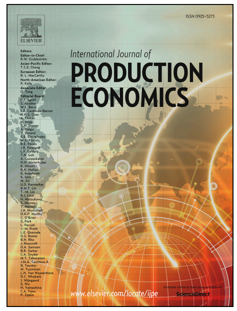

Correlation between strategic and operational risk mitigation strategies in supply networks

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Correlation between Strategic and Operational Risk Mitigation Strategies in Supply Networks

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Correlation between Strategic and Operational Risk Mitigation Strategies in Supply Networks

Abstract

A supply network's performance is affected by two types of risk: 1) risk of disruptions that distort the supply network's topology by inactivating certain production facilities or transportation lanes; and 2) risk of variations in a facility's performance that reduce the efficiency of the supply network's flow planning for fulfilling demands. In this paper, we demonstrate that strategic and operational risk mitigation strategies, which neutralize the impacts of disruptions and variations, respectively, are correlated. We consider "Robustness" and "Resilience" at a strategic level to mitigate disruptions and "Reliability" at an operational level to mitigate variations.

A mixed integer stochastic mathematical model is developed to simultaneously a) design a robust and resilient topology for supply networks; and b) plan a reliable flow throughout its topology. We solve the model using an example of a profit-based supply network that is constructed by relying on the assumptions that were primarily used in prior studies. A sensitivity analysis of the results from the model indicates that i) the correlation between robustness and resilience is negative; ii) the correlation between robustness and reliability is positive; and iii) the correlation between resilience and reliability is negative.

Keywords: Supply Network; Robust Design; Disruption; Variation; Reliability; Resilience; Flexibility.

1. Introduction

Supply networks (SNs) are crucial components of competitive and globalized markets. Companies improve their competitive advantage by working as parts of a SN, which results in lower production costs, higher product quality, and greater responsiveness with respect to the customers' rapidly changing needs and expectations (Chopra and Sodhi, 2004). Conversely, because SNs are globally distributed, they are vulnerable to risks in business and working environments (Schmitt and Snyder, 2010; Peng et al., 2011; Baghalian et al., 2013; Farahani et al., 2014). Therefore, risk management is critical for successful SNs because many different types of risks exist.

According to Sarkar et al. (2002), during the labor strike in 2002, 29 ports on the west coast of the United States were shut down, which led to the closure of the new United Motor manufacturing production factory (disruption in transportation facilities). During the destructive earthquake in Japan in 2011, the Toyota Motor Company ceased production in twelve assembly plants to repair production facilities, which resulted in a production loss of 140,000 automobiles (disruption in production facilities). In another instance, Ericsson lost 400 million Euros after their supplier's semiconductor plant was damaged due to a fire in 2000 (disruption in production facilities). The Taiwan earthquake of 1999 resulted in a supply shortage of DRAM chips for Apple that culminated in numerous order losses (variation in supply process). This supply variation has a cascading effect in multi-echelon SNs. For example[1], consider an apparel supply network, as follows: a small variation in machine performance at a thread manufacturing plant in India can cause a four-day delivery delay to a knitter in Malaysia, which can result in a seven-day delivery delay to a dyer in Hong Kong and finally lead to a 10-day delivery delay of trendy, new apparel at a clothing manufacturer in Europe and a loss of sales worth millions of dollars (variation propagation in supply process). Hendricks and Singhal (2005) quantify the negative effects of risks in SNs through empirical analysis. Their results demonstrate that risks result in 33 to 40% lower stock returns, a 107% decrease in operating income, 7% lower sales growth, and an 11% increase in cost.

Clearly, there are numerous sources of risk in SNs. In this paper, we demonstrate that risk mitigation strategies used by SNs for different risk sources (disruptions and variations) are not independent and important correlations exist among them. Therefore, compartmentalized decision making for the mitigation of variations and disruptions, as done in prior studies, results in suboptimal solutions.

2. Literature Review

Scholars have suggested numerous methods to classify the risks of SNs. Waters (2007) and Kar (2010) divide SN risk sources into ***internal risks*** and ***external risks*** based on their controllability. Internal risks are controllable and appear during normal operations, such as late deliveries, excess stock, poor forecasting, human error, and faults in IT systems. External risks are uncontrollable and come from outside of a supply network, such as earthquakes, hurricanes, industrial actions, wars, terrorist attacks, price increases, problems with trading partners, shortages of raw materials, and crime. Furthermore,

[1] http://www.decisioncraft.com/dmdirect/variability.htm

Chopra and Sodhi (2004) categorize potential supply chain risks into nine categories, as follows: (a) Disruptions (e.g., natural disasters, terrorism, war, etc.), (b) Delays (e.g., inflexibility of the supply source), (c) Systems (e.g., information infrastructure breakdown), (d) Forecast (e.g., inaccurate forecast, bullwhip effect, etc.), (d) Intellectual property (e.g., vertical integration), (e) Procurement (e.g., exchange rate risk), (f) Receivables (e.g., number of customers), (g) Inventory (e.g., inventory holding cost, demand and supply uncertainty, etc.), and (h) Capacity (e.g., cost of capacity). These classification schemes are not adequate to analyze correlations among the different risk mitigation strategies of SNs. Therefore, we identify and use a different classification. For this classification, risks are categorized into two groups based on the nature of the SNs' decisions that are affected, as follows:

Disruptions in a SN: Disruptions refer to rare and unexpected events that have a significant impact and distort the topology[2] of a SN by rendering certain facilities or connecting links inoperative. A SN's topology is determined by strategic level network design decisions (see Figure 1). Network design decisions are related to determining the number, location and capacity of the facilities (Schmidt and Wilhelm, 2010). We summarize certain recent studies that have been conducted in this domain. Tomlin (2006) investigates the unavailability of a supplier in a two-echelon SN that includes one manufacturer and two suppliers. Chopra et al. (2007) analyze the appropriate selection of mitigation strategies for a two-echelon SN that includes one buyer that is serviced by two suppliers. One of these suppliers is reliable and the other is unreliable but less expensive. Peng et al. (2011) develop a model to design a SN topology that performs well under normal conditions and performs relatively well when unreliable facilities are disrupted. Baghalian et al. (2013) and Mohammaddust et al. (2017) propose a path-based approach to design a robust SN topology for which there is a possibility of disruption in facilities and connecting links. Recently, certain scholars have extended the concept of disruption management from companies and SNs to communities and human societies that are in danger of natural and man-made disasters. For example, Gian et al. (2010) believe that communities and human societies should be able

[2] A SN's topology is the way its individual facilities are organized and connected, and the resulting network structure (http://www.personal.psu.edu/faculty/a/x/axk41/IEEE-Sys-Oct2010.pdf).

to mitigate danger and achieve a tolerable level of protection against disruptions and disasters. These scholars provide a framework to quantify these features for societies and present two applications of the methodology to healthcare facilities. According to Zobel (2011), two primary measures that are important for the disaster management of societies include the initial impact of a disaster event and the subsequent time for recovery. The author presents a new analytic approach to represent the relationship between these two characteristics. These studies only focus on employing risk mitigation strategies to preserve the performance of SNs or communities against disruptions. Risk mitigation strategies that are utilized to address disruptions are referred to as "Strategic Risk Mitigation (SRM)" strategies in this paper because they include strategic network design decisions (see Figure 1).

- ***Variations in a SN***: Variations refer to frequent and expected events with less significant impacts that only reduce the efficiency of flow planning in SNs (see Figure 1). Flow planning in a SN refers to the production quantities in the SNs' facilities and the quantities that are transported among the facilities (Schmidt and Wilhelm, 2010). Variations that occur in the performance of upstream facilities in a SN lead to changes in the quantities that flow from these facilities. This type of upstream variation is important because, in reality, the perfect production system does not exist. Furthermore, increasing the rate of production increases the likelihood of machinery and labor failures, which results in a higher rate of defective items that are produced (Sana, 2010). To the best of our knowledge, prior studies generally ignore variations in the performance of multi-echelon SNs (Rezapour et al., 2015).

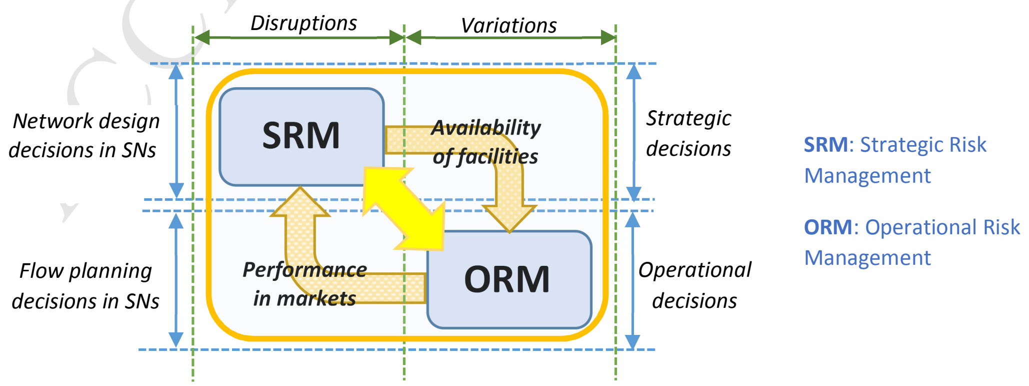


Figure 1. Disruptions and variations in SNs.

Downstream variations also occur for market demands; these can be modeled by defining scenarios (Pan and Nagi, 2010; Georgiadis et al., 2011; Leung et al., 2007; Lin and Wang, 2011; Hasani and Khosrojerdi, 2016) or considering demand as a random variable (Shen and Daskin, 2005; Santoso et al., 2005; Dada et al., 2007; Schmitt et al., 2010; Baghalian et al., 2013). This type of variation is critical for managing the flow and service level estimation in a SN. Prior studies only focus on downstream variations in demand and assume that the performance of the SNs' facilities is perfect. In this study, we demonstrate how upstream variations in the performance of facilities and their propagated impact should be managed in multi-echelon SNs. Prior studies only address variations and their corresponding risk mitigation strategies. Risk mitigation strategies that are utilized to address variations are referred to as "Operational Risk Mitigation (ORM)" strategies in this study because they include operational flow planning decisions (see Figure 1).

As illustrated in Figure 1, SRM and ORM strategies are not independent. SRM strategies preserve the availability of facilities in a SN's topology. Flow planning is conducted for the SNs' available facilities. In addition, ORM strategies increase the efficiency of flow planning in a SN and improve its performance in markets; this performance is used for the economic evaluation of SRM strategies. The existing literature mostly ignores this mutual impact between ORM and SRM approaches. Therefore, we contribute to the SN risk management literature by answering the following question: What correlations exist between SRM and ORM strategies? Risk mitigation that includes either redundancy or flexibility ensures that SNs are robust, resilient, and reliable. The standard use of redundancy includes holding safety stock of material and finished goods (You and Grossmann, 2008; Park et al., 2010; Schmitt, 2011) or multi-sourcing (Yu et al., 2009; Li et al., 2010; Schmitt and Snyder, 2010; Peng et al., 2011; Schmitt, 2011). Flexibility implies that facilities have adaptable capacities (Tomlin, 2006). In this study, we focus on redundancy in ORM strategies and flexibility in SRM strategies.

This study makes multifold contributions to the SN risk management literature as follows:

1) **Variation management:** For a multi-echelon SN's flow planning, we consider upstream variations in the performance of facilities in addition to downstream variations in market demands. We

demonstrate that local reliability as an ORM strategy should be assigned to each facility to control redundancy (extra production) in production systems against variation. In addition, we demonstrate that a SN's service level is a function of these local reliabilities. Finally, we develop a mathematical model to determine the optimal local reliabilities (ORM strategies) and service levels for the SN (in Section 4). Prior studies have ignored upstream variations in flow planning for SNs.

2) **Disruption management:** Considering flexibility as a SRM strategy, we demonstrate that the robustness of a SN's topology for maintaining acceptable performance during and after a disruption depends on its facilities' flexibility levels. The flexibility level of a facility indicates to what extent the capacity of that facility can be increased during a disruption. A SN's resilience is how quickly its performance can be returned to an acceptable level after a disruption; hence, we demonstrate that the resilience of a SN depends on the speed of flexibility in its facilities. The flexibility speed of a facility is how rapidly the capacity of that facility can be increased during a disruption. Finally, we develop a mathematical model to determine the optimal flexibility levels and speeds (SRM strategies) to ensure that a SN's facilities are robust and resilient against disruptions (in Section 5). In prior studies, the robustness and resilience of SNs against disruptions have been investigated separately.

3) **Integrated decision making for ORM and SRM strategies:** The final model we develop in Section 5 facilitates concurrent decision making about reliability (and the facilities' local reliabilities as ORM strategies), robustness and resilience (and the facilities' flexibility levels and speeds as SRM strategies). Therefore, a sensitivity analysis of this integrated model helps us to determine if correlations exist between ORM and SRM strategies and their corresponding reliability, robustness, and resilience (in Section 5.5). In prior studies, decisions regarding SRM and ORM strategies are made independently.

This paper is organized as follows. In Section 3, the details of the problem under normal (without disruption) and disrupted conditions are presented. The mathematical model, solution approach and computational results for a SN experiencing normal (without disruption) conditions are presented in

Section 4. In Section 5, the mathematical modeling, solution approach and computational results for a SN experiencing disrupted conditions are discussed. The paper is concluded with a summary in Section 6.

3. Problem Description

Without loss of generality, we consider a simple SN that produces and supplies a product to target markets. This SN includes two manufacturers, *M1* and *M2*, that produce products and four target markets that are serviced by these two manufacturers through retailers. *M1* fulfills the demands of the first and second markets through the first retailer, *R1*, while *M2* fulfills the demands of the third and fourth markets through the second retailer, *R2*. Two suppliers, *S1* and *S2*, provide the components required by these two manufacturers, *M1* and *M2*, respectively. In Figure 2, the existing network structure of the SN is illustrated. Product demand in a market is a stochastic function of the SN's marketing factors, e.g., price and service level (downstream variations in the SN's markets). Prior to the beginning of each sales period, retailers determine the quantities of the product that are required, and then issue orders to the corresponding manufacturers. The manufacturers receive these orders from the retailers and plan to produce the ordered products.

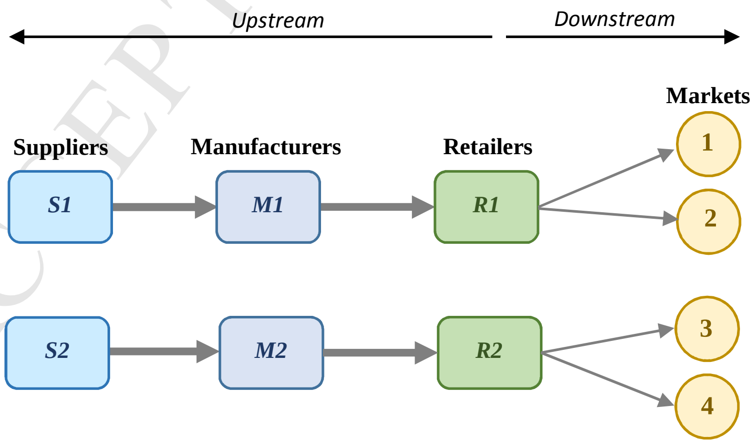


Figure 2. The network structure of the SN.

We assume the performance of the manufacturers' production systems are imperfect and they produce a stochastic percentage of defective units in their batches (upstream variations in the SN's manufacturers). To compensate for these defective units, the manufacturers plan to produce extra products. To assemble the products, the manufacturers order the required components from their corresponding suppliers. The suppliers' production systems (after initial setup) start producing components in an in-control state with almost zero defects. After a stochastic time, the suppliers' production systems deteriorate to an out-of-control state in which γ percent of output is nonconforming (upstream variations in the SN's suppliers). Similar to the manufacturers, the suppliers plan to produce some surplus components to compensate for the nonconforming output of their systems.

In the literature, two approaches are mainly used to model imperfect production systems. Some researchers assume that the performance of the production system is always accompanied by a stochastic defective production rate (e.g., Sana, 2010; Sana et al., 2007; Rezapour et al, 2016a, b). Some researchers consider the case in which the production machinery starts to operate in an in-control state after setting up. In the in-control state, all output is nearly perfect. After a stochastic time, the machinery deteriorates to an out-of-control state and starts to have a stochastically impaired production rate (e.g., Sarkar 2012; Rosenblatt and Lee, 1986; Lee and Rosenblatt, 1987). In this paper, we consider both types of imperfect production systems. Without loss of generality, the former is assumed for manufactures and the latter approach is assumed for suppliers. This shows that the approach developed in this paper is able to handle both types of imperfect production systems.

In a SN with multiple imperfect production facilities (multiple types of upstream variation), the conforming component/product quantity is reduced by moving from upstream to downstream in the SN. Modeling this flow reduction is necessary to quantify the conforming product volumes that can be supplied in the last echelon and to determine the best service level that balances the stochastic product demand (downstream variation) and product supply (upstream variation) in the most economical way. To preserve an appropriate service level in the markets, reliable flow planning throughout the SN is required to mitigate upstream and downstream variations. Increasing the reliability of facilities is an ORM strategy used to neutralize the impacts of variations in flow planning. In Section 4, we develop a mathematical model to plan the most profitable reliable flow through the SN. In this paper, reliability in the SNs' flow planning is defined as follows (see Figure 3):

Definition 1: Reliable flow planning in SNs employs appropriate ORM strategies to mitigate upstream and downstream variations and their propagation and preserves appropriate service levels for customers in markets.

In addition to variations that affect flow planning in a SN, we also consider the possibility that disruptions affect the availability of facilities in the SN. For the SN model in this study, *M1* is always available, but *M2* is prone to disruption. *M2* may be unavailable to fulfill *R2*'s orders. There may be several reasons that explain why this occurs, e.g., the failure of its machinery or the inability of its supplier (*S2*) to fulfill its order on time. In the event that *M2* is unavailable, the third and fourth markets cannot be served, and their sales are lost, which leads to a large loss in the SN's profitability and brand reputation. To avoid this possible loss, we redesign the SN's network (by adding extra capacity to its facilities) to simultaneously provide the following characteristics:

- **Robustness** against disruptions: A robust SN is able to appropriately manage disruptions and maintain service continuity. To have a robust SN, we must modify the production capabilities of its undisrupted facilities (*M1* and *S1*) to compensate for the unavailability of its disrupted facilities (*M2* and *S2*). For this purpose, the production capacities of *M1* and *S1* must be flexible enough to increase production, when needed, to compensate for the unavailability of disrupted facilities and decrease production when those facilities become available again. For this problem, we seek to determine the flexibility level that is required for the undisrupted facilities, *M1* and *S1*, to have a robust network. The flexibility level of a facility refers to the extent its capacity can be increased when it is needed.

- **Resilience** against disruptions: The resilience of a SN is how quickly disruptions can be managed by that SN and depends on the speed of its facilities to increase their capacities after disruptions, which are their flexibility speeds. The flexibility speed of a facility is how quickly its capacity can be increased when needed. Therefore, another important decision that must be made is to determine the optimal flexibility speeds for the undisrupted facilities, *M1* and *S1*, to maintain the SN's resilience.

Because numerous definitions exist for the robustness and resilience of SNs in the risk management literature, the definitions for these terms used in this study are as follows (see Figure 3):

Definition 2: A robust SN has appropriate SRM strategies to reduce its decrease in performance when it is affected by disruptions. In Figure 3, we show how a SN's robustness can be measured using this definition. After employing SRM strategies, if a SN's performance returns to its nominal value (and the performance decrease is zero), this means that the SN is completely robust. Since complete robustness can be very costly for SNs, a relatively robust SN is sometimes preferred, wherein the performance returns to an acceptable level with a finite performance decrease.

Definition 3: A resilient SN is able to rapidly use SRM strategies after disruptions to reduce the restoration time during which the SN's performance returns to the acceptable level that is defined by its robustness. In Figure 3, it is shown that a SN's resilience can be measured based on this definition. A SN's resilience is measured by its average restoration rate (ratio of restored performance to restoration time).

In this study, flexibility (including flexibility levels and speeds) in facilities is a SRM strategy that is used to neutralize the impacts of disruptions and to design a robust and resilient SN. Our definition of a facility's flexibility is as follows:

Definition 4: A flexible facility is able to increase its processing capacity when needed. The flexibility level of the facility is the maximum level to which its capacity can be increased. The flexibility speed of a facility is how quickly the capacity can be increased when needed.

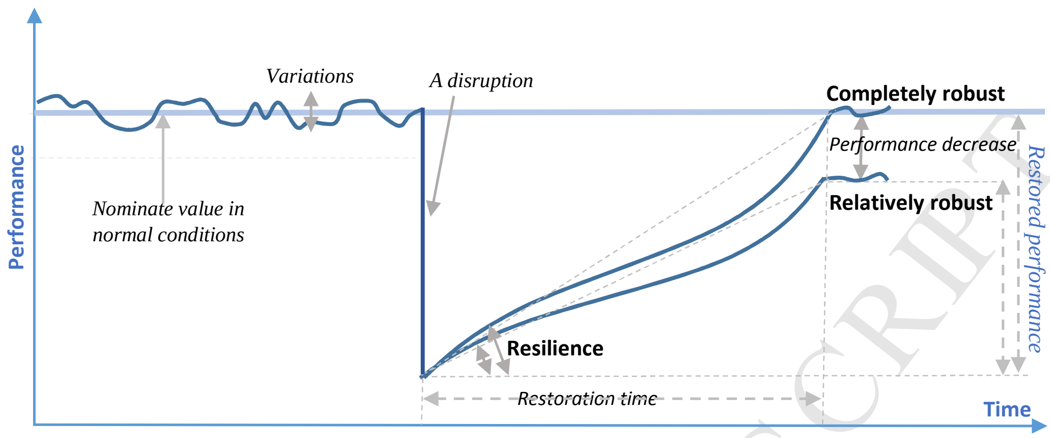


Figure 3. Reliability, robustness, and resilience in SNs.

Solving the problem of managing a SN's variations and disruptions is done in the following two steps: i) in the first step, we ignore disruptions in the SN and solely focus on flow planning and use an ORM strategy against variations (Section 4); and ii) in the second step, we add disruptions to the problem and use a SRM strategy to alleviate these disruptions (Section 5).

In this study, we consider a very simple SN with two supply paths: $[S1 \rightarrow M1 \rightarrow R1]$ and $[S2 \rightarrow M2 \rightarrow R2]$ (see Figure 2). We demonstrate what changes are needed in the first supply path, $[S1 \rightarrow M1 \rightarrow R1]$, to substitute for the second supply path, $[S2 \rightarrow M2 \rightarrow R2]$, when the latter is unavailable. We only consider two supply paths to simplify this analysis, but the problem is generalizable to more complicated SNs with more supply paths. For a SN with more supply paths, a subset of paths is unavailable during each disruption. To continue servicing customers, each unavailable path must be substituted by an available path and changes similar to those proposed in this study will need to be made in the available path.

4. Employing ORM Strategies

In conditions without disruption, all the facilities (*M1*, *M2*, *S1* and *S2*) are available. This SN case includes two product supply paths, as follows (Figure 2):

I) $[S1 \rightarrow M1 \rightarrow R1]$ represents the "first supply path," in which the flow of components begins with the first supplier, *S1*. These components then pass through the SN and become finished products at the first manufacturer, *M1* and are transported to the first retailer, *R1*, to supply the first and second markets and fulfill their demands.

II) $[S2 \rightarrow M2 \rightarrow R2]$ represents the "second supply path," in which the flow of components begins with the second supplier, *S2*. These components then pass through the SN and become finished products at the second manufacturer, *M2* and are transported to the second retailer, *R2*, to supply the third and fourth markets and fulfill their demands.

In this section, we discuss reliable flow planning of the first path against variations, in conditions without a disruption (in the second path, it is conducted in the same manner). In Section 5, we discuss how this flow planning changes during a disruption when the second supply path is unavailable.

The first path includes three types of facilities: the supplier (*S1*), the manufacturer (*M1*) and the retailer (*R1*). Each of these facilities faces a specific type of variation. The retailer faces a stochastic demand in the markets. The supplier and manufacturer encounter stochastic nonconforming units in their production batches. For each of these facilities, a desired local reliability must be determined to manage its corresponding variation. As will be demonstrated later, the service level provided by the supply path in the first and second markets is a function of these local reliabilities. We assume that $rl_{S1}^{WD}, rl_{M1}^{WD}$ and rl_{R1}^{WD} represent the local reliabilities of the first supply path's supplier, manufacturer and retailer, respectively, in conditions without any disruptions. In the remainder of this section, the performance of each facility when confronted with its corresponding variation is investigated from downstream to upstream along the supply path.

4.1. Retailer in the first supply path, R1

The first supply path services the first and second markets. The most important marketing factors in these markets are price, p, and service level, sl. The service level refers to the probability of fulfilling the realized demand from the retailer's on-hand product inventory. Therefore, the expected demand during each sale period of Market k ($k = 1$ and 2), $D_k(p, sl^{WD})$, is a function of these two factors. sl^{WD} represents the service level that is provided by the SN during normal conditions without disruptions. The retailer of the first supply path (*R1*) fulfills the total demand for the first and second markets. Therefore, the average demand for *R1* is $\sum_{k=1}^{2} D_k(p, sl^{WD})$. However, the actual demand is stochastic and varies around this mean value. This variation is treated as a random variable, ε, with a cumulative distribution function $G_{R1}(\varepsilon)$ (variation in *R1*'s demand). The actual demand of *R1* is $\sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon$. Without loss of generality, we assume $E(\varepsilon) = 1$, which implies $E\left[\sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon\right] = \sum_{k=1}^{2} D_k(p, sl^{WD})$ (Bernstein and Federgruen, 2004 and 2007).

Prior to the beginning of each sales period, a decision must be made about the quantity of *R1's* product stock, which is represented by x^{WD}, and an order must be issued to the corresponding manufacturer, *M1*. After realizing the actual demand, the unit holding cost, h^+, and unit shortage cost, h^-, are paid by the retailers for each unit of the end-of-the period for inventory and lost sales. Therefore, the expected total cost of *R1*, Π_{R1}^{WD}, should be minimized as in Equation (1), as follows:

$$MIN \quad \Pi_{R1}^{WD} = h^+ . E\left[x^{WD} - \sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon\right]^+ + h^- . E\left[\sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon - x^{WD}\right]^+ \tag{1}$$

$$S.T. \quad \Pr\left[\sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon \le x^{WD}\right] \ge rl_{R1}^{WD} \tag{2}$$

The constraint in Equation (2) preserves *R1*'s local reliability, which guarantees that in the rl_{R1}^{WD} percentage of time, *R1*'s product stock can fulfill the actual demand. The first term in the objective function, Equation (1), represents the expected end-of-period inventory holding cost for *R1* ($[\,]^+$ is used to compute the expected value of $x^{WD} - \sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon$ when it is positive). The second term in (1) is the expected cost of lost sales. $x^{WD} = \left[\sum_{k=1}^{2} D_k(p, sl^{WD})\right] . G_{R1}^{-1}\left(\frac{h^-}{h^- + h^+}\right)$ minimizes Π_{R1}^{WD} (see Appendix D for further evidence). Conversely, to satisfy the constraint in Equation (2), we must have $x^{WD} \ge \left[\sum_{k=1}^{2} D_k(p, sl^{WD})\right] . G_{R1}^{-1}(rl_{R1}^{WD})$ (see Appendix D for evidence). Accordingly, the quantity of product that must be ordered for *R1* is calculated as follows:

$$x^{WD} = \left[\sum_{k=1}^{2} D_k(p, sl^{WD})\right] . G_{R1}^{-1}\left(Max\left\{rl_{R1}^{WD}, \frac{h^-}{h^- + h^+}\right\}\right) \tag{3}$$

By substituting Equation (3) into (1), the least total cost for *R1*, ${\Pi_{R1}^{WD}}^*$, is as follows:

$$ {\Pi_{R1}^{WD}}^* = \left(h^+ . E\left[G_{R1}^{-1}\left(Max\left\{rl_{R1}^{WD}, \frac{h^-}{h^-+h^+}\right\}\right) - \varepsilon\right]^+ + h^- . E\left[\varepsilon - G_{R1}^{-1}\left(Max\left\{rl_{R1}^{WD}, \frac{h^-}{h^-+h^+}\right\}\right)\right]^+\right) \times \left[\sum_{k=1}^{2} D_k(p, sl^{WD})\right] \quad (4) $$

Ordering x^{WD} product units from *M1* enables *R1* to fulfill the product demand for the next sales period with a probability of rl_{R1}^{WD}. Maintaining local reliability, rl_{R1}^{WD} is an ORM strategy that is used by *R1* to manage demand variations. In Section 4.2, we demonstrate how *R1*'s order must be increased by moving backward to *M1*.

We assume that each facility either completely fulfills the order from its downstream facility or misses the order and sends nothing. This assumption is widely used in prior studies in the yield-uncertainty literature and is referred to as the Bernoulli supply process (Parlar et al., 1995; Swaminathan and Shanthikumar 1999; Dada et al., 2003; Tomlin and Wang, 2005).

4.2. Manufacturer in the first supply path, M1

M1 receives an order for x^{WD} product units from *R1*. *R1*'s order is produced by *M1* in O_{M1} production runs and includes y^{WD} items for each production batch (Figure 4). *M1*'s production system is not perfect and always includes an amount of waste. *M1*'s wastage ratio, α_{M1}, depends on the general conditions of its machinery and the skills of its labor force and is a random variable with cumulative distribution function G'_{M1} (a variation in *M*1's production system).

To compensate for waste in its production system, more products must be produced than *R1*'s order quantity (x^{WD}). This implies that *M1*'s extra production, represented by $O_{M1}.y^{WD} - x^{WD}$, should be positive. The batch size of each production run, y^{WD}, must be determined to preserve *M1*'s local reliability, rl_{M1}^{WD} (α_{M1}^i represents the value of random variable α_{M1} realized in production run $i = 1, 2, \ldots$, O_{M1}), as follows:

$$ rl_{M1}^{WD} = \Pr\left(\alpha_{M1}^1 . y^{WD} + \alpha_{M1}^2 . y^{WD} + \alpha_{M1}^3 . y^{WD} + \cdots + \alpha_{M1}^{O_{M1}} . y^{WD} \le O_{M1} . y^{WD} - x^{WD}\right) $$

(5)

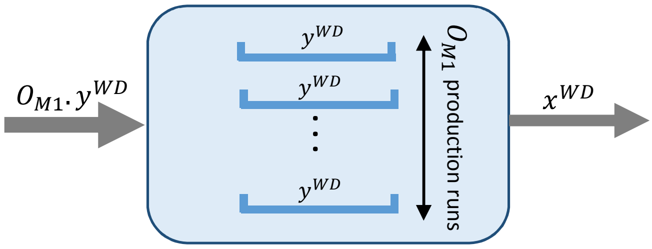


Figure 4. Production runs in *M1*.

To preserve rl_{M1}^{WD} local reliability for *M1*, the number of defective items in all production runs $(\alpha_{M1}^{1}.y^{WD} + \alpha_{M1}^{2}.y^{WD} + \alpha_{M1}^{3}.y^{WD} + \cdots + \alpha_{M1}^{O_{M1}}.y^{WD})$ must be less than the extra production volume $(O_{M1}.y^{WD} - x^{WD})$ with rl_{M1}^{WD} probability, as noted in Equation (5). Without loss of generality, we assume that to manufacture one unit of product, one unit of component is required. Because *M1* will produce $O_{M1}.y^{WD}$ product units, *M1* will issue an order for $O_{M1}.y^{WD}$ component units from it supplier, *S1*. This implies that *R1*'s order is increased to $O_{M1}.y^{WD} - x^{WD}$ units in *M1*. Maintaining local reliability, rl_{M1}^{WD}, is an ORM strategy that is used in *M1* to manage variations in its production system. In Section 4.3, it is shown that *M1*'s order is further amplified by moving backward to the supplier.

4.3. Supplier in the first supply path, S1

In the first supply path, *S1* receives an order for $O_{M1}.y^{WD}$ units of components from *M1*. To fulfill this order, O_{S1} production runs are performed by *S1* with z^{WD} items in each production batch. After setting up *S1*'s machines to produce z^{WD} items, all machines work in an in-control state and all the produced components are in perfect condition. Gradually, the machines deteriorate and after a stochastic time, they shift to an out-of-control state. γ_{S1} is the percentage of the produced components that are defective. The deterioration time of the machines is represented by t, which is a random variable with a G_{S1}'' cumulative distribution function. When the production system shifts to an out-of-control state, it remains in that state until the end of the batch production because interrupting the machines is prohibitively expensive (Rosenblatt and Lee, 1986; Lee and Rosenblatt, 1987). Cap_{S1}^{WD} represents the production capacity of *S1* during each production run with T time units. Therefore, the production rate of *S1* is $Cap_{S1}^{WD}/_{T}$ and it requires $T.z^{WD}/_{Cap_{S1}^{WD}}$ time units to produce each production batch. Before the production system deteriorates, all output units are sound, but after the production system deteriorates, γ_{S1} percent are defective. Therefore, the total number of defective units in the product batch *i* ($i = 1, 2, \dots, O_{S1}$) is equal to $\left(T.z^{WD}/_{Cap_{S1}^{WD}} - t_i\right).\left(\gamma_{S1}.Cap_{S1}^{WD}\right)$. t_i represents the value of random variable *t* in production run *i* ($i = 1, 2, \dots, O_{S1}$). To preserve the local reliability of *S1*, the following constraint is needed:

$$rl_{S1}^{WD} = \Pr\left(\sum_{i=1}^{O_{S1}}\left(T.z^{WD}/_{Cap_{S1}^{WD}} - t_i\right).\left(\gamma_{S1}.Cap_{S1}^{WD}\right) \le \left(O_{S1}.z^{WD}\right) - \left(O_{M1}.y^{WD}\right)\right)$$

$$= \Pr\left(\left(T.\gamma_{S1} - 1\right).O_{S1}.z^{WD} + O_{M1}.y^{WD} \le \gamma_{S1}.Cap_{S1}^{WD}.\sum_{i=1}^{O_{S1}} t_i\right) \quad (6)$$

Constraint (6) ensures that with rl_{M1}^{WD} probability, the total number of defective components produced by *S1* will be less than its surplus production quantity, $O_{S1}.z^{WD} - O_{M1}.y^{WD}$. The value of the z^{WD} variable must ensure that the local reliability of *S1* is preserved. Maintaining local reliability rl_{S1}^{WD} is an ORM strategy that is used to manage variations in *S1*'s production system.

The component production batch size (z^{WD}) satisfies Constraint (6) and ensures the ability of *S1* to fulfill *M1*'s entire order with rl_{S1}^{WD} probability. The production batch size (y^{WD}) satisfies Constraint (5) and guarantees the ability of *M1* to fulfill *R1*'s order with rl_{M1}^{WD} probability. The product stock quantity (x^{WD}) satisfies Constraint (3) and assures the ability of *R1* to fulfill the demand of the market during the next sale period with rl_{R1}^{WD} probability. In this case, the first supply path has a guaranteed probability of $rl_{S1}^{WD}.rl_{M1}^{WD}.rl_{R1}^{WD}$ to fulfill the markets' demand. In this problem, this probability of demand fulfillment is referred to as the service level.

$$sl^{WD} = rl_{S1}^{WD}.rl_{M1}^{WD}.rl_{R1}^{WD} \quad (7)$$

The relationship among the local reliabilities of the facilities in the first supply path and the SN's service level in the markets that are serviced by that path is shown in Equation (7).

In this problem, we assume that all variations (in *R1*'s demand, *M1*'s waste ratio, and *S1*'s deterioration time) are random variables with known distribution functions. Because these variations are related to the SNs' short-term operational decisions (either weekly or monthly), in practice it is possible to gather historical data to fit an appropriate distribution function. Several statistical methods, e.g., goodness-of-fit, can be used to analyze historical data and fit an appropriate distribution function for variations.

4.4. Mathematical model for ORM strategy selection during normal conditions without disruption

In this section, a mathematical model is presented for planning reliable flow in the SN's first supply path by using the analysis and relationships presented in Sections 4.1-4.3.

Maximize

$$\Psi^{WD} = \left(p - h^{+}.E\left[G_{R1}^{-1}\left(Max\left\{sl^{WD},\frac{h^{-}}{h^{-}+h^{+}}\right\}\right) - \varepsilon\right]^{+} - h^{-}.E\left[\varepsilon - G_{R1}^{-1}\left(Max\left\{sl^{WD},\frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right]^{+}\right) \times \left[\sum_{k=1}^{2} D_k(p, sl^{WD})\right] - c_{S1}.(O_{S1}.z^{WD}) - c_{S1,M1}.(O_{M1}.y^{WD}) - c_{M1}.(O_{M1}.y^{WD}) - c_{M1,R1}.(x^{WD}) \quad (8)$$

Subject to:

$$O_{S1}.z^{WD} \geq O_{M1}.y^{WD} \quad (9)$$

$$O_{M1}.y^{WD} \geq x^{WD} \quad (10)$$

$$x^{WD} = \left[\sum_{k=1}^{2} D_k(p, sl^{WD})\right].G_{R1}^{-1}\left(Max\left\{rl_{R1}^{WD},\frac{h^{-}}{h^{-}+h^{+}}\right\}\right) \quad (11)$$

$$rl_{M1}^{WD} = \Pr\left(\alpha_{M1}^{1}.y^{WD} + \alpha_{M1}^{2}.y^{WD} + \alpha_{M1}^{3}.y^{WD} + \cdots + \alpha_{M1}^{O_{M1}}.y^{WD} \leq O_{M1}.y^{WD} - x^{WD}\right) \quad (12)$$

$$rl_{S1}^{WD} = \Pr\left(\sum_{i=1}^{O_{S1}}\left(T.z^{WD}\Big/ Cap_{S1}^{WD} - t_i\right).\left(\gamma_{S1}.Cap_{S1}^{WD}\right) \le \left(O_{S1}.z^{WD}\right) - \left(O_{M1}.y^{WD}\right)\right) \quad (13)$$

$$sl^{WD} = rl_{S1}^{WD}.rl_{M1}^{WD}.rl_{R1}^{WD} \quad (14)$$

$$y^{WD} \le Cap_{M1}^{WD} \quad (15)$$

$$Z^{WD} \le Cap_{S1}^{WD} \quad (16)$$

$$0 \le rl_{S1}^{WD}, rl_{M1}^{WD} \text{ and } rl_{R1}^{WD} \le 1 \text{ and } x^{WD}, y^{WD} \text{ and } z^{WD} \ge 0 \quad (17)$$

The objective function, Equation (8), is used to maximize total profit during conditions without disruptions. The first term of Equation (8) is used to compute the capturable income after discarding the inventory holding cost for the end-of-period extra inventory and the shortage cost for end-of-period lost sales. The second term is the procurement and production cost of the components for *S1*. The third term is the cost of transporting the components from *S1* to *M1*. The fourth term is the cost of manufacturing products in *M1*. The fifth term represents the cost of transporting products from *M1* to *R1*. Based on the constraint in Equation (9), the number of components that are planned to be produced by *S1* should be more than *M1*'s order quantity. According to the constraint in Equation (10), the product production quantity in *M1* must be more than *R1*'s order quantity. The constraints in Equations (11), (12) and (13) represent the relationships between the order and production quantities in *R1*, *M1* and *S1* and their corresponding local reliabilities. The relationships between the service level during conditions without any disruptions and the local reliabilities of stochastic facilities are illustrated in Equation (14). Equations (15) and (16) are used to ensure that the production quantity for each run of *M1* and *S1* is less than its capacity, Cap_{M1}^{WD} and Cap_{S1}^{WD}, respectively. Equation (17) is used to ensure that facilities' local reliabilities are selected from the [0, 1] interval.

4.5. Solution procedure for ORM strategy selection during conditions without disruptions

The mathematical model proposed in Section 4.4 is nonlinear. The objective function and certain constraints in this model (such as Equations (11) and (14)) are highly nonlinear. In addition, this model includes two chance constraints, Equations (12) and (13). Because of these chance constraints, our model belongs to the category of a Chance Constrained Problem (CCP). CCPs were first introduced by Charnes, et al. (1958). For the theoretical background of CCPs, please refer to Prékopa (1995). From an application perspective, CCPs have been used for water management (Dupacová et al., 1991), chemical process optimization (Henrion et al., 2001; Henrion et al., 2003), and others. Although CCPs were introduced almost 50 years ago, little progress has been made to date. A CCP is extremely difficult to solve even in its linear form because it requires multidimensional integration (Pagnoncelli et al., 2009).

In prior studies, the two following approaches are used to solve CCPs: 1) in the first approach, the probability distribution of the chance constraints is discretized and the combinatorial problem thus obtained is solved sequentially (Dentcheva et al., 2000; Luedtke et al., 2008); and 2) in the second approach, the chance constraints are substituted by convex approximations (Nemirovski and Shapiro, 2006). A well-known approximation approach used to address the CCP is the sample average approximation (SAA). The SAA is also referred to as the Monte Carlo method, the Sample Path Optimization (SPO) method, and the Stochastic Counterpart (Robinson, 1996; Pagnoncelli et al., 2009; Atlason et al., 2008; and Luedtke and Ahmed, 2008). The SAA approach replaces the actual distribution in chance constraints by an empirical distribution that corresponds to a random sample. Refer to Rusczynski and Shapiro (2003) for a comprehensive review of this approach. We use the SAA to approximate chance constraints. Then, we linearize the model by discretizing reliability variables. The final model is a Mixed Integer Linear Programming (MILP) model that is solved by using CPLEX software (for more details about linearizing the model, please refer to Appendix B).

4.6. Computational result: Test Problem

In this section, we assume that in the first supply path, $[S1 \rightarrow M1 \rightarrow R1]$, the performances of the production systems for *M1* and *S1* are imperfect. After the equipment is set up in *S1*, the machinery works in an in-control state and all of the produced components are in perfect condition. After a stochastic time that follows an exponential distribution with $\mu = 2$, the machinery shifts to an out-of-control state and $\gamma_{S1} = 10\%$ of the output is defective. For *M1*, the product assembly process always includes a stochastic percentage of defective products. This percentage is a random variable with a uniform distribution in the interval $[0, \beta = 0.15]$. The total demand for the first and second markets that should be fulfilled by *R1* is a stochastic linear function of price, $p = \$14$, and service level, sl^{WD}: $\sum_{k=1}^{2} D_k(p, sl^{WD}) . \varepsilon = [1000 - 150 \times (p - 14) + 1000 \times (sl^{WD} - 0.85)]. \varepsilon$. ε is a normally distributed random variable with a mean of 1 and a variance of 1. Prior regression studies of historical sales data demonstrated that a linear demand function fits very well for $\left(\sum_{k=1}^{2} D_k, p, sl^{WD}\right)$ triples recorded for past sales periods (Bernstein and Federgruen, 2004, 2005, and 2007; Anderson and Bao, 2009). The biases of the real and estimated mean demand in these triples are analyzed by conducting a goodness-of-fit statistical test to determine the optimal distribution that represents these biases. The unit production cost for *S1* is \$1.40. The unit transportation cost for moving a component unit from *S1* to *M1* is \$0.50. The unit assembly cost for *M1* and the unit transportation cost from *M1* to *R1* are \$1.00 and \$0.60, respectively. The unit extra inventory and unit shortage costs for *R1* are \$0.10 and \$0.30, respectively. Demand for each period is fulfilled by $O_{S1} = 3$ and $O_{M1} = 4$ production runs.

Formulating and solving the mathematical model for this problem leads to the following results: the optimal service level for conditions without any disruptions is 80 percent (corresponding to the highest profit in Figure 5). In Figure 5, each point on Line AB corresponds to a service level $sl^{WD} = rl_{S1}^{WD}.rl_{M1}^{WD}.rl_{R1}^{WD} = 0.8$. Point A (red point) is the optimal $\left(rl_{S1}^{WD},\ rl_{M1}^{WD},\ rl_{R1}^{WD}\right)$ combination that maximizes the Model (Equations 8-17). Other points on Line AB (gray points) are feasible $\left(rl_{S1}^{WD},\ rl_{M1}^{WD},\ rl_{R1}^{WD}\right)$ combinations in the Model (Equations 8-17) that would result in a less than optimal profit for the SN. As illustrated in Figure 5, different combinations of local reliabilities for facilities can lead to the same service level, $sl^{WD} = rl_{S1}^{WD}.rl_{M1}^{WD}.rl_{R1}^{WD}$. For all points on line AB, the service level is 0.8, but these correspond to different local reliability combinations and significantly different profit levels. Therefore, for a supply path with multiple stochastic facilities, determining the optimal service level is not sufficient. We must also determine the least costly local reliability combination that supports the required service level. The mathematical model of this problem helps us to determine the optimal local reliability combination, which is calculated as $rl_{S1}^{WD} = 1$, $rl_{M1}^{WD} = 1$ and $rl_{R1}^{WD} = 0.8$. To preserve the local reliability of *R1*, its product order quantity from *M1* must equal $x^{WD} = 1748$. The optimal production quantity for each production run of *M1* is 496.15 which implies that *M1* produces 236.6 extra units ($4.y^{WD} - x^{WD} = 236.6$). This extra production preserves its local reliability, which is equal to 1. The optimal component production quantity for each production run of *S1* is 684.78. This production quantity leads to the extra production of 70 units for *S1* ($3.z^{WD} - 4.y^{WD} = 70$). This extra production assures a local reliability of 1 for *S1*.

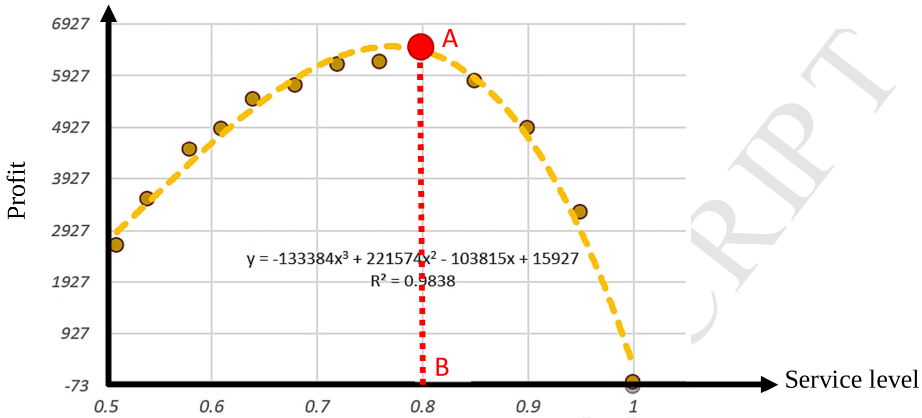


Figure 5. Profit of the first supply path with respect to the service level.

In the remainder of this section, we analyze the relationships among the local reliabilities of facilities in the supply path and the SN's profitability. For this purpose, we solve the model for different values of local reliabilities. The results are illustrated in the graphs of Figure 6. Based on these graphs, we conclude the following:

- For a given local reliability of *R1*, the patterns that determine the profit change with respect to *S1*'s local reliability are similar for all the local reliabilities of *M1*. This implies that for a given quantity of ordered product, the most profitable local reliabilities for *S1* and *M1* are almost independent. Therefore, determining the local reliability for these facilities separately leads to a workable and near-optimal solution. This feature significantly decreases the size and computational burden of the mathematical model. Therefore, it is necessary to consider this feature for large-scale problems to reduce their computational time.

- For a given local reliability of *R1*, the effects of the local reliabilities for *M1* and *S1* on the path's profit are similar. For instance, if reductions in *S1*'s local reliability lead to profit reductions for the path, reductions in *M1*'s local reliability also lead to profit reductions for the path and vice versa (see $rl_{R1}^{WD} = 1.00$ case in Figure 6). If reductions in *S1*'s local reliability first increase the path's profit and then reduce it, reductions in *M1*'s local reliability impose a similar pattern of changes on the path's profit (see $rl_{R1}^{WD} = 0.95$ case in Figure 6). Therefore, determining the optimal local reliability for one of these facilities provides a good estimate for the tentative local reliability of another facility. Using this feature significantly reduces the search interval for the local reliability of the other facility. Therefore, it is necessary that we consider this feature for large scale problems to reduce the computational time.

In this section, we develop a mathematical model to determine the most profitable local reliability (ORM strategy) for the SN's facilities against their variations. In Section 5, we consider the possibility of disruption and demonstrate how the model should be extended to incorporate SRM strategies.

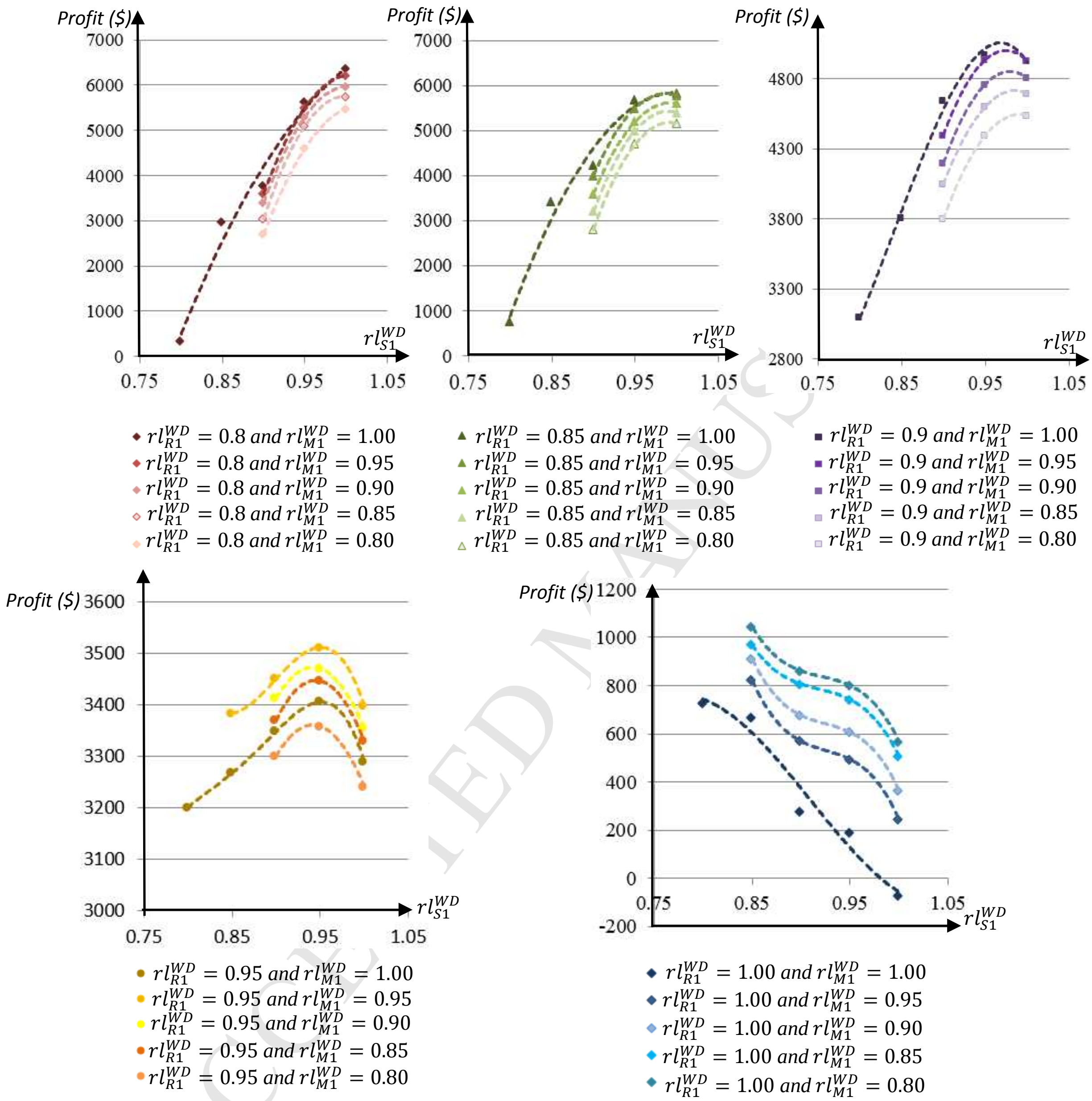


Figure 6. Relationships among the local reliabilities of the facilities in the supply path and their profitability.

5. Employing SRM Strategies

The SN is disrupted when *M2* or *S2* is unavailable. In this case, the second supply path, $[S2 \rightarrow M2 \rightarrow R2]$, is inoperative and unable to fulfill the demands of the third and fourth markets. Therefore, the only active supply path is $[S1 \rightarrow M1 \rightarrow R1]$, which can be used to fulfill the demands of all markets (Figure 7).

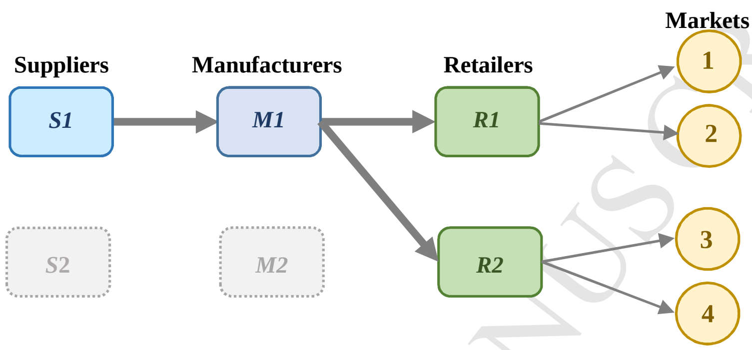


Figure 7. Network structure of the SN under disrupted conditions.

To address this disruption, the first supply path must not only serve the first and second markets but must also fulfill the demands of the third and fourth markets. For this purpose, its facilities, *S1* and *M1*, need flexible capacities. Following the onset of a disruption, the capacities of these facilities should increase to service both retailers and after the duration of the disruption, and they should decrease to only service *R1*. The measurement of the extent to which the capacity of a facility can be increased during disruptions is its flexibility level and the length of time that it takes to increase that capacity is its flexibility speed. The robustness of a SN is determined by its flexibility levels and the resilience of a SN is determined by its flexibility speeds. Determining the capacity of a production system is a strategic design problem and depends on factors such as the layout of its machinery. Adding capacity is generally a discrete process that involves adding machines to the system (Koren and Shpitalni, 2014). Figure 8 provides examples of *M1*'s flexibility speed. In this figure, it is assumed that one period that includes four production runs, $O_{M1} = 4$, is the maximum time that is available to increase capacity, and the flexibility level of *M1* is equal to Δ_{M1}. These flexibility speed options imply the following:

- ***For the first flexibility speed option, which is indicated by r^1_{M1} in Figure 8*:** an amount of time that is equal to three production runs is provided to *M1* to generate the extra capacity. In this extreme case, all of *M1*'s extra capacity, Δ_{M1}, is added at the beginning of the last (fourth) production run. The time pattern for this flexibility speed option is $r^1_{M1} = (r1^1_{M1} = 0, r2^1_{M1} = 0, r3^1_{M1} = 0, r4^1_{M1} = \Delta_{M1})$, which implies that the capacity increases during the first ($r1^1_{M1}$), second ($r2^1_{M1}$), and third ($r3^1_{M1}$) production runs are equal to 0 and for the last run ($r4^1_{M1}$), it is equal to Δ_{M1};
- ***For the second flexibility speed option, which is indicated by r^2_{M1} in Figure 8*:** $r^2_{M1} = (r1^2_{M1} = 0, r2^2_{M1} = 0, r3^2_{M1} = \Delta_{M1}/2, r4^2_{M1} = \Delta_{M1}/2)$;

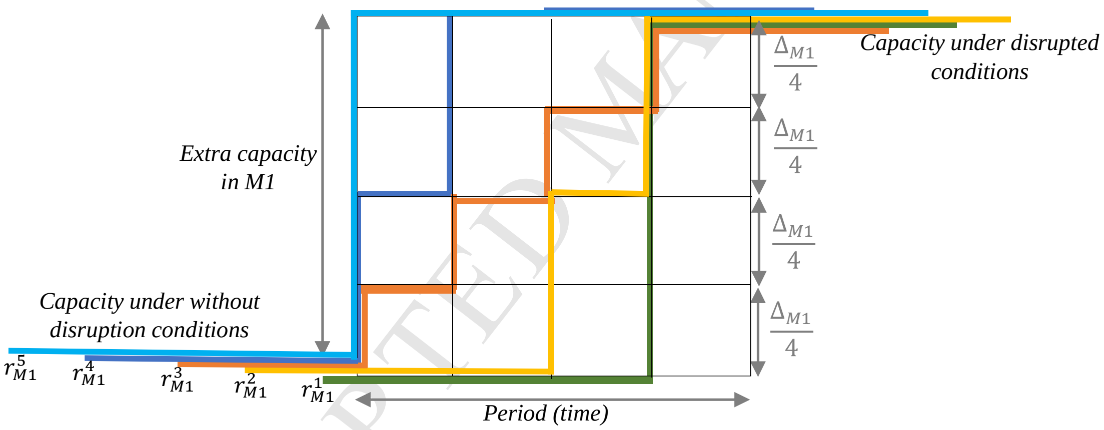


Figure 8. Sample resilience options for capacity ramp up in *M1*.

- ***For the third flexibility speed option, indicated by r^3_{M1} in Figure 8*:** $r^3_{M1} = (r1^3_{M1} = \Delta_{M1}/4, r2^3_{M1} = \Delta_{M1}/4, r3^3_{M1} = \Delta_{M1}/4, r4^3_{M1} = \Delta_{M1}/4)$;
- ***For the fourth flexibility speed option, indicated by r^4_{M1} in Figure 8*:** $r^4_{M1} = (r1^4_{M1} = \Delta_{M1}/2, r2^4_{M1} = \Delta_{M1}/2, r3^4_{M1} = 0, r4^4_{M1} = 0)$;
- ***For the fifth flexibility speed option, indicated by r^5_{M1} in Figure 8*:** $r^5_{M1} = \left(r1^5_{M1} = \Delta_{M1}, r2^5_{M1} = 0, r3^5_{M1} = 0, r4^5_{M1} = 0\right)$;

Therefore, we define a new set, $RO_{M1} = \{r_{M1}\}$, which includes all flexibility speed options that are available for *M1*. Providing extra production capacity costs more during early production runs following a disruption. Acquiring additional machinery and labor to increase the capacity over a short time can be difficult and costly. Conversely, an early increment in capacity leads to the availability of increased capacity during future production runs and, subsequently, more feasible production plans will be available to select from and more uniform production quantities during future production runs are possible. Therefore, we assume that the unit capacity increment cost is higher for early production runs. This assumption is consistent with observations in manufacturing systems (Koren and Shpitalni, 2014).

Assuming that parameter cap_{M1}^{i} $(i = 1, 2, \dots, O_{M1})$ represents the unit extra capacity cost for *M1*'s production run *i*, we have $cap_{M1}^{1} \geq cap_{M1}^{2} \geq cap_{M1}^{3} \geq \cdots \geq cap_{M1}^{O_{M1}}$. To determine the flexibility speed option, the binary variables, $w_{M1}^{r_{M1}}$ $(r_{M1} \in RO_{M1})$, are used. Variable $w_{M1}^{r_{M1}}$ is 1 if the flexibility speed option r_{M1} is selected for *M1*, and 0 otherwise. In the same manner, Δ_{S1} represents the flexibility level of *S1*, and different flexibility speed options are available that are included in the set, $RO_{S1} = \{r_{S1}\}$. Assuming that parameter cap_{S1}^{j} $(j = 1, 2, \dots, O_{S1})$ represents the unit extra capacity cost for *S1*'s production run *j*, we have $cap_{S1}^{1} \geq cap_{S1}^{2} \geq cap_{S1}^{3} \geq \cdots \geq cap_{S1}^{O_{S1}}$. To select the resilience option for *S1*, the binary variables, $w_{S1}^{r_{S1}}$ $(r_{S1} \in RO_{S1})$, are used. Variable $w_{S1}^{r_{S1}}$ is 1 if the resilience option r_{S1} is selected for *S1*, and 0 otherwise.

When a disruption occurs in the second supply path, the capacities of the first supply path's facilities, *M1* and *S1*, shifts from their without disruption values, Cap_{M1}^{WD} and Cap_{S1}^{WD}, to the capacity values that are suitable for the disrupted condition, Cap_{M1}^{D} and Cap_{S1}^{D}, based on the flexibility speed options that are selected. The time period in which the undisrupted capacity of a facility, Cap_{M1}^{WD} or Cap_{S1}^{WD}, shifts to its disrupted condition capacity, Cap_{M1}^{D} or Cap_{S1}^{D}, is referred to here as the ramp-up disruption period. The production capacities of *M1* and *S1* are not fixed during this ramp-up disruption period and may change for each production run. In Section 5.1, we elaborate on the production plan in the first supply path's facilities in the ramp-up disruption period. After the ramp-up period, capacities Cap_{M1}^{D} and Cap_{S1}^{D} are available for *M1* and *S1* for all production runs until the disruption dissipates. The disrupted periods that occur after the ramp-up period are referred to as normal-disruption periods. In Section 5.2, we elaborate on the production plan in the first supply path's facilities for a normal-disruption period. When the disruption ends, the extra capacity is not needed in the facilities of the first supply path. Therefore, the capacities of *M1* and *S1* must be reduced from Cap_{M1}^{D} and Cap_{S1}^{D} to Cap_{M1}^{WD} and Cap_{S1}^{WD}, respectively. The time period after the disruption is referred to as the ramp-down disruption period. The ramp-down disruption period is also the without disruption period; the only difference is that extra capacity is available. In Section 5.3, we elaborate on the production plan in the first supply path's facilities for a ramp-down disruption period. In Figure 9, we illustrate these periods for r_{M1}^{3} (one of the flexibility speed options illustrated in Figure 8) when the disruption lasts for only two periods. In this case, there is one ramp-up, one normal, and one ramp-down disruption period. For longer disruptions, more than one normal disruption period would occur.

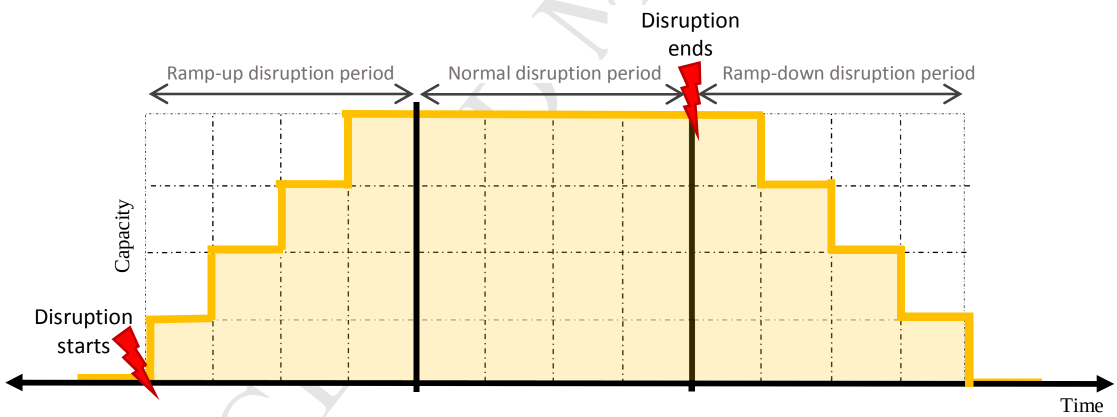


Figure 9. Ramp-up, normal disruption, and ramp-down periods for a disruption lasting for two periods.

5.1. Ramp-up disruption period (see Figure 9)

The capacities of the facilities in the first supply path (*S1* and *M1*) for each production run of the ramp-up disruption period depend on their selected flexibility speed options. Assume that the y_i^{RUD} and z_i^{RUD} variables represent the production quantities for the ramp-up disruption period's production run i of *M1* and *S1*, respectively.

During the ramp-up disruption period, each facility's production quantity for each production run must be less than its available capacity. Therefore, the following restrictions are imposed on these facilities:

$$y_i^{RUD} \leq Cap_{M1}^{WD} + \sum_{r_{M1}=1}^{|RO_{M1}|}\left(\sum_{j=1}^{i} rj_{M1}^{r_{M1}}\right).w_{M1}^{r_{M1}} \qquad (i = 1, 2, \ldots, O_{M1}) \tag{31}$$

$$z_i^{RUD} \leq Cap_{S1}^{WD} + \sum_{r_{S1}=1}^{|RO_{S1}|}\left(\sum_{j=1}^{i} rj_{S1}^{r_{S1}}\right).w_{S1}^{r_{S1}} \qquad (i = 1, 2, \ldots, O_{S1}) \tag{32}$$

It is clear that only one of the available options for the flexibility speed of each facility can be selected. Therefore:

$$\sum_{r_{M1}=1}^{|RO_{M1}|} w_{M1}^{r_{M1}} = 1 \tag{33}$$

$$\sum_{r_{S1}=1}^{|RO_{S1}|} w_{S1}^{r_{S1}} = 1 \tag{34}$$

During a disruption, the total product order received by *M1*, x^D, is calculated as follows:

$$x^D = x_1^D + x_2^D \tag{35}$$

$$x_1^D = \left[\sum_{k=1}^{2} D_k(p, sl^D)\right].G_{R1}^{-1}\left(Max\left\{rl_{R1}^D, \frac{h^-}{h^-+h^+}\right\}\right) \tag{36}$$

$$x_2^D = \left[\sum_{k=3}^{4} D_k(p, sl^D)\right].G_{R2}^{-1}\left(Max\left\{rl_{R2}^D, \frac{h^-}{h^-+h^+}\right\}\right) \tag{37}$$

In these equations, x_1^D and x_2^D represent the orders issued by *R*1 and *R*2, respectively. As explained in Section 4.1, Equations (36) and (37) are used to determine the ordering quantities of the retailers in a way that preserves their local reliabilities during disruptions rl_{R1}^D and rl_{R2}^D.

sl^D represents the service level that is provided by the SN during disruptions. To preserve the local reliabilities of *M1* and *S1* during a disruption, rl_{M1}^D and rl_{S1}^D, the following equations are necessary:

$$rl_{M1}^D = \Pr\left(\sum_{i=1}^{O_{M1}} \alpha_{M1}^i.y_i^{RUD} \leq \sum_{i=1}^{O_{M1}} y_i^{RUD} - x^D\right) \tag{38}$$

$$rl_{S1}^D =$$

$$\Pr\left((\gamma_{S1}.T - 1).\sum_{i=1}^{O_{S1}} z_i^{RUD} + \sum_{j=1}^{O_{M1}} y_j^{RUD} \leq \sum_{k=1}^{O_{S1}} \gamma_{S1}.t_k.\left(Cap_{S1}^{WD} + \sum_{r_{S1}=1}^{|RO_{S1}|}\left(\sum_{j=1}^{i} rj_{S1}^{r_{S1}}\right).w_{S1}^{r_{S1}}\right)\right)$$

(39)

Based on Equation (38), the sum of defective products for all production runs in the ramp-up disruption period is less than the added manufacturing quantity, $\sum_{i=1}^{O_{M1}} y_i^{RUD} - x^D$, with a probability of rl_{M1}^D. Equation (39) is used to ensure that the number of defective components for all production runs of *S1* during the ramp-up disruption period is less than the added production quantity with a probability of rl_{S1}^D. Equation (39) is a simplified version of the following equation, which is the modified version of Equation (13):

$$rl_{S1}^D = \Pr\left(\sum_{k=1}^{O_{S1}}\left(\frac{T.z_k^{RUD}}{Cap_{S1}^{WD}+\sum_{r_{S1}=1}^{|RO_{S1}|}\left(\sum_{j=1}^{k} rj_{S1}^{r_{S1}}\right).w_{S1}^{r_{S1}}} - t_k\right).\gamma_{S1}.\left(Cap_{S1}^{WD} + \sum_{r_{S1}=1}^{|RO_{S1}|}\left(\sum_{j=1}^{k} rj_{S1}^{r_{S1}}\right).w_{S1}^{r_{S1}}\right) \leq \left(\sum_{i=1}^{O_{S1}} z_i^{RUD}\right) - \left(\sum_{j=1}^{O_{M1}} y_j^{RUD}\right)\right) \quad (40)$$

Similar to the service level in conditions without disruption shown in Equation (14), the service levels provided by *R1* and *R2* to their markets during the ramp-up disruption period are $rl_{S1}^D.rl_{M1}^D.rl_{R1}^D$ and $rl_{S1}^D.rl_{M1}^D.rl_{R2}^D$, respectively. Without loss of generality, we assume that identical service levels are provided for all markets, which implies that $rl_{R1}^D = rl_{R2}^D$. Therefore, rl_R^D represents the local reliability of both retail facilities. Using a model that assumes similar service levels makes it easier to analyze the relationship between a SN's ORM and SRM strategies. Using this assumption, the service level for all markets under disrupted conditions is as follows:

$$sl^D = rl_{S1}^D.rl_{M1}^D.rl_R^D \quad (41)$$

The total profit that can be captured in the ramp-up disruption period is as follows:

$$\Psi^{RUD} = \Bigg\{\left(p - h^+.E\left[G_{R1}^{-1}\left(Max\left\{sl^D, \frac{h^-}{h^-+h^+}\right\}\right) - \varepsilon\right]^+ - h^-.E\left[\varepsilon - G_{R1}^{-1}\left(Max\left\{sl^D, \frac{h^-}{h^-+h^+}\right\}\right)\right]^+\right) \times \left[\sum_{k=1}^{2} D_k(p, sl^D)\right] + \left(P - h^+.E\left[G_{R2}^{-1}\left(Max\left\{sl^D, \frac{h^-}{h^-+h^+}\right\}\right) - \varepsilon\right]^+ - h^-.E\left[\varepsilon - G_{R2}^{-1}\left(Max\left\{sl^D, \frac{h^-}{h^-+h^+}\right\}\right)\right]^+\right) \times \left[\sum_{k=3}^{4} D_k(p, sl^D)\right]\Bigg\}$$

$$-c_{S1}.\left(\sum_{i=1}^{O_{S1}} z_i^{RUD}\right) - c_{S1,M1}.\left(\sum_{i=1}^{O_{M1}} y_i^{RUD}\right) - c_{M1}.\left(\sum_{i=1}^{O_{M1}} y_i^{RUD}\right)$$

$$- c_{M1,R1}.x_1^D - c_{M1,R2}.x_2^D$$

$$-\sum_{i=1}^{O_{M1}} cap_{M1}^i.\left(\sum_{r_{M1}=1}^{|RO_{M1}|} ri_{M1}^{r_{M1}}.w_{M1}^{r_{M1}}\right) - \sum_{j=1}^{O_{S1}} cap_{S1}^j.\left(\sum_{r_{S1}=1}^{|RO_{S1}|} rj_{S1}^{r_{S1}}.w_{S1}^{r_{S1}}\right)$$

$$-\sum_{i=1}^{O_{M1}} h_{M1}^i.\left(Cap_{M1}^{WD} + \sum_{r_{M1}=1}^{|RO_{M1}|}\left(\sum_{j=1}^{i} rj_{M1}^{r_{M1}}\right).w_{M1}^{r_{M1}} - y_i^{RUD}\right)$$

$$-\sum_{i=1}^{O_{S1}} h_{S1}^i.\left(Cap_{S1}^{WD} + \sum_{r_{S1}=1}^{|RO_{S1}|}\left(\sum_{j=1}^{i} rj_{S1}^{r_{S1}}\right).w_{S1}^{r_{S1}} - z_i^{RUD}\right) \quad (42)$$

Most of the terms in this function were explained in Section 4.4. However, the last four terms are new. The first two new terms represent the cost of adding capacity to the production runs of *M1* and *S1*. The last two new terms are related to the unused capacity costs for *M1* and *S1*.

5.2. The normal disruption period (see Figure 9)

A disruption that continues after the ramp-up disruption period results in at least one normal disruption period. The capacities of *M1* and *S1* for all production runs during a normal disruption period are calculated as $Cap_{M1}^D = Cap_{M1}^{WD} + \Delta_{M1}$ and $Cap_{S1}^D = Cap_{S1}^{WD} + \Delta_{S1}$, respectively. The total product order received by *M1* during a normal disruption period is similar to the ramp-up period.

$$x^D = x_1^D + x_2^D \quad (43)$$

$$x_1^D = \left[\sum_{k=1}^{2} D_k(p, sl^D)\right].G_{R1}^{-1}\left(Max\left\{rl_R^D, \frac{h^-}{h^- + h^+}\right\}\right) \quad (44)$$

$$x_2^D = \left[\sum_{k=3}^{4} D_k(p, sl^D)\right].G_{R2}^{-1}\left(Max\left\{rl_R^D, \frac{h^-}{h^- + h^+}\right\}\right) \quad (45)$$

Variables y^{ND} and z^{ND} represent the production quantities for the production runs during a normal disruption period for *M1* and *S1*, respectively. The amount of production for each run of these facilities must be less than their available capacities. Therefore, the following restrictions are imposed on the facilities:

$$y^{ND} \le Cap_{M1}^{WD} + \Delta_{M1} \quad (46)$$

$$z^{ND} \le Cap_{S1}^{WD} + \Delta_{S1} \quad (47)$$

As discussed in Section 5.1, it is assumed that rl_{S1}^{D}, rl_{M1}^{D} and rl_{R}^{D} represent the local reliabilities of the first supply path's supplier, manufacturer and retailers, respectively, during disruptions. To preserve these local reliabilities during normal disruption periods, the following equations become necessary:

$$rl_{M1}^{D} = \Pr\left(\sum_{i=1}^{O_{M1}} \alpha_{M1}^{i}.y^{ND} \leq O_{M1}.y^{ND} - x^{D}\right) \tag{48}$$

$$rl_{S1}^{D} = \Pr\left(\sum_{i=1}^{O_{S1}}\left(\frac{T.z^{ND}}{Cap_{S1}^{WD}+\Delta_{S1}} - t_i\right).\gamma_{S1}.\left(Cap_{S1}^{WD} + \Delta_{S1}\right) \leq \left(O_{S1}.z^{ND}\right) - \left(O_{M1}.y^{ND}\right)\right)$$

$$= \Pr\left((T.\gamma_{S1} - 1).O_{S1}.z^{ND} + O_{M1}.y^{ND} \leq \gamma_{S1}.\left(Cap_{S1}^{WD} + \Delta_{S1}\right).\sum_{i=1}^{O_{S1}} t_i\right) \tag{49}$$

The total profit that can be captured during the normal disruption period is calculated as follows:

$$\Psi^{ND} = \Bigg\{\left(p - h^{+}.E\left[G_{R1}^{-1}\left(Max\left\{sl^{D},\frac{h^{-}}{h^{-}+h^{+}}\right\}\right) - \varepsilon\right]^{+} - h^{-}.E\left[\varepsilon - G_{R1}^{-1}\left(Max\left\{sl^{D},\frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right]^{+}\right) \times$$

$$\left[\sum_{k=1}^{2} D_k(p, sl^{D})\right] +$$

$$\left(P - h^{+}.E\left[G_{R2}^{-1}\left(Max\left\{sl^{D},\frac{h^{-}}{h^{-}+h^{+}}\right\}\right) - \varepsilon\right]^{+} - h^{-}.E\left[\varepsilon - G_{R2}^{-1}\left(Max\left\{sl^{D},\frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right]^{+}\right) \times$$

$$\left[\sum_{k=3}^{4} D_k(p, sl^{D})\right]\Bigg\}$$

$$-c_{S1}.\left(O_{S1}.z^{ND}\right) - c_{S1,M1}.\left(O_{M1}.y^{ND}\right) - c_{M1}.\left(O_{M1}.y^{ND}\right)$$

$$-\, c_{M1,R1}.x_1^{D} - c_{M1,R2}.x_2^{D}$$

$$-\sum_{i=1}^{O_{M1}} h_{M1}^{i}.\left(Cap_{M1}^{WD} + \Delta_{M1} - y^{ND}\right)$$

$$-\sum_{i=1}^{O_{S1}} h_{S1}^{i}.\left(Cap_{S1}^{WD} + \Delta_{S1} - z^{ND}\right) \tag{50}$$

5.3. The ramp-down disruption period (see Figure 9)

During the ramp-down disruption period, the disruption is terminated, and the second supply path is available again to service its corresponding markets. During this period, the production plan is similar to normal periods that do not have a disruption, as discussed in Section 4. The only difference is that certain extra production capacities have been added to the non-disrupted facilities, *M*1 and *S*1. Therefore, the total profit during the ramp-down disruption period is calculated as follows:

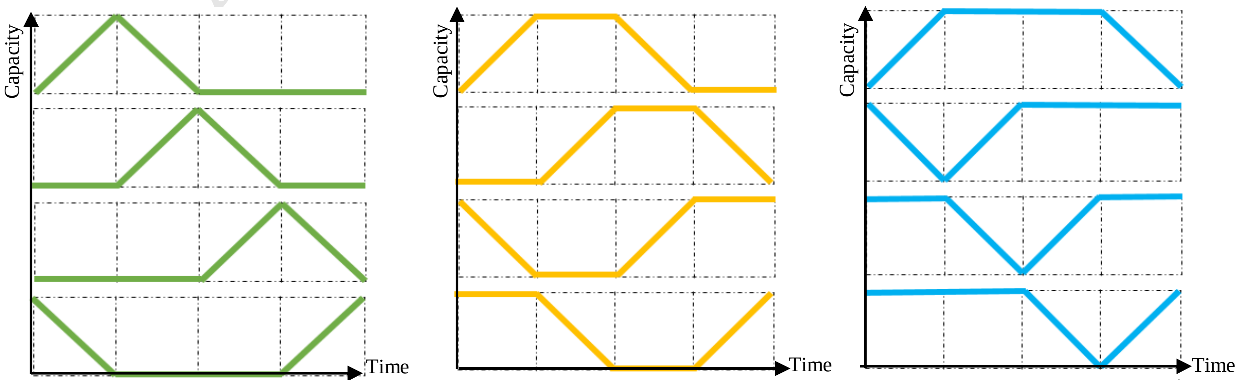
$$\Psi^{RD} = \Psi^{WD^*} - \sum_{i=1}^{o_{M1}} h_{M1}^{i} . \left[\sum_{r_{M1}=1}^{|RO_{M1}|} \left(\sum_{j=1}^{o_{M1}-(i-1)} rj_{M1}^{r_{M1}} \right) . w_{M1}^{r_{M1}} \right] - \sum_{i=1}^{o_{S1}} h_{S1}^{i} . \left[\sum_{r_{S1}=1}^{|RO_{S1}|} \left(\sum_{j=1}^{o_{M1}-(i-1)} rj_{S1}^{r_{S1}} \right) . w_{S1}^{r_{S1}} \right] \tag{51}$$

Ψ^{WD^*} is the solution of the model without disruption that is given in Equations 8-17 and represents the highest profit that can be achieved during each period that does not have a disruption. The second and third terms of Equation (51) represent the unused capacity costs for *M1* and *S1*, respectively.

5.4. Mathematical model for ORM and SRM strategy selection under disrupted conditions

We define different scenarios by the lengths of the disruptions. The number of normal disruption periods is different for each scenario. Set $SCE = \{s\}$ includes all possible scenarios. In Figure 10, set SCE is assumed to include four scenarios, i.e., $\{s_1, s_2, s_3, s_4\}$. Scenario s_1 represents the without disruption case. The remaining scenarios are described below.

- ✓ **In Scenario s_2:** the disruption continues for only one period. Therefore, there is no normal disruption period. In this case, the planning horizon spanning four sales periods has one ramp-up disruption, one ramp-down disruption, and two without disruption sales periods.
- ✓ **In Scenario s_3:** the disruption continues for two periods. Therefore, there is only one normal disruption period. In this case, the planning horizon includes one ramp-up disruption, one ramp-down disruption, one normal disruption, and one without disruption period.
- ✓ **In Scenario s_4:** the disruption continues for three periods and there are two normal disruption periods. In this case, the planning horizon includes one ramp-up disruption, one ramp-down disruption, and two normal disruption periods.



a) One period disruptions (s_2) b) Two period disruptions (s_3) c) Three period disruptions (s_4)

✓
✓
✓
✓

Figure 10. Sample scenarios for the length of disruption.

Each of the disruption scenarios, $s \in SCE$, occurs with a probability of pr_s. It is clear that,

$$\sum_{s=1}^{|SCE|} pr_s = 1 \quad (52)$$

Parameters num_s^{WD}, num_s^{RUD}, $num_s^{ND}$and num_s^{RD} show the number of without disruption, ramp-up disruption, normal disruption and ramp-down disruption periods in scenario $s \in SCE$, respectively. The flexibility level decisions (represented by the Δ_{M1} and Δ_{S1} variables) and flexibility speed decisions (represented by $w_{M1}^{r_{M1}}$ and $w_{S1}^{r_{S1}}$) for the first supply path's facilities should be made in a manner that maximizes the expected profit for all possible disruption scenarios. Therefore, the objective function of the SN under disrupted conditions is as follows:

Maximize $\Psi = \sum_{S=1}^{|SCE|} pr_s.\left[num_s^{WD}.\Psi^{WD^*} + num_s^{RUD}.\Psi^{RUD} + num_s^{ND}.\Psi^{ND} + num_s^{RD}.\Psi^{RD}\right]$ (53)

Subject to: (31-39), (41) and (46-49) (54)

$$\Delta_{M1}, \Delta_{S1}, y_i^{RUD}, z_j^{RUD}, y^{ND}, z^{ND}, x^D, x_1^D, x_2^D, sl^D, rl_{S1}^D, rl_{M1}^D, rl_R^D \geq 0$$

$$(i = 1, 2, \ldots, O_{M1} \quad and \quad j = 1, 2, \ldots, O_{S1}) \quad (55)$$

$$w_{M1}^{r_{M1}}, w_{S1}^{r_{S1}} \in \{0,1\} \qquad (\forall r_{M1} \in RO_{M1}, \forall r_{S1} \in RO_{S1}) \quad (56)$$

The mathematical model of disrupted conditions is a CCP similar to the model that was developed in Section 4 for normal conditions without any disruptions (the Model in Equations 8-17). The objective function of the model and the constraints that are presented in Equations 36, 37, and 41 are nonlinear. The constraints in Equations (38), (39), (48) and (49) are chance constraints. This model can be linearized using the approach described in Section 4.5. Appendix C provides more information about the size of the problems that can be solved by this model.

This model is used to simultaneously determine the most profitable (1) local reliabilities for the SN's facilities against their corresponding variations (ORM strategies) and (2) flexibility levels and speeds for its non-disrupted facilities that can compensate for the unavailability of its disrupted facilities and ensure the SN remains robust and resilient (SRM strategies). This concurrent determination makes it possible to determine what correlations exist between optimal ORM and SRM strategies in SNs. These correlations are investigated in Section 5.5.

5.5. Computational result: Extension of the Test Problem

In this section, we extend the test problem that was investigated in Section 4.6. We assume that disruption is possible in the second supply path, for which the total demand of the third and fourth markets is calculated as $\sum_{k=3}^{4} D_k(p, sl^D)\,.\,\varepsilon = [850 - 150 \times (p - 14) + 900 \times (sl^D - 0.85)].\,\varepsilon$, and should be fulfilled by the first supply path. ε is a normal random variable with a mean of 1 and a variance of 1. Four different scenarios for the length of disruption are possible in this problem, $SCE = \{s_1, s_2, s_3, s_4\}$. There is no disruption in Scenario s_1. Scenarios s_2, s_3 and s_4 represent disruptions with zero, one, and two normal disruption periods, respectively. The probabilities of these scenarios are as follows: $p_{s_1} = .83$, $p_{s_2} = .04$, $p_{s_3} = .10$ and $p_{s_4} = .03$.

The costs of adding extra capacity for each production run of *M1* are $cap_{M1}^1 = \$1$, $cap_{M1}^2 = \$0.8$, $cap_{M1}^3 = \$0.65$, and $cap_{M1}^4 = \$0.55$, respectively. The costs of adding extra capacity for the first, second, and third production runs of *S1* are $cap_{S1}^1 = \$1$, $cap_{S1}^2 = \$0.7$ and $cap_{S1}^3 = \$0.5$. The extra capacity cost for *S1* and *M1* in all production runs is $h_{S1}^i = h_{M1}^j = \$0.10$ ($i = 1, \dots, O_{S1}$ and $j = 1, \dots, O_{M1}$). The production and transportation cost components are similar to those in Section 4.6. The only new cost component is $c_{M1,R2} = \$0.70$ (the cost of transporting a unit of product from *M1* to *R2*). Based on the optimal production quantities that were determined for the production runs of the test problem in Section 4.6, we assume $Cap_{S1}^{WD} = 800$ and $Cap_{M1}^{WD} = 500$. Five different options for *M1*'s flexibility speed are assumed, as follows: $r_{M1}^1 = (r1_{M1}^1 = 0, r2_{M1}^1 = 0, r3_{M1}^1 = 0, r4_{M1}^1 = \Delta_{M1})$,

$r_{M1}^{2}=\left(r1_{M1}^{2}=0,\ r2_{M1}^{2}=0, r3_{M1}^{2}=\frac{\Delta_{M1}}{2}, r4_{M1}^{2}=\frac{\Delta_{M1}}{2}\right)$, $r_{M1}^{3}=\left(r1_{M1}^{3}=\frac{\Delta_{M1}}{4}, r2_{M1}^{3}=\frac{\Delta_{M1}}{4}, r3_{M1}^{3}=\frac{\Delta_{M1}}{4}, r4_{M1}^{3}=\frac{\Delta_{M1}}{4}\right)$, $r_{M1}^{4}=\left(r1_{M1}^{4}=\frac{\Delta_{M1}}{2}, r2_{M1}^{4}=\frac{\Delta_{M1}}{2}, r3_{M1}^{4}=0,\ r4_{M1}^{4}=0\right)$, and $r_{M1}^{5}=\left(r1_{M1}^{5}=\Delta_{M1}, r2_{M1}^{5}=0, r3_{M1}^{5}=0, r4_{M1}^{5}=0\right)$.

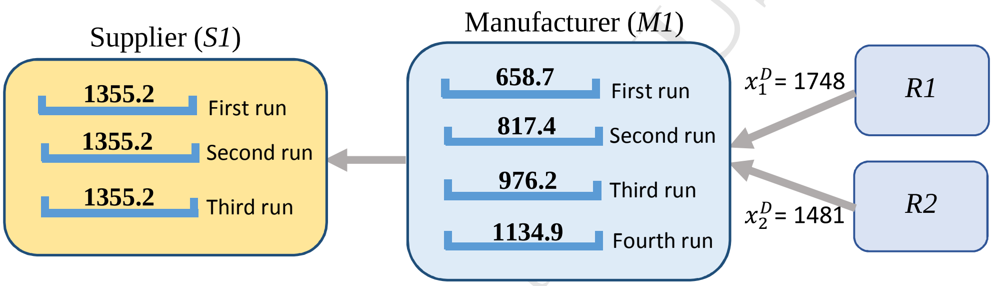


Figure 11. Flow dynamics in the first supply path during the ramp-up disruption period.

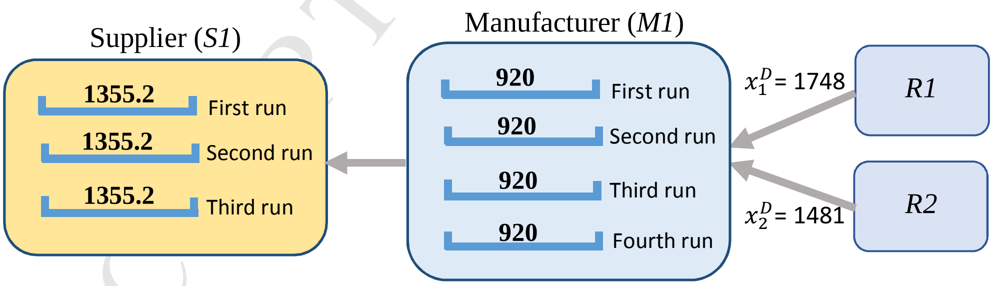


Figure 12. Flow dynamics in the first supply path during the normal disruption period.

In addition, for *S1*, five flexibility speed options are considered, as follows: $r_{S1}^{1}=(r1_{S1}^{1}=0,\ r2_{S1}^{1}=0,\ r3_{S1}^{1}=\Delta_{S1})$, $r_{S1}^{2}=\left(r1_{S1}^{2}=0,\ r2_{S1}^{2}=\frac{\Delta_{S1}}{3}, r3_{S1}^{2}=2\frac{\Delta_{S1}}{3}\right)$, $r_{S1}^{3}=\left(r1_{S1}^{3}=\frac{\Delta_{S1}}{3}, r2_{S1}^{3}=\frac{\Delta_{S1}}{3}, r3_{S1}^{3}=\right.$

$\frac{\Delta_{S1}}{3}\big)$, $r_{S1}^{4} = \left(r1_{S1}^{4} = 2\frac{\Delta_{S1}}{3}, r2_{S1}^{4} = \frac{\Delta_{S1}}{3}, r3_{S1}^{4} = 0\right)$ and $r_{S1}^{5} = \left(r1_{S1}^{5} = \Delta_{S1}, r2_{S1}^{5} = 0,\ r3_{S1}^{5} = 0\right)$. We solved this model using CPLEX Concert Technology on a Dell laptop computer with Windows 10, an Intel i7 processor, and 8 GB of installed RAM. The computational time was less than 6 minutes.

Solving the model of this problem leads to the following results. The optimal service level for the disrupted condition is 80 percent and the best supporting local reliability combination is $rl_{S1}^{D} = 1,\ rl_{M1}^{D} = 1$ and $rl_{R}^{D} = 0.8$. To preserve these local reliabilities, the required flexibility levels of *S1* and *M1* are $\Delta_{S1} = 555.2$ and $\Delta_{M1} = 634.9$, respectively. The optimal flexibility speed for *M1* is $w_{M1}^{3} = 1$, which implies that uniform capacity scalability is preferred for this facility. The optimal flexibility speed for *S1* is $w_{S1}^{5} = 1$, which implies that all extra capacity is added at the beginning of the first production run after disruption. The ordering and production quantities for the production runs of the first supply path's facilities during the ramp-up and normal disruption periods are represented in Figures 11 and 12, respectively.

The average profit of the first supply path, with respect to the disrupted condition's service level, is displayed in Figure 13. When comparing Figures 5 and 13, it can be noted that the profit reduction on both sides of the most profitable service level is less during the disrupted condition than for the condition without disruption. This gentler reduction is due to 1) the higher potential demand that is assigned to this path during the disrupted condition in which the first supply path services the first, second, third and fourth markets and 2) the decreased sensitivity of the third and fourth markets with respect to the service level (the service level sensitivity parameter in these markets is 900).

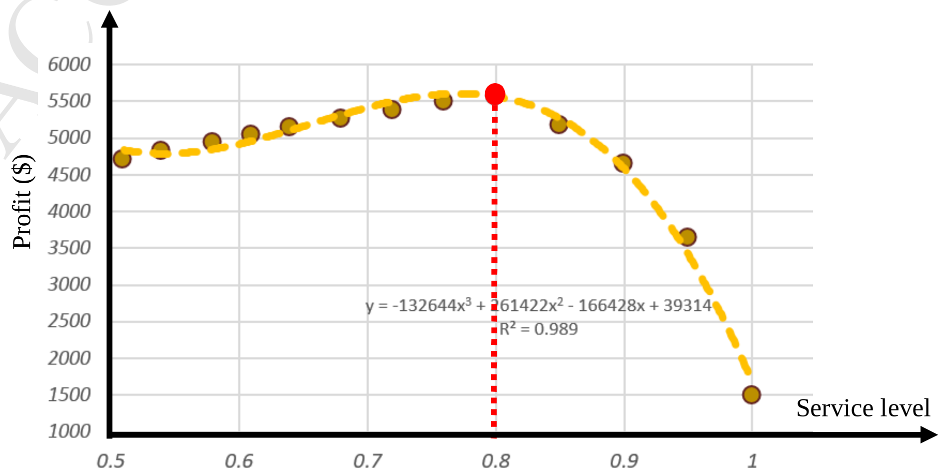


Figure 13. Average profit of the first supply path with respect to the service level under disruption.

This study's example was constructed using assumptions that are often used in prior studies regarding SNs. We tried to be as comprehensive as possible by considering different common options. Here, we summarize the assumptions (and their references) and options that were considered in our example.

- ***Density function for variations:*** we consider three different density functions for variations in the SN, as follows:
  - Normal distribution for the market's demands (Bernstein and Federgruen, 2004 and 2007; Santoso et al., 2005; Shen and Daskin, 2005; Baghalian et al., 2013; Rezapour et al., 2016a and 2016b; Mohammaddust et al., 2017);
  - Uniform distribution for the manufacturers' wastage ratio (Rezapour et al., 2015; Rezapour et al., 2016a and b); and
  - Exponential distribution for the suppliers' deterioration time (Rosenblatt and Lee, 1986; Lee and Rosenblatt, 1987).
- ***Demand functions:*** we assume that the markets' demand is a linearly decreasing function of price and a linearly increasing function of service level. This assumption is widely used in prior studies regarding SNs, such as Bernstein and Federgruen (2004 and 2007), Carr and Karmarkar (2005), Ha et al. (2003), Jiang and Wang (2009), Zhang and Rushton (2008), Rezapour and Farahani (2014), and Rezapour et al. (2016a, b, and c).
- ***Duration of the disruption:*** we consider different scenarios for the duration of the disruption in the SN (Schmitt, 2011; Klibi and Martel, 2012).
- ***Extra capacity costs:*** we assume that providing extra production capacity is costlier during the early production runs following a disruption. This assumption is consistent with observations in manufacturing systems (Koren and Shpitalni, 2014).

In this problem, there are three important risk mitigation strategies (one ORM and two SRM strategies) that determine the behavior of the SN when faced with variations and disruptions:

I) **Robustness** of the SN's structure/topology against disruptions: this feature of the SN's structure/topology depends on the levels of flexibility that are assigned to its facilities (SRM);

II) **Resilience** of the SN's structure/topology against disruptions: this feature of the SN's structure/topology depends on the flexibility speeds assigned to its facilities (SRM);

III) **Reliability** of the SN's flow planning against variations: this feature of the SN's flow dynamics depends on the local reliabilities assigned to its facilities (ORM).

In the remainder of this section, the correlations among the ORM and SRM strategies are investigated. For this purpose, we solve the model for our example and conduct a sensitivity analysis to analyze the correlations.

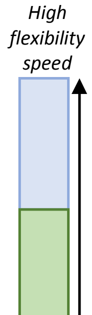
<u>Correlation between robustness and resilience</u>

First, we analyze the relationship between the two SRM strategies, i.e., the flexibility levels and flexibility speeds that are assigned to the SN's facilities (*M1* and *S1*). We solve the mathematical model (Model Equations 53-56) for 3 scenarios for the disruption duration and use 5 different values for the retailer's local reliability to change order quantities in the SN, 5 different values for the local reliability of *M1*, and 5 different values for the local reliability of *S1* to provide more variety in the markets' service levels. We solved 375 problems and summarize their results in Figures 14, 15, and 16. By increasing the local reliability of the retailer in the model, more products are ordered from the first supply path and, consequently, greater extra capacity or a higher flexibility level is needed in its facilities to address the disruptions. Therefore, the flexibility levels that are assigned to the facilities increase in the model's solution. In addition, we follow the trend of changes in the flexibility speeds that are assigned to the facilities to determine whether there is a correlation between the flexibility levels and flexibility speeds. These results are summarized in Figure 14.

Supplier (*S1*) Manufacturer (*M1*)

rl_R^D=1.00

rl_R^D=0.95



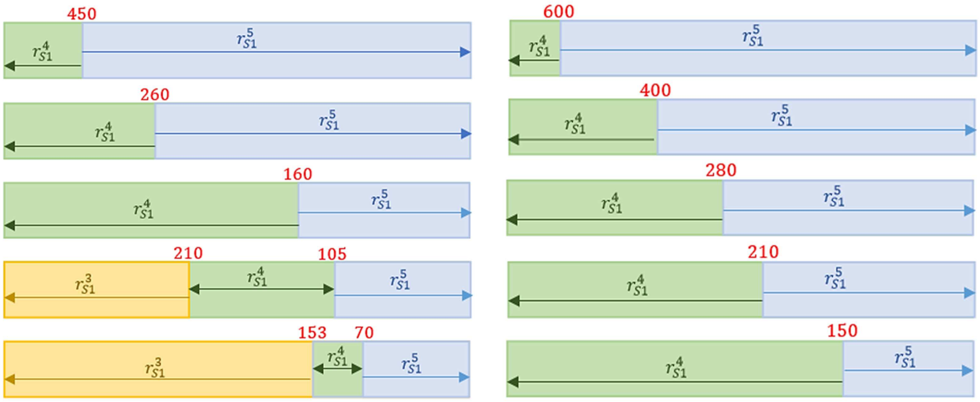


Figure 14. Correlation between flexibility levels and speeds.
(Each color corresponds to one flexibility speed option).

In Figure 14, the changes in the flexibility speeds of *S1* and *M1* with respect to their flexibility levels are shown for different values of rl_R^D. For instance, in $rl_R^D = 0.80$, when the flexibility level of *S1*, Δ_{S1}, is less than 70 (capacity units), the flexibility speed assigned by the model to this facility is rl_{S1}^5. This implies that the most rapid increase, or the highest flexibility speed, is selected for this facility. However, in the case that $70 \leq \Delta_{S1} < 153$, the flexibility speed of this facility reduces to rl_{S1}^4. By increasing Δ_{S1} to more than 153, the flexibility speed reduces further to rl_{S1}^3. The other bars of this figure can be interpreted similarly. Based on the results that are summarized in Figure 14, we conclude the following:

- For a given product order quantity (rl_R^D), when a facility's flexibility level is low, a high flexibility speed is generally preferred for that facility. This implies that when a low extra capacity is needed in a facility, it is primarily added during the early production runs after disruptions. However, when the required extra capacity increases, part of this increase should be postponed to later production runs to avoid high costs. Therefore, a negative correlation exists between the flexibility level and flexibility speed of each facility. For all the facilities in the SN, *higher robustness leads to lower resilience in* *profit-based SNs*. This tradeoff between robustness and resilience should be considered when designing/redesigning profit–based SNs.

- By increasing the product order quantity (caused by increasing rl_R^D), the flexibility level values differentiate two subsequent pairs of flexibility speed options. The red numbers in Figure 14 represent these differentiating flexibility level values. As an example, for $rl_R^D = 0.80$, the flexibility level value of *S1* that differentiates the rl_{S1}^5 and rl_{S1}^4 flexibility speed options is equal to 70 (capacity units). However, by increasing rl_R^D to 0.85, this differentiating flexibility level value increases to 105 (capacity units). This implies that high production rates stabilize the facilities' flexibility speeds against changes in their flexibility levels. To clarify, to reduce the flexibility speed of facilities, a greater increase in the flexibility level is required. For all the facilities in the SN, larger SNs with higher production rates are able to absorb greater levels of flexibility in their facilities without reducing their flexibility speeds. Greater flexibility levels and lower flexibility speeds in facilities result in higher robustness and lower resilience in the SN. Therefore, *the tradeoff between robustness and resilience is more stable in large SNs with high production rates*. This tradeoff is more fragile for low production rates.

<u>Correlations between Robustness, Resilience, and Reliability</u>

Figures 15 and 16 represent the flexibility levels of *M1* and *S1*, respectively, with respect to the local reliabilities of the first supply path's facilities. Analyzing Figures 15 and 16 leads to insights that are summarized below.

- Based on Figures 15 and 16, increasing the retailers' local reliabilities leads to higher flexibility levels in *M1* and *S1*. The retailers' increased reliabilities lead to an increase in the product ordering quantities in the first supply path. To fulfill these larger orders, greater capacities are needed in the supply path's facilities.

- Based on Figures 15 and 16, increasing the local reliabilities in *S1* and *M1* leads to higher flexibility levels for *M1* and *S1*. This implies that there is a positive correlation between the local reliability of each facility and the flexibility level of other factories.

Based on these results, we conclude that for SNs, there is a positive correlation between the flow reliability against variations (ORM) and the structural robustness against disruptions (SRM). We have presented evidence for a negative correlation between robustness and resilience; therefore, a negative correlation exists between the flow reliability against variations (ORM) and structural resilience against disruptions (SRM).

5.6. Extension to more complicated supply networks

In the previous subsections, we consider a very simple SN with only two supply paths, $[S1 \rightarrow M1 \rightarrow R1]$ and $[S2 \rightarrow M2 \rightarrow R2]$, to avoid unnecessary complication in modeling. We assumed that when Path $[S2 \rightarrow M2 \rightarrow R2]$ is disrupted, Path $[S1 \rightarrow M1 \rightarrow R2]$ substitutes for the disrupted path. Path $[S1 \rightarrow M1 \rightarrow R2]$ is selected for substitution because capacity expansion is needed in both $M1$ and $S1$. However, other substitutions are also possible. Path $[S2 \rightarrow M1 \rightarrow R2]$ can substitute for Path $[S1 \rightarrow M1 \rightarrow R1]$ if disruption only occurs in $M2$. In this case, capacity expansion would be needed only in $M1$. Path $[S1 \rightarrow M2 \rightarrow R2]$ can be used to substitute for Path $[S1 \rightarrow M1 \rightarrow R1]$ if disruption only occurs in $S2$. In this case, capacity expansion would be needed only in $S1$. By considering Path $[S2 \rightarrow M2 \rightarrow R2]$ for substitution, we consider the more complicated case when capacity expansion is needed in two facilities.

In this section, we show that the method for developing the model above can be used for more complicated SNs with a higher number of suppliers, manufacturers, retailers, and markets. Figure 17 shows a more complicated SN with 3 suppliers, 2 manufacturers, 7 retailers, and 14 markets.

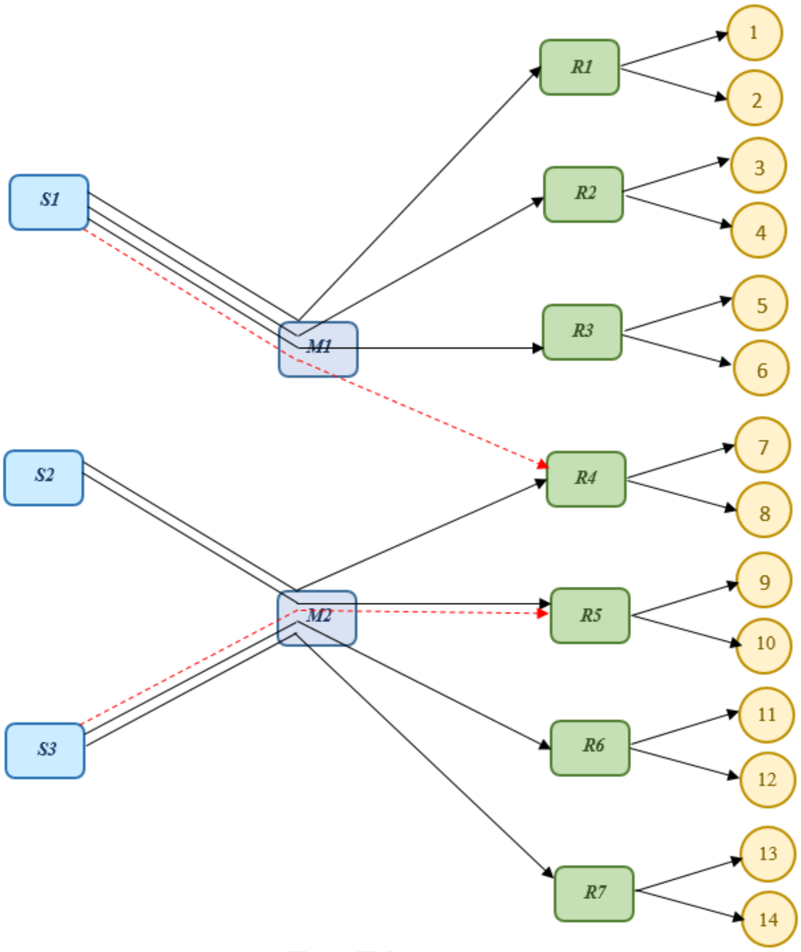


Figure 17. A more complicated SN with sample substitute paths.

The black paths are used in the normal condition to produce and supply products to the markets. However, if a disruption occurs in $S2$, Paths $[S2 \rightarrow M2 \rightarrow R4]$ and $[S2 \rightarrow M2 \rightarrow R5]$ would be inoperative for a while. There are four potential paths that can substitute for Path $[S2 \rightarrow M2 \rightarrow R4]$ and service Markets 7 and 8 after the disruption: Path $[S1 \rightarrow M1 \rightarrow R4]$, Path $[S1 \rightarrow M2 \rightarrow R4]$, Path $[S3 \rightarrow M2 \rightarrow R4]$ and Path $[S3 \rightarrow M1 \rightarrow R4]$ (substitute Path $[S1 \rightarrow M1 \rightarrow R4]$ is denoted by a dashed red arrow in Figure 17). To select the most profitable substitution, we define new binary variables, such as v_1, v_2, v_3 and v_4. Variable v_1 is 1 if the first potential path, Path $[S1 \rightarrow M1 \rightarrow R4]$, is selected to substitute for inoperative Path $[S2 \rightarrow M2 \rightarrow R4]$. Similarly, variable v_2, v_3 or v_4 is 1 if the second, third or fourth potential path is selected for substitution, respectively.

Additionally, there are four potential paths that can substitute for Path $[S2 \rightarrow M2 \rightarrow R5]$ and service Markets 9 and 10 after the disruption: Path $[S1 \rightarrow M1 \rightarrow R5]$, Path $[S1 \rightarrow M2 \rightarrow R5]$, Path $[S3 \rightarrow M2 \rightarrow R5]$ and Path $[S3 \rightarrow M1 \rightarrow R5]$ (substitute Path $[S3 \rightarrow M2 \rightarrow R5]$ is denoted by a dashed red arrow in Figure 17). To select the most profitable substitution, we define another set of new binary variables, such as $\acute{v}_1$, $\acute{v}_2$, $\acute{v}_3$ and $\acute{v}_4$. Variable $\acute{v}_1$ is 1 if the first potential path, Path $[S1 \rightarrow M1 \rightarrow R5]$, is selected to substitute for inoperative Path $[S2 \rightarrow M2 \rightarrow R5]$. Similarly, variable $\acute{v}_2$, $\acute{v}_3$ or $\acute{v}_4$ is 1 if the second, third or fourth potential path is selected for substitution, respectively.

Only one of the potential paths should be selected to substitute for each inoperative path: $\sum_{i=1}^{4} v_i = 1$ and $\sum_{j=1}^{4} \acute{v}_j = 1$. Function $\Psi^{v_i,\acute{v}_j}$ shows the average profit of the SN if the potential paths corresponding to Variables v_i and $\acute{v}_j$ are selected to substitute for the inoperative paths. The objective function of the SN would be as follows:

Maximize $\sum_{i=1}^{4} \sum_{j=1}^{4} \Psi^{*v_i,\acute{v}_j} . v_i . \acute{v}_j$ (57)

Subject to: $\sum_{i=1}^{4} v_i = 1$ (58)

$\sum_{j=1}^{4} \acute{v}_j = 1$ (59)

v_i and $\acute{v}_j \in \{0,1\}$ $(i = 1, 2, 3$ and $4)$ and $(j = 1, 2, 3$ and $4)$ (60)

In Model (57)-(60), $\Psi^{*v_i,\acute{v}_j}$ shows the optimal average profit that can be calculated using Model (53)-(56) after a few small modifications. Constraints (36) and (37) should be calculated for seven retailers instead of two. The total orders received by manufacturers $M1$ and $M2$ (x_{M1}^{D} and x_{M2}^{D}) should be revised as follows (Constraint (35)):

$$x_{M1}^{D} = x_{R1}^{D} + x_{R2}^{D} + x_{R3}^{D} + x_{R4}^{D}(v_1 + v_4) + x_{R5}^{D}(\acute{v}_1 + \acute{v}_4) \quad (61)$$

$$x_{M2}^{D} = x_{R6}^{D} + x_{R7}^{D} + x_{R4}^{D}(v_2 + v_3) + x_{R5}^{D}(\acute{v}_2 + \acute{v}_3) \quad (62)$$

The service levels, sl^D, in Markets 1-6 and Markets 11-14 are equal to $rl_{S1}^{D}.rl_{M1}^{D}.rl_{R}^{D}$ and $rl_{S3}^{D}.rl_{M2}^{D}.rl_{R}^{D}$, respectively. For the markets served by R4 and R5 (Markets 7-10), the service level is modified by the binary variables, as follows (Constraint (41)):

$$sl^D = (rl_{S1}^{D}.rl_{M1}^{D}.rl_{R}^{D}).v_1 + (rl_{S1}^{D}.rl_{M2}^{D}.rl_{R}^{D}).v_2 + (rl_{S3}^{D}.rl_{M2}^{D}.rl_{R}^{D}).v_3 + (rl_{S3}^{D}.rl_{M1}^{D}.rl_{R}^{D}).v_4$$

(63)

$$sl^D = (rl_{S1}^{D}.rl_{M1}^{D}.rl_{R}^{D}).\acute{v}_1 + (rl_{S1}^{D}.rl_{M2}^{D}.rl_{R}^{D}).\acute{v}_2 + (rl_{S3}^{D}.rl_{M2}^{D}.rl_{R}^{D}).\acute{v}_3 + (rl_{S3}^{D}.rl_{M1}^{D}.rl_{R}^{D}).\acute{v}_4$$

(64)

Additionally, the revenue of all retailers and the cost components of all SN entities should be added to Functions (42), (50) and (51), used to calculate $\Psi^{v_i,\acute{v}_j}$. Models (57)-(60) select the most profitable potential paths that should be substituted for the inoperative paths.

In more complicated SNs, more flexible facilities are usually needed to manage disruptions and variations. However, the correlations among the flexibility level, flexibility speed, and local reliability are similar for all the flexible facilities. Therefore, the size of the SN does not affect the correlations claimed in the paper.

6. Conclusions

In this study, we classify SN risks into two groups: 1) variations that affect flow planning decisions and 2) disruptions that affect the topology design decisions of the SN. We develop a model in Section 4 to plan a reliable flow for SNs to manage downstream and upstream variations by assigning the optimal local reliabilities to facilities (ORM strategies). We extend the model constructed in Section 4 in Section 5 by considering the possibility of disruptions. This model redesigns a robust and resilient network structure by adding flexibility levels and flexibility speeds to facilities (SRM strategies). Finally, we analyze the correlations among reliability, robustness, and resilience. Our results demonstrate that in profit-based SNs 1) the correlation between robustness and resilience is negative; 2) the correlation between robustness and reliability is positive; and 3) the correlation between resilience and reliability is negative.

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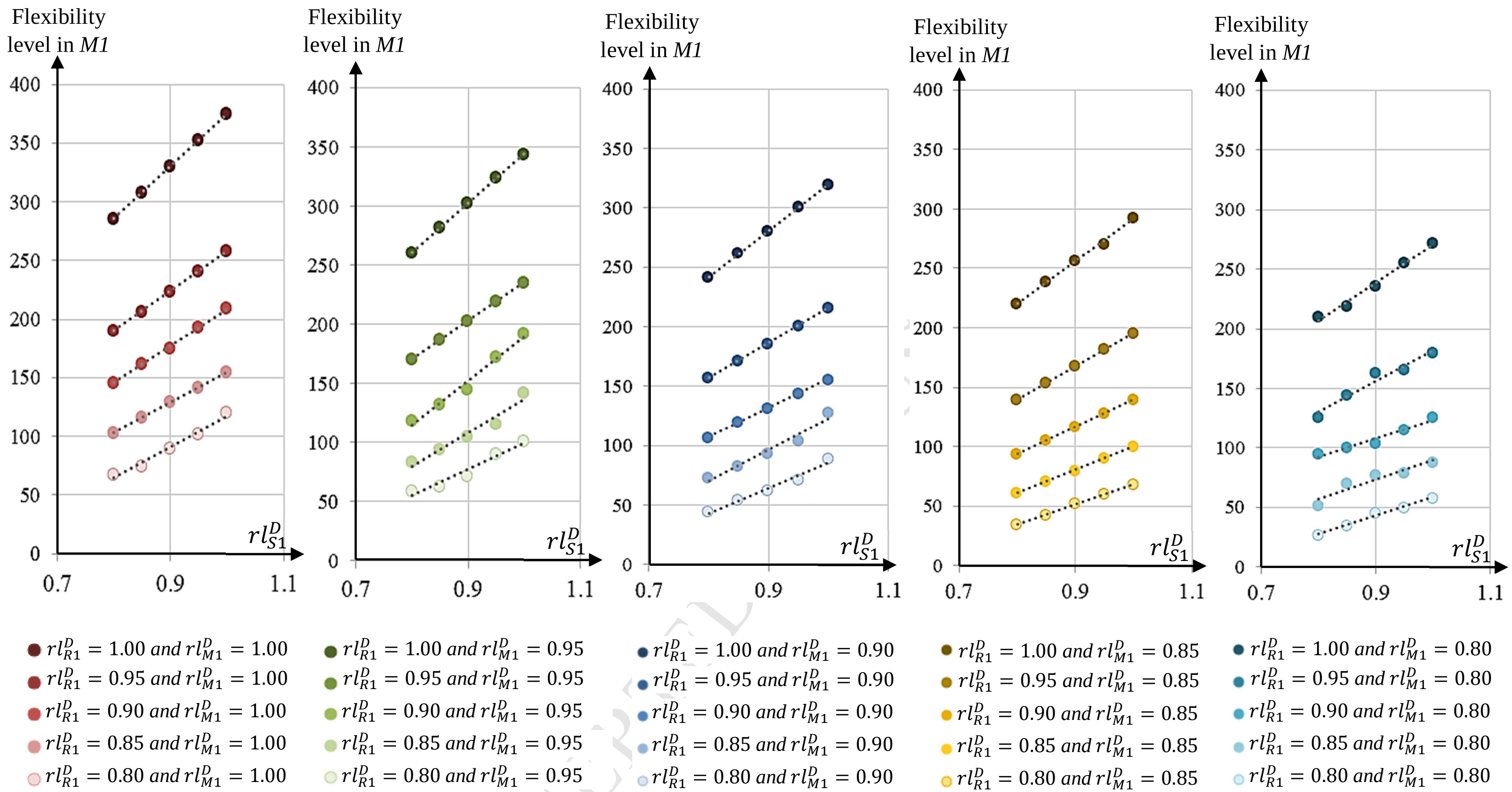

Figure 15. Flexibility level in *M1* with respect to the local reliabilities of facilities.

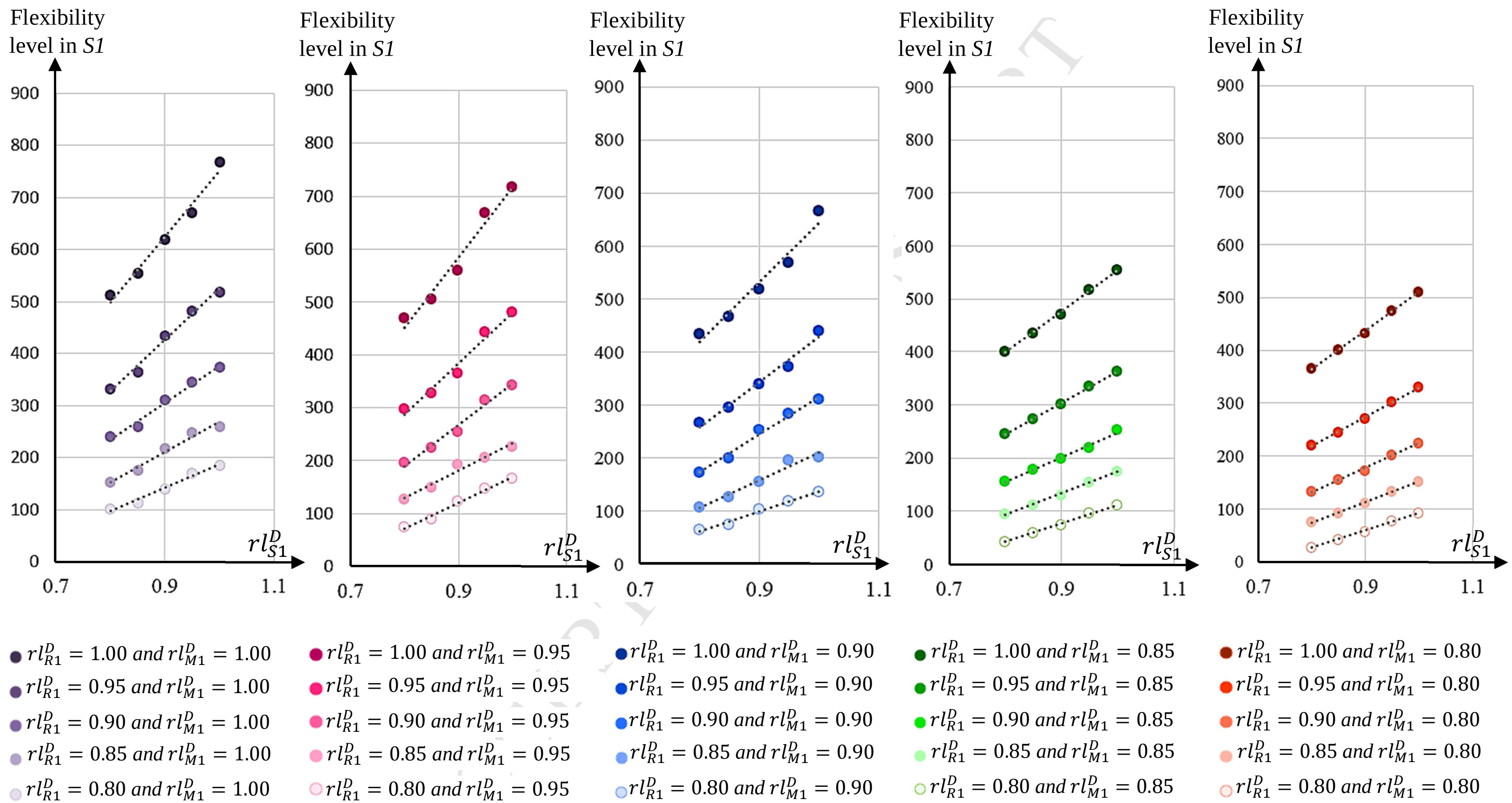

Figure 16. Flexibility level in *S1* with respect to the local reliabilities of facilities.

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Appendix A

Nomenclature.

Sets	
$J = \{j\}$	Set of samples used in SAA
$RL = \{rl\}$	Set of discretized values that can be taken by reliability variables
$RO_{M1} = \{r_{M1}\}$	Set of all flexibility speed options available for *M1*
$RO_{S1} = \{r_{S1}\}$	Set of all flexibility speed options available for *S1*
$SCE = \{s\}$	Set of all possible scenarios for the length of disruptions
Parameters	
p	Price
ε	Random deviation of the actual demand from its mean value
G_{R1}	Cumulative distribution function for ε in *R1*
h^+	Unit holding cost paid by the retailers for each unit of end-of-period extra inventory
h^-	Unit shortage cost paid by the retailers for each unit of lost sales
O_{M1}	Number of production runs in *M1*
α_{M1}	*M1*'s random wastage ratio
G'_{M1}	Cumulative distribution function for α_{M1}
α^i_{M1}	Value of random variable α_{M1} realized in production run $i = 1, 2, ..., O_{M1}$

O_{S1}	Number of production runs in *S1*
γ_{S1}	Nonconforming production rate when *S*1's machinery is in an out-of-control state
t	Random deterioration time in *S1*'s machinery
G''_{S1}	Cumulative distribution function for t
Cap_{S1}^{WD}	Production capacity of *S1* in each production run
Cap_{M1}^{WD}	Production capacity of *M1* in each production run
T	Number of time units in each production run
t_i	Value of random variable t realized in production run $i = 1, 2, ..., O_{S1}$
c_{S1}	Cost of procuring and producing a component unit in *S1*
$c_{S1,M1}$	Cost of transporting a component unit from *S1* to *M1*
c_{M1}	Cost of manufacturing a product unit in *M1*
$c_{M1,R1}$	Cost of transporting a product unit from *M1* to *R1*
cap_{M1}^{i}	Unit extra capacity cost in *M1*'s production run $i = 1, 2, \dots, O_{M1}$
cap_{S1}^{j}	Unit extra capacity cost in *S1*'s production run $j = 1, 2, \dots, O_{S1}$
$rj_{M1}^{r_{M1}}$	Capacity ramp-up quantity in production run j of *M1* if flexibility speed option r_{M1} is selected for it ($\forall r_{M1} \in RO_{M1}$ and $j = 1, 2, \dots, O_{M1}$)
$rj_{S1}^{r_{S1}}$	Capacity ramp-up quantity in production run j of *S1* if flexibility speed option r_{S1} is selected for it ($\forall r_{S1} \in RO_{S1}$ and $j = 1, 2, \dots, O_{S1}$)
h_{M1}^{i}	Unused capacity cost in *M1*'s production run $i = 1, 2, \dots, O_{M1}$
h_{S1}^{i}	Unused capacity cost in *S1*'s production run $i = 1, 2, \dots, O_{S1}$
pr_s	Occurrence probability of scenario $s \in SCE$
num_s^{WD}	Number of without disruption periods in scenario $s \in SCE$
num_s^{RUD}	Number of ramp-up disruption periods in scenario $s \in SCE$
num_s^{ND}	Number of normal disruption periods in scenario $s \in SCE$
num_s^{RD}	Number of ramp-down disruption periods in scenario $s \in SCE$
Variables	
sl	Service level
sl^{WD}	Service level provided by the SN under without disruption conditions
x^{WD}	Number of products ordered by *R1* from *M1*
rl_{R1}^{WD}	Local reliability for *R1* under without disruption conditions
y^{WD}	Number of products produced by *M1* in each production run
rl_{M1}^{WD}	Local reliability for *M1* under without disruption conditions
z^{WD}	Number of components produced by *S1* in each production run
rl_{S1}^{WD}	Local reliability for *S1* under without disruption conditions
$rl_{M1,j}^{WD}$	1 if the term $(O_{M1}.y^{WD} - x^{WD}) - y^{WD}.\sum_{i=1}^{O_{M1}} \alpha_{M1}^{i}$ is positive based on the realized values of α_{M1}^{i} ($\forall i = 1, \dots, O_{M1}$) in sample $j \in J$, and 0 otherwise
$rl_{S1,j}^{WD}$	1 if the term $\gamma_{S1}.Cap_{S1}^{WD}.\left(\sum_{i=1}^{O_{S1}} t_i\right) - O_{S1}.(\gamma_{S1} - 1).z^{WD} - O_{M1}.y^{WD}$ is positive based on the realized values of t_i ($\forall i = 1, \dots, O_{M1}$) in sample $j \in J$, and 0 otherwise
$\theta_{S1}^{WD,rl}$	1 if reliability option $rl \in RL$ is selected for *S1*, and 0 otherwise
$\theta_{M1}^{WD,rl'}$	1 if reliability option $rl' \in RL$ is selected for *M1*, and 0 otherwise
$\theta_{R1}^{WD,rl''}$	1 if reliability option $rl'' \in RL$ is selected for *R1*, and 0 otherwise
$\Theta_{S1,M1,R1}^{WD,rl,rl',rl''}$	1 if all three variables $\theta_{S1}^{WD,rl}, \theta_{M1}^{WD,rl'}$, and $\theta_{R1}^{WD,rl''}$ are equal to 1, and 0 otherwise
$w_{M1}^{r_{M1}}$	1 if flexibility speed option r_{M1} is selected for *M1*, and 0 otherwise ($r_{M1} \in RO_{M1}$)

$w_{S1}^{r_{S1}}$	1 if flexibility speed option r_{S1} is selected for *S1*, and 0 otherwise ($r_{S1} \in RO_{S1}$)
Δ_{M1}	Flexibility level in *M1*
Δ_{S1}	Flexibility level in *S1*
y_i^{RUD}	Production quantity in the ramp-up disruption period's production run i of *M1* ($i = 1, 2, \ldots, O_{M1}$)
z_i^{RUD}	Production quantity in the ramp-up disruption period's production run i of *S1* ($i = 1, 2, \ldots, O_{S1}$)
x^D	Total product order received by *M1* in disrupted periods
x_1^D	Order issued by *R1* from *M1* in disrupted periods
x_2^D	Order issued by *R2* from *M1* in disrupted periods
rl_{R1}^D	Local reliability for *R1* under disrupted conditions
rl_{R2}^D	Local reliability for *R2* under disrupted conditions
sl^D	Service level provided by the SN under disrupted conditions
rl_{M1}^D	Local reliability for *M1* under disrupted conditions
rl_{S1}^D	Local reliability for *S1* under disrupted conditions
rl_R^D	Local reliability for *R1* and *R2* if the same service level is provided to all markets under disrupted conditions
y^{ND}	Production quantity in the production runs of normal disruption period in *M1*
z^{ND}	Production quantity in the production runs of normal disruption period in *S1*
Functions	
$D_k(p, sl^{WD})$	Expected demand in each sale period in market *k* ($k = 1$ and 2) under without disruption conditions
$\sum_{k=1}^{2} D_k(p, sl^{WD})$	Average demand in *R1* under without disruption conditions
$\sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon$	Actual demand in *R1* under without disruption conditions
Π_{R1}^{WD}	Expected total cost in *R*1 under without disruption conditions
Ψ^{WD}	Total profit in the first supply path under without disruption conditions
$D_k(p, sl^D)$	Expected demand in each sale period in market *k* ($k = 1$ and 2) under disrupted conditions
$\sum_{k=1}^{2} D_k(p, sl^D)$	Average demand in *R1* under disrupted conditions
$\sum_{k=1}^{2} D_k(p, sl^D) \times \varepsilon$	Actual demand in *R1* under disrupted conditions
Ψ^{RUD}	Total profit that can be captured in the ramp-up disruption period
Ψ^{ND}	Total profit that can be captured in the normal disruption period
Ψ^{RD}	Total profit that can be captured in the ramp-down disruption period
Cap_{M1}^D	Capacity needed by *M1* in disrupted conditions
Cap_{S1}^D	Capacity needed by *S1* in disrupted conditions
Ψ	Expected profit of the SN under disrupted conditions

Appendix B: Linearizing Approach

The model (Equations 8-17) is linearized in three steps as follows:

- First, the chance constraints, Equations (12) and (13), are linearized by using the SAA approach,
- Second, the nonlinear constraints, Equations (11) and (14), and the objective functions, Equation (8), are linearized by discretizing reliability variables,

- Third, the multiplication of binary variables is linearized by defining a new variable.

The steps are explained in detail below.

Step 1: Chance constraints linearization

In this step, we explain how the SAA approach is used to approximate the chance constraints in the model (Equations 8-17). The SAA for Equation (12) is as follows:

$$rl_{M1}^{WD} = \Pr\left(\sum_{i=1}^{O_{M1}} \alpha_{M1}^{i}.y^{WD} \leq O_{M1}.y^{WD} - x^{WD}\right) = \frac{\sum_{j=1}^{J} rl_{M1,j}^{WD}}{J} \quad (57)$$

In Equation (57), the probability of the event that is defined as the "*left-hand side of the inequality in Equation (57) is less than or equal to its right-hand side (*$\sum_{i=1}^{O_{M1}} \alpha_{M1}^{i}.y^{WD} \leq O_{M1}.y^{WD} - x^{WD}$*)*" is replaced by the ratio of its occurrence in a sample that includes $J = \{j\}$ observations. Increasing the size of the sample, $|J|$, increases the accuracy of this statistical approximation. To determine the number of times in which term $(O_{M1}.y^{WD} - x^{WD}) - y^{WD}.\sum_{i=1}^{O_{M1}} \alpha_{M1}^{i}$ is positive, a new binary variable $rl_{M1,j}^{WD}$ is defined and the following constrains are added to the mode:

$$BM.\left(rl_{M1,j}^{WD} - 1\right) \leq (O_{M1}.y^{WD} - x^{WD}) - y^{WD}.\sum_{i=1}^{O_{M1}} \alpha_{M1}^{i} \leq BM.rl_{M1,j}^{WD}$$

$$(\forall j = 1, \dots, J \; and \; \forall i = 1, \dots, O_{M1}) \quad \left(\alpha_{M1}^{i} \sim G'_{M1}\right) \quad (58)$$

$$rl_{M1,j}^{WD} \in \{0,1\} \quad (\forall j = 1, \dots, J) \quad (59)$$

According to Constraint (58), variable $rl_{M1,j}^{WD}$ is 1 if the term $(O_{M1}.y^{WD} - x^{WD}) - y^{WD}.\sum_{i=1}^{O_{M1}} \alpha_{M1}^{i}$ is positive based on the realized values of α_{M1}^{i} ($\forall i = 1, \dots, O_{M1}$) in sample $j \in J$, and 0 otherwise (*BM* is a large constant value; refer to Appendix E for more information about the *BM* value). Increasing the accuracy of this approximation increases the number of these new variables. Therefore, selecting the smallest $|J|$ that ensures an acceptable accuracy is necessary.

The chance constraint in Equation (13) is approximated in the same manner. First, it is simplified algebraically and rewritten as follows:

$$rl_{S1}^{WD} = \Pr\left(\gamma_{S1}.Cap_{S1}^{WD}.\left(\sum_{i=1}^{O_{S1}} t_i\right) \geq O_{S1}.(\gamma_{S1} - 1).z^{WD} + O_{M1}.y^{WD}\right) \quad (60)$$

Then, it is approximated with the following constraints:

$$rl_{S1}^{WD} = \Pr\left(\gamma_{S1}.Cap_{S1}^{WD}.\left(\sum_{i=1}^{O_{S1}} t_i\right) \geq O_{S1}.(\gamma_{S1}-1).z^{WD} + O_{M1}.y^{WD}\right) = \frac{\sum_{j=1}^{J} rl_{S1,j}^{WD}}{J} \quad (61)$$

To determine the number of times in which term $\gamma_{S1}.Cap_{S1}^{WD}.\left(\sum_{i=1}^{O_{S1}} t_i\right) - O_{S1}.(\gamma_{S1}-1).z^{WD} - O_{M1}.y^{WD}$ is positive, a new binary variable, $rl_{S1,j}^{WD}$, is defined and the following constrains should be added to the model:

$$BM.\left(rl_{S1,j}^{WD}-1\right) \leq \gamma_{S1}.Cap_{S1}^{WD}.\left(\sum_{i=1}^{O_{S1}} t_i\right) - O_{S1}.(\gamma_{S1}-1).z^{WD} - O_{M1}.y^{WD} \leq BM.rl_{S1,j}^{WD}$$

$$(\forall j = 1,\ldots,J \text{ and } \forall i = 1,\ldots,O_{S1}) \quad (t_i \sim G''_{S1}) \quad (62)$$

$$rl_{S1,j}^{N} \in \{0,1\} \quad (\forall j = 1,\ldots,J)$$

(63)

Variable $rl_{S1,j}^{WD}$ is 1 if the term $\gamma_{S1}.Cap_{S1}^{WD}.\left(\sum_{i=1}^{O_{S1}} t_i\right) - O_{S1}.(\gamma_{S1}-1).z^{WD} - O_{M1}.y^{WD}$ is positive based on the realized values of t_i $(\forall i = 1,\ldots,O_{M1})$ in sample $j \in J$, and 0 otherwise. To verify the accuracy of this approximation and suggest appropriate values for the sample size, $|J|$, we conducted a numerical analysis and compute the average error of this approximation for different density functions. The results are summarized in Table B.1.

Table B.1 Average error for different density functions.

Normal Density Function				*Uniform Density Function*				*Exponential Density Function*			
J	***Average error***	***J***	***Average error***	***J***	***Average error***	***J***	***Average error***	***J***	***Average error***	***J***	***Average error***
1	0.220	**60**	0.034	**1**	0.230	**60**	0.033	**1**	0.210	**60**	0.032
5	0.129	**65**	0.033	**5**	0.134	**65**	0.031	**5**	0.131	**65**	0.030
10	0.084	**70**	**0.031**	**10**	0.082	**70**	**0.029**	**10**	0.085	**70**	0.030
15	0.065	**75**	0.029	**15**	0.067	**75**	0.028	**15**	0.064	**75**	0.029
20	0.061	**80**	0.028	**20**	0.061	**80**	0.028	**20**	0.060	**80**	0.028
25	**0.051**	**85**	0.028	**25**	**0.050**	**85**	0.027	**25**	0.050	**85**	0.027
30	0.048	**90**	0.026	**30**	0.049	**90**	0.025	**30**	0.048	**90**	0.026
35	0.045	**95**	0.025	**35**	0.043	**95**	0.025	**35**	0.043	**95**	0.026
40	**0.041**	**100**	0.025	**40**	0.041	**100**	0.024	**40**	0.039	**100**	0.025
45	0.039	**120**	0.023	**45**	**0.039**	**120**	0.022	**45**	0.037	**120**	0.023
50	0.038	**140**	**0.020**	**50**	0.036	**140**	**0.019**	**50**	0.036	**140**	0.022
55	0.035	**150**	0.019	**55**	0.034	**150**	0.018	**55**	0.034	**150**	0.019

Based on these results, when $|J|$ is in the $[25,30]$ interval, the average error of the approximations is less than or equal to 5 percent. To reduce the error to less than 4, 3, and 2 percent, $|J|$ should be selected from the $[40,45]$, $[65,70]$, and $[140,150]$ intervals, respectively.

Step 2: Nonlinear objective function and constraints linearization

To linearize the objective function in Equation (8) and the constraints in Equations (11) and (14), we discretize the facilities' local reliability variables, $rl_{S1}^{WD}, rl_{M1}^{WD}$ and rl_{R1}^{WD}. These variables only assume values in the [0, 1] interval. The service level, a function of these local reliabilities, generally assumes a value that is greater than or equal to 50 percent. By restricting the feasibility range of the local reliabilities to [0.8, 1], we ensure that the SN's service level for the markets is greater than 50 percent ($0.8^3 \geq 0.5$). This very restricted feasible range justifies the feasibility of their discretization (Rezapour et al., 2015). Set $RL = \{rl\}$ includes all discrete values that can be assumed by these variables. For example, if we use step size 0.05 to discretize the [0.8, 1] interval, we obtain $RL = \{0.80, 0.85, 0.90, 0.95, 1\}$ (the notation rl is used to represent the discretized values in Set RL). In this case, the variables $rl_{S1}^{WD}, rl_{M1}^{WD}$ and rl_{R1}^{WD} can only assume a value from Set $RL = \{0.80, 0.85, 0.90, 0.95, 1\}$ rather than assuming any value from [0.8, 1]. To select one of these reliability options for each facility, we define new binary variables, $\theta_{S1}^{WD,rl}$, $\theta_{M1}^{WD,rl}$ and $\theta_{R1}^{WD,rl}$. Variable $\theta_{S1}^{WD,rl}$ is 1 if the reliability option $rl \in RL$ is selected for *S1*, and 0 otherwise. Only one of the options available in RL can be selected for *S1*:

$$\sum_{rl=1}^{|RL|} \theta_{S1}^{WD,rl} = 1 \tag{64}$$

Variable $\theta_{M1}^{WD,\,rl'}$ is 1 if the reliability option $rl' \in RL$ is selected for *M1*, and 0 otherwise. Only one of the options available in RL can be selected for *M1*:

$$\sum_{rl'=1}^{|RL|} \theta_{M1}^{WD,rl'} = 1 \tag{65}$$

Variable $\theta_{R1}^{WD,rl''}$ is 1 if the reliability option $rl'' \in RL$ is selected for *R1*, and 0 otherwise. Only one of the options available in RL can be selected for *R1*:

$$\sum_{rl''=1}^{|RL|} \theta_{R1}^{WD,rl''} = 1 \tag{66}$$

By defining these new variables, the objective function, Equation (8), is rewritten as follows:

$$\boldsymbol{Max}\ \Psi^{WD} = \sum_{rl=1}^{|RL|}\sum_{rl'=1}^{|RL|}\sum_{rl''=1}^{|RL|}\theta_{S1}^{WD,rl}.\theta_{M1}^{WD,rl'}.\theta_{R1}^{WD,rl''}.\left[\left(p - h^{+}.E\left[G_{R1}^{-1}\left(Max\left\{rl.rl'.rl'',\frac{h^{-}}{h^{-}+h^{+}}\right\}\right) - \varepsilon\right]^{+} - h^{-}.E\left[\varepsilon - G_{R1}^{-1}\left(Max\left\{rl.rl'.rl'',\frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right]^{+}\right) \times \left[\sum_{k=1}^{2} D_k(p, rl.rl'.rl'')\right]\right]$$

(67)

After defining these new binary variables, the constraint in Equation (11) can be rewritten as follows:

$$x^{WD} = \sum_{rl=1}^{|RL|}\sum_{rl'=1}^{|RL|}\sum_{rl''=1}^{|RL|}\theta_{S1}^{WD,rl}.\theta_{M1}^{WD,rl'}.\theta_{R1}^{WD,rl''}.\left[\left(\sum_{k=1}^{2} D_k(p, rl.rl'.rl'')\right).G_{R1}^{-1}\left(Max\left\{rl'',\frac{h^{-}}{h^{-}+h^{+}}\right\}\right)\right] \quad (68)$$

The constraint in Equation (14) is rewritten:

$$sl^{WD} = \sum_{rl=1}^{|RL|}\sum_{rl'=1}^{|RL|}\sum_{rl''=1}^{|RL|}\theta_{S1}^{WD,rl}.\theta_{M1}^{WD,rl'}.\theta_{R1}^{WD,rl''}.\left[rl.rl'.rl''\right] \quad (69)$$

The accuracy of this linearization depends on the discretizing step of the reliability variables. To reduce complexity, we begin with a large step to determine a rough approximation of the optimal solution. Then, we can make the steps finer around the rough approximation to improve the solution's accuracy.

<u>Step 3: Linearizing multiplication of binary variables</u>

The objective function in Equation (67) and the constraints in Equation (68) and (69) are still nonlinear because there is a multiplication of binary variables in these equations. These multiplications can be easily linearized by defining a new binary variable, $\Theta_{S1,M1,R1}^{WD,rl,rl',rl''}$, and substituting as follows:

$$\Theta_{S1,M1,R1}^{WD,rl,rl',rl''} = \theta_{S1}^{WD,rl}.\theta_{M1}^{WD,rl'}.\theta_{R1}^{WD,rl''} \quad (70)$$

We must add the following constraints to the model to ensure that variable $\Theta_{S1,M1,R1}^{WD,rl,rl',rl''}$ is equal to 1, only if all three variables $\theta_{S1}^{WD,rl}$, $\theta_{M1}^{WD,rl'}$, and $\theta_{R1}^{WD,rl''}$ are equal to 1:

$$\theta_{S1}^{WD,rl} + \theta_{M1}^{WD,rl'} + \theta_{R1}^{WD,rl''} - 2 \le \Theta_{S1,M1,R1}^{WD,rl,rl',rl''} \le \frac{\theta_{S1}^{WD,rl}+\theta_{M1}^{WD,rl'}+\theta_{R1}^{WD,rl''}}{3} \quad (71)$$

$$\Theta_{S1,M1,R1}^{WD,rl,rl',rl''} \leq BM.\theta_{S1}^{WD,rl} \tag{72}$$

$$\Theta_{S1,M1,R1}^{WD,rl,rl',rl''} \leq BM.\theta_{M1}^{WD,rl'} \tag{73}$$

$$\Theta_{S1,M1,R1}^{WD,rl,rl',rl''} \leq BM.\theta_{R1}^{WD,rl''} \tag{74}$$

$$\Theta_{S1,M1,R1}^{WD,rl,rl',rl''} \in \{0,1\}$$

(75)

After these steps, our model becomes a MILP. The solution time of a MILP primarily depends on the number of binary variables that are equal to $|RL|^3 + 3.|RL| + 2.|J|$. We solve this model using CPLEX Concert Technology on a Dell laptop computer with Windows 10, an Intel i7 processor, and 8 GB of installed RAM. The computational time for the Test Problem (in Section 4.6) is less than 4 minutes.

Given that the test problem is not complicated, we verify the computational capability of the solution method by solving more complicated SNs with larger numbers of suppliers, manufacturers, and retailers. The features of these SNs and their computational times are summarized in Table B.2. In these problems, we assume that the local reliability of the facilities in each echelon is selected from set $RL = \{0.8, 0.85, 0.90, 0{,}95, 1\}$ and the sample size used in SAA is $|J| = 25$.

Table B.2. Computational capability for the model developed for without disruption conditions.

Problem	*Features of SN*				*Computational time (second)*
	Number of suppliers	*Number of manufacturers*	*Number of retailers*	*Number of paths*	
1	2	2	2	2	227
2	2	3	6	8	612
3	2	4	9	12	2990
4	2	5	12	16	10411
5	3	6	15	21	25965
6	3	7	18	26	57323
7	4	8	22	32	104711
8	4	9	26	38	> 172800

Decisions about the SN's risk mitigation strategies are types of strategic level decisions. These decisions are not made on a daily basis and do not need short computation times. As noted in Table B.2, for Problem 8 and problems larger than Problem 8, the computational time is more than 48 hours. Therefore, for this type of problem, we can use meta-heuristic approaches to solve the MILP and determine a good suboptimal solution in a rational computational time rather than the global optimum.

Appendix C: Computational Times

The model developed in Section 5.4 is a MILP. The solution time of a MILP primarily depends on the number of binary variables that are equal to $|RL|^3 + 3.|RL| + 2.|J| + |RO_{M1}| + |RO_{S1}|$. The computational time for the test problem (in Section 5.5) is less than 6 minutes. Given that the test problem is not complicated, we verify the computational capability of the model and solution method by solving the first 7 problems that are summarized in Table 2.B. The number of disrupted paths considered in these SNs and their computational times are summarized in Table C.1. In these problems, we assume that the numbers of flexibility speed options available for undisrupted suppliers and manufactures are 3 and 4, respectively. In addition, the facilities' local reliability in each echelon is selected from set $RL = \{0.8, 0.85, 0.90, 0{,}95, 1\}$ and the sample size used in the SAA is $|J| = 25$.

Table C.1. Computational capability for the model developed for disrupted conditions.

Problem	*Features of SN*					*Computational time (second)*
	Number of suppliers	*Number of manufacturers*	*Number of retailers*	*Number of paths*	*Number of disrupted paths*	
1	2	2	2	2	1	314
2	2	3	6	8	3	2439
3	2	4	9	12	4	5937
4	2	5	12	16	6	26462
5	3	6	15	21	8	58413
6	3	7	18	26	8	103847
7	4	8	22	32	9	> 172800

As noted in Table C.1, for Problem 7 and problems larger than Problem 7, the computational time is more than 48 hours. Therefore, for this type of problem, we suggest using meta-heuristic approaches to solve the MILP and determine a good suboptimal solution in a rational computational time rather than the global optimum.

Appendix D: Fubini's Theorem

According to Fubini's Theorem, it is possible to calculate the mean of a random function, such as $x^{WD} - \sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon$, using its cumulative distribution function (for more information about this relationship refer to Hajek (2015) – Equation (1.11)):

$$E\left[x^{WD} - \sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon\right]^+ = \int_0^{\frac{x^{WD}}{\sum_{k=1}^{2} D_k(p,sl^{WD})}} G_{R1}(\varepsilon).\, d\varepsilon \quad (76)$$

$$E\left(x^{WD} - \sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon\right)^+ - E\left(\sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon - x^{WD}\right)^+ = x^{WD} - \sum_{k=1}^{2} D_k(p, sl^{WD}) \quad (77)$$

Therefore, we can manipulate the objective function (1) as follows:

$$MIN \qquad \Pi_{R1}^{WD} = h^+ .\, E\left[x^{WD} - \sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon\right]^+ + h^- .\, E\left[\sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon - x^{WD}\right]^+$$
$$= h^+ .\int_0^{\frac{x^{WD}}{\sum_{k=1}^{2} D_k(p,sl^{WD})}} G_{R1}(\varepsilon).\, d\varepsilon + h^-\left[\sum_{k=1}^{2} D_k(p, sl^{WD}) - x^{WD} + .\int_0^{\frac{x^{WD}}{\sum_{k=1}^{2} D_k(p,sl^{WD})}} G_{R1}(\varepsilon).\, d\varepsilon\right] \quad (78)$$

Therefore, to compute the minimum x^{WD}, we should compute the derivative of Equation (78) with respect to x^{WD}, as follows:

$$\frac{\sigma \Pi_{R1}^{WD}}{\sigma x^{WD}} = h^+ .\, G_{R1}\left(\frac{x^{WD}}{\sum_{k=1}^{2} D_k(p,sl^{WD})}\right) + h^- .\left(-1 + G_{R1}\left(\frac{x^{WD}}{\sum_{k=1}^{2} D_k(p,sl^{WD})}\right)\right) = 0 \quad (79)$$

$$G_{R1}\left(\frac{x^{WD}}{\sum_{k=1}^{2} D_k(p,sl^{WD})}\right) = \frac{h^-}{(h^- + h^+)} \quad \rightarrow \quad x^{WD} = \sum_{k=1}^{2} D_k(p, sl^{WD})\, .\, {G_{R1}}^{-1}\left(\frac{h^-}{h^- + h^+}\right) \quad (80)$$

Furthermore, Constraint (2) can be simplified as follows:

$$\Pr\left[\sum_{k=1}^{2} D_k(p, sl^{WD}) \times \varepsilon \le x^{WD}\right] \ge rl_{R1}^{WD} \quad \rightarrow \quad \Pr\left[\varepsilon \le \frac{x^{WD}}{\sum_{k=1}^{2} D_k(p,sl^{WD})}\right] \ge rl_{R1}^{WD} \quad (81)$$

$$G_{R1}\left(\frac{x^{WD}}{\sum_{k=1}^{2} D_k(p,sl^{WD})}\right) \ge rl_{R1}^{WD} \quad (82)$$

$${G_{R1}}^{-1}(rl_{R1}^{WD}) \le \frac{x^{WD}}{\sum_{k=1}^{2} D_k(p,sl^{WD})} \quad \rightarrow \quad x^{WD} \ge \left[\sum_{k=1}^{2} D_k(p, sl^{WD})\right].\, {G_{R1}}^{-1}(rl_{R1}^{WD}) \quad (83)$$

Appendix E: *BM* Value

Equation $(O_{M1}.y^{WD} - x^{WD}) - y^{WD}.\sum_{i=1}^{O_{M1}} \alpha_{M1}^{i}$ is used to demonstrate the difference between the extra production in $M1$ $(O_{M1}.y^{WD} - x^{WD})$ and the total waste in its production runs $(y^{WD}.\sum_{i=1}^{O_{M1}} \alpha_{M1}^{i})$. The extra production in $M1$ cannot be greater than $O_{M1}.y^{WD}.Max(\alpha_{M1})$ and the total waste in $M1$ cannot be less than $O_{M1}.y^{WD}.Min(\alpha_{M1})$. Therefore, we can claim the following:

$$-1 \times \left(O_{M1}.y^{WD}\left(Max(\alpha_{M1}) - Min(\alpha_{M1})\right)\right) \leq (O_{M1}.y^{WD} - x^{WD}) - y^{WD}.\sum_{i=1}^{O_{M1}} \alpha_{M1}^{i} \leq O_{M1}.y^{WD}\left(Max(\alpha_{M1}) - Min(\alpha_{M1})\right) \quad (84)$$

Furthermore, the maximum value that can be assumed by y^{WD} corresponds to a case for which $O_{M1} = 1$ and at most is equal to $x^{WD}(Max(\alpha_{M1}) + 1)$. Because the maximum value for x^{WD} is $\left[\sum_{k=1}^{2} D_k(p, sl^{WD} = 1)\right].G_{R1}^{-1}\left(Max\left\{rl_{R1}^{WD} = 1, \frac{h^-}{h^- + h^+}\right\}\right)$, the BM in Equation (58) must satisfy the following inequality:

$$BM \geq O_{M1}.\left[\sum_{k=1}^{2} D_k(p, 1)\right].G_{R1}^{-1}\left(Max\left\{1, \frac{h^-}{h^- + h^+}\right\}\right).(Max(\alpha_{M1}) + 1).\left(Max(\alpha_{M1}) - Min(\alpha_{M1})\right) \quad (85)$$