ZDM Mathematics Education
DOI 10.1007/s11858-017-0837-9

COMMENTARY PAPER

Mathematical creativity and giftedness: perspectives in response

Gerald A. Goldin[1]

Accepted: 9 January 2017

Abstract The investigations described in the ten fascinating research studies contained in the current volume of *ZDM Mathematics Education* evoke some perspectives in response. I consider the articles thematically in relation to a suggested set of important or pressing questions about creativity and giftedness in mathematics education, grouped into four areas: (1) definitions and meanings, (2) learning environments, (3) teacher preparation, (4) educational policies and societal trends. The first three of these areas are addressed most specifically; the latter is given less attention, with much remaining to be researched. In the context of the topics most addressed, possible emphases in studying mathematical creativity and giftedness include cognitive, affective, conative (motivational), social, and behavioral aspects. The studies here focus mostly on the cognitive, social, and behavioral dimensions. I highlight some specific findings, offer comments, and suggest the need for greater future emphasis on affective and motivational variables. My response concludes with consideration of the value of prioritizing mathematical creativity and giftedness (or high ability), noting some troubling policy directions and societal trends that should be taken into serious account and studied.

1 Introduction

It is of the greatest importance that mathematics educators, together with those who set educational policies, prioritize the development of high ability, giftedness, and mathematical creativity. This issue of *ZDM Mathematics Education* presents ten evocative, in-depth research studies, most of them highly detailed, addressing a set of closely-related questions. The articles invite a broad, coherent response.

The comments that follow are organized around four broad areas of key research questions pertaining to mathematical creativity and giftedness:

- What do we mean by creativity and giftedness (or high ability) in mathematics?
- What learning environments or activities foster mathematical creativity and high ability?
- How should teacher education contribute to encouraging mathematically creative activity and nurturing giftedness?
- How do educational policies foster or inhibit mathematical creativity and the development of high-ability students, and how do societal trends or forces contribute toward or impede these goals?

In fact, I came to the task of responding to the articles in this volume having these areas of inquiry already in mind, as the most fundamental and/or most pressing in the field.

Let us consider the studies in the current volume in relation to each of these in turn. Sections 2, 3, and 4 elaborate on the first three areas of needed research, posing more specific questions and considering how the studies in the current volume address some of those questions. Discussions and findings are highlighted area by area, and comments are offered in response.

Section 5 addresses the fourth area, raising some points in connection with the *value* of prioritizing creativity and giftedness in mathematics education. This is a value I share with all of the present authors, but which is far from

✉ Gerald A. Goldin
geraldgoldin@dimacs.rutgers.edu

[1] Rutgers University, New Brunswick, NJ, USA

Published online: 22 February 2017

universally held. Section 6 offers some brief concluding remarks.

Besides commenting on findings in the articles, I shall point to some important issues and themes that are not discussed. The reader should understand that this is not intended as criticism—no article or collection of articles can possibly take everything into account. Rather it is a way to map out the landscape of overarching topics requiring attention, and to situate the research presented in this special issue within that landscape.

2 What are creativity and giftedness in mathematics?

2.1 Discussion questions

Evidently mathematical creativity and mathematical giftedness are complex constructs, involving many interacting dimensions. This area of investigation receives attention throughout the current volume. I take the following to be some of the most fundamental questions.

As a psychological process in context, how do we characterize the *cognitive, affective*, and motivational or *conative* aspects of mathematical creativity (or, respectively, of general creativity)? What are its *social* dimensions? What are its *behavioral* manifestations? In particular, should the evaluation of mathematical activity as "creative" (related evaluative terms might be "original" or "inventive") be based on psychological criteria or on social criteria—recognizing that these, of course, are not independent. That is, what does mathematically creative activity look like when it is occurring? And how can we characterize the *products* of mathematically creative activity: i.e., the "newness" of ideas, representations, patterns, structures, problems, problem solutions, conjectures, proofs, etc.? What makes such a product "new," and by what criteria should it judged as creative?

Should definitions distinguish the *trait* of mathematical (or general) creativity, a possibly long-term, relatively stable characteristic of the individuals, from the "in the moment" psychological and/or social *state* of creative activity, which may occur from time to time?

Likewise, what are the cognitive, affective, conative, social, and behavioral aspects of mathematical giftedness? How do we recognize them? How does "giftedness" relate to "high mathematical ability"—does the former refer to an innate trait, and the latter to one that can be developed through instruction?

How do creativity, giftedness, and high ability relate, respectively, to standard measures of high mathematical achievement in school?

2.2 Investigations and responses

Almost all the articles in the present volume include some characterization of creativity from a theoretical standpoint. The seminal work of Torrance (1974) assessing general creativity is cited repeatedly [see Cramond et al. (2005) for follow-up], together with a number of subsequent approaches in the mathematical domain. Noting that the field has reached only partial consensus, the authors of each article tend to adopt the perspective that seems to best further the research at hand. Then the qualitative empirical data help to illuminate the issues. Let us consider some representative examples.

Hershkowitz et al. (2017) point to creativity in school as "a relative phenomenon. In other words, students' ideas will be considered creative on the basis of their contribution to the mathematical knowledge of the class or the peer group …" They quote Leikin and Pitta-Pantazi (2013, p. 161): "Creative ideas are those that are considered by the reference social group as new and meaningful in a particular field." These ideas contribute to what Hershkowitz et al. call a "socio-cognitive" approach to examining creative reasoning in the course of classroom argumentation. The authors also make important use of a distinction described by earlier researchers (Lithner 2008; Granberg and Olsson 2015), between "imitative" reasoning and "creative" reasoning in mathematics—the former involving memorized or algorithmic steps, the latter involving a sequence of steps new to or re-created by the learner, plausible, and having foundation in relevant mathematical properties. These ideas enable them to investigate in great detail the complex shifts of knowledge that can occur during classroom discussions of challenging mathematical problems (see Sect. 3 below).

I would remark here that the feature of *newness to the learner* is fundamentally independent of whether or not the reasoning is *new to the group*. This distinction represents an important conceptual fork in the road. What I would call "inventive" mathematical activity (Goldin, 2009)—i.e., new to the learner—may take place in the context of immediate social interactions, or it may occur in relative isolation; and the social context may range from welcoming the individual's creation to squelching it. For example, a dismissive response by others, "We knew that already," may serve to inhibit the creative development of an idea new to the student when it is already familiar to some in his or her peer group. Alternatively, an idea already well-known to the learner may be quite new to others, and influence their subsequent thinking; should the term "creative" then apply?

Hershkowitz et al. also note the importance of the time frame over which phenomena occur. Creative reasoning in Lithner's sense facilitates the study of immediate, or "in the moment," acts of creation and the resulting knowledge shifts. Constructs such as "abstraction in context" resulting in knowledge new to the learner (Dreyfus et al. 2015; Hershkowitz et al. 2001), and "documenting collective activity" describing new normative activity in the group (Rasmussen and Stephan 2008) pertain to somewhat longer time frames.

The framework of Singer et al. (2017) for studying creativity focuses more exclusively on its cognitive aspects—specifically, "cognitive flexibility" which is seen as a composite of three dimensions to variable extents: "cognitive variety, cognitive novelty, and changes in cognitive framing." Newness is an important element in defining these components, as they are interpreted in evaluating students' behavior in a problem-posing context—e.g., posing a variety of new problems, or changing to a different mental frame in discovering new problems.

This strong focus by Singer et al. on the cognitive dimension of mathematical creativity contrasts with Hershkowitz et al.'s correspondingly strong focus on its social dimension. The cognitive emphasis is natural for Singer et al.'s study, as the authors' goal is to study creativity in relation to prospective teachers' cognitive styles in a problem-posing context (see Sect. 4 below). At the same time, Singer et al.'s framework is based on organizational theory, and thus it contains (albeit implicitly) a sociological perspective. The authors further note that the problems posed reflect not only the individual's cognition, but his or her attitude toward the mathematical content, and thus may provide information about affect as well as personality traits.

Tabach and Friedlander (2017) remark that "The essence and measurement of creative mathematical thinking are not defined in a generally accepted way." They build on a proposal of Leikin and Lev (2007) associating students' creativity with their problem-solving performance through the "originality, fluency, and flexibility expressed in solving a problem," and operationalize these characteristics by analyzing students' productions in the context of non-routine classroom activities involving equivalent expressions in early algebra (see Sect. 3 below). Thus their characterization of mathematical creativity is essentially cognitive. As in the article by Singer et al. (2017), the cognition is inferred from the work produced by the students in the study; but greater emphasis is placed by Tabach and Friedlander on the *product*. In fact, they note that "Sternberg and Lubart (1996) *define* [my emphasis added] creativity as an ability to produce unexpected, original, appropriate and useful pieces of work."

Other articles in the present volume employ variants of such characterizations of mathematical creativity, or elaborate on them. Hoth et al. (2017) quote Mann (2006, p. 243), "[creativity] entails incorporating experiences and conceptual understanding to solving authentic mathematical problems." They note several sub-abilities pertaining to problem-solving heuristics, such as formulating mathematical hypotheses and identifying patterns. Mhlolo (2017) also highlights the absence of consensus on definitions, and points to the trajectory of Kaufman and Beghetto (2009) from "mini-c" (referring to initial creative interpretations, accessible to many students) to "BIG-C" (referring to extraordinary professional-level creative accomplishment).

Taken as a whole, the ten studies provide valuable detail about cognitive, social, and behavioral aspects of giftedness and high ability. This is made possible by the *explicit incorporation of those dimensions* into theoretical frameworks for creativity adopted by the authors at the start. The appropriateness and adequacy of the frameworks that are adopted are borne out by the richness of interpretation within those frameworks of the qualitative empirical data.

What strikes me most vividly, however, is the relative absence of exploration of the affective dimension or the conative (motivational) dimension of creative mathematical activity, beyond occasional mention. Of interest here are the emotions experienced before, during, and after creative or inventive mathematical acts, the beliefs and attitudes of the individual that foster commitment, determination to succeed, or deep, intimate "in the moment" engagement with mathematical ideas, the shared affect of the social community within which creative activity is taking place, the aesthetic dimension of mathematical creativity, and so forth.

I want to suggest that if mathematical creativity itself is characterized from the outset as exclusively cognitive and/or social, with behavioral manifestations, then we will not sufficiently attend to its affective and conative aspects as we study it—and we shall lose important insights into the dynamics of creative processes. As educators, furthermore, it is essential to know how to encourage and motivate students to explore mathematics creatively and to develop their own mathematically creative or inventive processes, both in the immediate present and longer term. This would seem to suggest the central importance of understanding the affective and the conative in creative mathematical activity. I would therefore recommend *incorporating these dimensions from the outset* into the theoretical frameworks for discussing creativity.

With respect to the characterization of mathematical giftedness, Leikin et al. (2017) highlight the tension between different phenomena (giftedness, high ability, talent), noting, "The distinction between the constructs of mathematical giftedness and high mathematical ability is rooted in the debate between static and dynamic perspectives on reaching high achievements in mathematics (Leikin 2014)." They point to the construct of "mathematical promise," which incorporates not only mathematical ability, but motivation, self-efficacy beliefs, and learning opportunities. Motivation, in particular, is seen as essential to the development of mathematical ability. They further remark on the affective dimension of motivation, including its relation to emotional feelings of curiosity, excitement, courage, and joy. And they note, "the lack of a broadly accepted theoretical definition of mathematical giftedness is not a barrier to opening special schools, classes, and programs for supporting and nurturing 'mathematically promising' students." The explicit attention to motivation as a variable distinct from giftedness, but related to it, opens the door to the research direction they pursue (see Sect. 3).

Mhlolo (2017), following Gagné (2015), considers giftedness a matter of degree: students range from mildly and moderately gifted, to exceptionally and extremely gifted. The "mildly gifted" are taken to be, at least roughly, the highest-achieving 10% of a typical school mathematics class. In the view of Gagné, this population is or should be the main focus of "gifted and talented" programs, and it is the focus of Mhlolo's study.

It is very important that in Leikin et al.'s discussion, concepts such as "high ability" and "talent" (which can be influenced or developed greatly by education) and "mathematical promise" (a composite) feature prominently in the discussion of "giftedness." I am pleased that the dimensions of motivation and affect enter explicitly into "mathematical promise," but I would also like these to be considered from the outset as important components of both "giftedness" and "high ability."

It is not clear how "general giftedness," as defined by early IQ, fits into the picture. Mathematical giftedness—particularly when it occurs in the exceptional degrees mentioned by Mhlolo—is, like giftedness in fields such as music, art, creative writing, or chess—rather domain-specific. Theories of "multiple intelligences" (Gardner 1983) have advanced our understanding of distinct components of giftedness. The use of "general giftedness" as a population variable in Leikin et al.'s study seems to have been a practical choice, reflecting the way that school assignments are made in Israel, but may otherwise be of limited significance in reaching research-based conclusions.

In the present volume, Sheffield (2017) discusses the limitations of "general giftedness" from several perspectives, and notes that, "Even though research has repeatedly shown that a score on a single IQ test is not a reliable indicator of future student performance, it continues to be used to place students in gifted programs in many schools in the United States."

3 What learning environments foster mathematical creativity and high ability?

3.1 Discussion questions

What school or related environments, and what learning activities, foster creativity in mathematics and/or the mathematical development of gifted students? There are several dimensions to this question, running in parallel with the questions about the nature of creativity and giftedness:

3.1.1 Cognitive

What are optimal choices of curriculum content? What mathematical concepts, in what sequence, should be offered? What activities evoke creative mathematical behavior, and what learning pathways can be identified or associated with those activities? How can mathematical abilities commonly associated with "giftedness" be developed through teaching? How can classroom activity be tailored toward the different ability levels and distinct learning styles of students, so as to accommodate the highest-ability students and develop higher levels of ability in other students?

3.1.2 Social

What social environments develop mathematical ability, or encourage creative activity? What sociocultural classroom norms are effective or influential, and how can teachers establish these? What "in the moment" social dynamics are evocative of creativity in mathematics classrooms, and how does this happen? What are the influences of relationships with an important teacher or mentor?

3.1.3 Affective

What individual and shared affect, including emotions, attitudes, beliefs, and values, are associated with creative mathematical activity? How do longer-term emotional traits and affective structures, including powerful meta-affect (DeBellis and Goldin 2007; Goldin 2014), develop in gifted students as they pursue their mathematical education?

3.1.4 Conative

What kinds of incentives or motivational strategies (e.g., Middleton and Jansen 2011) encourage engagement in creative mathematical activity? What motivates high-ability students to develop their ability further, and to pursue mathematical study?

3.2 Investigations and responses

A valuable contribution of the current volume is the wealth of detail provided in the several qualitative studies focusing on creative mathematical activity of students in classroom contexts. The findings of these studies are necessarily specific to the contexts, the mathematical activities, and the teachers. Nevertheless, the rich descriptions of the complex social and psychological phenomena involved, and the measures developed in observing specific phenomena, provide us with a great resource.

For example, Hershkowitz et al. (2017) describe instances of how innovative ideas function to resolve earlier contradictions in the mathematical discussion, and how the creative idea of an individual serves as a turning point for the whole class. Fundamental to their analysis is the interesting notion of "knowledge agents" and "followers"—the latter influenced by or responding to ideas expressed by the former. As they trace the role of creative reasoning in the "knowledge shifts" that occur, they demonstrate vividly the rich social dynamic surrounding mathematical creativity in classroom contexts. This is a ground-breaking type of analysis, examining the interplay between the cognitive and the social dimensions of mathematical creativity as it plays out "in the moment." It deserves to be carried forward vigorously.

In contexts that include both algebraic and geometric explorations, with many episodes and exchanges described in rich detail, Leikin et al. (2017) analyze and classify the kinds of questions asked in two classes: a 10th-grade class for "generally gifted" students (having early high IQ) choosing to study advanced mathematics, and a 9th-grade class for highly motivated students, expressing the desire to devote additional time to studying mathematics. They find that in the discourse of the former group, elaboration questions stand out as distinctive, stemming from curiosity, the desire for sense-making, and reflecting the students' persistence. In the latter group, the predominant questions were clarification questions, which did less to raise the mathematical level of discussion.

Leikin et al.'s analysis of the different types of student questions provides an excellent window into the flow of mathematical ideas, and into the cultural norms pertaining to creative mathematical activity that had been established in each class. In their discussion, the authors highlight the role of curiosity as well as mathematical insight as influences on the kinds of questions asked—an observation with which I strongly agree. However I find myself skeptical of the attribution of greater "innate curiosity" to the students classified as generally gifted. More generally, I would question the attribution of the observed differences between the classes to the population selection characteristics. This was not a causal study, and many other relevant variables are clearly involved—including especially the different roles assumed by the teachers in each class (as the authors themselves remark), and the differing educational histories of the students in the two groups.

The many specific activities to foster creativity which are suggested and studied in this volume are likewise of real and lasting value—especially because we have richly detailed descriptions of the consequences of such activities, the way that students engage with them, and some of the events which ensue. These include the "Take-a-Quiz" and "Make-a-Quiz" activities involving equivalent algebraic expressions studied by Tabach and Friedlander (2017), where "originality" and "awareness of error" scores (as surrogates for creativity) are defined and applied to evaluate students' productions. They find the majority of students displaying some degree of originality as well as error awareness, pointing to the value of such activities in encouraging creative mathematical thinking.

A fascinating discussion is that of Sriraman and Dickman (2017). They develop the connection between creativity and "mathematical pathologies" (including, for instance, counterintuitive properties of mathematical examples and counterexamples), suggesting that consideration of such pathologies can generate creative activity by challenging preexisting perceptions and intuitions, motivating flexibility in mathematical definitions. The authors' characterization of pathologies is rather broad, including examples of partial patterns, misconceptions, and fallacies accessible to younger children, as well as historical counterintuitive discoveries ranging from irrational lengths (in the time of Pythagoras) to everywhere continuous but nowhere differentiable functions (in the time of Weierstrass). Their discussion of the "Lakatosian heuristic" suggests an interesting way to model some of the dynamics of the cognition underlying mathematical creativity—building on the now-classic tradition of Hadamard (1945) and Polya (1954, 1962, 1965).

The article by Nolte and Pamperien (2017) considers two challenging, exploratory mathematics problems developed for use with primary-school gifted students, of a kind that fall in the broad domain of discrete mathematics (e.g., DeBellis and Rosenstein 2004). The authors describe the consequences of using these problems in regular classes for average-ability students, comparing processes and results with a class of gifted students under similar conditions. They map out "patterns of action" (heuristics) associated with the problem activity. The findings of particular interest here include not only the relatively strong success levels of students in the average-ability classes, but also the high levels of motivation observed in these students. Far from being daunted by the complexity of the problems, the children seem to have engaged with considerable enthusiasm and persistence. The value of the article lies not only in the activities suggested and the learning processes described, but also in the policy directions toward which it points (see Sect. 5 below).

Taken as a whole, the empirical studies in this volume provide valuable focus on the cognitive, social, and behavioral dimensions of creative mathematical activity in classes of "gifted," "highly promising," and "average-ability" students. While some affective and motivational consequences are observed, these dimensions have not yet received the structured research attention given to the cognitive and social aspects.

4 How should teacher education contribute?

4.1 Discussion questions

How can teacher preparation foster teachers' ability to encourage the development of creativity in students? Evidently teachers need substantial mathematical content knowledge, pedagogical content knowledge, and a repertoire of good, exploratory problem activities, in order to work effectively with mathematically promising students. How can this knowledge be described, characterized, and developed in pre-service teachers? What skills do teachers presently have, and what do they lack? How can their knowledge be deepened through ongoing professional development activity?

What affective orientations do teachers bring to the classroom? In particular, what emotions to they associate with open-ended mathematical activity? Can they "connect" with mathematically promising students? What beliefs about mathematics, giftedness, creativity, and problem solving do they hold? How do their affective orientations influence the effectiveness of their teaching, particularly with respect to creative processes in the classroom and the development of high ability in students? How can counterproductive beliefs be changed?

What motivational strategies can and do teachers employ that are effective in encouraging students' "in the moment" mathematical creativity, and the longer-term commitment of promising students to mathematical development?

4.2 Investigations and responses

Several of the articles in the current volume consider teachers' professional competencies as they pertain to creative mathematical activity.

Mhlolo (2017) associates teachers' ability to support "mildly gifted" students with the representational fluency of the teachers, for which he creates a scoring rubric: "mathematically faulty," "correct but with no further justification," and "correct with further justification." In four classrooms in South Africa, he finds less than a third of teachers' mathematical representations at a level able to support the creativity of gifted students.

The report by Hoth et al. (2017) is a much broader-scale investigation of 131 German primary school teachers' professional capabilities with respect to teaching students of high ability and encouraging mathematical creativity. They explore the connection between the teachers' professional knowledge, and their relevant skills in specific situations. Their sources of data are the Teacher Education and Development Study in Mathematics (TEDS-M), an international study of the professional knowledge of mathematics teachers at the completion of their education, and a follow-up German study assessing teachers' skills during their first year.

This article give us a close-in look at the competencies of the teachers—particularly, their capacity to identify the responses of mathematically creative and high-ability students and to support these students appropriately in their learning. The findings suggest strong deficiencies in many of the teachers, which the authors are able to connect with aspects of the teachers' professional knowledge base (Mathematics Content Knowledge, and Mathematics Pedagogical Content Knowledge); for example, their understanding of mathematical structure. They conclude that "teachers need profound content knowledge in order to identify complex and creative students' solutions," and they point to the need to impart such knowledge in order to support and enhance teachers' ability to work with mathematically creative students.

The findings of Mhlolo and of Hoth et al. are not surprising. They tend to confirm long-standing observations that teachers, especially at the primary school level, are insufficiently prepared in mathematics; and they provide us with helpful additional details as to the nature of the underpreparation as it affects teachers' interactions with high-ability students. Both articles focus on mathematical and cognitive issues; they leave for future research the important questions of teachers' ability to motivate and to foster powerful affect in high-ability students of mathematics.

Zazkis (2017) adopts a "creativity lens" in looking at the dialogue created by prospective elementary school teachers of mathematics when they write "Lesson Plays" in response to prompts by the teacher educators. These are fictional plays describing imagined student–teacher interactions, and Zazkis examines a trove of such scripts. Although creativity was not an explicit requirement in the teachers' writing of the Lesson Plays, Zazkis identifies in the dialogues interesting instances of mathematical and pedagogical creativity (in the sense of being "original to the group"). In addition, she notes occurrences of creative thinking by the teacher educators in their design of new prompts.

These scripts are profoundly interesting in their own right, as a window into the thinking of prospective teachers.

In this article, it appears they also serve as a venue for creative possibility. I am convinced by the article that there is great potential for adapting the Lesson Play method as a tool for encouraging creative mathematical activity by teachers and students. The fictional nature of the play allows the teachers (as playwrights) to invent dramatic dialogue expressive of creative mathematical thinking, for which of course they first need to imagine such thinking themselves. Evidently appropriately-designed prompts can do still more to encourage Lesson Plays written to express creative mathematical ideas. In addition, the writing of the Lesson Play requires verbal description of mathematical concepts, and invites emotional and motivational expressions by both teachers and students—both of these extremely valuable aspects of mathematical creativity. I think this article should inspire mathematics educators to make use of such methods in working with prospective teachers at both elementary and secondary levels—indeed, I expect to do so myself.

Singer et al. (2017) create very detailed tools for assessing the cognitive/style of prospective teachers through their problem-posing in geometry. The problem-posing context is rich and complex. The authors suggest this construct as a way to characterize individuals' traits in the domain of mathematical competence. Among the criteria they consider in their analysis of the problems posed are the "geometric nature" of the problems (qualitative vs. metric) and what they term the "conceptual dispersion" of the problems (structured vs. entropic). In a sample of 13 self-selected students, they find that their basic indicator of creativity, cognitive flexibility, correlates inversely with a style in which metric problems and structured problems are predominant.

Of course, such a correlative observation in a small, self-selected sample can be no more than suggestive of a possibility for future investigation. The scales developed for evaluating the students' responses are complex and context-specific. For me the value of this study lies in the rich possibilities offered by the geometry problem-posing context for evoking diverse mathematical responses from the prospective teachers (individuals generated from 9 to as many as 50 problems), and for the opportunities it provides for creative mathematical activity. In addition, the problem posing venue suggests ways of exploring affective and motivational characteristics of the teachers, and offers the potential for enriching teachers' professional development.

While the first two of the four articles discussed in this section address possible deficiencies in mathematics teacher preparation (as pertaining to creativity and the education of mathematically promising or gifted students), the latter two explore possible directions for strengthening that preparation. The focus here in mainly cognitive; it is my hope that future research will similarly address affective and conative dimensions in the preparation of teachers.

5 What are the influences of educational policy and societal trends?

5.1 Discussion questions

School practices are heavily influenced by government policies. What policies foster or inhibit mathematical creativity and the development of high-ability students in schools, and how do they do so? What are the effects of large-scale curriculum standardization, as has occurred (for example) in the widespread adoption of the Common Core State Standards for Mathematics (CCSS-M) in the USA? How do the uses of standardized mathematics testing affect creative mathematical activity in schools?

What affordances and pitfalls are associated with ability tracking, particularly in the domain of gifted education in mathematics? What are optimal criteria to use in program design, for identifying mathematically gifted or highly promising students, and what flexibility should exist for students to change tracks at different ages or grade levels? What incentives and resources should be in place for students and teachers to encourage creative mathematics, both in school and in after-school activity?

What should be the balance of private and public educational opportunities for mathematically promising students?

How do societal trends or forces contribute toward or impede the twin goals of developing mathematical ability and encouraging creative mathematical activity in schools? What are the actual influences of prevailing beliefs about mathematics, mathematical ability, or the importance of mathematical creativity and giftedness for society?

5.2 Investigations and responses

The articles in this issue of *ZDM Mathematics Education* do not investigate policy issues explicitly, nor do they study the effects of educational policies. But some of the authors do explore mathematical creativity or giftedness in particular policy contexts, providing a look into how policy can work out in practice. Because the findings are so context-specific, however, broader generalization from these studies truly is not possible. Instead possible directions for future investigation are indicated.

The most salient issue discussed here seems to be the grouping of students by various criteria of ability. Leikin et al. (2017) mention that in Israel special schools in mathematics do not exist, but special classes are given for advanced students, and participation in university programs is an option. Thus their study compares a high school class formed using "general giftedness" as measured by an early IQ score above 130, motivated to study advanced mathematics, with a different class of students formed from motivational criteria alone. The direction of their findings suggests a higher level of creative mathematical activity in the gifted class, thus tending to favor (with many caveats) such criteria. Sheffield (2017), as previously noted, is critical of using IQ testing (or IQ testing alone) to identify gifted students, and is more inclined toward understanding mathematical ability as malleable—indeed, she characterizes the myth that "mathematics ability is genetically determined" as dangerous.

The study by Mhlolo (2017) suggests that South African teachers' representational fluency was not sufficient to meet the needs of mathematically gifted students in regular classrooms; thus inclusive education of gifted students may indeed result in inhibited creative activity. On the other hand, Nolte and Pamperien (2017) report a relatively high level of success and motivation on the part of children in regular classes challenged by problems designed for gifted children. Thus we see a spectrum of possibilities in this set of articles, depending on many different factors, without clear policy findings emerging.

The essay by Sheffield (2017) is extremely important, addressing possible deleterious effects of current societal beliefs and trends for mathematical creativity and the development of giftedness. She characterizes a set of five beliefs, many of them widespread (e.g., that mathematical ability is genetically determined, that mathematics is not creative, or that gifted mathematicians develop on their own), as "myths," citing various research sources to the contrary. The beliefs become "dangerous" when they (tacitly, if not overtly) become the basis of policy. For each one, she offers some general recommendations and sometimes more specific ones. For example, noting that the CCSS-M make no mention of creativity, she advocates an additional Standard specifying, "Solve problems in novel ways and pose new mathematical questions of interest to investigate" (Johnsen and Sheffield 2012, p. 16).

My comment here is simply that some of Sheffield's recommendations may not go far enough. The general ones from earlier reports are easy to support e.g., from *Preparing the Next Generation of STEM Innovators*: "Provide opportunities for excellence" and "Cast a wide net." But the specifics of implementation require resources, a level of political will, and most of all good research-based policy judgment that today is, in my opinion, mostly lacking—at least, among US policy makers. For example, I would be concerned that creating a Standard of creativity embodies a kind of contradiction. The CCSS-M and accompanying assessments, even as they have "raised the bar" of required mathematical content, have functioned to encourage routinization and test-focused teaching. A new Standard, if operationalized through commonly-used assessment methods, could merely result in an effort to routinize and test for "creative" mathematical activity. Perhaps the idea of standardizing mathematics education needs to be fundamentally re-thought.

5.3 The prioritization of creativity and giftedness in mathematics education

Why should people value the encouragement of mathematical creativity, the nurturing of giftedness, and the development high mathematical ability through education? Surely the answer has something important to do with the discoveries and inventions the next generations will make, the quality of their insights, and their potential contributions to enlightening humanity. What happens today in schools affects generations of children throughout the world, and their ability to contribute to that enlightenment. And much of the potential for scientific understanding and technological progress—as well as for a world view based on reason and understanding—lies in the efficacy, the power, and the beauty of mathematics.

As Mhlolo (2017) notes in his article,

> The intuitive thought has been that mathematically gifted individuals have the potential to become the critical human capital needed for driving modern day economies. While this assumption has only been intuitive, Terman's Genetic Studies (Friedman and Martin 2011) and the longitudinal Studies of Mathematically Precocious Youth—SMPY (Lubinski et al. 2014) are arguably among the most famous longitudinal studies in psychology to date that have tracked mathematically gifted youth over decades with the aim of confirming this intuitive thought. Results from these studies have confirmed beyond any reasonable doubt that mathematically talented males and females indeed became the critical human capital needed for driving modern day, conceptual economies.

We can point to past extraordinary achievements of brilliant mathematicians and scientists, and the benefits to humanity that have resulted. What we cannot point to, and can never know, are the breakthroughs *not* made—the cures not developed, the clean energy technology not invented, the prosperity not achieved—by the creative and gifted students whose potential was never recognized or developed through education.

I think that all of the authors in the present volume, myself included, are advocates for such higher goals, and consequently for encouraging and developing mathematical creativity, and prioritizing the development of mathematical giftedness in schooling. It is commendable to see these values expressed internationally, as all of the articles explicitly or implicitly take such goals to be worthy of major investments of resources. The research reported here has the potential to influence education positively through a better understanding of the phenomena we are studying. But this influence is only possible if our goals and priorities are shared widely among educators and in society.

Tabach and Friedlander (2017) assert, “The need to foster creative mathematical thinking in school mathematics nowadays is acknowledged by the mathematical research community (e.g., Leikin and Pitta-Pantazi 2013) and also, to some extent, by some educational policy documents.” Mhlolo (2017) notes that opportunities for gifted students have been recommended in South Africa, despite the fact that “stakeholders have been hostile to and resentful of gifted education programs.” Although these statements are qualified, they are in my opinion rather over-optimistic in their assessments. I think it is important to acknowledge that the value of fostering mathematical giftedness and high ability is *not* universally held, even by educational researchers. We must ask *why* that is so, and take the possible answers seriously.

Several factors contributing to the absence of a consensus, some of them discussed in the present set of articles, deserve attention.

5.3.1 Beliefs about mathematics

Widely-held beliefs about mathematics (Sheffield 2017), fostered through current educational practices, represent the field as intrinsically dry and rule-driven, offering creative possibilities only for the very few—beliefs which are in stark contrast with those generally held about reading, writing, music, or art. Thus Sheffield, in discussing five “dangerous myths,” notes: “The third myth, that mathematics is not creative, narrows students’ understanding of mathematics to a rotely-learned series of facts and algorithms, and denies them the opportunity to become engaged in the beauty and challenges of true mathematics.”

5.3.2 Anti-elitism

Focusing on mathematically gifted students is often taken to be “elitist” in a pejorative sense, even by researchers. Children who are so identified are seen as unfairly privileged—indeed, many develop their abilities in homes with parents who are themselves highly educated and thus in upper socioeconomic strata. Many educators and politicians, as well as the public at large, regard achieving basic mathematical competence in the general population of students—termed educational equity—as a far higher priority. That value can be perceived as threatened by according priority status to gifted students, by ability tracking, by devoting resources to the mathematically gifted, or by spending precious classroom time on less-directive, less assessment-oriented creative mathematical activity. Raising expectations for presently underachieving students, a central goal in fostering educational equity, does not address—and may even undercut—efforts to nurture high achievers.

5.3.3 Standardization and testing

Standards-based mathematics education (supported by many in the mathematics education research community), with accompanying assessments, has created *powerful incentives* for mathematics teachers to concentrate on training students to solve routine problems in core curriculum topics, and *powerful disincentives* for spending time on the exploratory, non-standard investigations and methods associated with creative mathematical activity. Such a policy environment, presently ascendant in the United States, reinforces the situation observed by Tabach and Friedlander (2017) in the domain of early algebra: “… simplifying symbolic expressions is usually perceived in middle school algebra as an algorithmic activity, which in many cases is achieved by performing sequences of short drill-and-practice tasks, which have little to do with conceptual learning or with creative mathematical thinking.”

5.3.4 Science denial

There is a growing public trend toward “science denial,” particularly in the domains of climate change and environmental science but extending to educational research. This trend, accompanied by a broad dismissal of expert opinion, is no longer on the fringe; it has been encouraged explicitly by the newly-elected president of the United States! It not only creates a high barrier to research findings influencing policy and practice, but potentially fuels skepticism as to the value of devoting public resources to the educational development of future mathematical and scientific experts.

Thus, as we reflect on the research in this volume of articles—which is, in a sense, advocacy research—it is important to keep in mind the broader political and policy environment, and the context of challenges within which the research takes place. I think research is needed that focuses on public attitudes toward the purposes of mathematics education in relation to giftedness, creative activity, and the higher goals discussed here, and the social dynamics behind the societal trends.

6 In conclusion

The articles in this volume of *ZDM Mathematics Education* are a treasure-trove of ideas for exploring students’ creativity in mathematics in school contexts, with considerable theoretical discussion, rich descriptions of creative activity, and detailed qualitative analyses of the complex fabric of such activity. The dimensions highlighted most are cognitive and social (or sociocultural), with of course their behavioral manifestations. I have tried here to situate this work in a landscape of wider fundamental or pressing questions in the study of mathematical creativity and giftedness.

Future research, building on the investigations reported here, should address some these issues further, especially: (1) incorporating students' and teachers' affect and motivation into theoretical conceptualizations of high ability, giftedness, and creativity, and into empirical investigations; (2) studying the effects of educational policies on mathematically creative activity as it occurs in schools, and on the development of mathematical talent; and (3) investigating further the attitudes, trends, and societal forces that can foster or impede visionary efforts to advance mathematical creativity and develop the capabilities of our most promising students.

References

Cramond, B., Matthews-Morgan, J., Bandalos, D., & Zuo, L. (2005). A report on the 40-year follow-up of the Torrance tests of creative thinking: Alive and well in the new millennium. *Gifted Child Quarterly, 49*(4), 283–291.

DeBellis, V. A., & Goldin, G. A. (2006). Affect and meta-affect in mathematical problem solving: A representational perspective. *Educational Studies in Mathematics, 63*, 131–147.

DeBellis, V. A., & Rosenstein, J. G. (2004). Discrete mathematics in primary and secondary schools in the United States. *ZDM, 36*(2), 46–55.

Dreyfus, T., Hershkowitz, R., & Schwarz, B. (2015). The nested epistemic actions model for abstraction in context—theory as methodological tool and methodological tool as theory. In A. Bikner-Ahsbahs, C. Knipping & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education: Examples of methodology and methods* (pp. 185–217). Dordrecht: Springer, Advances in Mathematics Education series.

Friedman, H. S., & Martin, L. R. (2011). *The longevity project: Surprising discoveries for health and long life from the landmark eight-decade study*. New York: Hudson Street Press.

Gagné, F. (2015). From genes to talent: The DMGT/CMTD perspective. *Revista de Educacion, 368*, 12–39.

Gardner, H. (1983). *Frames of mind: The theory of multiple intelligences*. New York: Basic Books.

Goldin, G. A. (2009). The affective domain and students' mathematical inventiveness. In R. Leikin, A. Berman & B. Koichu (Eds.), *Creativity in Mathematics and the Education of Gifted Students* (pp. 181–194). Rotterdam: Sense Publishers.

Goldin, G. A. (2014). Perspectives on emotion in mathematical engagement, learning, and problem solving. In R. Pekrun & L. Linnenbrink-Garcia (Eds.), *International handbook of emotions in education* (pp. 391–414). New York: Routledge.

Granberg, C., & Olsson, J. (2015). ICT-supported problem solving and collaborative creative reasoning: Exploring linear functions using dynamic mathematics software. *Journal of Mathematical Behavior, 37*, 48–62.

Hadamard, J. (1945). *The psychology of invention in the mathematical field*. Princeton: Princeton University Press.

Hershkowitz, R., Schwarz, B. B., & Dreyfus, T. (2001). Abstraction in context: Epistemic actions. *Journal for Research in Mathematics Education, 32*, 195–222.

Hershkowitz, R., Tabach, M., & Dreyfus, T. (2017). Creative reasoning and shifts of knowledge in the mathematics classroom. *ZDM Mathematics Education*. doi:10.1007/s11858-016-0816-6.

Hoth, J., Kaiser, G., Busse, A., Döhrmann, M., König, J., & Blömeke, S. (2017). Professional competences of teachers for fostering creativity and supporting high-achieving students. *ZDM Mathematics Education*. doi:10.1007/s11858-016-0817-5.

Johnsen, S., & Sheffield, L. J. (Eds.). (2012). *Using the common core state standards for mathematics with gifted and advanced learners*. Washington, DC: National Association for Gifted Children **(quoted in Sheffield (2017))**.

Kaufman, J. C., & Beghetto, R. A. (2009). Beyond big and little: The four C model of creativity. *Review of General Psychology, 13*, 1–12.

Leikin, R. (2014). Giftedness and high ability in mathematics. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 247–251). Dordrecht: Springer.

Leikin, R., Koichu, B., Berman, A., & Dinur, S. (2017). How are questions that students ask in high level mathematics classes linked to general giftedness? *ZDM Mathematics Education*. doi:10.1007/s11858-016-0815-7.

Leikin, R., & Lev, M. (2007). Multiple solution tasks as a magnifying glass for observation of mathematical creativity. In J. H. Woo, H. C. Lew, K. S. Park & D. Y. Seo (Eds.), *Proceedings of the 31st International Conference for the Psychology of Mathematics Education* (Vol. 3, pp. 161–168). Seoul: The Korea Society of Educational Studies in Mathematics.

Leikin, R., & Pitta-Pantazi, D. (2013). Creativity and mathematics education: The state of the art. *ZDM, 45*, 159–166.

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics, 67*, 255–276.

Lubinski, D., Benbow, C. P., & Kell, H. J. (2014). Life paths and accomplishments of mathematically precocious males and females four decades later. *Psychological Science, 25*(12), 2217–2232.

Mann, E. L. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted, 30*(2), 236–260.

Mhlolo, M. K. (2017). Regular classroom teachers' recognition and support of the creative potential of mildly gifted mathematics learners. *ZDM Mathematics Education*. doi:10.1007/s11858-016-0824-6.

Middleton, J. A., & Jansen, A. (2011). *Motivation matters and interest counts: Fostering engagement in mathematics*. Reston: National Council of Teachers of Mathematics.

Nolte, M., & Pamperien, K. (2017). Challenging problems in a regular classroom setting and in a special foster programme. *ZDM Mathematics Education*. doi:10.1007/s11858-016-0825-5.

Polya, G. (1954). *Mathematics and plausible reasoning, Vols. I, Vol. II*. Princeton: Princeton University Press.

Polya, G. (1962). *Mathematical discovery: On understanding, learning, and teaching problem solving, Vol. I*. New York: Wiley.

Polya, G. (1965). *Mathematical discovery: On understanding, learning, and teaching problem solving, Vol. II*. New York: Wiley.

Rasmussen, C., & Stephan, M. (2008). A methodology for documenting collective activity. In A. E. Kelly, R. A. Lesh & J. Y. Baek (Eds.), *Handbook of design research methods in education* (pp. 195–215). London: Taylor & Francis.

Sheffield, L. J. (2017). Dangerous myths about "gifted" mathematics students. *ZDM Mathematics Education*. doi:10.1007/s11858-016-0814-8.

Singer, F. M., Voica, C., & Pelczer, I. (2017). Cognitive styles in posing geometry problems: implications for assessment of mathematical creativity. *ZDM Mathematics Education*. doi:10.1007/s11858-016-0820-x.

Sriraman, B., & Dickman, B. (2017). Mathematical pathologies as pathways into creativity. *ZDM Mathematics Education*. doi:10.1007/s11858-016-0822-8.

Sternberg, R. J., & Lubart, T. I. (1996). Investing in creativity. *American Psychologist, 51*, 677–688.

Tabach, M., & Friedlander, A. (2017). Algebraic procedures and creative thinking. *ZDM Mathematics Education*. doi:10.1007/s11858-016-0803-y.

Torrance, E. P. (1974). *Torrance tests of creative thinking*. Lexington: Ginn & Co.

Zazkis, R. (2017). Lesson Play tasks as a creative venture for teachers and teacher educators. *ZDM Mathematics Education*. doi:10.1007/s11858-016-0808-6.