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Natural Limits of Wealth Inequality and the Effectiveness of Tax Policy

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Abstract
This article examines Thomas Piketty's thesis that there are no natural limits on the accumulation of wealth. We undertake our examination in the context of a simple general equilibrium model with infinitely lived dynasties. We show that extreme wealth accumulation does not happen in general equilibrium unless capital and labor are substitutes, an assumption which also leads to unbalanced growth. We also show that even with unbalanced growth, differences in rates of return and effective labor are not sufficient to cause unbounded inequality. Only permanent savings rate differences can lead to extreme wealth concentration. Finally, we show that while a flat wealth tax will not eliminate extreme wealth concentration, both a graduated wealth tax and a flat income tax will.

Keywords
inequality, wealth distribution, wealth tax, income tax, intergenerational transfers

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In this article, we study the factors that contribute to the stability or instability of the distribution of wealth across generations. In particular, we test the effects of heterogeneous savings rates, labor endowments, and rates of return on savings in environments of both balanced and unbalanced growth. We also test the effectiveness of a wealth tax and a progressive income tax in reducing inequality in these environments.

The study of wealth inequality has received increased attention in recent years, due in large part to advances in the collection of detailed time series data on the distribution of wealth from multiple countries. Also helpful have been advances in heterogeneous agent modeling techniques and advances in the availability of high-powered computing resources and capabilities.

In his recent work, Piketty (2014, 26) lays out extensive evidence that inequality is increasing in many countries across the globe. He attributes this increase to economic forces which build up the wealth of the already wealthy. He claims that the forces that tend toward convergence of wealth levels—namely, "diffusion of knowledge and investment in training and skills"—are insufficient to overcome the divergent forces that come into play when "growth is weak and the return on capital is high." Piketty (2014, 26) lays out the logic very clearly in his introductory chapter.

> When the rate of return on capital significantly exceeds the growth rate of the economy (as it did through much of history until the nineteenth century and as is likely to be the case again in the twenty-first century), then it logically follows that inherited wealth grows faster than output and income. People with inherited wealth need save only a portion of their income from capital to see that capital grows more quickly than the economy as a whole. Under such conditions, it is almost inevitable that inherited wealth will dominate wealth amassed from a lifetime's labor by a wide margin, and the concentration of capital will attain extremely high levels—levels potentially incompatible with the meritocratic values and principles of social justice fundamental to modern democratic societies.

Our goal in this article is to explore the conditions under which extreme wealth concentration is possible. We test the properties of various economic conditions in a fairly stylized theoretical framework. A host of recent papers address these issues in depth. For example, Acemoglu and Robinson (2014, abstract) argue that "the focus on the share of top incomes gives a misleading characterization of the key determinants of societal inequality" and show that "inequality dynamics are closely linked to institutional factors and their endogenous evolution, much more than the forces emphasized in Piketty's book." Krusell and Smith (2014, abstract) note that Piketty's "second fundamental law of capitalism," which states that in the long run, the wealth-to-income ratio equals $\frac{s}{g}$, where s is the economy's saving rate and g its growth rate, "rests on a theory of saving that is hard to justify." Semieniuk (2014) reviews Piketty's elasticity argument, which relies on a nonstandard capital definition. His estimation results cast doubt on Piketty's hypothesis of an elasticity of substitution greater than one.[1]

We ask a similar question to McCain (2014) who uses a three-factor model and finds that rising capital shares can be consistent with an elasticity of substitution less than one. Our modeling environment for studying the evolution of the distribution of wealth combines three main strands of literature. First, we build from a base of the standard Bewley model with heterogeneous infinitely lived dynasties.[2] We also focus the dynamics of our model on intergenerational transfers within dynasties. De Nardi (2004) studies the effect of both intended and unintended bequests on the distribution of wealth and provides a good survey of the empirical and theoretical literature. DeBacker et al. (2014) study an overlapping generations model with both intended and unintended bequests with a focus on the effects of a wealth tax and an income tax on the distribution of wealth. Lastly, we study the transition path to the ergodic distribution of wealth in a way that is most consistent with the survival literature dealing with distributions of types of traders with differing quality of information as in, inter alia Sandroni (2000), Kogan et al. (2006), Ya (2008), or Condie and Phillips (2016).

We address the issue of divergence by building and simulating a simple general equilibrium model. We consider infinitely lived dynasties that earn income from labor and return on capital. Some of this income is consumed and some is added to the dynasty's stock of wealth which is passed on to the next generation in the form of a bequest.

We show that having a return on capital that exceeds the growth rate is not sufficient for divergence to dominate. We also show that extreme wealth accumulation does not happen in general equilibrium unless capital and labor are substitutes, an assumption which also leads to unbalanced growth. Even with unbalanced growth, differences in rates of return and effective labor are not sufficient to cause unbounded inequality. Only permanent savings rate differences can lead to extreme wealth concentration. Finally, we show that while a flat wealth tax will not eliminate extreme wealth concentration, both a graduated wealth tax and a flat income tax will.

The rest of the article is organized as follows. The second section presents the theoretical framework of a dynastic, heterogeneous-agent Bewley model. We look at both balanced growth and unbalanced growth versions of the model. The third section tests the effectiveness of a wealth tax and an income tax on limiting the wealth inequality in the extreme case in which the wealth distribution exhibits extreme concentration in the long run. The fourth section concludes.

Model

We study the effects of three different types of exogenous heterogeneity on the ergodic distribution of wealth in settings of both balanced growth and unbalanced growth. The first type of heterogeneity studied is differences in labor endowment or productivity. The second type of heterogeneity is differences in real rates of return on investment. The last type of heterogeneity is differences in preferences or savings rates.

Our model is a simple deterministic version of the Bewley model in that there are only two types of households or dynasties indexed by $i = 1, 2$. The measure of type $i = 1$ agents is $\theta \in (0, 1)$, so that the measure of type $i = 2$ agents is $1 - \theta$, and the total population each period is 1. Each household lives for one period. In each period, the household inelastically supplies labor l_i and chooses how much to consume $c_{i,t}$ and how much to save and bequeath to their children $k_{i,t+1}$. We assume that both the household's period utility and the utility from bequests are logarithmic. Each household's optimization problem is, therefore, a two-period problem of the following form:

$$\begin{aligned} &\max_{k_{i,t+1}} \ln(c_{i,t}) + \gamma_{i,t}\ln(k_{i,t+1}) \\ &\text{subject to} \quad c_{i,t} = w_t l_i + (1 + r_{i,t})k_{i,t} - k_{i,t+1} \quad \forall i, t, \end{aligned} \tag{2.1}$$

where $\gamma_{i,t} \in (0, 1)$ is the relative weighting on the "warm glow" utility of bequests, w_t is the common real wage, and $r_{i,t}$ is a potentially heterogeneous real return on bequeathed wealth.[3] It is easy to show the standard log utility result that each household's optimal policy function for bequests is simply a fraction of disposable income, where the fraction is a function of $\gamma_{i,t}$:

$$k_{i,t+1} = \frac{\gamma_{i,t}}{1 + \gamma_{i,t}}[w_t l_i + (1 + r_{i,t})k_{i,t}]. \tag{2.2}$$

We note that our assumption of logarithmic utility means that there can be no differences in the rate of saving if all agents have the same tastes. This differs from Piketty's assumption that the very rich behave as though their marginal utility of consumption is zero; they save their income minus a threshold level of consumption. This is impossible in our model.

The supply side of the economy is characterized by a representative firm with a constant elasticity of substitution (CES) production technology and a deterministic productivity growth component to be specified later. The general form of the CES production function is the following:

$$Y_t = F(K_t, L, e^{gt}) \quad \forall t, \tag{2.3}$$

where Y_t is aggregate output, K_t is the aggregate capital stock, L is aggregate labor, and e^{gt} is the deterministic level of productivity which is growing at rate g. Profit maximization gives rise to the two standard first-order conditions for aggregate capital and labor demand, respectively:

$$r_t = F_K(K_t, L, e^{gt}) - \delta \quad \forall t, \tag{2.4}$$

$$w_t = F_L(K_t, L, e^{gt}) \qquad \forall t, \tag{2.5}$$

where δ is the depreciation rate and r_t is the average net real interest rate paid out to households for the rental of their bequeathed capital.

We allow household real returns to be exogenously different and define $\Delta = r_{1,t} - r_{2,t} > 0$. This definition, along with the requirement that $K_t = \theta k_{1,t} + (1-\theta)k_{2,t}$, implies that

$$r_{1,t} = r_t + \frac{\Delta(1-\theta)k_{2,t}}{K_t} \quad \forall t, \tag{2.6}$$

$$r_{2,t} = r_t - \frac{\Delta\theta k_{1,t}}{K_t} \qquad \forall t. \tag{2.7}$$

Exogenous differences in the rate of return are a parsimonious way to model the rate of return heterogeneity $\Delta > 0$ described in Piketty (2014, 430–35). First, wealthier individuals can employ more skilled financial advisors. Second, a wealthier investor can take more risks because he has more reserves. We impose this exogenously with $\Delta > 0$ and equations (2.6) and (2.7).

To close the model, we assume that aggregate supply equals aggregate demand in both the capital market and in the labor market:

$$K_t = \theta k_{1,t} + (1-\theta)k_{2,t} \quad \forall t, \tag{2.8}$$

$$L = \theta l_1 + (1-\theta)l_2. \tag{2.9}$$

The definition of equilibrium is the following:

Definition 1 (equilibrium in a dynastic model with intergenerational transfers): General equilibrium in the dynastic model with intergenerational transfers described in the second section is defined as allocations $k_{i,t+1}$ and prices w_t, r_t, and $r_{i,t}$ for all i and t, such that the following conditions hold:

i. households optimize according to equation (2.2),
ii. firms optimize according to equations (2.4) and (2.5),
iii. heterogeneous rates of return are determined by equations (2.6) and (2.7),
iv. markets clear according to equations (2.8) and (2.9).

Before studying this model in the balanced growth and unbalanced growth cases, we define two values that describe different aspects of wealth inequality in the model. Let κ_t be the ratio of type 2 wealth to type 1 wealth per capita in period t:

$$\kappa_{t+1} \equiv \frac{k_{2,t+1}}{k_{1,t+1}} = \frac{\frac{\gamma_{2,t}}{1+\gamma_{2,t}}\left[w_t l_2 + (1+r_{2,t})k_{2,t}\right]}{\frac{\gamma_{1,t}}{1+\gamma_{1,t}}\left[w_t l_1 + (1+r_{1,t})k_{1,t}\right]} \quad \forall t. \tag{2.10}$$

Let ω_t be the percent of total wealth held by all type $i = 1$ households in period t:

$$\omega_t \equiv \frac{\theta k_{1,t}}{K_t} \quad \forall t. \tag{2.11}$$

Both κ_t and ω_t are individually sufficient statistics for the distribution of wealth in this model with two types.

Balanced Growth Analysis

For the balanced growth case, we use a Cobb–Douglas production function with the productivity growth being labor augmenting technological change:

$$Y_t = AK_t^{\alpha}\left(e^{gt}L_t\right)^{1-\alpha} \quad \forall t. \tag{2.12}$$

In this model, the aggregate capital stock K_t and effective aggregate labor $e^{gt}L_t$ are growing at the same rate g.[4]

In our simulations below, we show that inequality is bounded for every parameterization we consider. While inequality is bounded in the balanced

Table 1. Long-run Inequality with Different Savings Rates.

s_1 (%)	s_2 (%)	s_1/s_2	$\bar{\omega}$ (%)	$\bar{\kappa}$ (%)
99	1	99.00	98.9	0.3
75	10	7.50	85.2	4.4
75	20	3.75	69.5	11.0
50	30	1.67	37.1	42.4
50	49	1.02	20.5	96.9
23.6	7.2	3.28	64.6	13.7

Note: The savings rates from equation (2.2) are defined as $s_i \equiv \frac{\gamma_i}{1+\gamma_i}$ for $i = 1, 2$. The values $\bar{\kappa}$ and $\bar{\omega}$ are the ergodic versions of equations (2.10) and (2.11), respectively.

growth case, this only means that the wealthy will never control 100 percent of the economy's wealth. They could, however, hold a large percentage of the wealth. We use simulations to determine how big this percentage is likely to be.

We consider three reasons for wealth concentration: (1) the wealthy save more, (2) the wealthy have higher earnings abilities or labor endowments, and (3) the wealthy earn higher returns at the expense of the nonwealthy.

For all our simulations, we use a Cobb–Douglas aggregate production function with a capital share parameter $\alpha = .35$. We choose an annual depreciation rate of 8 percent and an annual growth rate for technology of 2 percent, which translate to $\delta = .9180$ and $g = .8114$ in our generational model in which one period is thirty years. We normalize the effective labor endowment of type 2 dynasties to unity $l_2 = 1$.

Table 1 shows the effect of differential savings. We vary the values of γ_1 and γ_2 for both types of dynasties to achieve the desired savings rates. We set the percent of type $i = 1$ individuals to 20 percent $\theta = .2$ in order to match savings rate data by income quintile from Dynan, Skinner, and Zeldes (2004). For this set of simulations, we set effective labor of both types to the same value of unity ($l_1 = l_2 = 1$), and we let both types earn the same return on capital ($\Delta = 0$). Table 1 shows various savings rates and the long-run values of $\bar{\omega}$ (the percent of total wealth held by type 1 dynasties) and $\bar{\kappa}$ (the ratio of type 1 wealth per capita to type 2 wealth per capita).

Table 1 shows that differences in savings rates lead to long-run inequality differences and that inequality is increasing in the savings rate differential. The last row represents the calibration that most closely matches the estimated savings rate differential in the United States. Dynan, Skinner, and

Zeldes (2004) estimated that the savings rate is 23.6 percent for the top quintile of income earners corresponding to our type 1 households and is an average of 7.2 percent for the bottom four quintiles of income earners. This simulation shows that savings rate differences cannot cause income distribution degeneracy but do have the potential to cause significant wealth inequality in the long run. Quantitatively, our model suggests that the current difference in savings rates in the United States is likely to cause the wealth of the bottom quintiles to be about 14 percent of that of the top quintile. This compares to the ratio of average median net worth of householders in the 80/20 split of 10.9 percent in the United States in 2011 as reported by Vornovitsky, Gottschalck, and Smith (2014, table A1).

Note that savings differences could be innate, or they could be driven by a propensity to save that rises as relative wealth rises. Whatever the cause, dynasties that save more will control more wealth, albeit never 100 percent.[5] Dynan, Skinner, and Zeldes (2004) provide evidence that the savings rates of the wealthy differ significantly from those of the less wealthy and that it is difficult to explain these differences simply with rates of time preference. We note that our results differ from those in Becker (1980), which finds that households with the lowest time discount rates own all the economy's capital in the long run. This is due to the overlapping generations nature of our model as compared with the infinitely lived agents in Becker's. High discount rate households in his model live hand-to-mouth, consuming from wage income alone. In contrast, our framework forces low-savings households to pass on wage income to their heirs via intergenerational transfers of wealth in the form of capital.

For our second case, we consider differences between the effective endowments of labor. The type 2 household labor endowment is normalized to $l_2 = 1$. Intuitively, if this is the only difference, then labor endowments act as a scaling factor. Type 1 dynasties with x times the labor endowment of type 2 dynasties will have x times the wealth as well. Table 2 confirms this intuition. Here, we hold the savings rates constant for both types at 10.5 percent, which is the average savings rate estimated by Dynan, Skinner, and Zeldes (2004). And we set $\Delta = 0$, so that the rates of return for both types of households are equal. As in the previous example, $\theta = .2$. However, the results are not dependent on the value of the savings rates or rates of return. Note that for large concentrations of wealth to occur under this scenario, wealthy households must have many times the effective labor endowment of nonwealthy households.

Table 2. Long-run Inequality in Balanced Growth Model with Different Labor Endowments.

l_1	$\bar{\omega}$ (%)	$\bar{\omega}$
2	33.3	.5
5	55.6	.2
10	71.4	.1

Note: The labor endowment of type 2 households is normalized to $l_2 = 1$, so the value in column 1 is both l_1 and the ratio of l_1/l_2. The values $\bar{\kappa}$ and $\bar{\omega}$ are the ergodic versions of equations (2.10) and (2.11), respectively.

Table 3. Long-run Inequality with Different Returns on Savings.

Δ	$\bar{\omega}$ (%)	$\bar{\omega}$
.01	20.01	.999
.04	20.05	.997
.10	20.12	.993
.15	20.17	.989
.20	20.23	.986

Note: The difference in rates of return is $r_{1,t} - r_{2,t} = \Delta > 0$, where the respective interest rates are given in equations (2.6) and (2.7). The values $\bar{\kappa}$ and $\bar{\omega}$ are the ergodic versions of equations (2.10) and (2.11), respectively.

Our final scenario is where type 1 dynasties earn high rates of return at the expense of type 2 dynasties, as described in equations (2.6) and (2.7). We hold savings rates for both types constant at 10.5 percent and impose equal effective labor per capita $l_1 = l_2 = 1$. We set $\theta = .2$ and test values of the interest rate wedge between $\Delta = .01$ and .20. The intermediate wedge value of $\Delta = .04$ is in line with the estimates of Piketty (2014, table 12.1). Table 3 shows the results.

Intuitively, $\Delta > 0$ seems like a powerful mechanism for inequality. However, table 3 shows that even fairly large rate of return differences of .20 per annum make little difference in the long-run degree of inequality. That is, when savings rates and labor endowments are equal across households, rate of return differences do not generate large movements in wealth inequality. Recall that with the balanced growth Cobb–Douglas production function in equation (2.12) and with balanced growth in general, the share of capital income is fixed at α. Since our agents do not differ in labor endowments and we calibrate $\alpha = .35$, differences of 10 percent in the return on capital lead to a difference of only 3.5 percent in total income.

Hence, wealth inequality remains small for reasonable values of the capital share.

Unbalanced Growth Analysis

In this section, we show that unbounded wealth concentration in the long run depends crucially on the degree of substitution between capital and labor. If the two factors are sufficiently substitutable, wealth concentration can grow unchecked in the long run.

We are not the first to explore this point. Rognlie (2014, 2) discusses the concept in depth and notes the following: "When [the elasticity of substitution] is greater than one, a higher capital/income ratio is associated with a higher share of capital income; when the elasticity is less than one, the opposite is true." This has obvious implications for the accumulation of capital, which implications are born out in our model. Rognlie relies primarily on partial equilibrium analysis of production functions and is concerned with the share of capital in gross domestic product, not in wealth accumulation per se.

Earlier work by Smetters (2003) shows that this elasticity is key for the value of optimal savings rates over time. The results of the model presented here are consistent with his findings concerning when the savings rate can "overshoot" or "undershoot" the long run steady state. Smetters uses a Ramsey model and is concerned primarily with overshooting in savings rates.

In this section, we replace the Cobb–Douglas production function with labor augmenting technological change in equation (2.12) from Balanced Growth Analysis section with a general CES production function with total factor productivity growth:

$$Y_t = e^{gt}A[\alpha K_t^{\eta} + (1-\alpha)L^{\eta}]^{\frac{1}{\eta}}. \tag{2.13}$$

This production function results in the aggregate capital stock and aggregate output growing at the same rate while aggregate labor L is constant.[6] The parameter η is proportional to the elasticity of substitution between capital and labor $\frac{1}{1-\eta}$. This production function generates unbalanced growth when $\eta > 0$. Equivalently, this is when the elasticity of substitution is greater than one. When $\eta > 0$, capital and labor are substitutes.

Definition 1 characterizes equilibrium in this example. The CES production function (2.13) results in the following first-order conditions for capital and labor:

$$r_t = \frac{\alpha K_t^{\eta-1} Y_t}{\alpha K_t^{\eta} + (1-\alpha)L^{\eta}} - \delta, \tag{2.14}$$

Table 4. Long-run Inequality in Unbalanced Growth Model with Different Labor Endowments.

	$\eta = -1$		$\eta = 0$		$\eta = 2/3$	
l_1	$\bar{\omega}$ (%)	$\bar{\kappa}$	$\bar{\omega}$ (%)	$\bar{\kappa}$	$\bar{\omega}$ (%)	$\bar{\kappa}$
2	33.3	.500	33.3	.500	33.2	.502
5	55.6	.200	55.6	.200	55.4	.201
10	71.4	.100	71.4	.100	71.3	.100

Note: The labor endowment of type 2 households is normalized to $l_2 = 1$, so the value in column 1 is both l_1 and the ratio of l_1/l_2. The values $\bar{\kappa}$ and $\bar{\omega}$ are the ergodic versions of equations (2.10) and (2.11), respectively.

$$w_t = \frac{(1-\alpha)L^{\eta-1}Y_t}{\alpha K_t^{\eta} + (1-\alpha)L^{\eta}}. \tag{2.15}$$

It is useful to define $\Gamma_t \in [0, 1]$ as the relative product of capital to the production process:

$$\Gamma_t \equiv \frac{\alpha K_t^{\eta}}{\alpha K_t^{\eta} + (1-\alpha)L^{\eta}}. \tag{2.16}$$

Note that equation (2.16) implies that the relative product of labor is $1 - \Gamma_t$. We can write simplified expressions for the average return on capital and the real wage in the following way by substituting in Γ_t:

$$r_t = \Gamma_t \frac{Y_t}{K_t} - \delta, \tag{2.17}$$

$$w_t = (1 - \Gamma_t)\frac{Y_t}{L}. \tag{2.18}$$

We perform the same experiments as in Balanced Growth Analysis section on this model with unbalanced growth with the addition of a specification of the savings rate as an exogenous function of disposable wealth. Dynan, Skinner, and Zeldes (2004) find evidence that this is a realistic assumption. The results of these experiments are presented in Tables 4–6. We present the results of each experiment for three parameterizations of the CES production function (2.13): $\eta = -1$ (complements), $\eta = 0$ (Cobb–Douglas), and $\eta = 2/3$ (substitutes). Klump, McAdam, and Willman (2007) and Chirinko (2008) use short-run data model estimates and long-run data model estimates and find the most

Table 5. Long-run Inequality in Unbalanced Growth Model with Different Returns on Savings.

	$\eta = -1$		$\eta = 0$		$\eta = 2/3$	
Δ	$\bar{\omega}$ (%)	$\bar{\kappa}$	$\bar{\omega}$ (%)	$\bar{\kappa}$	$\bar{\omega}$ (%)	$\bar{\kappa}$
.01	20.01	.999	20.01	.999	20.01	.999
.04	20.03	.998	20.03	.998	20.05	.997
.10	20.08	.995	20.07	.995	20.14	.992
.15	20.11	.993	20.11	.993	20.21	.987
.20	20.15	.991	20.15	.991	20.27	.983

Note: The difference in rates of return is $r_{1,t} - r_{2,t} = \Delta > 0$, where the respective interest rates are given in equations (2.6) and (2.7). The values $\bar{\kappa}$ and $\bar{\omega}$ are the ergodic versions of equations (2.10) and (2.11), respectively.

Table 6. Long-run Inequality in Unbalanced Growth Model with Different Savings Rates.

			$\eta = -1$		$\eta = 0$		$\eta = 2/3$	
s_1 (%)	s_2 (%)	s_1/s_2	$\bar{\omega}$ (%)	$\bar{\kappa}$ (%)	$\bar{\omega}$ (%)	$\bar{\kappa}$ (%)	$\bar{\omega}$ (%)	$\bar{\kappa}$ (%)
99	1	99.00	96.3	1.0	98.9	0.3	100.0	0.0
75	10	7.50	65.8	13.0	85.1	4.4	100.0	0.0
75	20	3.75	48.9	26.1	69.3	11.1	100.0	0.0
50	30	1.67	29.6	59.6	37.0	42.6	100.0	0.0
50	49	1.02	20.3	98.0	20.5	96.9	100.0	0.0
23.6	7.2	3.28	45.2	30.3	64.5	13.8	100.0	0.0

Note: The savings rates from equation (2.2) are defined as $s_i \equiv \frac{\gamma_i}{1+\gamma_i}$ for $i = 1, 2$. The values $\bar{\kappa}$ and $\bar{\omega}$ are the ergodic versions of equations (2.10) and (2.11), respectively.

evidence for the elasticity of substitution between capital and labor being between 0.4 and 0.6. This is most similar to our specification of $\eta = -1$, which corresponds to an elasticity of substitution of 0.5. Notwithstanding, we present our results for the Cobb–Douglas case $\eta = 0$ and the substitutes case $\eta = 2/3$ to show the sensitivity of our results to varying η.[7] For $\eta = 2/3$, the initial value of $k_{1,t}$ and $k_{2,t}$ are set equal to 1 because the initial values slightly influence the resulting $\bar{\omega}$ and $\bar{\kappa}$ in this case.

Suppose, instead that $\eta > 0$. In this case, the capital contribution (Γ_t) goes to one in the long run. We rewrite equation (2.10) as below.

$$\kappa_{t+1} = \frac{\bar{s}_2\left(1-\delta+\frac{Y_t-\Delta\theta k_{1t}}{K_t}\right)\kappa_t}{\bar{s}_1\left(1-\delta+\frac{Y_t+\Delta(1-\theta)\kappa_t k_{1t}}{K_t}\right)}. \tag{2.19}$$

We first note that the effective labor endowments do not enter into equation (2.19). Hence, if the only difference across types is labor endowment, the equation gives $\frac{\kappa_{t+1}}{\kappa_t} = 1$ in the long run, and inequality is bounded.

We next consider the special case where the only difference between types is the savings rate, that is, $\Delta = 0$.

Proposition 1 (inequality is unbounded when $\eta > 0$ and savings rates differ): In this case, equation (2.19) reduces to:

$$\frac{\kappa_{t+1}}{\kappa_t} = \frac{\bar{s}_2}{\bar{s}_1} < 1.$$

Hence, inequality is increasing without bound in the long run if there is no difference in rates of return, but savings rates differ.

Lastly, we consider another special case where savings rates are identical, but there are differences in rates of return.

Proposition 2 (inequality is bounded when $\eta > 0$ and rates of return differ): Equation (2.19) becomes:

$$\frac{\kappa_{t+1}}{\kappa_t} = \frac{1-\delta+\frac{Y_t}{K_t}-\frac{\Delta\theta k_{1t}}{K_t}}{1-\delta+\frac{Y_t}{K_t}+\frac{\Delta(1-\theta)\kappa_t k_{1t}}{K_t}}.$$

We note now that as Γ_t goes to one, the output to capital ratio goes to infinity. However, the terms $\frac{\Delta\theta k_{1t}}{K_t}$ and $\frac{\Delta(1-\theta)\kappa_t k_{1t}}{K_t}$ are bounded between one and zero. Hence, to determine the limit, we must invoke L'Hospital's rule. This reveals that κ_{t+1}/κ_t approaches one in the limit.

Thus, while interest rate differences can be a force for inequality, in the long run, the effects disappear and inequality is bounded. This result is confirmed by simulation below.

In this section, we keep the same assumptions about dynasties as we made in Balanced Growth Analysis section, only we use equations (2.13) to (2.15) rather than the Cobb–Douglas equivalents.

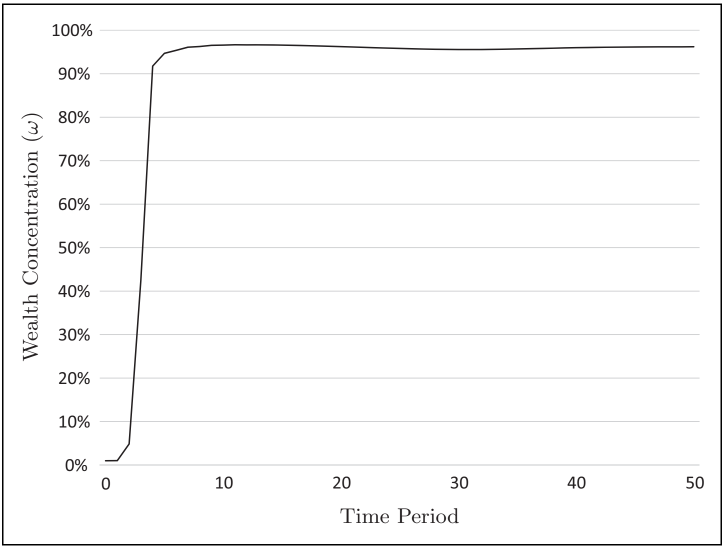

Figure 1. Wealth concentration over time: $\Delta = 20$ percent per annum.

Rates of Return Differ by Type

We first confirm our findings for the case where rates of returns differ by type. Figure 1 shows a simulation with equal starting wealth and savings rates of 75 percent. The difference between type 1 and type 2 returns is set to 20 percent per year.

For more modest differences, the divergence in wealth is much more slow. For example, Figure 2 shows wealth concentration over time for a simulation where the rate of return difference is two percent. However, as the previous section suggested, regardless of the rate difference, ω never goes to 100 percent in the limit.

Savings Rates Rise with Relative Wealth

We next consider the case where savings is a function of wealth. We know already that if savings rates differ by type, inequality is unbounded. We now consider what happens when savings rate differences arise endogenously because of a greater utility weight on bequests as relative wealth rises.

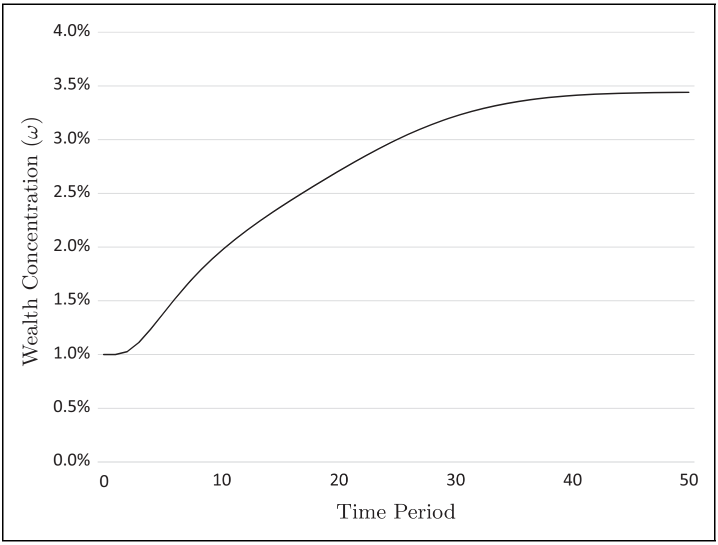

Figure 2. Wealth concentration over time: $\Delta = 20$ percent per annum.

The savings rate is $\frac{\gamma}{1+\gamma}$. We assume that γ is an increasing function of disposable wealth, $d_{it} \equiv w_t l_{it} + (1 + r_{it})k_{it}$, relative to the average, $\bar{d}_t$. In order to constrain the value of γ_t so that $\lim_{\tilde{d}\to-\infty} \gamma\left\{\frac{d_{it}}{\bar{d}_t}\right\} = \gamma_L$ and $\lim_{\tilde{d}\to\infty}\left\{\frac{d_{it}}{\bar{d}_t}\right\} = \gamma_H$, we use the following functional form

$$\gamma = \gamma_L + (\gamma_H - \gamma_L)\left(\tan^{-1}\left\{f + h\left(\ln\frac{d_{it}}{\bar{d}_t}\right)\right\}\frac{1}{\pi} + \frac{1}{2}\right).$$

This functional form gives $\lim_{\tilde{d}\to-\infty}\gamma = \gamma_L$ and $\lim_{\tilde{d}\to\infty}\gamma = \gamma_H$ and implies minimum and maximum savings rates of $s_{\min} = \frac{\gamma_L}{1+\gamma_L}$ and $s_{\max} = \frac{\gamma_H}{1+\gamma_H}$.

Figures 4 and 5 show the time paths of ω and the savings rates for both types from a baseline simulation. Here, the only difference between types is that type 1 dynasties start off with wealth that is 10 percent higher per capita. Type 1 dynasties comprise 1 percent of the population and we set the parameters in equation (2.20) so that $s_{\max} = 0.9$ and $s_{\min} = 0.1$. The savings rates as a function of relative wealth are plotted in Figure 3. For our baseline case, we use $f = -5$ and $h = 10$. The remaining parameters are set to $\eta = .1$, $\alpha = .35$, $\delta = .08$ (per annum), and $g = .01$ (per annum), and one period corresponds to twenty-five years.

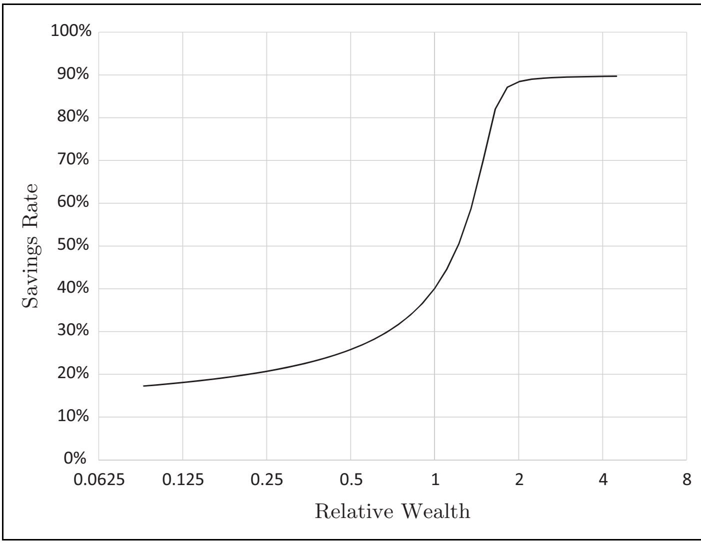

Figure 3. Savings rate as a function of relative wealth ($f = -5$ and $h = 10$): logarithmic horizontal scale.

We show this simulation to illustrate that it is possible for wealth concentrations to approach one in the limit. However, not every parameterization generates this. If the savings rate function is not steep enough, inequality can disappear in the limit. This is shown in figures 6 through 8 where we alter the γ function by setting $f = 5$. This makes savings less responsive when there are small deviations from relative wealth equal to one.

We summarize our results from the unbalanced growth case as follows:

i. Inequality will occur if one type of dynasty has a permanently higher savings rate than the other.
ii. It is possible, but not guaranteed that this can happen when savings rates differ across types because they rise with relative wealth.
iii. Wedges in rates of return cannot cause unbounded inequality, and the effects are so weak that the differences must be large enough to be equivalent to theft before significant wealth differences emerge.
iv. Differences in labor endowments cannot cause unbounded inequality.

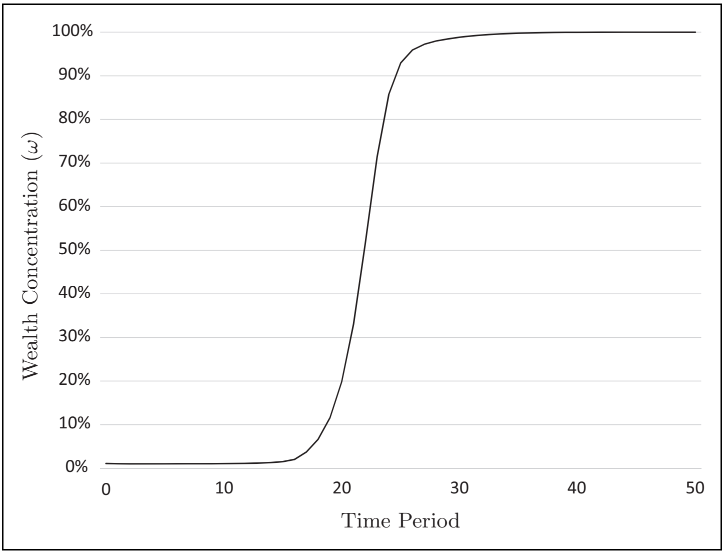

Figure 4. Wealth concentration over time ($f = -5$).

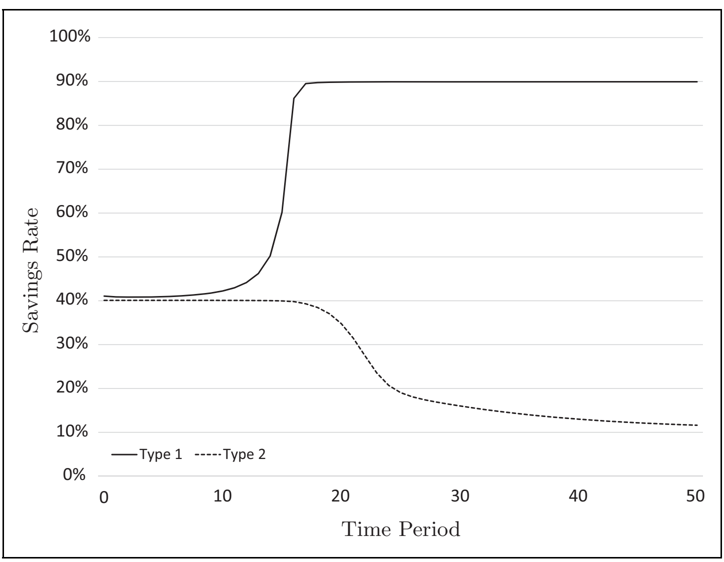

Figure 5. Savings rates over time ($f = -5$).

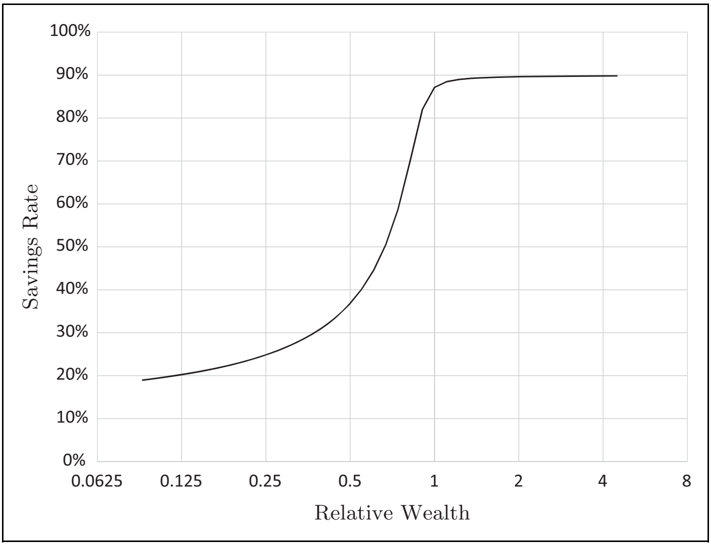

Figure 6. Savings rate as a function of relative wealth ($f = 5$ and $h = 10$): logarithmic horizontal scale.

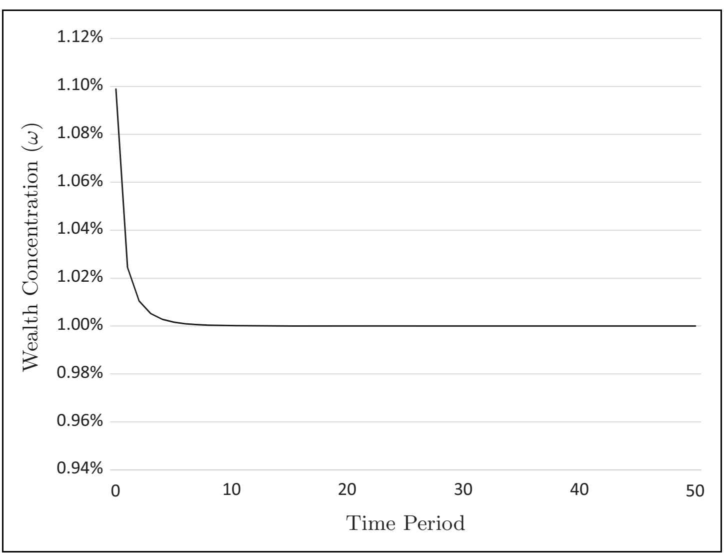

Figure 7. Wealth concentration over time ($f = 5$).

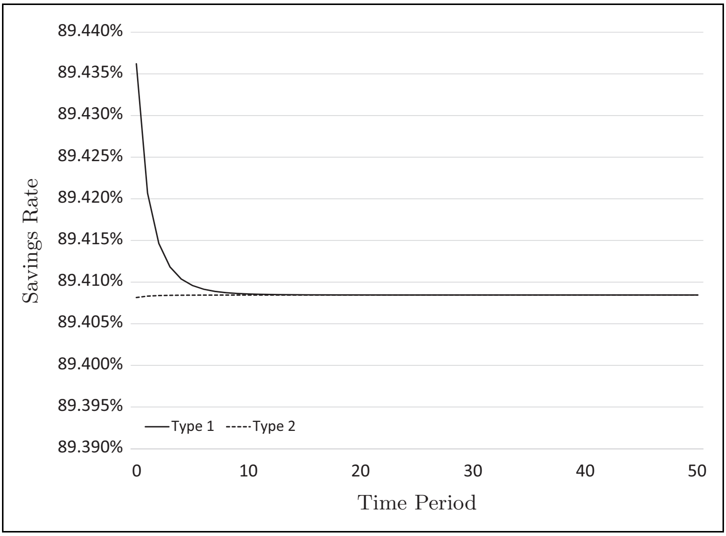

Figure 8. Savings rates over time ($f = 5$).

Overall, we find that in our model, it is very difficult to generate unbounded inequality. The only really viable mechanism is when savings rates across agents differ and capital and labor are substitutes in production. This said, we note that it is still possible to generate high but not unbounded levels of inequality when these conditions are not met.

Effectiveness of Wealth Tax and Income Tax

In this section, we consider the effects of taxation on inequality in the case in which inequality is unbounded in the limit. We test whether and how well a tax on wealth or income will put bounds on this otherwise boundless inequality. Since unbounded inequality occurs in the case of unbalanced growth, we focus on that model for this section. To see the effect of a wealth tax more clearly, we now distinguish between the bequest the old generation leaves to its heirs, which we denote b_{it}, and the amount of capital they actually receive after the tax is imposed, which we continue to denote as k_{it}. We can think of the wealth tax being imposed at the beginning of each period on bequests just received by the current generation. This would correspond closely to a wealth tax. Or equivalently, we could think of imposing the tax at the end of the period on the assets the current generation leaves to its heirs. This more closely matches an estate tax. In this context, of course, the two are identical, and we choose the former interpretation.

If we impose an income tax (τ^I) and a separate wealth tax (τ^W), we get the following. Equation (2.2) becomes equation (3.1),

$$k_{i,t+1} = \frac{\gamma_i}{1+\gamma_i}\{[w_t l_{it} + r_{it}k_{it}(1-\tau^W)](1-\tau^I) + k_{it}(1-\tau^W) + T_t\}. \quad (3.1)$$

The lump sum transfer to each dynasty (T_t) is given by equation (3.2):

$$T_t = N[\tau^I(w_t E_{1t} + r_{1t}k_{1t}) + \tau^W k_{1t}] + (1-N)[\tau^I(w_t E_{2t} + r_{2t}k_{2t}) + \tau^W k_{2t})]. \quad (3.2)$$

We run a series of simulations where the savings rate for the rich is 90 percent, while that for everyone else is 75 percent. We assume identical effective labor ($l_1 = l_2$) and returns ($\Delta = 0$). As before, we set $\eta = .1$, so that capital and labor are substitutes. We start our simulations with identical per capita wealth. In our unbalanced growth, model with no taxes wealth concentration goes to 100 percent. However, the imposition of either a wealth or income tax acts as a force for convergence and we end up with smaller steady-state wealth concentrations in these cases.

We consider both a flat wealth tax and a graduated tax with an upper limit. For the latter, we use the following functional form:

$$\tau^W = p\frac{k(W_i/\bar{W})}{k(W_i/\bar{W}) + m}, \quad (3.3)$$

where W_i is the wealth for dynasty i, $\bar{W}$ is the average dynasty wealth, p is the upper limit on the rate, and m is a constant. This function is plotted in figure 9.

Figures 10 through 12 show the time paths for ω under a series of increasing wealth and income taxes. A flat wealth tax delays inequality concentration but does not stop it. A graduated wealth tax and an income tax are more powerful forces for equalization than a flat wealth tax in our simulations. Note that a 2 percent tax each year on wealth as proposed by Piketty is roughly the same as a 40 percent tax over twenty-five years.

Our results are similar to those derived in Becker and Tomes (1979) and Davies (1986). Both find that a flat estate tax plus a lump-sum transfer cannot reduce wealth inequality in the long run. Davies

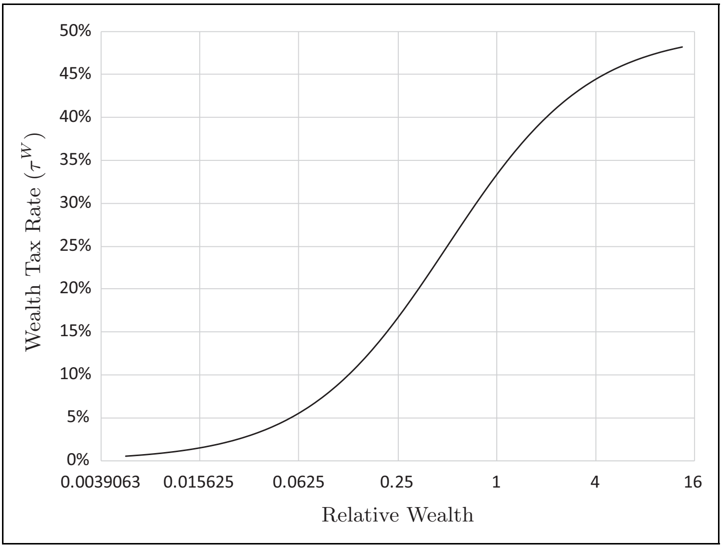

Figure 9. Graduated wealth tax function ($p = .5$, $k = 1$, $m = .5$): logarithmic horizontal scale.

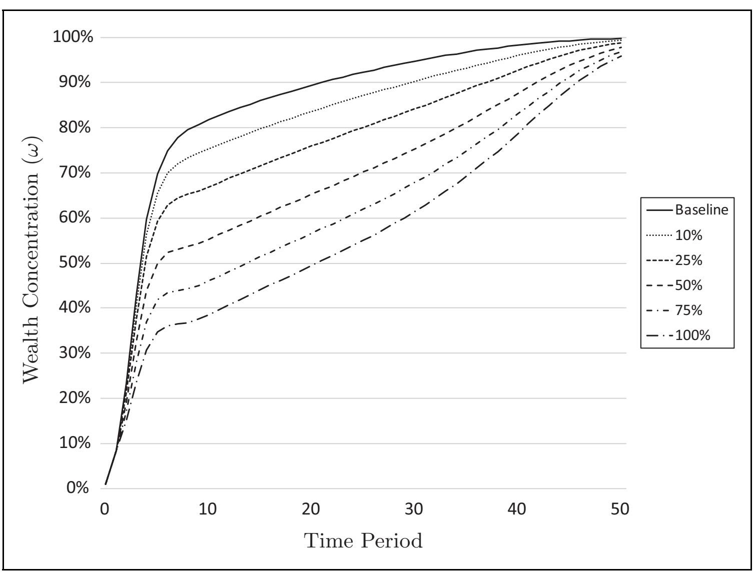

Figure 10. Simulations with various flat wealth taxes.

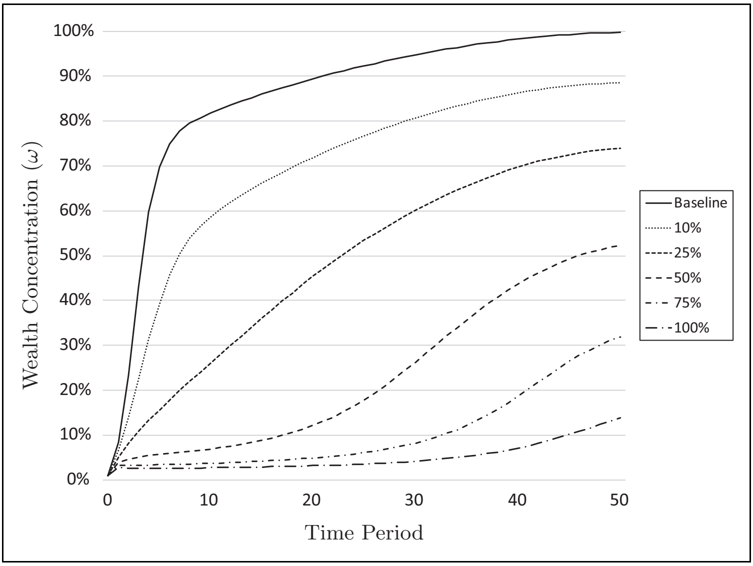

Figure 11. Simulations with various graduated wealth taxes ($k = 1$, $m = .5$).

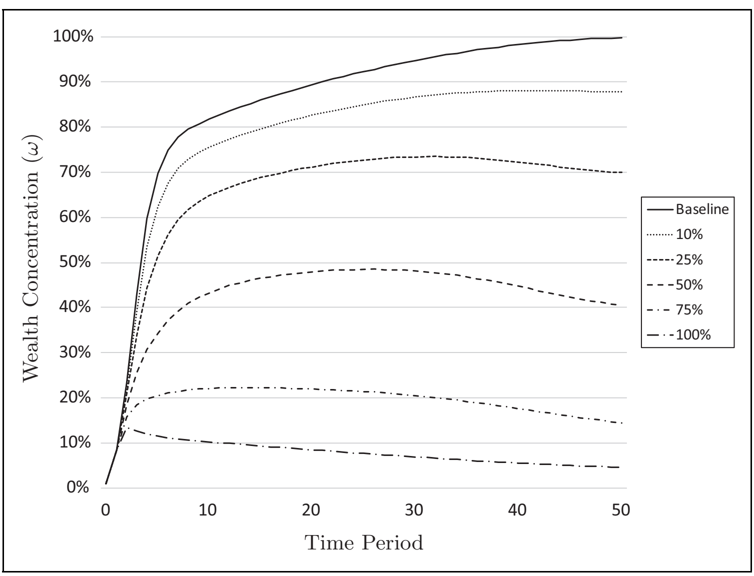

Figure 12. Simulation with various flat income taxes.

(1986) finds that a flat income tax plus a lump-sum transfer does reduce wealth inequality.

We find that both a flat income tax and graduated wealth tax eliminate unbounded inequality, while a flat wealth tax does not. However, a flat wealth tax does slow down the transition to unbounded inequality. Intuitively, this is because a flat wealth tax does not create additional disincentives for savings as wealth rises. A graduated wealth tax does and this will ultimately curtail savings by the wealthy as their wealth grows.

Conclusion

In this article, we have investigated the situations under which unbounded wealth concentration is possible. We have done so by building and simulating a simple general equilibrium model of long-run wealth accumulation through intergenerational bequests.

We find that two necessary conditions must be met for unbounded wealth concentration. First, capital and labor must be substitutes in production, and technical progress must not be labor augmenting. In this case, the importance of capital income rises over time and labor income becomes irrelevant in the limit. Second, the rich must save more than the rest of society. In our simple model, unbounded concentration will not occur if either of these conditions is absent.

We also show that other intuitive channels for wealth concentration are not sufficient. For example, differing rates of return on capital investment are insufficient even when there is unbalanced growth in favor of capital. The same is true for differing earnings abilities. Finally, we show that while a flat wealth tax will not eliminate extreme wealth concentration, both a graduated wealth tax and a flat income tax will.

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Notes

1. Less formal commentary that addresses substantive issues includes Posner and Weyl (2014), Rogoff (2014), and Sala-i-Martin (2014).
2. The Bewley's model terminology has become widely accepted for general equilibrium models with infinitely lived heterogeneous agents. See Bewley (1977), Aiyagari (1994), and Ljungqvist and Sargent (2012, 8–10).
3. Note that the household's problem is static despite the second term of the lifetime utility function including $k_{i,t+1}$. This is because the second term is the utility of bequests and not the utility of the household's children's consumption in the next period. This is a deviation from the Bewley model form.
4. With this Cobb–Douglas form, technical progress is both Hicks and Harrod neutral. Most of the empirical literature on CES assumes that technical progress is biased and tends to focus on manufacturing. Kennedy (1964) argues that bias is induced by market forces and that Harrod neutrality is the equilibrium condition. Piketty has consistently observed that intersectoral shifts are important and our balanced-growth model cannot capture these shifts. In Unbalanced Growth Analysis section, we consider a case of unbalanced growth and that model comes closer to capturing the shifts Piketty describes.
5. The one exception to this is if the nonwealthy has no savings, then trivially, all wealth belongs to type 1 dynasties.
6. The Cobb–Douglas production function is a nested case of this CES production function when $\eta = 0$. In the Cobb–Douglas case, aggregate capital, aggregate output, and effective labor are growing at rate $\frac{g}{1-\alpha}$.
7. A strand of research calibrates an elasticity of substitution between capital and labor greater than 1 ($\eta > 0$) in order to get local indeterminacy of equilibria (see Pintus 2006).

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