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Specimen size effect on compressive and flexural strength of high-strength fibre-reinforced concrete containing coarse aggregate

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A R T I C L E I N F O

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A B S T R A C T

It is a well-known fact that the size of the sample influences values of mechanical properties of concrete measured during the tests. However, the comparison of results of various researchers clearly shows that the importance of this effect varies for different types of concrete. The paper presents outcomes of extensive experimental study focused on determination of relations between the size of the specimen and results of mechanical tests of high-strength fibre-reinforced concrete (HSFRC) with coarse aggregate (maximum aggregate size 16 mm). The main observed parameter was compressive strength. Six different HSFRC mixes having the expected compressive strength between 100 and 175 MPa were investigated. Cube samples of four sizes – 40, 100, 150 and 200 mm – were examined. The results proved that the size dependence of compressive strength diminishes with increasing strength of concrete. For very high strength materials (more than 130 MPa), the results were almost size independent. Additionally, flexural strength of two high-strength concrete mixes was measured on prismatic specimens of two different sizes. The results indicated that flexural strength of HSFRC with coarse aggregate is size dependent without regard to expected compressive strength. Conversion factors for the tested type of material were proposed for both compressive and flexural strength measurements.

1. Introduction

Generally, there are two main types of specimens used for testing of compressive strength of concrete around the world – cylinders and cubes. Specimens of varying shapes and sizes are preferred in various countries. In the states using European standards (EN), 150 mm cubes and 150/300 mm (diameter/height) cylinders are the most commonly used types. In connection with the advancement of concrete technology and considering wider use of high-strength fibre-reinforced concretes (HSFRC), these types of specimens are often replaced by smaller samples, mainly by 100 mm cubes. The reason why cubes are preferred over cylinders is elimination of bottom and top surface preparation. Cylinder end grinding equipment is expensive, it is difficult to find a capping material with suitable properties for testing of HSFRC. Smaller specimens are also advantageous because of easier handling of the samples and lower material consumption. The main reason why smaller cubes are preferred is the effort to reduce testing machine capacity requirements. According to the experience of the authors, capacity of common testing machines usually does not exceed 3 MN, which means that the maximum possible strength of 150 mm cubes to be tested is 130 MPa. Such value is sufficient for majority of purposes, but not for research and application of the most advanced HSFRC. If the 100 mm cubes are used, the required force is theoretically 2.25 times lower. This enables testing of concretes exceeding 250 MPa compressive strength with the use of the same machine.

The situation is similar concerning the flexural strength which is the second most important parameter of HSFRC. Both 150 × 150 × 700 mm and 100 × 100 × 400 mm prisms are exploited in the states where European standards (EN) are adopted. In this case, capacity of testing machine is not an issue. Easier production and handling of the samples made 100 × 100 × 400 mm prisms more common in the recent years (the weight of the sample is approximately 10 kg compared to 40 kg in case of the bigger one), but 150 × 150 × 700 mm prisms are still employed occasionally as some specialists consider them to provide better representation of flexural behaviour of the material in real structure.

There is a general agreement on the fact that the results of both compressive strength and flexural strength tests are affected by the size of the sample to some extent. However, opinions regarding the quantification of the size effect are ambiguous. As will be shown in chapter 2.2, various researchers have come to miscellaneous conclusions when studying different types of concrete. In this paper, the authors focused on HSFRC with maximum aggregate size of 16 mm which is the object of their long-term research. They attempted to derive conversion factors

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for compressive and flexural strengths measured on specimens of different sizes made of 6 concrete mixes.

2. State of the art

2.1. Theory of size effect

The values of mechanical properties measured on different specimens made of the same material vary according to size of the specimen. This phenomenon, usually referred to as size effect, was mentioned already in 16th century by Leonardo da Vinci in association with strength of ropes. However, deeper examination did not take place until 1921, when Griffith measured significant increase in nominal strength of glass fibres after he decreased diameter of the fibres. Theory of size effect was later worked out in detail mainly by Weibull and Bažant [3].

2.1.1. Statistical size effect

Brittle materials like concrete follow weakest-link model – macro-fracture initiation from one representative volume element (RVE) causes the whole sample to fail. Since the material strength is random, the strength of the weakest element in a sample is likely to decrease with increasing size of the sample.

Statistical size effect can be best described by chain-link representation. In a chain, failure of one link causes failure of the whole chain. The more links we have in a chain, the higher the probability that one of the links will be defective. Similarly, with increasing size of the specimen, i.e. with increasing number of RVEs it contains, possibility of failure rises.

Until 1980s, statistical size effect was considered to be the only one with practical influence on real structures. The reason was that in neither elastic nor plastic classical material theories the nominal stress depends on the size of the sample. Nevertheless, later investigations have proved that size effect can have its origin also in material mechanics.

2.1.2. Deterministic (energetic) size effect

If a body is subjected to effects of stress, the most of deformation is concentrated in an area called localization band or fracture process zone (FPZ). The size of this area depends on the type of material, for concrete the width of FPZ w_c can be estimated as 2 to 3 times the diameter of maximum aggregate size [4,5]. For small-scale samples, width of FPZ is significant compared to the dimensions of the whole specimen, while for large-scale samples relative size of the zone is negligible (Fig. 1).

In FPZ, material exhibits plastic behaviour. When the size of FPZ is significant compared to the specimen size, stress redistribution in the sample due to damage in FPZ can be observed. On the other hand, if FPZ size is negligible, stress redistribution has no important effect on global behaviour of the sample. As a result, small specimens tend to behave according to laws of plasticity and therefore are able to absorb relatively higher fracture energy than larger ones. For large-scale samples, linear elastic fracture mechanics (LEFM) is more apposite.

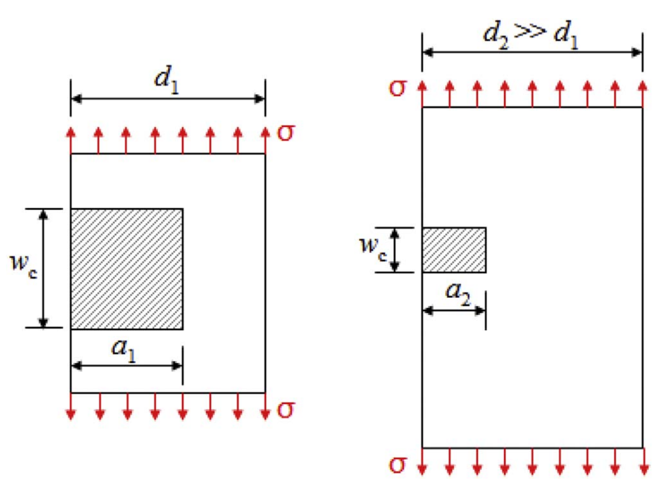

Fig. 1. Fracture process zone (FPZ) in small-scale (left) and large-scale (right) samples. Width of FPZ w_c is a material constant, i.e. w_c is the same in both cases.

Taking into consideration deterministic size effect, nominal strength σ_N for a specimen loaded by tensile stress can be calculated from Bažant's size-effect law (SEL) [4]:

$$\sigma_N = Bf_t \frac{1}{\sqrt{1 + \frac{d}{d_0}}} \tag{1}$$

in which B is a constant expressing shape of the specimen, f_t is tensile strength, d is characteristic dimension of the sample and d_0 is dimension of RVE. Proceeding strictly according to this law, calculated strengths should be close to zero for large samples, which is in contradiction with experimental results. Tests conducted on large-scale specimens (e.g. Ref. [18]) showed that nominal strength tends to a non-zero constant value related to uniaxial tensile strength. Therefore, Bažant [4] proposed modification of size effect law (MSEL) introducing size independent strength $\sigma_0 = \alpha f_t$:

$$\sigma_N = Bf_t \frac{1}{\sqrt{1 + \frac{d}{d_0}}} + \alpha f_t \tag{2}$$

in which α is an empirical constant less than unity. Abovementioned theory was originally derived for concrete in tension. Kim and Yi [13] proved that with properly defined parameters B and α, it can be considered valid also for concrete samples in compression, passing to a form of (f_c is compressive strength of the sample):

$$\sigma_N = Bf_c \frac{1}{\sqrt{1 + \frac{d}{d_0}}} + \alpha f_c \tag{3}$$

2.2. Size effect in concrete strength measurements in practice

2.2.1. Compressive strength

The theory of size effect described in the previous chapter clearly explains why different results are obtained when strength of a material on specimens of different size is measured. It also provides a mathematical model that can be used for prediction of strength measured on any size of the specimen, provided that sufficient set of experimental data is available for calibration of the model. In this chapter, results of various authors dealing with practical investigation of cube size effect on compressive strength of concrete are summarized in order to enable comparison and discussion of own results presented later. Studies from the past 20 years are cited as they are considered to be more relevant for currently used concretes, although the research of size effect on concrete strength dates back to the study conducted by Gonnermann as early as in 1925 [11]. The focus is solely on studies dealing with cube samples as cylinders were not tested in own experimental program.

In European standards, a correction factor is not used and 100, 150 or 200 mm cubes can be used without correction to the compressive strength obtained from the test when checking for strength class conformity [19].

Xincheng [21] gives a set of conversion factors to transform the strength measured on 100 mm cube $f_{c,100}$ to the strength of 150 mm cube $f_{c,150}$ (Table 1). The factors are taken from the Chinese codes. The value of the factor depends on the compressive strength, the difference is increasing with increasing strength.

Zhang [24] states that the results obtained on 100 mm cube should be multiplied by 0.95 and the results obtained on 200 mm by 1.05 to get $f_{c,150}$. The coefficients are valid for normal-strength concrete.

A comprehensive study on compressive strength on HSFRC was

Table 1
Factors K for conversion of $f_{c,100}$ to $f_{c,150}$ according to Xincheng [21].

$f_{c,100}$	K	$f_{c,100}$	K
≤55	0.95	76–85	0.92
56–65	0.94	86–95	0.91
66–75	0.93	≥96	0.90

Table 2
Compressive strength relative to $f_{c,100}$ according to Graybeal and Davis [12].

Mix no. ($f_{c,100}$ [MPa])	Cube size [mm]		
	50	70.7	100
A1 (198.1)	1.180	1.167	1.000
A2 (190.9)	1.132	1.132	1.000
A3 (189.1)	1.094	1.111	1.000
A4 (186.6)	1.103	1.095	1.000
A5 (170.8)	1.147	1.131	1.000
A6 (153.8)	1.026	1.016	1.000
A7 (141.5)	1.006	1.031	1.000
A8 (139.0)	1.005	1.021	1.000
A9 (120.2)	0.993	1.005	1.000
A10 (105.0)	1.026	1.043	1.000
A11 (84.2)	0.979	1.026	1.000

conducted by Graybeal and Davis [12]. At first, they compared many previous studies, coming to a conclusion that the compressive strength of concrete expressed by smaller specimens is expected to be slightly higher than the strength expressed by larger ones, and that strength differences will decrease at higher compressive strength levels. In their study they compared HSFRC in the strength range from 80 to 200 MPa, they used 50, 70.7 and 100 mm cube specimens. The concrete they used was made of a commercially available premix of a fine-grained nature, with the largest non-fibre constituent being a fine sand (< 0.6 mm diameter). Particular batches had slightly different composition, they were subjected to different modes of curing (air, humidity, heat treatment), the age of the premix varied. From the results in Table 2, it is clear that the strength was almost size-independent for mixes having the strength up to 155 MPa, without regard to curing conditions and premix age. For mixes demonstrating higher strength, the ratio between the strength measured on 100 mm cube and the other two sizes was approximately 0.9. This is in contradiction with the conclusions of their literature review.

An et al. [1] focused on size effect on compressive strength of reactive powder concrete (RPC). They compared 50, 70.7, 100 and 150 mm cubes containing 0, 1 or 2% of fibres by volume (mixes B1, B2 and B3, respectively). The maximum aggregate size in the mix was 1.25 mm. Table 3 shows the relative compressive strengths measured by An et al. No definite conclusion can be drawn regarding the relation between compressive strength level and the importance of size effect.

Dehestani et al. [9] studied size effect on self-consolidating concrete (SCC). Cube specimen sizes were 50, 75, 100, 125 and 150 mm, three different SCC mixes were compared. The results are summarized in Table 4. They came to a conclusion that the significance of size effect was almost the same for all the tested SCC mixes.

Del Viso et al. [10] measured compressive strength of high-strength

Table 3
Compressive strength relative to $f_{c,100}$ according to An et al. [1].

Mix no. ($f_{c,100}$ [MPa])	Cube size [mm]			
	50	70.7	100	150
B1 (91.8)	1.105	1.035	1.000	0.952
B2 (119.6)	1.155	1.052	1.000	0.932
B3 (137.1)	1.029	1.015	1.000	0.865

Table 4
Compressive strength relative to $f_{c,100}$ according to Dehestani et al. [9].

Mix no. ($f_{c,100}$ [MPa])	Cube size [mm]				
	50	70.7	100	125	150
C1 (61.2)	1.008	1.006	1.000	0.940	0.890
C2 (50.0)	1.074	1.044	1.000	0.912	0.896
C3 (44.2)	1.007	0.998	1.000	0.943	0.930

Table 5
Compressive strength relative to $f_{c,100}$ according to del Viso et al. [10].

Mix no. ($f_{c,100}$ [MPa])	Cube size [mm]			
	33	50	67	100
D1 (96.1)	1.144	1.085	1.066	1.000

concrete (HSC; around 100 MPa expected strength) on 33, 50, 67 and 100 mm cubes. The maximum aggregate size was 12 mm, w/c was 0.28, which means that the mix was very close to one of the mixes investigated in our research presented later, except the fact that no fibres were used. Results are given in Table 5.

Yi et al. [22] studied effect of specimen sizes, shapes and placement directions on compressive strength of concrete. They compared strength of 50, 100, 150 and 200 mm cubes made of four normal-strength concrete mixes (expected average cylinder strength after 28 days was 20, 40, 60 and 80 MPa). They came to a conclusion that compressive strength decreases with increasing specimen size, no clear dependence was found between the strength of concrete and importance of the size effect (Table 6).

Tokyay and Özdemir [20] did not confirm the decrease of compressive strength with increasing specimen size. They obtained the highest strengths on 150 mm cubes; for 75, 100 and 200 mm cubes the values they measured were lower. They compared three different normal strength mixes, see Table 7.

Zabihi and Eren [23] investigated normal-strength concrete mixes of two strength levels. They compared 100, 150 and 200 mm cubes. Similarly to Tokay and Özdemir [20], they found the compressive strength to be increasing with the size of the cube in most cases. The results for cubes cured in water are summarized in Table 8.

Fig. 2 clearly compares the results of all the authors mentioned in this chapter. Relatively high scatter of the results is apparent. Majority of the studies confirms the tendency of compressive strength to decrease with increasing size of the specimen, but some authors reported inverse behaviour. The relation between the compressive strength level and the importance of size effect is also equivocal.

2.2.2. Flexural strength

Whereas the studies dealing with compressive strength size effect are quite often, very little information is found in literature on investigation of size effect on flexural tensile strength of concrete. Situation is further complicated by variability of testing methods used. Dimensions of the samples are not fixed, three or four point bending

Table 6
Compressive strength relative to $f_{c,100}$ according to Yi et al. [22].

Mix no. ($f_{c,100}$ [MPa])	Cube size [mm]			
	50	100	150	200
E1 (32.2)	1.124	1.000	1.041	0.955
E2 (49.5)	1.250	1.000	0.901	0.934
E3 (73.1)	1.099	1.000	0.931	0.900
E4 (82.0)	1.148	1.000	0.925	–

Table 7
Compressive strength relative to $f_{c,100}$ according to Tokyay and Özdemir [20].

Mix no. ($f_{c,100}$ [MPa])	Cube size [mm]			
	75	100	150	200
F1 (47.0)	0.894	1.000	1.106	0.979
F2 (62.5)	0.928	1.000	1.120	1.024
F3 (66.5)	0.932	1.000	1.294	1.263

Table 8
Compressive strength relative to $f_{c,100}$ according to Zabihi and Eren [23].

Mix no. ($f_{c,100}$ [MPa])	Cube size [mm]		
	100	150	200
G1 (43.0)	1.000	1.163	1.186
G2 (49.0)	1.000	1.550	1.266

arrangement is used, specimens with and without notch are exploited.

In classic ASTM publication [17], Ozyildirim and Carino state based on the literature review that it is commonly agreed that as the size of the test specimen increases, flexural strength decreases.

Awinda et al. [2] compared flexural strength of 50, 100 and 150 mm deep HSFRC specimens (compressive strength of the mix was not specified by the authors). They employed three-point bending test arrangement for notched specimens. Little size effect on flexural strength was observed for 100 and 150 mm high beams. For 50 mm beams, larger difference was observed, see Table 9.

Nguyen et al. [16] studied size effect on flexural behaviour of ultra-high performance concrete (UHPC; 170–210 MPa compressive strength according to authors) on three types of beams. Four point bending test with unnotched specimens was used. They compared mixes with two different proportions of fibres (1% vol. twisted steel fibres +1% vol. smooth steel fibres and 1% twisted +0.5% smooth). Clear size effect was demonstrated, see Table 10.

Zhou et al. [26] investigated flexural strength of normal and lightweight HSC (115 MPa and 90 MPa compressive strength measured on 100 mm cubes, respectively). They used three-point bending test without notch, 50, 100 and 200 mm high beams were tested. Significant size dependence complying with basic Bažant size effect law (1) was found (Table 11).

Table 9
Flexural strength of beam specimens according to Awinda et al. [2].

Specimen no.	Height [mm]	Width [mm]	Span [mm]	Length [mm]	Notch size [mm]	Flexural strength [MPa]
H1	50	50	150	200	17	19.83
H2	100	50	300	350	33	15.70
H3	150	50	450	550	50	14.67

Table 10
Flexural strength of beam specimens according to Nguyen et al. [16].

Specimen no.	Height [mm]	Width [mm]	Span [mm]	Fibres	Flexural strength [MPa]
I1	50	50	150	1 + 1	14.26
I2	100	100	300	1 + 1	11.43
I3	150	150	450	1 + 1	10.63
J1	50	50	150	1 + 0.5	13.19
J2	100	100	300	1 + 0.5	10.89
J3	150	150	450	1 + 0.5	7.88

Table 11
Flexural strength of beam specimens according to Zhou et al. [2].

Specimen no.	Height [mm]	Width [mm]	Span [mm]	Length [mm]	Type	Flexural strength [MPa]
K1	50	100	200	300	Normal	3.43
K2	100	100	400	500	Normal	2.74
K3	200	100	800	840	Normal	2.19
L1	50	100	200	300	Lightweight	2.28
L2	100	100	400	500	Lightweight	1.52
L3	200	100	800	840	Lightweight	1.20

Zi et al. [25] investigated flexural strength of unnotched concrete beams subjected to four-point bending. They compared three different sizes of beams made of normal-strength concrete (33 MPa cylindrical compressive strength at 28 days). Specimen sizes and measured strengths are summarized in Table 12, the strength decreased with specimen size.

Although the data obtained in former studies are not directly comparable due to variability of testing approaches and specimen

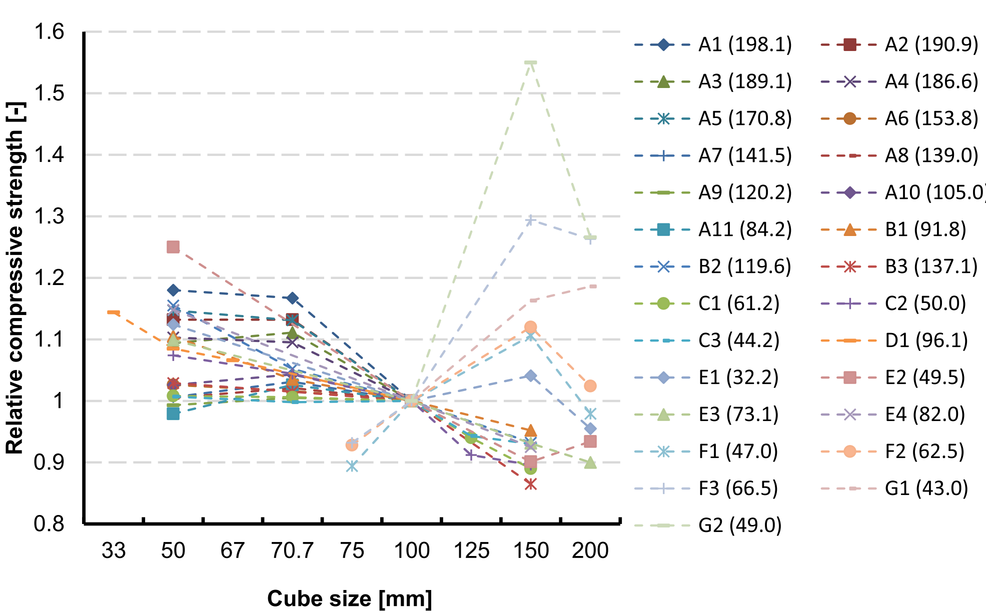

Fig. 2. Comparison of relative compressive strengths measured by various authors.

Table 12
Flexural strength of beam specimens according to Zi et al. [25].

Specimen no.	Height [mm]	Width [mm]	Span [mm]	Length [mm]	Flexural strength [MPa]
M1	30	30	90	190	5.43
M2	48	48	144	244	4.41
M3	75	75	225	325	4.24

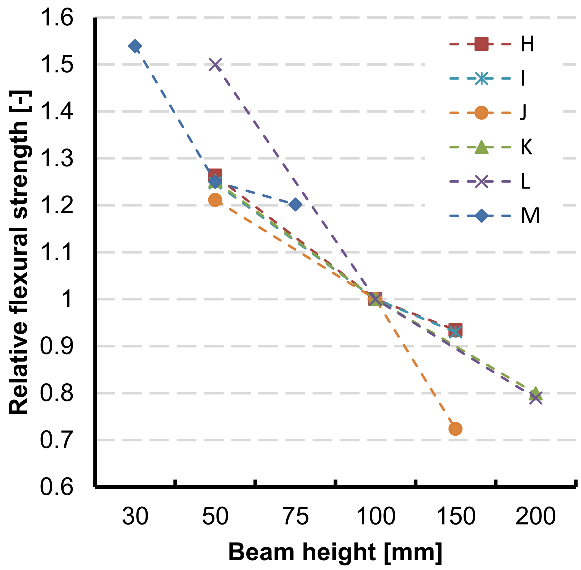

Fig. 3. Comparison of relative flexural strengths measured by various authors.

widths and spans, general opinion on the nature of specimen size effect on flexural strength can still be acquired. In Fig. 3, relative flexural strengths are compared. The strength of 100 mm high specimen is taken as a basis. The variance of the results is significantly smaller than in case of compressive strength, indicating clear size dependence of flexural strength.

3. Research significance

To facilitate testing of properties of HSC, use of smaller specimens is beneficial. On the other hand, for exploitation of the material in real structures it is important to be able to relate the test results to standard testing methods exploiting bigger specimens.

The comparison of many studies dealing with size effect on compressive strength (Fig. 2) clearly shows that the effect of specimen size varies significantly for different types of concrete. The only reasonable conclusion that can be made is that for each type of concrete, the relation between the size of the specimen and measured compressive strength has to be derived experimentally. Currently there is no available study on specimen size effect on HSFRC containing coarse aggregate (up to 16 mm). Therefore, experimental program was carried out to investigate this issue and to derive conversion factors enabling the use of smaller cubes for compressive strength tests. Mixes with different compositions and broad range of expected compressive strengths were tested to reach the maximum possible applicability of the results.

Size effect on flexural strength of the given type of HSFRC was also studied in smaller extent to verify the compliance with previous studies (Fig. 3).

4. Experimental program

The experimental program has been carried out as a part of long-term comprehensive research focused on optimization of production technology and investigation of material properties of HSFRC with maximum aggregate size of 16 mm. The main intention of the still continuing research [14] is to prepare competitive HSFRC for large scale civil engineering structures made solely from compounds that are commonly accessible on local market. The parameters of the material are reaching properties of commercially available fine-grained (up to 1 mm maximum grain size) UHPC premixes, but the unit production price is approximately four times lower.

Table 13
Overview of HSFRC mixes used in the study.
The meaning of steel fibres specification is following: fibre length [mm]/diameter [mm]/tensile strength [MPa]/Profiled or Straight ends of the fibre.

Compound [kg/m^3]	Mix					
	C100	C110	C120	C130	C145	C175
Cement 42.5 R	500	750	650	800	650	750
Water	147	225	138	150	155	150
w/c [−]	0.29	0.30	0.21	0.19	0.24	0.20
Aggregate type [−]	spilite	basalt	basalt	basalt	basalt	basalt
Aggregate 0/4 mm	800	668	845	1014	800	900
Aggregate 4/8 mm	250	1048	425	503	436	461
Aggregate 8/16 mm	700	–	424	–	350	424
Superplasticizer (polycarboxylate)	5	11	33	32	34	38
Microsilica	–	–	39	120	98	150
Steel fibres 30/0.55/1500/P	60	–	–	–	–	–
Steel fibres 13/0.20/2750/S	–	160	140	160	100	140
Steel fibres 25/0.50/450/P	–	–	140	–	100	–
Cut cord wires from recycled tyres	–	–	–	–	–	140

The study presented in this paper aims to determination of conversion factors for compressive and flexural strength measured on specimens of different sizes. During the three years period, six various HSFRC mixes were selected for the study. Compositions are listed in Table 13. The mixes were chosen so as to continuously cover wide range of compressive strengths from 100 to 175 MPa. The mixes were marked as C100 to C175 referring to the expected compressive strength (i.e. compressive strength that should be reached on 150 mm cube). The diversity of compositions of particular mixes should ensure that the results will be applicable to broad spectrum of HSFRC with coarse aggregate.

4.1. *Compressive strength*

Compressive strength was measured on cube specimens of four sizes according to ČSN EN 12390-3 [6]. The cube sizes were:

- 40 mm – represented the fragments of 40 × 40 × 160 mm beams used for testing of fine-grained mortars.
- 100 mm – represented the cube size suitable for testing of HSFRC.
- 150 mm – represented the standard cube size according to ČSN EN 12390-3 [6].
- 200 mm – represented the cube size that used to be standard in the Czech Republic before implementation of the European standards (according to ČSN 73 1317 [5]).

For each of the six mixes listed in Table 13, six samples of each size were prepared and tested (Fig. 4). In total, 144 cubes were examined. The samples were cured in water and tested after 28 days after concreting. The loading rate was 0.5 MPa/s. Two hydraulic loading machines were used according to Table 14. For smaller samples, 3 MN testing machine available in the laboratory of the authors was sufficient. For larger samples, unique 11 MN testing machine in Klokner institute in Prague was exploited.

Cylindrical compressive strength was not followed in the study as the authors do not use cylinders for compressive strength testing in

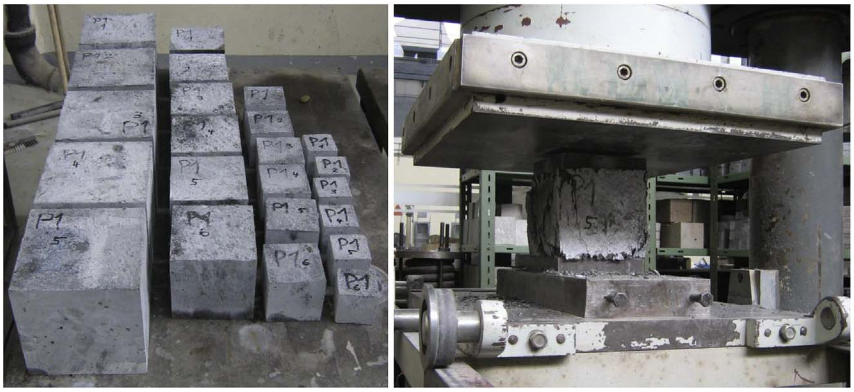

Fig. 4. One set of the specimens (left). 150 mm cube in 3 MN testing machine (right).

Table 14
Theoretical force required for crushing of the sample [kN]. Samples in blue color were tested using 3 MN testing machine, 11 MN testing machine was exploited for samples in red.

Mix	Cube size [mm]			
	40	100	150	200
C100	160	1000	2250	4000
C110	176	1100	2475	4400
C120	192	1200	2700	4800
C130	208	1300	2925	5200
C145	232	1450	3263	5800
C175	280	1750	3938	7000

their research. Finding exact relations between cylindrical and cube strength for the given material would require an independent study. For rough estimation, conversion factors found in the literature could be utilized. According to ČSN EN 206 [8], compressive strength measured on 150/300 mm cylinder can be transferred to the strength of 150 mm cube by multiplying by 1.15 in case of C100/115 class concrete without steel fibres. Kusumawardaningsih et al. [15] report the relation between 100/200 mm cylinder and 100 mm cube strength to be 1.12 for 170 MPa strength grade UHPC without fibres and 1.00 in case the fibres are employed in the material.

4.2. Flexural strength

Flexural strength was measured on two sizes of beam specimens according to ČSN EN 12390-5 [7]. The beam sizes were:

- 100 × 100 × 400 mm – represented the specimen size suitable for testing of HSFRC.
- 150 × 150 × 700 mm – represented the standard specimen size according to ČSN EN 12390-5 [7].

Flexural strength was measured for C100 and C145 mixes only. As

Fig. 6. 100 × 100 × 400 mm beam in testing machine.

Table 15
Overview of results of compressive strength tests.

Mix	Parameter	Cube size [mm]			
		40	100	150	200
C100	f_c [MPa]	138.3	116.3	98.5	75.2
	σ [MPa]	4.4	2.9	7.6	4.5
C110	f_c [MPa]	138.2	120.5	103.5	81.5
	σ [MPa]	5.7	4.6	6.8	6.5
C120	f_c [MPa]	138.0	123.8	114.5	95.3
	σ [MPa]	5.5	3.4	4.4	7.5
C130	f_c [MPa]	145.0	137.7	132.3	120.3
	σ [MPa]	7.8	4.5	3.1	8.3
C145	f_c [MPa]	153.5	144.2	142.0	144.5
	σ [MPa]	6.2	2.0	1.6	5.1
C175	f_c [MPa]	169.3	177.3	175.2	169.2
	σ [MPa]	7.7	7.1	2.9	5.4

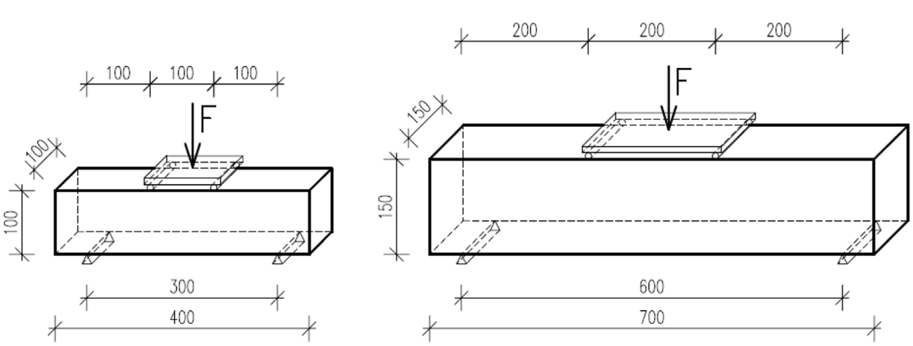

Fig. 5. Arrangement of four-point bending test of 100 × 100 × 400 mm beam (left) and 150 × 150 × 700 mm beam (right).

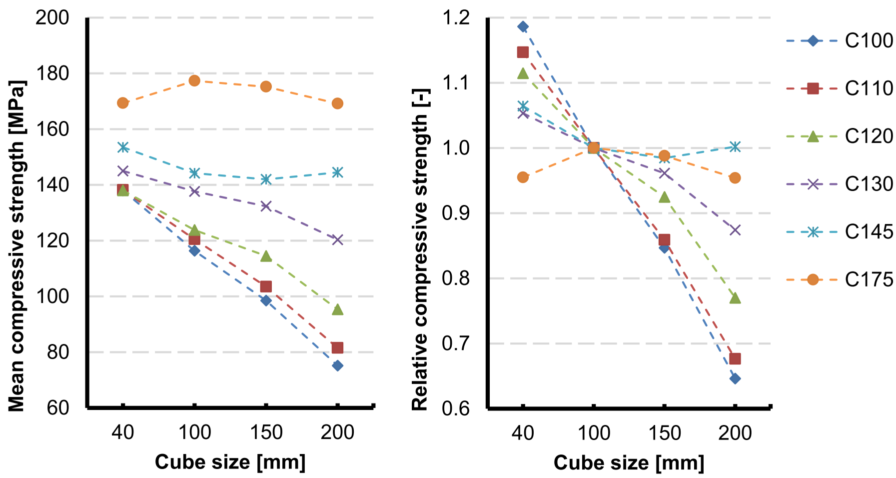

Fig. 7. Relation between mean measured compressive strength and cube specimen size (left). Compressive strength relative to $f_{c,100}$ measured on different cube specimens (right). The same legend applies to both plots.

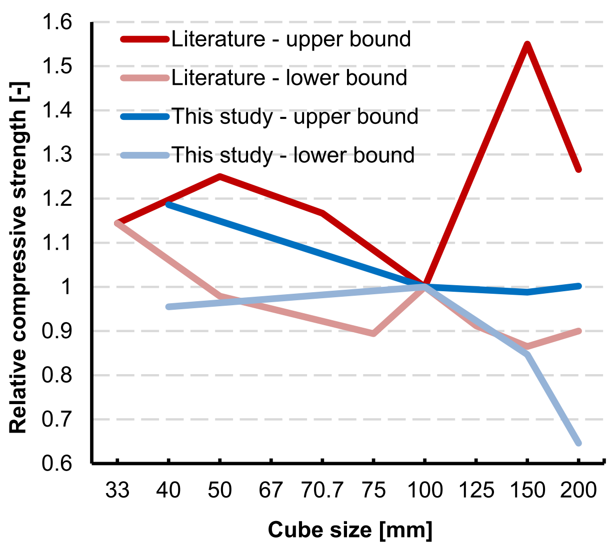

Fig. 8. Comparison of relative compressive strengths measured by other researchers ("Literature") and authors of this study.

Table 16
Overview of results of flexural strength tests.

Mix	Parameter	Beam height [mm]	
		100	150
C145	f_{fl} [MPa]	13.8	11.5
	σ [MPa]	0.8	0.1
C100	f_{fl} [MPa]	6.6	5.3
	σ [MPa]	0.4	0.4

flexural strength was not the main objective of the research, it was decided based on the results of literature review that there was no need for testing of the spectrum of mixes as wide as in case of compressive strength. For each mix, four specimens of each size were prepared and tested. Four-point bending arrangement without notch was used (Figs. 5 and 6). In total, 16 beams were examined. The samples were cured in water and tested in the age of 28 days after concreting. The loading rate was 0.2 mm/min. The same hydraulic loading machine with 100 kN capacity was used for all the samples.

5. Results and discussion

5.1. Compressive strength

The results of compressive strength tests are arranged in Table 15 and Fig. 7. All the values of compressive strength f_c were calculated as average from six tested samples. Standard deviation of measured values is denoted as σ. In the right part of Fig. 7, the results are plotted relatively to the compressive strength measured on 100 mm cube.

The importance of specimen size continuously decreases with increasing compressive strength of the material. For mixes up to C130, the results depend on the size of the specimen, higher strength was measured on smaller samples. The dependence is close to linear for all the tested mixes. For very-high strength mixes C145 and C175, the measured values were almost independent on the size of the cube. One possible reason is that w/c ratio of the higher-strength mixes was lower and fibre content was higher in most cases, leading to elimination of non-homogeneities in cement matrix and reduction of statistical part of the overall size effect with increasing strength of concrete.

The results measured on HSFRC with coarse aggregate are in contradiction with comprehensive study of Graybeal and Davis ([12], Table 2) which was focused on HSFRC made from fine-grained premix. These authors found the importance of size effect to be rising with increase of compressive strength. This indicates a fundamental difference between the size effect in coarse-grained and fine-grained HSFRC. Following hypothesis explaining this fact was formulated.

Fine-grained material is more homogeneous thanks to similar fineness of all the components of the mix. RVE size in fine-grained material is very small which means that just a negligible portion of the sample exhibits plastic behaviour at high loads. The influence of statistical and deterministic size effect is probably distinct in differently grained materials. In fine-grained concrete, the overall size effect is predominantly caused by statistical part due to the lack of plastic area. The material is more susceptible to local stress concentrations arising on the interface between cement matrix and aggregate grains. The stress concentrations in the instant of failure of the tested specimen increase with increasing strength of the material and therefore also the importance of size effect increases. On the other hand, in coarse-grained material relatively larger portion of the sample is in plastic state before the failure which means that the influence of local stress concentrations is eclipsed. Statistical size effect becomes less important and deterministic part predominates. With increasing strength of the material, size-independent part of equation (3) prevails over the size-dependent part and as a result, the overall size effect decreases with increasing concrete strength.

The results of other researchers mentioned in chapter 2.2 are hardly

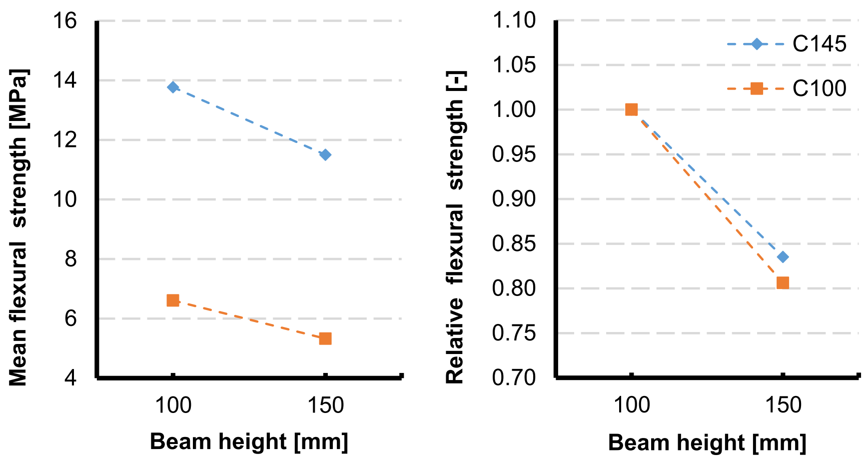

Fig. 9. Relation between mean measured flexural strength and beam height (left). Flexural strength measured on different beams relative to the strength of 100 × 100 × 400 mm beam (right). The same legend applies to both plots.

directly comparable with this study due to different nature of the tested material, distinct compressive strength range or too narrow extent of the results. However, there is a general agreement in the conclusion that the compressive strength has the tendency to decrease with increasing specimen size. Likewise, compressive strengths relative to $f_{c,100}$ measured in this study mostly lay in the range of relative compressive strengths observed by another authors, see Fig. 8. The only significant difference exists in case of 200 mm cubes. This is almost certainly caused by the fact that other researchers used 200 mm cubes only for concretes of lower strength (up to 70 MPa).

The authors must admit that the 40 mm cubes were probably not entirely suitable for most of the tested mixtures as it is usually agreed that the size of the mould should exceed three times the size of the biggest aggregate grain and twice the length of the longest fibre. However, the scatter of the results measured on 40 mm cubes was satisfactory and therefore the specimens were found to be acceptable for the purposes of this research.

5.2. Flexural strength

The results of flexural strength tests are arranged in Table 16 and Fig. 9. All the values of flexural strength f_{fl} were calculated as average from four tested samples. Standard deviation of measured values is denoted as σ. In the right part of Fig. 10, the results are plotted relatively to the flexural strength measured on 100 × 100 × 400 mm beam. The relation observed for HSFRC with coarse aggregate falls into the range of values determined earlier for another types of concrete by other researchers (Fig. 3).

The size effect was almost equal for both mixes. Higher strength was measured on smaller beams, possibly due to surface layer size effect. The surface layer effect is caused by the modifying influence of the mould on the surface of a specimen. The random orientation of the fibres is restricted close to the surface plane. Fibres on the moulded surface are be aligned parallel to it and therefore have a higher probability of bridging the cracks that form in the perpendicular direction. Hence the surface or skin layer has a higher strength than the inside of the specimen. The smaller the specimen the stronger this effect is as the surface layer constitutes larger proportion of the total cross-section.

6. Application of size effect law

The results of compressive strength tests were used to calibrate the size effect law formula (3) for the tested concrete mixes. RVE size d_0 was estimated as 2.5 times the maximum aggregate size for each mix in accordance with recommendations of Bažant [3] and Kim [13]. Coefficients B and α in Table 17 were calculated by regression analysis, Levenberg–Marquardt's least square method was applied.

Increasing value of α coefficient clearly shows that size-independent part of compressive strength prevails in mixes with higher compressive strength which is in accordance with experimental results. However, the R^2 values (coefficient of determination) are relatively low which indicates that the agreement of the model with experimental data is not very good. Therefore linear regression in the form of

$$\sigma_N = Pd + Q \tag{4}$$

was employed to approximate the experimental data. This approach yielded better results as can be seen in Table 18. For mixes up to C130, the compliance of the model is excellent ($R^2 > 0.95$). In case of very-high strength mixes C145 and C175, the R^2 value still shows poor agreement of the model with the experiments, but this is caused just by the way the R^2 value is calculated:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(\sigma_N^{test} - \sigma_N^{model})^2}{\sum_{i=1}^{n}(\sigma_N^{test} - \overline{\sigma}_N^{test})^2} \tag{5}$$

where n is the number of values, σ_N^{test} is an experimentally measured value of compressive strength, σ_N^{model} is a value of compressive strength predicted by equation (4) and $\overline{\sigma}_N^{test}$ is an average of all experimental values. As the results of C145 and C175 are almost size independent, the difference between experimental values and predicted values is relatively high compared to difference between experimental values and their average, leading to low values of R^2 without any real effect on the quality of prediction ability of the model. Therefore, it can be said that for the tested type of HSFRC, linear prediction model provides better approximation of experiments than classic MSEL model. The resulting curves are plotted in Fig. 10.

7. Conversion factors

Based on the values of relative compressive strength and considering the variability of test results, conversion factors in Table 19 were proposed by the authors as the final result of the whole study. The equivalent compressive strength of 150 mm cube $f_{c,150}$ can be calculated from the strength $f_{c,x}$ measured on cube of size x using formula:

$$f_{c,150} = C \cdot f_{c,x} \tag{6}$$

where C is the conversion factor based on the expected compressive strength of 150 mm cube $f_{c,exp}$. Considering the fact that linear regression was found to be the best approximation of experimental data, linear interpolation of C values is possible for intermediate cube sizes.

For flexural tensile strength, the authors propose to consider a conversion factor $C_f = 0.80$ to transform the strength measured on 100 × 100 × 400 mm prisms to the flexural strength of

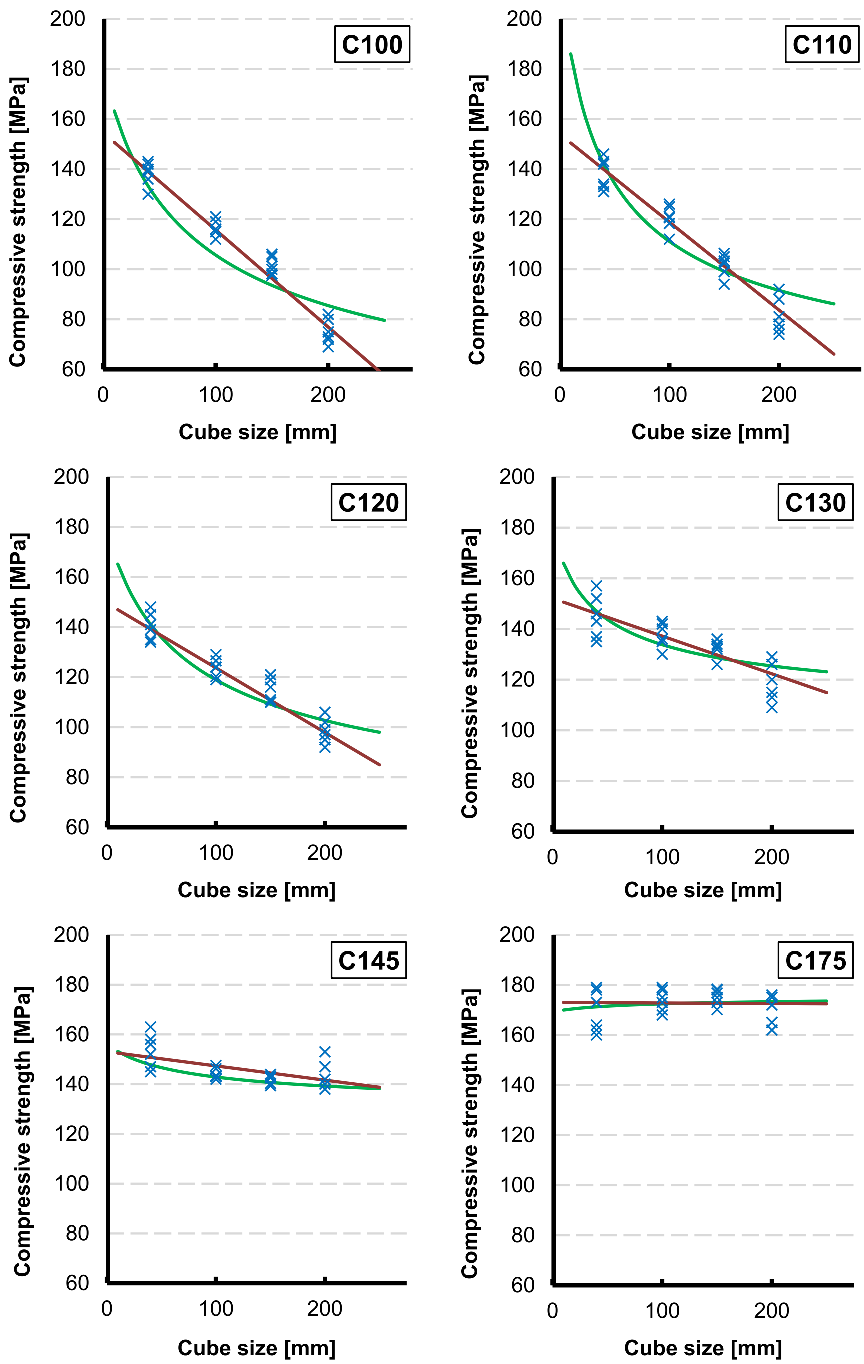

Fig. 10. MSEL curves (green) and linear regression curves (red) of the tested HSFRC mixes. Blue crosses represent the experimental data. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 17
MSEL parameters.

Mix	MSEL parameter				
	B [−]	f_c [MPa]	d_0 [mm]	α [−]	R^2 [-]
C100	1.600	100	40	0.202	0.650
C110	1.668	110	20	0.329	0.734
C120	1.071	120	40	0.419	0.452
C130	0.606	130	20	0.782	0.323
C145	0.198	145	40	0.880	0.446
C175	−0.039	175	40	1.006	0.060

Table 18
Linear regression parameters.

Mix	P [−]	Q [−]	R^2 [−]
C100	−0.389	154.590	0.996
C110	−0.352	153.970	0.991
C120	−0.258	149.530	0.977
C130	−0.149	152.070	0.963
C145	−0.057	153.060	0.592
C175	−0.002	173.030	0.001

Table 19
Proposed conversion factors C for HSFRC with coarse aggregate.

$f_{c,exp}$ [MPa]	Cube size [mm]			
	40	100	150	200
100–125	0.75	0.90	1.00	1.25
≥130	0.95	1.00	1.00	1.05

150 × 150 × 700 mm beams. The same value is applicable for all strength classes.

8. Conclusions

Based on the results of this investigation of compressive and flexural tensile strength exhibited by various size cubes and beams, the following conclusions are drawn:

- The results of previous studies dealing with size effect on concrete strength measurements have shown that the importance of size effect varies significantly for different mix compositions and strength levels. The only reasonable conclusion is that for each type of concrete, the relation between the size of the specimen and measured strength has to be derived experimentally.
- Compressive strength of HSFRC with coarse aggregate generally decreases with increasing cube size. However, the higher is the expected compressive strength of the material, the lower is the size effect. For very-high strength mixes (C145 and C175), the results were almost independent on the cube size, probably thanks to increase of homogeneity of the mix and reduction of statistical size effect.
- Comparison with results of Graybeal and Davis [12] showed that the relation between expected compressive strength and significance of size effect has opposite tendency in fine-grained and coarse-grained HSFRC, probably because of distinct rate of statistical and deterministic part of size effect as a result of different RVE size in differently grained materials.
- Size effect of flexural strength in HSFRC with coarse aggregate is very similar to other types of concrete. Smaller beams exhibit higher strength, possibly due to surface layer effect.
- Linear function proved to be better approximation of compressive strength experimental data than MSEL curve in case of tested HSFRC mixes.
- A set of conversion factors was proposed for transformation of compressive strength measured on cube of any size in the range of 40–200 mm to compressive strength of 150 mm cube. Thanks to extensiveness of the study, the factors can be used for wide range of HSFRC with coarse aggregate (up to 16 mm).
- Conversion factor was proposed for recalculation of flexural tensile strength measured on 100 × 100 × 400 mm beam to the strength measured on 150 × 150 × 700 mm beam.

The results of the study are useful for anyone dealing with research or application of HSFRC with coarse aggregate as they enable the exploitation of smaller specimens which facilitate preparation, handling and testing of the samples.

Acknowledgement

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