Probabilistic Methods for Maintenance

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Abstract: Many aspects of maintenance are affected by uncertainty. Observations obtained through inspection and monitoring are often not more than probabilistic indicators of the underlying complex deterioration processes. Therefore, analysis in support of maintenance must usually cope with incomplete information. Probabilistic methods are suggested for various aspects of maintenance analysis, such as inspection timing, flaw detection, failure rate prediction, reliability assessment, benefit cost analysis, and evaluation of decision alternatives. Examples demonstrate the relationship between life span, failure rates and reliability, and the probabilistic evaluation of decision criteria and decision alternatives. The probabilistic approach allows the inclusion of subjective experience and judgment in the quantitative analysis. This is shown to be especially important when analyzing risky decisions.

Introduction

All structures and their components are subject to aging, wear and tear in the performance of their functions, and deterioration by exposure to the operating environment. Left to themselves, they will eventually become inefficient, unreliable, and fail. Maintenance problems are ubiquitous in all areas of engineering and maintenance methods are rather generally applicable. Here, the probabilistic aspects of maintenance will be emphasized and some applications to hydraulic engineering works will be discussed.

Maintenance of existing structures is of continuing concern to owners and operators for economic, reliability and safety reasons. According to the definitions of the U.S. Army Corps of Engineers' repair, evaluation, maintenance, and rehabilitation (REMR) research program (Scanlon et al. 1983), maintenance is defined as action that prevents or delays damage or deterioration, or corrects deficiencies that would otherwise lead to early repair or need for rehabilitation. Repair is restoration of damaged or deteriorated elements of a structure to continuing service, while rehabilitation is a major modification of an existing structure to bring it up to prevailing operation requirements and standards. Here the word *maintenance* is used sometimes in a wider sense that spans these different concepts.

When a structure reaches a certain level of deterioration or obsolescence, economic or safety reasons may demand a slowdown, a halt, or reversal of the deterioration process. Maintenance usually can only reduce the deterioration rate but cannot eliminate or reverse it, as a structure can usually not maintain, let alone improve itself. Only repair or reconstruction (rehabilitation) can bring the structure or equipment back to an improved state or the as-good-as-new state. The timing and extent of this intervention must often be decided with incomplete knowledge of the actual state of the structure or equipment, its remaining strength, the loads acting on it, the true costs and benefits of rehabilitation alternatives, and other factors. One can deal with this uncertainty by allowing inputs to vary over a certain range and by considering various paths the improvement or deterioration processes

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Note. Discussion open until February 1, 1992. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on January 9, 1990. This paper is part of the ***Journal of Engineering Mechanics***, Vol. 117, No. 9, September, 1991. ©ASCE, ISSN 0733-9399/91/0009-2065/$1.00 + $.15 per page. Paper No. 26169.

can possibly take. This gives maintenance a probabilistic character, as success or failure can be predicted only with some probability. A probabilistic process can never be fully controlled, it can only be influenced by decisions to intervene in some way, e.g., through maintenance, repair, and rehabilitation.

The Maintenance Problem

Maintenance will increase in importance in the years to come. As structures and equipment reach or exceed their expected life, owners and operators will have to cope with an increasing incidence of wear-out problems. There is no letup in the services demanded from the existing water resources infrastructure. Actually, these demands will further increase and diversify, making water management all the more complex (Kelley 1990). Funds, as well as opportunities for major new construction, reconstruction, or expansion, are scarce and there is public pressure to keep down rates for traditional as well as expanded services. This situation calls for cost-effective capital investments in the public and private infrastructure sector. The use of new technology and materials and of advanced analysis methods can make a significant contribution here.

The magnitude of the maintenance and rehabilitation problem in the water resources sector nationwide has been analyzed in a national infrastructure study that among others stressed the need for innovative approaches (Schilling 1987). The U.S. Army Corps of Engineers estimates the number of existing dams in the United States at approximately 70,000. By the year 2000, some 20,000 of these dams will be 50-years old or older (*New Perspectives* 1983). Of the Corps' 600 major navigation, power, and multipurpose projects, about 40% will be 50-years old and older by the year 2000 (Scanlon et al. 1983). This means that a sizable portion of the existing water resources infrastructure will have reached or exceeded its expected life. In response to these needs, the Corps' civil works appropriations began to exceed new construction appropriations for the first time by the mid-1980s (Markow et al. 1989).

So far, outright failures of major dams, power plants, dikes, etc., have been rare, but the incidence of sudden failure is definitely not zero, as reported examples show (*Lessons from* 1988; *Re-assessing the* 1990; Jansen 1988). Keeping down or even reducing the relatively small probability of major failures can have an enormous effect, however, on the lives and property of many people. But even if the failure rate just stayed constant, reliability will decline, as will be shown herein. This trend must be addressed by an appropriate maintenance, repair, and rehabilitation program. Aside from keeping major failures in check, maintenance can contribute effectively to upholding efficiency in the day-to-day operation of existing projects. In the following paragraphs, some probabilistic concepts and methods will be presented that exemplify the probabilistic approach to maintenance.

Probabilistic Approach to Maintenance

The probabilistic approach is advocated here as a method that can bring logic and order to problems whose inputs and underlying processes are uncertain or incompletely known. Examples are the likely effect of maintenance on the future performance of a structure, the effect of an inspection program on unexpected failures, flaw detection in the field, flaw growth prediction, assessing the loads and remaining strength of structures, field data interpretation, maintenance scheduling, the choice among maintenance alternatives, and benefit-cost analysis. When confronted with incomplete or missing information, the practitioner may decide to abandon quantitative analysis altogether and resort to an unstructured approach based on experience, or he may decide to continue the analysis by substituting subjective information (assumptions) for the missing information. This latter approach is advocated here.

FLAW DETECTION

From time to time, structures are reported to have failed just hours, days, or weeks after they were inspected. Sometimes this occurs in the aftermath of an event, such as a flood or earthquake, but it also happens without any unusual exposure. Obviously, the inspection performed was inadequate, as it did not or could not recognize the impending failure. Such incidents are symptomatic of the uncertainty an inspection must deal with when it comes to recognizing design flaws, incipient deterioration symptoms, and the underlying deterioration process. Fig. 1 illustrates crack growth and its surveillance by occasional inspections. After several inspections, here at six-months intervals, a growth trend seems to emerge that keeps crack growth below the critical level of crack depth/wall thickness = 0.8 for some time to come. Nevertheless, sudden unexpected failure occurs within the next inspection interval. Obviously, the nature of the underlying deterioration process was not recognized by the previous inspections or not properly dealt with.

Successful inspection and monitoring depend on adequacy of flaw detection. Incipient flaws may be below detectable size for the technique used, so that their detection probability is zero. Fig. 1 can be interpreted as illustrating crack growth below the detection limit, e.g., 0.17, only to assume critical size almost as soon as it becomes detectable. Detection probability as function of flaw size is shown in Fig. 2 [Davidson (1973) and Rodrigues and Provan (1989)]. A good detection method is characterized by a low detection threshold and a rapid increase in detection probability, as soon as a flaw exceeds detectable size, while a poor detection method may have a

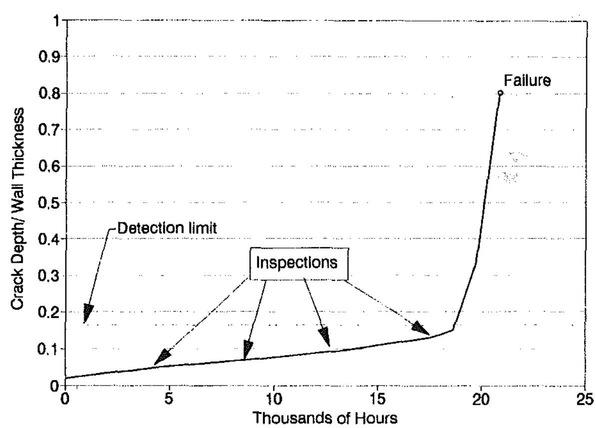

FIG. 1. Crack Growth and Inspection Schedule. Effect of Detection Limit on Inspection Results

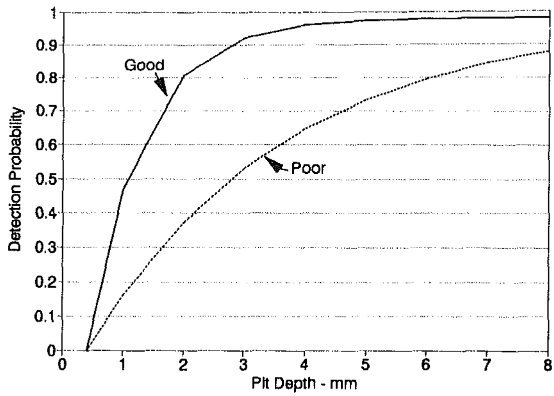

FIG. 2. Detection Probability as Function of Pit Depth. Detection Probability Is Good if It Has Low Detection Limit and Increases Rapidly with Pit Depth, and Poor if It Increases Slowly [Davison (1973) and Rodrigues and Provan (1989)]

high detection threshold and slow increase in detection capability with flaw size. Graham and Kahl (1989) report an incident where water leaking from a hydropower penstock revealed pinholes that had perforated the wall. While this corrosion damage did not fail the structure, further increase in pinhole size and the development of critical pinhole patterns, as well as secondary effects, such as undermining of foundations, could eventually lead to failure. The failure process is usually complex and makes the detected damage nothing more than a probabilistic symptom of a developing failure sequence.

INSPECTION INTERVAL

Successful inspection depends on several factors, such as accessibility (frequency of inspection may be low if extended shutdowns are required), observation environment (dangerous work area may make obtaining high-quality data difficult), flaw detection capability (inadequacy of tools), interpretation of observations (experience), observation error (oversight and misjudgment), and performance (observer's skill and reliability) (Meister 1982). Modern robotic techniques can overcome some of the inspection problems (Heffron 1990). But the layout of surveillance programs, data interpretation, and maintenance decisions remain dependent on human experience and judgment. The typically human functions can be supported, however, by the type of methods discussed here. Probabilistic inspection guidelines are presently under development with the support of various industries (Balkey, personal communication, 1989).

An inspection program's objective is to eliminate unexpected outages by preventive maintenance. There are usually many factors that influence the timing of inspections. A spillway inspection schedule will probably avoid the flood season and a hydropower inspection schedule will avoid the peak power demand period. While these considerations can serve as general guidelines, the inspection schedule must be related to the underlying deterioration processes. Suppose an indication of such a process is flaw growth. If the inspection interval is too short, the flaw is nondetectable, and the inspection is superfluous. If the inspection interval is too long, the flaw may reach failure size before the next inspection, as illustrated in Fig. 1. The

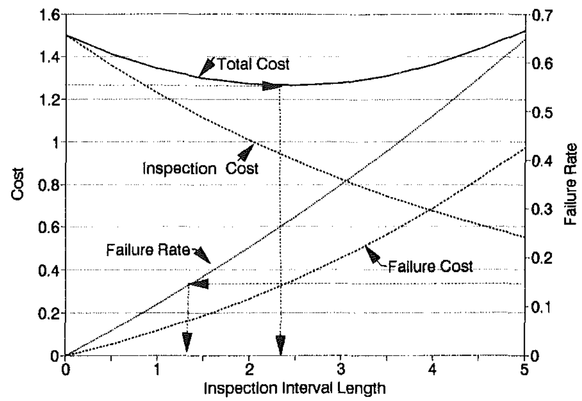

FIG. 3. Inspection Interval Timing Based on Economic Objective, Subject to Safety Constraint. Optimal Interval Is One that Meets Safety Constraint in Form of Limit on Failure Rate, Here Smaller Interval

optimal inspection interval is one that meets a stated objective, subject to stated constraints. For example, suppose the economic objective aims at minimum total cost. The corresponding inspection interval is the one for which the incremental failure cost increase is equal to the incremental inspection cost decrease. The interval at which the cost minimum occurs is the economic optimal interval, as illustrated in Fig. 3. An added safety requirement may, however, restrict the range in which economic optimality is acceptable. This is illustrated in Fig. 3 by a limit on the failure rate. As a result, the safety-constrained optimal interval is shown to be shorter and more expensive than the purely economic optimal interval.

Lifetime Failure Rates

The failure rate of a structure or component at any time of its life can be estimated as the sum of failure rates from various causes. At the initiation of service and in the early years, a structure is exposed to failures from design and construction defects and operation errors. If they can be eliminated by repair and learning, the structure or equipment survives the break-in period. During the subsequent normal service life, the failure rate usually drops to a relatively low and constant background level that accounts for failures by major random external events, such as extraordinary earthquakes or floods, which are rare. As the structure ages, the total failure rate starts again to grow because of wear failures. The resulting total lifetime failure rate function is known for its characteristic shape as "bathtub curve," and is illustrated in Fig. 4 (Harr 1987).

The Teton Dam failure in 1976 (Arthur 1977) was a typical design defect failure. The dam failed by piping along the right abutment, as the reservoir was filled for the first time. Eleven persons lost their lives and the damage was estimated at some 400,000,000 dollars. The Johnstown dam disaster of 1889 (Jansen, 1988) was a combined design and wear failure. An inadequate dam structure, with a spillway too small and inadequately maintained, was overtopped. The subsequent rapid breaching of the 36-year-old dam caused a flood that swept into Johnstown, killing 2,209 people. The sudden collapse of Malpasset arch dam in France by failure of the left abutment on first

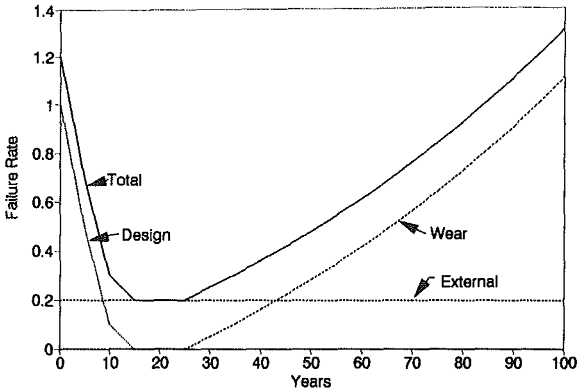

FIG. 4. Instantaneous Failure Rates over Project's Life. Sum of Design Defect Failure Rates, External Event Failure Rates, and Wear Failure Rates Produces "Bathtub Curve"

impoundment in 1959 (Bellier 1977) was another spectacular example of a design failure. Other examples of actual- and near-failure incidents have been reported (*Lessons from* 1983; Jansen 1988).

Wear failures that develop over time are of primary concern for maintenance. If the criterion for a structure's useful life is a permissible total failure rate, then maintenance can prolong the useful life by holding down the wear failure rate. Suppose $f(t)$ is the probability density function (PDF) of useful life, T, a random variable. Then the integral over $f(t)$ from zero to a time t, $F(t)$, is the probability that life will be less than or just equal to t, or $P(T \leq t)$. Hence, $R(t) = 1 - F(t)$ is the probability that the equipment will survive time t, or its reliability, $P(T > t)$. By definition, $dF(t) = f(t)dt$, where $dF(t)$ is the probability that equipment life will last until t but not until $t + dt$. Thus, $f(t)dt$ is the probability of life to end during the interval dt. Now suppose the conditional probability that the equipment will fail, given it has not failed up to now, is $h(t)dt$. Then the probability that the equipment will fail in the upcoming time increment dt can be written as $f(t)dt = h(t)dtR(t)$ or

$$h(t) = \frac{f(t)}{R(t)} \quad \text{(1)}$$

$h(t)$ = an instantaneous failure rate at time t, also called "hazard function" (Sidall 1972; Moan 1982).

To illustrate the use of $h(t)$, suppose $f(t)$ is a normal PDF of equipment life with mean $m = 50$ years and standard deviation $s = 10$ years. The normalized variate of this PDF is $t = (T - m)/s$. Then, the useful life, $T = 50$ years, is represented by $t = (50 - 50)/10 = 0$. The normal PDF is $f(t) = \exp(-t^2/2)/(s\sqrt{2\pi}) = 0.0399$, $F(t) = 0.5$ (according to definition), and $R(t) = 0.5$. The failure rate in the 51st year is then $h(t) = f(t)/R(t) = 0.0399/0.5 = 0.08$ incidences per year. After 10 more years, $T = 60$ years, $t = 1$, $f(t) = 0.0242$, $F(t) = 0.841$, $R(t) = 0.159$, and $h(t) = 0.0242/0.159 = 0.15$ incidences per year. This means that the instantaneous failure rate in the 61st year has about doubled from what it was in the 51st year.

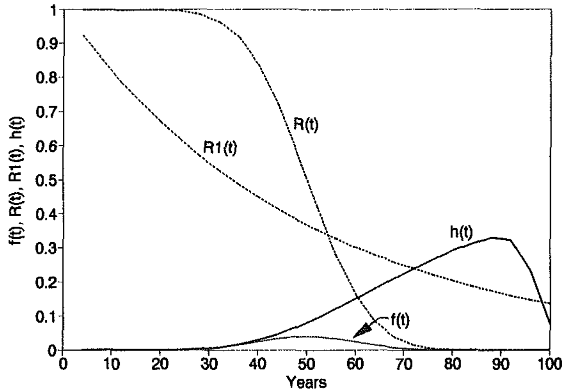

FIG. 5. Relationship between Life Length PDF, $f(t)$, Instantaneous Failure Rate (Hazard Function), $h(t)$, and Reliability $R(t)$. Reliability Associated with Constant Hazard Function Is $R1(t)$

It continues to increase with increasing age, but not unlimited, as $f(t)$ ultimately approaches zero. An illustration of $f(t)$, $R(t)$, and $h(t)$ is given in Fig. 5. The figure also shows that with rising failure rate the equipment reliability drops sharply.

The failure rate function, (1), provides a useful relationship between probability measures. If $f(t)$ can be estimated from data, $h(t)$ can be found, and vice versa, if $h(t)$ can be estimated, $f(t)$ or $R(t)$ can be found. For example, if the failure rate for a certain period is known or predictable, the corresponding reliability during this period can be calculated. From (1) and $dR(t) = -dF(t) = -f(t)dt$ follows $h(t)dt = -dR(t)/R(t)$. Integrating this latter expression yields

$$R(t_2) = R(t_1)\exp\left[-\int_{t_1}^{t_2} h(t)dt\right] \qquad (2)$$

$R(t_1)$ and $R(t_2)$ are the reliabilities at the start and end of the considered period, respectively. Suppose the failure rate of equipment over the 50-year expected life is estimated to be on the average one failure in a lifetime, $h(t) = h = 1/m$. Then, for $R(t_1) = 1$, $t_2 = m$ and $t_1 = 0$, one obtains $R(t_2) = \exp[-(ht_2 - ht_1)] = \exp(-1) = 0.368$. If maintenance over the expected life can reduce the failure rate to half, then $R(t_2) = \exp(-0.5) = 0.6$. The reliability for a constant failure rate, $R_1(t) = \exp(-ht)$, with $h = 1/50$, is shown in Fig. 5. It declines exponentially, even without any increase in the failure rate, but at a much smaller rate than the reliability associated with the increasing failure rate, $h(t)$.

Reliability Assessment

The evaluation of the decision of whether or not to embark on major maintenance may require an assessment of the actual loads on the structure, the remaining resistance or strength of the structure, and the costs and benefits of alternative maintenance or rehabilitation options. Both load (or total cost) and resistance (or total benefit) estimates are usually based on assumptions. The true load and resistance (or costs and benefits) will most likely never

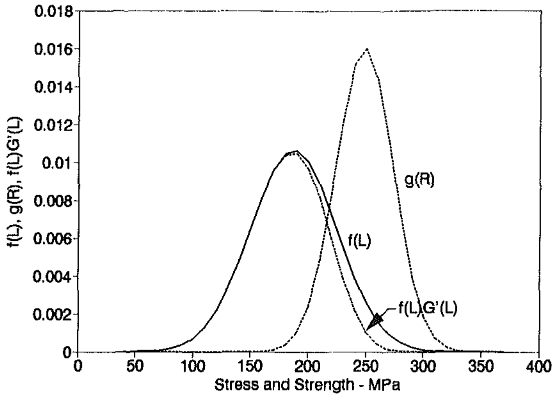

FIG. 6. Structural Reliability as Total Probability of Resistance Exceeding Load. Area under $f(L)G'(L)$ Is Reliability, 0.89 in This Case. Area between $f(L)$, $G'(L)$, and $f(L)$ Is Failure Probability, Here 0.11. Cross Section of Load Bearing Member Is Assumed to be Reduced 50% by Corrosion

be known. The conventional allowable stress design (ASD) recognizes this problem and uses a safety factor (SF), the ratio of a representative resistance and load that is specified to be a certain number greater than one. The load and resistance factor design method (LRFD) does not just use one safety factor but multiple load factors, one for each type of load, and a separate resistance factor (Galambos 1981).

The probabilistic approach goes a step further and establishes both load and resistance effects as probabilistic variables, represented by their PDFs, as shown in Fig. 6. For example, the load may be a probabilistic sum of subloads sampled from subload PDFs. If loads and resistances are independent of each other, a high load may occur with a low resistance and a low load with a high resistance, and so forth. The ratio or difference of two random variables is again a random variable. Arbitrarily specifying one safety factor amounts to selecting one out of many random realizations of possible safety factors. The probabilistic approach can actually test the reliability of an arbitrarily selected safety factor by answering the following question: Is the chosen safety factor (say SF = 1.5) sufficiently reliable? If yes it can possibly be lowered; if no, it needs to be raised (Yen 1986).

Fig. 6 illustrates the possible event of a load L occurring and the probability that it is exceeded by the resistance. The probability of a load L occurring is $f(L)dL$, with $f(L)$ the load PDF and dL a load increment, say 1 MPa. The probability of this load being exceeded by the resistance is

$$G'(L) = \int_{L}^{+\infty} g(R)dR \quad (3)$$

with $G'(L) = 1 - G(L)$; and $G(L)$ = the integral over the resistance PDF, $g(R)$, from $-\infty$ to L. The probability of one event $R > L$ is the product $G'(L)f(L)dL$. The sum of the probabilities of all (mutually exclusive) events (only one of them can actually occur at a time) can be expressed by the integral

$$P(R > L) = \int_{-\infty}^{+\infty} G'(L)f(L)dL \qquad (4)$$

where $P(R > L)$ = the probability that R exceeds L, or the reliability of the structure, which can also be written as $P(Z > 0)$, with $Z = R - L$ being the limit state equation. The limit state is the state that separates success from failure. Here, success is defined as $Z > 0$. In the numerical example of Fig. 6, the areas under $f(L)$ and $g(R)$ are 1 by definition. The area under $G'(L)f(L)$, as calculated by (4), is 0.89. This means that for the case examined, $P(Z > 0) = 0.89$. The difference in the areas $f(L)$ and $G'(L)f(L)$, as illustrated by the departure between the two respective curves, is the departure of the reliability from 1, here $P(Z \leqq 0) = 0.11$, which is the failure probability.

If the PDFs of the variables are unknown, reasonable assumptions on their ranges and frequency distributions can usually be made. A Monte Carlo method can be used to sample these distributions and construct the function Z (Hammersley and Handscomb 1967; Sidall 1972; White and Ayyub 1985). The analysis in Fig. 6 assumes a crack of 50% wall depth has reduced the stressed area to half. Similarly, the reliability for various other crack depths can be determined. Thus, the reliability as a function of crack depth can be derived. Furthermore, if crack growth can be related to time, the reliability decline versus time can be derived and the time when maintenance or rehabilitation should be initiated can be calculated. A reliability assessment to determine the maintenance need of corroding vertical lift gate members was described by Bryant and Mlakar (1987).

Probabilistic Benefit-Cost Analysis

A similar analysis can be used to assess the reliability of a positive net benefit. The limit state equation is again the difference of two random variables, benefit and cost, $Z = B - C$. Fig. 7 shows the PDFs for benefit and cost and for Z, directly derived by the Monte Carlo technique (Taylor and

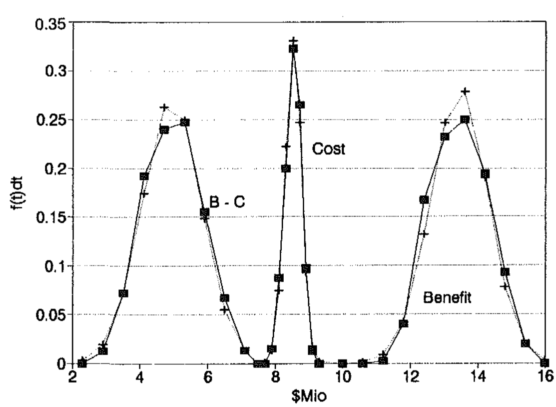

FIG. 7. Economic Reliability as Probability of Net Benefit Exceeding Zero. All PDFs Are Obtained by Monte Carlo Sampling. Straight Lines Connect Empirical Class Frequencies. Crosses Indicate Class Frequencies of Normal PDF Using Means and Standard Deviations of Samples [after Taylor and North (1975)]

North 1975). The data points represent frequencies of class intervals based on 400 evaluations. Both benefits and costs are sums of random variables and, according to the central limit theorem (Moan 1982), they approach normal distributions with increasing sample size, whatever their original distribution, and so does the PDF of Z. The normal PDFs, calculated for the same class intervals using the sample means and standard deviations (+ symbols in Fig. 7) confirm this useful property of random variables.

In contrast to Fig. 6, the PDFs of B and C in Fig. 7 do not intersect. Numerically this means that $G'(L)$ of (3) (with L representing C) is 1 for all L. Thus, also (4) (with R representing B) yields 1 because the area under the PDF of $f(L)$ is 1 by definition. Hence, $P(B > C) = P(Z > 0) = 1$, in other words, the probability of a positive economic safety margin is 100%. Furthermore, the empirical PDF of B-C shows, that also the threshold value of $2,000,000 has 100% probability of being exceeded. This result hinges, of course, on the assumptions made for the input variable ranges.

Maintenance Scheduling

Because of the probabilistic nature of the deterioration process, one never knows precisely in what state a structure or component will be found at the next inspection. It may have stayed the same, it may have deteriorated somewhat or severely, or it may have failed. If the transition to a new state between now and the next inspection only depends on the present state and the transition probabilities can be influenced by repair decisions, then the process can be construed as a Markov decision process (Hastings 1973; Hillier and Lieberman 1974). The possible transitions from the present state to possible future states at the end of the considered inspection interval are represented by the paths in Fig. 8. Each path has a probability attached to it. Now consider two decisions that can be made in the present state: decision 1, no repair; and decision 2, repair. Without repair, the structure can at best remain the same or further deteriorate, but usually it cannot improve itself. The possible paths under these decisions are depicted by the solid lines. With repair, it is assumed that the structure is restored to the "new" state, from where the deterioration process can again take its course. If the maintenance period is short compared to the inspection interval, this can be depicted by a path that first accesses the "new" state with a delay zero, and from there it can access all other possible states. These paths are depicted by the dashed lines. The probabilities that highly deteriorated states will again be accessed are assumed to be smaller, as a consequence of the repair.

The probabilities associated with the possible process paths may be obtainable from data, but often they have to be made up by judgement of experienced maintenance personnel. In other words, they may represent "quantified reasoning" of a maintenance supervisor or a group of experts. Benefits and costs are usually associated with each path, such as operation revenues, repair costs, replacement costs for lost production, and failure costs of an unplanned breakdown (on the path to the failure state). Benefits and costs may also be associated with the accessed states. For each decision, an expected net return is calculated as the probability-weighed returns of all paths. The decision that offers the maximum net return is the economically optimal decision. If several inspection intervals need to be considered, the procedure can be applied recursively backward in time, starting with the last interval.

The described probabilistic decision process can be expressed by a dy-

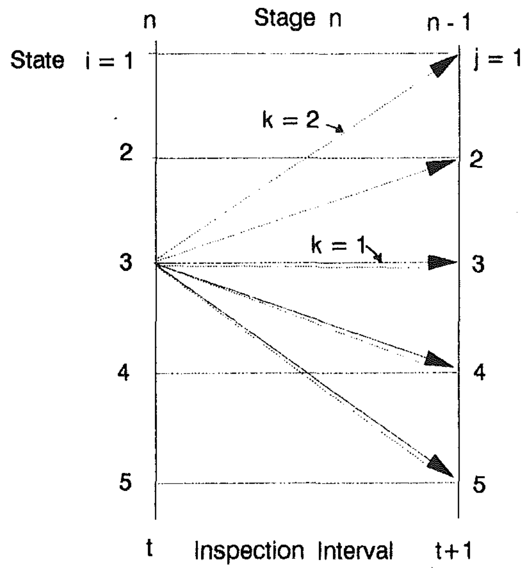

FIG. 8. Possible State Transitions (Paths) for Decisions without and with Maintenance in Stage (Inspection Interval) n: Solid Lines—without Maintenance (k = 1); Dotted Line—with Maintenance (k = 2)

namic programming formulation. Following the notations of Hastings (1973), the return of one transition can be expressed by

$$f(n,i) = \max_{k \text{ in } K} \text{sum } (j = 1, N)\{p(n,i,j,k)[r(n,i,j,k) + c(n)f(n-1,j,k)]\} \ldots (5)$$

where $f(n,i)$ = the maximum net return picked from the net returns of the k-decisions that can be made when the system is in state i at the start of stage n (with $n - 1$ additional stages left to the end of the planning period); K = the total number of alternative decisions in state (n,i), for example, $k = 1$ no repair, $k = 2$ repair, $K = 2$; $p(n,i,j,k)$ = the probability that the process will take path (i,j) during stage n under decisions k; $r(n,i,j,k)$ = the net return associated with path (i,j) in stage n under decision k; $f(n - 1, j, k)$ = the jth state accessed at the end of stage n under decision k; $c(n)$ = a discount factor for stage n. The sum is taken over the $j = 1, \ldots, N$ paths out of state (n,i), weighed by the path probabilities $p(n,i,j,k)$. The maximization is performed by picking the largest of the K returns, $f(n,i)$. The optimal decision sequence (over several stages) thus found is only valid, of course, for stage n. As new information becomes available, usually by the begin of the next stage, $n - 1$, the evaluation must be repeated. A spreadsheet implementation of (4) is described in Mays (1989).

COST EFFECTIVENESS UNDER RISK

The initial capital outlay is a primary hurdle for major maintenance and rehabilitation work. New construction techniques and materials offer ways to overcome it. Roller-compacted concrete spillways, rubber dams, and prestressed concrete pipes are examples of new techniques that have been used for drastically cutting initial capital investments ("RCC Saves" 1990; Post and Stussman 1989; Boyle 1990). But the use of new techniques and materials can also make the lifecycle costs of innovative designs highly probabilistic, in other words, the variances for individual cost items, such as maintenance costs, (premature) replacementn costs, outage costs, service replacement costs, and investment (construction) cost, may be large. The variance of the total cost, being the sum of all individual cost variances, is also increased, e.g., by the factor 2 if a cost of the same variance is added to a previous cost. It is readily seen from Fig. 7 that a widened range of the total-cost PDF will increase the intersection with the benefit PDF, unless the average cost is reduced so that the total-cost PDF is shifted significantly to the left. As soon as any intersection occurs, the reliability of $B - C > 0$, $R = P(Z > 0)$, drops below 100%. Thus, the reliability of the design could turn out to be unacceptably low while the average net benefit remains positive. This result supports the intuitive rule that the riskier a design the cheaper it must be on the average to achieve the same reliability level of an economic criterion, e.g., $P(Z > 0) = 100\%$.

The probabilistic analysis can, in an implicit way, also include the risk attitude of the decision maker through the choice of input variable and system parameter ranges and associated PDFs. For example, a very risk-averse decision maker may feel more comfortable with wider variable ranges than a less risk-averse one. In this way, the probabilistic approach allows a sensitivity analysis of assumptions that are characteristic of the decision maker's attitude toward risk. One important aspect of probabilistic analysis must always be kept in mind: While a value in the vicinity of the expected value has the highest probability of being realized, there is always the possibility of an extreme outcome actually happening and those responsible must be prepared to face it.

Summary

Probabilistic methods are suggested as tools for addressing various aspects of maintenance, such as inspection timing, flaw detection, reliability analysis, benefit-cost analysis, and decision analysis. There is no single answer to many of the problems encountered. The probabilistic approach can respond by producing multiple answers that cover the range of possible answers, with the most likely answer being the expected value. A typical example of a probabilistic approach is the calculation of the probability of exceedance of a specified limit state, which may be defined as net benefit being greater than zero, or as structural strength exceeding load stress. A probabilistic analysis is especially appropriate for analyzing risky designs, as it can include subjective variable ranges and probability density functions that are representative of the decision maker's risk perception. Omission of the fact that benefits and costs are probabilistic can actually lead to nonconservative results and wrong decisions. Also amenable to this type of analysis are deterioration processes that cannot be completely controlled but can be influenced by decisions that affect the transitions from one deterioration state to another. It is concluded that probabilistic methods have the potential of significantly contributing to the ultimate goal of maintenance, namely, minimizing failures and maintaining satisfactory operational efficiency, service reliability, and structural safety.

Appendix I. References

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APPENDIX II. NOTATION

The following symbols are used in this paper:

B = total benefit of maintenance alternative;
C = total cost of maintenance alternative;
$c(n)$ = discount factor for inspection interval n;
dt = increment of variable t;
$\exp(.)$ = exponential function;
$F(t)$ = probability distribution of t, $P(T \leqq t)$;
$f(L)$ = probability density function of load effect;
$f(n,i)$ = net return for state i at begin of stage n;
$f(n-1,j,k)$ = return associated with jth state that can be accessed from state i during stage n under decision k;
$f(t)$ = probability density function of variable t, has unit of 1/standard deviation;
$G(L)$ = $P(R \leq L)$, probability distribution of L;
$G'(L)$ = $P(R > L)$;
$g(R)$ = probability density function of resistance;
$h(t)$ = failure rate, unit of 1/time, e.g., 1/year;
i = structure or system state at start of stage n;
j = structure or system state at end of stage n;
K = total number of decisions that are possible when system is in state i;
k = decision number;
L = load stress on structure or component, computed as force/area, in MPa (1 MPa = 145 psi);
m = sample mean;
$\max_{k \text{ in } K} \text{sum}$ = "find the largest of K sums;"
N = total number of possible paths from state i to new state j;
n = stage, time interval between inspections or decisions, also time instant: start of stage n;
$P(.)$ = probability;
$p(n,i,j,k)$ = probability associated with process path (i,j), stage n and decision k;
R = resistance, computed as force/area, in MPa;
$R(t)$ = reliability of system at time t;
$R1(t)$ = reliability calculated for assumed failure rate;
$r(n,i,j,k)$ = return (net benefit) associated with path (i,j), stage n and decision k;
s = sample standard deviation, has same dimension as mean;
T = random variable representing project age at breakdown;
t = realization of random variable T, normalized as $t = (T - m)/s$ also period of time for exponential PDF;
t_1, t_2 = integration limits, here starting and ending time; and
Z = limit state variable, random variable.