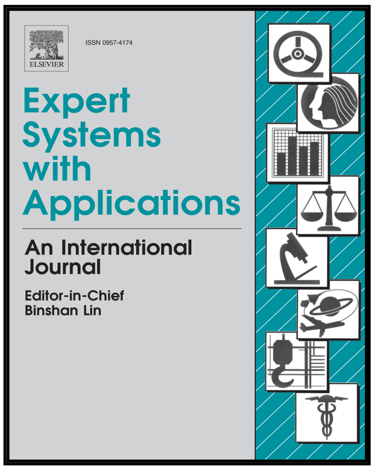

A Generalised Fuzzy TOPSIS with Improved Closeness Coefficient

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Highlights

- A versatile evaluation model suitable for fuzzy or interval-valued numbers
- Works with or without subjective weights of criteria defined by evaluators
- Improved closeness coefficient with the positive and negative distance weights
- Relevant examples of additive manufacturing technology and material selection
- Sensitivity analyses to assist managers in making more informed decisions

A Generalised Fuzzy TOPSIS with Improved Closeness Coefficient

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Abstract

In this paper, we propose a Generalised-Fuzzy-TOPSIS method as a versatile evaluation model. The model is suitable for different types of fuzzy or interval-valued numbers, with or without subjective weights of criteria being defined by evaluators. Additionally, we extend the final ranking step of the TOPSIS method, which is the calculation of closeness coefficient based on the separation from Negative Ideal Solution (NIS) and proximity to Positive Ideal Solution (PIS). Experiments show that with the same focus on PIS and NIS distances, our proposed ranking is identical to TOPSIS, and also performs very well when varying the distance weights. The applicability of the proposed method is demonstrated with relevant examples of technology and material selection in the context of additive manufacturing. Sensitivity analyses, based on subjective weights of criteria, degree of optimism,

evaluators' weights in group decision making, and distance weights, are presented to assist managers in making more informed decisions.

Keywords: Multiple criteria analysis; Fuzzy TOPSIS; Interval-valued intuitionistic fuzzy set; Closeness coefficient; Group decision making

1. Introduction

Multi-Criteria Decision Making (MCDM) methods are widely used to assist in decision making when there are different criteria and the best alternative is to be selected. Often one needs to make a best compromise choice from the available options, since finding the best alternative may not be practically feasible. Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is one of the most widely used MCDM methods (Zyoud & Fuchs-Hanusch, 2017) in such situations. TOPSIS works on the principle of finding the best compromise solution when compared to an *Ideal Solution*. However, in contemporary business situations such as technology selection, decisions are often taken in uncertain environments, and evaluators may feel more confident in expressing the ratings of alternatives for given criteria in fuzzy sets or interval-values (Durbach & Stewart, 2012). To address this challenge, researchers have presented different versions of Fuzzy-TOPSIS method for specific decision-making environments (Behzadian, Khanmohammadi Otaghsara, Yazdani & Ignatius, 2012; Joshi & Kumar, 2016; Mardani, Jusoh & Zavadskas, 2015; Walczak & Rutkowska, 2017). Further, it is also possible that there could be different types of fuzzy or interval-valued numbers, with or without subjective weights of criteria by evaluators, and the current approaches do not incorporate these flexibilities and uncertainties in a single method.

We therefore seek to enhance the decision making approach to incorporate all the above problem variants. Developing a generalised and flexible selection model is important, since an organisation or decision-maker may not be willing to invest unduly high time or money in the development of different types of selection models. Accordingly, in this paper we propose a Generalised-Fuzzy-TOPSIS (GFTOPSIS) method, a versatile evaluation model capable of incorporating different kinds of flexibilities and uncertainties in the decision making process.

The proposed approach is a generalised, flexible and intelligent fuzzy MCDM method, using Interval-Valued Intuitionistic Fuzzy Set (IVIFS) for preference rating, suitable for use in uncertain environments. TOPSIS method is modified to incorporate IVIFS preference rating along with Degree of Optimism (DOpt). The proposed GFTOPSIS method uses DOpt to derive the expected IFS matrix, and subsequent calculations are performed based on the expected IFS matrix and distance between two IFS. This method is different from the work of F. Ye (2010), who recommended TOPSIS based on distances between two IVIFS. The benefit of using DOpt is to help include the individual biases of an evaluator in the method. Thus, the evaluator may decide whether the interval values are inclined towards higher limits or lower limits or in-between. Additionally, weights of criteria are obtained by a combination of subjective weights given by evaluators, and fuzzy entropy or uncertainty weights. The entropy weights are derived based on input variability suggested by J. Ye (2010) for fuzzy decision making, so as to increase the intelligence in the model.

One limitation of the TOPSIS method is that it does not take care of different weights of Negative Ideal Solution (NIS) and Positive Ideal Solution (PIS) distances (Opricovic & Tzeng, 2004). We extend the TOPSIS method to include a situation where a decision-maker may decide final ranking with more focus either on PIS or on NIS. This is achieved by extending the final ranking step of TOPSIS method, which is the calculation of closeness coefficient, based on the separation from NIS and proximity to PIS. We also demonstrate that with the same focus on PIS and NIS distances, our proposed ranking is identical to TOPSIS, and also performs excellently when distance weights are varied. We also compare the experimental results of GFTOPSIS with earlier research on ranking steps.

The degree of generalisability of the model allows its application with several types of Fuzzy Set (FS) or Interval-Valued Fuzzy Set (IVFS), with different focuses on NIS and PIS distances. Specifically, GFTOPSIS method uses expert ratings in the form of IVIFS, and use DOpt to convert IVIFS into IFS. Further, the model is intelligent since it assigns weights based on the uncertainty levels in the ratings, and can work with or without subjective weights of criteria by evaluators. We also demonstrate the flexibility of the model with relevant examples and sensitivity analyses based on criteria with subjective weights, DOpt, evaluators' weights in group decision making, and distance weights.

We illustrate our proposed method using two cases of technology selection and material selection in the context of Additive Manufacturing (AM). AM, colloquially three-dimensional printing or '3D printing', is defined as "a process of joining materials to make objects from 3D model data, usually layer upon layer, as opposed to subtractive manufacturing methodologies" (ASTM Standard, 2012). AM is considered to have potential to disrupt the production and supply chain (D'Aveni, 2015; Jiang, Kleer & Piller, 2017). Recent developments in the AM technologies and AM materials have evoked interest among technologists to look for the best alternatives out of available processes and materials (Wohlers, 2016). Often, properties of the new technology's processes and materials are not available in crisp form, and this necessitates the expression of preference ratings as an appropriate fuzzy set. We also demonstrate how to use GFTOPSIS with different fuzzy inputs, and sensitivity analyses are presented to assist managers in choosing the best alternatives along with assessing the change in ranking with the change in the weights or DOpt. This provides users with a tool to facilitate making more informed decisions, while considering any particular condition which may have been missed by the evaluators while choosing input preference ratings. Such sensitivity analysis enhances the users' confidence in the results.

In Section 2 we present pertinent literature, basic concepts of fuzzy sets, and define DOpt. Detailed steps of GFTOPSIS are explained in Section 3. The extension in the final ranking step is described in Section 4. The proposed method is used to demonstrate some instances for Additive Manufacturing (AM) technology and material selection in Section 5, followed by the conclusions and future directions in Section 6.

2. Background

Considerable literature is available on applications of MCDM methods in the selection of technology, suppliers, and materials. Different MCDM methods have been proposed in the literature to assist in decision making with multiple criteria and uncertain situations. TOPSIS, proposed by Hwang & Yoon (1981) is one of the widely used methods to derive ranks of the candidate alternatives (Zyoud & Fuchs-Hanusch, 2017). Preferences or rating of alternatives are often vague or underspecified, and defining them by crisp numbers may be difficult in many real world situations. C.T. Chen (2000) extended TOPSIS method to the fuzzy environment, in which multiple evaluators use triangular fuzzy numbers to define preference ratings. Jahanshahloo et al. (2006) extended TOPSIS with interval data. F. Ye (2010) used IVIFS for preference rating in fuzzy-TOPSIS, for virtual enterprise partner selection. Recently, Yang et al. (2017) applied IVFS in fuzzy-TOPSIS method, to derive preference ranking for material selection for design for remanufacturing.

Atanassov (1986) proposed IFS, an extension and generalisation of the basic fuzzy set theory introduced by Zadeh (1965). As per fuzzy sets concept, membership of an element to a fuzzy set may be between 0 and 1, and non-membership value is taken as the difference of membership value from 1. In real life situations, the degree of non-membership may be between 0 and the difference of membership value from 1, accounting for the degree of hesitation (Atanassov, 1986). Thus, the sum of degrees of membership, non-membership, and hesitation is equal to 1 in IFS. Further, it is often challenging to express these degrees with single numbers, due to complexity and uncertainties. Thus, these degrees may be defined by interval-valued numbers as IVIFS (Atanassov & Gargov, 1989).

Although different studies are available specific to the application contexts, individuals or practitioners often lack the expertise to choose a method which is best for a particular selection process. Learning and applying several types of methods or software packages require significant time, effort and money. To help such users, we develop generalised methods to choose the best alternative in an uncertain environment.

The theoretical background of IVIFS and related concepts are briefly presented as follows.

Intuitionistic Fuzzy Sets (IFS) (Atanassov, 1986)

Let A be an IFS in a universe of discourse $X = \{x_1, x_2, \dots, x_m\}$. A may be represented as $A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle | x_i \in X\}$ where $\mu_A(x_i)$ and $\nu_A(x_i)$ are membership degree and non-membership degree of the element x_i to the IFS A, such that $\mu_A(x_i), \nu_A(x_i) \in [0,1]$, and $0 \le \mu_A(x_i) + \nu_A(x_i) \le 1$.

Degree of hesitation $\pi_A(x_i)$ of the element x_i to A is defined as $\pi_A(x_i) = 1 - \left(\mu_A(x_i) + \nu_A(x_i)\right)$. One can see that $\pi_A(x_i) \in [0,1]$ and if $\pi_A(x_i) = 0$, the IFS A is similar to a fuzzy set.

Distance $d(A, B)$ between two IFS, say A and B, may be calculated as proposed by Szmidt & Kacprzyk (2000).

$$d(A,B) = \sqrt{\frac{1}{2}\sum_{i=1}^{m}\left\{\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(\nu_A(x_i) - \nu_B(x_i)\right)^2 + \left(\pi_A(x_i) - \pi_B(x_i)\right)^2\right\}}$$

Interval-Valued Intuitionistic Fuzzy Sets (IVIFS) (Atanassov & Gargov, 1989)

Let A be an IVIFS in a universe of discourse X. A may be represented as $A = \{\langle x_i, [\mu_{AL}(x_i), \mu_{AU}(x_i)], [\nu_{AL}(x_i), \nu_{AU}(x_i)]\rangle | x_i \in X\}$, where $[\mu_{AL}(x_i), \mu_{AU}(x_i)]$ and $[\nu_{AL}(x_i), \nu_{AU}(x_i)]$ are interval-valued (lower and upper) degrees of membership and non-membership of the element x_i to the IVIFS A, such that $\mu_{AL}(x_i), \mu_{AU}(x_i), \nu_{AL}(x_i), \nu_{AU}(x_i) \in [0,1]$, and $0 \le \mu_{AL}(x_i) + \mu_{AU}(x_i) \le 1$, $0 \le \nu_{AL}(x_i) + \nu_{AU}(x_i) \le 1$, $0 \le \mu_{AU}(x_i) + \nu_{AU}(x_i) \le 1$.

Degree of Optimism (DOpt)

DOpt, $\lambda \in [0,1]$, is defined to incorporate human biases in the rating process. If an evaluator is optimistic for the ratings, the value of λ will be 0.5 to 1, and if s/he is pessimistic for the ratings, s/he will assign the value of λ between 0 and 0.5; if s/he is normal in ratings, $\lambda = 0.5$ will be chosen. Calculation of expected membership $E(\mu_A)$ and non-membership $E(\nu_A)$ values of IFS of an IVIFS using a DOpt is as follows:

$$E(\mu_A) = (1 - \lambda)\mu_{AL} + \lambda\mu_{AU}$$

$$E(\nu_A) = \lambda\nu_{AL} + (1 - \lambda)\nu_{AU}$$

Intuitionistic fuzzy entropy (Burillo & Bustince, 1996):

Intuitionistic entropy of IFS A is expressed in term of degree of hesitation, $\pi_A(x_i)$, as follows:

$$E(A) = \sum_{i=1}^{m} \left(1 - \mu_A(x_i) - \nu_A(x_i)\right) = \sum_{i=1}^{m} \pi_A(x_i)$$

3. GFTOPSIS Method

The proposed GFTOPSIS methodology is summarised in Figure 1. Inputs need to be specified before calculation steps. First, evaluators explore available alternatives and criteria on which the alternatives will be ranked. From the available candidates or alternatives, we select the alternatives which pass the minimum requirement for evaluation. Initial screening is necessary to reduce the assessment efforts later. Further, criteria are defined for evaluation. It is also preferred to define the subjective importance weight for each criterion, on a scale of 0 to 1. If an evaluator is more comfortable in expressing the criteria weights on 1 to 10 scale, s/he may choose to do so. Alternatively, weights of the criteria may also be determined by using other MCDM techniques such as AHP (Vaidya & Kumar, 2006). In the next step, we prepare IVIFS performance matrix $\tilde{R}^k = \left[\tilde{r}_{ij}^k\right]_{n\times m}$ where $\tilde{r}_{ij}^k = \langle\left[a_{ij}^k, b_{ij}^k\right], \left[c_{ij}^k, d_{ij}^k\right]\rangle$ is an IVIFS, given by evaluator $k \in \{1, \cdots, K\}$ to an alternative X_i, with $i \in \{1, \cdots, m\}$, against each criterion A_j, and $j \in \{1, \cdots, n\}$. Here $\left[a_{ij}^k, b_{ij}^k\right]$ represents the degrees of lower and upper membership, with $\left[c_{ij}^k, d_{ij}^k\right]$ being the degrees of lower and upper non-membership, of the alternative X_i for the criterion A_j by the evaluator k. Evaluators need to specify the degree of optimism $\lambda \in [0,1]$ as defined in Section 2 above. The calculation steps of the proposed GFTOPSIS method are as follows:

- Estimate IFS performance matrix from IVIFS performance matrix using DOpt
- Calculate entropy weight of each criterion
- Calculate final weight of each criterion
- Calculate PIS and NIS
- Calculate weighted Euclidean distances for each candidate material from PIS and NIS
- Calculate the extended closeness coefficient and rank order of alternatives

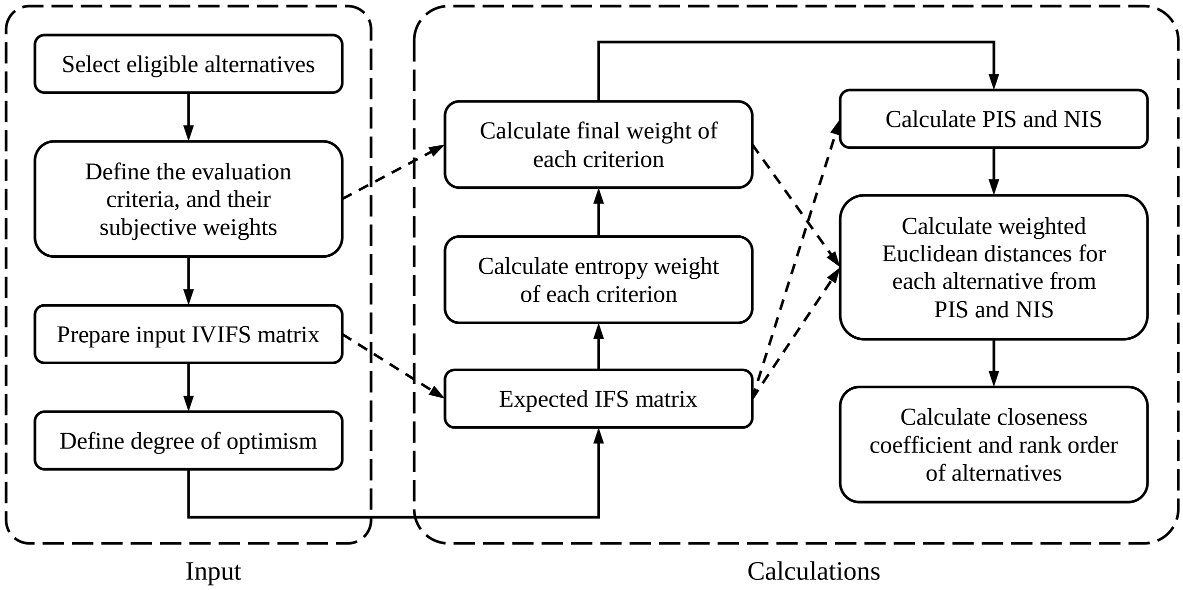


Figure 1: Proposed GFTOPSIS Methodology

Expected IFS performance matrix

We propose the use of DOpt λ to calculate expected IFS performance matrix $\tilde{S}^k = \left[\tilde{s}_{ij}^k\right]_{n\times m}$, where $\tilde{s}_{ij}^k = \langle \mu_{ij}^k, \nu_{ij}^k \rangle$ is the expected IFS of IVIFS $\tilde{r}_{ij}^k = \langle \left[a_{ij}^k, b_{ij}^k\right], \left[c_{ij}^k, d_{ij}^k\right] \rangle$ of the performance matrix $\tilde{R}^k = \left[\tilde{r}_{ij}^k\right]_{n\times m}$. Here μ_{ij}^k is the expected membership value and ν_{ij}^k is the expected non-membership values of the IFS. Expected IFS performance matrix $\tilde{S}^k$ may be computed as follows:

$$\mu_{ij}^k = (1-\lambda)a_{ij}^k + \lambda b_{ij}^k$$

$$\nu_{ij}^k = \lambda c_{ij}^k + (1-\lambda) d_{ij}^k$$

Based on $\tilde{s}_{ij}^k = \langle \mu_{ij}^k, \nu_{ij}^k \rangle$, the degree of hesitation the alternative X_i on the criterion A_j is defined as follows:

$$\pi_{ij}^k = 1 - \left(\mu_{ij}^k + \nu_{ij}^k\right)$$

Calculate PIS and NIS

Positive Ideal Solution (PIS): $\tilde{S}^{k+} = \left[\tilde{s}_j^{k+}\right]_{n\times 1}$

Where IFS $\tilde{s}_j^{k+} = \langle \mu_j^k, \nu_j^k \rangle = \langle \max_i \mu_{ij}^k, \min_i \nu_{ij}^k \rangle$; $i \in \{1, \cdots, m\}; j \in \{1, \cdots, n\}; k \in \{1, \cdots, K\}$

For IFS $\tilde{s}_j^{k+}$ degree of hesitation is $\pi_j^{k+} = 1 - \left(\mu_j^{k+} + \nu_j^{k+}\right)$.

Negative Ideal Solution (NIS): $\tilde{S}^{k-} = \left[\tilde{s}_j^{k-}\right]_{n\times 1}$

Where IFS $\tilde{s}_j^{k-} = \langle \mu_j^k, \nu_j^k \rangle = \langle \min_i \mu_{ij}^k, \max_i \nu_{ij}^k \rangle;\ i \in \{1, \cdots, m\}; j \in \{1, \cdots, n\}; k \in \{1, \cdots, K\}$

For IFS $\tilde{s}_j^{k-}$ degree of hesitation is $\pi_j^{k-} = 1 - \left(\mu_j^{k-} + \nu_j^{k-}\right)$.

Calculate the weight factor

All decision-making criteria may not be equally important, due to specific application requirements. Accordingly, we need to assign higher weights to criteria which are more important than others. To derive an appropriate weight w_j for a criterion j, we use a combination of subjective weights and entropy weights for each criterion. Subjective weight α_j^k for a criterion j will be assigned by the evaluator k are normalised subjective weight such that their sum is equal to one for each evaluators. Intuitionistic fuzzy entropy weight β_j^k are calculated as the total of the degrees of hesitation of the alternatives against the criterion j by the evaluator k. Final weight, w_j^k of the criterion j for the evaluator k may be calculated as follows:

$$w_j^k = \frac{\alpha_j^k \beta_j^k}{\sum_{j=1}^{n} \alpha_j^k \beta_j^k}$$

A similar method for weight calculation by a combination of subjective weights and entropy weights based on probability theory is presented by Yang et al. (2017), in the context of material selection for remanufacturing using fuzzy sets. Earlier, J. Ye (2010) proposed a fuzzy MCDM method applying entropy weights in IFS concept. Burillo & Bustince (1996) introduced the concept of intuitionistic fuzzy entropy, relating it to uncertainty or hesitation degrees of all the alternatives for a criterion. A lower total uncertainty for a criterion means lower value of entropy, which indicates more useful information for evaluators or decision-makers. Thus, a lower entropy value must result in higher entropy weight. We propose calculation of entropy weights, based on the works of J. Ye (2010) and Burillo & Bustince (1996), as follows.

Intuitionistic fuzzy entropy weight $\boldsymbol{\beta_j^k}$ criterion $\boldsymbol{j}$

$$\beta_j^k = \frac{\left|1 - E_j^k\right|}{\sum_{j=1}^{n} \left|1 - E_j^k\right|}$$

Where $E_j^k = \frac{1}{m} E\left(A_j^k\right) = \frac{1}{m} \sum_{i=1}^{m} \left(1 - \left(\mu_{ij}^k + \nu_{ij}^k\right)\right) = \frac{1}{m} \sum_{i=1}^{m} \pi_{ij}^k$

and $\beta_j^k \in [0,1]$, such that $\sum_{j=1}^{n} \beta_j^k = 1$, $0 \le E_j^k \le 1$ and $i \in \{1, \cdots, m\}; j \in \{1, \cdots, n\}; k \in \{1, \cdots, K\}$.

Calculate weighted Euclidean distances for each alternative from PIS and NIS

Weighted Euclidean distances for each alternative from PIS

$$d_i^{k+} = d\left(W^k \tilde{S}^k, W^k \tilde{S}^{k+}\right) = \sqrt{\frac{1}{2}\sum_{j=1}^{n}\left[\left(w_j^k\right)^2\left\{\left(\mu_{ij}^k - \mu_j^{k+}\right)^2 + \left(\nu_{ij}^k - \nu_j^{k+}\right)^2 + \left(\pi_{ij}^k - \pi_j^{k+}\right)^2\right\}\right]}$$

Weighted Euclidean distances for each alternative from NIS

$$d_i^{k-} = d\left(W^k \tilde{S}^k, W^k \tilde{S}^{k-}\right) = \sqrt{\frac{1}{2}\sum_{j=1}^{n}\left[\left(w_j^k\right)^2\left\{\left(\mu_{ij}^k - \mu_j^{k-}\right)^2 + \left(\nu_{ij}^k - \nu_j^{k-}\right)^2 + \left(\pi_{ij}^k - \pi_j^{k-}\right)^2\right\}\right]}$$

Group weighted Euclidean distances for each alternative from ideal and NIS, where γ_k is the weight given to the evaluator k.

$$D_i^+ = \sum_{k=1}^{K} \gamma_k d_i^{k+}$$

$$D_i^- = \sum_{k=1}^{K} \gamma_k d_i^{k-}$$

Calculate closeness coefficient and rank order of alternatives

A new Closeness Coefficient (CC), as a $CC_{GFTOPSIS}$, is proposed based on PIS distance, its weight w^+, NIS distance, and its weight w^-, where $w^+ \in [0,1]$ and $w^- \in [0,1]$. $CC_{GFTOPSIS}$ may be calculated as follows:

$$CC_{GFTOPSIS\_i} = \frac{(D_i^-)^{w^-}}{\left(D_i^-\right)^{w^-} + \left(D_i^+\right)^{w^+}}$$

Based on the $CC_{GFTOPSIS}$, the final ranking of the alternative may be determined. It is evident from the formula that if CC is high for an alternative, it is near PIS and far from NIS. Thus, a higher CC is rightly connected with a higher rank.

4. New $CC_{GFTOPSIS}$

In TOPSIS, CC_{TOPSIS} is calculated as follows:

$$CC_{TOPSIS\_i} = \frac{D_i^-}{D_i^- + D_i^+} = \frac{1}{1 + \left(D_i^+ / D_i^-\right)}$$

Here, $D_{i1}^+/D_{i1}^- < D_{i2}^+/D_{i2}^- \leftrightarrow CC_{TOPSIS_{i1}} > CC_{TOPSIS\_i2}$, thus rank is implicitly decided based on the reverse order of ratio (D_i^+/D_i^-).

The most obvious way to add the relative importance of the distances from PIS and NIS is to take distance multiplied by weights, as follows:

$$CC_{\widehat{TOPSIS}\_i} = \frac{w^- D_i^-}{w^- D_i^- + w^+ D_i^+} = \frac{1}{1 + (w^+/w^-)\left(D_i^+/D_i^-\right)}$$

Here $CC_{\widehat{TOPSIS}\_i1} > CC_{\widehat{TOPSIS}\_i2} \leftrightarrow (w^+/w^-)(D_{i1}^+/D_{i1}^-) < (w^+/w^-)(D_{i2}^+/D_{i2}^-) \leftrightarrow D_{i1}^+/D_{i1}^- < D_{i2}^+/D_{i2}^- \leftrightarrow CC_{TOPSIS_{i1}} > CC_{TOPSIS\_i2}$. Thus, we may conclude that directly multiplying the weights to distances or weight ratio (w^+/w^-) to distance ratio (D_i^+/D_i^-) does not change the order or rank of coefficients. This weighted CC is not useful because ranking based on it will be same as ranking based on CC_{TOPSIS}, for any possible value of w^- or w^+.

Another method to derive ranks can be along the lines of the index based on difference between weighted distances from NIS and PIS (Doukas, Karakosta, & Psarras, 2010; Kuo, 2017). An index similar to $(w^- D_i^- - w^+ D_i^+)$, called Index-DKP hereafter, was proposed by Doukas, Karakosta, & Psarras (2010), and we will compare this index with $CC_{GFTOPSIS}$. In the method by Kuo (2017), $D_i^-$and D_i^+, are further normalised. In our proposed GFTOPSIS method, normalisation is already done in many steps such as preference input scale, criteria weights, and distance calculation. Here, preference input are IVIFS with all degrees between 0 and 1, and Weighted Euclidean distances means normalised distances since weights are already normalised and their sum is equal to 1. So we only compare GFTOPSIS ranking with the ranking by Index-DKP. The proposed $CC_{GFTOPSIS}$ may be written as follows:

$$CC_{GFTOPSIS\_i} = \frac{(D_i^-)^{w^-}}{\left(D_i^-\right)^{w^-} + \left(D_i^+\right)^{w^+}} = \frac{1}{1 + \left(\left(D_i^+\right)^{w^+} / \left(D_i^-\right)^{w^-}\right)}$$

It is easy to see that with equal $w^-$and w^+, proposed $CC_{GFTOPSIS}$ and CC_{TOPSIS} will be same, i.e. $w^- = 1, w^+ = 1 \rightarrow CC_{GFTOPSIS\_i} = \frac{1}{1+\left(D_i^+/D_i^-\right)} = CC_{TOPSIS\_i}$.

If $w^- = 1, w^+ = 0$, it means that the evaluator wants final ranking only based on D_i^-, which is rightly reflected in the formula as $CC_{GFTOPSIS\_i}$ is reduced to $\frac{1}{1+\left(1/D_i^-\right)}$, thus ranking will only depend on the order of D_i^-.

If $w^- = 0, w^+ = 1$, it means that the evaluator wants final ranking only based on D_i^+, which is rightly reflected in the formula as $CC_{GFTOPSIS\_i}$ is reduced to $\frac{1}{1+D_i^+}$, thus ranking will only depend on the reverse order of D_i^+.

The hypothetical case of $w^- = 0, w^+ = 0$ means that final ranking is not based on any of the distances, which is appropriately reflected since $CC_{GFTOPSIS\_i}$ is reduced to 0.5 for all alternatives resulting in the same ranks for all alternatives irrespective of distances from NIS or PIS.

For cases when weights are between 0 and 1: with decreasing weight w^-, the difference between two$(D_{i1}^-)^{w^-}$and$(D_{i2}^-)^{w^-}$will decrease and result in lesser differentiation based on D_i^-. In the extreme case, when $w^- = 0$, all $(D_i^-)^{w^-}$ would become 1, and have no role in differentiation of the $CC_{GFTOPSIS}$ or final ranking. Similarly, when $w^+ = 0$, all $(D_i^+)^{w^+}$ would be 1, and have no role in differentiation of the $CC_{GFTOPSIS}$ or final ranking. With the help of experiments, we have shown that in these two cases ($w^+ = 0$ or $w^+ = 0$), ranking of the alternatives using $CC_{GFTOPSIS}$ are same as ranking by Index-DKP, and for the case ($w^- = 1, w^+ = 1$), the ranking of the alternative by $CC_{GFTOPSIS}$ is same as CC_{TOPSIS}. For cases when weights are between 0 and 1, the results lie between the results of these two rankings methods.

A comparison of the results of the two ranking methods is summarised in Figure 2, for visual validation. Contour maps are shown for ranking, to visualise the effect of different distances from PIS and NIS. Alternatives which are on the same curve have the same rank. Rank or index/coefficient values improve on moving from lower left corner to right top corner, and a contour's colour changes from green to red. Comparing two calculation steps, one may conclude that proposed $CC_{GFTOPSIS}$ is similar to the ranking by Index-DKP. However, we must point out that there is a difference between the ranking by $CC_{GFTOPSIS}$ and Index-DKP. The $CC_{GFTOPSIS}$ results in more radial curves near PIS and NIS than the Index-DKP. In real situations, if an alternative has higher criteria ratings, an evaluator will focus more on its proximity from PIS; if an alternative has lower ratings on criteria, an evaluator will tend to see how far it is from NIS. For alternatives closer to PIS or NIS, same ranks due to $CC_{GFTOPSIS}$ are observed on a circular curve, rather than a relatively flat curve due to Index-DKP. Alternatives which are at near equal distances from PIS and NIS, lie on straight lines in solution space for both the methods. This suggests that GFTOPSIS method is more intuitive because when an alternative is near PIS, its ranking depends more on distance from PIS; while when an alternative is near NIS, its ranking depends more on NIS distance.

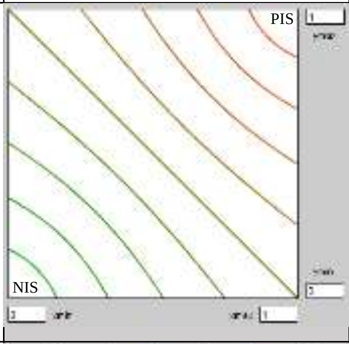
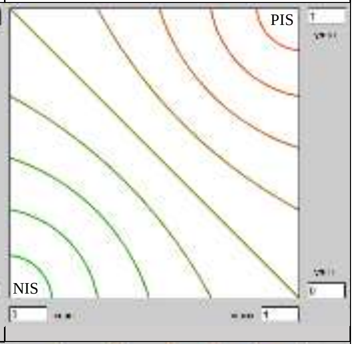
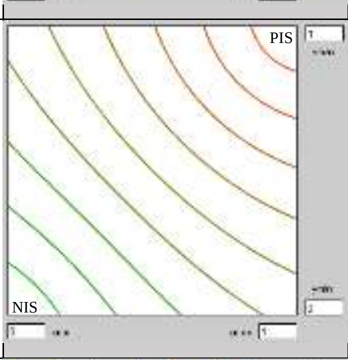
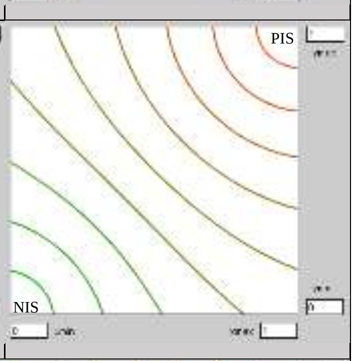
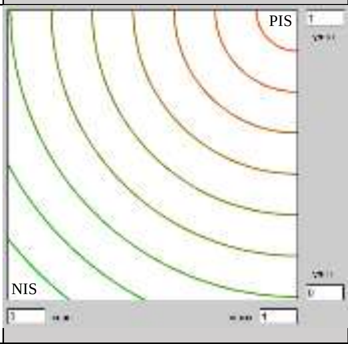
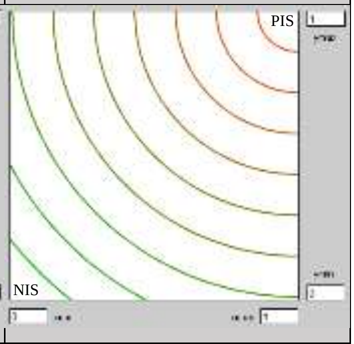
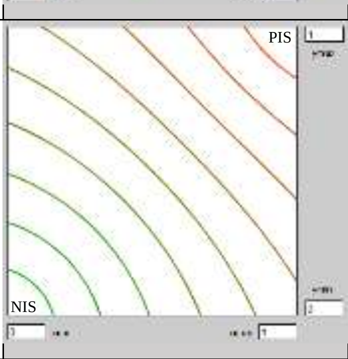
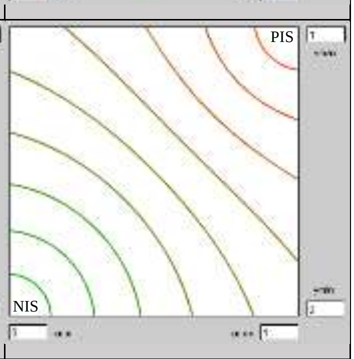
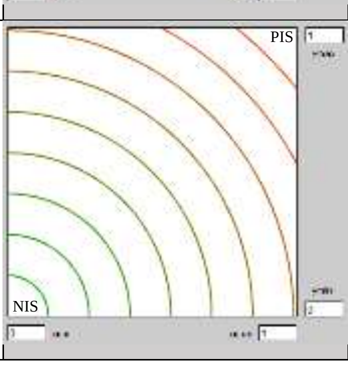
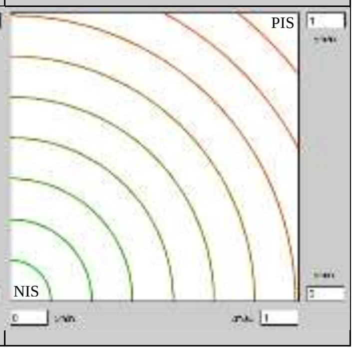
Distance Weights	$(w^- D_i^- - w^+ D_i^+)$	$(D_i^-)^{w^-} / \left((D_i^-)^{w^-} + (D_i^+)^{w^+}\right)$
$w^- = 1, w^+ = 1$		
$w^- = 0.5, w^+ = 1$		
$w^- = 0, w^+ = 1$		
$w^- = 1, w^+ = 0.5$		
$w^- = 1, w^+ = 0$		

Figure 2: Visual Comparison of Distance Weights and Ranks

We have also performed experiments to check the consistency rate of GFTOPSIS method. For this, we generated uniformly distributed random numbers for three criteria with PIS (1,1,1) and NIS (0,0,0). We generated 1000 points for the alternatives in this solution space. By generating alternatives in the solution space, after fixing the PIS and NIS, we are taking care of abnormality in the distance values from PIS and NIS. For example, values of (D_i^-, D_i^+) cannot be $(0.457, 0.504)$ and $(0.142, 0.143)$ within one experiment. Maximum ratio of two different values of $(D_i^- + D_i^+)$ can be $\sqrt{2} = 1.4142$ in two criteria decision and with equal criteria weights, where $\min(D_i^- + D_i^+)$ will be for an alternative on the diagonal connecting PIS and NIS, and $\max(D_i^- + D_i^+)$ will be for an alternative on any other extreme corner of the solution space. It can be mathematically shown that the maximum ratio of two $(D_i^- + D_i^+)$ values will converge to 1, with the increase in difference between criteria weights.

With the increase in number of criteria, assuming we have equal weights for all criteria, $\min(D_i^- + D_i^+)$ will be for an alternative on the diagonal connecting PIS and NIS, and $\max(D_i^- + D_i^+)$ will be for an alternative on one of the extreme corners of the solution space. Thus for n-dimensional hypercube space, $\min(D_i^- + D_i^+) = \sqrt{n}$ and $\max(D_i^- + D_i^+) = \sqrt{\lfloor n/2 \rfloor} + \sqrt{\lceil n/2 \rceil}$, where $\lfloor n/2 \rfloor$ is the rounded down integer value of $n/2$, and $\lceil n/2 \rceil$ is the rounded up integer value of $n/2$. Thus for even values of n, the maximum ratio of two different values of $(D_i^- + D_i^+)$ will be $\sqrt{2} = 1.4142$ in n criteria decision situations with equal criteria weights, while for odd values of n, the maximum ratio determined as above converges to $\sqrt{2}$.

If the weights of criteria are different, the shape of solution space will change from square to rectangle, and in extreme case when weight of one criterion is much higher than others, it will tend to result in uni-dimensional solution space, in which $\min(D_i^- + D_i^+)$ and $\max(D_i^- + D_i^+)$ will be equal and maximum ratio will become 1.

Because of this reason, we first generated alternatives in the solution space, and then calculated (D_i^-, D_i^+), to further calculate different coefficients for final ranking, as opposed to the experiments in (Kuo, 2017).

The experiment and visual comparison of methods lead to following key points:

- Proposed GFTOPSIS method incorporates the two distance weights from NIS and PIS.
- $CC_{GFTOPSIS}$ closeness coefficient is more intuitive than Index-DKP.
- When $w^- = w^+ = 1$, ranking based on $CC_{GFTOPSIS}$ is same as ranking based on CC_{TOPSIS}.

- When w^{-} or $w^{+} = 0$, ranking based on $CC_{GFTOPSIS}$ is same as ranking based on Index-DKP.
- For other cases $0 < w^{-} < 1$ and $0 < w^{+} < 1$, ranking based on $CC_{GFTOPSIS}$ is balanced between rankings based on CC_{TOPSIS} and Index-DKP.

5. Application Examples

We have explored applications of the GFTOPIS approach in solving real life problems. In a practical situation, managers or technologists are often comfortable in expressing the preference rating of alternatives on chosen criteria in different forms. For example, supplier or technology selection decisions require more information, and an individual may have both positive inclination as well as negative inclination for a particular alternative, so IVIFS is most appropriate for this type of application. On the other hand, preference ratings in AM material selection decisions are more appropriately expressed in IVFS. Two examples from these situations are used to demonstrate the applicability and to validate the correctness of results when different types of inputs are used in GFTOPSIS. Next, we elaborate on these examples, one on technology supplier selection using IVIFS preference rating, and one on material selection using IVFS, based on experimental input data. Several types of sensitivity analyses are also demonstrated with each example to assist the decision-maker to make more informed choices.

Input and Data Set

Basic data for technology processes and material data set is taken for reference from material safety data sheet available online from one of the leading material providers in the AM sector, and an industry report (“Material Safety Data Sheets | Stratasys”; Wohlers, 2016). Input from industry experts is considered in selecting properties for evaluation criteria. Finally, weights for individual criteria and performance matrix are prepared based on input from experts, in order to begin the calculation process and subsequently arrive at ranked candidates based on weighted criteria. Final results are also verified by the experts and empirically tested to validate generalisation of the model.

Technology Supplier Selection Example

A numerical example for technology selection is shown here to clarify and demonstrate the method proposed in the above section. We take evaluators’ input in the form of IVIFS performance matrix shown in Table 1. Four technology alternatives (X1 to X4) are eligible for evaluation against five criteria (accuracy, finish, strength, cost, and build time) by two evaluators. For each alternative, the columns named a, b, c, and d present degrees of lower membership, upper membership, lower non-membership, and upper non-membership respectively. For simplicity, we consider different subjective weights of criteria from the two evaluators shown under the columns titled A1 and A2, but the same input for IVIFS performance matrix. Weights (γ_k) assigned to the first and the second evaluators are 2/3 and 1/3 respectively. Both the evaluators have expressed that they have normal degree of optimism.

Based on DOpt $\lambda = 0.5$, expected IFS performance matrix is derived as shown in Table 2. For each alternative, expected degrees of membership, non-membership and uncertainty are presented under columns titled μ, ν and π respectively in Table 2. Derived criteria weights and ideal solutions are presented in Table 3. Normalised subjective weights assigned by the two evaluators are presented in columns titled α^1 and α^2 in Table 3. Entropy values and intuitionistic fuzzy entropy weights are shown in columns titled E and β respectively. Final weights of criteria are derived by combining subjective weights and entropy weights, are shown under columns titled w^1 and w^2 respectively. Degrees of membership, non-membership and uncertainty for PIS and NIS values are shown in columns titled μ, ν and π respectively in Table 3. Distances from PIS and NIS for the two evaluators, combined distances, CC value, and rank for each alternative are provided in Table 4.

Table 1: IVIFS performance matrix

Alternative	X_1				X_2				X_3				X_4					
Criterion	a	b	c	d	a	b	c	d	a	b	c	d	a	b	c	D	A1	A2
Accuracy	0.30	0.45	0.40	0.45	0.30	0.70	0.20	0.25	0.60	0.65	0.25	0.30	0.30	0.40	0.50	0.55	8	10
Finish	0.50	0.65	0.25	0.30	0.45	0.50	0.40	0.45	0.05	0.05	0.80	0.90	0.20	0.25	0.65	0.70	5	8
Strength	0.85	0.90	0.00	0.05	0.30	0.35	0.50	0.60	0.15	0.25	0.65	0.75	0.70	0.85	0.05	0.10	2	5
Cost	0.70	0.80	0.15	0.30	0.40	0.55	0.30	0.40	0.60	0.70	0.10	0.25	0.70	0.75	0.10	0.25	5	5
Build Time	0.15	0.30	0.55	0.60	0.60	0.70	0.25	0.25	0.35	0.40	0.45	0.55	0.85	0.85	0.05	0.10	9	6

Table 2: Expected IFS performance matrix

	X_1			**X_2**			**X_3**			**X_4**		
	μ	ν	π	μ	ν	π	μ	ν	π	μ	N	π
Accuracy	0.375	0.425	0.200	0.500	0.225	0.275	0.625	0.275	0.100	0.350	0.525	0.125
Finish	0.575	0.275	0.150	0.475	0.425	0.100	0.050	0.850	0.100	0.225	0.675	0.100
Strength	0.875	0.025	0.100	0.325	0.550	0.125	0.200	0.700	0.100	0.775	0.075	0.150
Cost	0.750	0.225	0.025	0.475	0.350	0.175	0.650	0.175	0.175	0.725	0.175	0.100
Build Time	0.225	0.575	0.200	0.650	0.250	0.100	0.375	0.500	0.125	0.850	0.075	0.075

Table 3: Criteria Weights, PIS, NIS

	Criteria Weights						**PIS**			**NIS**		
Criteria	α^1	α^2	E	β	w^1	w^2	μ	ν	π	μ	ν	π
Accuracy	0.276	0.294	0.700	0.190	0.263	0.294	0.625	0.225	0.150	0.350	0.525	0.125
Finish	0.172	0.235	0.450	0.204	0.177	0.235	0.575	0.275	0.150	0.050	0.850	0.100
Strength	0.069	0.147	0.475	0.203	0.070	0.147	0.875	0.025	0.100	0.200	0.700	0.100
Cost	0.172	0.147	0.475	0.203	0.176	0.147	0.750	0.175	0.075	0.475	0.350	0.175
Build Time	0.310	0.176	0.500	0.201	0.314	0.176	0.850	0.075	0.075	0.225	0.575	0.200

Table 4: PIS and NIS distances and CC

	d^{1+}	d^{1-}	d^{2+}	d^{2-}	D+	D-	CC	Rank
X_1	0.190	0.119	0.121	0.173	0.167	0.137	0.450	3
X_2	0.091	0.158	0.106	0.145	0.096	0.154	0.615	1
X_3	0.180	0.086	0.187	0.082	0.182	0.085	0.317	4
X_4	0.101	0.191	0.123	0.147	0.108	0.176	0.619	2

Sensitivity Analysis

Sensitivity analysis for final ranking may be performed by changing the value of DOpt. The CC value is also based on the value of DOpt. If an evaluator is optimistic, it suggests that expected IFS should be towards the higher level of IVIFS, and value of λ would be 0.5 to 1. If the evaluator is pessimistic for the ratings, s/he will assign the value of λ between 0 and 0.5, and if s/he is normal in ratings, $\lambda =$ 0.5 would be chosen. Based on the sensitivity analysis, the decision-maker may choose the value of λ, and select the best alternative. In this example, for $\lambda \geq 0.5$ candidate X_2 is ranked one, while for $\lambda <$ 0.5 candidate X_4 is top ranked.

Sensitivity analysis assists in quantifying the effect of different types of fuzzy numbers. We have taken input in terms of higher and lower degrees of memberships and non-memberships. If input preference is towards the higher limit, for example in the form of a right triangular fuzzy number, one may select the best alternative with a higher value of λ (≥ 0.5). If one alternative clearly dominates in the sensitivity analysis, this means that evaluators need not be concerned about the input fuzzy number format or DOpt level.

Further sensitivity analysis is performed to check the effect of criteria weights on ranking. This analysis is done for the example by doubling each criterion weight. Thus, if a manager still thinks that strength, cost, and build time are more important than the other two criteria, s/he will select candidate X_4 even if it is slightly less in the final ranking in Table 4. We elaborate more on criteria weight sensitivity analysis in the next example.

Similar to the above sensitivity analyses, the decision-maker may change the distance weights $(W+, W-)$ and evaluators' weights (γ_1, γ_2), to check their effect and select the best alternative (Table 5). The two highlighted columns in bold-italic fonts are those for which the previous calculations had been performed.

Table 5: Sensitivity Analysis for distance weights and evaluators' weights

	Rank with Distance Weights (W+,W-)					Rank with Evaluators Weights (γ_1, γ_2)			
	(1,0)	(1,0.5)	***(1,1)***	(0.5,1)	(0,1)	(1,0)	***(2/3,1/3)***	(1/3,2/3)	(0,1)
X_1	3	3	***3***	3	3	3	***3***	3	1
X_2	1	1	***1***	2	2	2	***1***	1	2
X_3	4	4	***4***	4	4	4	***4***	4	4
X_4	2	2	***2***	1	1	1	***2***	2	3

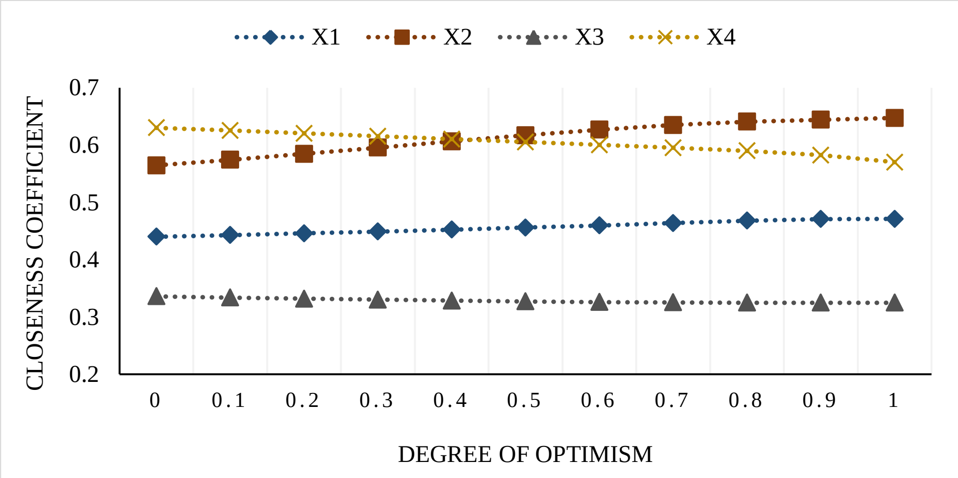


Figure 3: Sensitivity Analysis DOpt Technology Selection

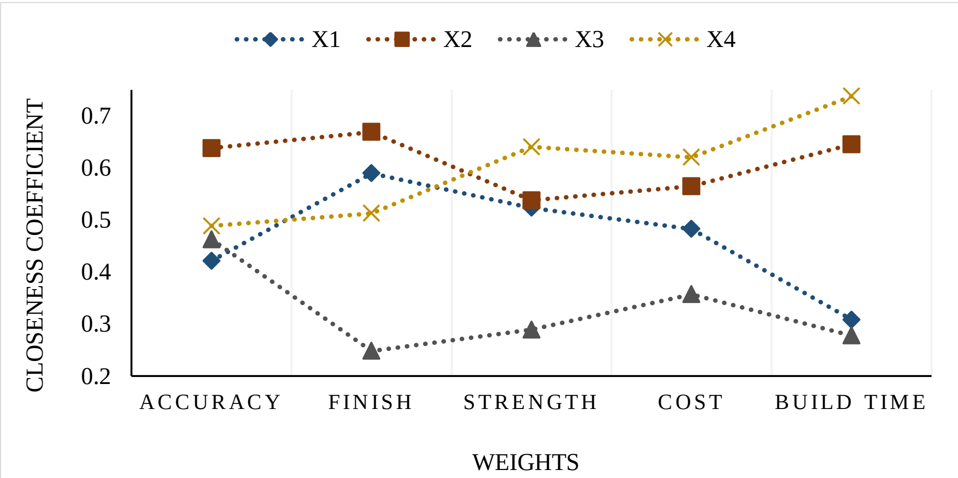


Figure 4: Sensitivity Analysis Criteria Weights in Technology Selection

Material Selection Example

As discussed earlier, growth in AM application is based on recent developments in the materials used in AM. However, there may still be some uncertainty involved in the case of new and possibly expensive materials, which could hinder the widespread use of such AM technologies. Here, we present the AM material selection example, with consideration of cost, emissions and other performance factors which are required in the final part being produced. We have taken four candidate materials for performance evaluation on seven criteria. As per discussion with industry experts, it was found that rating of new material can be most explicitly explained in the interval values. Therefore, the input is taken in the form of IVFS. From the datasheet of AM materials available on material suppliers web sources, material properties are also given in the interval form.

Input																	
Materials	PP				ABS				HDPE				Nylon				Weights
Criteria	a	b	c = 1-b	d = 1-a	a	b	c	d	a	b	c	d	a	b	c	d	Subjective
Dimension stability	0.8	0.9	0.1	0.2	0.9	1	0	0.1	0.6	0.8	0.2	0.4	0.9	1	0	0.1	9
Appearance	0.7	0.8	0.2	0.3	0.8	0.9	0.1	0.2	0.6	0.7	0.3	0.4	0.4	0.5	0.5	0.6	6
Tensile strength	0.7	0.8	0.2	0.3	0.8	0.9	0.1	0.2	0.5	0.6	0.4	0.5	0.9	1	0	0.1	5
Impact	0.5	0.6	0.4	0.5	0.7	0.8	0.2	0.3	0.4	0.6	0.4	0.6	0.8	0.9	0.1	0.2	5
Recyclability	0.7	0.8	0.2	0.3	0.3	0.5	0.5	0.7	0.8	0.9	0.1	0.2	0	0.1	0.9	1	5
Energy	0.4	0.6	0.4	0.6	0.4	0.5	0.5	0.6	0.7	0.9	0.1	0.3	0.4	0.3	0.7	0.6	5
Cost	0.7	0.9	0.1	0.3	0.2	0.5	0.5	0.8	0.6	0.8	0.2	0.4	0.1	0.2	0.8	0.9	10
Output																	
	D+	D-	CC	Rank	D+	D-	CC	Rank	D+	D-	CC	Rank	D+	D-	CC	Rank	
	0.065	0.181	0.735	1	0.132	0.100	0.432	3	0.072	0.185	0.719	2	0.202	0.066	0.245	4	

Table 6: IVFS GFTOPSIS Material Selection Example

Input data, in the form of IVFS and the subjective weight of each criterion, is shown under *Input* heading in Table (6). The calculations steps have not been described here. DOpt may be changed to check the sensitivity of the model, as illustrated in the previous technology selection example. Here, sensitivity analysis is presented by considering doubled weight for each criterion weight, to demonstrate the effect on a particular criterion on CC or rank.

Sensitivity Analysis

Here we describe only criteria weight-based sensitivity analysis since other sensitivity analyses are similar to those in the previous example and have been already explained. In this example, seven criteria are used to derive the final ranking of the candidate materials. These criteria are assigned weights, based on the subjective inputs from the experts. To analyse the effect of weights on CC or rank, we increase the value of one weight at a time. When focus is on one particular criterion, we have doubled its weight and then normalised the weights of all criteria for further calculations. Thus, a decision-maker may analyse the effect of a particular criterion on the ranking of alternatives. For example, if the management focus is more on the environmental effects, then recyclability and energy criteria would be more important, and HDPE material is likely to be selected. However, with the economic focus based on cost criterion, PP is the most appropriate choice.

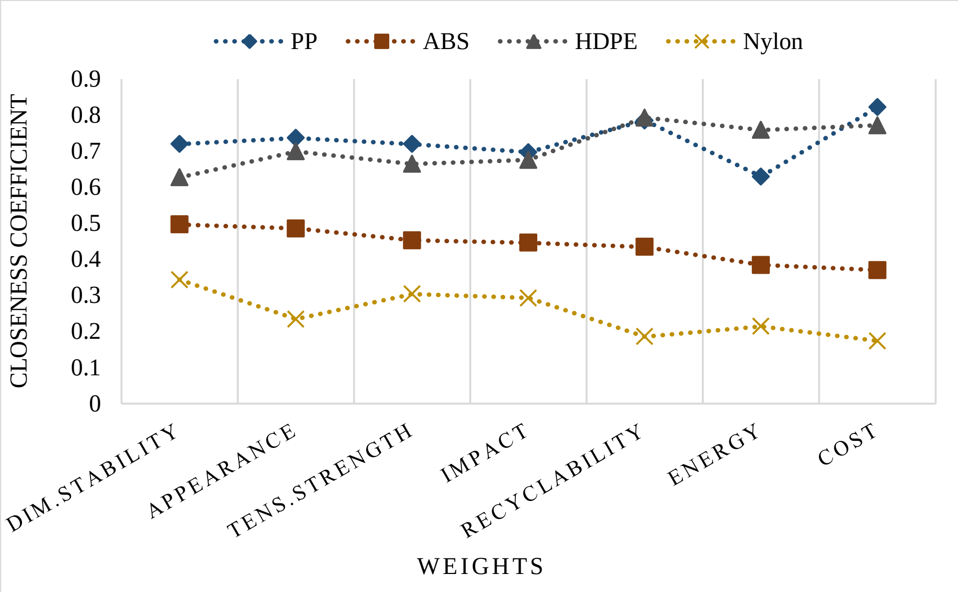


Figure 5: Sensitivity Analysis Criteria Weights in Material Selection

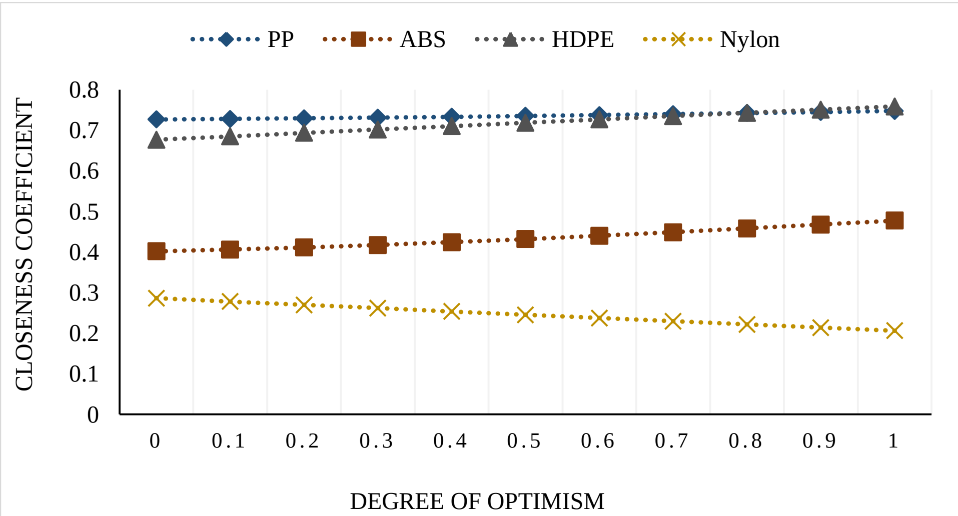


Figure 6: Sensitivity Analysis DOpt in Material Selection

6. Discussion and Conclusion

The study, to the best of the authors' knowledge, is a novel generalisable model to select the best alternative in an uncertain environment. Additionally, we have shown that IFS, IVFS, FS are special cases of the proposed method based on IVIFS. We have demonstrated how to use different preference numbers with two practical examples of AM: technology and material selections. We have combined the uncertainty entropy weights with subjective weights, to present an intelligent and flexible model. The study further introduces the degree of optimism, which gives a choice to the evaluator to check performance sensitivity and ascertain its effect on ranking. This study, therefore, addresses the criticism by Opricovic & Tzeng (2004), that TOPSIS method does not take care of different weights of NIS and PIS distances, by extending the calculation of closeness coefficient to incorporate the different distance weights.

The application of GFTOPSIS method is demonstrated for AM technology selection and material selection. AM is an emerging technology which has attracted the attention of researchers and practitioners, for finding solutions to problems arising in newer and potential applications of AM in manufacturing and supply chain management. The increase in the usage of AM has been facilitated by the development of the technology and input materials. Business forecasts place AM as one of the potentially disruptive technologies, and it is becoming evident that manufacturing and supply chains of the future will be changed due to ongoing developments in the AM technology with different materials and newer applications. Technology selection and material selection are major decisions involved in using AM.

The potential business implications of AM, and uncertainties involved in the newer materials and technologies, motivated us to devise an appropriate and comprehensive GFTOPSIS method to assist in the technology selection and the material selection decision processes. In both the application examples, different types of input preference numbers based on experts' preferences are applied. This demonstrates the generalisability of the proposed GFTOPSIS method, which makes it useful for practical situations. We have also used subjective weights of criteria and DOpt, so that a user may analyse the changes in these two parameters and their effect on the ranking of alternatives. The flexibility in GFTOPSIS model is shown through the application of sensitivity analyses. This flexibility makes GFTOPSIS a suitable decision support tool, where the user is the main decision-maker with more informed choices. The model incorporates a degree of intelligence through the use of inputs in fuzzy format. Additionally, it automatically uses the entropy or uncertainty based criteria weights. Thus, even if an evaluator does not know the weights of criteria in an uncertain environment, the model is intelligent enough to assign criteria weights, based on the variability in the preference ratings.

Based on the proposed GFTOPSIS selection model, a user may work in different situations without having to invest in multiple decision support systems. Flexibility, in terms of subjective weights and sensitivity based on the degree of optimism and weights, assists managers in verifying the robustness of candidates in case of interval data and the impact of criteria weights in ranking and making more informed decision. For example, if one alternative is ranked best when high weight is assigned to cost, and another alternative is preferred when high weight is assigned to emissions, then the final choice may be the alternative which aligns closely with the overall economic/ environmental focus of the organisation as a whole.

The proposed method can be used in numerous ways to assist different evaluation processes, for example in project selection, R&D investments, outsourcing decision, recruitment, employee performance appraisals, and benchmarking of firms' performance across the industry. Researchers and practitioners may apply the proposed method with an appropriate input form, to evaluate candidates in new decision situations. However, decision-makers still need to choose which type of input preference numbers to use in different scenarios. The future scope of the study includes the application of other MCDM methods on similar lines, in order to assist decision-makers in various situations.

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