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PII: S0377-2217(16)31078-5
DOI: 10.1016/j.ejor.2016.12.044
Reference: EOR 14175

To appear in: *European Journal of Operational Research*

Received date: 17 February 2016
Revised date: 12 October 2016
Accepted date: 27 December 2016

Please cite this article as: Dariusz Walczak, Aleksandra Rutkowska, Project rankings for participatory budget based on the fuzzy TOPSIS method, *European Journal of Operational Research* (2016), doi: 10.1016/j.ejor.2016.12.044

**Highlights**

- TOPSIS method for ranking of participatory budget projects is proposed.
- Inexact and vague projects descriptions are modeled as fuzzy variables.
- PIS selection is modified due to relative preferences of the individual participants.
- Distance measure for category classification is proposed.
- An example of the ranking is presented.

# Project rankings for participatory budget based on the fuzzy TOPSIS method

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**Abstract**

In this study, a fuzzy technique is proposed for order preference based on the similarity to an ideal solution for the personalized ranking of projects in a participatory budget (PB). A PB is a group decision-making process where citizens distribute public resources among a set of city investment proposals. The dynamic growth in the popularity of PB during the last 10 years has been due to a significant increase in the number of projects submitted and the demonstrable weakness of the traditional majority vote. The rationality of decision-makers is restricted by the large number of possible options from which voters can choose only a few within a limited amount of time, and thus there is no opportunity to review all of the projects. Appropriate decision support tools can assist with the selection of the best outcome and help to address the growth of PB processes. The ranking of PB projects is a specific problem because multi-criteria comparisons are based on non-quantitative criteria, i.e., nominal and fuzzy criteria. The "Technique for Order Preference by Similarity to Ideal Solution" (TOPSIS) method aims to minimize the distance to the ideal alternative while maximizing the distance to the worst. In a fuzzy extension of TOPSIS, the ratings of alternatives and the weights of the criteria are fuzzy numbers or linguistic variables. The major modification required to the TOPSIS method for PB is that the perfect objective solution does not exists among the maximum and minimum

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values for the criteria. Thus, the subjective choice is the ideal solution for the decision maker and the negative ideal solution is the most dissimilar solution. This study describes the application of fuzzy TOPSIS with a modification for PB based on an empirical example from a Poznan PB project (Poland).

*Keywords:* Community operational research; Participatory budget; project ranking; fuzzy TOPSIS

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## 1. Introduction

A participatory budget (PB) is a group decision-making process where citizens distribute public resources among a set of proposed projects. PB is highly beneficial for multiple parties because: it enables people to shape the local budget, municipalities obtain clear information about social priorities, it helps to integrate local communities and motivates them to cooperate, it educates citizens about costs, and it constrains local investments. All of these benefits have helped PB to grow in terms of the number of processes and budget limits. The present study investigated Polish PBs. Based on this study, we can describe a typical PB in Poland according to four steps: (1) a city announces the PB; (2) citizens propose projects; (3) the city verifies the proposals and formulates the final ballots; and, finally, (4) the citizens vote for projects. We found that major Polish cities included more than 100 projects in their ballots and people only had to choose 3-7, so the winners were usually selected by majority rule. However, this method causes high dispersion of the votes among multiple alternatives, where large numbers of people may vote for less popular projects and the process is completed without any project winning. Despite those issues majority rule has great advantage - it is easy to understand and scale. Complicated decision support systems could solve money distribution problem but people would lost trust to the system. We see our solution as a recommendation system that helps people with information overload during the voting. According to Malhotra [1], negative effects start with 10 or more options while in PB we have around 100 options. Recommendation system helps people to get familiar with potentially interesting projects instead of scanning all titles. Final solution should rank projects by different criteria: category, potential beneficiaries, location and cost. Final decision belongs to the participant.

In order to build such a system for PB, an algorithm is essential for ranking projects, which was the focus of the present study. Thus, we propose automated comparisons of PB projects using the "Technique for Order Preference by Similarity to Ideal Solution" (TOPSIS) method. The ranking of PB projects is a specific problem because multi-criteria comparisons are based on non-quantitative criteria, i.e., nominal and fuzzy criteria such as topic, location, and beneficiaries. The TOPSIS method minimizes the distance to the ideal alternative while maximizing the distance to the worst. In a fuzzy extension of TOPSIS, the ratings of alternatives and the weights of the criteria are fuzzy numbers or linguistic variables. The major modification of the TOPSIS method required for PB is that the objective perfect solution does not exist among the maximum and minimum values for the criteria. Thus, the subjective choice is the ideal solution for the decision maker and the negative ideal solution is the most dissimilar solution.

The remainder of this paper is organized as follows. First, we briefly describe the PBs. Next, we present an overview of DSS systems and fuzzy TOPSIS with preliminary definitions. In Section 4, we describe the application of the modified TOPSIS method to PB projects. We then present examples based on the Poznan PB project set. In the final section, we discuss the results obtained.

## 2. Participatory budgets

### *2.1. Development*

PB has its origins in Latin America but it has recently become widespread. Dias [2] identified five stages of PB growth: trial period (local experiments in Brazil, 1989–1997); Brazilian PB (140 municipalities adopted PB, 1997–2000); Latin American and European expansion (2000–2007); national and international PB networks (2007–2008); and "jumping off the scale" (after 2008). At present, we are in the last stage where PB has become part of more complex participatory systems. The development of PB in Europe was described by Sintomer et al. [3], who described 13 cases of PB in 2001 and more than 200 in 2009. In Poland, the first PB project occurred in Sopot during 2011 and 80 cities were involved three years later [4], while there are now more than 100 PBs in Poland. PB has also grow in terms of the amount of money spent each year, where the highest amount in Poland is PLN 40 million ($10 million) distributed in Łódź[1]. In the current year, Warsaw is planning to allocate PLN 58 million ($15 million) [2].

In the present study, we investigated the process regulations, projects descriptions, and voting methods for PB in 67 Polish cities. This data set is incomplete due to a lack of publicly available information for some of the PBs. We identified several problems with the process, particularly the following two issues.

**Project submission and verification:** In 17 of 67 cities, the proposals were collected only on paper, whereas the others allowed participants to submit them via the Internet. In all cases, the project descriptions were processed by humans. The most common approach used for verification was to appoint a group of people who accepted or rejected the proposals without following any other regulations. In some cases, the host imposed additional acceptance criteria on the projects. Some of the cities decide on all of the propositions based on a final ballot with additional city comments. The lack of clear rating criteria and a transparent process for proposal acceptance meant that many projects reached the final voting phase.

**Voting:** Citizens had to select from a large number of potential alternatives. We found that there could be 100–200 projects in some ballots and people could only choose 3-7. The projects were described according to multiple criteria but there were no standards. Each city described the potential

[1] III edition, http://budzet.dlalodzi.info, access 2016-01-10

[2] http://twojbudzet.um.warszawa.pl/w-dzielnicach, access: 2016-01-10

investment in a different manner. The data provided according to each criterion also differed, e.g., the location was only part of the title in some cases, such as *parking places near the stadium*, although in some cases, long text was provided with an address so it could be a found on a map.

| No. | attributes |
|---|---|
| 62 | title |
| 55 | cost, location |
| 23 | project id |
| 20 | authors |
| 14 | motivation |
| 7 | additional comments |
| 6 | attachments, category |
| 5 | beneficiaries, municipality comments |
| 3 | future costs |
| 2 | status, citizen comments |
| 1 | others[3] |

Table 1: Number of cities that used different project attributes in descriptions.

These two issues meant that votes were distributed among a large number of projects and the winners were selected by a small fraction of people because the majority of participants chose less popular projects with no chance of implementation. We analysed anonymous votes from 2014 for one of the biggest PBs in Poland (in terms of budget, number of projects, and participants). We divided the voters into three groups: people who chose winners and all of their selections were going to be implemented; people who chose at least one but less than all of the projects that were going to be implemented; and, finally, people who did not chose any winners. Figure 1 shows the distribution of the votes among these three groups.

Thus, most of the voters probably fell disappointed with the results of this process and from a long-term perspective, this might stop them from voting in

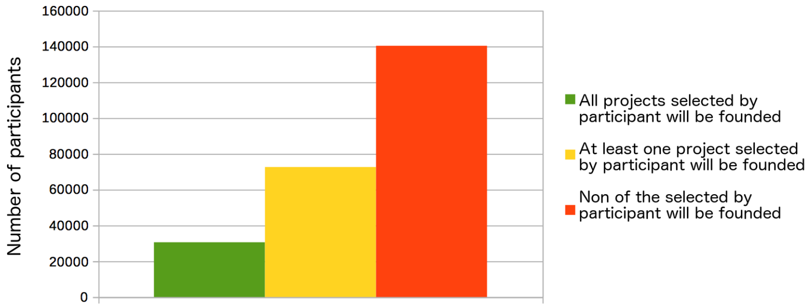


Figure 1: Voting success among participants (source: own research).

further PBs. This situation is also negative from the organizer's perspective because most people did not express their preferences for the winning projects.

### *2.2. Participatory DSSs*

Splitting budgets in a participatory manner is a group multiple criteria decision-making problem, which could be supported by computers. Theoretical studies have focused on communication, deliberation, and decision making [5], modeling under uncertainty [6], designing general frameworks [7], and experimental solutions [8], [9], but most of these existing solutions only consider support for administrative tasks related to PBs, rather than the actual decision-making process. We found no implementations of a DSS for PBs in Poland or elsewhere throughout the world. The popularity of DSS in participatory systems is much greater in environmental decision making [10] or city planning [11], [12], [13], [14]. Indeed, planners have their own participatory systems called public participatory geographic information systems, and the rise and development of this movement was described in [15]. There are also Web-based public participation systems [16], social DSSs [17], e-negotiation systems called generic negotiation platforms [18], negotiation systems [19], generic negotiation of contracts APIs[20], and policy analysis systems [21]. Examples include voting support systems such as "Opinions-Online," which is a platform for global participation in voting, surveys, and group decisions [4]. More examples have been described previously [22]. Thus, public participation involves DSSs in multiple fields, but PB still lacks any state-of-the-art implementations. Regardless of the stage when the PB process might be implemented, the key issue in these system is the ranking of PB projects and finding similar projects.

## 3. Multi-criteria decision analysis based on the TOPSIS method

One of the most widely used multi-criteria decision analysis methods is the TOPSIS method, which was proposed by Hwang and Yoon in 1981 [23], and extended by Yoon in 1987 [24], as well as by Hwang et al. in 1993 [25]. In the TOPSIS method, the optimal alternative is nearest to the positive ideal solution (PIS) and farthest from the negative ideal solution (NIS). A comparison of different methods for the multiple criteria decision problem can be found in [26]. The TOPSIS process is conducted as follows.

1. Decision matrix construction.

   Let us assume that there are $k$ decision makers, $n$ possible alternatives called $A = \{A_1, A_2, \ldots, A_n\}$, which are evaluated against $m$ criteria $C = \{C_1, C_2, \ldots, C_m\}$. For every decision maker, the decision matrix $D_{n\times m}$ contains performance ratings for each alternative $A_i (i = 1, \ldots, n)$ with respect to criteria $C_j (j = 1, ..., m)$, which are denoted as $x_{ij}$. Thus, for every decision maker, the decision matrix is expressed as follows.

| | $C_1$ | $C_2$ | $\cdots$ | $C_m$ |
|---|---|---|---|---|
| $A_1$ | $x_{11}$ | $x_{12}$ | $\cdots$ | $x_{1m}$ |
| $A_2$ | $x_{21}$ | $x_{22}$ | $\cdots$ | $x_{2m}$ |
| | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $A_n$ | $x_{n1}$ | $x_{n2}$ | $\cdots$ | $x_{nm}$ |

[4] www.opinions.hut.fi

2. Normalized decision matrix construction.

   The matrix $D_{n\times m}$ is converted into the matrix $R_k = (r_{ij})_{n\times m}$, using the normalization method $r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}, i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$

3. Weighted normalized decision matrix construction.

   Depending on the purpose and decision-makers, various evaluation criteria have different weights, so we need to calculate the weighted normalized decision matrix. Let the criteria weights be denoted by $w_j(j = 1, ..., m)$. Then, $T = (t_{ij})_{n\times m} = (w_j r_{ij})_{n\times m}, i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$

4. Determine the PIS and NIS.

   The PIS is that with the best level for all of the attributes considered. The NIS is that with the worst attribute values. Let $J_+$ be the set of benefit criteria (more is better) and let $J_-$ be the set of negative criteria (less is better).

$$PIS_i = \left\{\min\left(t_{ij}|i = 1, 2, ..., n\right)|j \in J_-, \max\left(t_{ij}|i = 1, 2, ..., n\right)|j \in J_+\right\}, \tag{1}$$

$$NIS_i = \left\{\max\left(t_{ij}|i = 1, 2, ..., n\right)|j \in J_-, \min\left(t_{ij}|i = 1, 2, ..., n\right)|j \in J_+\right\}. \tag{2}$$

5. Calculate the distance for each alternative to the PIS and NIS.

$$d_{iPIS} = \sqrt{\sum_{j=1}^{m}(t_{ij} - PIS_{ij})^2}, i = 1, 2, \ldots, n \tag{3}$$

$$d_{iNIS} = \sqrt{\sum_{j=1}^{m}(t_{ij} - NIS_{ij})^2}, i = 1, 2, \ldots, n \tag{4}$$

6. Calculate the relative proximity based on the similarity to the best alternative.

$$s_i = \frac{d_{iNIS}}{d_{iNIS} + d_{iPIS}}, 0 \leq s_{iNIS} \leq 1, i = 1, 2, \ldots, n \tag{5}$$

   It should be noted that $s_i = 1$ if and only if the $i$-th solution is the PIS and $s_i = 0$ if and only if the $i$-th solution is the NIS.

7. Rank the alternatives according to $s_i(i = 1, 2, \ldots, n)$.

TOPSIS minimizes the distance to the ideal alternative while maximizing the distance to the worst. According to previous studies and applications, various specific procedures can be used to develop the weights (step 3) and to select the PIS and NIS (step 4), but particularly for choosing the distance measure (step 5).

### *3.1. Fuzzy TOPSIS*

When handling inexact and vague information, particularly modelling human judgments, it is more realistic and intuitive to use linguistic assessments instead of numerical evaluations. Thus, in many previous studies, the TOPSIS method was used in conjunction with fuzzy logic. Numerous fuzzy TOPSIS methods and applications have been developed since the 1990s, e.g., for supplier selection [27] [28], finance [29] [30], power industry [31] [32], and negotiation problems [33]. In our study, we employ a fuzzy extension of the TOPSIS method presented by Chen [34].

#### *3.1.1. Preliminary definitions of fuzzy data*

Zadeh first introduced the fuzzy set theory [35] to deal with the vagueness of human reasoning. Later, Zadeh developed this method using a mathematical framework for processing linguistic values based on linguistic variables [36].In the following, we briefly review some basic definitions of fuzzy set theory [35],[36], [37].

**Definition 1.** *Let $X$ be a nonempty set. A fuzzy set $\tilde{A}$*[5] *in $X$ is characterized by its membership function $\mu_A(x) : X \to [0, 1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element $x$ in fuzzy set $A$ for each $x \in X$.*

[5] A ˜ above a symbol denotes a fuzzy set

A fuzzy set $\tilde{A}$ determined by the triplet $\tilde{A}(a, b, c)$ of crisp number with $a < b < c$ and with a membership function given by

$$\mu_A = \begin{cases} \frac{x-a}{b-a} & \text{if } a \leq x < b \\ \frac{c-x}{c-b} & \text{if } b \leq x < c \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

is called a triangular fuzzy set.

**Definition 2.** *Let $A$ be a fuzzy subset of $X$. The support of $A$ denoted by supp$(A)$ is the crisp subset of $X$ with elements that all have nonzero membership grades in $A$. supp$(A) = \{x \in X | \mu_A(x) > 0\}$.*

**Definition 3.** *A fuzzy set $A$ is called normal if an $x \in X$ exists such that $\mu_A(x) = 1$; otherwise, $A$ is subnormal.*

**Definition 4.** *A fuzzy number $A$ is a fuzzy set of the real line with a normal, convex, and continuous membership function of bounded support.*

Given any two triangular fuzzy numbers, $\tilde{A}$ and $\tilde{B}$, then according to the extension principle [35], the main operation can be expressed as follows.

$$\begin{gathered} \mu_C(z) = \max_{z=x \circ y} \{\min [\mu_A(x), \mu_B(y)]\}, \\ \text{where } \circ \in \{+, -, \times, \div\} \end{gathered} \tag{7}$$

**Definition 5.** *$\tilde{D}$ is called a fuzzy matrix if at least one entry in $\tilde{D}$ is a fuzzy number.*

A linguistic variable is a variable with values that are linguistic terms.

**Definition 6.** *A linguistic variable is characterized by a quintuple $(\gamma, T(\gamma), U, G, A)$, where*

- *$\gamma$ is the name of the variable,*
- *$T$ is the set of terms of $\gamma$,*
- *$U$ is the universe of discourse,*

- *$G$ is a syntactic rule for generating the labels in the terms set, and*
- *$A$ is the semantic rule for associating each element of $T(\gamma)$ with its meaning.*

*3.1.2. TOPSIS steps with fuzzy criteria value*

In the fuzzy extension of TOPSIS, the ratings of alternative $A_i$ with respect to criterion $C_j$ and the weights of the criteria $w_j$ are fuzzy numbers ([38]) or linguistic variables ([39]). Thus, the decision matrix is a fuzzy decision matrix $\tilde{D}$ and the weight vector is a fuzzy vector $\tilde{W} = (\tilde{w_1}, \tilde{w_2}, \ldots, \tilde{w_n})$.
To calculate the normalized decision matrix $\tilde{R}$ (step 2), the following transformation is used.

$$\begin{aligned} c_j^+ &= \max_i c_{ij}, & j \in J_+; \\ \tilde{r_{ij}} &= \left(\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+}\right), & j \in J_+; \\ a_j^- &= \min_i a_{ij} & j \in J_-; \\ \tilde{r_{ij}} &= \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right), & j \in J_-; \end{aligned} \tag{8}$$

Next, steps (3–6) are unchanged, but the operations are performed on fuzzy numbers. Due to the fuzziness of the variables, measuring the distance between alternatives is a separate problem. Many distance measurement functions have been proposed in previous studies and overviews can be found in [40], [41], and [42]. Following Chen ([34]), due to its effectiveness and simplicity, we use the vertex method to calculate the distance between two triangular fuzzy numbers.

**Definition 7.** *Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers, then the distance between them is*

$$d\left(\tilde{A}, \tilde{B}\right) = \sqrt{\frac{1}{3}\left[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2\right]}. \tag{9}$$

It should be noted that if $\tilde{A}$ and $\tilde{B}$ are real numbers, then the distance measurement is proportional to the Euclidean distance.

## 4. TOPSIS for ranking PB projects

In this section, we present a modified fuzzy TOPSIS method for ranking PB projects. The main components for decision making are as follows.

**The Goal** is to rank participatory budget projects according to participant first choice (PIS).

**Decision-makers** are city residents and temporarily resident citizens such as students.

**Alternatives** are different projects that could be implemented. The proposals are submitted by citizens and described in a rather general manner.

**Criteria** It is difficult to determine the exact set of decision-making criteria without conducting broad social studies. However, each city decides the attributes for describing the projects during their process. Only this information is available to the participants. Therefore, it can be assumed that the decision criteria are equivalent to description attributes.

**Weights** represent the internal preferences of each individual, where the values of the weights are unknown.

**Outcomes** comprise a set of selected projects and the satisfaction of decision-makers, which plays a key role in decisions about participation in subsequent PBs and the development of the PB process.

### *4.1. Decision support algorithm*

Constructing the PB decision matrix is difficult because of vague evaluation criteria as well as the fuzzy and qualitative rather than quantitative values of these criteria. Thus, we modified the order of the steps in the algorithm. Therefore, after modification for PB applications, the TOPSIS process can be conducted as follows.

1. Determine the PIS.
2. Determine distance from the PIS.

3. Create the distance measures matrix.
4. Determine the NIS.
5. Normalize the decision matrix.
6. Weight the normalized decision matrix with linguistic weights.
7. Calculate distance from the NIS.
8. Calculate the closeness coefficient for each project.
9. Rank the projects.

*4.1.1. Determination of the PIS*

The primary modification of the TOPSIS method is that we do not seek the perfect solution among the maximum and minimum values of the criteria because the PIS must be chosen by a participant. Thus, if the decision-maker chooses project $A_j$, then $PIS = A_j$ and $PIS_i = x_{ji}$.

*4.1.2. Determining the distances and creating the distance measures matrix*

Based on a number of project attributes (see Table 1) used in previous PB processes, we selected several for use as evaluation criterion. Some were defined clearly but others were identified from the project descriptions. They could be divided into two groups: nominal criteria and fuzzy criteria. Some values were quantitative such as distances or costs, but voters still treated them as fuzzy, e.g., exact locations were less important then information about whether places could be reached by car or on foot. It was impossible to determine which value was greater, or whether a higher or lower value was better, so we considered their distances from the PIS and calculated them for fuzzy values according to (9).

There were no official categories or beneficiary groups for the PB projects, so we decided to construct our own classification (see Appendix A). As a basis for the project categories, we used legal regulations[6] that list the duties of municipalities and the scope of their responsibilities. We created a hierarchical classification

[6] Act of 8th March, 1990 about municipal government.

based on this act. We proposed our own classification for beneficiary criteria according to empirical data gathered from polish PBs, e.g., age, commuting method, and interests. Projects could be assigned to multiple categories. Thus, the labelled projects could be described as a subset of our reference classification system, which is why we decided to build a distance measure based on the semantic similarity measure (SSM) proposed by Alani & Brewster [43]. The original SSM is used to calculate the closeness of classes from two ontologies. Our measure comprises the average of the shortest paths between labels in classification trees. Let us assume that there are two projects $p_1, p_2$, where each described with labels $c_i \in \{c_1, \ldots, c_n\}, c_j \in \{c_1, \ldots, c_m\}$, $c_i \rightsquigarrow c_j$ is a minimal path $p \in P$ among the paths between $c_i$ and $c_j$.

$$SSM - MIN(p_1, p_2) = \frac{1}{n} \sum_{i=1}^{n} min_j [c_i \rightsquigarrow c_j] \tag{10}$$

This measure is asymmetric because projects could have different numbers of category or beneficiary labels. For example, if a skate park is for skaters and a fountain in the city center is for all citizens, then skaters could benefit from the fountain but not all citizens would be interested in the skate park.

*4.1.3. Determining the NIS*

Thus, for every criterion $i$, the $NIS_i$ value is that with the greatest distance from the PIS: $NIS_i = \{\max_j (d_{jPIS})\}$.

An example showing the two last steps for two criteria and four projects is presented in Figure (2). Project $C$ is the $PIS$ and the $NIS$ is created with the greatest distance from the $PIS$ for all criteria.

*4.2. Normalization and weighting the decision matrix*

All of the distance values were normalized. If the values were fuzzy, normalization was performed according to eq. (8).

Assessing criteria weights is separate discussion problem. Many different methods have been proposed in literature ([44], [45], [46], [47]). The importance

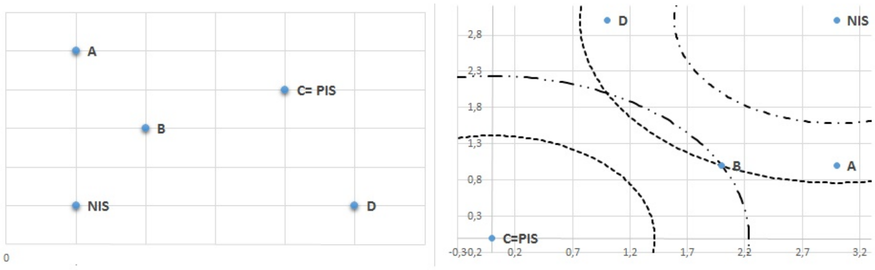


Figure 2: Example of modified TOPSIS for two criteria and four projects: $A$, $B$, $C$, and $D$.

weight for each criterion can be obtained by direct assignment or based on pair-wise comparisons ([48]). In case of PB the weights are subjective values and can be different for each citizen, so DSS should allow individual decision-makers to determine the weights. In order to facilitate this task, we used linguistic variables, which are much more intuitive for users. Another possible approach is to establish a common weights by the city council. The city council could decide which criterion is more important on the basis of budgetary targets eg. distribution of projects throughout the city (bigger weights for localization), or social justice - so the benefits to the broadest group of residents, which can be achieved by assigning greater weight to the beneficiaries.

For the purposes of the experiment weights were chosen, so as to trace the algorithm is working properly. We determined the weights arbitrarily by using the linguistic variables proposed by Chen [34]:

- $\gamma$ = importance,
- $T(\gamma) = \{$very low, low, medium low, medium, medium high, high, very high$\}$,
- $U = \{x : x \in (0, 1)\}$
- Membership function for all of the labels are given in Table 2

*4.3. Calculating the distance from the NIS, closeness coefficient, and ranking*

The final three steps were performed according to the classical TOPSIS algorithm. It is worth mentioning that the choice of PIS does not depend

| Terms | Fuzzy number |
|---|---|
| Very low | (0,00;0,00;0,10) |
| Low | (0,00;0,10;0,30) |
| Medium low | (0,10;0,30;0,50) |
| Medium | (0,30;0,50;0,70) |
| Medium high | (0,50;0,70;0,90) |
| High | (0,70;0,90;1,00) |
| Very high | (0,90;1,00;1,00) |

Table 2: Linguistic variables for the importance weights.

on the measure of distance. PIS is a real project selected by the user. NIS while an artificial project, having the worst possible outcome for each criterion independently (as in 4.1.3). A measure of distance depends on the criterion. We are considering 4 criteria: category, beneficiaries, location, cost. In case 1 and 2, the distance is calculated by SSM. For location we chose fuzzy approach, because the decision maker does not consider the precise distance, draws attention to the projects are within walking distance, short drive or long. The distance of cost is the absolute value of the difference in the cost of the two projects.

## 5. Examples from the Poznan project set

We tested the performance of the algorithm based on examples from recent PB projects in Poznan (PO2016). In this process, people proposed 267 (120 citywide and 147 district) projects. The vote count was (total) 73 136, which comprised 52 997 (72.46%) electronic and 20 139 (27.54%) paper votes. The number of valid votes was 66 124 (90.41%). The budget was 15 million PLN[7] and it was divided as follows: 5 million PLN for citywide projects and 2 million PLN $\times$ five district projects = 10 million. The three winning citywide projects received 9439, 7309, and 6024 votes with 14.27%, 11.05%, and 9.11% of the

[7]3.49 mln EUR, calculated at 2016-01-10

valid votes, respectively.

### *5.1. Data*

All of the projects were described by multiple attributes, including the title, estimated cost, full project description, location, potential beneficiaries, motivation, and future costs. The process was highly transparent and city published the results of its internal verification. Unfortunately, no categories and beneficiaries were defined. Thus, in order to extract the categories and beneficiaries, we had to read the descriptions and label the projects. To simplify this example, we selected projects from the Grunwald district (24 projects). In the evaluation, we employed the following criteria: beneficiaries, category, location, cost.

After cleaning, geotagging, and labelling, we obtained input data with the id, cost, category, geographical coordinates, and beneficiaries. The PB project categories and beneficiaries could be described using at least one label. We identified all of the possible categories and beneficiaries group labels (see Appendix A).

### *5.2. Results*

To evaluate the approach, we calculated multiple rankings with different weight vectors that responded to the following criteria: category, beneficiaries, location, and cost. Three cases are presented, i.e., first two simple cases to evaluate whether the results were reasonable and one other to illustrate a more realistic case.

**[VH, L, L, L ]** A case where the most important criterion was the category of the project. We attached a *very high* weight to the category and *low* weights to beneficiary, distance, and cost.

**[L, L, VH, L ]** A case where the most important criterion was the location.

**[VH, H, M, VL ]** This was the most realistic case and the most difficult to assess, with a *very high* weight for category, *high* weight for beneficiaries, *medium* weight for location, and *very low* weight for cost.

Below we describe 5 selected projects, which are important for example (see table 3). Full list can be found in table B.7.

| PIS | Cost | Category | Beneficiaries | Description |
|---|---|---|---|---|
| A1 | 5000 | 2.2 | city,tourist | securing routes for pedestrians and bikers, organizing space |
| A2 | 200000 | 10.1.1, 12.1 | Grunwald, pupil | organizing space in park, new educational routs |
| A3 | 66000 | 10.1.1 | Grunwald, pupil | building 3 fields for petanque |
| A4 | 80000 | 10.1.1 | Grunwald north, enthusiasts, tennis | pupil tennis court renovations |
| A15 | 90000 | 6.1,8.1 | disabilities | classroom for disabilities |
| A22 | 20500 | 2.2,2.5,4.1 | city | bus stop |

Table 3: Selected projects B.7

We chose A1 and A22 to check TOPSIS and SSM results for asymmetric similarity[8]. Notice A1 is described by 1 category, while A22 by 3; A1 is similar to A22 - both share the same category but A22 is less similar to A1 while it has only one common category. A3 and A4 are equal in terms of categories, very similar in terms of cost and beneficiaries. A15 is very specific project for whole district, there was only one project similar in terms of categories (A14). A2 was a random selection.

In the following, we describe the results obtained by our algorithm. We decide to show only first three and the last positions from each ranking. Detailed information about projects categories, costs, distance can be found in the appendix.

#### *5.2.1. [VH, L, L, L]*

Table 4 presents partial rankings for selected projects. The results of the algorithm correspond to the weight configuration - the categories have the biggest influence on the result. We can notice asymmetric similarity between A1 and A22 and symmetric between A3 and A4. A15 is similar to A14 but the second

[8]Similarity of two object (a, b) is symmetric if and only if value of similarity measure for a and b is equal to the value of similarity measure for b and a

| PIS | 1st | | 2nd | | 3rd | | last | |
|---|---|---|---|---|---|---|---|---|
| | A | d | A | d | A | d | A | d |
| A1 | A22 | 0,875 | A6 | 0,869 | A7 | 0,847 | A16 | 0,138 |
| A2 | A10 | 0,897 | A14 | 0,7022 | A4 | 0,698 | A1 | 0,0219 |
| A3 | A4 | 0,960 | A14 | 0,958 | A13 | 0,958 | A1 | 0,019 |
| A4 | A3 | 0,929 | A13 | 0,922 | A14 | 0,919 | A1 | 0,019 |
| A15 | A14 | 0,979 | A3 | 0,536 | A12 | 0,535 | A1 | 0,058 |
| A22 | A1 | 0,797 | A6 | 0,795 | A7 | 0,795 | A16 | 0,165 |

Table 4: Rankings with similarity coefficients for [VH, L, L, L]

and third closeness coefficients are very low. For A2 the highest similarity gets A10 with two common categories, A14 is the second rank with one common category.

The category was hard to evaluate because projects could be attached to multiple categories, e.g., A2 and A10 shared two common categories so they had first place in ranking; A14 shared one common category with A2 but two others were completely different; A2 shared one common category with A16 and A14 but A14 is very close to A2 in terms of the second category while A16 is very far.

*5.2.2. [L, L, VH, L]*

Table 5 presents the rankings for the same projects but with different weights ([L, L, VH, L]). It should be noted that the score was higher when the distance between project locations was smaller. Precise geographical distances between projects are presented in table B.8 but for TOPSIS we have used fuzzy value, which may be equivalent to the terms: walking distance, public transport distance etc.. In all cases first rank is within the scope of walking distance (below 2km). Second rank, A20 for A15 is located further (2.47km) than third one, A17 (1.72km) but it shared category and beneficiaries. For A22 first rank is located slight further (1.35km) than the second one (1.27) but the difference has no meaning as both alternatives are within walking distance.

| PIS | 1st | | 2nd | | 3rd | | last | |
|---|---|---|---|---|---|---|---|---|
| | A | d | A | d | A | d | A | d |
| A1 | A5 | 0.837 | A16 | 0.653 | A22 | 0.528 | A21 | 0.200 |
| A2 | A13 | 0.821 | A3 | 0.791 | A17 | 0.779 | A1 | 0.200 |
| A3 | A14 | 0.840 | A4 | 0.816 | A13 | 0.815 | A1 | 0.199 |
| A4 | A3 | 0.820 | A2 | 0.796 | A13 | 0.770 | A1 | 0.219 |
| A15 | A14 | 0.792 | A20 | 0.734 | A17 | 0.721 | A9 | 0.286 |
| A22 | A6 | 0.726 | A7 | 0.705 | A10 | 0.695 | A24 | 0.281 |

Table 5: Rankings with similarity coefficients for [L, L, VH, L]

*5.2.3. [VH, H, M, VL]*

In the final example, we assigned very high weights to category, high weights to beneficiaries, medium weights to distance, and very low weights to cost. According to the results (Table 6), the rankings were different than in the previous examples. This confirms the importance of assigned weights.

| PIS | 1st | | 2nd | | 3rd | | last | |
|---|---|---|---|---|---|---|---|---|
| | A | d | A | d | A | d | A | d |
| A1 | A6 | 0,784 | A22 | 0,757 | A17 | 0,7029 | A13 | 0,363 |
| A2 | A10 | 0,869 | A14 | 0,734 | A3 | 0,731 | A1 | 0,396 |
| A3 | A13 | 0,918 | A2 | 0,902 | A4 | 0,900 | A1 | 0,370 |
| A4 | A16 | 0,7935 | A3 | 0,783 | A13 | 0,778 | A1 | 0,360 |
| A15 | A20 | 0,695 | A14 | 0,596 | A24 | 0,590 | A1 | 0,251 |
| A22 | A1 | 0,797 | A17 | 0,772 | A19 | 0,761 | A3 | 0,390 |

Table 6: Rankings with similarity coefficients for [VH, H, M, VL]

## 6. Summary

The importance of PBs has increased significantly in the last 10 years. However, the sudden and dynamic growth of PBs has highlighted the need for DSSs in this area. The key problem with PBs is that the ranking methods used for projects do no employ quantitative assessment criteria. In this study, we proposed a modified fuzzy TOPSIS method for PBs, which we illustrated using real-world data from Poznan. The application of TOPSIS to PBs required some major changes to the algorithm, i.e., the transition to relative values (distance) depends on the initial choice of the voter and the criteria are modelled as fuzzy or categorical values, where the distances are calculated basis on the proposed classifications. At present, the ranking results are being tested by the test group. In further research, we plan to extend the classification to include the degree of membership for each class and we aim to apply the algorithm in a new voting process.

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## Appendix A. Project classifications

Figure A.3 shows some of the two classifications used in this study. The categories are shown the left and the potential benefits of the project on the right.

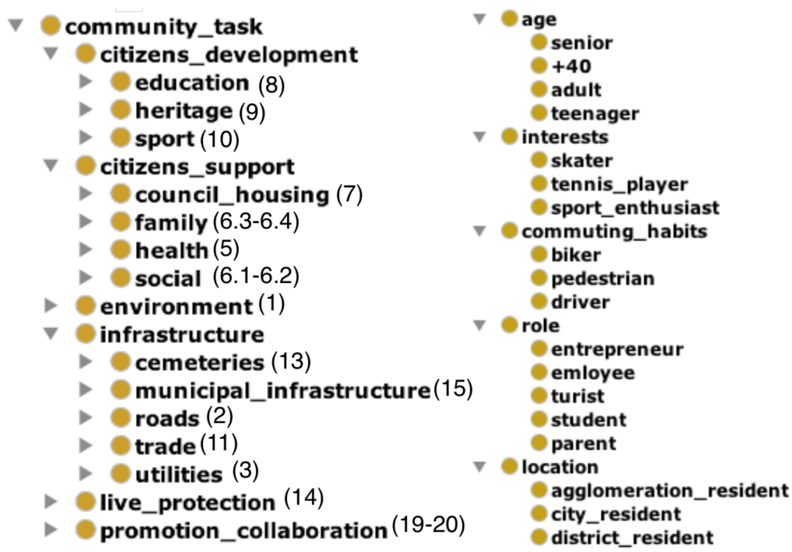


Figure A.3: Project classifications (source: own research).

## Appendix B. Example projects

Description of the project from Poznań used in the example.

| PIS | Cost | Category | Beneficiaries | Description |
|---|---|---|---|---|
| A1 | 5000 | 2.2 | city,tourist | securing routes for pedestrians and bikers, organizing space |
| A2 | 200000 | 10.1.1, 12.1 | Grunwald, pupil | organizing space in park, new educational routs |
| A3 | 66000 | 10.1.1 | Grunwald, pupil | building 3 fields for petanque |
| A4 | 80000 | 10.1.1 | Grunwald north, enthusiasts, tennis | pupil tennis court renovations |
| A5 | 49000 | 2.4, 10.1.1 | Lawica, Bajkowe, Edwardowo, pupils | playground |
| A6 | 58000 | 2.2, 3.5 | Kopernika, tourists | historic path renovation |
| A7 | 75000 | 2.2 | Popieluszki, Grunwald north, bikers | set of public bicycle parking |
| A8 | 110000 | 12.1 | Lazarz | planting trees |
| A9 | 52700 | 10.1.1 | city, Kwiatowe, adults, seniors | open air gym |
| A10 | 65000 | 2.4, 10.1.1, 12.1 | Grunwald, pupil | outdoor classroom |
| A11 | 43000 | 2.2 | Grunwald, bikers, walkers | modernization of parking places |
| A12 | 240000 | 10.1.1 | pupil, parent, Gorczyn | several playgrounds modernization |
| A13 | 224800 | 10.1.1 | pupil, teenagers, adults, Grunwald | multidisciplinary court with artificial grass |
| A14 | 340000 | 8.1,9.1, 10.1.1 | city, pupil, teenagers, seniors | house of culture |
| A15 | 90000 | 6.1,8.1 | disabilities | classroom for disabilities |
| A16 | 485267 | 10.1.1,10.1.2 | city, Junikowo, team sports, citizen, pupil, enthusiasts | courts for volleyball, basketball, badminton, jump, treadmill etc. |
| A17 | 450000 | 2.2,12.1 | city, outsiders, pupil, adult | square renovation |
| A18 | 180000 | 6.3 | Grunwald, pupil, parent | additional equipment for playground |
| A19 | 12000 | 2.2 | city, tourist | additional bicycle paths |
| A20 | 40000 | 6.1 | law victims | office for law victims with free layer service |
| A21 | 84735 | 12.1 | city, tourist | aerators for park ponds |
| A22 | 20500 | 2.2,2.5,4.1 | city | bus stop |
| A23 | 418000 | 2.2 | parent | street renovation - children protection |
| A24 | 16400 | 12.1 | municipality | planting trees |

Table B.7: Proposed projects used in the example case study

Citizens described the location of each project using text. We extracted the geographical coordinates of these projects and calculated the distances between their locations. We did not enter the geographical coordinates in Table B.7 because they had no use without a map. We employed a distance matrix (see Table B.8).

| id | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 5,38 | 5,74 | 4,87 | 0,08 | 4,4 | 5,12 | 7,13 | 2,73 | 4,48 | 6,99 | 6,37 | 6,3 | 5,15 | 5 | 2,57 | 6,67 | 7,98 | 7,96 | 7,47 | 7,99 | 3,96 | 6,06 | 8,16 |
| 2 | 5,38 | 0 | 1,18 | 1,05 | 5,42 | 1,27 | 1,36 | 1,76 | 4,81 | 1,36 | 1,61 | 1,74 | 0,93 | 1,35 | 1,16 | 3,64 | 1,56 | 2,73 | 2,61 | 2,57 | 2,73 | 1,56 | 0,69 | 2,82 |
| 3 | 5,74 | 1,18 | 0 | 0,87 | 5,76 | 2,28 | 0,68 | 1,8 | 5,67 | 2,42 | 1,86 | 2,75 | 1,26 | 0,64 | 0,73 | 4,45 | 1,01 | 2,31 | 2,42 | 1,75 | 2,31 | 1,8 | 1,34 | 2,6 |
| 4 | 4,87 | 1,05 | 0,87 | 0 | 4,9 | 1,67 | 0,45 | 2,44 | 4,86 | 1,84 | 2,4 | 2,79 | 1,7 | 0,46 | 0,2 | 3,63 | 1,81 | 3,13 | 3,17 | 2,62 | 3,14 | 0,93 | 1,61 | 3,37 |
| 5 | 0,08 | 5,42 | 5,76 | 4,9 | 0 | 4,45 | 5,14 | 7,17 | 2,81 | 4,53 | 7,03 | 6,43 | 6,34 | 5,18 | 5,03 | 2,65 | 6,7 | 8,01 | 8 | 7,49 | 8,02 | 3,99 | 6,11 | 8,2 |
| 6 | 4,4 | 1,27 | 2,28 | 1,67 | 4,45 | 0 | 2,11 | 2,94 | 3,54 | 0,2 | 2,72 | 2,02 | 2,13 | 2,13 | 1,87 | 2,4 | 2,83 | 3,98 | 3,81 | 3,83 | 3,98 | 1,35 | 1,84 | 4,02 |
| 7 | 5,12 | 1,36 | 0,68 | 0,45 | 5,14 | 2,11 | 0 | 2,44 | 5,27 | 2,29 | 2,47 | 3,09 | 1,8 | 0,03 | 0,25 | 4,03 | 1,68 | 2,98 | 3,09 | 2,35 | 2,99 | 1,27 | 1,78 | 3,27 |
| 8 | 7,13 | 1,76 | 1,8 | 2,44 | 7,17 | 2,94 | 2,44 | 0 | 6,46 | 2,96 | 0,33 | 1,97 | 0,84 | 2,41 | 2,4 | 5,34 | 0,93 | 1,13 | 0,87 | 1,57 | 1,13 | 3,23 | 1,11 | 1,07 |
| 9 | 2,73 | 4,81 | 5,67 | 4,86 | 2,81 | 3,54 | 5,27 | 6,46 | 0 | 3,5 | 6,21 | 5,03 | 5,67 | 5,29 | 5,06 | 1,24 | 6,36 | 7,52 | 7,33 | 7,34 | 7,52 | 4,02 | 5,36 | 7,53 |
| 10 | 4,48 | 1,36 | 2,42 | 1,84 | 4,53 | 0,2 | 2,29 | 2,96 | 3,5 | 0 | 2,73 | 1,9 | 2,17 | 2,3 | 2,04 | 2,39 | 2,91 | 4,02 | 3,83 | 3,93 | 4,02 | 1,55 | 1,87 | 4,04 |
| 11 | 6,99 | 1,61 | 1,86 | 2,4 | 7,03 | 2,72 | 2,47 | 0,33 | 6,21 | 2,73 | 0 | 1,64 | 0,72 | 2,44 | 2,4 | 5,12 | 1,13 | 1,45 | 1,13 | 1,88 | 1,44 | 3,14 | 0,92 | 1,33 |
| 12 | 6,37 | 1,74 | 2,75 | 2,79 | 6,43 | 2,02 | 3,09 | 1,97 | 5,03 | 1,9 | 1,64 | 0 | 1,64 | 3,08 | 2,9 | 4,12 | 2,53 | 3,07 | 2,69 | 3,46 | 3,07 | 3,09 | 1,44 | 2,86 |
| 13 | 6,3 | 0,93 | 1,26 | 1,7 | 6,34 | 2,13 | 1,8 | 0,84 | 5,67 | 2,17 | 0,72 | 1,64 | 0 | 1,78 | 1,7 | 4,53 | 0,91 | 1,85 | 1,69 | 1,91 | 1,85 | 2,43 | 0,31 | 1,9 |
| 14 | 5,15 | 1,35 | 0,64 | 0,46 | 5,18 | 2,13 | 0,03 | 2,41 | 5,29 | 2,3 | 2,44 | 3,08 | 1,78 | 0 | 0,26 | 4,06 | 1,65 | 2,95 | 3,06 | 2,32 | 2,95 | 1,3 | 1,77 | 3,24 |
| 15 | 5 | 1,16 | 0,73 | 0,2 | 5,03 | 1,87 | 0,25 | 2,4 | 5,06 | 2,04 | 2,4 | 2,9 | 1,7 | 0,26 | 0 | 3,82 | 1,72 | 3,03 | 3,1 | 2,47 | 3,04 | 1,09 | 1,65 | 3,29 |
| 16 | 2,57 | 3,64 | 4,45 | 3,63 | 2,65 | 2,4 | 4,03 | 5,34 | 1,24 | 2,39 | 5,12 | 4,12 | 4,53 | 4,06 | 3,82 | 0 | 5,17 | 6,37 | 6,21 | 6,14 | 6,37 | 2,78 | 4,24 | 6,42 |
| 17 | 6,67 | 1,56 | 1,01 | 1,81 | 6,7 | 2,83 | 1,68 | 0,93 | 6,36 | 2,91 | 1,13 | 2,53 | 0,91 | 1,65 | 1,72 | 5,17 | 0 | 1,31 | 1,42 | 1,03 | 1,32 | 2,72 | 1,19 | 1,59 |
| 18 | 7,98 | 2,73 | 2,31 | 3,13 | 8,01 | 3,98 | 2,98 | 1,13 | 7,52 | 4,02 | 1,45 | 3,07 | 1,85 | 2,95 | 3,03 | 6,37 | 1,31 | 0 | 0,5 | 1 | 0,01 | 4,03 | 2,16 | 0,5 |
| 19 | 7,96 | 2,61 | 2,42 | 3,17 | 8 | 3,81 | 3,09 | 0,87 | 7,33 | 3,83 | 1,13 | 2,69 | 1,69 | 3,06 | 3,1 | 6,21 | 1,42 | 0,5 | 0 | 1,43 | 0,49 | 4,03 | 1,98 | 0,2 |
| 20 | 7,47 | 2,57 | 1,75 | 2,62 | 7,49 | 3,83 | 2,35 | 1,57 | 7,34 | 3,93 | 1,88 | 3,46 | 1,91 | 2,32 | 2,47 | 6,14 | 1,03 | 1 | 1,43 | 0 | 1,01 | 3,55 | 2,21 | 1,49 |
| 21 | 7,99 | 2,73 | 2,31 | 3,14 | 8,02 | 3,98 | 2,99 | 1,13 | 7,52 | 4,02 | 1,44 | 3,07 | 1,85 | 2,95 | 3,04 | 6,37 | 1,32 | 0,01 | 0,49 | 1,01 | 0 | 4,03 | 2,16 | 0,49 |
| 22 | 3,96 | 1,56 | 1,8 | 0,93 | 3,99 | 1,35 | 1,27 | 3,23 | 4,02 | 1,55 | 3,14 | 3,09 | 2,43 | 1,3 | 1,09 | 2,78 | 2,72 | 4,03 | 4,03 | 3,55 | 4,03 | 0 | 2,25 | 4,23 |
| 23 | 6,06 | 0,69 | 1,34 | 1,61 | 6,11 | 1,84 | 1,78 | 1,11 | 5,36 | 1,87 | 0,92 | 1,44 | 0,31 | 1,77 | 1,65 | 4,24 | 1,19 | 2,16 | 1,98 | 2,21 | 2,16 | 2,25 | 0 | 2,18 |
| 24 | 8,16 | 2,82 | 2,6 | 3,37 | 8,2 | 4,02 | 3,27 | 1,07 | 7,53 | 4,04 | 1,33 | 2,86 | 1,9 | 3,24 | 3,29 | 6,42 | 1,59 | 0,5 | 0,2 | 1,49 | 0,49 | 4,23 | 2,18 | 0 |

Table B.8: Distances between the locations of projects/alternatives (in kilometres)