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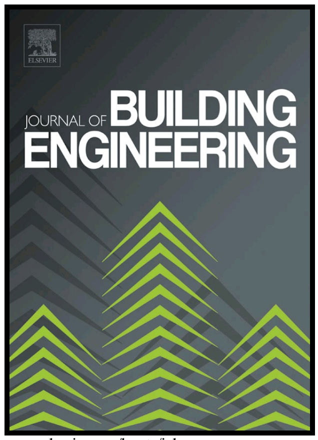


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# Direct Design Method and Design Diagrams for Reinforced Concrete Columns and Shear walls

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## ABSTRACT

Design of reinforced concrete columns and shear walls is an iterative process. The capacity of an assumed section is checked using interaction diagrams and the procedure continues until a satisfactory section is found. This study introduces a “Direct Design” method and “Design Diagrams”. The direct design method is an analytical approach by which the required area of reinforcement for short reinforced concrete columns or shear walls is determined directly without using an interaction diagram. This method provides a fitted solution for a reinforced concrete section; the capacity of the section is equal to the demand from the applied loads and moments. For each column or shear wall, many fitted sections with different sizes and bar arrangements could be employed. A design diagram shows all possible fitted sections for a particular column or shear wall. This study provides an algorithm for making design diagrams.

**Keywords:** direct design; design diagram; reinforced concrete column; shear wall

## INTRODUCTION

The accepted approach for design of concrete columns under combined axial load and bending moments is using the Short Column Interaction Diagrams along principal axes of symmetry[1]. Interaction diagrams for columns are generally computed by assuming a series of strain distributions, each corresponding to a particular point on the interaction diagram, and computing the corresponding values of $P$ and $M$[2].

The design procedure starts by assuming a column cross section and checking its capacity using the corresponding interaction diagram. If the selected section capacity does not satisfy applied load and moments, a new section is assumed. The procedure continues until an appropriate section is found. The column section founded by conventional approach is not necessarily a fitted section. Finding a fitted section, which its capacity is exactly equal to applied load and moments requires more trial-and-error. In addition, for any set of applied axial load and moments, it is possible to find several fitted sections with different section sizes, bar arrangements, and bar areas. The main objective of this paper is to present a general procedure by which all fitted solutions for a column or shear wall are found and presented on a design diagram.

This paper also proposes the direct design method for designing concrete columns. A system of equations is prepared based on the section shape and bar arrangement. By solving this system of equations, the required area of reinforcement bars is determined directly without using diagrams or tables. The resulting solution represents a fitted section since its capacity is equal to the applied load and moments. The provided procedure is not limited to any cross section shape or any particular stress-strain diagram for concrete and reinforcement.

## RESEARCH SIGNIFICANCE

An efficient and straightforward method for designing reinforced concrete columns and shear walls is presented in this paper. This method creates a design diagram for a short column or shear wall based on the applied load and moments. The provided design diagram shows all possible fitted sections for the column. Design diagram is a practical tool by which design of column would be much faster, easier, and efficient, since the designers have all practical fitted solutions in one diagram.

## PREVIOUS RESEARCHES ON RC COLUMN DESIGN

The design of reinforced concrete columns has been investigated by numerous researchers. Whitney[3], introduced an equivalent compression zone. Chu and Pabarcius[4], studied the ultimate strength of biaxially loaded reinforced concrete columns. Bresler[5], developed a reciprocal interaction equation used in ACI 318 commentary. Fleming and Werner[6], developed design aids for columns subjected to biaxial bending. Hsu and Mirza[7], studied acceptable strength for biaxial bending and compression. Marin[8], developed design aids for L-Shaped reinforced concrete columns. Hsu[9], presented theoretical and experimental results for biaxially loaded L-Shaped reinforced concrete columns. Hsu[10], proposed a design aid relationship considering the nominal axial load and balanced axial load ratio. Hsu[11], reported T-Shaped column under biaxial bending and axial compression.

Many of above researchers tried to develop simple design relationships or design aids. Availability of powerful and inexpensive personal computers, changed the type and direction of reinforced concrete column researches. Dinsmore[12], developed a program for column analysis with a programmable calculator. Brondum-Nielsen[13] and Yen[14], introduced methods for flexural capacity of cracked arbitrary concrete sections under axial load combined with biaxial bending. Barzegar and Erasito[15], developed interactive spreadsheets for concrete sections analysis under biaxial bending. Zenon[16] et al., introduced a method for designing reinforced concrete short-tied columns using the optimization technique. Rodriguez and Dario Aristizabal-Ochoa[17], developed a computer algorithm for biaxial interaction diagrams for short RC column of any cross section. Wang-Hong[18], Used the reciprocal load method for evaluating the capacity of reinforced concrete columns of high strength concrete. Bonet[19] et al., proposed an analytical approach for calculating failure surfaces in rectangular reinforced concrete column cross sections with symmetrical reinforcement. Cedolin[20] et al., developed an approximate analytical solution of the failure envelope of rectangular reinforced concrete columns. Paultre[21] et al., presented new equations for design of confinement reinforcement for rectangular and circular columns. Rodrigues[22] et al., studied the behavior of reinforced concrete column under biaxial cyclic loading. Lequesne-Pincheira[23], Proposed revisions to the strength reduction factor for axially loaded members.

## ACCEPTED COLUMN DESIGN PROCEDURE

The widely accepted approaches for design of RC column could be classified as follow:

### Uniaxial interaction diagram

An “Interaction diagram” can be generated by plotting the design axial load strength $\phi Pn$ against the corresponding design moment strength $\phi Mn$; this diagram defines the “usable” strength of a section at different eccentricities of the load[24]. Any combination of loading that falls inside the curve is satisfactory, whereas any combination falling outside the curve represents failure, see **Fig. 4**.

### Uniaxial column load capacity tables

The column load capacity tables provide capacities for bending about both major and minor axes. These tables give the factored usable capacity for usual range of sizes of square, rectangular, and round columns. The appropriate table is entered with values of the factored load and moment, and the column dimensions and reinforcement are obtained[25]. **Table 1** shows a sample column capacity table.

**Table 1.** A sample Column Capacity Table

| SQUARE TIED COLUMNS 16" x 16" | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Short columns - no sidesway<br>Bars symmetrical in 4 faces | | | | | | | | | f'c = 4,000 psi<br>$\phi M_n$ in inch-kips | | | f'y = 60,000 psi<br>$\phi P_n$ in kips | | | |
| BARS | RHO | Max Cap | | 0% $f_y$ | | 25% $f_y$ | | 50% $f_y$ | | 100% $f_y$ | | $\varepsilon_t = 0.005$ | | Zero Axial Load |
| | | $\phi M_n$ | $\phi P_n$ | $\phi M_n$ | $\phi P_n$ | $\phi M_n$ | $\phi P_n$ | $\phi M_n$ | $\phi P_n$ | $\phi M_n$ | $\phi P_n$ | $\phi M_n$ | $\phi P_n$ | $\phi M_n$ |
| 4#8 | 1.23 | 866 | 546 | 1232 | 468 | 1483 | 392 | 1625 | 332 | 1782 | 239 | 2064 | 189 | 1092 |
| 4#9 | 1.56 | 898 | 570 | 1321 | 481 | 1590 | 402 | 1752 | 338 | 1950 | 237 | 2255 | 179 | 1344 |
| 4#10 | 1.98 | 936 | 602 | 1432 | 499 | 1724 | 415 | 1912 | 345 | 2159 | 234 | 2492 | 166 | 1660 |
| 4#11 | 2.44 | 963 | 636 | 1547 | 514 | 1853 | 425 | 2059 | 351 | 2322 | 224 | 2653 | 141 | 1963 |
| 4#14 | 3.52 | 1044 | 717 | 1806 | 561 | 2169 | 459 | 2435 | 372 | 2783 | 210 | 3161 | 98 | 2698 |
| 4#18 | 6.25 | 1225 | 924 | 2412 | 681 | 2914 | 546 | 3289 | 419 | 3823 | 166 | 4254 | -35 | 4222 |
| 8#6 | 1.38 | 831 | 556 | 1170 | 481 | 1416 | 402 | 1550 | 339 | 1686 | 243 | 1968 | 160 | 1217 |
| 8#7 | 1.88 | 867 | 594 | 1275 | 503 | 1543 | 418 | 1700 | 349 | 1885 | 240 | 2205 | 136 | 1602 |
| 8#8 | 2.47 | 907 | 639 | 1395 | 530 | 1690 | 438 | 1875 | 361 | 2119 | 238 | 2477 | 107 | 2043 |
| 8#9 | 3.13 | 949 | 688 | 1525 | 560 | 1849 | 459 | 2065 | 374 | 2371 | 235 | 2765 | 73 | 2513 |
| 8#10 | 3.97 | 1001 | 752 | 1689 | 598 | 2049 | 487 | 2304 | 391 | 2686 | 230 | 3122 | 28 | 3056 |
| 8#11 | 4.88 | 1041 | 820 | 1851 | 635 | 2237 | 512 | 2524 | 405 | 2932 | 214 | 3370 | -40 | 3332 |
| 8#14 | 7.03 | 1164 | 982 | 2232 | 733 | 2709 | 583 | 3088 | 447 | 3625 | 191 | 4136 | -175 | 3976 |

### Load contours

In this method, the failure surface is approximated by a family of curves corresponding to constant values of $P_n$. These curves, may be regarded as "load contours".

**3D interaction diagram**

A uniaxial interaction diagram defines the load-moment strength along a single plane of a section under an axial load *P* and a uniaxial moment *M*. The biaxial bending resistance of an axially loaded column can be presented schematically as a surface formed by a series of uniaxial interaction curves drawn radially from the *P* axis. **Fig. 1** shows a biaxial interaction surface.

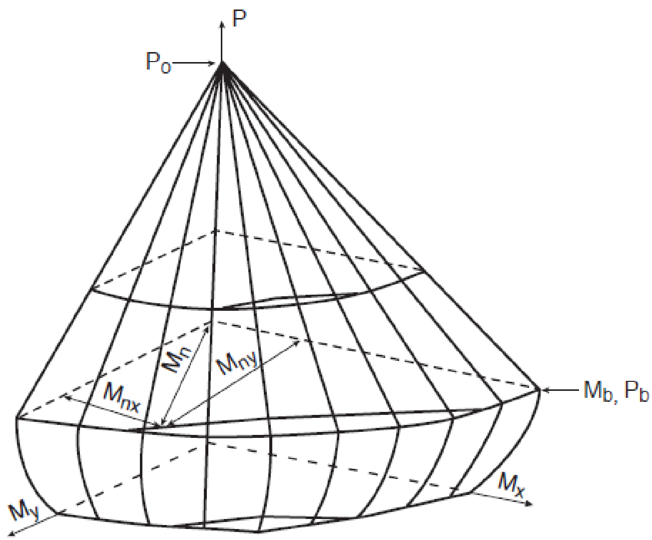


**Fig. 1.** Biaxial Interaction Surface



**Computer programs**

Some of the available computer programs for research and practice are listed in this section. spColumn (StructurePoint) is a software for the design and investigation of reinforced concrete sections subjected to axial and flexural forces. The section can be rectangular, round or irregular, with any reinforcement layout or pattern[1]. CSiCOL (CSi) is a software package used for analysis and design of columns. The program can carry out the design of reinforced concrete, or composite cross-section[26]. Response 2000 (Bentz et al.) is a sectional analysis program that calculates the strength and ductility of a reinforced concrete cross-section subjected to shear, moment, and axial load based on the modified compression field theory[27]. OpenSees (Fenves et al.) the open system for earthquake engineering simulation, is an object-oriented, open source software framework. It allows users to create finite element computer applications for simulating the response of structural systems in element, section, and fiber levels[28]. BIAX (Wallace et al.) is a general-purpose computer program to evaluate uniaxial and biaxial strength and deformation of reinforced concrete sections. The program can be used to compute strength or moment-curvature relations for monotonic loading[29].

## PROBLEM STATEMENT

Design of reinforced concrete columns and shear walls is a trial-and-error procedure. If the factored loads and moments are known and it is necessary to select a cross section to resist them, the procedure is referred to as design or proportioning. A design problem is solved by guessing a section, analyzing whether it will be satisfactory, revising the section, and reanalyzing it[30]. The analysis portion of the problem for column section design is mostly carried out via interaction diagrams. These diagrams are the result of analyzing the cross section for assumed strain distributions. **Fig. 2** shows a sample interaction diagram and some of the assumed strain distributions used to create the interaction diagram. The corresponding point for each assumed strain distribution is shown on the interaction diagram

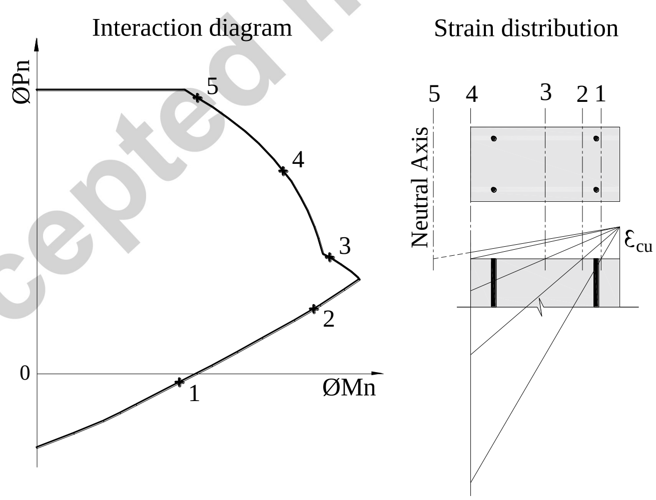


**Fig. 2.** Sample interaction diagram

The traditional design procedure is time consuming and does not necessarily lead to a fitted section. For example, assume that it is required to select a cross section for a short square reinforced concrete column subjected to the following factored load and moment:

$Pu \quad = 2180 \quad kN \quad (490\, kips)$ : Applied axial load.

$Mu_x = 190 \;\; kN\text{-}m \;\; (140 \; k\text{-}ft)$ : Applied moment about x-axis

**Fig. 3** shows an assumed section for the above column. Size and reinforcement arrangement of the section are known. It is required to investigate the assumed section with different bar areas $A_b$ to find whether any of them would satisfy the applied loads. **Fig. 4** shows a series of interaction diagrams, created for different bar areas $A_b$ for the column section shown in **Fig. 3**.

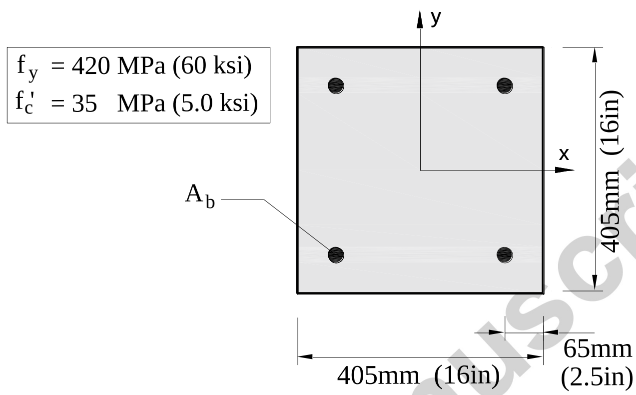


**Fig. 3.** Cross section of concrete column

Point 1 in **Fig. 4** that represents the above applied loads is located on the plot for $A_b$=500 mm$^2$ (0.8in$^2$). It shows that the capacity of the section with $A_b$=500 mm$^2$ is equal to the applied loads. Other sections with $A_b$>500 mm$^2$ are over designed and sections with $A_b$<500 mm$^2$ are not adequate. The calculated bar area is an acceptable value otherwise the designer should revise the section dimension or bar arrangement and repeat the procedure.

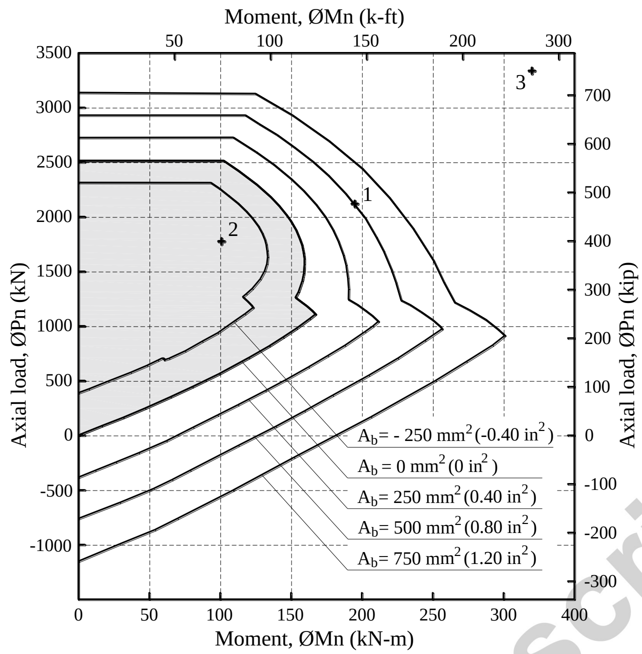


**Fig. 4.** Interaction diagrams for section in **Fig. 3**

For a short rectangular reinforced concrete column subjected to the above factored load and moment, a 405 mm square section with four bars with $A_b$=500 mm$^2$ (#8 bar) is a fitted section. It is possible to find other fitted sections for the above column with different dimensions, bar arrangements, and bar areas. For example, a 355 mm (14in) square section with 12 #9 bar is also another fitted solution. However, finding all fitted solutions for a column or shear wall using traditional methods is time consuming and impractical.

This paper proposes a general and efficient procedure by which all possible fitted solutions for a column or shear wall are found and presented on a design diagram.

## BACKGROUND THEORY ASSUMPTION AND METHOD DEVELOPMENT

For making a design diagram, it is necessary to have an efficient method for calculating the required area of bars. In this study, direct design method is employed for efficient and fast calculation of required area of reinforcement.

The formulation developed for direct design method in this paper is according to ACI 318-14 Building Code[31]. It is assumed that maximum allowable strain on concrete is $\varepsilon_{cu}=0.003$ and the strains in reinforcement and concrete are directly proportional to their distance from the neutral axis. Also, concrete stress of $0.85f'_c$ is uniformly distributed over the compression zone bounded by the cross section and a straight line parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain, where $c$ is distance from the neutral axis to the fiber of maximum compressive strain (see **Fig. 5**). Reinforcement is assumed elastic-perfectly plastic. However, this procedure allows the use of any stress-strain diagram for both the concrete and the reinforcement. The allowable area of longitudinal reinforcement for non-composite compression members is considered not less than *0.01Ag* or more than *0.08Ag***.**

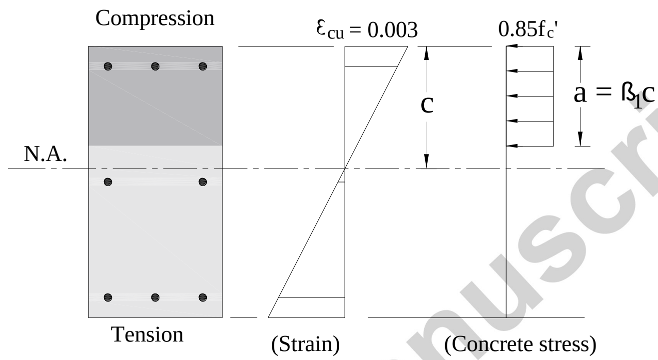


**Fig. 5.** Strain and stress distribution according to ACI 318

Consider the general cross section in **Fig. 6**. The coordinate system is referred to the centroid of the concrete section. The location of neutral axis is defined by variables $c$, $\theta$. Where $\theta$ is the angle of neutral axis with the x-axis.

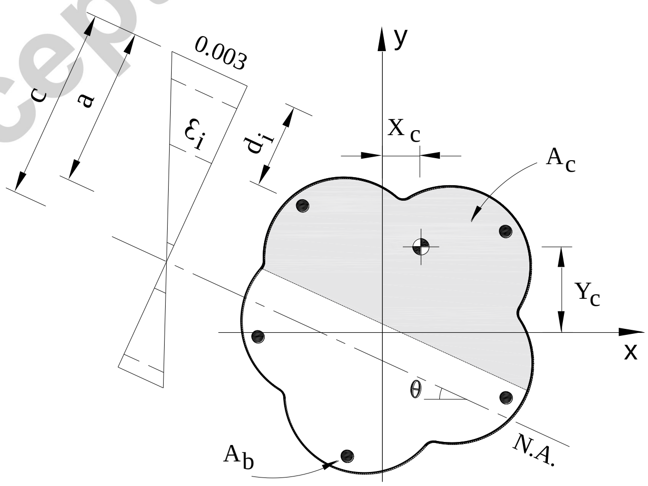


**Fig. 6.** A generic column cross section



## Contribution of concrete

Assume following functions calculate the contribution of concrete to the nominal axial strength, nominal flexural strengths about the x-axis and y-axis respectively.

$Pn_c\ (c,\theta)$ : Concrete nominal axial strength

$Mnx_c\ (c,\theta)$ : Concrete nominal flexural strengths about the x-axis

$Mny_c\ (c,\theta)$ : Concrete nominal flexural strengths about the y-axis

The contribution of concrete to the nominal strength of cross section is:

$$Pn_c(c,\theta) = 0.85f'_c * Ac\ (c,\theta) \quad (1)$$

$$Mnx_c(c,\theta) = 0.85f'_c * Ac\ (c,\theta) * Yc\ (c,\theta) \quad (2)$$

$$Mny_c(c,\theta) = 0.85f'_c * Ac\ (c,\theta) * Xc\ (c,\theta) \quad (2)$$

Where:

$Ac\ (c,\theta)$ : Area of compression zone

$Xc\ (c,\theta)$ : X coordinate of the centroid of $Ac\ (c,\ \theta)$

$Yc\ (c,\theta)$ : Y coordinate of the centroid of $Ac\ (c,\ \theta)$

**Contribution of reinforcement**

From the maximum allowable strain in concrete $\varepsilon_{cu}= 0.003$, the strain compatibility condition and the location of neutral axis, $c$ the strain $\varepsilon s_i$ at the i$^{th}$ bar is:

$$\varepsilon s_i = 0.003(1- d_i /c) \quad (4)$$

Where $d_i$ is distance of i$^{th}$ bar from extreme fiber in compression in the direction perpendicular to neutral axis.

Stress in each bar is determined from the stress-strain diagram of the reinforcement. The contribution of reinforcement to the nominal strength of cross section is:

$$Pn_s(c,\theta,A_b) = \Sigma\ Fs_i(c,\theta,A_b) \quad (5)$$

$$Mnx_s(c,\theta,A_b) = \Sigma\ Fs_i(c,\theta,A_b) * Ys_i \quad (6)$$

$$Mny_s(c,\theta,A_b) = \Sigma\ Fs_i(c,\theta,A_b) * Xs_i \quad (7)$$

Where:

$Fs_i(c,\theta,A_b)$ : force at the i$^{th}$ bar

$Xs_i$ : X coordinate of the i$^{th}$ bar

$Ys_i$ : Y coordinate of the i$^{th}$ bar

The strength reduction factor $\phi\ (c,\ \theta)$ is defined by the tensile strain in the extreme bar in tension at nominal strength.

The nominal factored strength of section is:

$$\phi Pn\ (c,\theta,A_b\ ) = \phi(c,\theta) * [Pn_c\ (c,\theta) + Pn_s\ (c,\theta,A_b)] \tag{8}$$

$$\phi Mn_x\ (c,\theta,A_b\ ) = \phi(c,\theta) * [Mnx_c\ (c,\theta) + Mnx_s\ (c,\theta,A_b)] \tag{9}$$

$$\phi Mn_y\ (c,\theta,A_b\ ) = \phi(c,\theta) * [Mny_c\ (c,\theta) + Mny_s\ (c,\theta,A_b)] \tag{10}$$

The ideal design for a column is when the factored strengths $\phi Pn$ **,** $\phi Mn_x$ and $\phi Mn_y$ in Eq. (8) - (10) are equal with the externally applied load and moments $P_u$, $M_{ux}$ and $M_{uy}$ respectively.

$$\phi Pn\ (c,\theta,A_b) - Pu = 0 \tag{11}$$

$$\phi Mnx(c,\theta,A_b) - Mux = 0 \tag{12}$$

$$\phi Mny(c,\theta,A_b) - Muy = 0 \tag{13}$$

The final system of equations is:

$$P\ (c,\theta,A_b) = \phi Pn(c,\theta,A_b) - Pu = 0 \tag{14}$$

$$Mx\ (c,\theta,A_b) = \phi Mn_x(c,\theta,A_b) - Mu_x = 0 \tag{15}$$

$$My\ (c,\theta,A_b) = \phi Mn_y(c,\theta,A_b) - Mu_y = 0 \tag{16}$$

Solving the system of Eq. (14) to (16), determines the three unknown variables $c,\ \theta,\ A_b$. The location of N.A. is defined by $c$ *and* $\theta$, and $A_b$ is the required area of each bar.

Equations (14) to (16) make up a nonlinear system of equations. There is no closed form solution for this nonlinear system of equations. Therefore, the solution depends on numerical iteration techniques, like Newton's method. Eq. (17) shows Newton's method for nonlinear systems, and it is generally expected to give quadratic convergence[32].

$$X^{(k+1)} = X^{(k)} - J\left(X^{(k)}\right)^{-1} F\left(X^{(k)}\right) \qquad k \geq 0 \tag{17}$$

Newton's method could be written in terms of equations (14) to (16) as follows (for simplicity $A_b$ has replaced by $A$):

$$\begin{bmatrix} c_{k+1} \\ \theta_{k+1} \\ A_{k+1} \end{bmatrix} = \begin{bmatrix} c_k \\ \theta_k \\ A_k \end{bmatrix} - J\left(X^{(k)}\right)^{-1} \begin{bmatrix} P(c_k,\theta_k,A_k) \\ Mx(c_k,\theta_k,A_k) \\ My(c_k,\theta_k,A_k) \end{bmatrix} \tag{18}$$

Where

$$J\left(X^{(k)}\right) = \begin{bmatrix} \frac{\partial}{\partial c_k} P(c_k,\theta_k,A_k) & \frac{\partial}{\partial \theta_k} P(c_k,\theta_k,A_k) & \frac{\partial}{\partial A_k} P(c_k,\theta_k,A_k) \\ \frac{\partial}{\partial c_k} Mx(c_k,\theta_k,A_k) & \frac{\partial}{\partial \theta_k} Mx(c_k,\theta_k,A_k) & \frac{\partial}{\partial A_k} Mx(c_k,\theta_k,A_k) \\ \frac{\partial}{\partial c_k} My(c_k,\theta_k,A_k) & \frac{\partial}{\partial \theta_k} My(c_k,\theta_k,A_k) & \frac{\partial}{\partial A_k} My(c_k,\theta_k,A_k) \end{bmatrix} \quad (19)$$

And

$c_k,\ \theta_k,\ A_k$ : value for unknown variables at $K^{\text{th}}$ iteration

$c_{k+1},\ \theta_{k+1},\ A_{k+1}$ : value for unknown variables at $(K+1)^{\text{th}}$ iteration

The following steps are required for solving the non-linear system of Equations (14) to (16) by implementing Newton's method:

1- Assume initial value for unknown variables $C, \theta, A_b$.

2- Calculate the Jacobean matrix $\boldsymbol{J}$ using equation (19).

3- Use equation (18) to find the new values for unknown variables $C, \theta, A_b$.

4- Repeat the process until the calculated section capacity is close enough to the applied load and moments.

The following guidelines could help to have appropriate initial assumption for unknown variables.

- For doubly symmetric sections $0^\circ \leq \theta \leq 90^\circ$ and $\theta$ can be initially approximated as $\theta \approx$ atan$(Mu_x/\ Mu_y)$.
- $c$ > 0, and for large values of c there would be no significant changes in nominal strengths of section.
- The bar area could be initially approximated as $A_b$ = 650 mm$^2$(1.0in$^2$)
- The acceptable range of $A_b$ is 200 mm$^2$ (0.31in$^2$) $\leq A_b \leq$ 2580 mm$^2$ (4in$^2$) for bars No. 5 to No. 18 respectively (nine bars sizes), and the total area of reinforcement is limited from 1% to 8% of the gross section of concrete.

## SPECIAL CASES, REAL AND UNREAL SOLUTIONS

As mentioned earlier, in accepted approach for designing RC column the capacity of an assumed section is checked using interaction diagrams. In practice, interaction diagrams are generated based on real and reasonable sections.

In direct design method, the appropriate section is found by solving a non-linear system of equations. In many cases, a negative (unreal) value is found for unknown variables *C, θ*, $A_b$.

1- *C* is distance from the neutral axis to the fiber of maximum compressive strain. A negative value for *C* is not acceptable. Whenever the solution contains a negative value for *C*, the iteration should start from another initial assumption.
2- *θ* is the angle of neutral axis with the x-axis. A negative value for *θ* is acceptable.
3- $A_b$ is the required area of reinforcement. Determining the validity of a negative value for $A_b$ is complicated.

**Acceptable Negative value for $A_b$**

In some situations, the theoretical capacity of the column section without contribution of reinforcement is greater than the applied load and moments. In these cases, solving the non-linear system of equations will lead to a negative value for $A_b$. In fact, the negative value for $A_b$ indicates that a smaller section is required to satisfy the applied load and moments.

For example, consider the square column section shown in

**Fig.** 7. The column is subjected to the following load and moment:

$Pu = 400\ \ kips$ : Applied axial load.

$Mu_x = 75\ \ k\text{-}ft$ : Applied moment about x-axis.

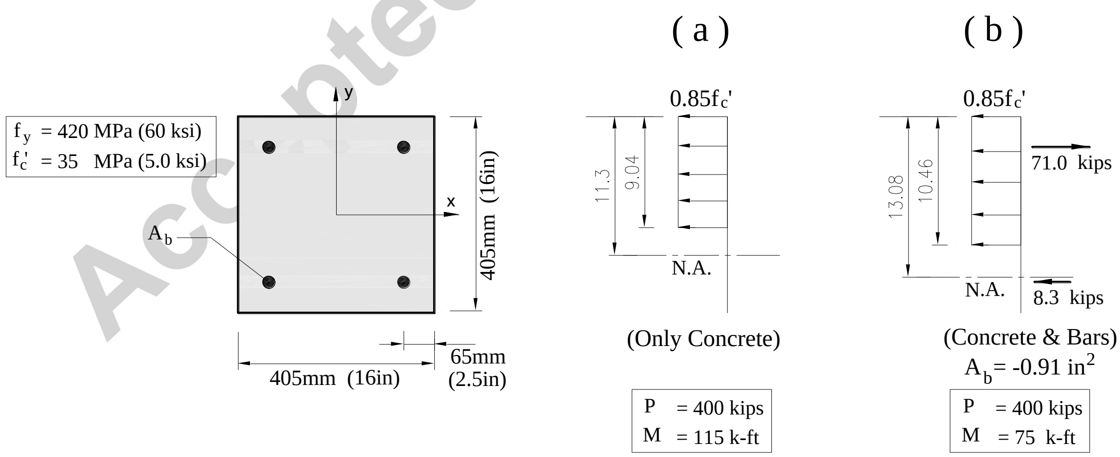


**Fig. 7.** Acceptable Negative Reinforcement Area

**Fig.** 7(a) shows the stress diagram of concrete without considering reinforcement bars. The neutral axis depth is *C*=11.3in. As it is shown in

**Fig.** 7(a), the corresponding capacity of the column section without steel contribution is:

$\phi Pn = 400 \ \ kips$ : Applied axial load.

$\phi Mn_x = 115 \ \ k\text{-}ft$ : Applied moment about x-axis.

It means that the capacity of concrete without steel contribution is more than the applied load. In

**Fig.** 7(b) the neutral axis depth is *C*=13.08in, $A_b$= -0.91in$^2$. The capacity of the section shown in

**Fig.** 7(b) is exactly equal to applied load and moment. In other word, Negative $A_b$ reduces the section capacity and makes it equal to the applied loads. For calculation of factored load and moments the strength reduction factor is considered $\phi$=0.65.

In general, whenever the capacity of a column section without contribution of steel is greater than the capacity of the section with negative reinforcement, the negative calculated value for bars is acceptable. In this case, there is no need to start a new iteration to find another solution.

**Non-Acceptable Negative value for $A_b$**

In some cases, there are two set of solutions for the non-linear system of equations. One set of answer with positive $A_b$ and one set with negative $A_b$. For example, consider the square column section shown in **Fig. 8**. The column is subjected to the following load and moment:

$Pu = 400 \ \ kips$ : Applied axial load.

$Mu_x = 129 \ \ k\text{-}ft$ : Applied moment about x-axis.

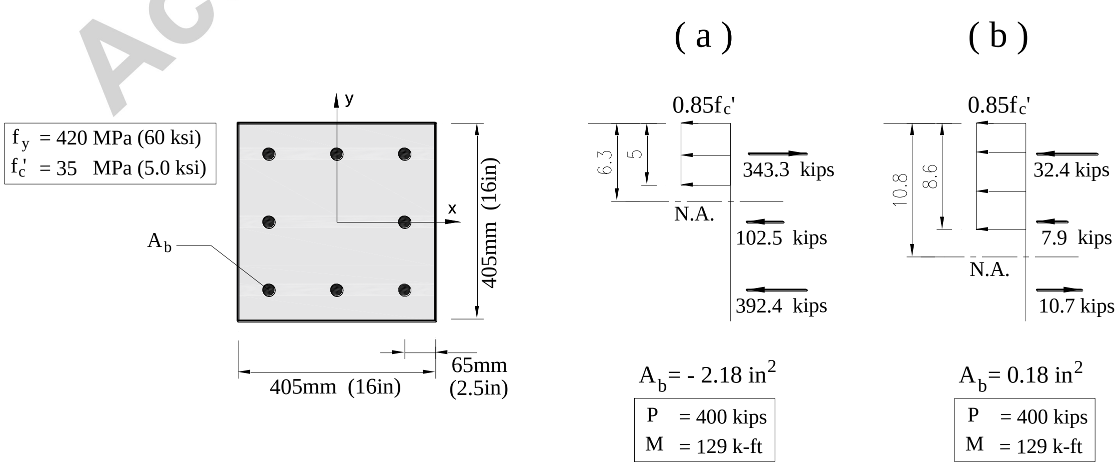


**Fig. 8.** Non-Acceptable Negative Reinforcement Area

In **Fig. 8** (a) $C$=6.3in, $A_b$=-2.18in$^2$, $\phi$=0.76 and in **Fig. 8** (b) $C$=10.8in, $A_b$=0.18in$^2$, $\phi$=0.65. Both stress distributions have the same capacity. In this case, the negative $A_b$ is not acceptable and the solution shown in **Fig. 8** (b) is the acceptable solution.

## DIRECT-DESGN PROGRAM

A computer program based on direct design method has been developed by the authors for creating design diagrams. The program solves the system of Eq. (14) to (16) by implementing Newton's method. Solving a nonlinear system of equations by Newton's method is an iterative process and should start from an initial assumption for the unknown variables. Newton's method is accurate and converges to the solution very fast; however, the calculations may not converge if the initial assumption is not close to the solution[33, 34].

The developed program uses the technique and guidelines previously explained in this paper for solving the nonlinear system of equations by starting from appropriate initial assumption. In addition, since the program uses the Direct Design method, it is able to calculate and show the location of neutral axis, shape and location of compression zone, and stress in reinforcement bars for each set of applied load and moments (see **Fig. 15**).

## RESULT INTERPRETATION

**Table 2** shows the design results for the section in **Fig. 3** for four different set of applied loads and moments. The last column displays the computed required areas for each of the reinforcing bars. Load cases 1-3 represent points 1-3 in **Fig. 4** respectively.

**Table 2.** Design results for section shown in **Fig. 3**

| Load case | Input | | | Result |
|---|---|---|---|---|
| | Pu kN (kip) | $Mu_x$ kN-m (k-ft) | $Mu_y$ kN-m (k-ft) | $A_b$ mm$^2$ (in$^2$) |
| 1 | 2180 (490) | 190 (140) | 0 | 504 (0.78) |
| 2 | 1780 (400) | 102 (75) | 0 | -586 (-0.91) |
| 3 | 3335 (750) | 339 (250) | 0 | 2268 (3.51) |
| 4 | 2225 (500) | 135 (100) | 102 (75) | 468 (0.73) |

For load case 1, the computed area of reinforcement is an acceptable value. It shows that the section dimensions and bars arrangement are appropriate. Use #8 bars, $A_b$ = 510 mm$^2$ (0.79 in$^2$) that has the closest area to the computed value. In load case 2, the calculated area of bars is negative. The point corresponding to load case 2 falls into the highlighted region of **Fig. 4** which bounded by the $A_b$ = 0 diagram. Theoretically, the concrete section without contribution of reinforcement can carry any load case that falls into this region. In this case, the designer shall provide the minimum required reinforcement, or reduce the size of the section. In load case 3, the calculated area of bars is so large that it exceeds #18 bars, $A_b$ = 2580 mm$^2$ (4.0 in$^2$) the largest available bar sizes. Therefore, a larger cross section is required for the concrete strength and bar grade assumed. In load case 4, the concrete section is subjected to axial load and biaxial moments. The computed area of reinforcement is an acceptable value. Use #8 bars, $A_b$ = 510 mm$^2$ (0.79 in$^2$).

The direct design method is not limited to rectangular sections with simple bar arrangement. This method could be used to calculate the required area of reinforcement for any column or shear wall section with any arbitrary bar arrangement. In addition, designing a section using the direct design method would provide the following information:

1- Location of neutral axis.
2- Shape and location of compression zone.
3- Stress in each reinforcement bar.

Traditional design method usually cannot provide above information.

## RESULTS VALIDATIONS

Several design examples are given in **Table 3** to show the validity of direct design method. These examples are based on column capacities presented in Values in **Table 3** are calculated for 405x405mm (16x16in) square tied column. Bars clear cover is considered 48mm (1.875in) and $f'_c$ = 28 MPa (4.0 ksi), $f_y$ = 420 MPa (60 ksi).

**Table 3.** Validation design examples

| Design example | CRSI (2008) | | | Direct Design | |
|---|---|---|---|---|---|
| | Bars | $\phi$Mn kN-m (k-in) | $\phi$Pn kN (kip) | $A_b$ mm$^2$ (in$^2$) | Bar size |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 4#8 | 139 (1232) | 2081 (468) | 509 (0.79) | #8 |
| 2 | 4#8 | 168 (1483) | 1744 (392) | 509 (0.79) | #8 |
| 3 | 8#7 | 192 (1700) | 1552 (349) | 387 (0.60) | #7 |
| 4 | 8#7 | 213 (1885) | 1067 (240) | 387 (0.60) | #7 |
| 4 | 12#10 | 228 (2016) | 3104 (698) | 819 (1.27) | #10 |
| 4 | 12#10 | 314 (2782) | 1966 (442) | 819 (1.27) | #10 |

## DESIGN DIAGRAM – BASIS AND DEVELOPMENT

For each column or shear wall, it is possible to find many sections that their capacities are equal to the applied load and moments. A design diagram shows all these possible practical fitted sections for a column or shear wall.

### Theory and Assumption

For making a design diagram, it is required to calculate the required area of bars for several section dimensions and bar arrangements. It is not practical to make a design diagram using traditional design method. So, creating a design diagram is highly dependent on the direct design method. In this research, design diagrams are developed based on ACI318-14 provisions. However, creating a design diagram is not limited to any structural standard.

### Design Diagram development

A design diagram is created by investigating the required area of reinforcement for different section sizes and bar arrangements. The procedure starts from minimum acceptable dimension and minimum acceptable number of bars for the section. The number of bars increases step-by-step and the required area of reinforcement for each step is calculated using direct design method. By increasing the number of bars, the required area of each bar decreases and the process continues until the calculated area for each bar is smaller than minimum available bar size. By repeating the same approach for different section dimensions, a matrix of required bar areas for different section sizes and bar arrangements is created, and then design diagram is made by interpolating the points in the matrix. The lines corresponding to the bar numbers are calculated by interpolation between the saved areas for each point in the matrix, and the reinforcement percentage lines are calculated with a similar approach by interpolating between reinforcement percentages. The procedure is shown by the flowchart in **Fig. 9**.

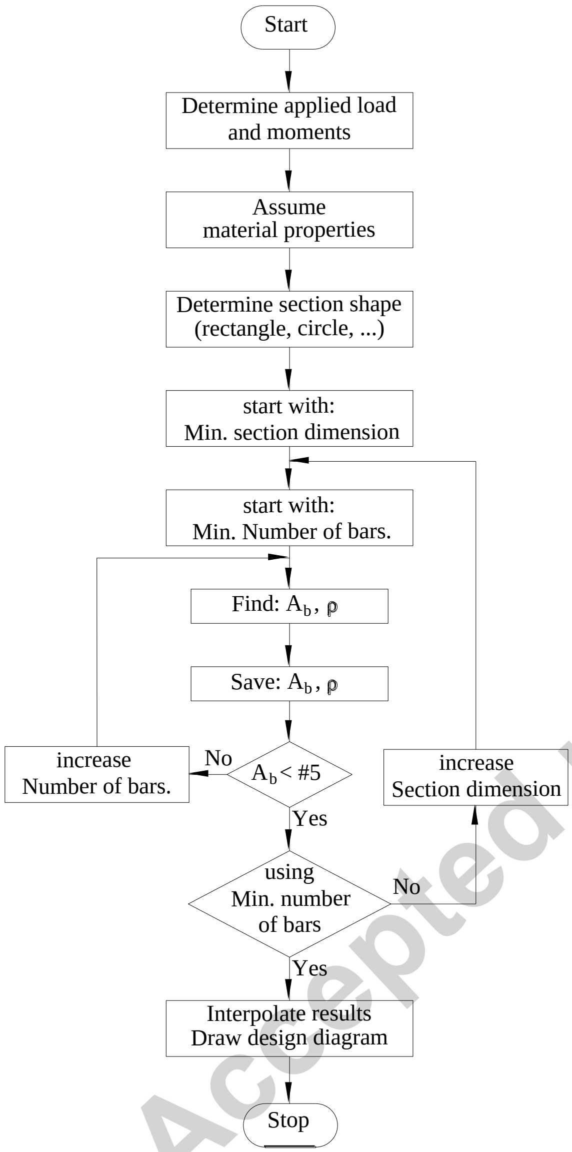


**Fig. 9.** Design diagram flowchart



**Fig. 10** shows design diagram generated for a square reinforced concrete column, subjected to the following applied axial load and moment. Cover to the center of bars is considered 65mm (2.5in) and $f'_c$ = 35 MPa (5.0 ksi), $f_y$ = 420 MPa (60 ksi).

$Pu \quad = 2180 \quad kN \quad (490\ kips)$

$Mu_x = 190 \quad kN\text{-}m \ \ (140\ k\text{-}ft)$

$Mu_y = 0$

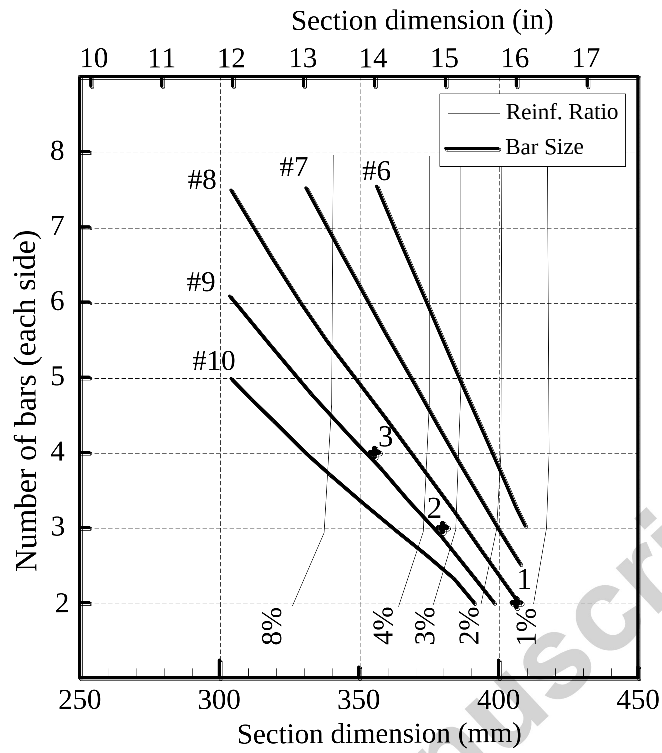


**Fig. 10.** Design diagram for a square section

The design diagram shown in **Fig. 10** is created by interpolation between calculated area of bars for several square sections, from 280mm (11in) to 430mm (17in), and from two bars to eight bars at each side for each section size.

Any point on a design diagram represents a section, with its capacity equal to applied loads. For example, any of the points 1, 2 and 3 in the design diagram shown in **Fig. 10** represents a fitted section. Point 1 represents a 405mm (16in) square section, which has 2 #8 bars at each side. Point 2 shows a 380mm (15in) square section which has 3 #9 bars at each side (no bar is available between #8 and #9 so, designer should choose the bigger size). Point 3 shows a 355mm (14in) square section, which has 4 #9 bars at each side. **Fig. 11** shows corresponding cross sections for points 1-3 in **Fig. 10**.

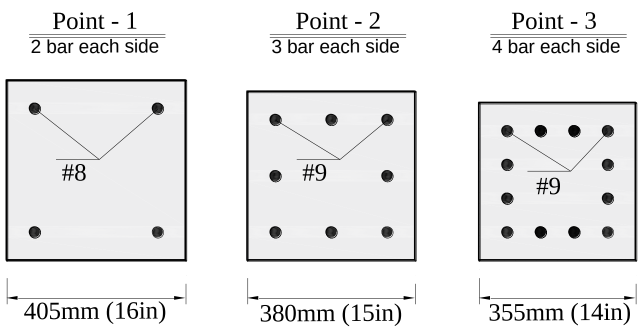


**Fig. 11.** Selected points in **Fig. 10**

In addition, design diagrams show the acceptable range for section dimension and bar arrangement. For example, **Fig. 10** shows that for the above applied loads and assumed material properties, it is not possible (reinforcement ratio greater than 8%) to have a square section smaller than 330mm (13in), or a 350mm (13.8in) square section, should have at least 4 #9 bar at each side.

## EXAMPLE 1. DESIGN DIAGRAM FOR CIRCULAR COLUMN

Create the design diagram for the circular section shown in **Fig. 12**. Assume that $f'_c$ = 28 MPa (4.0 ksi), $f_y$ = 420 MPa (60 ksi), bars clear cover is 50 mm (2.0 in), and section is subjected to the following load and moments.

$$Pu \quad = 900\ kN \qquad (202\ kips)$$
$$Mu_x = 400\ kN\text{-}m\ \ (295\ k\text{-}ft)$$
$$Mu_y = 200\ kN\text{-}m\ \ (147\ k\text{-}ft)$$



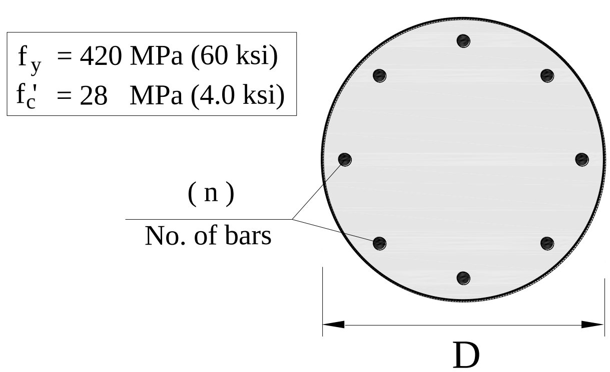

**Fig. 12.** Circular section, Example 1

***Solution***

For creating the design diagram according to the flowchart given in **Fig. 9**, it is required to investigate the required area of reinforcement for different section dimensions and bar arrangements. **Table 4** shows the design results for some sample points. The input data are section diameter *D*, and number of bars in section *n*. The outputs are the angle of neutral axis with x-axis, distance from the neutral axis to the fiber of maximum compressive strain *c*, and required area of each bar $A_b$.

**Table 4.** Design results for circular section in **Fig. 12**

| | Input | | Results | | |
|---|---|---|---|---|---|
| | D | n | Angle | c | $A_b$ |
| Points | mm (in) | - | deg. | mm (in) | $mm^2$ ($in^2$) |
| 1 | 600 (23.6) | 8 | 27.6 | 205 (8.0) | 505 (0.78) |
| 2 | 600 (23.6) | 11 | 26.9 | 205 (8.0) | 355 (0.55) |
| 3 | 550 (21.6) | 10 | 27.5 | 230 (9.1) | 705 (1.09) |

By repeating the sample calculation shown in **Table 4** for other section diameters, and bar arrangements a matrix for output results would be prepared. The design diagram shown in **Fig. 13** is created by interpolating the above results. Points 1-3 in **Fig. 13** represent the corresponding points in **Table 4**.

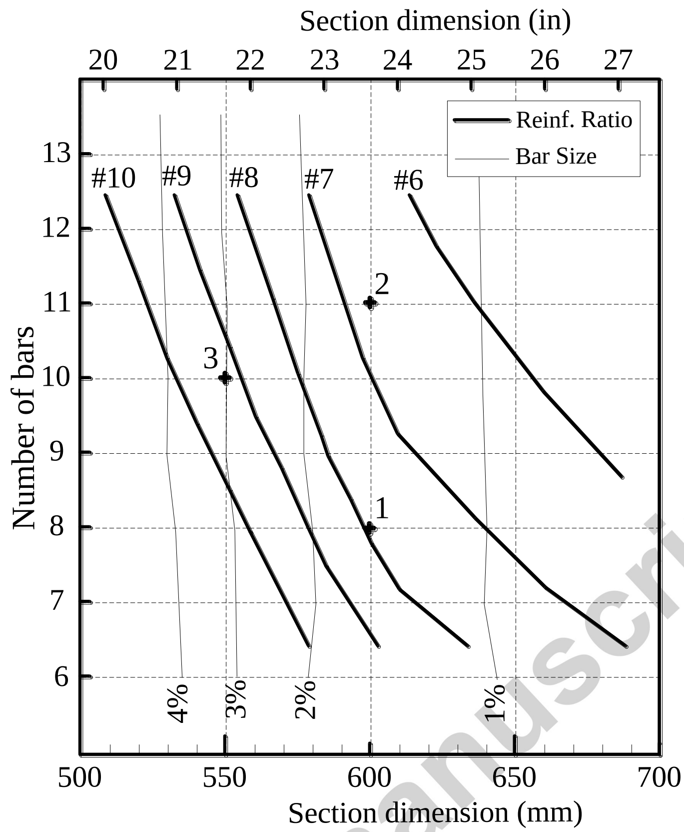


**Fig. 13.** Design diagram for Example 1



**EXAMPLE 2. DESIGN DIAGRAM FOR L-SHAPED SHEAR WALL**

Create the design diagram for the L-Shaped shear wall shown in **Fig. 14**. Assume that $f'_c$ = 28 MPa (4.0 ksi), $f_y$ = 420 MPa (60 ksi), bars clear cover is 50 mm (2.0 in), and section is subjected to the following load and moments.

$Pu$ = 4,500 kN (1010 kip)

$Mu_x$ = -10000 kN-m (-7375 k-ft)

$Mu_y$ = 250 kN-m (185 k-ft)

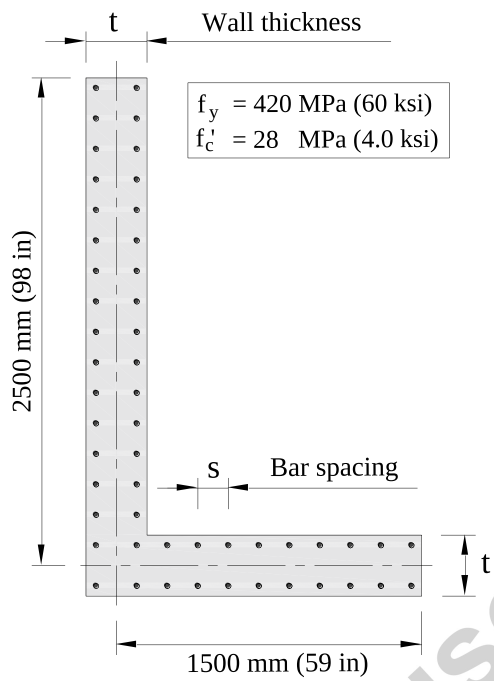


**Fig. 14**. L-Shaped shear wall, Example 2

***Solution***

**Table 5** shows design results for some sample points. The input data are wall thickness *t*, and bar spacing *s*. The output results are similar to example 1.

**Table 5.** Design results for L-Shaped section in **Fig. 14**

| Points | Input | | Results | | |
|---|---|---|---|---|---|
| | t<br>mm (in) | s<br>mm (in) | Angle<br>deg. | c<br>mm (in) | $A_b$<br>$mm^2$ ($in^2$) |
| 1 | 350 (13.8) | 100 (4.0) | 128.5 | 780 (30.8) | 330 (0.51) |
| 2 | 350 (13.8) | 150 (6.0) | 128.7 | 795 (31.3) | 510 (0.79) |
| 3 | 300 (11.8) | 150 (6.0) | 130.2 | 895 (35.3) | 720 (1.12) |

**Fig. 15** shows the required bar area, neutral axis, compression zone, and bar stresses for point 3 of **Table 5** calculated by the direct design method. **Fig. 16** shows the design diagram for the L-Shaped shear wall. Points 1-3 in **Fig. 16** represent the corresponding points in **Table 5**.

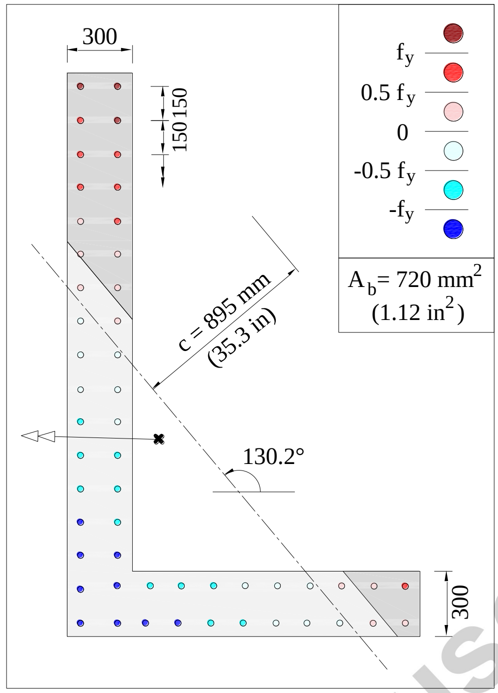


**Fig. 15.** Design result for point 3 of **Table 5**

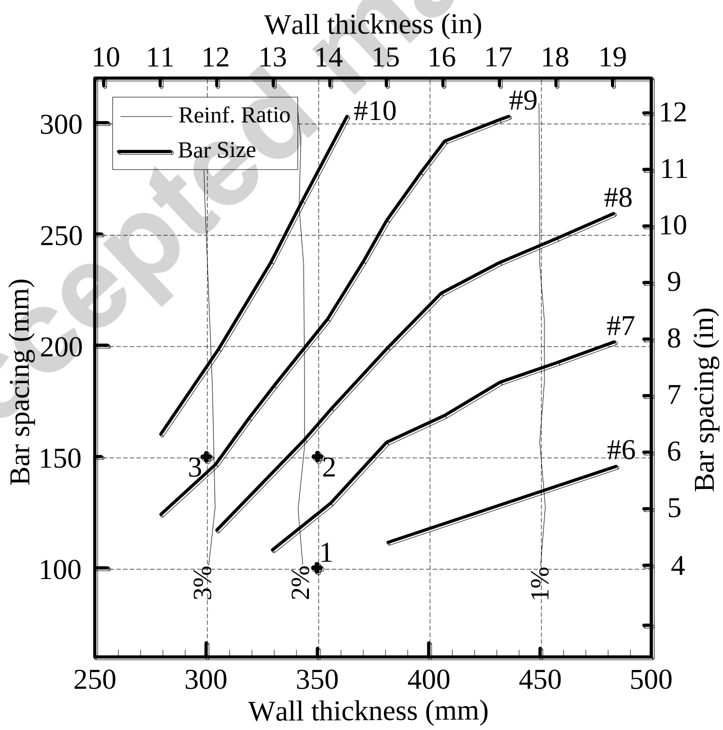


**Fig. 16.** Design diagram for Example 2

## CONCLUSION

Direct design method is an analytical method by which the required area of reinforcement for short reinforced concrete columns or shear walls is directly calculated without using interaction diagrams. Some other advantages of this method are:

- It could be used for any column or shear wall section with any arbitrary bar arrangement.
- It directly provides a fitted section without going through a trial-and-error procedure.
- It is not limited to any particular stress-strain diagram for concrete and reinforcement.
- The numerical solution of this method is very efficient, accurate, and fast in computer calculations.

Efficiency of direct design method provides a practical way for making design diagrams. The advantages of design diagram are:

- Shows all possible fitted column or shear wall cross sections in one diagram.
- Eliminates trial-and-error procedure from column design procedure.
- Shows the acceptable limitation for section dimension and bar arrangement.
- ACI and most of the other concrete codes, consider slenderness effect by magnifying the applied moments. Moment magnification is highly dependent on loads, column boundary conditions and many other factors. Considering all these additional factors in the presented method would produce more complexity and will be incorporated in a future study.
- In order to apply the direct design method to slender columns, the applied moments on the columns should be first magnified according to ACI 318.



## NOTATION

$A_b$ = area of each bar;
$a$ = depth of compression zone at nominal flexural strength;
$c$ = neutral axis depth at nominal flexural strength;
$d_i$ = distance of $i^{th}$ bar from extreme fiber in compression;
$fc'$ = compressive strength of concrete;
$Fs_i$ = force at $i^{th}$ bar;
$fy$ = yield stress of reinforcing steel;
$Mu_x$ = applied moment about x axis;
$Mu_y$ = applied moment about y axis;
$Pu$ = applied axial load;
$\theta$ = angle between neutral axis and x-axis;

$\varepsilon_{cu}$ = ultimate concrete compressive strain;
$\varepsilon s_i$ = strain in $i^{th}$ bar;
$\phi$ = strength reduction factor;
$\phi Mn_x$ = nominal factored flexural strength about x-axis;
$\phi Mnx_c$ = concrete nominal factored flexural strength about x-axis;
$\phi Mnx_s$ = reinforcement nominal factored flexural strength about x-axis;
$\phi Mn_y$ = nominal factored flexural strength about y-axis;
$\phi Mny_c$ = concrete nominal factored flexural strength about y-axis;
$\phi Mny_s$ = reinforcement nominal factored flexural strength about y-axis;
$\phi Pn$ = nominal factored axial strength;
$\phi Pn_c$ = concrete nominal factored axial strength;
$\phi Pn_s$ = reinforcement nominal factored axial strength;

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