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Numerical modelling method for inelastic and frequency-dependent behavior of shallow foundations

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ABSTRACT

This paper presents a novel framework with which the inelastic behavior and the frequency-dependent dynamic characteristics of soil-foundation system can be represented with a computationally efficient numerical model. The inelastic behavior of soil in the vicinity of a shallow foundation is represented with a macro-element which is based on the classical plasticity theory. The frequency-dependent property of soil-foundation system is represented with a recursive parameter model. The framework allows integration of both models such that both the inelastic behavior and the frequency-dependent characteristics can be captured. The proposed method is verified against FE analysis of a shallow foundation in the two dimensional parametric space of frequency and inelasticity. The verification shows that the model using the proposed framework can fully represent the inelastic cyclic behavior at low frequency excitation and the dynamic response at high frequency excitation. The method provides an approximate solution for the cases in-between, e.g. a foundation subjected large amplitude high-frequency excitation. As an application example, the method is applied to an analysis of a bridge pier subjected to earthquake loading.

1. Introduction

Performance-Based Seismic Design (PBSD) approach embraces explicit assessment of the response of structural components with target building performance objectives. As shallow foundation exhibits inelastic behavior at the interface of the soil-foundation system upon excessive load, realistic assessment of cyclic inelastic rocking response of foundation is recommended [1,2]. Two mechanisms of nonlinearity take place between the soil and foundation; geometric nonlinearity (i.e. rocking response) and material nonlinearity (i.e. yielding of soil).

The rocking response of shallow foundations has been one of the key research areas that has gained interest in recent years. Various experimental studies, such as a large-scale shaking table test of a bridge column [3], a small-scale shaking table test of a 3-storey building [4] and a centrifuge modelling of rocking-isolated inelastic RC bridge piers [5], have captured the rocking of the shallow foundation and found that this behavior reduces the residual drift and seismic demand of the structure. However, rocking shallow foundation may also experience large differential settlement during excessive cyclic loads which mainly result from yielding of near-field soil. On this basis, rocking of the foundation and yielding of soil should be carefully analyzed in the PBSD of shallow foundations [4,5].

There exists literature which provides guideline on modelling of shallow foundations. For example, ASCE 41-13 [6] provides a component action table with modelling parameters and acceptance criteria for nonlinear and linear analysis of shallow foundations. The values in the component action tables for nonlinear analysis procedures are based on the analysis of rocking shallow foundation, which was observed from experimental model tests. For the linear analysis procedure, the empirical coefficient, m-factor, is revised to reflect the allowable rotation of the rocking foundation from the nonlinear analysis procedure [7]. Unlike ASCE 41-06 [8] where rocking foundation and yielding at the soil-foundation interface are uncoupled and checked separately, ASCE 41-13 [6] considers the coupled behavior of foundation rocking and yielding of the soil. This approach is more realistic as the failure of the foundation is governed by stiffness degradation and yielding of the soil [9]. Kutter et al. [7] provided a rationale for the revisions made in ASCE 41-13 for rocking shallow foundation and validated these revisions with extensive experimental results [7,10].

The numerical modelling of dynamic soil-structure interaction (SSI) between an inelastic soil domain and a structural model for shallow foundation is a complicated task. The model entails an infinite soil domain, interface property between the structural foundation and soil, and verification and benchmark analysis. The Finite Element (FE) analysis is a rigorous method which is able to model the infinite soil domain with an arbitrary geometry and diverse soil layers. However,

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Nomenclature

c	cohesive strength of soil
$\mathbf{c}$	damping matrix of foundation-soil system
$\mathbf{C_j}$	recursive damping matrix at $t=t-t_j$
$\mathbf{C_0}$	instantaneous damping matrix of soil
D	characteristic dimension of strip or circular footing (i.e. width of strip or diameter of circular footing)
E_d	dissipated energy
E_{So}	elastic restoring energy
$\boldsymbol{F}$	force vector of foundation-soil system
$\boldsymbol{F}_i$	force vector of foundation DOF ($i=f$) or soil DOF ($i=s$)
h_0	Initial plasticity parameter in macro-element
$\mathbf{k}$	static stiffness matrix of foundation- soil system
$\mathbf{k}_{ij}$	static stiffness matrix corresponding to foundation DOF(f) or soil DOF(s)
$\mathbf{K}_{el,uplift}$	stiffness parameter with uplift coefficient of the foundation
$\mathbf{K}_{ij}$	dynamic stiffness matrix corresponding to foundation DOF (f) or soil DOF (s)
K_{ij}	ij= N, V, M , normalized elements of the stiffness parameters for macro-element
$\mathbf{K_j}$	recursive stiffness matrix at $t=t-t_j$
$\mathbf{K_0}$	instantaneous stiffness of soil
$\mathbf{K_{pl}}$	the plastic stiffness calculated using mapping rule
N	vertical force on the footing
N_{max}	maximum bearing capacity of footing
$\boldsymbol{m}$	mass matrix of foundation-soil system
M	moment applied on the footing
$\mathbf{M_0}$	instantaneous mass matrix of the soil
p_1	plasticity parameter in macro-element
$\boldsymbol{q}_{el}$	elastic response of macro-element
$\boldsymbol{q}_{pl}$	plastic response of macro-element
$q_i,\ i=N, V, M$	normalized displacement parameter
$Q_i,\ i=N, V, M$	normalized force parameter
$Q_{M,O}$	Uplift moment initiation for macro-element
$\boldsymbol{r}_D$	restoring force vector from dynamic response of soil-foundation system
$\boldsymbol{r}_f$	restoring force vector from the overall response of soil-foundation system
$\boldsymbol{r}_S$	restoring force vector from quasi-static response of soil-foundation system
t_j	occurrence time (time delay) of the reflection reaction
$\boldsymbol{u}$	displacement vector of foundation-soil system
$\boldsymbol{u}_i$	displacement vector of foundation DOF ($i=f$) or soil DOF ($i=s$)
u_x	horizontal displacement of the footing
u_z	vertical displacement of the footing
V	horizontal force on the footing
x, y, z	cartesian coordinates
Greek	
ν	Poisson's ratio
θ_y	rotation angle
ω	frequency
ζ	equivalent damping ratio

special measures should be taken to accurately model the boundaries of the numerical model. The scattered waves from a structure should be dissipated or absorbed at the boundaries in order to avoid the wave reflection. This means that the numerical domain should be large enough to avoid the negative effects of the reflected wave to the structural responses. If the inelastic dynamic behavior of the foundation is of interest, it is necessary to model soil as a nonlinear material and include the soil-foundation interface. This type of analysis, however, takes enormous computing time as presented in Kabanda et al. [11] where a large inelastic soil domain and a structure were modelled with FE method. Due to these challenges, shallow foundations have been modelled using simplified methods. There are mainly four categories of simplified modelling techniques commonly used in the research and engineering practice; the uncoupled lumped spring approach as presented in ASCE 41-13 [7,10,12]; beam-on-a-nonlinear Winkler foundation (BNWF) [1,13]; simplified nonlinear model with springs and dashpots [2,14]; and a macro-element method with plasticity formulation [15–17].

These simplified models have their own strengths and drawbacks in simulating inelastic behavior of shallow foundations. One major drawback of these methods is the replacement of a dynamic soil-foundation system with an equivalent static or lumped element which ignores the frequency-dependent properties of soil domain. As the simplified model replaces the soil model with equivalent linear or nonlinear springs, the frequency-dependent stiffness and damping components of the soil-foundation system are ignored. This leads to inaccurate representation of wave propagation from structure to soil domain as the inertia and energy dissipation from radiation damping of soil is neglected. It has been well documented that the nature of soil-structure interaction is frequency-dependent [12,18,19]. Mylonakis et al. [12] have compiled studies on frequency-dependent stiffness and damping of soil-foundation system and provided a guideline for engineers to include frequency-independent properties of soil. In their study, it has concluded that soil foundation system exhibits frequency-dependent behavior even for low magnitude cyclic load. Also, Lesgidis et al. [19] have investigated the influence of frequency-dependent characteristics of soil-foundation system on the fragility curves established for RC bridges. It was found that there is a meaningful correlation between the frequency content of the earthquake and the numerical error introduced with the use of frequency-independent approach in establishing the fragility curves.

The objective of this paper is to propose a framework, which can couple a frequency-dependent model of soil-foundation system with a frequency-independent inelastic model. The framework aims to provide a practically accurate modelling approach which can capture the behavior of nonlinear soil-structure interaction problem subjected to dynamic load. The proposed framework is a generalized approach such that it can be used to couple any frequency-dependent model with any inelastic model. As an implementation example, the recursive parameter model by Nakamura [20,21] and the macro-element by Chatzigogos et al. [17] are adopted to model frequency-dependent and inelastic behavior, respectively.

The overall framework is presented in Section 2.1. This framework is used to integrate macro-element and a recursive parameter model, which are introduced in Section 2.2 and Section 2.3, respectively.

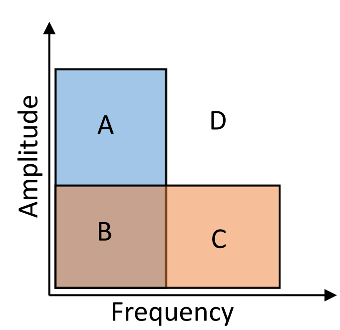


Fig. 1. Domain of dynamic loads.

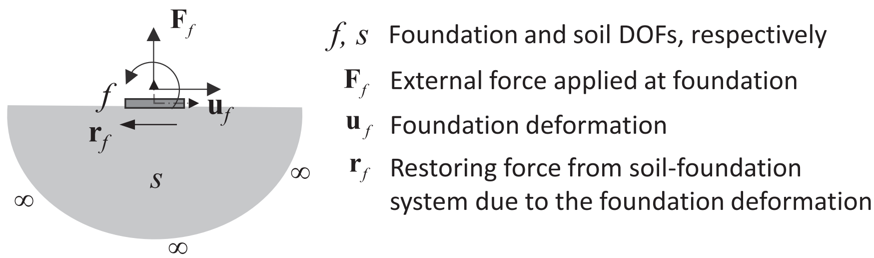


Fig. 2. Illustration of soil-foundation system subjected to dynamic load at the foundation.

Section 2.4 presents details on how the two models are integrated. In Section 3, the integrated model is verified against extensive FE analysis of a shallow foundation in two dimensional parametric space of frequency and inelasticity. This verification illustrates the ability of the proposed model to capture the frequency dependency as well as the inelastic behavior of the soil-foundation system. A practical application example of the proposed method is presented in Section 4, which is followed by the conclusion in Section 5.

2. Integration method for inelastic and frequency-dependent models

2.1. Proposed framework

Dynamic loads applied to a soil-foundation system can be characterized in terms of frequency and amplitude as illustrated in Fig. 1. When the frequency of excitation is relatively small (i.e. domain A and B in Fig. 1), the dynamic characteristics of the soil-foundation system does not largely influence the structural response. In this case, the numerical model of the soil-foundation system should capture the quasi-static inelastic hysteretic behavior which results from yielding of soil and/or uplifting or sliding of foundation. The inelastic hysteretic behavior may also be affected by coupling between axial force, shear force, and moment. A finite element (FE) model, nonlinear springs, or macro-elements can be used to capture the response of soil foundation systems subjected to loads in domain A and B.

On the other hand, when the amplitude of excitation is relatively small (i.e. domain B and C), then inelasticity of a soil-foundation system can be ignored. Then, the numerical model should focus on capturing correct dynamic characteristics of a soil-foundation system. The dynamic characteristics of a soil-foundation system is frequency-dependent, thus special approach is necessary in numerical modelling such as a set of calibrated lumped springs, masses, and dampers; FE model with fine mesh and energy absorbing (or wave transmitting) boundary; recursive parameter models, etc. To simplify the modelling process, the frequency dependent characteristics are often simplified as frequency independent lumped springs or dashpots, whose properties are determined at the predominant frequency of excitation or at the natural frequency of the system.

In the event of an earthquake, however, the excitation may encompass all regions in Fig. 1(i.e. domains A–D). Thus, ideally, a numerical model should be able to capture both the inelasticity and frequency dependency. Several approximate approaches have been proposed to meet this requirement. For example, Pecker et al. [15] and Chatzigogos et al. [16] used a dashpot damper where the property of the damper was selected at the predominant frequency of excitation or natural frequency of the soil-structure system [22]. Because the damping value is selected at a specific frequency of excitation, the model needs to be recalibrated for a structure with different natural frequency or for input excitation with different frequency contents.

The dynamic response of soil-foundation system, which is illustrated in Fig. 2, subjected to external force can be found by solving the following equation:

$$\mathbf{m}\ddot{\boldsymbol{u}}+\mathbf{c}\dot{\boldsymbol{u}}+\mathbf{k}\boldsymbol{u}=\boldsymbol{F} \tag{1}$$

where m, c, and k are the mass, damping, and stiffness matrices of the soil-foundation system, and F is the excitation force. The system of equation may include dashpot components for energy absorbing boundary. In the frequency domain, the equation can be expressed as

$$(\mathbf{k}-\omega^2\boldsymbol{m}+i\omega\mathbf{c})\boldsymbol{u}(\omega)=\boldsymbol{F}(\omega)$$

$$\text{or, } \mathbf{K}(\omega)\boldsymbol{u}(\omega)=\boldsymbol{F}(\omega) \tag{2}$$

where $\mathbf{K}(\omega)=\mathbf{k}-\omega^2\mathbf{m}+i\omega\mathbf{c}$. Eq. (2) can be decomposed into two domains; one for foundation degrees of freedoms (DOFs), f, and the other one for soil DOFs, s.

$$\begin{bmatrix}\mathbf{K}_{ff} & \mathbf{K}_{fs}\\ \mathbf{K}_{sf} & \mathbf{K}_{ss}\end{bmatrix}\begin{bmatrix}\boldsymbol{u}_f\\ \boldsymbol{u}_s\end{bmatrix}=\begin{bmatrix}\boldsymbol{F}_f\\ \boldsymbol{F}_s\end{bmatrix} \tag{3}$$

The notation (ω) is dropped for simplicity. Because the dynamic force from a structure is applied to the foundation DOFs only (i.e. $\boldsymbol{F}_s=0$) and because the foundation's response, $\mathbf{u}_f$ is of interest, Eq. (3) can be condensed as below.

$$(\mathbf{K}_{ff}-\mathbf{K}_{fs}\mathbf{K}_{ss}^{-1}\mathbf{K}_{sf})\mathbf{u}_f=\boldsymbol{F}_f$$

$$\text{or } \mathbf{K}_f\mathbf{u}_f=\boldsymbol{F}_f \tag{4}$$

where $\mathbf{K}_f=\mathbf{K}_{ff}-\mathbf{K}_{fs}\mathbf{K}_{ss}^{-1}\mathbf{K}_{sf}$. The term $\mathbf{K}_f$ is the dynamic stiffness of the soil-foundation system which is a function of the excitation frequency, ω, and considers energy dissipation from wave propagation to infinite viscous medium. When the excitation frequency approaches zero (i.e. $\omega\to0$) or when the mass and stiffness matrices are negligible, the dynamic stiffness matrix, $\mathbf{K}_f$ is equivalent to the condensed static stiffness matrix $\mathbf{k}_f$ where $\mathbf{k}_f=\mathbf{k}_{ff}-\mathbf{k}_{fs}\mathbf{k}_{ss}^{-1}\mathbf{k}_{sf}$. The numerical evaluation of $\mathbf{K}_f$ requires significant amount of computing time as it requires a large soil domain and fine mesh. Thus, formulas and charts are available for typical foundation geometry [23].

To run analysis of soil-foundation-structure system in time domain, Eq. (4) needs to be transformed to an equivalent model suitable for direct time stepping method. Several methods are available for the transformation such as lumped parameter models [10–17] and recursive parameter models [20,21,24,25]. Assuming that Eq. (5) is the time domain representation of Eq. (4) using any one of the available methods gives the following expression,

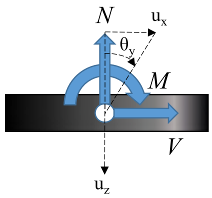
$$\boldsymbol{r}_D(\boldsymbol{u}_f)=\boldsymbol{F}_f \tag{5}$$

where $\mathbf{r}_D$ is a restoring force vector from soil-foundation system which is a function of displacements of the foundation DOFs, $\mathbf{u}_f$. The subscript D denotes that the restoring force vector is from dynamic response of the soil-foundation system. When mass and damping terms can be ignored, or when the excitation frequency approaches zero, the restoring force vector is equivalent to the condensed static stiffness matrix multiplied to the displacement vector.

$$\boldsymbol{r}_D(\boldsymbol{u}_f)=\mathbf{k}_f\boldsymbol{u}_f=\boldsymbol{F}_f \tag{6}$$

For inelastic quasi-static response, the restoring force from the soil-foundation system, $\mathbf{r}_s$ is a nonlinear function of the deformation due to given quasi-static deformation, $\mathbf{u}_f$.

$$\boldsymbol{r}_S(\boldsymbol{u}_f)=\boldsymbol{F}_f \tag{7}$$



$$\boldsymbol{Q}=\begin{bmatrix}Q_N\\Q_V\\Q_M\end{bmatrix}=\begin{bmatrix}\dfrac{N}{N_{max}}\\\dfrac{V}{N_{max}}\\\dfrac{M}{DN_{max}}\end{bmatrix},\quad \boldsymbol{q}=\begin{bmatrix}q_N\\q_V\\q_M\end{bmatrix}=\begin{bmatrix}\dfrac{u_z}{D}\\\dfrac{u_x}{D}\\\theta_y\end{bmatrix}$$

Fig. 3. Generalized force and displacement diagram for shallow foundation in macro-element.

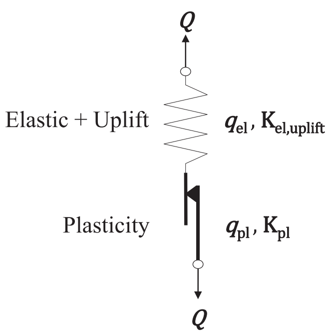


Fig. 4. Schematic diagram of the macro-element model with elasticity and plasticity.

Table 1
Soil material properties for FE model.

Components	Properties
Foundation	10 m wide rigid strip foundation
Interface	With/without uplift
Soil	ρ= 1.6 t/m^3, G = 65,000 kPa
	ν=0.25, c = 30 kPa
	100 m in height, 100 in width soil domain; cohesive soil

Table 2
Numerical parameters for the proposed method.

Element	Numerical parameters	Comments
Macro-element (Chatzigogos et al. [16,17])	B=10 m	Width of foundation
	Nmax=1.6×10^6 N	Maximum bearing capacity of footing
	$\mathbf{K}_{NN}$=8.9×10^7 N/m	Vertical stiffness
	$\mathbf{K}_{VV}$=8.6×10^7 N/m	Horizontal stiffness
	$\mathbf{K}_{MM}$=3.2×10^7 Nm/rad	Rotational stiffness
	$\boldsymbol{h}_0$=0.1$\mathbf{K}_{NN}$	Initial plasticity parameter calibrated from FE model
	$\boldsymbol{p}_1$=3	Plasticity parameter calibrated from FE model.
	$Q_{V,\,max}=0.3$	Maximum horizontal load capacity in bounding surface
	$Q_{M,\,max}=1.72$	Maximum moment capacity in bounding surface
	$Q_{M,O}=\pm\frac{Q_N}{4}\exp(-5Q_N)$	Moment of uplift initiation of footing on elastoplastic soil
Recursive parameter model (Nakamura [20,21])	Dynamic stiffness	Dynamic stiffness defined at 0.5 Hz interval (0.5, 1.0, ..., 20 Hz). Dynamic stiffness at beyond 20 Hz was extrapolated based on Duarte et al. [29]
	Impulse response	Following recursive parameters are defined at 0.001 s interval based on the dynamic stiffness. $\mathbf{M}_0$, $\mathbf{K}_0$, $\mathbf{K}_1$, $\mathbf{K}_2$, ..., $\mathbf{K}_{1000}$, $\mathbf{C}_0$, $\mathbf{C}_1$, $\mathbf{C}_2$, ..., $\mathbf{C}_{999}$

where the subscript S denotes that the restoring force vector only considers static response. The restoring force is path dependent, and may show broad range of hysteretic behaviors depending on the constitutive model of soil or uplifting and sliding of foundation. Most simplified nonlinear spring models or macro-elements attempt to replicate the restoring force vector,$\boldsymbol{r}_S$. If the amplitude of displacement is sufficiently small, then the soil can be assumed to be elastic. Then,

$$\boldsymbol{r}_S(\boldsymbol{u}_f)=\mathbf{k}_f\boldsymbol{u}_f=\boldsymbol{F}_f \tag{8}$$

It is worth noting that Eqs. (6) and (8) are identical, and represent the restoring force of soil-foundation system when both amplitude and frequency of excitation is small, which is domain B in Fig. 1.

In order to capture both dynamic elastic and quasi-static inelastic responses (domains A, B, and C in Fig. 1), the following expression is proposed.

$$\boldsymbol{r}_f(\boldsymbol{u}_f)=\boldsymbol{r}_D(\boldsymbol{u}_f)+\boldsymbol{r}_S(\boldsymbol{u}_f)-\mathbf{k}_f\boldsymbol{u}_f \tag{9}$$

Eq. (9) clearly shows that when the dynamic response can be ignored (i.e. low excitation frequency or negligible damping and mass matrices, domain A and B in Fig. 1), then $\boldsymbol{r}_D(\boldsymbol{u}_f)$ and $\mathbf{k}_f\boldsymbol{u}_f$ cancel each other, which results in Eq. (7). In the same way, when the amplitude is small or the system is linear elastic (i.e. domain B and C in Fig. 1.), then Eq. (9) is identical to Eq. (5). Thus, Eq. (9) can approximate the inelastic dynamic response of a soil-foundation system. Extensive parametric study with a FE model proves the efficacy of this method. This method, however, provides approximate result for the domain D in Fig. 1.

To implement the framework for soil-structure-interaction problem, it is necessary to select suitable models which can evaluate the restoring force vector, $\boldsymbol{r}_S$ and $\boldsymbol{r}_D$, for quasi-static inelastic response and dynamic elastic response, respectively. In this paper, for two-dimensional shallow foundation, the macro-element proposed in Chatzigogos et al. [17] is adopted to calculate the restoring force vector, $\boldsymbol{r}_S$, and the recursive parameter model in Nakamura [20,21] is used to calculate the vector $\boldsymbol{r}_D$. The proposed model for the restoring force vectors ($\boldsymbol{r}_S$and $\boldsymbol{r}_D$.) are summarized in the following sections.

2.2. Inelastic cyclic behavior modelled with a macro-element

The macro-element is a simplified element where the behavior of soil-structure interaction is captured in a single node. The initial concept of macro-element was introduced by Nova and Montrasio [26] for rigid shallow foundation on sand with generalized force-displacement relationship. Paolucci [27] has applied the macro-element to structures subjected to earthquake excitations. Then, Cremer et al. [15] has introduced uplift condition of the foundation coupled with the plasticity of the soil. Chatzigogos et al. [17] have extended the macro-element to dynamic loading application with frictional soil, taking into consideration of the coupled uplift and sliding of the foundation. More details on the macro-element can be found in Chatzigogos et al. [17]. The following is a high level summary of the element.

In the macro-element, a single node is placed at the center of the rigid foundation. This node has horizontal, vertical, and rotational DOFs to represent the response of footing in two dimensional problems. The normalized force and displacement parameters are used in the formulation as shown in Fig. 3.

For analysis purposes, the force parameters are normalized with respect to the maximum bearing capacity of the foundation, N_{max}, and the displacement parameters are normalized with the characteristic dimension of the footing, D (i.e. width of the strip foundation). Using these parameters, the generalized force-displacement relationship is represented by a generalized stiffness matrix given by:

$$\begin{bmatrix}\dot{Q}_N\\\dot{Q}_V\\\dot{Q}_M\end{bmatrix}=\begin{bmatrix}K_{NN} & K_{NV} & K_{NM}\\K_{VN} & K_{VV} & K_{VM}\\K_{MN} & K_{MV} & K_{MM}\end{bmatrix}\begin{bmatrix}\dot{q}_N\\\dot{q}_V\\\dot{q}_M\end{bmatrix} \tag{10}$$

In the macro-element formulation, the generalized force and

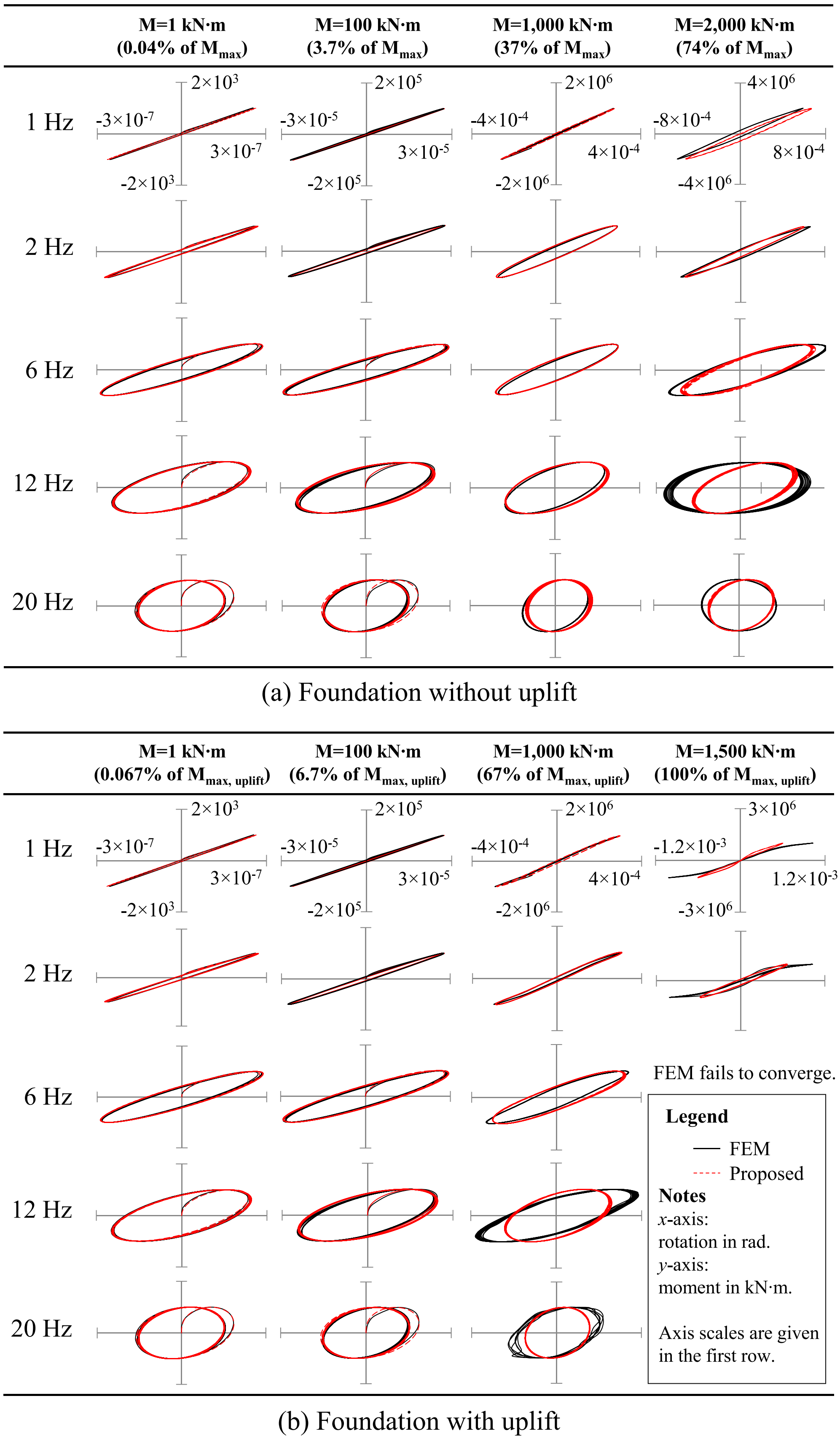


Fig. 5. Parametric study of varying frequency and amplitude.

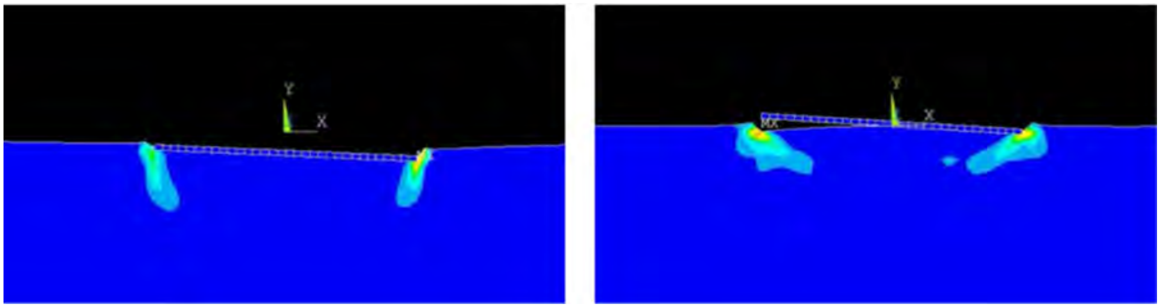


Fig. 6. 1000 kN m moment applied at the foundation at 4 Hz excitation.

displacements are expressed in the incremental format, denoted by dots on each force and displacement variables as shown in Eq. (10). The static stiffness matrix for various soil and foundation types are provided by Mylonakis et al. [12]. Based on this formulation, the uplift of the foundation is captured by the coupling terms of vertical and rotational DOFs. More details on the formulation of this matrix and each parameters used for uplift of the foundation are available in Chatzigogos et al. [17].

The mechanism of soil material yielding in the vicinity of the footing is described by the bounding surface through a hypoplastic model. For a simplified macro-element model, ellipsoidal shape bounding surface at the origin is used [17]. This bounding surface defines the maximum capacity of the foundation in each DOFs. The role of the bounding surface is to define the case of pure loading, neutral loading and unloading; where pure loading and reloading are accompanied by the development of plastic displacements and unloading response is purely elastic. The mapping rule is used where each point in the interior of the bounding surface is projected onto the specific point on the bounding surface. The hyperplastic formulation is discussed in details in Chatzigogos et al. [17]. Once the elastic and plastic response of the element is analyzed, they are superimposed to represent the overall behavior of the soil-foundation as shown in Eq. (11).

$$\boldsymbol{q}=\boldsymbol{q}_{\mathbf{el}}+\boldsymbol{q}_{\mathbf{pl}} \tag{11}$$

Fig. 4 shows the schematic diagram of this element accounting for two nonlinear mechanisms. Using the hypoplastic constitutive law, the plasticity is defined by the ratio of the force experienced by the footing to the bounding surface of the footing. Uplift, on the other hand, is defined as a condition in the elastic response when the moment exceeds certain moment (referred to as 'uplift initiation moment') which initiates detachment of the foundation from soil. Then, coupling effect of the uplift condition and inelasticity of the soil is triggered. This formulation consists of mathematical expressions and numerical parameters that are explained in Chatzigogos et al. [17].

The elastic response of the macro-element is formulated based on the static stiffness and uplift coefficients of the foundation. Then, the elastic and plastic response of the soil-foundation system is combined at each force increments. The restoring force of the foundation, $\boldsymbol{Q}$, occurring from macro-element can be calculated based on the displacement vector, $\boldsymbol{q}$:

$$\boldsymbol{Q}=[\mathbf{K}_{\mathbf{el,uplift}}{}^{-1}+\mathbf{K}_{\mathbf{pl}}{}^{-1}]^{-1}\boldsymbol{q}=\boldsymbol{r}_s \tag{12}$$

where $\mathbf{K}_{\mathbf{el,uplift}}$ is the stiffness parameter from macro-element with uplift coefficient of the foundation, and $\mathbf{K}_{\mathbf{pl}}$ is the plastic stiffness calculated using mapping rule. The response of force can be represented with $\boldsymbol{r}_s$ which is the restoring force of the foundation with nonlinearity of the soil and uplift of the foundation for the respective DOFs as shown in Fig. 3.

2.3. Frequency-dependent stiffness represented with a recursive parameter model

In order to capture the frequency dependent behavior of soil in a time domain analysis, it is necessary to transform the dynamic impedance function from frequency domain to time domain. The most straightforward method for this conversion was proposed by Wolf [28], where it was suggested to use the inverse Fourier transform to convert the impedance function into an impulse response function. This method, however, is susceptible to numerical instability [29]. Şafak [25] formulated a recursive parameter model that relies on implementing an infinite impulse response filter. This transformation method generates reaction forces that are dependent on the values of the reaction force at previous time steps in addition to being dependent on the foundation displacement history. Nakamura [20] formulated a recursive parameter model where the restoring force at any time step is dependent on the past displacement and velocity histories. Nakamura [21] later improved the method by introducing an instantaneous mass component that would make the restoring force dependent on the current acceleration as well. More details on Nakamura's recursive parameter model can be found in Nakamura [20,21]. A method to assess numerical stability of the recursive parameter models is proposed by Duarte et al. [29]. The study showed that the recursive parameter model in Nakamura [20,21] is stable when it is integrated with a time integration scheme. Thus, in this study, the model in Nakamura [20,21] is used to represent frequency-dependent characteristics. However, the proposed framework in Section 2.1 can adopt any other time-domain representation of frequency-dependent behavior.

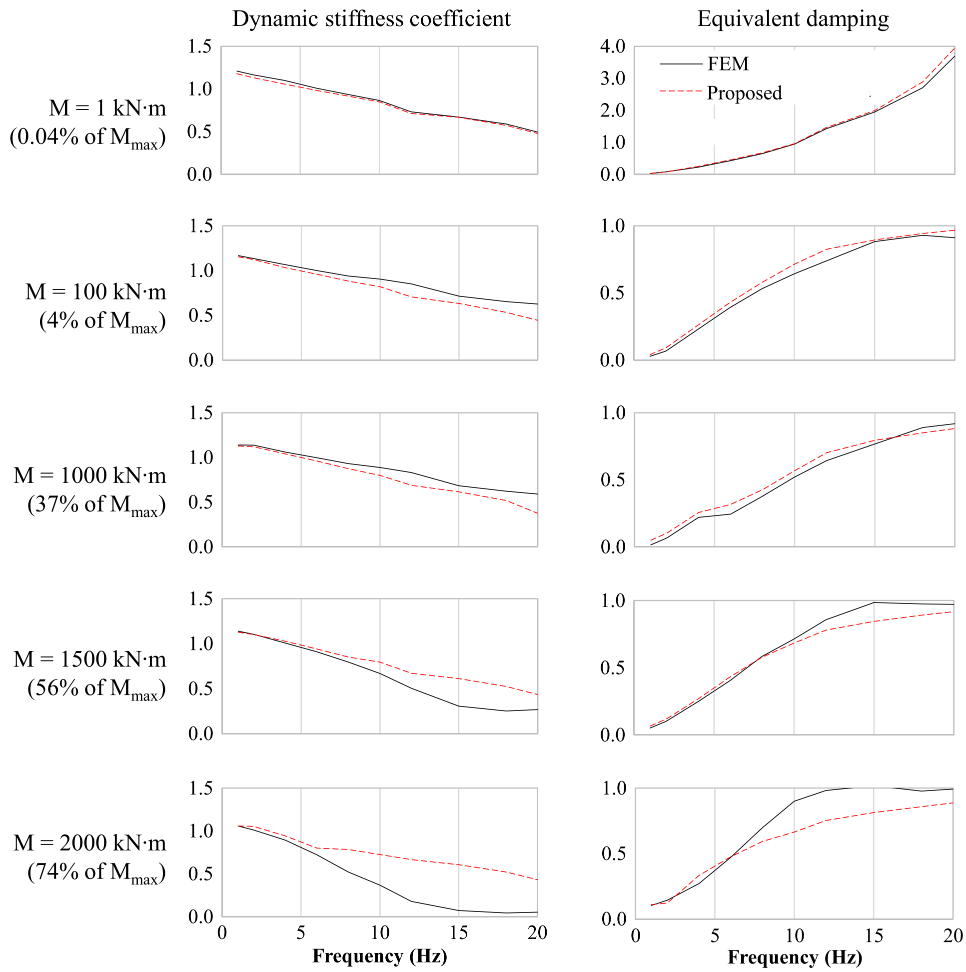
In the improved Nakamura's model [21], the restoring force resulting from the dynamic response of soil-foundation system is defined using stiffness, damping, and mass terms as shown in Eq. (13). Nakamura [23] proposed a transformation method with which a dynamic impedance function can be directly transformed into a set of stiffness, damping, and mass coefficients in Eq. (13).

$$\boldsymbol{r}(t)=\mathbf{K}_0\boldsymbol{u}(t)+\mathbf{C}_0\dot{\boldsymbol{u}}(t)+\mathbf{M}_0\ddot{\boldsymbol{u}}(t)+\left\{\sum_{j=1}^{N-1}\mathbf{K}_{\mathbf{j}}\boldsymbol{u}(t-t_j)+\sum_{j=1}^{N-2}\mathbf{C}_{\mathbf{j}}\dot{\boldsymbol{u}}(t-t_j)\right\}=\boldsymbol{r}_{\mathbf{D}}(t) \tag{13}$$

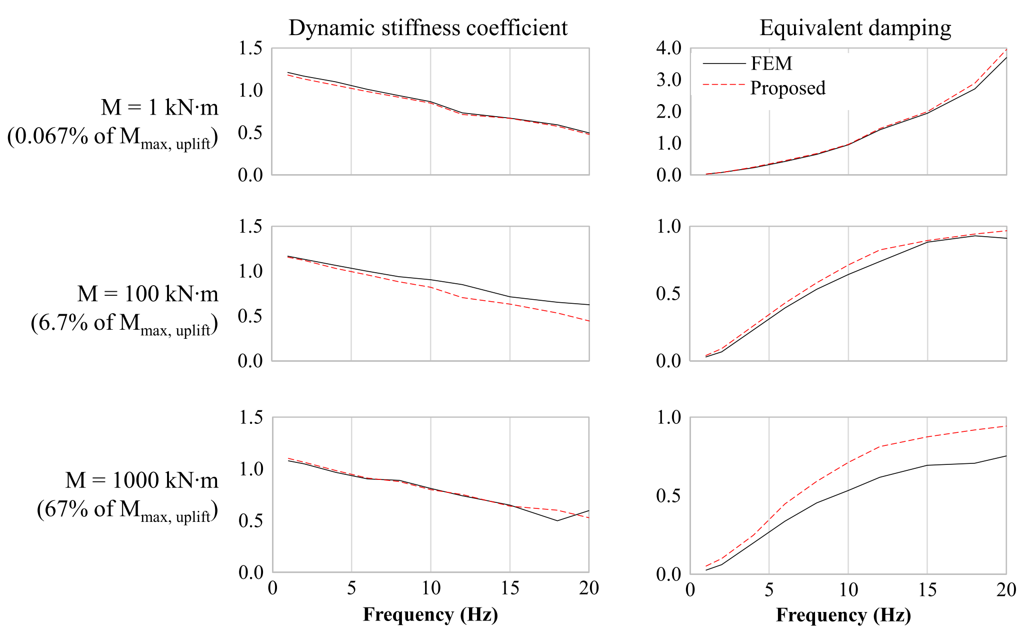
where $\boldsymbol{r}(t)$ is the restoring force occurring from the soil at time t. $\mathbf{K}_0$, $\mathbf{C}_0$ and $\mathbf{M}_0$represent the instantaneous stiffness, damping, and mass of soil foundation system at time t, respectively. The instantaneous mass of the soil takes into account the inertial force of the soil. $\mathbf{K}_{\mathrm{j}}$ and $\mathbf{C}_{\mathrm{j}}$ represent the recursive parameters of the past displacement and velocity terms. Eq. (13) can be integrated into a typical numerical time integration scheme, such as Newmark's method.

2.4. Integration of macro-element and recursive parameter model

The restoring force from macro-element Eq. (12) in Section 2.2 and



(a) Numerically evaluated dyanmic impedance of foundation without uplift



(b) Numerically evaluated dyanmic impedance of foundation with uplift

Fig. 7. Dynamic impedance with uplift (uplift initiates at M=1000 kN m).

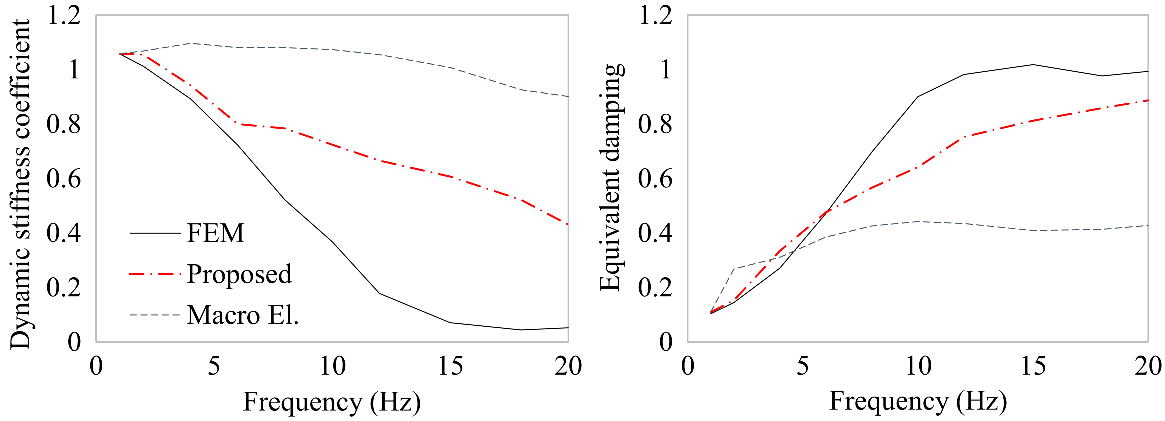


Fig. 8. Dynamic impedance without uplift for FE, macro-element and the proposed method with 2000 kN m load (macro-element and Nakamura's model).

Table 3
Measured computational time for different methods.

	Nakamura	Macro-element	Nakamura and Macro-element	2D FE model
Computational time (s)	20	10–15	30	5400–7200

the restoring force from the recursive parameter model Eq. (13) in Section 2.3 can be integrated using Eq. (9) to capture both inelasticity and frequency-dependency of soil-foundation system. The combined expression is given in Eq. (14) which shows calculation of restoring forces for the DOFs of the soil-foundation system (i.e. vertical, horizontal, and rotational DOFs of foundation). This formulation includes the three DOFs at the foundation shown in Fig. 3 with dynamic impedance for each DOFs

$$\mathbf{r}_f=\boldsymbol{r}_D+\boldsymbol{r}_S-\mathbf{k}_f\boldsymbol{u}_f=\mathbf{K}_0\boldsymbol{u}_f(t)+\mathbf{C}_0\dot{\boldsymbol{u}}_f(t)+\mathbf{M}_0\ddot{\boldsymbol{u}}_f(t)+\left\{\sum_{j=1}^{N-1}\mathbf{K}_j\boldsymbol{u}_f(t-t_j)+\sum_{j=1}^{N-2}\mathbf{C}_j\dot{\boldsymbol{u}}_f(t-t_j)\right\}+\left[[\mathbf{K}_{\mathrm{el,uplift}}{}^{-1}+\mathbf{K}_{\mathrm{pl}}{}^{-1}]\boldsymbol{u}_f(t)-\mathbf{k}_f\boldsymbol{u}_f(t)\right] \tag{14}$$

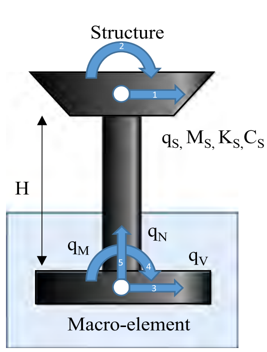
The impedance terms used for shallow foundation include the vertical, horizontal, rotational DOFs and a coupling term of horizontal to rotational DOFs. The cross coupling of horizontal-rotation impedance is usually negligibly small in shallow foundations [12], but it is included in this paper for completeness. $\boldsymbol{r}_S$ is the restoring force of macro-element Eq. (12), and $\boldsymbol{r}_D$ is the dynamic impedance terms represented as a restoring force from the recursive parameter terms Eq. (13). Note that $\mathbf{K}_0,\mathbf{C}_0,\mathbf{M}_0$ are the instantaneous terms from the dynamic impedance functions of soil, and are different from static stiffness, damping, and mass terms of the soil foundation. $\mathbf{k}_f$ is the condensed static stiffness of the foundation as discussed in Eq. (9). The restoring force at each time increment is analyzed from both of the models. This allows the restoring force from macro-element with uplift of the foundation and plasticity of soil to be combined with the recursive parameter model.

3. Verification of the proposed model

The nonlinear dynamic response of soil foundation model is analyzed with a broad range of frequency as well as a broad range of amplitude to cover various cases of dynamic loads as presented in Fig. 1. The purpose of this analysis is to create parametric space of frequency and amplitude of load where the model exhibits inelastic and frequency dependent behavior. The model description and material properties are provided in Table 1.

FE model has been used to construct a soil model that is 100 m wide and 100 m deep. FE mesh consists of 1 m by 1 m four-node quadrilateral elements. The von Mises failure criterion is used in this specific example. A 10 m wide rigid strip foundation is placed at the top center of the soil domain. The Lysmer and Kuhlemeyer [30] boundary is applied to the FE model to dissipate energy propagation toward the boundaries. FE analysis software, RS2 [31] is used for analyzing the foundation without uplift. OpenSees [32] is used for analyzing the foundation with uplift.

The constant vertical force of 385.5 kN is applied to the foundation which is the 25% of the maximum bearing capacity of the foundation for this soil foundation system using FE analysis. The required parameters for macro-element are obtained using the static analysis results from the FE model. The maximum capacity of foundation in each DOFs is determined with monotonic analysis using the FE model. For the recursive parameter model, the dynamic impedance of the soil-foundation model is obtained using FE model with sinusoidal sweep analysis in each DOFs at the foundation (i.e. horizontal, vertical, and rotational DOFs). Details on obtaining dynamic impedance function of



Numerical parameters	Properties
Structure	M_s = 5 x 10[4] kg; J_s = 1.25 x 10[6] kg m[2]; H = 20m; S = 1.6 m[2], I = 2.13 m[4]; E = 35 x 10[9] Pa
Foundation	B = 10m; e = 0.7 m; L = 1.0m; ρ = 18 x 10[3] kg/m[3]; E → ∞
Interface	Uplift allowed; no sliding
Soil	ρ = 1.6 ton/m[3], G = 65,000 kPa ν = 0.25, c = 30 kPa

*Note: For strip foundation, unit length of 1 meter is applied.

Fig. 9. Dynamic analysis example with realistic bridge pier and footing dimension [16].

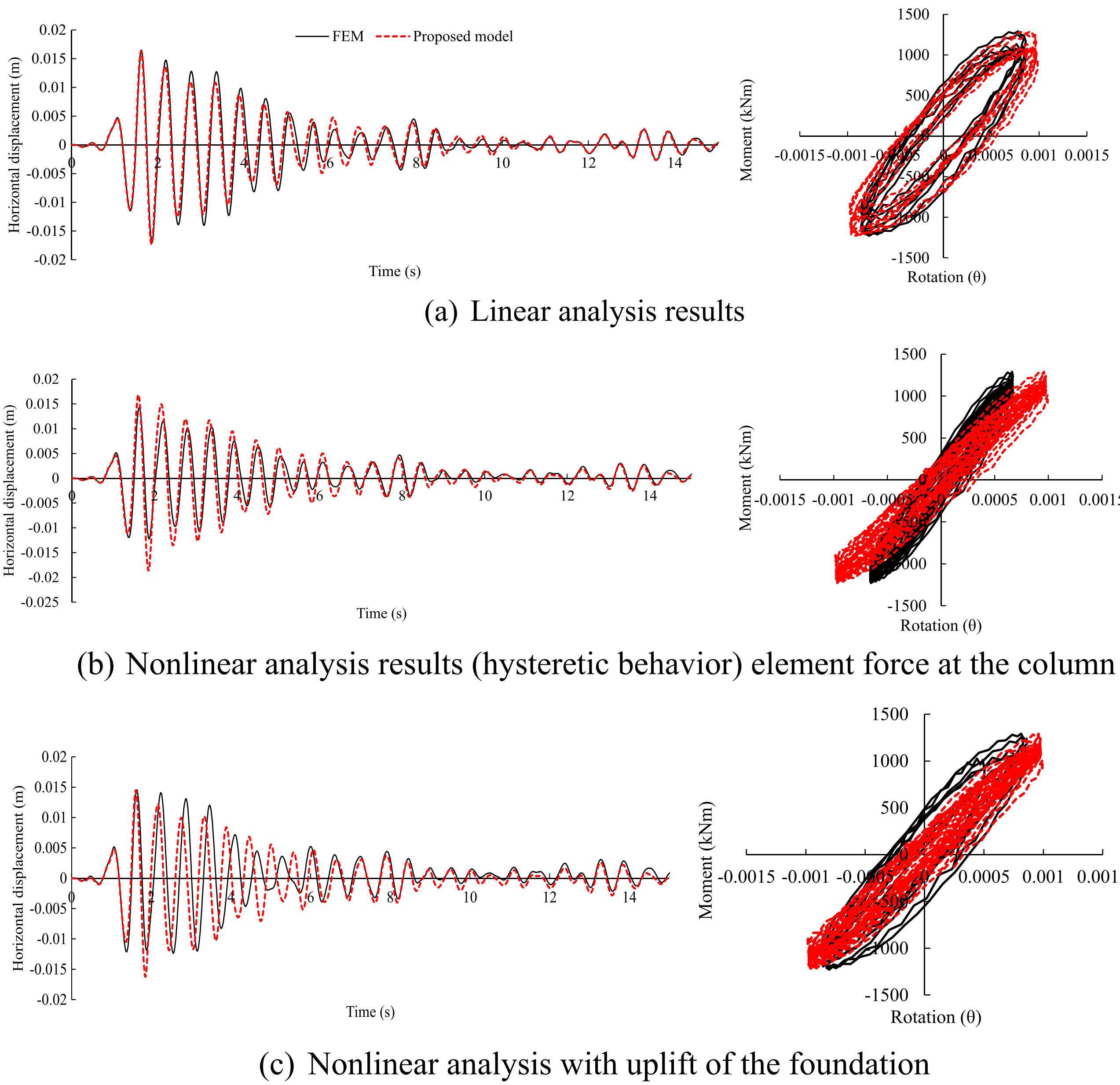


Fig. 10. Time history analysis results for Hollister excitation.

soil domain using FE analysis is provided in Zhang and Tang [33]. The required calibration parameters are obtained directly from the FE soil-foundation model. The calibrated values are provided in Table 2 for both macro-element and Nakamura recursive parameter model. For the analysis, sinusoidal moment is applied with varying frequencies and amplitudes at the center of the foundation. Two cases are considered: a) foundation without uplift, and b) foundation with uplift. For each case, an extensive FE analysis is carried out with the following parameters for verification of the proposed model.

1. Amplitude of excitation: M=100, 1000, 1500, 2000 kN m
   (Note: maximum moment capacity of the foundation is M_{max}≈2700 kN m)
2. Frequency of excitation: 1–20 Hz

The hysteretic loops showing the cyclic moment excitation versus rotation at the center of the foundation are shown in Fig. 5(a) for foundation without uplift and Fig. 5(b) for foundation with uplift. Also, in order to check the inelastic behavior of the foundation with uplift, the FE contour plots with excitation at 2 Hz frequency and moment amplitude of 1,000 kN m applied to the foundation is shown in Fig. 6(a) for case without the uplift and Fig. 6(b) for the uplift of the foundation.

For the case with moment of 1500 kN m applied to the foundation with uplift, the foundation is detached from the soil in excessive manner to a point where the center of the foundation detaches from the soil. Thus, the analysis beyond 2 Hz of excitation with magnitude of 1500 kN moment fails to converge for the uplift foundation model.

The results are in good agreement for low frequency range with all magnitudes of the moment as shown in Fig. 5(a) and (b) in the first two rows (1 Hz and 2 Hz) of the results. Thus, the quasi-static loading scenario is well captured with the proposed model. Also, at low magnitude of cyclic moment as shown in Fig. 5(a) and (b) in the first two columns (M=1 kN m and M=100 kN m) of the results, the proposed model agrees well with FE model, which shows that frequency-dependent property of the soil is well captured as well. The difference in the results is apparent when the foundation is subjected to high intensity with high frequency of excitation (i.e.

domain D from Fig. 1). There are still limitations with the proposed model in capturing a full nonlinear frequency dependent behavior of soil using the simplified model for high excitation frequencies. However, the proposed model can capture the inelastic behavior of foundation with wide range of excitation frequencies with small loss of accuracy.

In order to summarize the overall analysis results, dynamic impedance function has been used to plot all of the result findings in frequency domain. Within the hysteretic graph, apparent secant dynamic stiffness can be obtained by taking the slope of the hysteretic loop while energy dissipation can be calculated as a dynamic damper of the system. To normalize the dynamic impedance values, the dynamic stiffness terms are normalized with rotation static stiffness of the foundation. The equivalent damping terms are normalized with area under maximum force and displacements as shown in Eq. (16). More details on this formulation can be found in [22].

$$\zeta=\frac{E_d}{2\pi E_{So}}=\frac{\text{Energy dissipated}}{2\pi\left(\frac{f_{max}u_{max}}{2}\right)} \tag{16}$$

Fig. 7(a) shows the results of foundation without uplift for the proposed model and FE model at different magnitude of moment applied to the foundation, and Fig. 7(b) for the foundation with uplift in frequency domain. Note that the magnitude of loads and excitation frequency values are chosen for this specific example with a defined geometric and material properties of soil and foundation. The proportion of the load to the maximum load capacity of the foundation is also shown in the results.

As shown in the figure, the results are in good agreement from load intensity of 1 kN m to 100 kN m because uplift of the foundation has not occurred at these loads. At the magnitude of 1000 kN m, the FE and proposed model do have similar dynamic stiffness but the equivalent damping is slightly different as the frequency increases. The foundation uplift, plasticity of the soil and high excitation of frequency in numerical model is quite difficult to capture. The proposed model provides a satisfactory agreement with these nonlinearities as shown in Fig. 7(b).

In order to compare the proposed method with the existing model for macro-element with frequency independent soil, macro-element analysis has been carried out with frequency independent soil. A specific damping coefficient of soil is defined at 10 Hz of the soil impedance function in the analysis. The results are compared for foundation with 2000 kN m applied to foundation as shown in Fig. 8.

The macro-element model with frequency independent soil does not capture the full dynamic characteristics of soil at different excitation frequency. The proposed model, however, provides results that are in good agreement with the FE model by incorporating frequency-dependent properties of soil.

In order to highlight the computational efficiency of the proposed method, the measured computational time to complete the case study examples are presented in Table 3. The analysis has been performed using a processor with Intel ® Core ™ i7-4810MQ CPU @2.8 Ghz, and 8.00 GB of RAM.

As it can be observed from the table, FE model takes far more computational demand in comparison with the simplified modelling approach. While not quantitatively measured, the FE model also requires significant time to develop a model. Although a full three dimensional FE model provides can capture relatively realistic response, it is not a feasible option in most routine engineering practice due to the extensive modelling and computation time [1,11]. Therefore, the proposed model provides efficient way to model soil foundation system with inelastic soil and frequency dependency of soil.

4. Application example

To demonstrate how the proposed framework Eq. (9) and its implementation with a macro element and a recursive parameter model Eq. (14) can be used in a structural analysis, a simple soil-foundation-structure model is used as shown in Eq. (15). The system consists of total five DOFs where horizontal and rotational DOF are located at the structure and horizontal, rotational, and vertical DOFs are assigned at the foundation. It is assumed that the masses are lumped at the nodal points where the diagonal terms in the mass matrix are zeros. Also, the column is assumed to be axially rigid in the analysis. The equation of motion of the overall soil-foundation-structure system is presented in Eq. (15) and the diagram illustrates the model in Fig. 9. In this example, an effective seismic load is applied directly to the structure only in horizontal direction, F_x. The general matrix formulation allows load to be applied in any DOF desired.

$$\begin{bmatrix} M_{str} & 0 & 0 & 0 & 0 \\ 0 & J_{str} & 0 & 0 & 0 \\ 0 & 0 & M_f & 0 & 0 \\ 0 & 0 & 0 & J_f & 0 \\ 0 & 0 & 0 & 0 & M_{str}+M_f \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \\ \ddot{u}_4 \\ \ddot{u}_5 \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C}_{st\,4\times 4} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \\ \dot{u}_4 \\ \dot{u}_5 \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_{st\,4\times 4} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ \boldsymbol{r}_{f\,3\times 1} \end{Bmatrix} = \begin{Bmatrix} F_x \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{15}$$

As shown in the Eq. (15), matrix representation of the structure and foundation is presented. The combination of the DOFs with structure and foundation allow the model to analyze the seismic response of the desired DOFs in the time domain. The M_{str} *and* M_f terms refer to mass of the structure and foundation, while I_{str} and J_f are the moment of inertia of the structure and foundation respectively. $\boldsymbol{C}_{st}$ represents the damping matrix of the structure, while $\boldsymbol{K}_{st}$ represents stiffness matrix of the structure. The $\boldsymbol{r}_{f_{3\times 1}}$refers to the restoring force occurring from the soil foundation system as discussed in Eq. (14). This formulation allows the proposed model to simultaneously analyze the structure and foundation response with inelastic behavior of foundation including frequency-dependent properties of the soil. A bridge pier example has been analyzed using the proposed method as shown in Fig. 9.

This example illustrates the capacity of the model to analyze SSI effect in bridge piers with earthquake load where the system exhibits nonlinear soil behavior and the foundation uplift. The results are verified with FE model. The same material properties and soil domain size are used from the model in Section 3, and the structure and foundation material properties are provided from the example provided in Chatzigogos et al. [17]. The model is created in OpenSees. The details of the model material properties are provided in Fig. 9. Dynamic impedance function of soil model is obtained prior to this example with the FE model using sinusoidal sweep analysis as discussed in Section 3 with frequency range of 1–20 Hz.

The model is subjected to acceleration time history recorded in City Hall recording station during Hollister earthquake (USA, 1974), which is applied to the structure in horizontal direction as an effective force in this analysis. The horizontal displacement of the structure is recorded, and rotation of the foundation is recorded as well. The results from the proposed model is compared with Opensees model in linear elastic analysis (Fig. 10(a)), nonlinear soil domain analysis (material nonlinearity) (Fig. 10(b)) and nonlinear soil domain with uplift of the foundation (material and geometric nonlinearity) (Fig. 10(c)).

On the right side of Fig. 10, the hysteretic loop of the foundation rotation with moment is recorded when the structure is excited with harmonic load of 1 Hz with amplitude equivalent to the peak magnitude of the Hollister ground motion. As shown in Fig. 10(a), the proposed model is in good agreement with FE model linear elastic analysis. In Fig. 10(b), as the nonlinearity of the soil is introduced to the system, there is a slight difference at the peak magnitude of the displacement along the time history results. By comparing the hysteretic behavior of the system, the proposed method seems to calculate a higher rotation at the foundation than the FE model. For the case of nonlinear soil with foundation uplift, the results are in good agreement for the peak horizontal displacement of the structure. Although the horizontal displacement from the proposed method is slightly shifted from FE model, the hysteretic behavior of the proposed method for rotation of the foundation shows comparable results from the FE model.

Therefore, the proposed model is in satisfactory agreement in capturing the nonlinear soil behavior with frequency dependent behavior of soil foundation system. For this specific example, macro-element and Nakamura's recursive parameters are used to represent the soil foundation system which accurately captures the response of overall structure when subjected to a seismic excitation. Not only does this model greatly simplify the modelling approach with few calibrated parameters from users, but it also reduces significant amount of time to analyze the model. The proposed method can handle various types of analysis at a computationally efficient speed.

5. Conclusion

The paper presents a new framework to integrate the nonlinear model of soil foundation system with frequency dependent properties of soil in dynamic loading application. The nonlinearity of the soil and foundation system is captured with a computationally efficient macro element by Chatzigogos [17] while the frequency dependent behavior of soil-foundation system is captured with a recursive parameter model in Nakamura [20,21]. The proposed framework, however, is very general such that any other nonlinear element or frequency-dependent model can be integrated.

The results from the proposed framework have shown good agreement with FE model at low amplitude excitation with a broad range of frequencies. Also, the model can capture the nonlinear behavior of soil foundation system with uplift of the foundation at high amplitude excitation with low frequency. However, the accuracy of the model decreases when both the excitation frequency and magnitude are high. Thus, there exist still limitation in capturing a full inelastic frequency dependent behavior of soil foundation system. This limitation might be one of the shortcomings of the proposed framework in application to structures with significant higher mode effects.

It is also worthwhile to mention that while the framework was implemented for shallow foundations, the framework can be applied to embedded or deep foundations as long as reliable inelastic model and dynamic impedance functions are available. In addition, depending on the characteristics of the analyzed structure, the impact of the nonlinearity or frequency-dependency of soil-foundation system is expected to be different. Thus, it is equally important to have good understanding on structural characteristics when determining the modelling approach for soil-foundation system.

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