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The analysis of bank business performance and market risk—Applying Fuzzy DEA

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ABSTRACT

In the fast changing financial circumstances of nowadays, in avoiding the crisis of closing down, financial institutions are concerned about the efficiency and risk strictly in the meantime. Therefore, efficiency and risk management are goals for a financial institution administrator. Data Envelopment Analysis (DEA) is a non-parameter approach to evaluate the performance of DMU's efficiency and the variables used in the DEA are all accurate values. However, when the input or output variables are fuzzy, the performance of DMUs must proceed by the Fuzzy-DEA. On the basis of risk uncertainty, this research plans to apply the expanding model of Fuzzy Slack-Based Measurement (Fuzzy SBM). The efficiency scores estimated by Fuzzy SBM model are subordinate to functional form, which provides efficiency value region in different degrees of confidence, conforms to the characteristic of risk anticipation, and estimates the management achievement of Taiwan banking under market risk.

1. Introduction

As financial institutions around the world become more internationalized and globalized, the trading activities of the financial industry continue to rise. The market structure is further complicated due to the diversity and innovativeness of products available. Therefore, the risk of investment for financial institutions likewise increases. With such changes in the economic state, banks no longer have the sole role of being the purely monetary intermediary. They must now develop a whole range of investment channels in order to survive under such conditions. However, bearing the objective of profit-making in mind, banks will naturally increase their investments in high-risk products or increase leveraged trading, which means that the high potential profits mask the high risks involved and increase the probability of a bank's bankruptcy due to poor management. For this reason, more attention must be paid to the high risks attached to the high potential profits. The topic of Risk Adjusted Performance Measurement has, in recent years, gained increasing awareness and has become more widely discussed as people place more importance on risk management.

From the perspective of efficiency measurement, Data Envelopment Analysis (DEA) takes into consideration both inputs and outputs. The mathematical method therefore provides a fair measurement of efficiency. Since this analytical model was first proposed, it has been widely applied in a whole range of industries. Most studies to date on bank efficiency have focused mainly upon the economies of scale and scope (Berger and Humphrey, 1991; Berger et al., 1987; Hunter and Timme, 1986; McAllister and McManus, 1993), total productivity (Aly et al., 1990; Favero and Papi, 1995; Fukuyama et al., 1999; Grabowski et al., 1993; Schaffnit et al., 1997; Zaim, 1995), and the efficiency effect (Barr et al., 1994; Casu and Molyneux, 2003; Cebenoyan et al., 1993; Chang, 1999; DeYoung and Hasan, 1998; Elyasiani et al., 1994). The fact that increasing importance is gradually being placed on risk management means that more attention is also given to DEA models that include risk in their equations. There are two issues concerning banks' efficiency and risk. One issue treats risk as exogenous in order to analyze efficiency effects (Ataullah et al., 2004; Barr et al., 1994; Berger and DeYoung, 1997; Chang and Chiu, 2006; Cebenoyan et al., 1993; Elyasiani et al., 1994; Pastor, 2002). The above results show that the efficiency level is significantly correlated with the risk indicators. The other issue treats risk as endogenous in order to analyze banks' efficiency (Altunbas et al., 2000; Chang, 1999; Chiu and Chen, 2008; Drake and Hall, 2003; Girardone et al., 2004; Hughes, 1999; Hughes et al., 2001; Mester, 1996; Pastor, 1999). However, the majority of literatures adopt the overdue loan ratio as the substitute variable for risks, which does not reflect the characteristic of uncertainty that risks display.

Risk is defined as the presence of the characteristic uncertainty, and the degree of risk varies with the asset value fluctuation and the manager's attitude toward risk. Risk may therefore either bring profit or loss to the asset value. The basic function of capital in this

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context is to help bear the possible loss incurred by taking risks. The appropriate provision of capital is therefore key to a stable financial structure, which can help prevent a situation of an inability to make payments. In 2002, the Basel Committee on Banking Supervision (BCBS) proposed the New Basel Capital Accord (Basel II), which sets out guidelines for international banks in terms of taking risks, and therefore, to prevent financial crises. In the section on minimum capital requirement outlined in Basel II, the internal rating uses Value at Risk (VaR) as the basis to estimate the maximum potential loss of the portfolio selection. In simple terms, the VaR 'uses a single value to represent the maximum potential loss of an investment portfolio during a period of time, with a certain confidence level'. Hence, VaR is a prediction interval that provides different estimates according to the different confidence intervals, and therefore takes into account the characteristic of uncertainty that risks displays.

While VaR is widely used to represent the level of risks entailed, the input and output values of the original DEA models are considered crisp values. This is a reoccurring issue encountered when using VaR to estimate the efficiency values of banks. Considering both domestic and foreign literatures, there have been none that have combined these two issues and provided an analytical discussion on the topic. Therefore, this paper seeks to combine the Slack-Based Measure of Efficiency (SBM) as proposed by Tone (2001), with the Fuzzy Measure Theory, and develops the non-radial Fuzzy Slack-Based Measure of Efficiency model (Fuzzy-SBM).

2. Literature review

DEA was a method first proposed by Charnes et al. (1978); then Banker et al. (1984) developed the method for variable returns to scale, called the BCC model. Both the CCR and BCC models considered the weighting of inputs and outputs, and used linear programming to estimate efficiency values. Tone (2001) proposed the Slack-Based Measure of Efficiency (SBM). This model adopts a non-radial method of estimation, while considering the input and output slacks. Therefore, when the efficiency value of a Decision Making Unit (DMU) equals 1, the DMU displays no slacks in either its input or output.

DEA is widely used to estimate the efficiency values of various organizations and industries, and the SBM model solves the issue presented in efficiency ranking. However, these traditional DEA models assume crisp input and output values. If these values are fuzzy numbers, the traditional DEA models cannot accurately measure the efficiency values. For this reason, scholars (Cooper et al., 1999; Despotis and Smirlis, 2002; Guo and Tanaka, 2001; Jahanshahloo et al., 2004; Kao and Liu, 2000) developed the Fuzzy-DEA model, which has the fuzzy measure characteristic. Fuzzy-DEA was originally proposed by Sengupta (1992), in which Sengupta proposed the fuzzy goal-oriented and constraint-based technique based on Zimmermann's (1976) method. This provided the results of Fuzzy-DEA, although the technique is limited to analyzing efficiency with multiple inputs and a single output. Kao and Liu (2000a,b) argued that when fuzzy data exists or there is missing data, it is necessary to adopt the fuzzy measurement concept and the Extension Principle as proposed by Zadeh (1965) to transform the Fuzzy-DEA model into a traditional DEA model with parameters of the level α. Subsequently, Saati et al. (2002) and Lertworasirikul et al. (2003) proposed respectively the Fuzzy-CCR model with asymmetric triangular fuzzy numbers, and the Fuzzy-BCC model that uses probability to conduct analysis. They used a-cut to transform the Fuzzy-DEA model into a linear structure model. Kao and Liu (2004) published a research paper, which was the first research based on financial institutions in Taiwan, with research using the Fuzzy-DEA model to evaluate the efficiency values of those banks. Unfortunately, this paper did not take into account the risks faced by the banks, and in the ever changing market conditions, this was a limitation.

The Basel Capital Accord was created due to the increasing importance placed on risk management. The VaR approach detailed in the Basel Capital Accord has been, so far, the most popular method employed in risk management. Beder (1995) used the Historical Simulation and the Monte Carlo Simulation methods to estimate the VaR of three simulated investment portfolios. Research carried out by Hendricks (1996) concluded that there is no one particular risk valuation model that was more superior to other models under every set of performance criteria. Alexander and Leigh (1997) believed that because the Historical Simulation method tends to use data collected over a few years to evaluate the market variants and the distribution of profits and losses, it was therefore a better model as no distribution hypothesis is required to estimate the VaR. Jackson et al. (1997) stressed that there can be significant differences in VaR values due to the different types of portfolios, and that the simulation method is able to provide more precise tail probabilities than the parameter formula. Looking at all the literatures mentioned and their empirical results, it is evident that the use of VaR models to measure adequacy for market risk is very popular.

When evaluating risk characteristics and estimating efficiencies, it is suitable to employ the Fuzzy-DEA model where the input and output are non-particular values. The above method suggested by this research paper is different from the traditional DEA models proposed in past literatures. Another unique aspect of this paper is to have developed the Fuzzy-DEA model into the Fuzzy-SBM model, which is also different from the current Fuzzy-BCC and Fuzzy-CCR models. To summarize, this research considers the operational risk of banks and uses the VaR values as fuzzy numbers to estimate the business performance of banks in Taiwan.

3. Research methodology

Tone (2001)proposed that Slack-Based Measure of Efficiency model utilizes non-radial estimation method. It also takes the slacks of investment and production items into consideration. Due to the employment of the non-radial method to estimate the efficiency value, the issues such as infeasible would not occur. Thus, this study tends to base on the SBM model, utilizing Fuzzy DEA (Kao and Liu, 2000) to further develop Fuzzy SBM based on the concepts of fuzzy numbers. In the following, the SBM model will be first illustrated, then the derivation of Fuzzy SBM.

3.1. Slack-based measure of efficiency

Tone (2001) proposed the SBM model which is in a manner of a non-ray efficiency of the estimated value, and it will not incur a problem that cannot be estimated. Assume there are n DMUs, m inputs, and s outputs. The production possibility set is defined as $P=\{(x,y)| x\geq X\lambda, y\leq Y\lambda, \lambda\geq 0\}$ in which $X=(x_{ij})\in R^{m\times n}$ is the input matrix and $Y=(y_{rj})\in R^{s\times n}$ is the output matrix. The index δ_j for the DMU_j is from (x_0, y_0) so as to average distances $(\overline{x},\overline{y})\in\overline{P}$ (x_0, y_0). The SBM is as follows.

$$\min_{\delta_j,\lambda_1,\lambda_2,\ldots,\lambda_n} \quad \delta_j=\frac{\frac{1}{m}\sum_{i=1}^{m}\overline{x}_{ij}/x_{i0}}{\frac{1}{s}\sum_{r=1}^{s}\overline{y}_{rj}/y_{r0}} \tag{1}$$

$$\begin{aligned} s.t.\quad & \overline{x}_j\geq\sum_{k=1}^{n}\lambda_k x_k \\ & \overline{y}_j\leq\sum_{k=1}^{n}\lambda_k y_k \\ & \overline{x}_j\geq x_0 \text{ and } \overline{y}_j\leq y_0 \\ & \overline{y}_j\geq 0, \lambda_k\geq 0. \end{aligned}$$

3.2. Fuzzy slack-based measure of efficiency (Fuzzy-SBM)

Assuming that the $\tilde{X}_{ij}$ and $\tilde{Y}_{ij}$ are inputs and outputs characterized by uncertainty of the jth *DMU* respectively, and these can be represented by membership functions $\mu_{\tilde{X}}ij$ and $\mu_{\tilde{Y}}ij$ in the convex fuzzy set. In the fuzzy environment, the Fuzzy-SBM formula can therefore be written as:

$$\begin{aligned}
&\text{Min } \tilde{\delta}_k = q - \frac{1}{m}\sum_{i=1}^{m} S_i^- / \tilde{X}_{ik}\\
&s.t.\quad 1 = q + \frac{1}{s}\sum_{r=1}^{s} S_r^+ / \tilde{Y}_{rk}\\
&\qquad q\,\tilde{X}_{\mathrm{ik}} = \sum_{j=1}^{n} \tilde{X}_{ij}\lambda_j' + S_i^- \quad \mathrm{i}=1,\ldots,\mathrm{m},\\
&\qquad q\,\tilde{Y}_{\mathrm{rk}} = \sum_{j=1}^{n} \tilde{Y}_{rj}\lambda_j' - S_r^+ \quad \mathrm{r}=1,\ldots,\mathrm{s},\\
&\qquad \sum_{j=1}^{n}\lambda_j' = q\\
&\qquad \lambda_j' \ge 0,\ \mathrm{j}=1,\ldots,\mathrm{n},\ S_i^- \ge 0,\quad \mathrm{i}=1,\ldots,\mathrm{m},\ S_r^+ \ge 0,\quad \mathrm{r}=1,\ldots,\mathrm{s},\ \ \mathrm{q}>0.
\end{aligned} \tag{2}$$

In formula (2) all inputs and outputs are assumed as fuzzy data. If any input or output number is an exact value, the exact data can be expressed as degenerated membership functions. Therefore, only one value is present in the range. We then make $S(\tilde{x}_{ji})$ and $S(\tilde{y}_{jr})$ the support of $\tilde{x}_{ji}$ and $\tilde{y}_{jr}$, where the definition of support is the set of elements with membership functions larger than 0. The α-cut of $\tilde{x}_{ji}$ and $\tilde{y}_{jr}$ is defined as:

$$\begin{aligned}
(X_{ji})_\alpha &= \left\{x_{ji}\in S(\tilde{X}_{ji}) \,\middle|\, \mu_{\tilde{X}}ji(x_{ji}) \ge \alpha\right\}, \quad \forall j,i\\
(Y_{jr})_\alpha &= \left\{y_{jr}\in S(\tilde{Y}_{jr}) \,\middle|\, \mu_{\tilde{Y}_{jr}}(y_{jr}) \ge \alpha\right\}, \quad \forall j,r.
\end{aligned} \tag{3}$$

The $(X_{ji})_\alpha$ and $(Y_{jr})_\alpha$ here are a crisp set. Therefore when using a-cut, input and output can both be expressed as the crisp intervals of various level α standards. The set of level α standards defined in the formula above can be expressed as:

$$\begin{aligned}
(X_{ji})_\alpha &= \left\{x_{ji}\in S(\tilde{X}_{ji}) \,\middle|\, \mu_{\tilde{X}_{ji}}(x_{ji}) \ge \alpha\right\} = \left[(X_{ji})_\alpha^L, (X_{ji})_\alpha^U\right]\\
&= \left[\min_{x_{ji}}\left\{x_{ji}\in S(\tilde{X}_{ji}) \,\middle|\, \mu_{\tilde{X}_{ji}}(x_{ji}) \ge \alpha\right\}, \max_{x_{ji}}\left\{x_{ji}\in S(\tilde{X}_{ji}) \,\middle|\, \mu_{\tilde{X}_{ji}}(x_{ji}) \ge \alpha\right\}\right]\\
(Y_{jr})_\alpha &= \left\{y_{jr}\in S(\tilde{Y}_{jr}) | \mu_{\tilde{Y}_{jr}}(y_{jr}) \ge \alpha\right\} = \left[(Y_{jr})_\alpha^L, (Y_{jr})_\alpha^U\right]\\
&= \left[\min_{y_{jr}}\left\{y_{jr}\in S(\tilde{Y}_{jr}) \,\middle|\, \mu_{\tilde{Y}_{jr}}(y_{jr}) \ge \alpha\right\}, \max_{y_{jr}}\left\{y_{jr}\in S(\tilde{Y}_{jr}) \,\middle|\, \mu_{\tilde{Y}_{jr}}(y_{jr}) \ge \alpha\right\}\right].
\end{aligned} \tag{4}$$

In the situation of various α levels for $\{(X_{ji})_\alpha | 0<\alpha\le 1\}$ and $\{(Y_{jr})_\alpha | 0<\alpha\le 1\}$, the Fuzzy-DEA model can be converted into the Crisp-DEA model. According to the Extension Principle (Yager, 1981; Zadeh, 1965; Zimmerman, 1976), the efficiency membership function for the jth *DMU* can be defined as:

$$\mu_{\tilde{E}_k}(z) = \sup_{x,y} \min\left\{\mu_{\tilde{X}_{ji}}(x_{ji}), \mu_{\tilde{Y}_{jr}}(y_{jr}), \forall j,r,i | z = E_k(x,y)\right\}. \tag{5}$$

In this case, $E_k(x,y)$ is the efficiency value calculated using the traditional SBM model under a set of inputs and outputs. According to formula (5), for any efficiency value with the combination x_{ji}, y_{jr} of z, its minimum degree of membership equals to the membership of $\tilde{E}_k$ on point z.

According to the Pareto-optimal solution, the lower and upper bounds of a-cut under $\mu_{\tilde{E}_k}$ can be converted into a traditional one-step programming model in order to obtain the solution.

$$\begin{aligned}
&\mathrm{Min}(\delta_k)_\alpha^U = q - \frac{1}{m}\sum_{i=1}^{m} (S_i^-)^L / (x_{ik})_\alpha^L\\
&s.t.\quad 1 = q + \frac{1}{s}\sum_{r=1}^{s} \left(S_r^+\right)^U / (y_{rk})_\alpha^U\\
&\qquad q(\mathrm{x_{ik}})_\alpha^L = \sum_{j=1,\ne k}^{n} (x_{ij})_\alpha^U \lambda_j' + (\mathrm{x_{ik}})_\alpha^L \lambda_k' + (S_i^-)^L \quad \mathrm{i}=1,\ldots,\mathrm{m},\\
&\qquad q(\mathrm{y_{rk}})_\alpha^U = \sum_{j=1,\ne k}^{n} (y_{rj})_\alpha^L \lambda_j' + (\mathrm{y_{rk}})_\alpha^U \lambda_k' - \left(S_r^+\right)^U \quad \mathrm{r}=1,\ldots,\mathrm{s},\\
&\qquad \sum_{j=1}^{n}\lambda_j' = q\\
&\qquad \lambda_j' \ge 0,\ \mathrm{j}=1,\ldots,\mathrm{n},\ (S_i^-)^L \ge 0,\quad \mathrm{i}=1,\ldots,\mathrm{m},\ \left(S_r^+\right)^U \ge 0,\quad \mathrm{r}=1,\ldots,\mathrm{s},\ \ \mathrm{q}>0
\end{aligned} \tag{6a}$$

$$\begin{aligned}
&\mathrm{Min}(\delta_k)_\alpha^L = q - \frac{1}{m}\sum_{i=1}^{m} (S_i^-)^U / (x_{ik})_\alpha^U\\
&s.t.\quad 1 = q + \frac{1}{s}\sum_{r=1}^{s} \left(S_r^+\right)^L / (y_{rk})_\alpha^L\\
&\qquad q(\mathrm{x_{ik}})_\alpha^U = \sum_{j=1,\ne k}^{n} (x_{ij})_\alpha^L \lambda_j' + (\mathrm{x_{ik}})_\alpha^U \lambda_k' + (S_i^-)^U \quad \mathrm{i}=1,\ldots,\mathrm{m},\\
&\qquad q(\mathrm{y_{rk}})_\alpha^L = \sum_{j=1,\ne k}^{n} (y_{rj})_\alpha^U \lambda_j' + (\mathrm{y_{rk}})_\alpha^L \lambda_k' - \left(S_r^+\right)^L \quad \mathrm{r}=1,\ldots,\mathrm{s},\\
&\qquad \sum_{j=1}^{n}\lambda_j' = q\\
&\qquad \lambda_j' \ge 0,\ \mathrm{j}=1,\ldots,\mathrm{n},\ (S_i^-)^U \ge 0,\quad \mathrm{i}=1,\ldots,\mathrm{m},\ \left(S_r^+\right)^L \ge 0,\ \mathrm{r}=1,\ldots,\mathrm{s},\ \ \mathrm{q}>0.
\end{aligned} \tag{6b}$$

Similarly, this model has the limitation of the maximum relative efficiency value being 1, which makes it difficult for the ranking process because the efficiency values are expressed as intervals. For this reason, this research further develops the Fuzzy Slack-Based Measure of Super-Efficiency model.

3.3. Fuzzy slack-based measure of super-efficiency in DEA

Andersen and Petersen (1993) proposed the supper-efficiency model to solve the ranking problem. Assuming again that a set of DMUs' input and output display the characteristic of uncertainty, we use $\tilde{X}_{ij}$ and $\tilde{Y}_{ij}$ to denote the input and output of the jth *DMU* respectively, and these can be represented as membership functions $\mu_{\tilde{X}_{ij}}$ and $\mu_{\tilde{Y}_{ij}}$ in the convex fuzzy set. In the fuzzy environment, the Fuzzy Super SBM formula can therefore be written as:

$$\begin{aligned}
&\tilde{\tau}_k = \mathrm{Min}\ \frac{1}{m}\sum_{i=1}^{m} \bar{x}'_i / \tilde{X}_{ik}\\
&s.t.\quad 1 = \frac{1}{s}\sum_{r=1}^{s} \bar{y}'_r / \tilde{Y}_{rk}\\
&\qquad \bar{x}'_i \ge \sum_{j=1,\ne k}^{n} \tilde{X}_{ij}\lambda_j' \quad \mathrm{i}=1,\ldots,\mathrm{m},\\
&\qquad \bar{y}'_r \le \sum_{j=1,\ne k}^{n} \tilde{Y}_{rj}\lambda_j' \quad \mathrm{r}=1,\ldots,\mathrm{s},\\
&\qquad \sum_{j=1,\ne k}^{n} \lambda_j = q\\
&\qquad \lambda_j' \ge 0,\ \mathrm{j}=1,\ldots,\mathrm{n},\ne\mathrm{k},\ \bar{x}'_i \ge q\tilde{X}_{ik},\quad \mathrm{i}=1,\ldots,\mathrm{m},\ \bar{y}'_r \le q\tilde{Y}_{rk}\\
&\qquad \bar{y}'_r \ge 0,\quad \mathrm{r}=1,\ldots,\mathrm{s},\quad q>0.
\end{aligned} \tag{7}$$

Based on the definitions (4) and (5), the upper and lower bounds of a-*cut* under $\mu_{\tilde{E}_k}$ can be determined. The two-step mathematical programming model can be transformed into a traditional one-step programming model using the Pareto-optimal solution. Therefore, the Fuzzy Super SBM model can be transformed into:

$$
\begin{aligned}
&(\tau_k)_\alpha{}^U = Min\ \frac{1}{m}\sum_{i=1}^{m}\left(\bar{x}'_i\right)^L/(X_{ik})_\alpha{}^L\\
&s.t.\quad 1=\frac{1}{s}\sum_{r=1}^{s}\left(\bar{y}'_r\right)^U/(Y_{rk})_\alpha{}^U\\
&\qquad \left(\bar{x}'_i\right)^L\geq\sum_{j=1,\neq k}^{n}(X_{ik})_\alpha{}^L\lambda_j{}'\quad \mathrm{i}=1,\dots,\mathrm{m},\\
&\qquad \left(\bar{y}'_r\right)^U\leq\sum_{j=1,\neq k}^{n}(Y_{rk})_\alpha{}^U\lambda_j{}'\quad \mathrm{r}=1,\dots,\mathrm{s},\\
&\qquad \sum_{j=1,\neq k}^{n}\lambda_j{}'=q\\
&\qquad \lambda_j{}'\geq 0,\ \mathrm{j}=1,\dots,\mathrm{n},\neq\mathrm{k},\ \left(\bar{x}'_i\right)^L\geq q(X_{ik})_\alpha{}^L,\\
&\qquad \mathrm{i}=1,\dots,\mathrm{m},\ \left(\bar{y}'_r\right)^U\leq q(Y_{rk})_\alpha{}^U\\
&\qquad \left(\bar{y}'_r\right)^U\geq 0,\quad \mathrm{r}=1,\dots,\mathrm{s},\quad q>0
\end{aligned}
\tag{8a}
$$

$$
\begin{aligned}
&(\tau_k)_\alpha{}^L = Min\ \frac{1}{m}\sum_{i=1}^{m}\left(\bar{x}'_i\right)^U/(X_{ik})_\alpha{}^U\\
&s.t.\quad 1=\frac{1}{s}\sum_{r=1}^{s}\left(\bar{y}'_r\right)^L/(Y_{rk})_\alpha{}^L\\
&\qquad \left(\bar{x}'_i\right)^U\geq\sum_{j=1,\neq k}^{n}(X_{ik})_\alpha{}^U\lambda_j{}'\quad \mathrm{i}=1,\dots,\mathrm{m},\\
&\qquad \left(\bar{y}'_r\right)^L\leq\sum_{j=1,\neq k}^{n}(Y_{rk})_\alpha{}^L\lambda_j{}'\quad \mathrm{r}=1,\dots,\mathrm{s},\\
&\qquad \sum_{j=1,\neq k}^{n}\lambda_j{}'=q\\
&\qquad \lambda_j{}'\geq 0,\ \mathrm{j}=1,\dots,\mathrm{n},\neq\mathrm{k},\ \left(\bar{x}'_i\right)^U\geq q(X_{ik})_\alpha{}^U,\\
&\qquad \mathrm{i}=1,\dots,\mathrm{m},\ \left(\bar{y}'_r\right)^L\leq q(Y_{rk})_\alpha{}^L\\
&\qquad \left(\bar{y}'_r\right)^L\geq 0,\quad \mathrm{r}=1,\dots,\mathrm{s},\quad q>0.
\end{aligned}
\tag{8b}
$$

The relative efficiency values calculated using Eqs. (8a) and (8b) differ from the crisp values calculated by the traditional DEA method in that they are fuzzy numbers. It is therefore difficult to rank the DMUs being evaluated according to their efficiency values. Furthermore, because the efficiency values in this research are the upper and lower bounds of the relative efficiency values calculated under various α levels, the membership functions of the efficiency values are unknown. According to Chen and Klein (1997), in the situation of unknown membership functions the interval values obtained by using α-cut can be used in the Area Measurement Method to rank the fuzzy numbers. To this end, make h the maximum height for the membership function such that $k=1,\dots,n$. Assume that h is divided into m intervals with m approaching infinitive, so that $\alpha_i=ih/m, i=0,\dots,m$. The following index can then be used to rank the fuzzy numbers (Chen and Klein, 1997; Kao and Liu, 2000):

$$
I(\tilde{E}_k,R)=\frac{\sum_{i=0}^{m}\left[(E_k)_{\alpha_i}^U-c\right]}{\sum_{i=0}^{m}\left[(E_k)_{\alpha_i}^U-c\right]-\sum_{i=0}^{m}\left[(E_k)_{\alpha_i}^L-d\right]},m\to\infty. \tag{9}
$$

In this case, $c=\min_{i,k}\left\{(E_k)_{\alpha_i}\right\}$ and $d=\max_{i,k}\left\{(E_k)_{\alpha_i}\right\}$. The bigger the fuzzy ranking index $I(\tilde{E}_k,R)$, the better the rank for the DMU.

4. Empirical results

4.1. Data source and data handling

This research uses 30 banks in Taiwan recorded in 2008 as the sample and uses trigonometry as the basis for calculating the upper and lower bound intervals for each value of α. The source of data and the process for handling the data are detailed as follows.

(1) Source of data used to calculate VaR.
VaR is a quantifiable measurement for market risks. The Bank for International Settlements (BIS) requires banks to calculate VaR on a daily basis at a 99% confidence level, a ten-day holding period, with the minimum sample period of one year. The main advantage of VaR is that a straightforward numeric value captures the concept of risk and allows risks to be easily comparable.
Market risk refers to the possible loss caused by irregular fluctuations of the financial asset value during a specific period of time, as a result of market price changes due to the interest rate, foreign exchange rate, equity security, commodity prices and so on. Market risk is broadly categorized into interest rate risk, equity risk, foreign exchange risk and commodity risk. Due to the scarcity of detailed data, this research only takes into account equity risk and foreign exchange risk. The annual VaR of each bank calculated using Historical Simulation is shown in Table 1.
(2) Input and output variables in efficiency estimation.
The illustrations of the production process of banking are unclear in the reference articles. The definitions of the production process of banking can be found in the production approach and the intermediary approach proposed by Miller and Noulas (1996). In the production approach, banks are considered tools utilizing capital, labor and facility to generate and provide deposits and loans (Berger et al., 1987; Ferrier and Lovell, 1990; Parkan, 1987); on the other hand, intermediary approach considers that the functions of banks lie in providing the service of financial agents; that is, banks employ labors and invest resources in order to absorb savings and funds, also providing money to those who need it and transferring it to capital with interests. This research emphasizes estimating efficiencies after risks are taken into consideration, and treats risks as input variables. Furthermore, using the Intermediation Approach(Berger and Humphrey, 1991; Hughes and Mester, 1993; Kaparakis et al., 1994; Siems, 1992; Yeh, 1996; Yue, 1992), three output variables and four input variables were included. The output variables include total loans, total investments, and handling fees and commissions. The input variables include the number of staff, total deposits, total fixed assets, and VaR values. Of these variables, the VaR values are interval-valued fuzzy numbers, and the VaR and original risk of holdings are triangular membership functions.

4.2. Empirical results

This research firstly estimates the VaR values of banks in Taiwan, and uses the Fuzzy-SBM model to estimate the efficiency values of the sample banks. The results obtained differ from that calculated by traditional DEA models as the efficiency values calculated using the Fuzzy-SMB are membership functions.

In this research, the VaR is a triangular membership function that uses the a-cut concept to estimate the upper and lower bounds of the efficiency values. The assumption values of a-cut used are 0, 0.3, 0.5, 0.7 and 1. When the a-cut value equals 0, it means there is high risk volatility and the change is that the VaR lies within a 99% confidence interval. Theoretically, the difference between the upper and lower bounds of the efficiency values in this situation

Table 1
Data for the 30 commercial banks in Taiwan in 2008.

DMU	(I) Staff (person)	(I) Total fixed assets (NT dollar)	(I) Total deposits (NT dollar)	(I) VaR (NT dollar)	(O) Total loans (NT dollar)	(O) Total investments (NT dollar)	(O) Handling fees and commissions (NT dollar)
1	6357	1,326,533,000	1,050,190,000	(70,529,416.955; 78,807,369.000; 84,939,256.998)	974,943,000	243,653,636,000	24,964,000
2	7087	1,710,707,000	1,287,330,000	(106,858,247.144; 119,478,198.000; 128,818,105.258)	1,152,060,000	347,215,787,000	35,068,000
3	7054	1,719,297,000	1,318,371,000	(125,572,158.135; 140,237,214.000; 151,133,746.121)	1,114,366,000	339,244,517,000	37,643,000
4	619	302,961,000	29,834,000	(36,615,960.067; 40,599,569.000; 43,536,997.803)	78,758,000	117,615,019,000	6,013,000
5	5103	1,951,405,000	1,289,290,000	(119,208,049.328; 132,442,637.000; 142,214,397.413)	1,303,503,000	262,799,160,000	39,554,000
6	4554	486,452,000	390,918,000	(60,148,426.724; 67,459,310.000; 72,441,632.297)	309,643,000	162,408,129,000	29,925,000
7	2057	263,525,000	240,894,000	(2,690,746.149; 3,006,908.000; 3,249,640.602)	201,832,000	15,489,524,000	11,217,000
8	8660	1,495,246,000	1,100,243,000	(140,833,824.365; 128,880,642.000; 166,447,808.328)	838,473,000	464,253,083,000	132,530,000
9	5910	1,287,367,000	1,020,416,000	(63,920,629.897; 71,386,010.000; 76,937,362.099)	809,587,000	383,290,422,000	37,837,000
10	6259	1,144,145,000	835,647,000	(64,333,945.741; 71,857,095.000; 77,470,993.458)	752,384,000	135,871,482,000	44,377,000
11	5109	1,127,815,000	945,385,000	(23,138,413.488; 25,843,708.000; 27,887,187.847)	878,770,000	143,826,345,000	13,244,000
12	887	156,853,000	130,526,000	(1,218,610.301; 1,362,854.000; 1,475,856.728)	136,244,000	22,504,931,000	4,562,000
13	3388	363,637,000	286,768,000	(4,840,110.110; 5,416,305.000; 5,868,685.009)	178,254,000	99,894,696,000	13,826,000
14	4986	999,939,000	811,336,000	(19,730,865.362; 22,043,240.000; 23,773,976.241)	628,204,000	190,743,070,000	21,153,000
15	3956	793,935,000	621,534,000	(72,253,195.559; 80,909,553.000; 87,255,243.926)	532,833,000	137,988,571,000	27,546,000
16	2865	345,832,000	273,644,000	(17,059,173.302; 19,022,491.000; 20,468,053.950)	235,411,000	62,967,373,000	8,955,000
17	7029	922,248,000	730,199,000	(29,130,040.429; 32,501,226.000; 34,952,896.035)	517,193,000	82,864,223,000	51,064,000
18	2327	360,972,000	281,299,000	(15,070,948.811; 16,806,072.000; 18,088,254.187)	210,523,000	33,423,633,000	11,590,000
19	2662	314,171,000	240,961,000	(7,918,030.617; 8,842,479.000; 9,513,124.336)	218,440,000	15,382,449,000	11,830,000
20	2057	282,356,000	214,779,000	(10,455,868.813; 11,676,926.000; 12,605,004.466)	68,862,000	47,285,483,000	7,405,000
21	3264	385,703,000	329,084,000	(18,750,447.446; 21,063,506.000; 22,551,675.739)	278,853,000	45,843,179,000	11,400,000
22	2459	244,797,000	210,391,000	(2,049,796.528; 2,294,035.000; 2,487,135.188)	164,816,000	14,130,703,000	4,059,000
23	2267	601,748,000	422,033,000	(33,692,235.857; 37,616,815.000; 40,523,263.768)	326,869,000	205,799,438,000	15,672,000
24	1974	250,195,000	214,344,000	(7,406,408.591; 8,280,655.000; 8,928,622.163)	136,151,000	8,176,279,000	5,893,000
25	977	103,845,000	90,772,000	(2,176,761.992; 2,413,018.000; 2,582,166.537)	80,928,000	4,459,217,000	4,840,000
26	1167	103,575,000	93,359,000	(2,873,551.616; 3,158,796.000; 3,367,802.933)	77,650,000	5,089,629,000	4,622,000
27	8792	2,440,706,000	1,998,654,000	(74,570,485.906; 83,266,015.000; 89,697,570.698)	1,823,898,000	175,139,720,000	26,579,000
28	8219	3,087,269,000	2,509,014,000	(97,328,763.805; 108,717,690.000; 117,299,074.842)	1,981,786,000	354,095,568,000	23,412,000
29	1459	171,934,000	146,151,000	(1,968,698.956; 2,199,182.000; 2,369,657.752)	119,632,000	4,529,515,000	3,215,000
30	398	41,241,000	35,808,000	(3,244,061.428; 3,622,866.000; 3,903,951.098)	26,916,000	5,345,923,000	146,000

should be the highest. On the contrary, when the a-cut value is set at 1, there is no risk volatility and the risk of the holdings is stable. Therefore, there should be no difference between the upper and lower bounds of the efficiency values when the a-cut value is 1. The use of the Fuzzy-SBM model for estimating efficiency values not only represents the characteristic of uncertainty with the upper and lower bounds of the efficiency values, it also presents the potential effect of risk volatility on efficiency values by using different a-cut values in the calculation.

Empirical results show that, when a-cut is set at 0, both the upper and lower bounds of the nine DMUs (DMU3, 15, 16, 18, 20, 21, 24, 26, 29) being smaller than 1. What this means is that these DMUs are inefficient DMUs, and while the risk variable affects the efficiency value, it does not alter the conclusion that these DMUs are inefficient. The upper and lower bounds of the efficiency values of DMU10, DMU19 and DMU22 all exceed 1, which means that depending on the risk variable, these three DMUs may be either inefficient or efficient DMUs. In terms of the difference between their upper and lower bounds of the efficiency value, the difference displayed for DMU10 is approximately 0.2761, for DMU19 it is approximately 0.5449, and for DMU22 the difference is approximately 0.5492. The significance of these differences is that the efficiency values of these DMUs can be greatly affected by the risk variable. Finally, for the remaining 18 DMUs, namely DMU1, 2, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 17, 23, 25, 27, 28 and 30, their upper and lower bounds of the efficiency value are all greater than 1, meaning they are all efficient DMUs. While the risk variable may affect the efficiency value, it does not alter the conclusion that these DMUs are efficient.

From the empirical research results shown in Table 2, it can be seen that apart from DMU1, 4, 5, 6, 8 and 30, the difference between the upper and lower bounds of the efficiency value for all other DMUs are the biggest when a-cut is set at 0. Conversely, the estimated efficiency values are fixed when a-cut is equal to 1, such that the values for the upper and lower bounds of efficiency are the same. This finding conforms to the model's hypothesis. Furthermore, when a-cut is set at 0.5, the upper and lower bounds of efficiency for DMU10 and DMU22 change from greater than 1 to being smaller than 1. For DMU19, this change happens when a-cut is set at 0.7. From the

Table 2
The empirical results.

DMU	SBM (non-risk)	Rank	Super-SBM (non-risk)	Rank	Fuzzy-SBM							
						α = 0	α = 0.3	α = 0.5	α = 0.7	α = 1	Fuzzy-SBM	Rank
1	1	1	1.010876	14	L	1.010876	1.010876	1.010876	1.010876	1.010876	0.392204	18
					U	1.010978	1.010876	1.010876	1.010876	1.010876		
2	1	1	1.012370	13	L	1.01237	1.01237	1.01237	1.01237	1.01237	0.393232	17
					U	1.014691	1.013368	1.01237	1.01237	1.01237		
3	0.760788	16	0.760788	16	L	0.714766	0.718712	0.721441	0.724253	0.728635	0.254169	21
					U	0.780447	0.759866	0.750451	0.741442	0.728635		
4	1	1	2.460382	2	L	2.18291	2.18291	2.18291	2.18291	2.18291	1	1
					U	2.18291	2.18291	2.18291	2.18291	2.18291		
5	1	1	1.1000196	5	L	1.093618	1.093618	1.093618	1.093618	1.093618	0.435105	12
					U	1.093618	1.093618	1.093618	1.093618	1.093618		
6	1	1	1.0305215	10	L	1.022891	1.022891	1.022891	1.022891	1.022891	0.398427	16
					U	1.022891	1.022891	1.022891	1.022891	1.022891		
7	0.496357	23	0.4963572	23	L	1.141832	1.156551	1.166031	1.17654	1.193894	0.488979	6
					U	1.260653	1.239034	1.225421	1.2124	1.193894		
8	1	1	1.4239116	3	L	1.423912	1.423912	1.423912	1.423912	1.423912	0.606392	4
					U	1.423912	1.423912	1.423912	1.423912	1.423912		
9	1	1	1.0522678	7	L	1.076371	1.081469	1.083308	1.084793	1.087133	0.435919	11
					U	1.1208	1.112951	1.106444	1.099264	1.087133		
10	0.721241	17	0.7212409	17	L	0.729654	0.733883	0.736808	0.739822	0.745866	0.302437	19
					U	1.005763	1.002148	0.790793	0.771664	0.745866		
11	1	1	1.0135138	12	L	1.036586	1.046362	1.054316	1.062043	1.07323	0.425956	14
					U	1.111176	1.09909	1.091411	1.083988	1.07323		
12	1	1	1.0764688	6	L	1.489473	1.518279	1.538248	1.558862	1.591067	0.68441	3
					U	1.712006	1.672821	1.648157	1.624572	1.591067		
13	0.585279	20	0.5852791	20	L	1.283883	1.298294	1.307406	1.314814	1.328235	0.557434	5
					U	1.389831	1.370965	1.3585	1.345872	1.328235		
14	0.561888	21	0.5618883	21	L	1.047402	1.064706	1.076678	1.088557	1.106331	0.445547	10
					U	1.181622	1.157867	1.142907	1.128138	1.106331		
15	0.806053	15	0.8060528	15	L	0.713482	0.714752	0.715631	0.716536	0.717948	0.248113	22
					U	0.760602	0.74677	0.738069	0.729754	0.717948		
16	0.547762	22	0.5477624	22	L	0.571087	0.570841	0.570665	0.570478	0.570154	0.186199	25
					U	0.674129	0.645566	0.625353	0.604118	0.570154		
17	0.446168	24	0.4461684	24	L	1.099378	1.103683	1.106362	1.1087	1.110214	0.453921	8
					U	1.184512	1.167044	1.153699	1.138982	1.110214		
18	0.441941	25	0.441941	25	L	0.515099	0.521463	0.526175	0.526848	0.526732	0.152588	27
					U	0.575564	0.565597	0.558686	0.546404	0.526732		
19	0.362956	26	0.3629557	26	L	0.460273	0.464445	0.468979	0.474199	0.482726	0.226698	24
					U	1.005172	1.00242	1.00027	0.536209	0.482726		
20	0.35685	27	0.3568501	27	L	0.444449	0.444918	0.445242	0.445577	0.446099	0.108604	28
					U	0.485595	0.475423	0.466582	0.458124	0.446099		
21	0.594348	19	0.5943483	19	L	0.579712	0.580468	0.58099	0.581526	0.582358	0.181664	26
					U	0.641872	0.619186	0.608052	0.597424	0.582358		
22	0.278693	28	0.2786935	28	L	0.465875	0.485443	0.498987	0.513102	0.535426	0.237525	23
					U	1.015089	1.00574	0.576198	0.559232	0.535426		
23	1	1	1.033213	9	L	1.039986	1.049225	1.056128	1.062843	1.07322	0.426015	13
					U	1.108241	1.097801	1.090809	1.083793	1.07322		
24	0.187704	30	0.1877038	30	L	0.254601	0.256523	0.25762	0.258751	0.260513	0.00475	30
					U	0.267138	0.265091	0.263756	0.262417	0.260513		
25	1	1	1.0284205	11	L	1.07645	1.090524	1.100224	1.11019	1.125664	0.453043	9
					U	1.174286	1.161739	1.153031	1.142254	1.125664		
26	0.660516	18	0.6605158	18	L	0.711621	0.727174	0.738311	0.750129	0.769275	0.27923	20
					U	0.855858	0.831984	0.815515	0.797945	0.769275		
27	1	1	1.0412508	8	L	1.041001	1.041321	1.041515	1.041696	1.049361	0.417523	15
					U	1.093209	1.080378	1.071672	1.062843	1.049361		
28	1	1	1.1540496	4	L	1.15405	1.15405	1.1546	1.155767	1.157315	0.472109	7
					U	1.189358	1.181061	1.17447	1.166785	1.157315		
29	0.216144	29	0.2161442	29	L	0.277225	0.280267	0.282371	0.284539	0.287917	0.020488	29
					U	0.301093	0.296827	0.294141	0.29157	0.287917		
30	1	1	2.5083725	1	L	2.131279	2.131279	2.131279	2.131279	2.131279	0.973225	2
					U	2.131279	2.131279	2.131279	2.131279	2.131279		

change in the efficiency values, it is clear that the degree of a-cut affects the estimation result. The bigger the volatility is, the bigger the difference between the upper and lower bounds of the efficiency value. In other words, risk volatility affects the efficiency value.

Subsequently, regardless of the a-cut is set, the upper and lower bounds of the efficiency values of DMU1, 4, 5, 6, 8 and 30 are equal. In such a situation, risk volatility has no effect on the efficiency value and the risk variable also does not affect a bank's efficiency performance.

Summarizing the above, this research uses the Fuzzy-SBM model to estimate the efficiency value to represent the characteristic of uncertainty in risks, and varying the a-cut value to demonstrate the effect of risk volatility on the efficiency value.

There are two issues concerning banks efficiency and risk. One treats risk as exogenous in order to analyze efficiency effects (Ataullah et al., 2004; Barr et al., 1994; Berger and DeYoung, 1997; Chang and Chiu, 2006; Cebenoyan et al., 1993; Elyasiani et al., 1994; Pastor, 2002). The above results show that the efficiency level is significantly correlated with the risk indicators. The other issue treats risk as endogenous in order to analyze banks efficiency (Altunbas et al., 2000; Chang, 1999; Chiu and Chen, 2008; Drake and Hall, 2003; Girardone et al., 2004; Hughes, 1999; Hughes et al., 2001; Mester, 1996; Pastor, 1999). However, the majority of literatures adopt the overdue loan ratio as the substitute variable for risks, which does not reflect the characteristic of uncertainty that risks display.

5. Conclusion

With the increasing frequency of financial disasters and its devastating impact over the recent years, countries around the world have begun to pay much more attention to financial risk management. The fact that the financial industry is the supporting structure to a country's economic. This research firstly derives models from the theories, using trigonometry as the basis to develop the Fuzzy-SBM model for the empirical study. Subsequently, banks in Taiwan were used as the sample for the study, with the research carrying out interval estimation of VaR values for use as the input variables. We then used the Fuzzy-SBM model to estimate the upper and lower bounds of the efficiency value for the banks, while varying the a-cut value to represent the effect of risk volatility on the efficiency value. Empirical results from the research show that: 1) The performance of most DMUs varies according to the risk factor. 2) The a-cut value affects the efficiency value, and therefore risk volatility affects the efficiency value. The higher volatility leads to a greater difference between the upper and lower bounds of the efficiency value, while conversely, no volatility in risk means that the efficiency value is fixed. 3) For some DMUs, regardless of the a-cut value, their upper and lower bounds of efficiency value are equal, meaning that risk volatility does not affect their efficiency values and the risk variable does not affect their efficiencies. 4) The risk variable is a factor in the estimation of efficiency values and in the determination of the ranking of efficiencies.

The Fuzzy-SBM model derived in this research paper uses trigonometry as the basis to estimate efficiency values as triangular membership functions. This model conforms to the characteristic of forecasting VaR and differs from traditional DEA models in that the results it produces can better demonstrate the implications of risk and the effects of risk on efficiency. The majority of literatures adopt the overdue loan ratio as the substitute variable for risks proxy variable, which does not reflect the characteristic of uncertainty that risks display (Altunbas et al., 2000; Chang, 1999; Chiu and Chen, 2008; Drake and Hall, 2003Girardone et al., 2004; Hughes, 1999; Hughes et al., 2001; Mester, 1996; Pastor, 1999). Thus, the main contribution of this article consists of the utilization of VaR as risk variables to reflect the features of risk fluctuation encountered by banks. Besides, in this article, the efficiency values calculated using the Fuzzy-SMB are membership functions, so the evaluated efficiency value is in fact an interval value with the function of anticipating the prospective efficiency performance. The results obtained differ from that calculated by traditional DEA models as a constant.

Data Envelopment Analysis (DEA) is a non-parameter approach to evaluate the performance of DMU's efficiency and the variables used in DEA are all accurate values. However, when the input or output variables are fuzzy, the performance of DMUs must proceed by the Fuzzy-DEA. The limitations of this study consist in the risk assessment method suggested by Basel II. The method is able to produce the approximate estimations of the VaR of the banks in Taiwan; nevertheless, influenced by the limitations of incomplete data, this study simply can evaluate a part of the risk conditions. In addition, due to the different financial environment in each nation and the great diversity of banks' investment deployment, the situations of Taiwanese banks may not reflect the actual financial environment in other countries. This article, therefore, provides a more applicable DEA assessment method for reference.

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