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# Seismic response control with inelastic tuned mass dampers

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ABSTRACT

An elasto-plastic spring is utilized in a tuned mass damper (TMD) with eliminating its viscous damper to establish a new seismic response control system. A novel method to find the most appropriate parameters of the proposed elasto-plastic TMD (P-TMD) including its initial stiffness/frequency and yield strength is presented so as to reduce the seismic response of the main system with the P-TMD to a level of that obtained with a previously suggested optimum TMD. The parameters are used to compute the responses of several main structures in the form of single-degree of freedom systems with the proposed P-TMD under different earthquake excitations. To evaluate the effectiveness of the proposed device and tuning method, maximum displacements and accelerations are compared to those of optimum TMD systems as well as those obtained from uncontrolled ones. The numerical results show that the proposed device, when using the introduced procedure for selecting its design parameters, reduces the seismic responses significantly and can be used instead of the optimum TMD without the need for a viscous damper.

## 1. Introduction

The tuned mass damper (TMD) is a typical passive control device attached to the main system with the goal of reducing vibrations of mechanical and structural systems under the action of external loads. This device consists of a mass, a spring and a viscous damper which all should be selected properly according to the properties of the main system and the applied loads. Because of its simple and reliable implementation, TMD has been widely used and studied. Effectiveness of TMDs depends on their properties, such as mass ratio, frequency and damping ratios, hence various studies have been carried out to obtain the optimal parameters of these devices.

Den Hartog [1] derived closed-form expressions for the optimum TMD parameters for undamped single-degree-of-freedom (SDOF) main systems under harmonic external forces. Simple expressions for optimum TMD parameters are also derived by Warburton [2] for undamped main systems subjected to external forces or support accelerations in the forms of harmonic and white noise random excitations. The effect of light damping in the main system on the optimum parameters of the TMD has also been investigated in [2] for random excitations and in [3,4] for harmonic force excitations. Tsai and Lin [5] developed a numerical searching procedure to find the optimum tuning frequency and damping ratio of the TMD for minimizing steady-state response of damped main systems subjected to harmonic support motions. In addition to harmonic excitations, the effectiveness of TMDs for wind loads has also been confirmed by several investigations [6–8]. Performance of single TMD systems, however, in structures subjected to earthquake loads, which possess many frequency components, is expected to be different and to depend on the ground motion properties [9–11]. While effectiveness of a TMD will be greatest when a real structure with a number of degrees of freedom oscillates around a predominant mode, this device does not reduce the structural response to a great extent when several modes contribute significantly to the main system response. Nevertheless, several successful studies have been devoted to improve the seismic performance of TMDs in different structural systems [12–16].

The well-known high modal damping criterion was used by some researchers such as Sadek et al. [12] and Miranda [13,14] to determine the optimum parameters of TMDs for the purpose of seismic response reduction. Some other criteria (or objective functions) have also been considered for this purpose [15–18].

Most of the studies on TMDs including those discussed above have employed the devices with elastic springs and linear behavior. However, the nonlinear behavior in TMDs has been considered for more effective control of unwanted vibrations in some investigations. The nonlinearity can be achieved with some simple implementation of practical engineering options such as combined action of several elastic and linear springs coming into action sequentially [19], equipping with friction-spring elements [20], using nonlinear viscous damping elements [21], and using Duffing spring for stiffness element of TMD [22,23]. Contrary to these sources of nonlinearity, here the nonlinear behavior of a TMD arising from inelastic behavior of its spring is of

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**Nomenclature**

| | |
|---|---|
| $a_{pb}$ | acceleration ratio of the P-TMD, $= \ddot{u}_{b0}/a_y$ |
| $a_y$ | yield acceleration of the P-TMD, $= F_y/m_d = \omega_p{}^2 u_y$ |
| $c$ | damping coefficient of the main system |
| $c_d$ | damping coefficient of the TMD |
| $f$ | frequency (tuning) ratio of the TMD, $= \omega_d/\omega$ |
| $f_p$ | frequency (tuning) ratio of the P-TMD, $= \omega_p/\omega$ |
| $F_y$ | yield strength of the P-TMD, $= k_p u_y$ |
| $k$ | stiffness of the main system |
| $k_d$ | stiffness of the TMD |
| $k_{eff}$ | secant stiffness of the P-TMD at $u_0$ |
| $k_p$ | initial stiffness of the P-TMD |
| $m$ | mass of the main system |
| $m_d$ | mass of the mass damper (TMD and/or P-TMD) |
| $T$ | natural period of the main system, $= 2\pi/\omega$ |
| $u_0$ | resonant response amplitude of the mass damper (TMD and/or P-TMD) |
| $u_y$ | yield deformation of the P-TMD |
| $\ddot{u}_{b0}$ | acceleration amplitude in the base of the P-TMD |
| $\gamma$ | mass ratio of the mass damper (TMD and/or P-TMD), $= m_d/m$ |
| $\zeta$ | damping ratio of the main system, $= c/2m\omega$ |
| $\zeta_d$ | damping ratio of the TMD, $= c_d/2m_d\omega_d$ |
| $\omega$ | natural circular frequency of the main system, $= (k/m)^{0.5}$ |
| $\omega_d$ | natural circular frequency of the TMD, $= (k_d/m_d)^{0.5}$ |
| $\omega_p$ | natural circular frequency of the P-TMD vibrating within its linearly elastic range, $= (k_p/m_d)^{0.5}$ |

particular interest.

Jaiswal et al. [24] examined the effectiveness of an elasto-plastic TMD in controlling the seismic response of SDOF systems. For such a TMD, they used the same parameters that had already been suggested for the optimum elastic TMD with a limited parameter study on that TMD. It was seen that such an elasto-plastic TMD became more efficient than the elastic one only in certain frequency range of base excitations, while it became less effective in the other frequency ranges. However, they did not propose an approach to identify the optimum parameters of this TMD for seismic applications and highlighted the need for more rigorous studies on this issue. Moreover, to the best of our knowledge, any approach to determine optimum parameters of an elasto-plastic TMD has not previously been reported in the literature. Here, as a first attempt to establish a framework for utilizing the elasto-plastic behavior of the spring of TMDs, we try to present a novel method for estimating the most appropriate parameters of the elasto-plastic TMD (hereafter referred to as P-TMD) under seismic loads and to assess the effectiveness of the proposed technique. The proposed P-TMD consists of a mass and an elasto-plastic spring without the need to employ a supplementary viscous damper which is essential in the traditional/optimal TMDs.

It is important to note that the inelastic behavior of the TMD not only is proposed to be utilized in a new passive control device in the present study, but also can be activated in traditional TMDs undergoing large displacements during severe earthquake events. From this point of view, inelastic response analysis of TMDs is also of practical interest in structural and earthquake engineering.

## 2. Mathematical model of the tuned mass dampers

A SDOF system with a TMD as shown in Fig. 1(a) is modeled by two masses, springs and viscous dampers where $m$, $k$, and $c$ are the mass, stiffness and damping coefficient of the main system. For this system, $\omega = (k/m)^{0.5}$ and $\zeta = c/2m\omega$ are the natural frequency and damping ratio of the main system, respectively. The parameters of the TMD are the mass, $m_d$, stiffness, $k_d$, and damping coefficient, $c_d$. The natural frequency and damping ratio of the TMD are $\omega_d = (k_d/m_d)^{0.5}$ and $\zeta_d = c_d/2m_d\omega_d$, respectively. A TMD is usually characterized in terms of mass ratio $\gamma = m_d/m$, frequency (tuning) ratio $f = \omega_d/\omega$, and damping ratio $\zeta_d$.

The P-TMD suggested in this paper consists of only a mass and an elasto-plastic spring as shown in Fig. 1(b). The idealized elastic-perfectly plastic behavior of the spring is shown in Fig. 2. The spring has an initial stiffness of $k_p$, a yield deformation of $u_y$ and a yield strength of $F_y = k_p u_y$.

The spring of the proposed P-TMD with an idealized elastic-perfectly plastic behavior is representative of a structural element having elasto-plastic behavior with a negligible hardening. A well-known class of such devices has already been used as metallic-yielding dampers in different structural systems. There are several simple and economical types of these devices with large deformation capacities and good low cycle fatigue performances such as U-shaped steel strips [25–27] and crawler steel damper [28] among others. The energy dissipating steel elements of such devices can be easily calibrated to obtain the desired initial elastic stiffness and the plastic threshold, as well as to undergo as large as desired displacements. Furthermore, the elastic-perfectly plastic behavior can be achieved with a combination of a conventional linear spring in series with a friction element. Such friction elements have wide structural applications such as those used as friction dampers in series with the bracing elements in building structures.

It should be noted that both above-mentioned ideas to achieve the elasto-plastic behavior have already been used in the base isolation systems. Of course, other practical suggestions can be made to achieve this behavior in the theoretical proposition of the P-TMD.

The natural frequency of the P-TMD vibrating within its linearly elastic range is $\omega_p = (k_p/m_d)^{0.5}$. Assuming $F_y = m_d a_y$, the yield acceleration of the proposed device becomes $a_y = \omega_p{}^2 u_y$ which can be interpreted as the acceleration of the mass $m_d$ to produce the yield force similar to that defined in a general inelastic SDOF system [29]. Therefore, the proposed P-TMD can be characterized in terms of non-dimensional parameters of mass ratio $\gamma = m_d/m$, frequency (tuning)

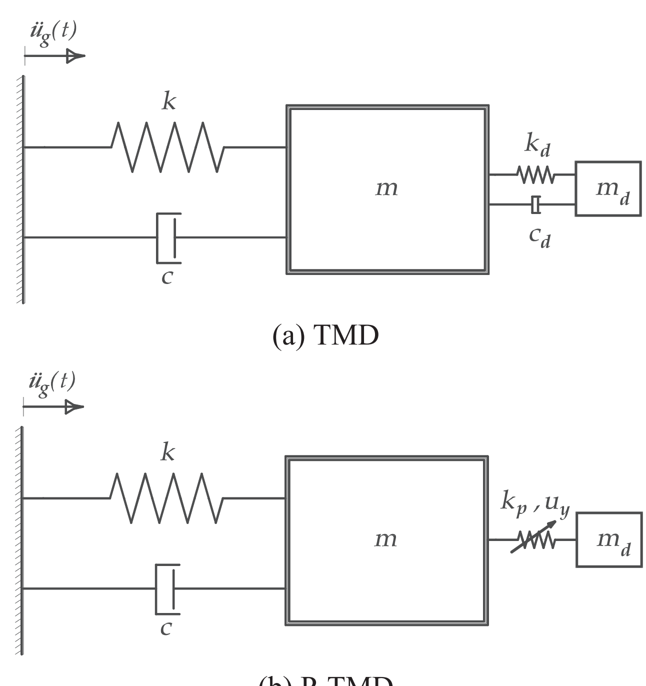



**Fig. 1.** Mass dampers attached to main systems.

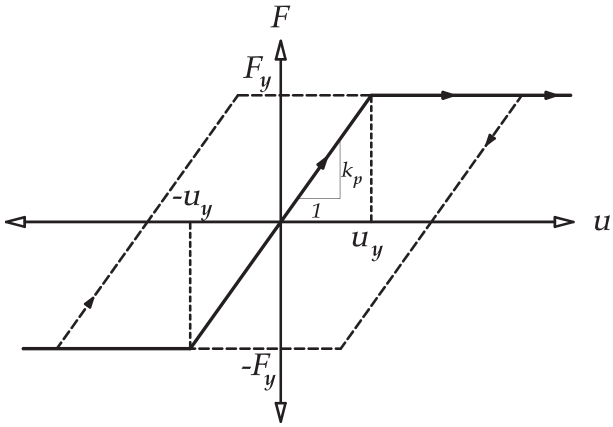



**Fig. 2.** Elasto-plastic behavior of P-TMD spring.

ratio $f_p = \omega_p/\omega$, and acceleration ratio $a_{pb} = \ddot{u}_{b0}/a_y$, where $\ddot{u}_{b0}$ is the acceleration amplitude in the base of the proposed mass damper (see Fig. 3). The non-dimensional parameter $a_{pb}$ represents the ratio between the base acceleration and a measure of the yield strength of the proposed mass damper. It is noted that using such a non-dimensional ratio is often unavoidable in the parametric study of the seismic response of inelastic systems [29].

For the new device, energy dissipation through inelastic deformations of the elasto-plastic spring is considered to be a replacement for the contribution of the eliminated viscous damper. It should be noted that the main structure is considered to remain elastic in both above mentioned systems.

## 3. A method of estimating parameters of proposed P-TMD

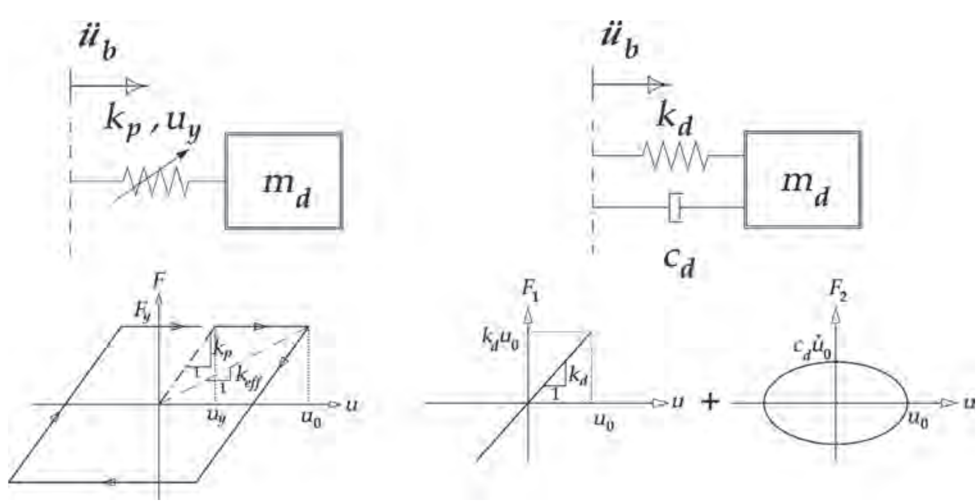
There exist several methods to determine the equivalent viscous damping for a yielding structure. Jennings [30] has addressed and examined in detail some of these methods based on the assumption of harmonic response behavior. The application of the concept of equivalent viscous damping to seismic response of yielding structures is also examined by him. In the current study, an inverse procedure is proposed in which the elsto-plastic system called P-TMD (Fig. 3(a)) can be obtained as an equivalent to the elastic TMD with viscous damper (Fig. 3(b)). It is evident that yielding response is different from linear response, and therefore only limited success can be obtained by using such approximate methods; however, it will be shown that the adopted method, i.e. Geometric Stiffness (GS) method, leads to quite satisfactory results for the purpose of this study. In most of the other linearization methods, the maximum equivalent viscous damping is usually limited

**Table 1**
Optimum TMD parameters suggested by Sadek et al. [12].

| Mass ratio γ | $\zeta$ = 0.00 | | $\zeta$ = 0.02 | | $\zeta$ = 0.05 | |
|---|---|---|---|---|---|---|
| | $f$ | $\zeta_d$ | $f$ | $\zeta_d$ | $f$ | $\zeta_d$ |
| 0.005 | 0.9950 | 0.0705 | 0.9936 | 0.0904 | 0.9915 | 0.1199 |
| 0.01 | 0.9901 | 0.0995 | 0.9881 | 0.1193 | 0.9852 | 0.1488 |
| 0.02 | 0.9804 | 0.1400 | 0.9776 | 0.1596 | 0.9735 | 0.1889 |
| 0.03 | 0.9709 | 0.1707 | 0.9676 | 0.1900 | 0.9626 | 0.2190 |
| 0.04 | 0.9615 | 0.1961 | 0.9578 | 0.2153 | 0.9521 | 0.2440 |
| 0.05 | 0.9524 | 0.2182 | 0.9482 | 0.2372 | 0.9420 | 0.2656 |
| 0.06 | 0.9434 | 0.2379 | 0.9389 | 0.2567 | 0.9322 | 0.2848 |
| 0.07 | 0.9346 | 0.2558 | 0.9298 | 0.2744 | 0.9226 | 0.3022 |
| 0.08 | 0.9259 | 0.2722 | 0.9209 | 0.2906 | 0.9133 | 0.3181 |
| 0.09 | 0.9174 | 0.2873 | 0.9122 | 0.3056 | 0.9042 | 0.3329 |
| 0.10 | 0.9091 | 0.3015 | 0.9036 | 0.3196 | 0.8954 | 0.3466 |
| 0.11 | 0.9009 | 0.3148 | 0.8952 | 0.3328 | 0.8867 | 0.3595 |
| 0.12 | 0.8929 | 0.3273 | 0.8870 | 0.3451 | 0.8782 | 0.3716 |
| 0.13 | 0.8850 | 0.3392 | 0.8790 | 0.3568 | 0.8699 | 0.3831 |
| 0.14 | 0.8772 | 0.3504 | 0.8710 | 0.3679 | 0.8618 | 0.3939 |
| 0.15 | 0.8696 | 0.3612 | 0.8633 | 0.3785 | 0.8538 | 0.4042 |

**Table 2**
Proposed P-TMD parameters.

| Mass ratio γ | $\zeta$ = 0 | | $\zeta$ = 0.02 | | $\zeta$ = 0.05 | |
|---|---|---|---|---|---|---|
| | $f_p$ | $a_{pb}$ | $f_p$ | $a_{pb}$ | $f_p$ | $a_{pb}$ |
| 0.005 | 1.0551 | 0.1410 | 1.0727 | 0.1808 | 1.1005 | 0.2398 |
| 0.01 | 1.0779 | 0.1990 | 1.0961 | 0.2386 | 1.1255 | 0.2976 |
| 0.02 | 1.1100 | 0.2800 | 1.1294 | 0.3192 | 1.1608 | 0.3778 |
| 0.03 | 1.1349 | 0.3414 | 1.1552 | 0.3800 | 1.1885 | 0.4380 |
| 0.04 | 1.1559 | 0.3922 | 1.1774 | 0.4306 | 1.2124 | 0.4880 |
| 0.05 | 1.1748 | 0.4364 | 1.1971 | 0.4744 | 1.2339 | 0.5312 |
| 0.06 | 1.1921 | 0.4758 | 1.2154 | 0.5134 | 1.2540 | 0.5696 |
| 0.07 | 1.2084 | 0.5116 | 1.2327 | 0.5488 | 1.2729 | 0.6044 |
| 0.08 | 1.2238 | 0.5444 | 1.2491 | 0.5812 | 1.2912 | 0.6362 |
| 0.09 | 1.2385 | 0.5746 | 1.2650 | 0.6112 | 1.3091 | 0.6658 |
| 0.10 | 1.2530 | 0.6030 | 1.2805 | 0.6392 | 1.3266 | 0.6932 |
| 0.11 | 1.2671 | 0.6296 | 1.2958 | 0.6656 | 1.3439 | 0.7190 |
| 0.12 | 1.2810 | 0.6546 | 1.3108 | 0.6902 | 1.3611 | 0.7432 |
| 0.13 | 1.2948 | 0.6784 | 1.3258 | 0.7136 | 1.3785 | 0.7662 |
| 0.14 | 1.3082 | 0.7008 | 1.3406 | 0.7358 | 1.3957 | 0.7878 |
| 0.15 | 1.3221 | 0.7224 | 1.3558 | 0.7570 | 1.4131 | 0.8084 |




(a) P-TMD (b) TMD

**Fig. 3.** A sketch showing P-TMD and TMD characteristics.

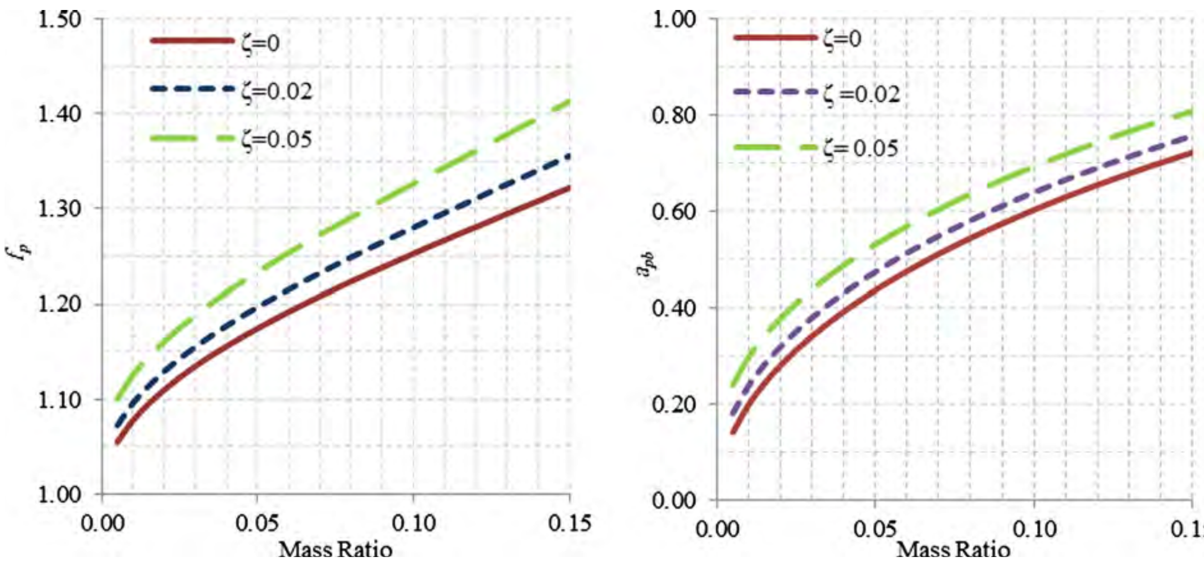



**Fig. 4.** Estimated P-TMD parameters for different mass and damping ratios.

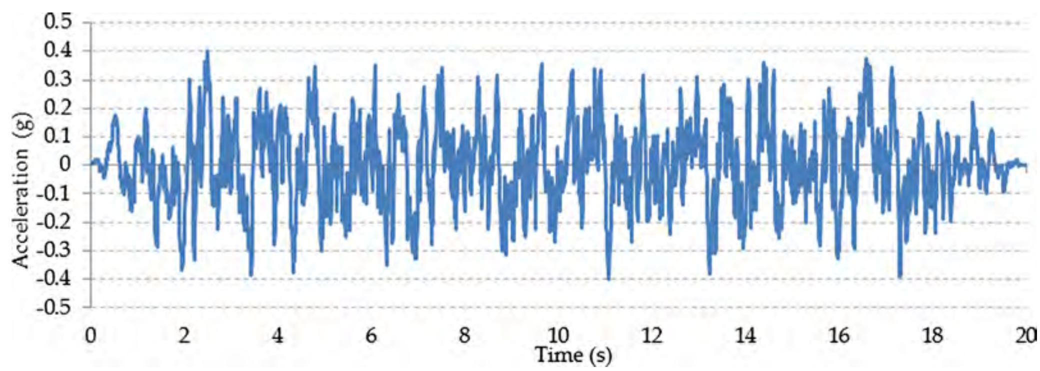



**Fig. 5.** Acceleration time history of the artificial record used in this study.

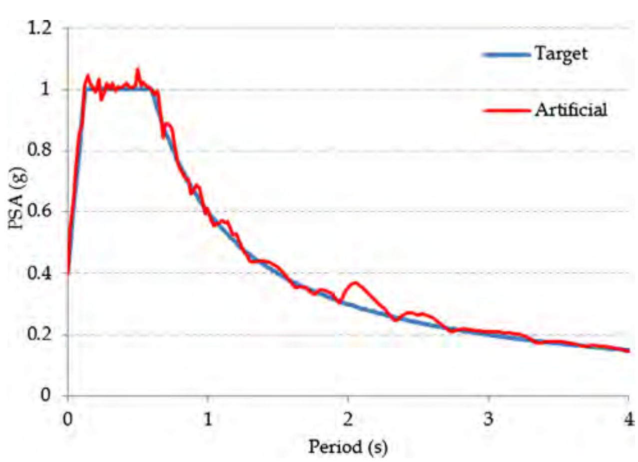



**Fig. 6.** Response spectrum of the artificial record used in this study.

while being less than the optimum damping of TMDs. They are thus not appropriate for the aim of this study.

According to the GS method, the following four conditions should be applied:

(i) The masses of the elasto-plastic system (P-TMD) and associated linear system (TMD) are taken to be equal (i.e. $m_d$).

(ii) The resonant response amplitudes of the two systems are also assumed to be the same (i.e. $u_0$) which can be written as

$$u_0 = \frac{\ddot{u}_{b0}}{2\omega_d^2\zeta_d} \tag{1}$$

(iii) The stiffness of the associated linear system (i.e. $k_d$) is fixed by the geometry of the hysteresis loops of the elasto-plastic system. This is accomplished by using the secant stiffness of the elasto-plastic system at $u_0$, as the effective linear system stiffness, i.e. $k_d = k_{eff}$. This gives

$$k_d = \frac{u_y}{u_0}k_p \quad \rightarrow \quad k_p = \frac{u_0}{u_y}k_d \tag{2}$$

(iv) The energies dissipated per cycle by the two systems are equated at resonance. With the aid of Eq. (2), this condition simplifies to

$$\zeta_d = \frac{2}{\pi}\left(1-\frac{u_y}{u_0}\right) \quad \rightarrow \quad u_y = u_0\left(1-\frac{\pi}{2}\zeta_d\right) \tag{3}$$

Now, we can obtain a closed-form solution for the proposed P-TMD parameters as follows:

Substituting Eq. (3) into Eq. (2) leads to an expression for the initial stiffness of the P-TMD in terms of the equivalent linear TMD parameters. Then, the natural frequency ($\omega_p = \sqrt{k_p/m_d}$) and the non-dimensional tuning frequency ($f_p = \omega_p/\omega$) of the P-TMD are respectively given as

$$\omega_p = \frac{\omega_d}{\sqrt{1-\frac{\pi}{2}\zeta_d}} \tag{4}$$

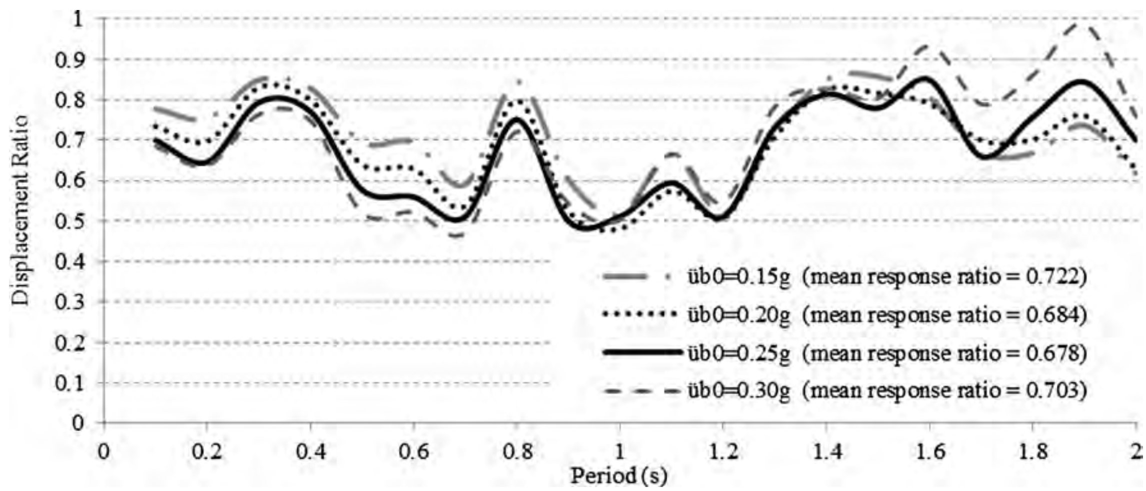



**Fig. 7.** Reduction effect of P-TMD on a system with $\zeta = 0.02$ and $\gamma = 0.1$ under the artificial record using different values for $\ddot{u}_{b0}$.

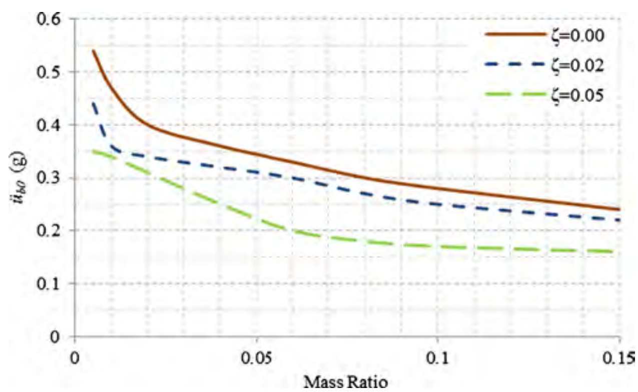



**Fig. 8.** Proposed values of $\ddot{u}_{b0}$ for a system with different mass and damping ratios subjected to the considered design motion.

$$f_p = \frac{f}{\sqrt{1-\frac{\pi}{2}\zeta_d}} \tag{5}$$

It now remains to determine the second non-dimensional parameter of the P-TMD, i.e. acceleration ratio, $a_{pb}$. Substituting Eq. (1) into Eq. (3) leads to an expression for the yield deformation of the P-TMD in terms of the equivalent linear TMD parameters and base acceleration amplitude as

$$u_y = \frac{\ddot{u}_{b0}}{2\omega_d^2\zeta_d}\left(1-\frac{\pi}{2}\zeta_d\right) \tag{6}$$

Using Eq. (4), Eq. (6) can be rewritten as

$$u_y = \frac{\ddot{u}_{b0}}{2\omega_p^2\zeta_d} \tag{7}$$

Thus, the acceleration ratio ($a_{pb} = \ddot{u}_{b0}/a_y = \ddot{u}_{b0}/\omega_p{}^2u_y$) is given as

$$a_{pb} = 2\zeta_d \tag{8}$$

Therefore, as a final outcome of the above procedure, the non-dimensional parameters of the proposed P-TMD, i.e. $f_p$ and $a_{pb}$, are obtained as Eqs. (5) and (8) in terms of the elastic TMD parameters, i.e. $f$ and $\zeta_d$. These two equations indeed allow us to use the proposed P-TMD as an equivalent to any given elastic TMD. It is self-evident that the most proper P-TMD will correspond to an optimum TMD. Hereafter, the optimum TMD suggested by Sadek et al. [12] for seismic applications is considered as a reference TMD; however, one may consider any other previously suggested TMD.

The optimum TMD parameters suggested by Sadek et al. [12] are listed in Table 1, while the parameters of the equivalent P-TMD according to Eqs. (5) and (8) are presented in Table 2 for different mass ratios, $\gamma$, between 0.005 and 0.15, and three different damping ratios of the main system, $\zeta = 0$, 0.02 and 0.05. The proposed parameters of the P-TMD are also plotted in Fig. 4 as functions of mass ratio for the three damping ratios of the main system. It is seen that the higher the main system's damping, the higher both the tuning ratio, $f_p$, and acceleration ratio, $a_{pb}$, of the P-TMD. Fig. 4 also reveals that increasing the mass ratio, $\gamma$, requires high values of $f_p$ and $a_{pb}$.

Unlike the optimum values for the tuning ratio of the linear TMD which are less than unity (see Table 1), the estimated ratios for the P-TMD appear to be always greater than one. This implies that a P-TMD typically requires a spring with a higher initial stiffness compared to the

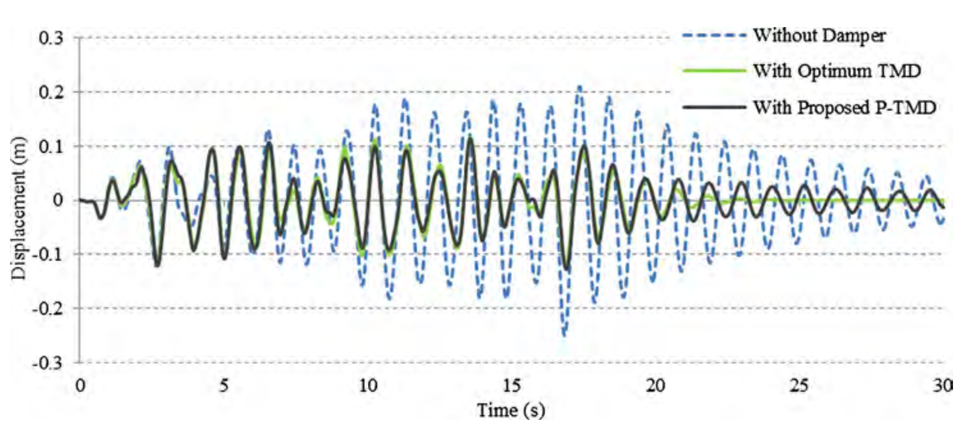



**Fig. 9.** Displacement responses of a main structure with $\zeta = 0.02$, $\gamma = 0.1$ and $T = 1$ s under the artificial record.

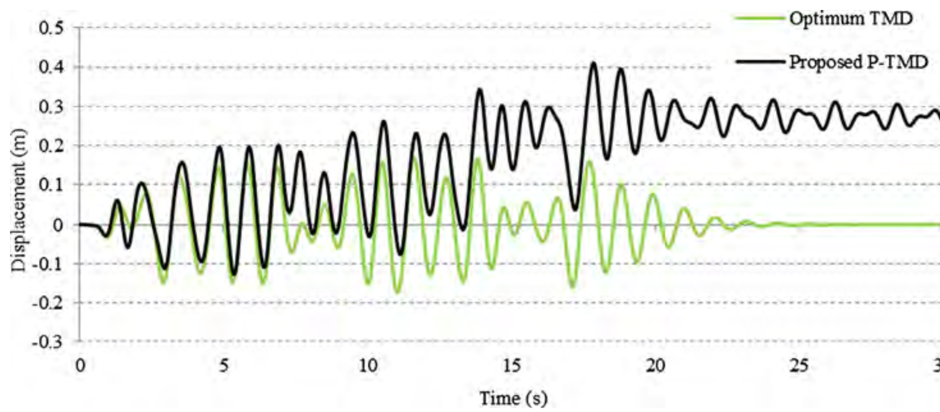



**Fig. 10.** Displacement responses of mass dampers relative to the main structure with $\zeta$ = 0.02, $\gamma$ = 0.1 and $T$ = 1 s under the artificial record.

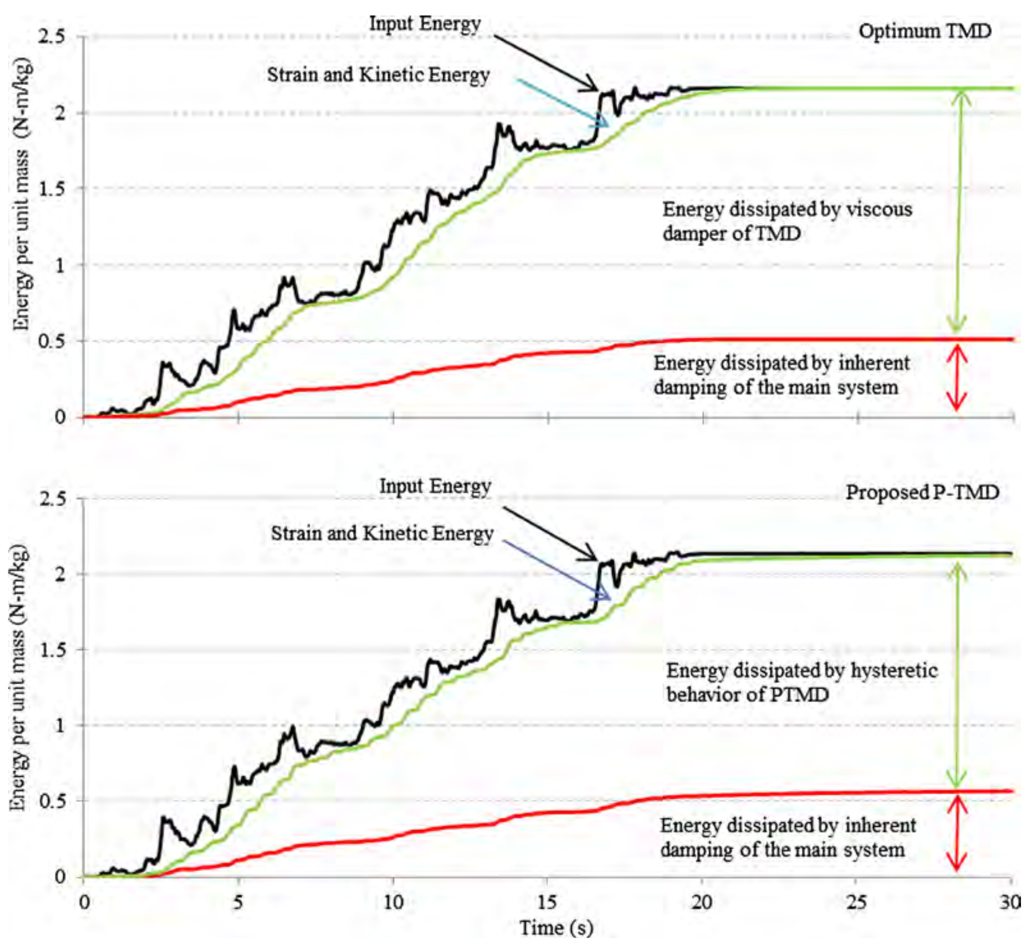



**Fig. 11.** Energy responses of a system with $\zeta$ = 0.02, $\gamma$ = 0.1 and $T$ = 1 s under the artificial record, equipped with optimum TMD and proposed P-TMD.

corresponding linear TMD. The overall reason for this stems from the fact that we obtained the P-TMD (with elasto-plastic behavior) as an equivalent to the elastic TMD: since the effective (secant) stiffness of the P-TMD is assumed to be equal to the stiffness of the corresponding elastic TMD, the initial stiffness of the P-TMD will be greater than the stiffness of the optimum TMD.

In addition to the tabulated results, simple equations were derived through a curve fitting procedure to represent the optimum TMD parameters in [12]. Substituting those equations into Eqs. (5) and (8) leads to the following relations for the proposed P-TMD parameters, $f_p$ and $a_{pb}$, in terms of $\gamma$ and $\zeta$, which give very close approximations to the values in Table 2.

$$f_p = \frac{1-\zeta\sqrt{\frac{\gamma}{1+\gamma}}}{(1+\gamma)\sqrt{1-\frac{\pi}{2}\left(\frac{\zeta}{1+\zeta}+\sqrt{\frac{\gamma}{1+\gamma}}\right)}} \tag{9}$$

$$a_{pb} = \frac{2\zeta}{1+\gamma} + 2\sqrt{\frac{\gamma}{1+\gamma}} \tag{10}$$

After estimating the non-dimensional parameters for a given set of mass and damping ratios of the main system, initial stiffness of the P-TMD spring can be directly obtained from the value of the first parameter, i.e. $f_p$; however, to determine its yield level from the value of the second parameter, i.e. $a_{pb}$, the acceleration amplitude in the base of the

**Table 3**
Earthquake records used for numerical studies.

| Earthquake | Date | Station | Component | Significant Duration (s) | Arias Intensity (m/s) | Housner Intensity (m) | Specific Energy Density ($m^2$/s) | PGA (g) | PGV (m/s) | Scale Factor |
|---|---|---|---|---|---|---|---|---|---|---|
| W. Washington | 1949 | 325 | N04W | 25.8 | 0.75 | 0.75 | 0.169 | 0.17 | 0.18 | 2.74 |
| | | | N86E | 18.1 | 1.13 | 0.80 | 0.104 | 0.28 | 0.17 | |
| Eureka | 1954 | 022 | N11W | 14.2 | 0.34 | 0.84 | 0.133 | 0.17 | 0.30 | 1.74 |
| | | | N79E | 9.9 | 0.71 | 0.87 | 0.102 | 0.26 | 0.30 | |
| San Fernando | 1971 | 241 | N00W | 16.6 | 1.28 | 1.55 | 0.369 | 0.25 | 0.30 | 1.96 |
| | | | S90W | 21.7 | 0.68 | 1.18 | 0.301 | 0.13 | 0.24 | |
| San Fernando | 1971 | 458 | S00W | 26.4 | 0.55 | 1.06 | 0.394 | 0.12 | 0.30 | 2.22 |
| | | | S90W | 28.2 | 0.52 | 1.25 | 0.288 | 0.11 | 0.29 | |
| Loma Prieta | 1989 | Gilroy 2 | 90 | 9.9 | 1.23 | 1.84 | 0.179 | 0.32 | 0.39 | 1.07 |
| | | | 0 | 11.0 | 1.20 | 1.20 | 0.092 | 0.35 | 0.33 | |
| Loma Prieta | 1989 | Hollister | 90 | 29.7 | 0.79 | 1.09 | 0.443 | 0.18 | 0.31 | 1.46 |
| | | | 0 | 16.4 | 2.22 | 2.52 | 0.660 | 0.37 | 0.63 | |
| Landers | 1992 | Yermo | 360 | 21.4 | 0.69 | 0.91 | 0.216 | 0.15 | 0.29 | 1.28 |
| | | | 270 | 19.4 | 0.94 | 1.49 | 0.515 | 0.25 | 0.51 | |
| Landers | 1992 | Joshua | 90 | 28.2 | 2.41 | 1.64 | 0.395 | 0.28 | 0.43 | 1.48 |
| | | | 0 | 30.8 | 1.68 | 1.27 | 0.205 | 0.27 | 0.27 | |
| Northridge | 1994 | Moorpark | 180 | 14.9 | 0.94 | 0.78 | 0.063 | 0.29 | 0.20 | 2.61 |
| | | | 90 | 17.0 | 0.78 | 0.82 | 0.069 | 0.19 | 0.20 | |
| Northridge | 1994 | Century | 90 | 13.6 | 1.19 | 1.05 | 0.116 | 0.26 | 0.21 | 2.27 |
| | | | 360 | 14.7 | 0.76 | 0.98 | 0.083 | 0.23 | 0.25 | |

mass damper, $\ddot{u}_{b0}$, needs to be known *a priori*. In other words, in order to specify the value of $u_y$ (or $F_y$) for this device, it seems that the intensity level of the input motions should be prescribed, which may introduce a difficulty in the design of this device in comparison to the design of traditional TMDs. It is, however, noticeable that the seismic excitation is applied at the base of the main system but not at the base of the device; hence the value of $\ddot{u}_{b0}$ will be a function of both input motion and system properties. Therefore, a trial and error approach is required to determine $\ddot{u}_{b0}$ for a system subjected to the specified earthquake ground motion. For design purposes, one may obtain the desired seismic response of the main system and the reduction effect of the P-TMD for a reasonable range of values of $\ddot{u}_{b0}$ under the design earthquake motion which may be represented by a single or a set of real or artificial earthquake record(s) compatible with the design spectrum. Then, a value is chosen for $\ddot{u}_{b0}$ so as to make the seismic response of the P-TMD system a minimum.

## 4. A practical example for validation of the method

To illustrate the proposed approach with a practical example, an artificial earthquake record compatible with the design spectrum of ASCE7-10 [31] for Site Class D, with $S_{DS}$ of 1.0g and $S_{D1}$ of 0.6g is generated using the software SIMQKE [32]. The acceleration time history of this artificial record is shown in Fig. 5 and the response spectrum of the record for 5% damping is compared with ASCE's design spectrum in Fig. 6. Here, displacements of the main systems with natural periods over the range of 0.1–2.0 s are considered as the responses to be reduced as much as possible. Thus, the seismic response ratio due to the utilization of P-TMD can be computed as the ratio of the peak displacement response of the main system with P-TMD to the peak response without P-TMD. For example, in Fig. 7, this effect is shown for four different values of $\ddot{u}_{b0}$ in a system with damping ratio of 0.02 and mass ratio of 0.1. This figure also shows that the effectiveness of the P-TMD is not very sensitive to small variations in $\ddot{u}_{b0}$ as long as the values do not lie in a physically unreasonable range. However, for a specified system, one can find the best value among the examined values of $\ddot{u}_{b0}$. Fig. 8 shows the obtained values for the acceleration amplitude in the base of the mass damper, $\ddot{u}_{b0}$, which lead to the best reduction effects of P-TMDs on the mean displacement responses of the main systems with different natural periods over the range of 0.1–2.0 s subjected to the above-considered earthquake. In a same manner, similar diagrams can be obtained for other design spectrums.

Now, we can show and assess the time history results obtained from the proposed method of estimating design parameters of the P-TMD for the above example. Fig. 9 shows the displacement time history of a main system with the natural period of 1.0 s in three different cases: without mass damper, with P-TMD, and with optimum TMD suggested by Sadek et al. [12]. As can be seen from this figure, the mass dampers, both TMD and P-TMD, are quite effective in reducing the seismic response of the main system and the responses match reasonably together as expected. The maximum response of the main structure decreases from 0.250 m to 0.120 m when using the optimum TMD, and to 0.128 m when using the proposed P-TMD, indicating ~50% reduction effect for both mass dampers in this example. Displacements of the mass dampers relative to the main structure (stroke lengths) are compared in Fig. 10. Since the proposed P-TMD has elasto-plastic behavior, the permanent deformation in the mass damper occurs, as observed in Fig. 10. This may require replacement of the P-TMD spring after severe earthquake events. Energy responses of the considered system equipped with the TMD and P-TMD are shown in Fig. 11. It is seen that the input energies imposed on the system by the artificial earthquake are almost the same when using either TMD or equivalent P-TMD. This figure also indicates that the hysteretic behavior of P-TMD can dissipate remarkable seismic energy which is equivalent to the energy dissipated by the viscous damper of TMD. Therefore, this example indeed confirms the concept of an equivalent elasto-plastic TMD to a previously suggested optimum TMD.

## 5. Numerical studies

To examine the effectiveness of the proposed device (P-TMD) in reducing the structural responses, SDOF structures with natural periods between 0.1 and 2.0 s with increments of 0.1 s are considered as the main systems. The structures are assumed to have damping ratios $\zeta = 0$, 0.02 and 0.05 and the mass ratios are taken to be $\gamma = 0.01$, 0.02, 0.04, 0.06, 0.08 and 0.10. The seismic responses of the uncontrolled systems and controlled ones by the proposed P-TMDs as well as by the optimum TMDs are obtained and then for the purpose of comparison, the

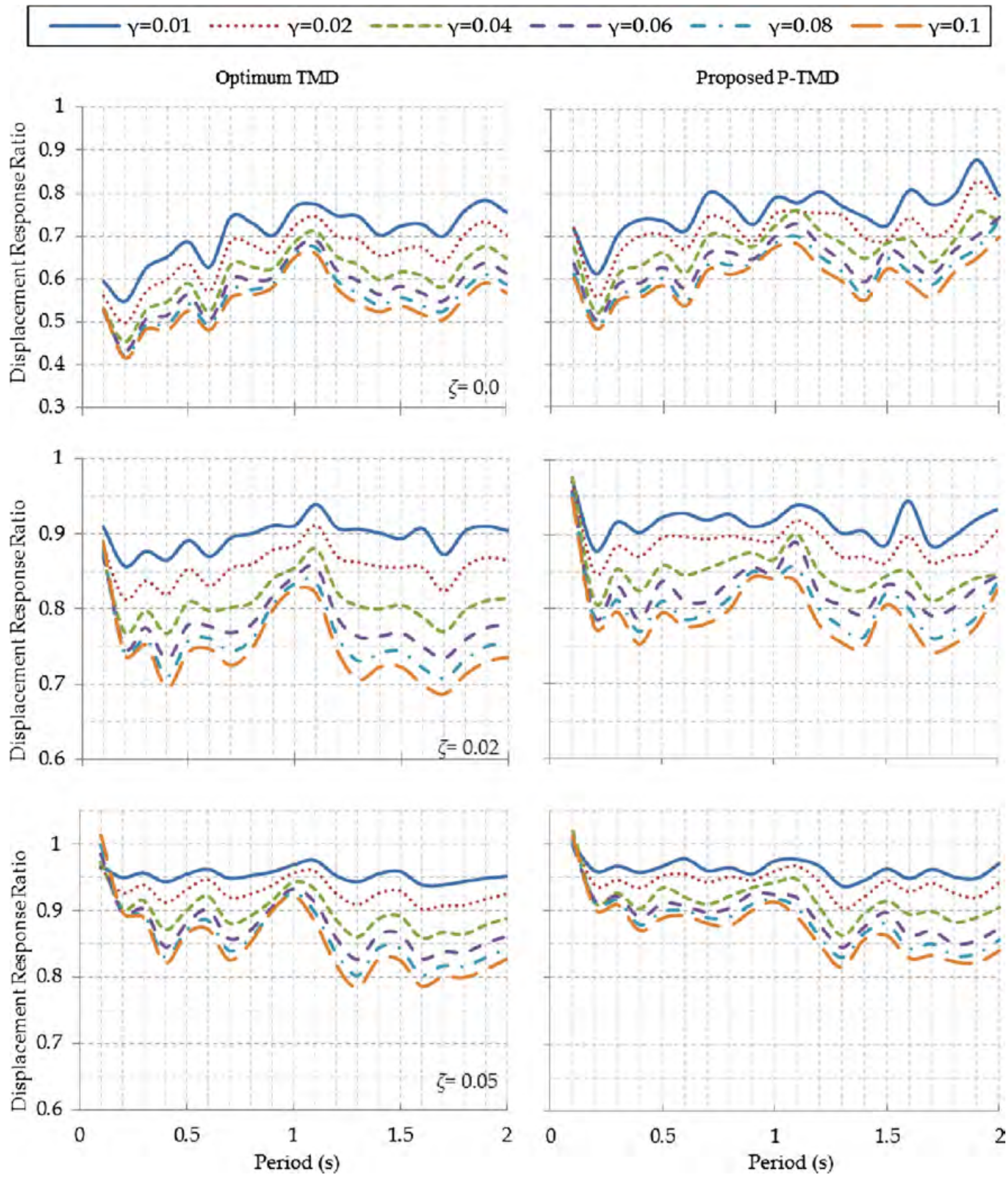



**Fig. 12.** Mean displacement response ratios of structures with TMDs and P-TMDs.

displacement (relative to the ground) and acceleration (absolute) response ratios are computed as the ratio of the peak response of the structure with mass damper to the peak response without mass damper. The TMD and P-TMD parameters used are those presented in Tables 1 and 2, respectively.

### 5.1. Response to design basis earthquake (DBE) ground motions

The twenty horizontal components of the ten earthquake motions used by Tsopelas et al. [33] are adopted here for linear and nonlinear dynamic analyses. Each of these earthquakes has a magnitude larger than 6.5, an epicentral distance between 10 and 20 km, and site conditions of soft rock to stiff soil. The ground motions were scaled by Tsopelas et al. [33] so that the average of the 20 scaled records represented well the design spectrum, which was 1994 NEHRP spectrum for soil type of C-D. This target spectrum is similar to the aforementioned design spectrum of ASCE7-10, hence the curves of Fig. 8 can be used to estimate $\ddot{u}_{b0}$ for these input motions. The complete list of the earthquake records is given in Table 3.

The mean displacement response ratios of the 20 scaled records are shown in Fig. 12 for the three different damping ratios of the main system. The effectiveness of the proposed P-TMD in reducing displacement demands can be compared with that of the optimum TMD in this figure. Although less displacement of the main system is the main purpose of using mass dampers, less acceleration would be desirable. Thus, similar comparisons are made for acceleration demands in Fig. 13. It is seen from Figs. 12 and 13 that reductions in displacement and acceleration demands can be achieved by the proposed P-TMD for structural systems with different values of $\zeta$, $T$ and $\gamma$. It is also evident in these figures that the effectiveness of the P-TMD in reducing the seismic

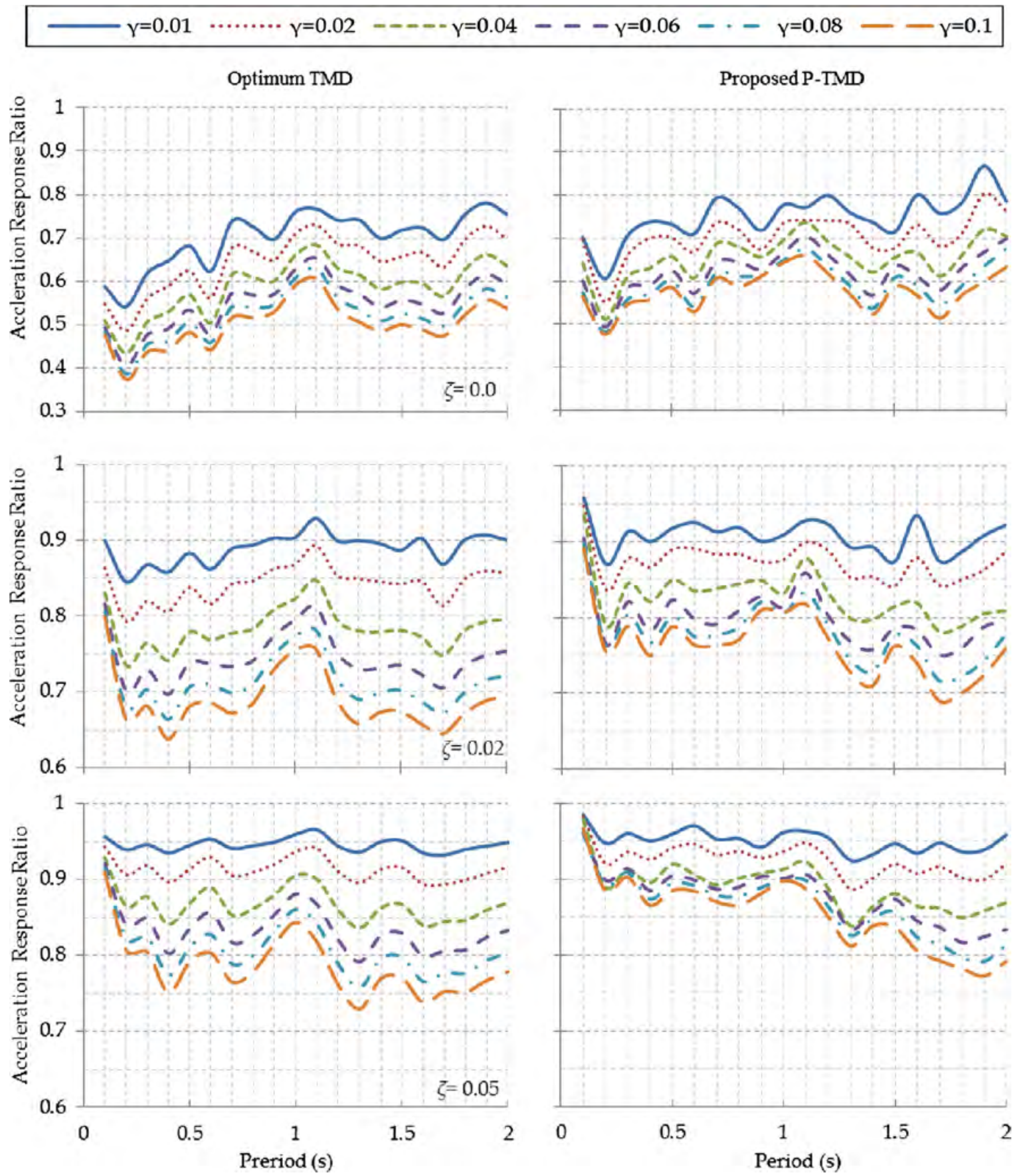



**Fig. 13.** Mean acceleration response ratios of structures with TMDs and P-TMDs.

responses is about the same as that of the optimum TMD. This is, of course, the expected outcome of the proposed method for determining the P-TMD parameters. Therefore, observations on the seismic performance of the P-TMD in terms of system properties are similar to those reported by Sadek et al. [12] for the optimum TMD. The most important ones are: (a) increase of the mass ratio can reduce the displacement and acceleration responses of the main system, and consequently, can enhance the effectiveness of the proposed mass damper; (b) the effectiveness of the mass damper also increases as the damping ratio of the main system decreases; (c) for damped structures with very low periods, the mass dampers are no longer effective.

As a quantitative confirmation of the above findings, the mean response ratios (displacement and acceleration) averaged over the considered period range (i.e. 0.1–2.0 s) are calculated and compared for the analyzed systems. To simplify this discussion, we only consider the maximum and minimum averages in the following. When using either the optimum TMD or the proposed P-TMD it is found that: (i) the maximum reduction effect in displacement response is achieved in the undamped main system and in the maximum assumed mass ratio (i.e. $\gamma = 0.1$) with averaged response ratio of 0.54 for the TMD and 0.59 for the P-TMD; (ii) the lowest reduction is observed in $\zeta = 5\%$ and $\gamma = 0.01$ with averaged response ratios of 0.95 and 0.96 for the TMD and P-TMD, respectively. Reduction effects of both mass dampers on the acceleration response are also similar to those found in the displacement response. The averaged acceleration response ratios for utilizing the optimum TMD and the proposed P-TMD are 0.50 and 0.58, respectively, at the maximum reduction level, whereas they are 0.95 for both of TMD and P-TMD, at the minimum level.

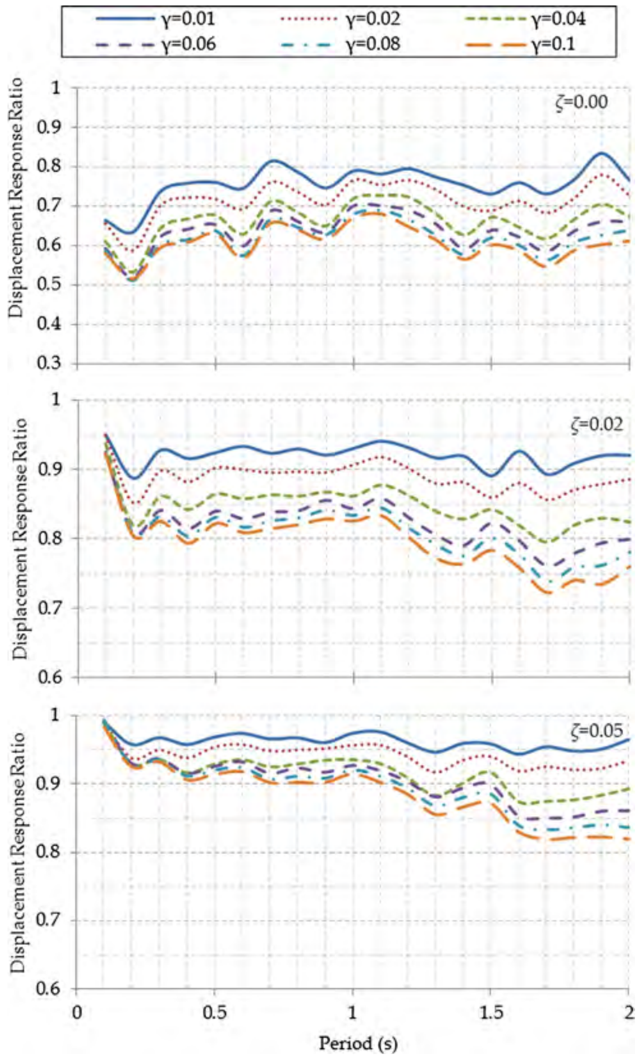



**Fig. 14.** Mean displacement response ratios of structures with P-TMDs under MCE ground motions.

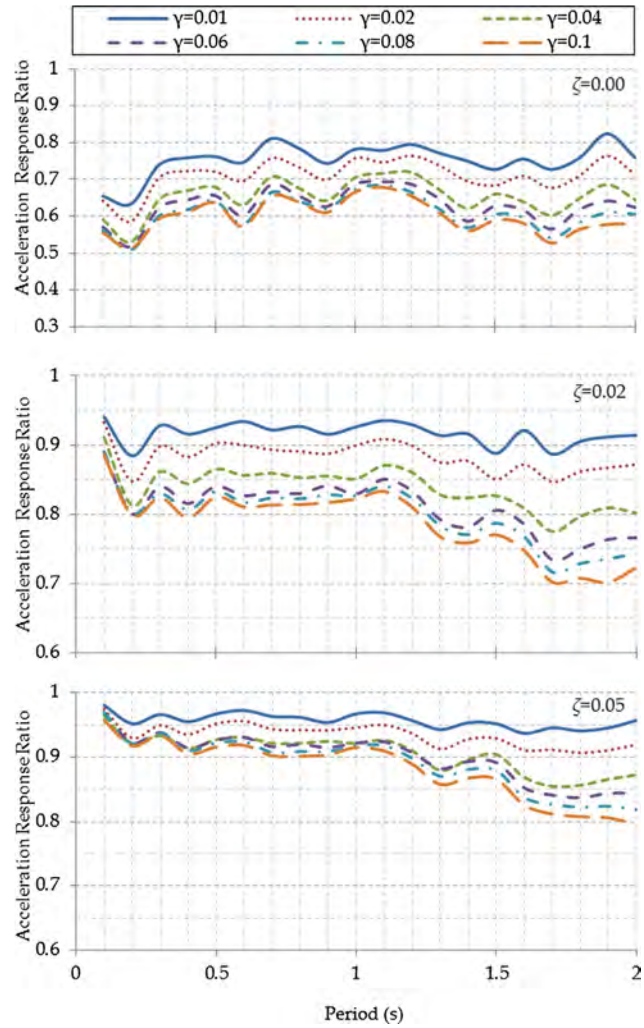



**Fig. 15.** Mean acceleration response ratios of structures with P-TMDs under MCE ground motions.

### 5.2. *Response to maximum considered earthquake (MCE) ground motions*

As mentioned before, the acceleration amplitude in the base of the mass damper, $\ddot{u}_{b0}$, which should be known to determine the yield strength of the P-TMD spring, can be best estimated by a trial and error process, as a function of input motion and system properties. For the purpose of practical application, however, it is useful to evaluate the effectiveness of the above-designed P-TMDs (Section 5.1) in controlling the responses of structures at a higher seismic hazard level such as MCE. The ground acceleration data of the MCE ground motions are considered to be 1.5 times that of the DBE records according to ASCE7-10 [31]. The previous structural models with and without P-TMDs have been analyzed again under these MCE ground motions and the mean displacement and acceleration response ratios are presented in Figs. 14 and 15, respectively.

It is seen that the P-TMDs designed for DBE ground motions, are also effective in reducing both displacement and acceleration responses at the MCE seismic hazard level. These response reduction effects are similar to those obtained for DBE records; that is, the maximum reduction effect on both displacement and acceleration responses is achieved in the undamped main system and in the maximum assumed mass ratio (i.e. $\gamma = 0.1$) with averaged response ratio of 0.61 for the displacement and 0.60 for the acceleration, whereas the minimum reduction is observed in $\zeta = 5\%$ and $\gamma = 0.01$ with averaged response ratio of 0.96 for both displacement and acceleration.

## 6. Conclusions

The present study can be summarized as follows:

- A new approach for seismic response control of structures with the elasto-plastic TMD (P-TMD), which consists of a mass and an elasto-plastic spring without the need for a viscous damper, has been presented in this paper.
- A simple yet effective method to estimate the design parameters of the device was developed and applied to the structural systems subjected to the seismic excitations. The proposed procedure has resulted in the explicit formulas to compute the tuning and acceleration ratios of the device for a given set of mass and damping ratios of the main system so that the inelastic behavior of the P-TMD can be equivalent to the visco-elastic behavior of a given optimum TMD.
- For the purpose of comparative analysis, the suggested optimum TMD by Sadek et al. [12] was then considered as a reference TMD. Comparison of the energy responses of a structural system equipped with P-TMD and optimum TMD shows that hysteretic behavior of P-TMD dissipates nearly the same energy as that dissipated by the viscous damper of TMD.
- In order to evaluate the effectiveness of the P-TMD when designed

according to the proposed method, several SDOF structures with and without P-TMDs as well as with optimum TMDs subjected to a number of earthquake excitations, each scaled to the DBE and MCE levels, were analyzed. The numerical results indicate that using the proposed P-TMD parameters reduces displacement and acceleration responses considerably at both seismic hazard levels.

- The effectiveness of the P-TMD in reducing the seismic responses is similar to the optimum TMD, which increases as the mass ratio increases and decreases as the main system's damping increases.
- As an alternative to the traditional/optimum TMDs, the main advantage of our proposed device in the current form is in the elimination of the expensive viscous damper. However, if further reductions of structural responses are desired, P-TMD with an additional viscous damper can be investigated.

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