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Analysis of the lag effect of embankment dam seepage based on delayed mutual information

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ABSTRACT

Environmental variables, such as the upstream level, significantly affect the embankment dam seepage lag time. The lag effect should, therefore, be given adequate consideration when determining the saturation line and establishing a mathematical model for seepage. At present, however, the lag time is mainly estimated qualitatively. A method for analyzing the seepage lag effect is presented herein based on the mutual information principle of information science theory, wherein the upstream water level is considered an information source and water level of the seepage pressure is considered an information function point. The method explores application of the directed information transfer index (*DITI*) of mutual information in order to establish an information transfer model between seepage levels of measurement points and upstream levels of the embankment dam. Furthermore, an extreme value of the *DITI* function is adopted as the judging criterion to determine the lag time of the influence of the upstream level on corresponding seepage levels. Results obtained through analysis of an engineering case study, demonstrate the effectiveness and feasibility of the proposed method.

1. Introduction

Following the impoundment of an embankment dam, the seepage field of the dam body is seen to gradually stabilize. At this point, seepage within the dam body is primarily influenced by environmental variables, such as the upstream level, which demonstrate a certain lag effect. As one of the main problems encountered in data analysis of embankment dam monitoring, lag-effect analysis of dam-body seepage plays an important and central role in health diagnosis and safety monitoring of embankment dams. For example, the lag time recorded at each measurement point must be known when using monitoring data to determine the saturation line, thereby determining the relationship between seepage pressure and upstream level. Lag-effect analysis is also required in selecting preset factors when establishing seepage models.

The causes and mechanism of the seepage lag effect of embankment dams are complicated. Gu et al. (2005) summarized the mechanism as follows. Variations in the upstream level alter the dam seepage field from one stable condition to another, and the transition process occurs over a certain time period. This transition is the main cause of the seepage lag effect. Delayed transmission of water pressure in the dam body is another factor contributing to the lag effect. Further, response times of seepage monitoring sensors, such as piezometric tubes and osmometers, with regards to variations occurring in the seepage state could also contribute to the lag effect.

At present, only a few specialized studies are being conducted with focus on the seepage lag effect in embankment dams. The basic method used in these studies involves comparison of the process line of the measured water pressure with those of the corresponding upstream water level. This technique serves to qualitatively determine the lag time of the seepage in terms of the appearance time of "peaks" and "valleys" in the process line. Gu et al. (2005) utilized the mean value of the upstream water level over several time periods as the precession factor for the hydraulic-pressure component in their seepage monitoring statistic model, thereby considering the hysteresis effect of seepage. In addition, several scholars have investigated seepage characteristics of embankment dams while also inadvertently studying aging characteristics of seepage. Ozer and Bromwell (2012) used the finite element method and measured seepage data to simulate seepage and aging characteristics of embankment dams. Simeoni (2012) investigated the relationship between the lag time and pore-water pressure using the piezometer experimental method. Wang et al. (2013) studied the influence of the crack self-healing phenomenon on the

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permeability of dams via 12 simulation tests. Lee et al. (2007) explored the tracer experiment method combined with specific engineering to study the concentrated seepage path of embankment dams. However, a common drawback of the studies mentioned above concerning determination of the seepage lag time is the exclusive utilization of qualitative estimations without quantitative measurements. It is, therefore, necessary to apply new ideas and methods to accurately quantify the seepage lag effect.

Seepage monitoring data reflect changes in the dam seepage field. In information science theory, any substance can be considered as an information source. This information source constantly transfers typical data to the surrounding medium through the information field, thereby altering the characteristics of space and time to generate an information effect. Based on this theory, the upstream water level can be considered as an information source while variations in the upstream water level could be considered typical information conveyed by the information source. Seepage information is transmitted to each monitoring point (information function point) through the seepage field in the embankment dam body, which in turn, alters the water level of the seepage pressure. The concept of mutual information in the information entropy theory can be used to quantitatively describe shared data that the source transfers to the function point (Zhang and Liu, 2000). The paper introduces this concept of mutual information into the study of the seepage lag effect, and proposes a new method for determining the lag time between the seepage pressure in embankment dams and upstream water level.

2. Delayed mutual information

2.1. Mutual information

The domain of the continuous random variables X and Y was set as S; their marginal distributions were represented by $f_X(x)$ and $f_Y(y)$, respectively, and their joint distribution were given by $f_{X,\ Y}(x,y)$. The information entropy $H(X)$ and $H(Y)$ of the continuous random variables X and Y could, thus, be defined as (Resconi et al., 2013) follows.

$$H(X) = -\int_S f_X(x) log f_X(x),\quad H(Y) = -\int_S f_Y(y) log f_Y(y) \tag{1}$$

Further, the mutual information of the two-dimensional joint distribution of X and Y could be defined as (Khademi et al., 2017) follows.

$$I(X;Y) = \iint_S f_{X,Y}(x,y)\, log \frac{f_{X,Y}(x,y)}{f_X(x) f_Y(y)} dxdy \tag{2}$$

Mutual information represents the amount of related information that is jointly owned by two or more attributes. A considerably large amount of mutual information indicates a close relationship between the concerned attributes or variables.

2.2. Directed information transfer index (DITI)

Movement is an important feature of information. Once a certain information is produced by the source, a flow is set up through its transmission, and any point in time and space of the structure (information function point) is affected, thereby resulting in different degrees of information benefits. *DITI* combines the information transfer theory with information entropy. Information entropy represents the quantity of directional transmission of specific information from the source in order to determine the degree of mutual influence between the source and the function point of information.

Assuming that X and Y refer to the information source and information function point in the information field, respectively, *DITI* $(X;Y)$ of the information source X to information function point Y can be represented as follows (Zhang and Liu, 2000).

$$DITI(X;Y) = \frac{I(X;Y)}{H(Y)} \tag{3}$$

DITI exhibits two important characteristics—(1) it measures the information transfer capability between the two information (X and Y); (2) it describes the degree of coupling between X and Y.

2.3. Delayed mutual information

The original data sequence was set to $X = \{x_1, x_2, \cdots, x_n\}$ and viewed as a source of information. The length of this data sequence was n. With an X delay considered as t, the reconstructed data sequence Y could be obtained. If Y be set as the information function point, then the length of the data sequence becomes $n - t$. Thus, $Y = \{y_1, y_2, \cdots, y_{n-t}\} = \{x_{1+t}, x_{2+t}, \cdots, x_n\}$. The mutual information between X and Y is, therefore, defined as delayed mutual information (Albers and Hripcsak, 2012).

The delay time t is theoretically desirable for either value, and is usually selected based on the characteristics of research questions. Since t is an unknown quantity, a series of reconstructed data sequences $\{Y_i, i = 1 \sim m\}$ could be obtained by setting t different delay times $t = \{t_1, \cdots t_i, \cdots, t_m\}$. That is,

$$Y = \{y_{i1}, y_{i2}, \cdots, y_{i(n-t_i)}\} = \{x_{1+t_i}, x_{2+t_i}, \cdots, x_n\} \tag{4}$$

where Y_i is the reconstructed data sequence when the delay time $t = t_i$.

Takens' theorem (Takens, 1981) indicates that when the delay time t covers the actual delay between X and Y, a reconstructed data sequence $Y_i(t_i = \tau)$ equivalent to the original data sequence X in the topological sense always exists. The mutual information between the original sequence X and reconstructed sequences $Y_i(\mathrm{i} = 1, 2, \cdots, \mathrm{m})$ is then calculated and analyzed. Subsequently, one reconstructed sequence $Y_i(t_i = \tau)$ that is equivalent to the original data sequence X in the topological sense could be obtained using Eq. (4) in order to determine the actual delay time, τ.

3. Determining seepage flow lag time based on delayed mutual information

3.1. Basic principles

Let us assume the data sequence of dam environment variables (such as upstream water level) to be $X = \{x_1, x_2, \cdots, x_n\}$, and the data sequence of dam effect variables (such as the water level of the seepage pressure at the measurement point) are correspondingly represented by $Y = \{y_1, y_2, \cdots, y_n\}$. Given that the environmental variables influence the monitoring effect variables of the dam, the environment variable sequence X is considered as the information source while the effect variable sequence Y is regarded as the information function point.

For seepage monitoring in embankment dams, several monitoring cross sections are usually selected, wherein several measurement points are arranged. A piezometric tube or an osmometer is buried at each measurement point to facilitate regular monitoring of the water level of seepage pressure, and the saturation line of each monitoring cross section is obtained using the monitored data. The water level of the seepage pressure at measurement points in the dam body is primarily influenced by the upstream water level, demonstrating varying degrees of hysteresis. The concept of delayed mutual information in Eq. (4) is extended to two variables with correlations X and Y. The dam monitoring effect variable $Y = \{y_1, y_2, \cdots, y_n\}$ is considered as the corresponding data sequence of the environmental variable sequence $X = \{x_1, x_2, \cdots, x_n\}$ after delay time t. The actual delay time τ of each measurement point is unknown; thus, a suitable method must utilize time to determine the actual delay time τ from amongst a set of assumed delay times $t_i(i = 1, 2, \cdots, m)$. Consequently, the approach proposed in this paper for determining the embankment dam seepage lag time is based on information entropy and delayed mutual information. Further, the method utilizes *DITI* as its parameter and an extreme solution as its criterion.

3.2. Modeling method

It was assumed that the upstream water level is X, and the water level of the seepage pressure was $Y_0 = \{y_1, y_2, \cdots, y_n\}$. The reconstructed data sequence of Y_0 was set as Y_i. Therefore, $Y_i = \{y_{1+t_i}, y_{2+t_i}, \cdots, y_n\}$ could be obtained by setting the delay time as $t_i (i = 1, 2, \cdots, m)$. Given that the marginal distribution of Y_i is $f_{Y_i}(y_i)$, based on Eq. (1), the information entropy of Y_i could be expressed as

$$H(Y_i) = -\int_s f_{Y_i}(y_i) \log f_{Y_i}(y_i). \tag{5}$$

Given that $f_{X,Y_i}(x, y_i)$ refers to the joint distribution of X and Y_i, the mutual information of the two-dimensional joint distribution of X and Y_i could be expressed as follows.

$$I(X; Y_i) = \iint_S f_{X,Y_i}(x, y_i) \log \frac{f_{X,Y_i}(x, y_i)}{f_X(x) f_{Y_i}(y_i)} dxdy \tag{6}$$

The *DITI* of X to Y_i, therefore could be written as

$$DITI(X; Y_i) = \frac{I(X; Y_i)}{H(Y_i)} \tag{7}$$

Function $DITI(t)$ for delay time $t_i (i = 1, 2, \cdots, m)$ was obtained using Eq. (7). The principle of information transmission (Albers and Hripcsak, 2012) indicates that when the information transfer index function *DITI* (t) attains the first extreme maximum value, the coupling between the two sets of information is the highest, and the amount of directional information transmission attains maxima. Here, the corresponding time $t_i = \tau$ was considered the actual seepage lag time τ. The actual lag time, therefore, must satisfy Eq. (8).

$$\frac{\partial DITI(t)}{\partial t} = 0 \; (t \geq 0) \tag{8}$$

Upon obtaining $DITI(X; Y_i)$ in different delay times t_i, the process line of $DITI(X; Y_i)$ could be drawn. The actual lag time τ between the seepage pressure and upstream water level can be estimated, and regression could be used to obtain the function $DITI(t)$. By employing Eq. (8), the extremum of the function can be solved. Therefore, the actual lag time τ between seepage pressure and corresponding upstream water level can be quantitatively obtained.

A schematic of the process of modeling the lag-effect analysis of seepage based on delayed mutual information is depicted in Fig. 1.

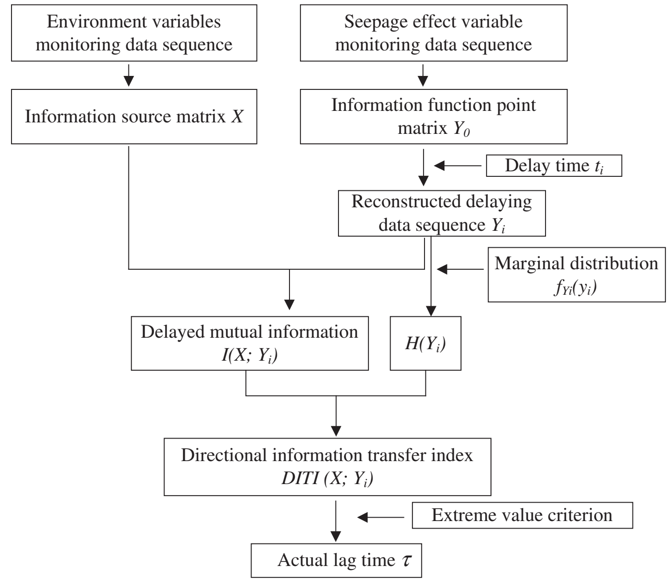

Fig. 1. Modeling process schematic of the lag effect of seepage analysis based on delayed mutual information.

4. Application

4.1. General situation

A case study was performed by considering a dam with the following parameter values—peak dam height, 51.0 m; dam crest elevation, 63.0 m; total top length, 630.0 m. The slope ratio of the upstream slope was 1:3.0, whereas the slope ratios of the downstream slope were 1:2.5, 1:2.75, and 1:3.0. The dam filling comprised sandy loam with gravel with an average permeability coefficient of 4.3×10^{-3} cm/s. The anti-seepage comprised a clay core wall with an average permeability coefficient of 1.6×10^{-6} cm/s. Rockfill prism drainage was arranged for at the dam toe.

Four seepage monitoring cross sections were arranged within the body of the embankment dam (cross sections A, B, C, and D, respectively), and within each monitoring cross section, four piezometric tubes buried with an osmometer in order to accurately monitor the seepage pressure of the dam in an automated manner. The observation frequency was set as once a day. In this study, the observed water level of the seepage pressure at the four measurement points (B1, B2, B3, and B4, respectively) was selected from the monitoring section B located in the middle of the embankment dam. The method for lag-effect analysis of seepage proposed in this paper was applied in this case study. The monitoring layout of cross section B is depicted in Fig. 2. Measurement point B1 is located at the core wall, while other measurement points are located behind the core wall. Process lines of the seepage pressure water level corresponding to each measurement point as well as upstream water levels measured between 2012 and 2014 are depicted in Fig. 3.

4.2. Establishment of the model

Without loss in generality, the selected upstream water level corresponded to that of the seepage pressure at the measurement point B2 observed from January 2012 to December 2014. Given that the upstream water level was considered the information source X while the seepage pressure water level of B2 was considered the information function point Y, the proposed method was applied to determine the actual delay time τ between the water level of the seepage pressure at B2 and upstream water level.

The upstream water level and seepage pressure at each measurement point were automatically recorded once every day, as previously mentioned. Thus, when the delay-time interval t was set as one day (i.e., delay time $t_i = 1, 2, \cdots, m$ (days)), the reconstructed delay data sequence of the water level of the seepage pressure at B2 was $Y_i (i = 1, 2, \cdots, m)$.

Based on the upstream water level X and the reconstructed delay data sequence of the water level of the seepage pressure at B2, $Y_i (i = 1, 2, \cdots, m)$ from January 2012 to December 2014 was obtained. In accordance with Eqs. (5)–(7), $DITI(t)$ of the upstream water level to the reconstructed delay data sequence of the water level corresponding to the seepage pressure at B2, $Y_i (i = 1, 2, \cdots, m)$ under different delay times t could be obtained and represented in the form of the data sequence $DITI_{B2}(t)$ $(i = 1, 2, \cdots, m)$.

The above calculation process could be performed through use of a computer program with the following specific calculation steps (for delay time $t = 22d$).

(1) Marginal distribution is the basis of mutual information calculation. In the joint distribution composed of multidimensional random variables, the probability density function of a single random variable itself is representative of the marginal distribution. With

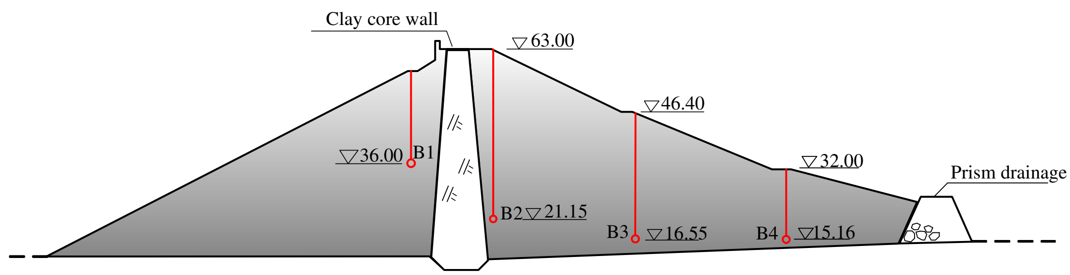

Fig. 2. Layout of the monitoring section B.

regards to dam-safety monitoring, the water level of the seepage pressure and the upstream water level are considered random variables, which could constitute a joint distribution function, but the marginal distribution of the water level of the seepage pressure and the upstream water level are unknown. In engineering, if the marginal distribution is unknown, the Gaussian kernel function method is usually applied to obtain an estimate of the marginal distribution.

We, therefore, used the Gaussian kernel function (Esmaeilbeigi and Garmanjani, 2017) to calculate the marginal distribution $f_{Y_{22}}(y_{B2})$ representing the data sequence for the water level corresponding to the seepage pressure Y_{22} at B2 after a delay time $t = 22d$. The expression for the standard Gaussian kernel function $K(t)$ is as follows.

$$K(t) = e^{-\frac{t^2}{2}} \tag{9}$$

The corresponding equation for function estimation of the marginal distribution $f_{Y_{22}}(y_{B2})$ representing the data sequence of the water level corresponding to the seepage pressure at B2 after a delay of 22d could be written as

$$\hat{f}_{Y_i}(y_{B2}) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{y_{B2} - y_{B2i}}{h}\right) \tag{10}$$

where n is the total number of samples, and h represents the scale parameter of the kernel function, which is usually controlled by the asymptotic mean square error (AMISE). When AMISE is at its minimum value, the optimal value of scale parameter h is given as follows (Thomas et al., 2014).

$$h = \left(\frac{4}{t+2}\right)^{1/(c+4)} n^{-1/(c+4)} \tag{11}$$

where n is the total number of samples. c refers to the space dimension, i.e., the number of random variables. In this example, $c = 2$. t is the delay time represented in days. In this example, $t = 22d$.

In accordance with Eqs. (9)–(11), the scale parameter h is 0.2401 when $t = 22d$. Thus, the marginal distribution $\hat{f}_{Y_{22}}(y_{B2})$ of each data point in the data sequence of the water level of the seepage pressure of B2 after a delay of 22d is given by

$$\hat{f}_{Y_{22}}(y_{B2}) = \frac{1}{1073 \times 0.2401}\sum_{i=1}^{1073} K\left(\frac{y_{B2} - y_{B2i}}{0.2401}\right)$$

Similarly, the marginal distribution estimation $\hat{f}_x(x)$ of each data point in the upstream water level set X is given by

$$\hat{f}_x(x) = \frac{1}{1073 \times 0.2401}\sum_{i=1}^{1073} K\left(\frac{x - x_i}{0.2401}\right)$$

According to Eq. (5), the information entropy of the water level corresponding to the seepage pressure at B2 could be calculated using

$$H(Y_{22})_{B2} = -\sum_{i=1}^{n} f_{Y_{22}}(y_{B2}) \log f_{Y_{22}}(y_{B2}) = 0.7036.$$

(2) The two-dimensional joint distribution of random variables X and Y_{22} could also be solved using the Gaussian kernel function. The joint distribution estimation of X and Y_{22}, $\hat{f}_{x,Y_{22}}(x, y_{B2})$, is expressed as follows:

$$\hat{f}_{x,Y_{22}}(x, y_{B2}) = \frac{1}{\mathrm{n}}\sum_{i=1}^{n} K_x\left(\frac{x - x_i}{h_x}\right) K_y\left(\frac{y - y_{B2i}}{h_y}\right) \tag{12}$$

where h_x and h_y represent scale parameters of X and Y_{22}, respectively. The space dimension of h_x and h_y is set as $c = 2$. Therefore, based on Eq. (11), the scale parameter of X was equal to that of Y_{22}; i.e., $h_x = h_y = 0.2401$. The joint distribution estimation $\hat{f}_{x,Y_{22}}(x, y_{B2})$ for X and Y_{22} could be expressed as

$$\hat{f}_{x,Y_{22}}(x, y_{B2}) = \frac{1}{1073}\sum_{i=1}^{1073} K_x\left(\frac{x - x_i}{0.2401}\right) K_y\left(\frac{y - y_{B2i}}{0.2401}\right)$$

Thus, the delayed mutual information of X and Y_{22} can be calculated using Eq. (7) as follows:

$$I(X; Y_{22}) = \sum_{i=1}^{nx}\sum_{i=1}^{ny} \hat{f}_{X,Y_{22}}(x, y_{B2}) \log \frac{\hat{f}_{X,Y_{22}}(x, y_{B2})}{\hat{f}_X(x)\hat{f}_{Y_{22}}(y_{B2})} dxdy = 0.1968$$

(3) Based on Eq. (7), $DITI(t)$ for X and Y_{22} is given by

$$DITI(X; Y_{22}) = \frac{I(X; Y_{22})}{H(Y_{22})_{B2}} = \frac{0.1968}{0.7036} = 0.2797$$

Similarly, $DITI_{B2}(t)(i = 1, 2, \cdots, m)$ of the upstream water level to the reconstructed delay data sequence of the water level corresponding to the seepage pressure at B2, under different delay times, $t_i(i = 1, 2, \cdots, m)$, can be obtained using the method above.

Based on the data sequence of $DITI_{B2}(t)(i = 1, 2, \cdots, m)$, the process

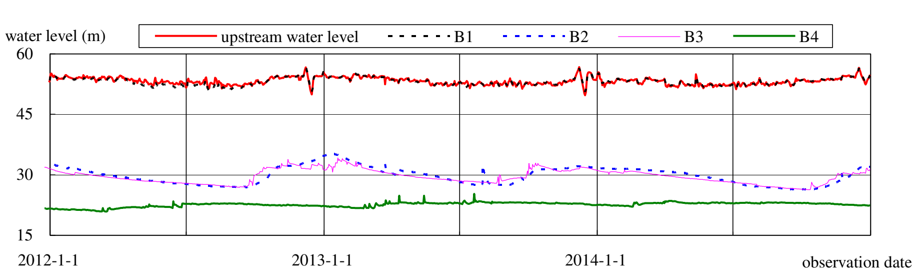

Fig. 3. Process lines of the water level of seepage pressure for each measurement point and upstream water levels between 2012 and 2014.

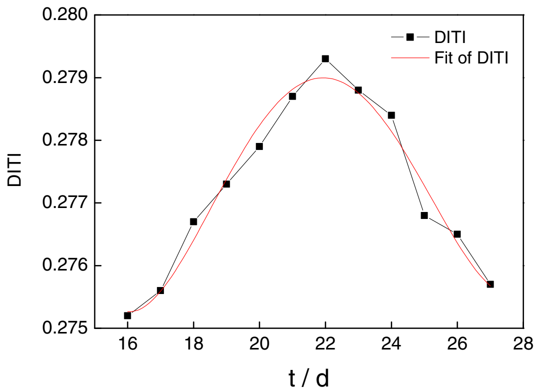

Fig. 4. *DITI* process lines between the upstream water level and B2.

line for $DITI_{B2}$ is plotted as depicted in Fig. 4. In the figure, the maximum value of $DITI_{B2}$ appears at $t = 22d$. It is possible to qualitatively determine that the actual lag time between the water level of the seepage pressure and the upstream water level is about 22 days.

The fitting curve of $DITI_{B2}(t)$ was obtained using the polynomial regression fitting method available in MATLAB.

$$DITI_{B2}(t) = 0.223 + 5.110 \times 10^{-3}t - 1.168 \times 10^{-4}t^2 \quad (13)$$

This fitting curve for $DITI_{B2}(t)$ is also plotted in Fig. 4. The maximum value of Eq. (13) is obtained based on Eq. (8). Thus, it can be quantitatively determined that the actual lag time for the upstream water level corresponding to the water level at the measurement point B2 is 22.0 days.

Similarly, $DITI_{B1}(t)$, $DITI_{B3}(t)$, and $DITI_{B4}(t)$ $(i = 1, 2, \cdots, m)$ of the upstream water level to the reconstructed delay data sequences corresponding to the water level of the seepage pressure at B1, B2, and B3, respectively, $Y_i(i = 1, 2, \ldots, m)$, under different delay times $t_i(i = 1, 2, \cdots, m)$, could be obtained using the above method.

With the data sequences of $DITI_{B1}(t)$, $DITI_{B3}(t)$, $DITI_{B4}(t)$ $(i = 1, 2, \cdots, m)$, process lines corresponding to $DITI_{B1}$, $DITI_{B3}$, and $DITI_{B4}$ could be plotted, as depicted in Figs. 5–7. As shown in the figures, the actual lag time τ of the water level at the B1 measurement point is approximately zero days for the upstream water level; that at point B3 is approximately 25 days while the corresponding delay time at B4 is approximately 28 days.

The respective fitting curve equation of Trend-line equations for $DITI_{B1}(t)$, $DITI_{B3}(t)$, and $DITI_{B4}(t)$ can be obtained using MATLAB.

$$DITI_{B1}(t) = 1.385 - 4.732 \times 10^{-1}t + 7.490 \times 10^{-2}t^2 - 4.030 \times 10^{-3}t^3 \quad (14)$$

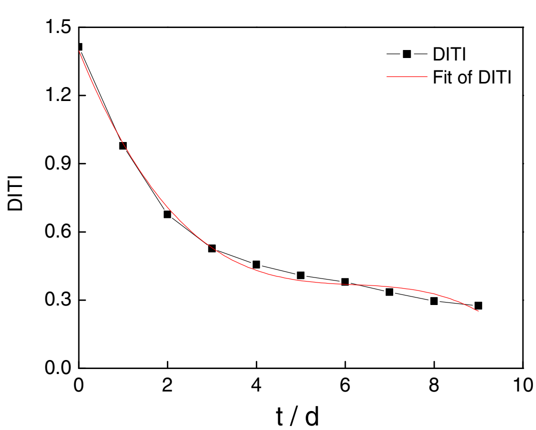

Fig. 5. *DITI* process lines between the upstream water level and B1.

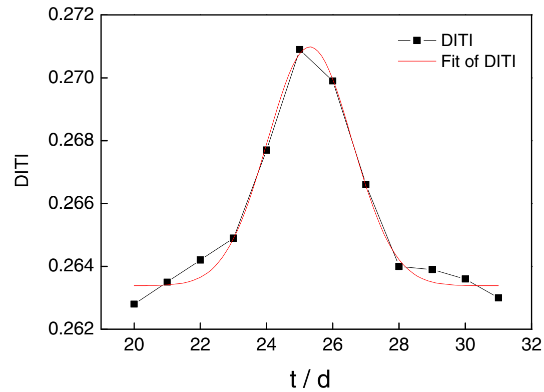

Fig. 6. *DITI* process lines between the upstream water level and B3.

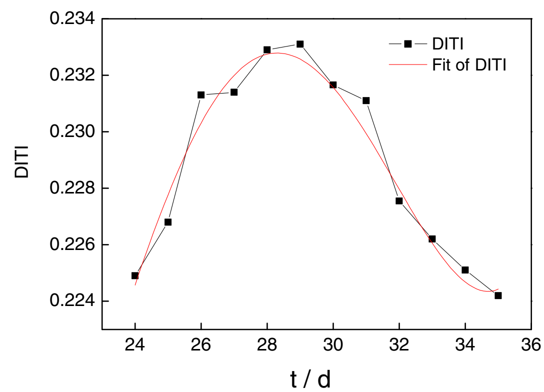

Fig. 7. *DITI* process lines between the upstream water level and B4.

$$DITI_{B3}(t) = 5.414 - 0.842t + 5.109 \times 10^{-2}t^2 - 1.360 \times 10^{-3}t^3 + 1.342 \times 10^{-5}t^4 \quad (15)$$

$$DITI_{B4}(t) = -0.740 + 0.093t - 2.910 \times 10^{-3}t^2 + 2.986 \times 10^{-5}t^3 \quad (16)$$

Extreme values of Eqs. (14)–(16) could be solved for by employing Eq. (8), and it can be quantitatively determined that the actual lag time of the water level at B1, B3, and B4 is zero days, 25.3 days, and 28.2 days, respectively. The lag times obtained via direct estimation of DITI process lines were found to be fundamentally similar to lag times obtained via quantitative solutions and fitting curve. The former method is simpler while the latter is more accurate.

Measurement point B1 is located ahead of the core wall. Therefore, the water level of the seepage pressure contributes to no lag in the upstream water level. Points B2, B3, and B4 are located behind the core wall. Thus, the lag effect between the water level corresponding to the seepage pressure and upstream water level is obvious. The downstream body of this dam is relatively uniform; consequently, the difference in lag times recorded at B2, B3, and B4 is not significant.

The lag time of the water level of the seepage pressure to the upstream water level is difficult to obtain by intuitional judgment according to the process line depicted in Fig. 3. However, the proposed method is capable of quantitatively estimating the lag time. The proposed method, therefore, provides a new approach for quantitatively analyzing the seepage lag effect in embankment dams.

4.3. Validation of modeling results

To verify the rationality of the proposed method, the finite element engineering simulation software ABAQUS was used to calculate the theoretical infiltration line of monitoring section B of the embankment dam under normal water levels.

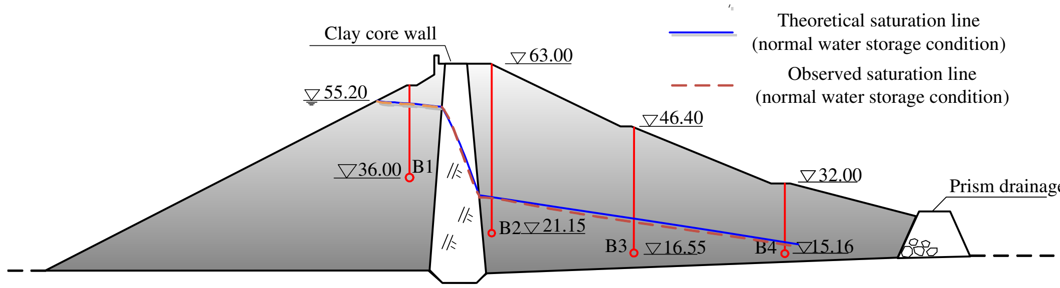

Fig. 8. Comparison between the saturation line calculated theoretically and that determined using the proposed method.

Fig. 8 presents a comparison between the theoretical saturation line of monitoring section B under normal water level conditions, determined through finite-element calculations, and the observed saturation line determined using the proposed method.

From Fig. 8, it is clear that the saturation line determined by the proposed method concurs with the theoretical saturation line of monitoring section B determined via finite-element calculations. This finding confirms the effectiveness and feasibility of the proposed method for analysis of seepage lag in embankment dams based on delayed mutual information.

5. Conclusion

The lag effect in the seepage of an embankment dam is influenced by environmental variables, such as the upstream water level. This lag effect has an important influence on health diagnosis and safety monitoring of embankment dams. However, the lag time is primarily estimated qualitatively at present because of a lack of quantitative analysis methods. This paper proposes an analysis method for seepage lag based on DITI estimated by exploring the mutual information principle with the upstream water level considered as an information source and the water level of the seepage pressure considered as an information function point. The proposed method provides a new approach for quantitatively determining lag times of the water level of the seepage pressure in embankment dams with respect to the upstream water level. The proposed method has obvious advantages, particularly in the case of a weak lag that typically exists between the water level of the seepage pressure and upstream water level. Our analysis of the model demonstrates that the proposed method is reasonably feasible and effective for use in the said application.

In addition to determining the lag time of the seepage in embankment dams, the proposed method can also be employed for lag effect analysis of other related variables with hysteric characteristics.

Marginal distribution is the basis of mutual information calculation. In this study, the Gaussian kernel function was used to estimate the marginal distribution of the water level of the seepage pressure and the upstream water level. However, determination of the actual marginal distribution of seepage pressure and upstream water level require further research.

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