IFPR R_3. For IFPR R_3, go to step 6; for other IFPRs, go to step 7.

Step 6: Repair the inconsistent IFPR R_3 according to Algorithm I.

$$\bar{\mu}_{13}=\frac{\sqrt{\mu_{12}\mu_{23}}}{\sqrt{\mu_{12}\mu_{23}}+\sqrt{(1-\mu_{12})(1-\mu_{23})}}=0.85$$

$$\bar{v}_{13}=\frac{\sqrt{v_{12}v_{23}}}{\sqrt{v_{12}v_{23}}+\sqrt{(1-v_{12})(1-v_{23})}}=0.1$$

Repair the inconsistent IFPR R_3 according to Eq. (14) and Eq. (15). It follows that

$$\tilde{\mu}_{13}=\frac{(\mu_{13})^{1-\sigma}(\bar{\mu}_{13})^{\sigma}}{(\mu_{13})^{1-\sigma}(\bar{\mu}_{13})^{\sigma}+(1-\mu_{13})^{1-\sigma}(1-\bar{\mu}_{13})^{\sigma}}\quad(i=1,k=3)$$

$$\tilde{v}_{13}=\frac{(v_{13})^{1-\sigma}(\bar{v}_{13})^{\sigma}}{(v_{13})^{1-\sigma}(\bar{v}_{13})^{\sigma}+(1-v_{13})^{1-\sigma}(1-\bar{v}_{13})^{\sigma}}\quad(i=1,k=3)$$

We set $\sigma=1$, so $\tilde{r}_{13}=(\tilde{\mu}_{13},\tilde{v}_{13})=(0.85,0.1)$. The repaired IFPR $\tilde{R}_3$ can be given as:

$$\tilde{R}_3=\begin{pmatrix}(0.5,0.5) & (0.85,0.1) & (0.85,0.1)\\ (0.1,0.85) & (0.5,0.5) & (0.85,0.1)\\ (0.0122,0.9698) & (0.1,0.85) & (0.5,0.5)\end{pmatrix}$$

For the repaired IFPR $\tilde{R}_3$, we can use the OPO model in Step 4 to calculate its local priority vector.

$$\max\quad \tau$$
$$s.t.\ \begin{cases}1\text{-}[(0.85\text{-}1)\omega_1+0.85\omega_2]\ge\tau\\ 1\text{-}[(0.85\text{-}1)\omega_1+0.85\omega_3]\ge\tau\\ 1\text{-}[(0.85\text{-}1)\omega_2+0.85\omega_3]\ge\tau\\ 1\text{-}[0.1\omega_1+(0.1\text{-}1)\omega_2]\ge\tau\\ 1\text{-}[0.0122\omega_1+(0.0122\text{-}1)\omega_3]\ge\tau\\ 1\text{-}[0.1\omega_2+(0.1\text{-}1)\omega_3]\ge\tau\\ 0\le\omega_1\le1,0\le\omega_2\le1,0\le\omega_3\le1\\ \omega_1+\omega_2+\omega_3=1\end{cases}$$

Solving the optimization problem, we can acquire $\tau_{\tilde{R}_3}=0.9751$ and the priority vector $\omega_3^{'}=(0.7754,0.1661,0.0585)^T$, which implies that the repaired IFPR $\tilde{R}_3$ is consistent. Thus, we go to step 7.

Step 7: Construct the unweighted supermatrix with local priority vectors and adjust it to the weighted supermatrix. The unweighted supermatrix of actors and times of brand is listed in Table III in Appendix according to the local priorities calculated in Step 6. For factors and times of brand, the two clusters are equally important. The weighted supermatrix can be listed in Table IV in Appendix.

Step 8: Raise the column stochastic supermatrix to $2k+1$ powers and k is an arbitrarily number, which stops at the moment of power $2k+1$ equals to power $2k$. The convergent supermatrix is shown in Table V in Appendix.

Step 9: Calculate the final priority vector and rank the alternatives according to it. The priorities of factors and times of brand are listed in each column of Table V. The evaluation matrix of alternatives on times of brand and measures could be listed in Table X in Appendix. The data means that among product image, Price $\succ$ Taste $\succ$ Public praise $\succ$ Quality, between origin image, Natural condition $\succ$ Custom, among firm image, Social responsibility $\succ$ Firm's history $\succ$ Scale. For example, we can find that the highest and lowest scores are 0.8408 and 0.0715, respectively, which means that Wu Liangye in scale is largely advantageous and Tuopai Shede in price is quite disadvantageous, respectively. The final scores of alternatives are the sum of each column in Table XI and XII in Appendix. From Table XI, the final ranking of the six golden flowers is Wu Liangye or $\succ$ Jian Nanchun $\succ$ Luzhou liquor $\succ$ Lang liquor $\succ$ Tuopai Shede $\succ$ Quanxing Daqu.

VII. Comparative Analysis

A. Compared with IFAHP

IFAHP [11] is a useful means and its judgement scale is in the form of IFV. IFAHP collapses the problem to linear levels and then furnishes the comparative judgements among each level. Weight vectors are executed from every judgement matrix of a level, and final valuations of alternatives are calculated by the linear multiply operation among the relevant weight vectors. In IFAHP model, the elements in each level should be independent, which forbids the existence of feedback or dependence. For brand management of six golden flowers in Sichuan liquor, using IFAHP forbids the existence of

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dependence or feedback among the actors and the times of brand.

For the sake of comparison, here we ingore the actual relationship among the actors and the times of brand. So the weight vector among times of brand and factors is $\omega_A = (0.2504, 0.1651, 0.0845, 0.1002, 0.074, 0.0566, 0.0196, 0.1106, 0.0369, 0.0681, 0.0189, 0.0151)^T$. Because there lacks dependence or feedback among the times of brand and factors, we do not need to construct the supermatrix as in the other three methods. The final scores calcualted by the linear operations of IFAHP can be seen in Table XII in Appendix.

B. Compared with FANP

FANP [4] uses fuzzy pairwise judgement matrixes to obtain the crisp priorities vectors and the other steps are similar to those of the ANP process. For this example, using FANP can only take membership into consideration and we have to ignore the non-membership degrees given by the DMs, i.e., simplify the IFPRs to the FPRs without considering the non-membership degree of each comparative judgement. The local priority vector calculated by FANP is $\omega_B = (0.3133, 0.1029, 0.0838, 0.2115, 0.1683, 0.1202)^T$ and the final results are shown in Table XII in Appendix.

C. Compared with the intuitionistic fuzzy value function based method

Another similar method is using the intuitionistic fuzzy value function based method [19], which converts the IFPRs to the crisp values by Eq. (2). The local priority vector calculated by the intuitionistic fuzzy value function based method is $\omega_B = (0.1996, 0.1649, 0.1355, 0.1980, 0.1701, 0.1319)^T$ and the final results are listed in Table XII in Appendix.

Table XII shows the final scores of the alternatives which are calculated by four methods, IFAHP, FANP, the intuitionistic fuzzy value function based method and IFANP, respectively. The rankings of the alternatives with respect to the four methods are viewable as follows:

① Luzhou liquor $\succ$ Tuopai Shede $\succ$ Quanxing Daqu $\succ$ Lang liquor $\succ$ Jian Nanchun $\succ$ Wu Liangye.

② Luzhou liquor $\succ$ Tuopai Shede $\succ$ Lang liquor $\succ$ Quanxing Daqu $\succ$ Jian Nanchun $\succ$ Wu Liangye.

③ Jian Nanchun $\succ$ Wu Liangye $\succ$ Luzhou liquor $\succ$ Lang liquor $\succ$ Tuopai Shede $\succ$ Quanxing Daqu.

④ Wu Liangye $\succ$ Jian Nanchun $\succ$ Luzhou liquor $\succ$ Lang liquor $\succ$ Tuopai Shede $\succ$ Quanxing Daqu.

Fig. 4 and Fig. 5 further illustrate the differences of the results yielded by the diffferent methods. From Fig. 4 and Fig. 5, we can see that the rankings over the alternatives between IFAHP and IFANP are extremely different due to the feedback existed among factors and times of brand. While in real life quite a lot of MCDM problems have abundant feedbacks or dependences, it is better to use IFANP rather than IFAHP although the calculation of IFANP is more complicated. If we take the making wine history of firm and origin into consideration at the same time, the IFAHP are not capable to takcle the depence of the two factors which inherit property from the AHP, while the IFANP could easily cope with this by using the supermatrix to transmit this information to the final priorities.

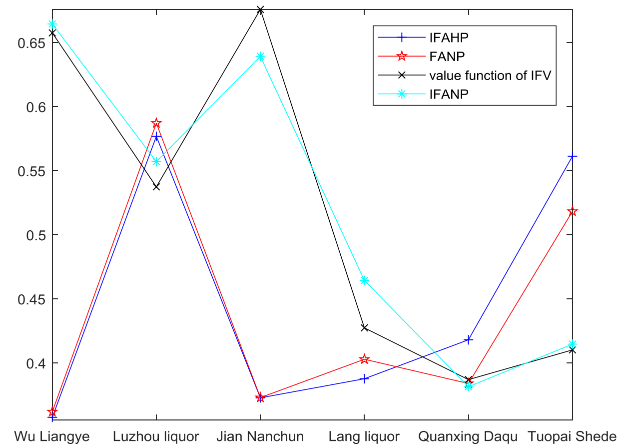

Fig. 4. The final scores of the six golden flowers in Sichuan liquor regarding to different methods

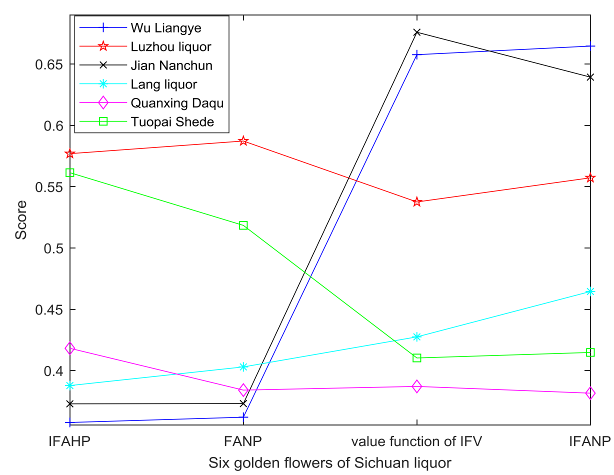

Fig. 5. The final scores of the six golden flowers in Sichuan liquor regarding to different methods

Furthermore, we observe the difference between FANP and IFANP: Using the same data, the different rankings of alternatives with the FANP and the IFANP are 0.5872>0.5184>0.403>0.3841> 0.3731>0.362, which implies Luzhou liquor $\succ$ Tuopai Shede $\succ$ Lang liquor $\succ$ Quanxing Daqu $\succ$ Jian Nanchun $\succ$ Wu Liangye, and 0.6646>0.6393>0.5572>0.4645>0.4148>0.3816, which implies Wu Liangye $\succ$ Jian Nanchun $\succ$ Luzhou liquor $\succ$ Lang liquor $\succ$ Tuopai Shede $\succ$ Quanxing Daqu, respectively. We can see the difference occurs among the six alternatives when comparing by IFANP. Actually, the pairwise judgements given in fuzzy numbers require the DMs owning the overall knowledge about the decision making problem. In case that the DMs do not have accurate judgments, using the non-membership degree and hesitancy degree of IFVs can express more information than the traditional fuzzy numbers. Hence, IFVs are more general and flexible representation form of the DM's judgements and the following process to solve the problem may include more valuable information.

In addition, the difference between the IFANP and the value function of IFV using in the ANP shoule be emerged. Using the same data, the rankings of alternatives with the value function of IFV and the IFANP are 0.6759>0.6576>0.5375>0.4275>

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0.4103>0.3871, which implies Jian Nanchun $\succ$ Wu Liangye $\succ$ Luzhou liquor $\succ$ Lang liquor $\succ$ Tuopai Shede $\succ$ Quanxing Daqu, and 0.6646>0.6393>0.5572>0.4645>0.4148>0.3816, which implies Wu Liangye $\succ$ Jian Nanchun $\succ$ Luzhou liquor $\succ$ Lang liquor $\succ$ Tuopai Shede $\succ$ Quanxing Daqu, respectively. The results implies that only two ranks of the adjoint alternatives, Jian Nanchun and Wu Liangye, are inverse, while the others are the same. The value function of IFV is usually used in ranking the order of any two IFVs. Although it is easy to calculate, the value function of IFV has disadvantages. For example, two different IFVs could produce the same value and the value cannot reprent all degrees that the DMs want to rate. On the other hand, we can also see the advantages of IFVs, which own multi-dimensional degrees that contribute to reserve the oringal information given by the DMs.

In summary, the IFANP has quite imcomparable advantages compared to other methods while the only disadvantage is its complicated calculation of priority vectors, and this can be overcome by the optimization reads variables.

VIII. Conclusion

In this paper, we have developed the range of ANP application in the form of IFVs. The IFV is benificial to the DMs especially when tackling the MCDM problems. The IFV could express the membership degree, non-membership degree and the hestiancy degree, which almost show the value that the DMs want to give. We have put forward the procedure of the IFANP, and developed a new priority determining method from the IFPR to spare us from the complicated calculation which results from the there-dimensional degrees of IFVs.

It should be noted that Zhu et al. [34] introduced the generalized analytic network process, which can deal with intervals characterized by all distributions and the interval values and mathematically equivalent to the IFV. However, the underlying foundations between IFS and IVFS are quite different. The IFS uses two different indicators to represent the membership degree and non-membership degree; while the IVFS can only be used to represent the membership degree with intervals. Since Atanassov developed the IFS in 1986, it attracts many scholars' interests and fruitful achievements can be seen in references. This shows that IFS has very good practical application potientials. Thus, it is important to investigate the IFANP paradigm to build an integrated framework of IFAHP method.

In the future, more priority deriving methods will be done and we will adapt the IFANP to solve other MCDM problems, such as R&D project selection, SWOT anlaysis, logistics service provider selection, production planning, and so forth. Furthermore, other information representational forms may be used in the pairwise judgements to be calculated in the ANP, for example, the hesitant fuzzy linguistic analytical network process.

Appendix

The proof of Theorem 1.

Proof. It is easy to note that the feasible area Ω of the OPO model is on the simple hyperplane denoted as:

$$W^{n-1}=\left\{(\omega_1,\omega_2,\cdots,\omega_n)^T \mid 0\le\omega_i\le 1, i=1,2,\cdots,n, \sum_{i=1}^{n}\omega_i=1\right\} \tag{17}$$

Based on Eq. (8), the membership function of the feasible area Ω can be denoted as the intersection of all the individual membership functions, $m_\Omega(\omega)$. That is

$$m_\Omega(\omega)=\{\min_{\substack{i=1,2,\cdots,n;\\ j=2,\cdots,n-1}}\{m(P_1\omega), m(P_2\omega),\cdots, m(P_l\omega),\cdots, m(P_m\omega)\} \mid \omega\in W^{n-1}\} \tag{18}$$

Since Eq. (7) is a linear function with respect to ω, it is easy to know that Ω is convex. Thus, we can always find the optimal solution ω^* on the hyperplane W^{n-1}, which maximizes the objective function value. ■

The IFPRs among actors and times of brand are

$$R_4=\begin{pmatrix}(0.5,0.5)&(0.6,0.1)&(0.6,0.2)\\(0.1,0.6)&(0.5,0.5)&(0.7,0.1)\\(0.2,0.6)&(0.1,0.7)&(0.5,0.5)\end{pmatrix},\ \omega_4=\begin{pmatrix}0.5229\\0.3395\\0.1376\end{pmatrix}$$

$$R_5=\begin{pmatrix}(0.5,0.5)&(0.6,0.3)&(0.7,0.2)\\(0.3,0.6)&(0.5,0.5)&(0.6,0.1)\\(0.2,0.7)&(0.1,0.6)&(0.5,0.5)\end{pmatrix},\ \omega_5=\begin{pmatrix}0.5273\\0.3091\\0.1636\end{pmatrix}$$

$$R_6=\begin{pmatrix}(0.5,0.5)&(0.7,0.2)&(0.5,0.2)\\(0.2,0.7)&(0.5,0.5)&(0.8,0.1)\\(0.2,0.5)&(0.1,0.8)&(0.5,0.5)\end{pmatrix},\ \omega_6=\begin{pmatrix}0.5926\\0.2963\\0.1111\end{pmatrix}$$

IFPR R_7-R_{12} focus on the elements of times of brand:

$$R_7=\begin{pmatrix}(0.5,0.5)&(0.6,0.3)&(0.6,0.3)\\(0.3,0.6)&(0.5,0.5)&(0.7,0.2)\\(0.3,0.6)&(0.2,0.7)&(0.5,0.5)\end{pmatrix},\ \omega_7=\begin{pmatrix}0.4751\\0.3484\\0.1765\end{pmatrix}$$

$$R_8=\begin{pmatrix}(0.5,0.5)&(0.4,0.3)&(0.7,0.2)\\(0.3,0.4)&(0.5,0.5)&(0.6,0.1)\\(0.2,0.7)&(0.1,0.6)&(0.5,0.5)\end{pmatrix},\ \omega_8=\begin{pmatrix}0.5000\\0.3333\\0.1667\end{pmatrix}$$

$$R_9=\begin{pmatrix}(0.5,0.5)&(0.6,0.1)&(0.5,0.2)\\(0.1,0.6)&(0.5,0.5)&(0.6,0.2)\\(0.2,0.5)&(0.2,0.6)&(0.5,0.5)\end{pmatrix},\ \omega_9=\begin{pmatrix}0.5273\\0.3091\\0.1636\end{pmatrix}$$

$$R_{10}=\begin{pmatrix}(0.5,0.5)&(0.5,0.4)&(0.7,0.2)\\(0.4,0.5)&(0.5,0.5)&(0.5,0.4)\\(0.2,0.7)&(0.4,0.5)&(0.5,0.5)\end{pmatrix},\ \omega_{10}=\begin{pmatrix}0.4775\\0.3153\\0.2072\end{pmatrix}$$

$$R_{11}=\begin{pmatrix}(0.5,0.5)&(0.6,0.3)&(0.5,0.4)\\(0.3,0.6)&(0.5,0.5)&(0.6,0.2)\\(0.4,0.5)&(0.2,0.6)&(0.5,0.5)\end{pmatrix},\ \omega_{11}=\begin{pmatrix}0.4286\\0.3214\\0.2500\end{pmatrix}$$

$$R_{12}=\begin{pmatrix}(0.5,0.5)&(0.7,0.2)&(0.6,0.2)\\(0.2,0.7)&(0.5,0.5)&(0.6,0.3)\\(0.2,0.6)&(0.3,0.6)&(0.5,0.5)\end{pmatrix},\ \omega_{12}=\begin{pmatrix}0.5963\\0.2477\\0.1560\end{pmatrix}$$

TABLE III
INITIAL SPUERMATRIX OF ACTORS WITH RESPECT TO TIMES OF BRAND

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	c_{11}	c_{12}	c_{13}	c_{21}	c_{22}	c_{23}
c_{11}	0.4400	0.5400	0.7754	0.5229	0.5273	0.5926
c_{12}	0.3600	0.2400	0.1661	0.3395	0.3091	0.2963
c_{13}	0.2000	0.2200	0.0585	0.1376	0.1636	0.1111
c_{21}	0.4751	0.5000	0.5273	0.4775	0.4286	0.5963
c_{22}	0.3484	0.3333	0.3091	0.3153	0.3214	0.2477
c_{23}	0.1765	0.1667	0.1636	0.2072	0.2500	0.1560

TABLE IV
WEIGHTED SPUERMATRIX OF ACTORS WITH RESPECT TO TIMES OF BRAND

	c_{11}	c_{12}	c_{13}	c_{21}	c_{22}	c_{23}
c_{11}	0.2200	0.2700	0.3877	0.2615	0.2637	0.2963
c_{12}	0.1800	0.1200	0.0831	0.1698	0.1546	0.1482
c_{13}	0.1000	0.1100	0.0293	0.0688	0.0818	0.0556
c_{21}	0.2376	0.2500	0.2637	0.2388	0.2143	0.2982
c_{22}	0.1742	0.1667	0.1546	0.1577	0.1607	0.1239
c_{23}	0.0883	0.0834	0.0818	0.1036	0.1250	0.0780

TABLE V
SPUERMATRIX CONVERGENCE OF ACTORS WITH RESPECT TO TIMES OF BRAND

	c_{11}	c_{12}	c_{13}	c_{21}	c_{22}	c_{23}
c_{11}	0.2657	0.2657	0.2657	0.2657	0.2657	0.2657
c_{12}	0.1533	0.1533	0.1533	0.1533	0.1533	0.1533
c_{13}	0.0810	0.0810	0.0810	0.0810	0.0810	0.0810
c_{21}	0.2439	0.2439	0.2439	0.2439	0.2439	0.2439
c_{22}	0.1604	0.1604	0.1604	0.1604	0.1604	0.1604
c_{23}	0.0956	0.0956	0.0956	0.0956	0.0956	0.0956

TABLE VI
WEIGHT VECTORS OF MEASURES WITH RESPECT TO Product

	c_{31}	c_{32}	c_{33}	c_{34}	ω_{11}
c_{31}	(0.5,0.5)	(0.6,0.1)	(0.5,0.2)	(0.7,0.2)	0.4000
c_{32}	(0.1,0.6)	(0.5,0.5)	(0.6,0.2)	(0.6,0.1)	0.2957
c_{33}	(0.2,0.5)	(0.2,0.6)	(0.5,0.5)	(0.8,0.1)	0.2261
c_{34}	(0.2,0.7)	(0.1,0.6)	(0.1,0.8)	(0.5,0.5)	0.0783

TABLE VII
WEIGHT VECTORS OF MEASURES WITH RESPECT TO ORIGIN

	c_{41}	c_{42}	ω_{12}
c_{41}	(0.5,0.5)	(0.7,0.2)	0.75
c_{42}	(0.2,0.7)	(0.5,0.5)	0.25

TABLE VIII
WEIGHT VECTORS OF MEASURES WITH RESPECT TO FIRM

	c_{51}	c_{52}	c_{53}	ω_{13}
c_{51}	(0.5,0.5)	(0.8,0.1)	(0.7,0.2)	0.6667
c_{52}	(0.1,0.8)	(0.5,0.5)	(0.6,0.2)	0.1852
c_{53}	(0.2,0.7)	(0.2,0.6)	(0.5,0.5)	0.1481

TABLE IX
PRIORITYS OF TIMES OF BRAND AND MEASURES

c_{21}	0.2439	c_{34}	0.0208
c_{22}	0.1604	c_{41}	0.1150
c_{23}	0.0956	c_{42}	0.0383
c_{31}	0.1063	c_{51}	0.0540
c_{32}	0.0786	c_{52}	0.0150
c_{33}	0.0601	c_{53}	0.0120

TABLE X
EVALUATIONS OF ALTERNATIVES WITH RESPECT TO TIMES OF BRAND AND MEASURES

	c_{61}	c_{62}	c_{63}	c_{64}	c_{65}	c_{66}
c_{21}	0.6987	0.4068	0.5921	0.8687	0.5653	0.6133
c_{22}	0.7287	0.5333	0.6810	0.0591	0.0818	0.5931
c_{23}	0.4450	0.7183	0.8693	0.1348	0.0605	0.2311
c_{31}	0.8408	0.7419	0.1196	0.5815	0.1292	0.0715
c_{32}	0.7929	0.9607	0.6734	0.5614	0.1581	0.0345
c_{33}	0.8760	0.8599	0.8690	0.1894	0.7899	0.3696
c_{34}	0.0468	0.8425	0.2183	0.4329	0.4763	0.8916
c_{41}	0.3123	0.2391	0.7189	0.5738	0.4888	0.5621
c_{42}	0.5575	0.6219	0.6879	0.0723	0.5344	0.1913
c_{51}	0.9678	0.2563	0.9470	0.5496	0.9879	0.2685
c_{52}	0.6662	0.4327	0.8823	0.2796	0.0492	0.1295
c_{53}	0.8327	0.7224	0.5654	0.1095	0.8762	0.7092

TABLE XI
FINAL SCORES OF THE ALTERNATIVES

	c_{61}	c_{62}	c_{63}	c_{64}	c_{65}	c_{66}
c_{21}	0.1704	0.0992	0.1444	0.2119	0.1379	0.1496
c_{22}	0.1169	0.0855	0.1092	0.0095	0.0131	0.0951
c_{23}	0.0425	0.0687	0.0831	0.0129	0.0058	0.0221
c_{31}	0.0894	0.0788	0.0127	0.0618	0.0137	0.0076
c_{32}	0.0623	0.0755	0.0529	0.0441	0.0124	0.0027
c_{33}	0.0526	0.0517	0.0522	0.0114	0.0475	0.0222
c_{34}	0.0010	0.0175	0.0045	0.0090	0.0099	0.0185
c_{41}	0.0359	0.0275	0.0827	0.0660	0.0562	0.0646
c_{42}	0.0214	0.0238	0.0264	0.0028	0.0205	0.0073
c_{51}	0.0523	0.0138	0.0511	0.0297	0.0534	0.0145
c_{52}	0.0100	0.0065	0.0132	0.0042	0.0007	0.0019
c_{53}	0.0100	0.0087	0.0068	0.0013	0.0105	0.0085
Final Score	**0.6646**	**0.5572**	**0.6393**	**0.4645**	**0.3816**	**0.4148**

TABLE XII
FINAL SCORES OF ALTERNATIVES

Score	c_{61}	c_{62}	c_{63}	c_{64}	c_{65}	c_{66}
IFAHP	0.3577	0.5769	0.3729	0.3878	0.4182	0.5614
FANP	0.3620	0.5872	0.3731	0.4030	0.3841	0.5184
Value function of IFV	0.6576	0.5375	0.6759	0.4275	0.3871	0.4103
IFANP	0.6646	0.5572	0.6393	0.4645	0.3816	0.4148

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