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Positioning–pricing problem of heterogeneous duopoly with uncertain consumer preferences

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Abstract Small- and medium-sized firms in China have competed passively with advantaged enterprises since 2000, and the prosperity indexes have always been in the recession. This paper aims to find the optimal positioning–pricing strategies for disadvantaged small- and medium-sized firms and explore the complex influence of consumer preference uncertainty in the optimal strategies in duopoly market. In order to do so, the classic Hotelling model is modified to be a heterogeneous duopoly game model under consumer preference uncertainty. By using the backward recurrence algorithm, we first get the optimal equilibrium and then figure out the expected equilibrium solution through mathematical expectation method. The computational results show that disadvantaged enterprises could survive themselves in fierce competition as long as they choose appropriate strategies. In addition, consumer preference uncertainty plays a significant role in affecting the optimal strategy decisions of positioning and pricing in duopoly since producers can hardly obtain complete and perfect information about the market. Specifically, the increased uncertainty of consumer preference generally raises the expected equilibrium price and profit of duopoly enterprises. However, the expected equilibrium profit of the advantaged enterprise in a duopoly decreases with consumer preference uncertainty increasing under certain conditions.

Keywords Heterogeneous duopoly game · Product differentiation · Random variables · Expected equilibrium

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1 Introduction

With the acceleration of economic globalization, an increasing number of strong enterprises are expanding their market share through their advanced technologies and almost perfect products. Faced with the competitive pressures coming from powerful opponents, many low-tech enterprises (disadvantaged ones) intend to take a series of countermeasures, for instance, lowering prices to resist the invasion. However, most of them ended in failure because of their inappropriate competitive strategies. The issue of how disadvantaged enterprises can survive in such an uncontrollable fierce competitive market has become an urgent problem to be solved.

In previous literature, there is an agreement that disadvantaged and advantaged enterprises can coexist in duopoly market. For example, the Cournot model describes how two non-cooperative manufacturers set production quantities separately to maximize their own profits and achieve equilibrium. Then, pointed out that the Cournot model failed to give a proper explanation of how prices were determined in the oligopoly market. He proposed the so-called Bertrand model in which price was taken to be the decision variable. In the Bertrand equilibrium, the price of products was equivalent to marginal cost, namely no firms could obtain positive profit which is the "Bertrand paradox." In order to deal with this problem, in 1897, Edgeworth introduced "capacity constraints" to the Bertrand model and proved that the equilibrium of the Bertrand model does not necessarily exist. However, all of the three models ignored the production differentiation, thereby coming to the conclusion that enterprises would choose to set the same price in equilibrium (if it exists). To bridge this gap, Hotelling (1929) established a linear location model to explain why the same products had different prices in reality. He added transportation cost into the spatial competition model and found that firms in duopoly would gather in the middle of the line segment in equilibrium, known as "principle of minimum differentiation." D'aspremont et al. (1979) continued the setting of the Hotelling model and revised its hypothesis. He regarded the transportation cost as a quadratic function of distance and got a completely different result from Hotelling. That is, two firms in duopoly would locate at two ends of a line segment in equilibrium. "Principle of maximum differentiation" is exactly derived from his study. Both "principles" have their theoretical basis and it is not important to debate which one makes more sense (Netz and Taylor 2002; Gao 2010), but one thing that is for sure is that differentiation enables disadvantaged enterprises to find their niches in an oligopoly market.

Differentiation is generally reflected in the quality and price of products and drives consumers' purchasing decisions (Karmarkar and Pitbladdo 1997; Tang et al. 2006; Wang et al. 2006). Consumers purchase a product based on both its price and its quality level (Anderson and Parker 2002). Therefore, it is important for firms to wisely compete in determining their prices and quality levels to maximize their profits. Consider Chinese frozen food as an example, although high-class quick-frozen dumplings are comparatively pricey than low-end ones, their producers still have healthy profits because of the products' high degree of differentiation (nutrition, taste, etc.; Lu et al. 2005).

The classic Hotelling model has always been used to study differentiation strategy problems. However, the Hotelling model is not in full compliance with reality mainly because it does not take any uncertainties into account and concludes that duopoly firms with the same unit cost will finally concentrate on the center of the market. Most relevant research on the Hotelling game model lacked consideration of product cost and put forward the hypothesis that products' homogeneities are between two enterprises (namely same per-unit cost). Under the uncertainty of consumer preference, this paper modified the classic Hotelling model and analyzed the positioning–pricing problem in a duopoly context.

This paper has two main purposes: The first is to investigate how disadvantaged enterprises compete with strong enterprises by appropriate positioning and pricing strategies. The second is to explore the complex impact of consumer preference uncertainty on expected equilibrium position and price. To achieve this, a two-stage game of heterogeneous duopoly with uncertain consumer preference was established. In this model, product differentiation is reflected with the quality of products instead of spatial location, and consumer demand and production costs are characterized by independent uncertainty variables. The calculation process is divided into two steps in this paper. First, backward recurrence algorithm has been used to solve the optimal equilibrium. Second, with the uncertainty of consumer preference, uncertain theory has been applied to figuring out the expected equilibrium solution through mathematical expectation method.

The rest of this paper is organized as follows: In Sect. 2, we briefly review some recent related literature about positioning–pricing problems and consumer preference uncertainty; then, the modified Hotelling model under consumer preference uncertainty is introduced in Sect. 3. In Sect. 4, we present the computational results and the implications. Finally, discussion and conclusion are proposed in Sect. 5 and Sect. 6, respectively.

2 Literature review

The literature on the two-stage positioning–pricing competition is quite diverse since consumers purchase a product normally based on both price and quality level (Economides 1999; Syam et al. 2005; Liu and Serfes 2005; Tang et al. 2006; Wang et al. 2006). Many prior studies established creative models with different demand functions and cost functions to deal with quality and price problems in oligopoly market. (Banker et al. 1998; Balasubramanian and Bhardwaj 2004; Matsubayashi 2007; Kim and Chhajed 2002; Chi 2007; Matsubayashi 2007).

The Hotelling model is widely known as a classic spatial competition model and always be used to study positioning–pricing competition in an oligopoly market. A number of studies further extended the Hotelling model and introduced additional forces to investigate how the two-stage competition changes with the new factors, like advertising and promotions (Bloch and Manceau 1999; Shaffer and Zhang 2002). Based on the Hotelling model, there are also some related work focused on the impact of consumer heterogeneity on the positioning–pricing problem (Rhee 1996; Matsubayashi and Yamada 2008; Coibion et al. 2007; Sun et al. 2013). In this case, consumers' loyalty, earning and the sensitivity of commodity properties can be used to define the heterogeneity and largely affect the outcome of price and quality competition.

However, in many areas of related research, enterprises make decisions with perfect and complete information (Eston and Lipsey 1975; Salop 1979; Chen and Riordan 2007; Diao et al. 2009). Consumer preference uncertainty on products has become an important factor which cannot be ignored in positioning and pricing decisions. K.J. Arrow, the Nobel laureate, pointed out that uncertainty is one of the basic characteristics of economic behaviors such that no decision can be made without substantial uncertainty. Therefore, as a universal differentiation force, uncertainty has attracted attention gradually and its related theory, which has been applied more often by research of heterogeneous oligopoly games (Ronald and Lázló 1996). By reviewing the literature concerning on the application of uncertain theory on oligopoly model, it is easy to find much research which has revised the assumption of consumer preference in the Hotelling model. For example, Anderson et al. (1992) proposed a random preference model by introducing extra dimensions of product heterogeneity and pointed out that the randomness of consumer purchasing behavior hindered enterprises' capture of consumer demand and that duopoly enterprises would concentrate on the center of segment. Takatoshi and Thisse (1995) relaxed the assumption that firms must be located inside the market space and found out two asymmetric equilibriums in the case of symmetric triangular density. Harter (1997) proposed a duopoly game with uncertain consumer preference distribution. According to his model, firms in oligopoly market hardly found market niches and would fail to find a sufficiently large pool of potential entrants. Gu et al. (2002) modeled a two-stage Hotelling duopoly game to analyze how agglomerative demand affects strategies of product differentiation. He set the consumer preference as trigonometric function form and concluded that the competitive enterprises always moved to the center of the city (or information center) for profit maximization. These studies relaxed the assumption of consumer preference and provided interesting views on location and pricing strategies of differentiation in oligopoly spatial competitions.

3 Heterogeneous duopoly game model

3.1 Classic Hotelling model

The classic theoretical models of oligopoly competition before the Hotelling model (such as the Cournot model, Bertrand model, Edgeworth model) have almost completely neglected the importance of space, taking the market as a "point" for granted whereas Hotelling expands market into a linear space, in which consumers are uniformly distributed. Consumers' demands for commodities are completely inelastic, and the commodity differentiation between two manufacturers is not reflected in the physical and chemical properties of commodities,[1] but in the expenses that consumers purchase the commodities such as transportation costs.

The Hotelling model can be standardized into a static two-stage game (Aguirre and Arroyuelos 2001). In the first stage, duopoly enterprises choose the best location simultaneously and set the proper price to maximize their profits in the second stage. The concept of a sub-game perfect Nash equilibrium

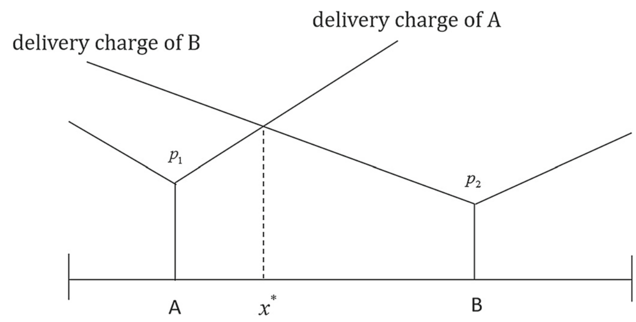

Fig. 1 Hotelling schematic diagram of double linear model

can be used to derive the Nash equilibrium solution by backward recurrence algorithm (Fig. 1).

Formulation of the classic Hotelling model is as follows: There are two firms producing homogeneous goods at constant and equal marginal costs. Consumers are uniformly distributed on a line segment [0, 1], with two manufacturers positioning two sections of the space. Homogeneous commodities are sold in price p_1 and p_2, respectively, and unit transportation cost is t. Assuming the commodities bought by someone is located at x from two manufacturers are non-differentiated, we have $p_1 + tx = p_2 + t(1 - x)$, and then the manufacturer demand function is satisfied

$$D_i(p_i, p_j) = \frac{(p_j - p_i + t)}{2t}$$

Profit function can be represented as

$$\pi_i(p_i, p_j) = \frac{(p_i - c)(p_j - p_i + t)}{2t}$$

Let $\frac{\partial \pi_i(p_i, p_j)}{\partial p_i} = 0$, we get equilibrium price $p_1^* = p_2^* = c+t$ to maximize profit, and equilibrium profit is $\pi_1^* = \pi_2^* = \frac{t}{2}$. In this case, manufacturers obviously have no way to sell products with price exceeding cost, and their profit would consequently be equal to zero. Therefore, it is reasonable to believe that differentiation enables enterprises to achieve more profits.

This two-stage model is modified from the Hotelling model where each firm's product should be located in a specific location in the geographic or commodity space. In the Hotelling model, a closer link in geographic or feature space of two products means higher product substitutability. The farther a company is from the consumer, the higher the cost of consumer purchase. In this paper, the product differentiation is reflected with the quality of product instead of spatial differentiation. The strong enterprise produces almost perfect products. The product position is determined by the failure rate, and the indifference equilibrium point can be sought through calculating consumer surplus. For maximiz-

[1] Hotelling assumed the firms producing a homogeneous good at a constant and equal marginal cost.

ing profit, expected equilibrium solution shall be figured out with uncertain theory.

3.2 Hypothesis

As Fig. 2 shows, the horizontal axis represents the quality of commodities, including quality and performance indices. The vertical axis represents cost and price. From the angle of cost, advantaged enterprise F's fixed cost is relatively high. While it provides high-quality commodities to the market, the variable cost has no obvious rises because of its advanced technology. In comparison, the disadvantaged enterprise L's cost is usually very low. However, once it upgrades the product quality, the variable cost goes up sharply (such as the import of foreign components and purchase of advanced technology). The product pricing of two companies is flexible. In our model, it is assumed that the quality of the advantaged enterprise F's commodity is infinitely close to perfect. Hence, the only way for L to stay in the market is to find the balance between product quality and costs.

It is considered that consumers may only pursue perfect quality products regardless of the price, but never pursue the lowest price above the quality. This is realistic because a minority of affluent consumers are very likely to pursue perfect quality of commodities (although it may not exist in reality). While the majority of consumers would like to make some measurements on price–cost performance before purchasing, some consumers pay more attention to quality and are willing to spend more money to get high-quality commodities. These consumers can be called risk-averse because they are less willing to take the risk of getting the faulty goods. The arc p_h in Fig. 2 shows the price curve buyers are willing to pay. Vendors with good reputation and customer service are a main attraction for this kind of users; however, there are some risk-pursuing consumers who pay more attention to the price, and arc p_l shows the price curve buyers are willing to pay. Manufacturers of promotional activities have significant influence on the purchasing behavior (Gronroos 1983). These three types of buyers construct the consumer model in this research.

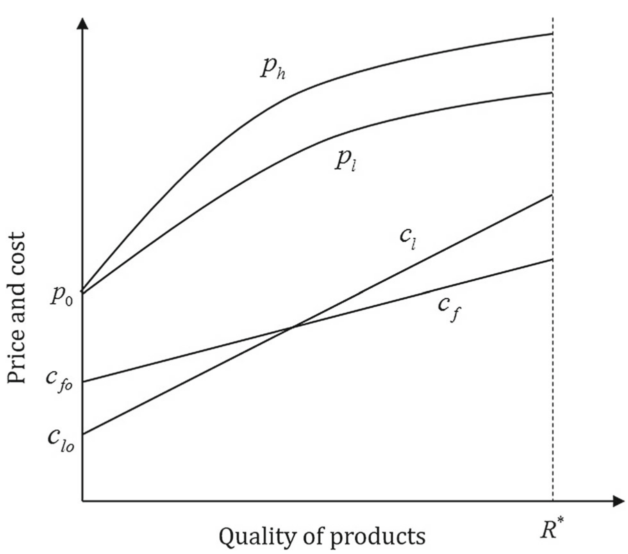

Fig. 2 disadvantaged firm's target customers

3.3 Game model

Assuming that the quality of the commodity in the advantaged enterprise F is high, the commodity failure rate (PFR) x_f is extremely low (supposed $x_f = 0$ for simplified analysis). Similarly, supposing commodity quality of the enterprise in a disadvantaged position L is relatively poor, failure rate is higher. The loss caused by the commodities produced by L is supposed to be w. Based on the hypothesis of Hotelling, x_f and x_l can be regarded as the orientation of linear spatial oligopoly, and consumer's preference for commodity quality is distributed uniformly in the $[0, x_m]$, satisfying $x_m \in (0, 1)$. It means no one will buy such a commodity with failure rate beyond x_m. When $x_l \notin (0, x_m)$, the enterprise profit becomes 0, while enterprise F monopolizes the market. The profits of duopoly can be described as follows:

$$\pi_l = \begin{cases} (p_l - c_l)q_l & x_l \in (0, x_m) \\ 0 & \text{other} \end{cases} \tag{1}$$

$$\pi_f = \begin{cases} (p_f - c_f)q_f & x_l \in (0, x_m) \\ p_f - c_f & \text{other} \end{cases} \tag{2}$$

Note that $p_i (i = l, f)$ is commodity price, $c_i (i = l, f)$ represents commodity cost per unit, $q_i (i = l, f)$ means market share. According to the hypothesis of heterogeneous duopoly, we consider the commodity cost per unit c as a function of failure rate x, which is contracted as follows:

$$c_l(x_l) = a + b(1 - x_l) \tag{3}$$

Formula (3) indicates that the unit cost of the commodity gets higher with the increase in product qualified rate (PQR).[2]

The model established in this paper can be seen as a two-stage game: First, the duopoly competition in commodity position carried out simultaneously based on consumer preference distribution. Then the duopoly pricing competition begins in second stage.

[2] The unit cost c in this paper can be regarded as the enterprise's "location cost", similarly the cost to achieve specific positioning.

4 Expected equilibrium in the backward recurrence algorithm

4.1 Consumer indifference equilibrium Point

When firms' product positions are set up at x_l and $x_f (x_f = 0)$, the marginal consumer, who is indifferent between purchasing from either firm, is located at x^*. Then we obtain:

$$(x_l - x^*)w + p_l = p_f \tag{4}$$

The formula (4) can be comprehended as follows: The left side of equation represents the reduction of consumer surplus occurred when the commodity with a high failure rate is being purchased ($x = x_l \neq x^*$). w means the loss to consumers caused by the accident destruction of products. The right side is relative to the reduction of consumers' surplus when the users buy high-quality commodities ($x = x_f = 0$). When $x = x_l = x^*$, the result of pricing game as in Bertrand game.

It is important to emphasize that the assumption in formula (4) originates from the classic "1surplus function" hypothesis on commodity feature which is commonly retrieved in market research. It indicates that commodities always contain excess needless functions. For example, it is known that the goods' failures always lower the efficiency or effectiveness. If the failure occurred when the user just does not need high efficiency, the reduction of failure rate x would not increase the consumer surplus.

Because of that, the right side of formula (4) failed to reflect the change in consumer surplus when the failure rate is less than x^*.

It follows that

$$x^* = x_l - \frac{p_f - p_l}{w} \tag{5}$$

Obviously, the users whose taste is located in $x < x^*$, their rational choice is to purchase from F. Whereas for the other consumers whose requirement for the commodity is lower (satisfying $x_m > x > x^*$), they would like to purchase from L. Now it is clear that the problem we are discussing has become a standard Hotelling price game matter.

Through normalizing the distribution of consumer taste, the market share in duopoly can be obtained (demand function):

$$q_l = \frac{\int_{x^*}^{x_m} \mathrm{d}x}{x_m} = \frac{x_m - x^*}{x_m} \tag{6}$$

$$q_f = \frac{\int_{x_f}^{x^*} \mathrm{d}x}{x_m} = \frac{x^*}{x_m} \tag{7}$$

4.2 Equilibrium solution

Assuming the profit functions are differentiable and the first-order conditions for equilibrium prices are given by

$$\frac{\partial \pi_l}{\partial p_l} = \frac{1}{x_m w}(x_m w - x_l w + p_f - 2p_l + c_l)$$
$$\frac{\partial \pi_f}{\partial p_f} = \frac{1}{x_m w}(x_l w + p_l - 2p_f + c_f)$$

Let $\frac{\partial \pi_i}{\partial p_i} = 0$, then we get the Nash equilibrium solution of commodity pricing as follows:

$$p_l^* = \frac{1}{3}[(2x_m - x_l)w + c_f + 2c_l] \tag{8}$$

$$p_f^* = \frac{1}{3}[(x_m - x_l)w + 2c_f + c_l] \tag{9}$$

We obtain the duopoly equilibrium profit after substituting (8) and (9) into profit function:

$$\pi_l^* = \frac{1}{9x_m w}(2x_m w - x_l w - c_l + c_f)^2 \tag{10}$$

$$\pi_f^* = \frac{1}{9x_m w}(x_m w + x_l w - c_f + c_l)^2 \tag{11}$$

Since x_f has been regarded as fixed and the value is zero, we can get the optimal product position of L (namely the equilibrium failure rate x_l^* for L). Let $\frac{\partial \pi_l^*}{\partial f_l} = 0$, then it yields

$$x_l^* = \frac{2x_m w + c_f - a - b}{w - b} \tag{12}$$

This equation must be satisfied $0 < x_l^* < 1$ such that the disadvantaged enterprise L would be possible to find a place in duopoly market.

According to the analysis above, given $\frac{\partial p_l^*}{\partial x_m} > 0$, $\frac{\partial p_f^*}{\partial x_m} > 0$, $\frac{\partial p_l^*}{\partial x_l} < 0$, $\frac{\partial p_f^*}{\partial x_l} < 0$, we conclude that the duopoly enterprises' commodity pricing is in direct proportion to the maximum failure rate x_m which users can accept, but inversely proportional to disadvantaged enterprise L's failure rate x_l. The results demonstrate that since the users' minimum requirements on the quality of the commodity declined, producers have many reasons to raise prices. In other words, with the same prices as before, consumers can only access the commodities of inferior quality because the producers clearly understand users will not refuse such commodities. When the failure rate of L is reduced, which means the quality of L is improved, both companies will similarly raise prices on the commodity. While the enterprise L 's purpose in raising price is to cover the extra cost on quality improvement, F's purpose is to sustain profits in the competition. Further, from

$\frac{\partial p_l^*}{\partial x_l} = \frac{1}{3}(-w-2b)$, $\frac{\partial p_f^*}{\partial x_l} = \frac{1}{3}(-w-b)$,we can see the extent of L's price raising is higher than F's.

4.3 Expected equilibrium value

In this section, we introduced the uncertain consumer preference. During the process of the enterprises' development on new commodities or product positioning, it is common to be unable to grasp and capture consumer preference on commodity completely because of the variability and complexity of market. Assuming that L and F enterprises have the same uncertainty on consumer preference information in the duopoly market, and both subjectively perceive x_m as a random variable, which uniformly distributed in the interval $(0, \varepsilon)$, satisfying $0 < \varepsilon < 1$. Hence the formula (10) can be used to calculate the expected profit function of L

$$E\pi_l^* = \int_0^\varepsilon \frac{1}{9x_m w}(2x_m w - x_l w - c_l + c_f)^2 \frac{1}{\varepsilon} \mathrm{d}x$$

Thus, we can draw the following conclusions with the derivation of the equilibrium solution:

Theorem 1 *When enterprise F fixed its failure rate x_f equal to zero under the uncertainty of consumer preference, the equilibrium position of L 's commodities is*

$$x_l^{**} = \frac{\frac{2w\varepsilon}{\ln\varepsilon} + c_f - a - b}{w - b}$$

The expected equilibrium price is

$$Ep_l^* = \frac{1}{3}\left[-(w+2b)x_l^{**} + w\varepsilon + c_f + 2a + 2b\right]$$
$$Ep_f^* = \frac{1}{3}\left[-(w+b)x_l^{**} + \frac{w\varepsilon}{2} + 2c_f + 2a + 2b\right]$$

The expected equilibrium profit is

$$E\pi_l^* = \frac{2}{9}w\varepsilon - \frac{4}{9}\left[(w-b)x_l^{**} - c_f + a + b\right] + \frac{\ln\varepsilon}{9w\varepsilon}\left[(w-b)x_l^{**} - c_f + a + b\right]^2$$
$$E\pi_f^* = \frac{1}{18}w\varepsilon + \frac{2}{9}\left[(w-b)x_l^{**} - c_f + a + b\right] + \frac{\ln\varepsilon}{9w\varepsilon}\left[(w-b)x_l^{**} - c_f + a + b\right]^2$$

Corollary 1 *When the product failure rate of an advantaged firm F is fixed as zero, and the uncertainty variable of consumer preference is uniformly distributed in $(0, \varepsilon)$, provided $0 < \varepsilon < 1$, the optimal product positioning difference between a disadvantaged firm L and advantaged firm F shows a negative correlation with consumer preferences uncertainty. Specifically, a smaller range of the random variable leads the L's equilibrium product position x_l^{**} closer to x_f. That is to say, less uncertainty would have the disadvantaged enterprise able to upgrade product quality and produce better commodities.*

Proof The following formula could be derived by the function of $E\pi_l^*$

$$\frac{\partial x_l^{**}(\varepsilon)}{\partial\varepsilon} = \frac{2w(\ln\varepsilon - \varepsilon)}{\ln^2\varepsilon}$$

Let $f(\varepsilon) = \ln\varepsilon - \varepsilon$, then we get $\frac{\partial f(\varepsilon)}{\partial\varepsilon} = \frac{1}{\varepsilon} - 1$, which is greater than zero constantly. Clearly, $f(\varepsilon)$ is strictly monotone increasing in interval (0, 1). Due to $f(0) < 0$, $f(1) < 0$, we get $f(\varepsilon) < 0$. As $\ln^2\varepsilon > 0$, we have $\frac{\partial x_l^{**}(\varepsilon)}{\partial\varepsilon} < 0$.

The theorem is proved. □

Corollary 2 *When the product failure rate of an advantaged firm F is fixed as zero, and the uncertainty variable of consumer preference is uniformly distributed in $(0, \varepsilon)$, provided $0 < \varepsilon < 1$, the expected equilibrium prices for the products of both enterprises F and L are positively correlated with the uncertainty variable. That is, larger uncertainty would increase the expected equilibrium prices for both duopoly and vice versa.*

Proof The following formulas could be derived by the function of Ep_l^* and Ep_f^*

$$\frac{\partial Ep_l^*(\varepsilon)}{\partial\varepsilon} = -(w+2b)\frac{\partial x_l^{**}(\varepsilon)}{\partial\varepsilon} + w$$
$$\frac{\partial Ep_f^*(\varepsilon)}{\partial\varepsilon} = -(w+b)\frac{\partial x_l^{**}(\varepsilon)}{\partial\varepsilon} + \frac{w}{2}$$

Since $\frac{\partial x_l^{**}(\varepsilon)}{\partial\varepsilon} < 0$, $w, b > 0$, it is clear that $\frac{\partial Ep_l^*(\varepsilon)}{\partial\varepsilon} > 0$, $\frac{\partial Ep_f^*(\varepsilon)}{\partial\varepsilon} > 0$.

The theorem is proved. □

Corollary 3 *When the product failure rate of an advantaged firm F is fixed as zero, and the uncertainty variable of consumer preference is uniformly distributed in $(0, \varepsilon)$, provided $0 < \varepsilon < 1$, the expected equilibrium profit of a disadvantage firm L shows a negative correlation with consumer taste uncertainty. While the relationship between expected equilibrium profit of advantage firm F and the uncertainty of consumer taste is contingent on the value of ε: If ε satisfies $\ln\varepsilon < -8 - 4\sqrt{5}$, expected equilibrium profit of the advantaged firm F is positively correlated with the uncertainty of consumer taste. On the contrary, if ε satisfies $\ln\varepsilon > -8 - 4\sqrt{5}$, expected equilibrium profit of F is negatively correlated with consumer preferences uncertainty. That is to say, the expected equilibrium profit of the* *advantaged firm F increases with the random variable when consumer taste uncertainty is in, respectively, low level. However, when the uncertainty of consumer taste becomes large enough, expected equilibrium profit of F decreases with the random variable.*

Proof The following formulas could be derived by the function of $E\pi_l^*$ and $E\pi_f^*$

$$\frac{\partial E\pi_l^*(\varepsilon)}{\partial \varepsilon} = \frac{2}{9}w + \frac{4w(1-\ln\varepsilon)}{9\ln^2\varepsilon}$$
$$\frac{\partial E\pi_f^*(\varepsilon)}{\partial \varepsilon} = \frac{1}{18}w - \frac{8w(1-\ln\varepsilon)}{9\ln^2\varepsilon}$$

As $0 < \varepsilon < 1$, then $\ln\varepsilon < 0$, obviously $\frac{\partial E\pi_l^*}{\partial\varepsilon} > 0$

By deriving of $\frac{\partial E\pi_f^*}{\partial\varepsilon}$, we have $\frac{\partial E\pi_f^*}{\partial\varepsilon} = \frac{w[(\ln\varepsilon+8)^2-80]}{18\ln^2\varepsilon}$. Through calculation, we obtain that when $\ln\varepsilon < -8 - 4\sqrt{5}$(another solution is $\ln\varepsilon > -8 + 4\sqrt{5}$, dropped), $\frac{\partial E\pi_f^*}{\partial\varepsilon} > 0$; when $\ln\varepsilon > -8 - 4\sqrt{5}$(another solution is $\ln\varepsilon < -8 + 4\sqrt{5}$, constantly satisfied), $\frac{\partial E\pi_f^*}{\partial\varepsilon} < 0$.

The theorem is proved. □

5 Discussion

This work established a product positioning–pricing game model of duopoly to address the position–price dual problem, and highlighted the uncertainty of consumer taste. Based on this model, the expected equilibrium of the two-stage game had been obtained and the significant influence of uncertain consumer preference on enterprises' decision making had been proved as well.

The two-stage duopoly game model, similar to the Hotelling model, included both positioning game and pricing part and the analysis was started by finding out the indifference point. Compared with classic Hotelling, the model in this paper has two obvious differences: (1) Product position in this paper does not mean spatial localization but the level of quality.[3] (2) This paper searches the indifference point neither through the increase amount of consumer utility nor taking account of transportation cost. Besides, the conception of "superfluous function" is introduced, and therefore the equilibrium position for disadvantaged enterprise can be easily obtained through calculating the reduction of consumer utility. Finally, equilibrium solution of positioning–pricing game is calculated.

In the part of the equilibrium solution, we found that product quality positioning of a disadvantaged enterprise was related to the cost c_l and c_f of the duopoly, the loss w of the product defective to consumers and the maximum failure rate x_m that consumers could accept. When $0 < x_l^* < 1$, the disadvantaged enterprise can find its niche in the duopoly market. Under the uncertainty of consumer preference, the expected equilibrium price of both enterprises' products and the expected equilibrium profit of the disadvantaged enterprise L increased when ε increased (the price of the disadvantaged enterprise L was more greatly improved than that of the advantaged enterprise F). However, the expected equilibrium profit of the advantaged enterprise L decreases with ε when $\ln\varepsilon > -8 - 4\sqrt{5}$.

The computational results and analysis indicated that uncertain consumer preference largely complicated the positioning and pricing of competition in a duopoly. For disadvantaged enterprises, it is necessary to reduce their product failure rates in order to obtain maximum profits when the random variable of consumer preference increase, while the expected equilibrium price and profit increase simultaneously. As for advantaged firms, the increased uncertainty will possibly decrease the expected equilibrium profit, although it could raise the expected equilibrium price.

Returning to the frozen dumpling example, when the dumpling enterprises are unable to produce high-class quick-frozen dumplings and fall in a disadvantaged situation, they can shift their products to low-end market and reduce the price in order to survive. While the consumer preference uncertainty is reduced in this case, a better way for disadvantaged enterprises to stay in the market is to improve product quality or they will be driven out.

6 Conclusion

The flaws of the classic Hotelling model are pointed out in this paper, and a modified Hotelling model is constructed. Compared with classic Hotelling, this modified model in this paper has two obvious differences: (1) Product position in this paper does not mean spatial localization but the level of quality. (2) This paper searches the indifference point neither through the increased amount of consumer utility nor taking account of transportation cost. The conception of "superfluous function" is introduced, and therefore the equilibrium position for disadvantaged enterprise can be easily obtained through calculating the reduction of consumer utility. Finally, equilibrium solution of positioning–pricing game is calculated.

This modified Hotelling model is closer to real market than the classic one because of the relaxed assumptions and the addition of uncertain consumer preference. In the classic model, the equilibrium locations of duopoly are both in the center of the market and the equilibrium prices are both equal to marginal costs, which means no one could make profits in that case. The modified model in this paper reasonably

[3] In this paper, product quality level is reflected on the failure rate. Specifically, the lower failure rate, the higher level of product quality.

explains why different enterprises have different positioning–pricing strategies in reality from the view of mathematics. In addition, considering that consumer preference uncertainty is largely affecting enterprises' competitive strategies, this proposal could be very helpful to address real problems in which it is required to improve the firms' competitive power without complete and perfect information about consumer preference.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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