Optimal Disclosure and Fight for Attention*

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Abstract

This paper shows that managers can compete for speculators' scarce attention by withholding private information about their firm. By raising speculators' uncertainty regarding the future payoff, a firm becomes a more attractive target for speculators who allocate more attention to it, in turn. Each firm has an incentive to do this because more attention implies more informative prices, which allows firm managers to extract more information and to invest more efficiently. In a setting with endogenous capital, the firms' fight for attention is inefficient, i.e. all firms would be better off if they instead disclosed their private information to speculators. Surprisingly, giving each firm manager a myopic contract that rewards increases in the short-run asset price, encourages disclosure and implements the first-best outcome.

Keywords: attention allocation, firm investment, disclosure, cost of capital, managerial compensation.

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1 Introduction

In reality, financial market participants are overwhelmed with information like data releases, earnings reports or public announcements. Clearly, a rational trader with unlimited resources would like to thoroughly read and process all of this information to make the optimal trading decision. However, in actual markets even professional traders face capacity constraints. They have to decide how to allocate their limited time (or resources) between different tasks, firms or sectors. A natural question to ask in such a world of *limited attention capacity* is: how can firms influence the decision of market participants to follow their firm and when is this fight for attention optimal? In this paper, I focus on one major firm decision, its disclosure policy and show that (and how) it can be used to attract speculators' limited attention.

How firms should structure their disclosure policy is clearly an important question. First, there is a substantial policy debate about disclosure regulations or mandatory disclosure. Second, the existing literature has highlighted several costs and benefits associated with firm disclosure (see e.g. Leuz and Wysocki (2016) and Kanodia and Sapra (2016)). At the same time, there is ample evidence that investors regularly shift attention from one firm (or industry) to the other (see e.g. Barber and Odean (2008) and Da et al. (2011)).[1] Nevertheless, we still know very little about the way in which firms can actively control (or influence) this reallocation of attention. This paper shows theoretically that a firm's disclosure policy is a powerful tool to "fight for attention," a mechanism that has received recent support from the empirical literature (see e.g. Cunat and Groen-Xu (2016) and Edmans et al. (2017)).

In this paper, I theoretically study the economic consequences of corporate disclosure decisions in the presence of speculators who optimally allocate their attention between firms. I show that managers can increase the attractiveness of their firm by choosing *not* to disclose information to the financial market. In a setup with multiple firms, managers therefore perform a "race to the bottom" such that in equilibrium all firms choose to withhold their information. I discuss the consequences for the informational content of prices, cost of capital, and economic efficiency.

[1]See also Dellavigna and Pollet (2009) and deHaan et al. (2015) for empirical evidence of investors' limited attention.

The model features two firms that are run by benevolent managers who are in charge of two decisions: disclosure and capital investment. The return on the firms' assets in place are traded in a financial market by a continuum of informed traders ("speculators"). These speculators can acquire private information about the firms' fundamentals depending on the amount of attention allocated to each firm. Thus, speculators' attention should be interpreted as time or effort allocated to research or analysis of a certain firm. More extensive research leads to a more precise signal that allows the speculator to invest more efficiently.

The model has four periods. In $t = 1$, the firm managers decide whether to disclose their private signal about the fundamental to the financial market participants. In $t = 2$, speculators choose how to allocate their scarce attention between the two firms. This decision determines the precision of their private signals about the firm fundamentals. In $t = 3$, the endogenously informed speculators trade the two risky assets and the two equilibrium prices are determined. Due to noisy supply, the two prices are imperfect signals of the true fundamentals. In $t = 4$, the firm managers decide on their investment in a growth opportunity. Since the return on this investment is correlated with the return on the assets in place, the managers partially base their investment decision on the equilibrium asset price which creates a "feedback effect" from the financial market to firm decisions.

In equilibrium, each firm's asset price depends on the fundamental, which is revealed through the speculators' aggregate private signal. However, due to the noisy supply shock, asset prices are only imperfect signals about the true fundamental. If the managers chose to disclose their information, the asset prices are also affected by the disclosed signal which serves as a public signal for all speculators. As a result, both firm managers can extract additional information from the asset prices to improve their knowledge about the firm's fundamental shock. This additional price signal then allows the managers to invest more efficiently which increases both firm's ex ante value.

A crucial feature of the model is the speculators' endogenous choice of the signal precisions. As in Kacperczyk et al. (2016), each speculator decides on the optimal split of attention between the two firms. The speculators' aggregate attention choice then determines the informational content of the equilibrium asset prices in the following period which, in turn, affects investment efficiency in the last period.

An important difference to the setup in Kacperczyk et al. (2016) is that the speculators' attention allocation decision is a function of the firm managers' disclosure decision in the previous period. Thus, the two managers can actively control the speculators' signal precisions and thus the informational content of their firm's price. In particular, the managers have an incentive to maximize the amount of outside information in the price because this allows them to extract as much information as possible in order to invest optimally in the growth opportunity.

Section 3 discusses the disclosure equilibrium in the baseline model. Driven by the incentive to fight for the speculators' scarce attention, the two firm managers compete in a "race to the bottom" and choose not to disclose their private information. To understand the intuition behind this result, consider the counterfactual outcome of full disclosure by both firms. In this case, speculators would be indifferent between both firms and allocate 50% of their attention capacity to each firm. However, this outcome is not stable as both firms have an incentive to deviate and choose not to disclose. By doing so, each firm can attract the speculators' entire attention because their firm is now the more attractive target. As a result, the firm's asset price becomes more informative and the firm manager can increase investment efficiency. Thus, in equilibrium both firms choose not to disclose.

Furthermore, I show that this fight for attention has important (and novel) consequences for the firms' cost of capital and price efficiency. Each firm's cost of capital is proportional to the speculators' conditional payoff variance. Interestingly, this variance is affected by the firms' disclosure policy through two different channels. First, if a firm chooses to disclose, this decision provides the speculators with an additional public signal about the payoff and reduces their payoff uncertainty. Second, if a firm chooses to disclose, this renders the firm a *less* attractive target for speculators such that their average precision about the firm's fundamental decreases. As a result, the firm's asset price becomes less informative. Taken together, this second (indirect) channel increases the conditional variance and thus the firm's cost of capital. Therefore, this analysis leads to the surprising result that disclosure does not necessarily reduce a firm's cost of capital. The net effect crucially depends on the disclosure decisions of the firm's competitor and the impact on the speculators attention allocation decision. The informational content of the firms' asset price, however, is identical in the two symmetric outcomes of full disclosure and non-disclosure. Intuitively, this efficiency measure only depends on the amount of attention allocated to each firm and in any symmetric outcome, this capacity is split equally between both firms. As a consequence, the model highlights the delicate difference between the total amount of price information (reflected in the firm's cost of capital) and the amount of novel (or outside) information (reflected in the firm's investment efficiency).

Next, I analyze the impact of managerial incentives on the firms' optimal disclosure decisions. In particular, I assume that both managers' compensation contract includes a long-run component (as before) and a short-run component that rewards the manager for increases in the firm's (short-run) asset price. I show that the managers' objective functions under this contract can be written as a weighted average of the manager's and the speculators' conditional payoff variance. Interestingly, the second variance captures the impact of non-disclosure on the firm's cost of capital. As a result, by increasing the managers' myopic preferences over a fixed threshold, both managers can be incentivized to disclose their private information in equilibrium. As a byproduct, an increase in myopic preferences also reduces the speculators' payoff uncertainty and the cost of capital.

In Section 4, I study an extension of the baseline model and endogenize the scale of the firms' assets in place. In particular, I allow the firms to chose this scale ($\overline{X}_j$) depending on the firms' expected cost of capital. As a result, the two firms grow in expectation of decreasing capital cost and vice versa. This extension is useful because it highlights a potential inefficiency associated with the no-disclosure equilibrium. If a higher cost of capital translates into less installed capital, the non-disclosure equilibrium might be inefficient ex ante. Intuitively, the firm managers might not be able to commit to full-disclosure if the issuance decisions is made a priori. As a consequence, the speculators' payoff uncertainty (and so the firms' cost of capital) remains inefficiently high which leads to a decrease in the scale of the firms' assets in place (and their ex ante value). As a straightforward implication, it follows that both firms can be made better off under mandatory disclosure. Intuitively, the informational content of the asset prices remains the same, but the firms' cost of capital decreases which raises the scale of the firms' assets in place. Moreover, the analysis in Section 3 shows that myopic incentives for the two managers can implement the full disclosure equilibrium. Therefore, the extended model implies that it is *efficient* to give both managers sufficiently strong myopic incentives, e.g. through rewards for increases in the short-run asset price.

This paper is related to three literatures. First, the accounting (and finance) literature on the real effects of disclosing accounting information (reviewed in Kanodia (2007)). Second, the finance literature on the real effects of financial markets (reviewed in Bond et al. (2012)). Third, the economics literature on limited attention and endogenous information acquisition (reviewed in Veldkamp (2011)).

There is a substantial literature in accounting and finance on the real effects of disclosing information. Recent contributions are Gao (2008), Gao (2010) and Cheynel (2013). These papers study the impact of corporate disclosure on the firm's cost of capital and price efficiency.

One main difference with respect to this literature is the assumption that financial markets have spillover ("feedback") effects to the real economy. This feedback effect is modeled through the informational role of the two asset prices, as e.g. in Subrahmanyam and Titman (1999) and Goldstein et al. (2013). Other papers in the feedback literature that discuss firms' optimal disclosure are Goldstein and Yang (2015), Kurlat and Veldkamp (2015) and Edmans et al. (2016). Gao and Liang (2013) also discuss a feedback model with optimal disclosure. In their paper, disclosure crowds out private information production, reduces price informativeness and harms managerial learning. Thus, my results are complementary to their findings because in my paper information disclosure affects price efficiency through a different channel, namely the speculators' attention allocation problem. I model the speculators' attention allocation decisions as in Kacperczyk et al. (2016) or van Nieuwerburgh and Veldkamp (2010) and add an initial stage in which firm managers optimally choose how much information to disclose to the financial market. Importantly, this disclosure decision is highly entangled with the speculators attention allocation problem because *more* disclosure reduces the learning score (i.e. the firm's "attractiveness") and vice versa.

Closely related is an early paper by Fishman and Hagerty (1989). Their paper assumes that it is costly for traders to process the disclosed information. Therefore, firms compete for traders' attention over the disclosed signal because traders have to choose which signal to pay attention to. Even though this mechanism might look similar to the one in my paper, it is quite different. In my paper, the disclosed public signal does not require any attention from the speculators. It does, however, have an important impact on the speculators' information acquisition decision regarding their *private* information. In particular, in my model the lack of disclosure is a useful tool to render a firm a more "attractive" target for speculators. As a result, the equilibrium in the two models are also fundamentally different. While in Fishman and Hagerty (1989), both firms have an incentive to disclose in equilibrium, I show that under limited attention *withholding* information (or disclosing as little information as possible) is the natural equilibrium outcome.

Overall, the main contribution of this paper is to study equilibrium (and efficient) disclosure in a setting with feedback effects and limited attention. In particular, I show that these two additional features have important implications for the disclosure equilibrium and the efficient allocation. Moreover, they also change the real effects of disclosure more generally and highlight novel advantages of myopic incentives in managerial contracts. For instance, I show that under certain conditions the efficient contract includes a myopic component that makes sure that the firm manager chooses to disclose. Furthermore, I derive novel implications for the relationship between disclosure and the firm's cost of capital by showing that disclosure might *reduce* this cost if it is accompanied by a loss in attention.

The remainder of this paper is organized as follows: Section 2 describes the basic model. Section 3 solves for the firms' optimal disclosure decisions in equilibrium and shows the implications for cost of capital and price efficiency. Section 4 studies an extended model with endogenously determined assets in place and Section 5 concludes.

2 Basic Model

In this section, I develop a model to understand the incentives of firm managers to disclose payoff-relevant information to the financial market. Importantly, the financial market is populated by speculators who have to decide how much attention they want to pay to each of the two firms. This attention choice then determines the precision of their private signal and, in turn, the informational content of each firm's price. The model builds on the n-asset attention allocation model in Kacperczyk et al. (2016) and adds an initial stage in which the two firm managers can decide whether to disclose their private information about their firm's shock (the "fundamental").[2]

2.1 Setup

The model has four periods, indexed by $t \in \{1, \ldots, 4\}$. At $t = 1$, firm managers make their disclosure decision and issue a (potentially uninformative) public signal. At $t = 2$, traders choose how to allocate their scarce attention between the two firms. This decision determines the informational content of both prices (realized in the following period). At $t = 3$, trading takes place and at $t = 4$, the managers' investment decisions determine the final firm values and the asset payoffs are realized. Importantly, the managers' investment decisions depend, in part, on the realized prices which aggregate the speculators' private information. This dependence represents the "feedback effect" in this paper. Figure 1 summarizes the sequence of events.

[Insert Figure 1 here]

There are two types of agents: (i) a continuum of informed traders ("speculators") and (ii) two firm managers. Each manager runs a single firm $j \in \{A, B\}$ and makes two choices: how much to invest in a growth opportunity (at $t = 4$) and whether to disclose his private information to the financial market (at $t = 1$).

At $t = 4$, each manager invests an endogenous amount K_j in a growth opportunity at a private cost $\frac{1}{2}K_j^2$. Together with the random return on assets in place (in constant supply $\overline{X} > 0$), these two

[2]Note that I analyze the simplest case of two firms ($n = 2$) to focus on the economic mechanism. The main results of the paper are, however, applicable to a setting with more than two firms ($n > 2$).

components determine the firm's future value V_j:

$$V_j = (\mu_z + z_j)\overline{X} + (\mu_z + z_j)K_j - \frac{1}{2}K_j^2. \tag{1}$$

For simplicity, I assume that the return on the growth opportunity is perfectly correlated with that on the assets in place which resembles the functional form in Bai et al. (2016).[3] In particular, $\mu_z > 0$ denotes the constant, expected return while $z_j \sim N\left(0, \pi_z^{-1}\right)$ represents the (partially predictable) random component of this return.

Each manager acts benevolently and chooses capital investment to maximize the expected firm value:

$$\max_{K_j} E\left[V_j | \mathcal{I}_{j,4}^M\right] \tag{2}$$

where $\mathcal{I}_{j,4}^M$ denotes manager j's information set at $t = 4$ (specified in detail in Section 2.2).

The model features one riskless asset (with a normalized net return of zero) and two risky assets. As in Subrahmanyam and Titman (1999), Foucault and Gehrig (2008) and Gao and Liang (2013) risky assets are claims to each firm's assets in place, $(\mu_z + z_j)$, rather than the sum of assets in place and the growth opportunity. This assumption substantially simplifies the analysis and implies that the investment decision is affected by the security price, but the security price does not depend on this decision.[4] If the asset was also a claim to the growth opportunity, the asset payoff would no longer be normally distributed and the signal extraction problem becomes intractable.[5] Each asset is in noisy supply, $\overline{X} - x_j$, where $x_j \sim N\left(0, \pi_x^{-1}\right)$ is independent across firms. As in most noisy rational expectations models, this random component is necessary to prevent the two equilibrium prices from fully revealing the speculators' private information.

There is a continuum of informed speculators indexed by $i \in [0, 1]$. Each speculator is endowed with initial wealth W_0. They have mean-variance preferences over terminal wealth W_i, with risk aversion coefficient $\rho > 0$. Therefore, speculator i chooses his holdings in both assets $\{q_{iA}, q_{iB}\}$ to

[3] I could add a random, unpredictable component $\tilde{z}_j$ to either return without changing the main results of the paper.

[4] Importantly, the managers are able to learn information about the return on the growth opportunity from the asset price because the two returns are (perfectly) correlated.

[5] Other learning models use other assumptions to circumvent this intractability: e.g. Goldstein et al. (2013) assume that the payoff is net of the quadratic investment cost and Leland (1992) assumes that the returns from investment go entirely to new shareholders, not existing ones.

maximize

$$U_{3i} = \rho E[W_i|\mathcal{I}_{i,3}^S] - \frac{\rho^2}{2} V[W_i|\mathcal{I}_{i,3}^S] \tag{3}$$

where $E[\cdot]$ and $V[\cdot]$ denote the speculator's conditional expectation and variance, respectively. Each speculator's budget constraint is given by: $W_i = W_0 + q_{iA}(\mu_z + z_A - p_A) + q_{iB}(\mu_z + z_B - p_B)$, where $\{p_A, p_B\}$ denote the risky assets' equilibrium prices.

These two prices are determined by market clearing:

$$\int_0^1 q_{ij} di = \overline{X} - x_j \tag{4}$$

where $j \in \{A, B\}$. The left hand side represents the speculators' aggregate demand for asset j, while the right hand side represents the noisy supply.

At $t = 2$, speculators have to allocate their limited attention between the two risky assets. In particular, they choose the precision of the two private signals about z_j, the asset's payoff: $s_{ij} = z_j + \varepsilon_{ij}$ where $\varepsilon_{ij} \sim N\left(0, \pi_{\varepsilon,ij}^{-1}\right)$. Speculators choose the precisions of these two signals to maximize their $t = 2$ expected utility:

$$U_{2i} = E\left[\rho E\left[W_i|\mathcal{I}_{i,3}^S\right] - \frac{\rho^2}{2} V\left[W_i|\mathcal{I}_{i,3}^S\right] |\mathcal{I}_{i,2}^S\right] \tag{5}$$

subject to the following constraints. First, each speculator's budget constraint holds. Second, $\pi_{\varepsilon,iA} + \pi_{\varepsilon,iB} \leq \widehat{\pi_\varepsilon}$, i.e. the sum of both precisions cannot exceed the common capacity constraint. Thirds, the chosen precisions are non-negative, i.e. $\pi_{\varepsilon,iA}, \pi_{\varepsilon,iB} \geq 0 \; \forall i \in [0, 1]$. Thus, the basic attention allocation problem resembles that in Kacperczyk et al. (2016). However, the setup differs in one important dimension: the speculators' attention allocation decision is affected by the precision of the firms' disclosed public signals (made in the previous period, $t = 1$).

At $t = 1$, both managers receive a private signal about the return on their firm's growth opportunity, z_j: $y_j = z_j + \xi_j$ where $\xi_j \sim N\left(0, \pi_y^{-1}\right)$. Managers can choose to reveal this signal to the financial market at no cost, i.e. they can choose to disclose ($D_j = 1$) or choose not to disclose ($D_j = 0$).[6] Each manager makes his disclosure decision to maximize the $t = 1$ expected firm value:

$$\max_{D_j \in \{0,1\}} E\left[V_j|\mathcal{I}_{j,1}^M\right]. \tag{6}$$

[6]One could, of course, also allow the managers to partially reveal their signal, i.e. to release a public signal of the form $\tilde{y}_j = y_j + u_j$.

Of course, the two managers' take the impact of their disclosure decision on all subsequent actions into account. In particular, they understand that their disclosure decision influences the speculators' optimal attention allocation decision (at $t = 2$) and their trading behavior (at $t = 3$).

If manager j chooses to disclose, all speculators have access to an additional informative signal (y_j) and can use this information to improve their trading decisions. Importantly, the disclosure decision also changes each speculator's payoff uncertainty and thus, the incentive to pay attention to the respective firm. In particular, the managers understand this dependence and have an incentive to choose D_j in such a way that speculators allocate attention away from the other firm and towards their own firm. Intuitively, more attention for a given firm leads to more precise private signals about this firm and so to a more informative asset price. As a consequence, the firm manager can learn more additional (outside) information from the price and invest more efficiently in the growth opportunity.[7]

2.2 Information Sets

At $t = 4$, when firm managers make their investment decision, they can condition on their asset price and their private signal about z_j: $\mathcal{I}^M_{j,4} = \{p_j, y_j\}$.[8] At $t = 3$, when speculators trade, they have access to their private signals, both asset prices, and up to two public signals (depending on the firms' disclosure decisions): $\mathcal{I}^S_{i,3} = \{s_{iA}, s_{iB}, p_A, p_B, D_A \times y_A, D_B \times y_B\}$. At $t = 2$, when speculator allocate their attention between both firms, they can only condition on the public signals (if firms choose to disclose): $\mathcal{I}^S_{i,2} = \{D_A \times y_A, D_B \times y_B\}$. At $t = 1$, when the managers decide whether to disclose their private information, their information set just contains their private signal: $\mathcal{I}^M_{j,1} = \{y_j\}$.

2.3 Solution

I solve the model backwards: I start with the firms' optimal investment decision (at $t = 4$) and the financial market equilibrium (at $t = 3$). Then, I characterize the speculators equilibrium attention allocation (at $t = 2$) and the two firms' optimal disclosure decisions (at $t = 1$). The

[7]Note, that speculators do not have to pay attention to both types of public signals (y_j and p_j) in order to process this type of information. This convention follows the approach in Kacperczyk et al. (2016). See Fishman and Hagerty (1989) for an alternative approach.

[8]Due to the independence of both firms, the other firm's price and the potentially disclosed signal are not useful for predicting z_j.

solution method is therefore similar to that employed in Kacperczyk et al. (2016). The two crucial additional features are the investment stage ($t = 4$) and the disclosure stage ($t = 1$).

2.3.1 Financial Market Equilibrium and Equilibrium Investment

I first guess that each asset's price is linear in the three random variables that can potentially affect equilibrium demand and supply. The true payoff (which is revealed by the average private signal of all speculators, $z_j = \int_0^1 s_{ij} di$), the potentially disclosed private signal of the manager, and the random supply shock, $\{z_j, y_j, x_j\}$:

$$p_j = \alpha_{0,j} + \alpha_{1,j} z_j + \alpha_{2,j} y_j + \alpha_{3,j} x_j. \tag{7}$$

Naturally, the α coefficients are functions of the two decisions that are made prior to trading, i.e. the speculators' collective attention allocation decision and the firms' disclosure decision. In particular, we know that if firm j chooses not to disclose ($D_j = 0$), then $\alpha_{2,j} = 0$ and the firm manager's private signal (y_j) cannot affect the equilibrium price.

From equation (7) it follows that each speculator can infer additional information from the unbiased price signal $\eta_{p,j} \equiv \frac{p_j - \alpha_{0,j} - \alpha_{2,j} y_j}{\alpha_{1,j}} = z_j + \frac{\alpha_{3,j}}{\alpha_{1,j}} x_j$ at $t = 3$. This is an endogenous signal about the asset's payoff (z_j) that is clouded by the stochastic supply shock (x_j). The signal to noise ratio $\left(\frac{\alpha_{1,j}}{\alpha_{3,j}}\right)$ determines the precision of this signal which is given by $\pi_{p,j} = \left(\frac{\alpha_{1,j}}{\alpha_{3,j}}\right)^2 \pi_x$. Thus, each speculator optimally uses this signal together with the private and public signal (if $D_j = 1$) to trade more efficiently. As a result, the optimal asset demands q_{ij} are functions of the vector $\{s_{ij}, D_j \times y_j, \eta_{p,j}\}$.

Similarly, both managers can use the price signal $\eta_{p,j}$ (in addition to their private signal y_j) to improve the efficiency of their investment in the growth opportunity. In particular, equation (1), implies that firm j's equilibrium investment is given by $K_j = \mu_z + E[z_j | \mathcal{I}_{j,4}^M]$, i.e. the manager's conditional expectation of the return on the growth opportunity. Due to the normality of the learning problem, this conditional expectation is linear in the available signals y_j and $\eta_{p,j}$ (or alternatively p_j). The following Lemma formalizes these results.

Lemma 1 *There exists a unique linear financial market equilibrium. The asset price for firm* $j \in \{A, B\}$ *is given by:*

$$p_j = \alpha_{0,j} + \alpha_{1,j} z_j + \alpha_{2,j} y_j + \alpha_{3,j} x_j. \tag{8}$$

The equilibrium investment decisions are given by:

$$K_j = \beta_{0,j} + \beta_{1,j} y_j + \beta_{2,j} p_j. \tag{9}$$

All coefficients are given in the Appendix (as functions of the firms' disclosure and the speculators' attention allocation decisions).

Proof: *See Appendix A.1.1.*

The equilibrium asset prices and investment decisions in Lemma 1 are similar to those in the existing literature, like e.g. Subrahmanyam and Titman (1999). Note, however, that the speculators' collective attention allocation decision has a subtle impact on each firm's value (V_j) through its impact on real investment (K_j). For instance, if all speculators pay more attention to the signal about firm A, then this firm's price (p_A) becomes more informative about the future payoff (z_A). Therefore, we expect the manager's weight on the price signal ($\beta_{2,A}$) to increase. In particular, the Appendix shows that each asset's price informativeness is equal to $\pi_{p,j} = \left(\frac{\pi_{\varepsilon,j}}{\rho}\right)^2 \pi_x$. Thus, the informational content of each firm's price increase if all speculators pay more attention to this firm and increase the average precision of the private signal about it ($\pi_{\varepsilon,j}$). Moreover, this increase in attention should also increase the firm's (ex ante) value because its manager is able to invest more efficiently, i.e. increase K_j when z_j is high (on average) and vice versa.

2.3.2 Attention Allocation Equilibrium

Next, I solve for the speculators' optimal attention allocation between the two firms. At $t = 2$, each speculator has to decide how much of his limited capacity he wants to allocate to each asset. This decision, in turn, determines the precision of his private signal about the firms' future payoff. Given the equilibrium prices in Lemma 1, I can compute the expected utility of speculator i at $t = 2$

as $U_{2i} \equiv E\left[U_{3i}|\mathcal{I}_{i,2}^S\right]$ which can be simplified to

$$U_{2i} = W_0 + \sum_j \frac{V\left[E[z_j - p_j|\mathcal{I}_{i,3}^S]|\mathcal{I}_{i,2}^S\right] + E\left[\left(E[z_j - p_j|\mathcal{I}_{i,3}^S]\right)^2|\mathcal{I}_{i,2}^S\right]}{2V\left[z_j|\mathcal{I}_{i,3}^S\right]}$$

as in Kacperczyk et al. (2016).

Each speculator then chooses the precisions of both private signals ($\{\pi_{\varepsilon,iA}, \pi_{\varepsilon,iB}\}$) to maximize this objective function. In the Appendix, I show that this optimization problem can be rewritten as $\max_{\pi_{\varepsilon,iA},\pi_{\varepsilon,iB}} \sum_j \lambda_j \pi_{\varepsilon,ij}$ + constant as in Kacperczyk et al. (2016). Therefore, each speculator optimally assigns all capacity to one firm if $\lambda_A \neq \lambda_B$ or 50% of the capacity to both otherwise. This result is formalized in Lemma 2.

Lemma 2 *Each speculator optimally assigns all capacity to the firm with the higher learning score (λ_j), defined in Appendix A.1.2. If $\lambda_A = \lambda_B$, then speculators are indifferent and allocate half of their capacity to each firm, $\pi_{\varepsilon,ij} = \frac{1}{2}\widehat{\pi_\varepsilon}$.*
Proof: *See Appendix A.1.2.*

Note that firm j's learning score (λ_j) depends on the parameters associated with the respective firm, see Figure 2. Most importantly however, the score also depends on the firm's disclosure decision (D_j). The next section shows that each firm can increase its learning score (or "attractiveness") by *not disclosing* private information to the speculators. Technically, $\lambda_j(D_j = 0) > \lambda_j(D_j = 1)$ which mirrors the finding in Kacperczyk et al. (2016) that speculators pay more attention to shocks with a higher prior uncertainty. Thus, withholding information about a certain shock has a similar effect as increasing its prior variance. The underlying economic intuition for both results is that in this setup (with mean-variance preferences) a marginal unit of information about a certain shock is more valuable when the prior uncertainty about it is higher.

[Insert Figure 2 here]

Figure 2 plots this learning score (λ_j) against several model parameters. As in Kacperczyk et al. (2016) a certain firm (or risk factor) becomes less attractive for higher values of π_ε (average precision) or π_z (prior uncertainty). Similarly, assets are particularly attractive targets if they are in large supply ($\overline{X}$).

3 Fight for Attention through Optimal (Non-)Disclosure

In this section, I solve for the firm managers' optimal disclosure policy at the initial stage ($t = 1$). As a first step, however, it is useful to discuss a natural benchmark economy in which speculators do not rationally allocate their attention between the two firms. This could be because speculators are informationally constrained in the short run and so it requires time to shift attention towards another signal. In this benchmark economy, firm managers cannot change the precision of the speculators' private signal through their disclosure decision. As a consequence, the managers are not able to influence the informational content of their price and so the ex ante value of their firm with their disclosure policy. Intuitively, the firms' ex ante value only depends on the efficiency of their investment in the growth opportunity, which is not directly affected by disclosure. Thus, both firm managers are indifferent between disclosure ($D_j = 1$) or non-disclosure ($D_j = 0$) in the benchmark economy.

In the setup with attention allocation, however, firm managers can actively influence the speculators' behavior with their decision to disclose their private information at $t = 1$. Intuitively, a firm's disclosure of information changes each speculator's uncertainty about the asset's final payoff. As a result, it also affects their decision to acquire information about this source of uncertainty by paying attention to a signal about a particular shock (z_A or z_B). The following Lemma formalizes this motive of both firm managers to "fight" for the speculators' scarce attention.

Lemma 3 *Each firm's expected value (at $t = 1$) strictly increases in the speculators' average attention ($\pi_{\varepsilon,j} = \int_0^1 \pi_{\varepsilon,ij} di$) and the learning score (λ_j) is higher if the firm chooses not to disclose ($D_j = 0$).*
Proof: *See Appendix A.1.3.*

Lemma 3 shows two important results. First, each firm's ex ante expected value is higher if speculators pay more attention to this firm. This is intuitive because more collective attention leads to more outside information in the firm's asset price. As a result, firm j's manager is able to infer more information from the financial market and this increased knowledge about the return on the growth opportunity allows the manager to invest more efficiently. Thus, less resources are wasted (due to the quadratic investment cost) because the manager can, on average, invest more when the return is high and vice versa. Second, firm managers can render their firm a more "attractive" target to speculators by choosing not to disclose their private information to the financial market. Technically, the learning score λ_j is higher if firm j chooses not to withhold information because a marginal unit of private information is more useful for speculators if their ex ante uncertainty about the respective shock is higher.

Taken together, Lemma 3 implies that firm managers have an incentive to compete for the speculators' scarce attention by choosing not to disclose their private information. The lemma also confirms that both components are crucial for this result: the speculators' attention allocation and the feedback effect from prices to investment. Proposition 1 formalizes this insight and represents the first major result of the paper.

Proposition 1 *In equilibrium, both firm managers' choose to withhold their private information, i.e. they choose $D_j = 0$.*

Proof: *See Appendix A.1.4.*

Proposition 1 shows that firm managers fight for the speculators' scarce attention by choosing not to disclose their private information. Through this choice firms appear riskier from the speculators' perspective and so a marginal unit of attention (and thus more precise private information) is worth more to them. In essence, the two firms perform a "race to the bottom" in terms of their disclosure decision: by the symmetric nature of the setup, both firms will always attract an equal amount of attention in equilibrium. Importantly, this amount is always equal to half of the overall capacity that speculators can allocate, no matter how much information the firms disclose. However, each firm has an incentive to slightly lower the precision of the disclosed signal because this deviation allows the firm to attract the entire attention of all speculators (because $\lambda_j > \lambda_{-j}$ in this case). This intuition also implies that it is not crucial to restrict the firm managers to have a binary choice between full disclosure and non-disclosure. Even if the managers were allowed to send out noisy signals about their private information, the described race to the bottom will always make sure that both firms disclose as little information as possible.

3.1 Cost of Capital and Price Efficiency

In this section, I discuss the implications of the "fight for attention" described above for the firms' cost of capital and price efficiency.

From the results in Lemma 1 it follows that the cost of capital for firm j is proportional to the conditional variance, i.e.: $E_1[\mu_z + z_j - p_j] \propto \widehat{\sigma}_j\left(D_j, \pi_{\varepsilon,j}\right)$. Here $\widehat{\sigma}$ represents the average conditional variance about the payoff given the three signals $\{s_{ij}, D_j \times y_j, p_j\}$, i.e. the private signal, the possibly disclosed public signal, and the price signal. Clearly, this variance (and so the firm's cost of capital) depends on its disclosure decision (D_j) and the average precision of the private signal ($\pi_{\varepsilon,j}$), which in turn depends on *both* firms' disclosure decisions. Intuitively, even the disclosure decision of the other firm affects the average precision about firm j because the speculators rationally allocate their scarce attention across firms. For example, we know from the discussion above that if firm A were to choose to disclose ($D_A = 1$), but firm B chooses not to disclose ($D_B = 0$), then all speculators would pay attention solely to the shock about firm B. As a consequence, firm A's price would not reflect any outside information because the speculators do not pay any attention to this firm such that their private signal about its shock has zero precision ($\pi_{\varepsilon,A} = 0$).

Next, I analyze the implications of the firm's disclosure decision on the firms' conditional variance and cost of capital, in more detail. In particular, I start with the equilibrium outcome of non-disclosure ($D_A = D_B = 0$) and analyze the impact of the off-equilibrium outcomes of unilateral or bilateral disclosure. This analysis is interesting because it helps to understand how firm managers can be incentivized to move to the disclosure equilibrium. Moreover, it highlights potential costs of the here described race to the bottom.

Figure 3 plots firm j's conditional variance against the fixed capacity of attention ($\overline{\pi}_\varepsilon$) for four different scenarios: (i) the equilibrium outcome of non-disclosure for both firms, (ii) mandatory disclosure for both firms, (iii) mandatory disclosure for the other firm, and (iv) mandatory disclosure for firm j. The plot shows that firm j's cost of capital (or similarly its conditional variance) is *lowest* for scenarios (ii) and (iii). This result reflects the inherent trade-off associated with disclosure. On the one hand, disclosing information reduces the speculators' uncertainty about the future payoff which reduces the firm's cost of capital. On the other hand, unilateral disclosure also implies that speculators allocate their entire attention to the other firm which reduces price efficiency (and thus increases uncertainty) about firm j. Figure 3 confirms that the second effect is particularly costly if speculators have to allocate a large capacity of attention, i.e. for high values of $\overline{\pi}_{\varepsilon}$. Intuitively, firms face a potentially larger loss in attention if this maximum capacity is higher. Vice versa, firm j's cost of capital is *highest* under scenarios (i) and (iv), i.e. either if both firms choose not to disclose or if firm j discloses unilaterally. This analysis thus challenges the conventional wisdom that firm disclosure necessarily reduces a firm's cost of capital. The model highlights that disclosure might as well *increase* a firm's cost of capital if it leads to a loss in investor attention (compare the orange dashed line and the solid blue line in Figure 3).

[Insert Figure 3 here]

Next, I discuss the implications of firm j's disclosure policy on the informational content of its stock price ($\pi_{p,j}$). In particular, $\pi_{p,j}$ measures the reduction in prior variance about z_j that is achieved by learning from the endogenous price signal $\eta_{p,j}$ defined above.

Figure 4 plots this measure of price efficiency against the speculators' attention capacity. The plot emphasizes the firms' motive for withholding their private information by choosing $D_j = 0$. Starting from the symmetric case of full disclosure by both firms (solid blue line), firm j can attract more attention by choosing non-disclosure ($D_j = 0$) and increase the informational content of its price (dotted orange line). Of course, in equilibrium firm j's competitor acts the same way and also chooses not to disclose information, which brings both firms back to the initial level of price informativeness. Therefore, the firms' fight for attention through non-disclosure does not change the amount of outside information in their prices. It should be noted, however, that the non-disclosure equilibrium always features a higher cost of capital for both firms (see Figure 3).

[Insert Figure 4 here]

This analysis, thus, emphasizes an important difference between the *overall* amount of information in the price and the amount of *outside* information. The first measure determines the firms' cost of capital, which is heavily affected by the firms' race to the bottom (see Figure 3). The second measure determines the amount of outside information in the firms' prices and is identical in both symmetric equilibria ($D_j = 0$ and $D_j = 1$). Bond et al. (2012) terms the first measure of price efficiency *FPE* (forecasting price efficiency) and the second measure *RPE* (revelatory price efficiency). Edmans et al. (2016) show that the latter matters for firms' investment decisions because it determines the amount of outside information, i.e. information not already known by the real decision maker (manager).

3.2 Managerial Incentives

In this section, I analyze the impact of managerial incentives (through their compensation package) on the optimal disclosure decision. In particular, I assume that now firm j's manager no longer only maximizes the expected firm value, but chooses firm disclosure to maximize a sum of long and short-run incentives:

$$\max_{D_j \in \{0,1\}} E\left[V_j + \omega_j p_j | \mathcal{I}_{j,1}^M\right] \tag{10}$$

where $\omega_j \geq 0$ measures the importance of the firm's short-run value in the manager's incentive package as e.g. in Edmans et al. (2016). For simplicity, I consider the symmetric case, where $\omega_A = \omega_B = \omega$.

First, note that the firm's investment decision is still given by the expression in Lemma 1. Intuitively, both managers take the asset prices (p_j) as given when they choose investment in the growth opportunity (K_j). Similarly, the equilibrium asset prices are also still given by the expressions in Lemma 1. However, the alternative contract above can have an impact on the α coefficients in the pricing rule through the implicit dependence of the coefficients on equilibrium disclosure and attention allocation. The speculators' attention allocation problem still obeys the rule in Lemma 2 but can of course be affected if the alternative incentives change the firms' disclosure decisions.

In contrast to the benchmark setup with a benevolent manager, the compensation package in equation (10) also includes the short-run asset price p_j. Importantly, this price is closely related to the firm's cost of capital ($E_0[\mu_z + z_j - p]$) which, in turn, is negatively affected by the firms' fight for attention (see the discussion in the previous section and Figure 3). The appendix shows that the managers' objective function (U_{1j}^M) at $t = 1$ under the alternative contract can be written as:

$$U_{1j}^M = \text{const} + \frac{1}{2}\widehat{\sigma}_j^M + \omega\rho\overline{X}\widehat{\sigma}_j^S.$$

Thus, the managers' $t = 1$ objective function depends on two conditional variances $\widehat{\sigma}_j^M$ and $\widehat{\sigma}_j^S$. The first conditional variance represents the manager's conditional variance at $t = 3$ and determines investment efficiency at $t = 4$. Importantly, this variance depends on both firms' disclosure decisions because it depends on the amount of information revealed through firm j's price. The second term corresponds to the speculators' conditional variance at $t = 3$ and determines the firms' cost of capital. Interestingly, this term is affected by the firms' disclosure decision in two ways. First, firm disclosure *directly* affects this variance because it leads to more information for all speculators. Second, it also affects this variance *indirectly* by changing the speculators' attention allocation decision and thus the informational content of the firms' prices.

Proposition 2 *There exists a threshold $\overline{\omega} > 0$, such that both firms choose to disclose in equilibrium ($D_A = D_B = 1$) if $\omega \geq \overline{\omega}$ given that the overall attention capacity is sufficiently high ($\overline{\pi}_\varepsilon > 2\pi_y$).*
Proof: *See Appendix A.1.5.*

Proposition 2 shows that myopic incentives for the two managers can implement a disclosure equilibrium. Intuitively, myopic incentives incentivize the managers to take their effect on the firms' cost of capital into account as well. Therefore, the managers internalize that by choosing no disclosure they reduce the *total* amount of information for the speculators which leads to an increase in the cost of capital. This increase, in turn, reduces to expected price level and so each manager's objective function (U_{1j}^M) if $\omega > 0$. Therefore, the alternative contract not only focusses on the amount of *novel* information in prices (RPE), but also the amount of *total* information (FPE).

Corollary 1 *Increasing both manager's myopic incentives (ω) weakly reduces the speculators' payoff uncertainty and the firms' cost of capital if the overall attention capacity is sufficiently high ($\overline{\pi}_\varepsilon > 2\pi_y$).*

Proof: *See Appendix A.1.6.*

Corollary 1 connects the managers' compensation contract to the firms' cost of capital and the speculators' uncertainty about the future payoff. As a result of Proposition 2, increases in the managers' myopic incentives (ω) give them a motive to also care about the firm's cost of capital when making their disclosure decision. Thus, increasing ω can lead to a change in the disclosure equilibrium (from $D_j = 0$ to $D_j = 1$ if $\omega \geq \overline{\omega}$) and so to a reduction in the firms' cost of capital (as shown in Figure 3 and the previous section).

4 Extension: Inefficient Fight for Attention

So far, the managers' disclosure decisions had no impact on economic efficiency as measured by the firms' ex ante expected value $E_0[V_j]$. Intuitively, as long as both firms make the same disclosure decisions, the overall attention capacity ($\overline{\pi}_\varepsilon$) is equally split between both firms, such that the amount of outside information is identical under both policies (see Figure 4). In this section, I extend the baseline model in one dimension: I allow the firms to endogenously determine the initial scale of their assets in place ($\overline{X}$). In particular, each firm chooses this quantity at $t = 0$ to maximize the revenue from selling $\overline{X}_j$ at price p_j, similar to the extension in Farboodi and Veldkamp (2017):

$$\max_{\overline{X}_j} E_0 \left[\overline{X}_j p_j - c\left(\overline{X}_j\right) \right]$$

here $c(\cdot)$ denotes the (private) issuance cost and to keep the model tractable, I assume a simple quadratic form, $c\left(\overline{X}_j\right) = \frac{c}{2}\overline{X}_j^2$. It follows that the scale of assets in place for each firm is given by: $\overline{X}_j = \frac{1}{c}E_0[p_j]$. Intuitively, the firms optimally choose to scale up their assets in place if the expected cost of capital ($\mu_z - E_0[p_j]$) and the issuance cost (c) is low.

It follows that all decisions following the issuance decision remain unchanged. In particular, the expressions in Lemma 1 and Lemma 2 are the same (with $\overline{X}_j$ replacing $\overline{X}$). The endogenous choice of $\overline{X}$ does, however, strongly affect the *level* of prices (on average) and, in particular, the expected firm value.

Proposition 3 *In the extended model with endogenous assets in place, the non-disclosure equilibrium is inefficient as long as c is finite.*

Proof: *See Appendix A.1.7.*

Proposition 3 shows that there is an efficiency gain from disclosure in the extended model. If both firms choose to reveal their private information to the financial market, speculators face lower uncertainty about the final payoff which, in turn, reduces the firms' cost of capital. A lower cost of capital allows the firms to invest more in their assets in place such that $\overline{X}_j$ increases for both firms. Consequently, the expected value of the firms increases which benefits both managers. Corollary 2 formalizes this result.

Corollary 2 *In the extended model, both firms can be made better off under mandatory disclosure and myopic incentives ($\omega \geq \omega^*$).*

Proof: *See Appendix A.1.8.*

Corollary 2 emphasizes that both firms can be made better off by forcing their managers to disclose their private information. As shown before, one tool to achieve this disclosure equilibrium is through myopic incentives in the managers' compensation contract. Thus, if the weight on the short-run stock price is sufficiently high ($\omega \geq \omega^*$), the first-best outcome with full disclosure can be implemented in equilibrium.

[Insert Figure 5 here]

Figure 5 plots the threshold value for ω against the issuance cost parameter c. The figure shows that the managers' have to be endowed with more myopic incentives for *lower* values of c. Intuitively, in these cases the efficiency loss from the benchmark equilibrium without disclosure is particularly high.

5 Conclusion

Corporate disclosure can have important consequences on speculators' (limited) attention and the informational content of asset prices. The baseline model shows that withholding information renders firms a more attractive target for speculators, which increases the informational content of the asset price. Therefore, if firm managers can learn some additional information from the financial market, they have an incentive to disclose less information than the other firms. In a setting with multiple firms competing for scarce attention, I show that this behavior leads to a race to the bottom. In equilibrium, all firms choose to withhold their private information.

In a setting with an endogenous amount of assets in place, this fight for attention is inefficient. Thus, mandatory disclosure or an increase in the managers' myopic incentives can be welfare improving. The latter gives the managers an incentive to take the effect on the firms' cost of capital into account which implies that the manager not only cares about the amount of outside information but also the amount of total information in prices.

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A Appendix

A.1 Proofs

A.1.1 Proof of Lemma 1

I first compute each speculator's demand for asset j as:

$$q_{ij} = \frac{\mu_z + E\left[z_j | \mathcal{I}_{i,3}^S\right] - p_j}{\rho V\left[z_j | \mathcal{I}_{i,3}^S\right]}$$

which follows from the standard mean-variance objective function. The two conditional moments $\widehat{\mu}_{ij}(D_j, \pi_{\varepsilon,ij}) \equiv E\left[z_j | \mathcal{I}_{i,3}^S\right]$ and $\widehat{\sigma}_{ij}(D_j, \pi_{\varepsilon,ij}) \equiv V\left[z_j | \mathcal{I}_{i,3}^S\right]$ naturally depend on the firm's disclosure decision and the speculators' attention allocation decision. The exact expressions follow from simple Bayesian updating:

$$\begin{aligned}
\widehat{\mu}_{ij}(D_j, \pi_{\varepsilon,ij}) &= D_j \times \left(\frac{\pi_{\varepsilon,ij} s_{ij} + \pi_y y_j + \pi_{p,j} \eta_{p,j}}{\pi_z + \pi_{\varepsilon,ij} + \pi_y + \pi_{p,j}}\right) + (1 - D_j)\left(\frac{\pi_{\varepsilon,ij} s_{ij} + \pi_{p,j} \eta_{p,j}}{\pi_z + \pi_{\varepsilon,ij} + \pi_{p,j}}\right) \\
\widehat{\sigma}_{ij}(D_j, \pi_{\varepsilon,ij}) &= \frac{D_j}{\pi_z + \pi_{\varepsilon,ij} + \pi_y + \pi_{p,j}} + \frac{1 - D_j}{\pi_z + \pi_{\varepsilon,ij} + \pi_{p,j}}
\end{aligned}$$

Next, I solve the market clearing condition for each asset $\int_0^1 q_{ij} = \overline{X} - x_j$ for the asset price p_j:

$$p_j = \mu_z - \rho \widehat{\sigma}_j \overline{X} + \int_0^1 \widehat{\mu}_{ij} di + \rho \widehat{\sigma}_j x_j$$

where $\widehat{\sigma}_j$ denotes the average conditional variance across speculators. Plugging in the expressions for the conditional moments gives:

$$p_j = \alpha_{0,j} + \alpha_{1,j} z_j + \alpha_{2,j} y_j + \alpha_{3,j} x_j$$

with

$$\begin{aligned}
\alpha_{0,j} &= \mu_z - \frac{D_j \times \rho \overline{X}}{\pi_z + \pi_{\varepsilon,j} + \pi_y + \pi_{p,j}} - \frac{(1 - D_j) \times \rho \overline{X}}{\pi_z + \pi_{\varepsilon,j} + \pi_{p,j}} \\
\alpha_{1,j} &= \frac{D_j \times \left(\pi_{\varepsilon,j} + \pi_{p,j}\right)}{\pi_z + \pi_{\varepsilon,j} + \pi_y + \pi_{p,j}} + \frac{(1 - D_j) \times \left(\pi_{\varepsilon,j} + \pi_{p,j}\right)}{\pi_z + \pi_{\varepsilon,j} + \pi_{p,j}} \\
\alpha_{2,j} &= \left(\frac{D_j \times \pi_y}{\pi_z + \pi_{\varepsilon,j} + \pi_y + \pi_{p,j}}\right) \\
\alpha_{3,j} &= \frac{D_j \times \rho\left(1 + \frac{\pi_{p,j}}{\pi_{\varepsilon,j}}\right)}{\pi_z + \pi_{\varepsilon,j} + \pi_y + \pi_{p,j}} + \frac{(1 - D_j) \times \rho\left(1 + \frac{\pi_{p,j}}{\pi_{\varepsilon,j}}\right)}{\pi_z + \pi_{\varepsilon,j} + \pi_{p,j}}
\end{aligned}$$

where $\pi_{p,j} = \left(\frac{\pi_{\varepsilon,j}}{\rho}\right)^2 \pi_x$.

From each manager's objective function to maximize V_j, it follows that optimal investment is given by $K_j = \mu_z + E\left[z_j | \mathcal{I}_{j,4}^M\right]$. This conditional expectation, in turn, is linear in the two unbiased signals y_j and $\eta_{p,j}$:

$$E\left[z_j | \mathcal{I}_{j,4}^M\right] = \frac{\pi_y}{\pi_z + \pi_y + \pi_{p,j}} y_j + \frac{\pi_{p,j}}{\pi_z + \pi_y + \pi_{p,j}} \eta_{p,j}$$

Replacing $\eta_{p,j}$, leads to:

$$K_j = \beta_{0,j} + \beta_{1,j} y_j + \beta_{2,j} p_j$$

with $\beta_{0,j} = \mu_z - \frac{\alpha_{0,j}}{\alpha_{1,j}} \times \frac{\pi_{p,j}}{\pi_z+\pi_y+\pi_{p,j}}$, $\beta_{1,j} = \frac{\pi_y}{\pi_z+\pi_y+\pi_{p,j}} - \frac{\alpha_{2,j}}{\alpha_{1,j}} \times \frac{\pi_{p,j}}{\pi_z+\pi_y+\pi_{p,j}}$ and $\beta_{2,j} = \frac{1}{\alpha_{1,j}} \times \frac{\pi_{p,j}}{\pi_z+\pi_y+\pi_{p,j}}$. Replacing the α coefficients by the expressions derived above gives all β coefficients in terms of model parameters and the two equilibrium choices made prior to trading (attention allocation and disclosure).

A.1.2 Proof of Lemma 2

Note that in the expression for U_{2i}, the only variable that depends on i is $\pi_{\varepsilon,ij}$ in $V_3[z_j]$. Using this result and dropping constant terms implies that the objective function is proportional to:

$$\widehat{U}_{2i} = \sum_j \pi_{\varepsilon,ij} \lambda_j$$

where $\lambda_j \equiv V\left[E[z_j - p_j | \mathcal{I}_{i,3}^S] | \mathcal{I}_{i,2}^S\right] + E\left[\left(E[z_j - p_j | \mathcal{I}_{i,3}^S]\right)^2 | \mathcal{I}_{i,2}^S\right] \geq 0$. Then, plugging in the equilibrium expression for p_j from Lemma 1 yields:

$$\lambda_j = D_j \left(\frac{\rho^6 + \rho^4 \pi_x \left(\rho^2 \overline{X}^2 + \pi_y + \pi_z + 2\pi_{\epsilon,j}\right) + \rho^2 \pi_x^2 \pi_{\epsilon,j}^2}{\pi_x \left(\pi_x \pi_{\epsilon,j}^2 + \rho^2 \left(\pi_y + \pi_z\right) + \rho^2 \pi_{\epsilon,j}\right)^2} \right) + (1 - D_j) \left(\frac{\rho^6 + \rho^4 \pi_x \left(\rho^2 \overline{X}^2 + \pi_z + 2\pi_{\epsilon,j}\right) + \rho^2 \pi_x^2 \pi_{\epsilon,j}^2}{\pi_x \left(\pi_{\epsilon,j} \left(\rho^2 + \pi_x \pi_{\epsilon,j}\right) + \rho^2 \pi_z\right)^2} \right)$$

Note that $\lambda_j(D_j = 0)$ corresponds to the value in Kacperczyk et al. (2016).

A.1.3 Proof of Lemma 3

Each firm's value at $t = 1$ is given by $E\left[V_j | \mathcal{I}_{j,1}^M\right]$. Plugging in the definition of V_j and simplifying leads to:

$$E\left[V_j | \mathcal{I}_{j,1}^M\right] = \frac{1}{2}\mu_z^2 + \left(\mu_z + \frac{\pi_y}{\pi_z + \pi_y} y_j\right) \overline{K} + \mu_z \frac{\pi_y}{\pi_z + \pi_y} y_j + \frac{1}{2}\pi_z^{-1} - \frac{1}{2}\left(\pi_z + \pi_y + \pi_{p,j}\right)^{-1}$$

Given that $\pi_{p,j} = \left(\frac{\pi_{\varepsilon,j}}{\rho}\right)^2 \pi_x$ it follows that the expected firm value increases in $\pi_{\varepsilon,j}$.

It is straightforward to show that $\lambda_j(D_j = 0) > \lambda_j(D_j = 1)$, such that firm j's attention score is always higher under no disclosure.

A.1.4 Proof of Proposition 1

This result directly follows from Lemma 3: firm managers choose D_j to maximize the expected firm value at $t = 1$. This firm value increases in the speculators' collective attention paid to this firm, which in turn is higher if the manager chooses not to disclose. Technically, the $\{D_A = 0, D_B = 0\}$ outcome is the only stable equilibrium because each firm would be worse off by deviating.

A.1.5 Proof of Proposition 2

Plugging in the possible disclosure decisions and the implied attention choice according to Lemma 2, it follows that:

$$
\begin{aligned}
U_{1j}^M\left(D_j = 1, D_{-j} = 1\right) &= \frac{\rho\overline{X}\omega}{\frac{\overline{\pi}_\varepsilon^2\pi_x}{4\rho^2} + \frac{\overline{\pi}_\varepsilon}{2} + \pi_y + \pi_z} + \frac{1}{2\left(\frac{\overline{\pi}_\varepsilon^2\pi_x}{4\rho^2} + \pi_y + \pi_z\right)} \\
U_{1j}^M\left(D_j = 0, D_{-j} = 1\right) &= \frac{\rho\overline{X}\omega}{\frac{\overline{\pi}_\varepsilon^2\pi_x}{\rho^2} + \overline{\pi}_\varepsilon + \pi_z} + \frac{1}{2\left(\frac{\overline{\pi}_\varepsilon^2\pi_x}{\rho^2} + \pi_y + \pi_z\right)} \\
U_{1j}^M\left(D_j = 0, D_{-j} = 0\right) &= \frac{\rho\overline{X}\omega}{\frac{\overline{\pi}_\varepsilon^2\pi_x}{4\rho^2} + \frac{\overline{\pi}_\varepsilon}{2} + \pi_z} + \frac{1}{2\left(\frac{\overline{\pi}_\varepsilon^2\pi_x}{4\rho^2} + \pi_y + \pi_z\right)} \\
U_{1j}^M\left(D_j = 1, D_{-j} = 0\right) &= \frac{2\rho\overline{X}\omega + 1}{2\left(\pi_y + \pi_z\right)}.
\end{aligned}
$$

Then it is straightforward to show that no firm has an incentive to deviate from $D_A = D_B = 1$ if $\overline{\pi}_\varepsilon > 2\pi_y$ and if ω is large enough. Moreover, under the same conditions $D_A = D_B = 0$ is not stable as both firms have an incentive to deviate.

A.1.6 Proof of Corollary 1

As shown before, the firms' cost of capital is proportional to the speculators' conditional payoff variance which is given by:

$$\widehat{\sigma}_j = \left(\pi_z + \pi_{\varepsilon,j} + D_j\pi_y + \pi_{p,j}\right)^{-1}$$

As a result, this conditional variance depends on both firms' disclosure decisions and takes on the following two values under $D_j = 0$ and $D_j = 1$, respectively:

$$\widehat{\sigma}_j\left(D_A = D_B = 0\right) = \left(\pi_z + \frac{\overline{\pi}_{\varepsilon,j}}{2} + \left(\frac{\overline{\pi}_{\varepsilon,j}}{2\rho}\right)^2 \pi_x\right)^{-1}$$

$$\widehat{\sigma}_j\left(D_A = D_B = 1\right) = \left(\pi_z + \frac{\overline{\pi}_{\varepsilon,j}}{2} + \pi_y + \left(\frac{\overline{\pi}_{\varepsilon,j}}{2\rho}\right)^2 \pi_x\right)^{-1}$$

Clearly, $\widehat{\sigma}_j\left(D_A = D_B = 0\right) < \widehat{\sigma}_j\left(D_A = D_B = 1\right)$ if $\pi_y > 0$. This result, together with Proposition 2, shows that increasing ω weakly reduces the firms' cost of capital and the speculators' conditional variance.

A.1.7 Proof of Proposition 3

In the extended model, the unconditional expected firm value for firm j is given by:

$$V_{0,j} \equiv E_0[V_j] = \frac{\mu_z}{c} E_0[p_j] + \mu_z E_0[K_j] + E_0[z_j K_j] - \frac{1}{2} E_0\left[K_j^2\right]$$

From Lemma 1 and Lemma 2 it follows that $E_0[p_j] = \alpha_{0,j} = \frac{\mu_z}{1+\rho\widehat{\sigma}_j}$ and $E_0[K_j] = \mu_z$. Moreover, $K_j = \mu_z + E[z_j|\mathcal{I}_{j,4}^M]$ which implies that $E_0[K_j^2] = \mu_z^2 + E_0\left[E[z_j|\mathcal{I}_{j,4}^M]^2\right] = \mu_z^2 + \pi_z^{-1} - \widehat{\sigma}_j^M$, where $\widehat{\sigma}_j^M$ denotes the manager's conditional expectation of z_j at $t = 4$. It follows that the ex ante firm value can be written as:

$$V_{0,j} = \frac{\mu_z^2}{c\left(1 + \rho\widehat{\sigma}_j\right)} + \frac{1}{2}\mu_z^2 + \frac{1}{2}\left(\pi_z^{-1} - \widehat{\sigma}_j^M\right)$$

By plugging in the conditional variances, simple algebra shows that the difference between $V_{0,j}$ in the disclosure outcome ($D = 1$) and that of non-disclosure ($D = 0$) is given by:

$$V_{0,j}\left(D = 1\right) - V_{0,j}\left(D = 0\right) = \frac{16\mu_z^2\rho^5\pi_y}{c\left(\overline{X}_j^2\pi_x + 2\rho^2\left(\overline{X}_j + 2\rho + 2\pi_z\right)\right)\left(\overline{X}_j^2\pi_x + 2\rho^2\left(\overline{X}_j + 2\rho + 2\pi_y + 2\pi_z\right)\right)}$$

which is strictly positive as long as c is finite.

A.1.8 Proof of Corollary 2

The result that mandatory disclosure strictly increases both firm's ex ante firm values ($V_{0,j}$) follows directly from Proposition 3. Moreover, from Proposition 2 it follows that this more efficient disclosure equilibrium can be implemented if the weight on $p_j(\omega)$ in each manager's incentive package is sufficiently high ($\omega \geq \omega^*$). In particular, the optimal threshold value solves

$$U_{1j}^{M}\left(D_j = 1, D_{-j} = 1\right) = U_{1j}^{M}\left(D_j = 0, D_{-j} = 1\right)$$

such that no firm manager has an incentive to deviate from this equilibrium outcome. Simple algebra shows that this threshold is given by:

$$\omega^* = \frac{\frac{1}{\frac{\overline{\pi}_{\varepsilon}^2 \pi_x}{\rho^2}+\pi_y+\pi_z} - \frac{1}{\frac{\overline{\pi}_{\varepsilon}^2 \pi_x}{4\rho^2}+\pi_y+\pi_z}}{2\left(\frac{4\mu_z^2\rho^3}{c\left(\overline{\pi}_{\varepsilon}^2\pi_x+2\rho^2\left(\overline{\pi}_{\varepsilon}+2\rho+2\pi_y+2\pi_z\right)\right)} - \frac{c\rho^3\left(\overline{\pi}_{\varepsilon}^2\pi_x+\rho^2\left(\overline{\pi}_{\varepsilon}+\rho+\pi_z\right)\right)}{\left(\overline{\pi}_{\varepsilon}^2\pi_x+\rho^2\left(\overline{\pi}_{\varepsilon}+\pi_z\right)\right)^2}\right)}$$

A.2 Figures and Tables

$\underline{t=1}$ managers disclose public signal	→	$\underline{t=2}$ speculators allocate limited attention	→	$\underline{t=3}$ speculators trade both assets	→	$\underline{t=4}$ managers invest payoffs realized

Figure 1: Timeline for the basic model.

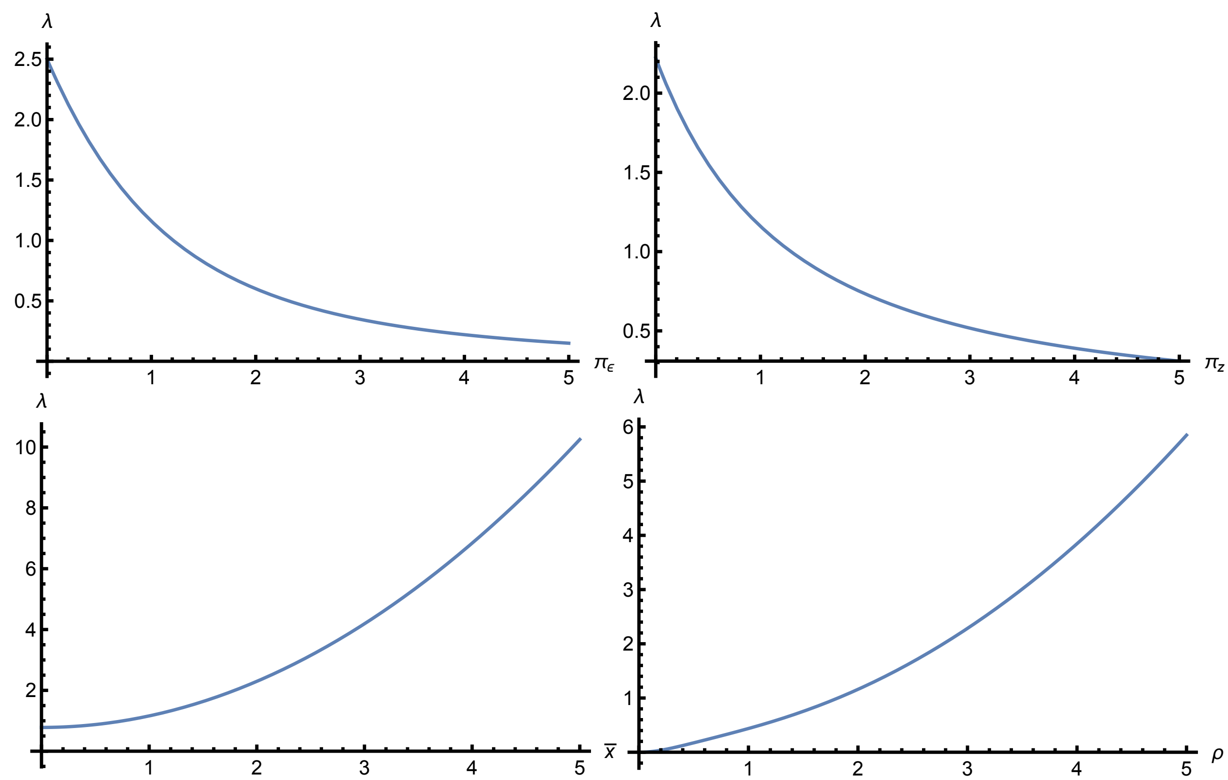

Figure 2: Comparative statics for λ_j. Parameter values (if not on axis): $\overline{x} = 1$, $\rho = 2$, $\pi_z = \pi_x = \pi_\varepsilon = \pi_y = 1$.

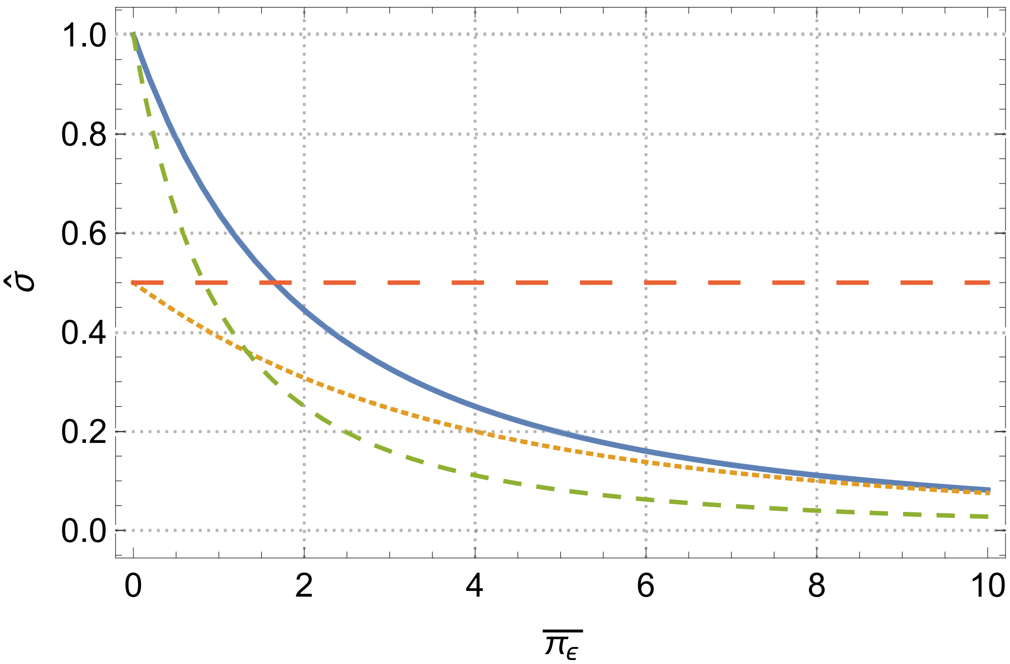

Figure 3: This figure plots the conditional variance for firm j against the fixed attention capacity. Parameters: $\pi_z = \pi_x = 1$, $\rho = 2$. Blue solid line: $D_j = D_{-j} = 0$, orange dotted line: $D_j = D_{-j} = 1$, green dashed line: $D_j = 0, D_{-j} = 1$, orange dashed line: $D_j = 1, D_{-j} = 0$.

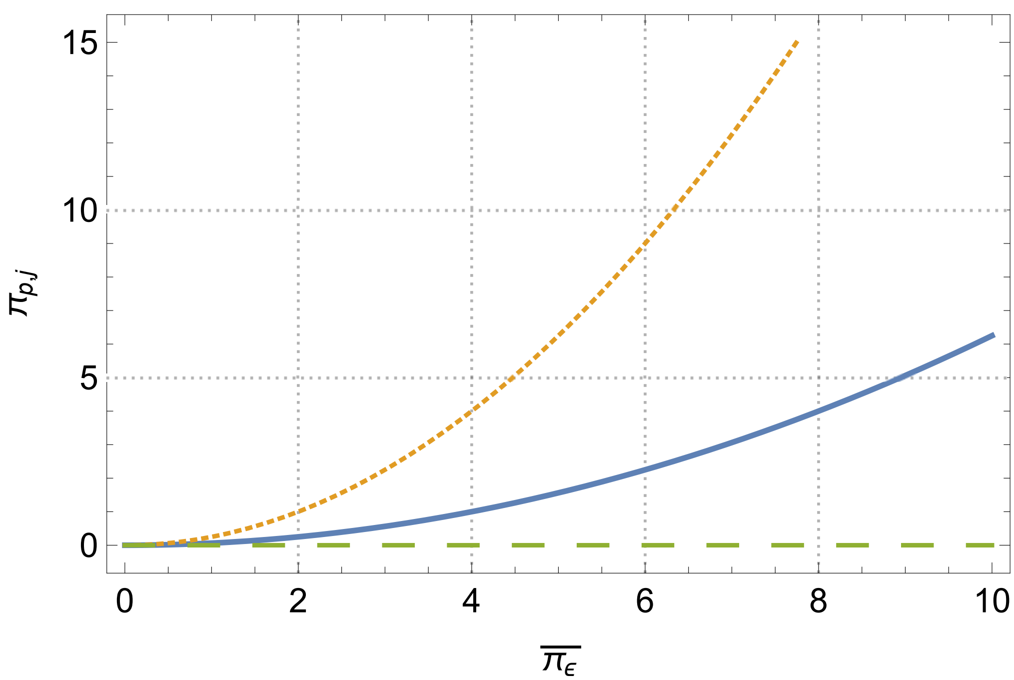

Figure 4: This figure plots price informativeness for firm j against the fixed attention capacity. Parameters: $\pi_z = \pi_x = 1$, $\rho = 2$. Blue solid line: $D_j = D_{-j} = 0$ and $D_j = D_{-j} = 1$, orange dotted line: $D_j = 0, D_{-j} = 1$, green dashed line: $D_j = 1, D_{-j} = 0$.

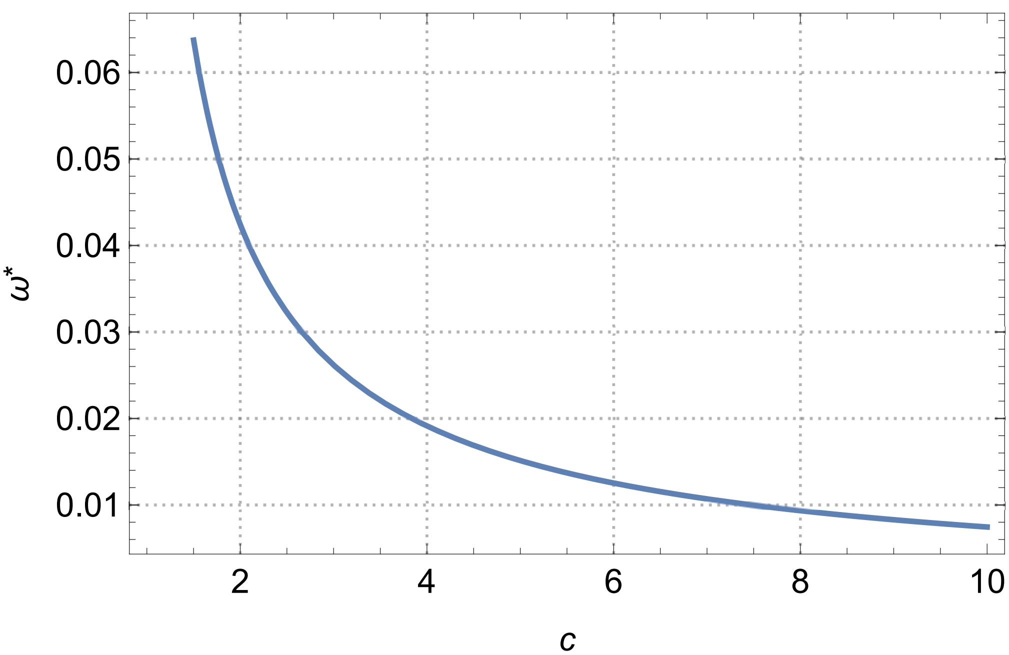

Figure 5: This figure plots the threshold value for ω against the issuance cost c. Parameters: $\pi_z = \pi_x = \pi_y = 1$, $\rho = 2$, $\mu_z = 1$, $\overline{\pi}_\varepsilon = 2$.