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Mobility and volatility: What is behind the rising income inequality in the United States

Huixuan Wu [a,*], Yao Li [b]

[a] Department of Aerospace Engineering, University of Kansas, Lawrence, KS, 66045, USA
[b] Ash Center for Democratic Governance and Innovation, Harvard University, Cambridge, MA, 02138, USA

HIGHLIGHTS

- This paper helps bridge the gap between econophysics and social sciences.
- We re-define the terms in Fokker–Planck equation as income mobility and volatility.
- Rising volatility is a major contributor to the surge in U.S. income inequality.
- The probability of income growth has become lower for all American households.

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ABSTRACT

Inequality of family incomes in the United States has increased significantly in the past four decades. This is largely interpreted as a result of unequal mobility, e.g., the rich can get richer at a faster pace than the rest of the population. However, using nationally representative data and the Fokker–Planck equation, our study shows that income mobility in the United States has remained stable. Instead, we find another factor – income volatility, which measures the instability of incomes – has increased considerably and caused the surge of income inequality. In addition, the rising volatility is associated with the plummeting of income-growth opportunity, creating the feeling that the American Dream is in decline. Volatility has often been overlooked in previous studies on inequality, partially because mobility and volatility are usually studied separately. By contrast, the Fokker–Planck equation takes both mobility and volatility into consideration, making it a more comprehensive model.

1. Introduction

Since the 1970s, the inequality of household incomes in the United States has increased considerably [1–4]. This trend can be caused by changes in income mobility. For instance, if the affluent can become wealthier at a faster pace, while the growth-rate of the middle class and the poor stays constant, this will lead to the increase of income inequality. Indeed, the income share of the top 1% U.S. households doubled from 1979 to 2007 owning largely to the growth of CEO and executive compensation [5,6]. On the other hand, the surge of inequality can also be a result of rising income volatility [7,8], a measure of the instability of household income. A few studies have found that U.S. household incomes became more volatile during the preceding decades [9–13]. Yet, because these studies focus primarily on detecting the trend of volatility, the causality between volatility and inequality has received limited attention. Moreover, much of the existing research on

* Corresponding author.
E-mail addresses: hwu@ku.edu (H. Wu), Yao_Li@hks.harvard.edu (Y. Li).

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income distribution uses cross-sectional data; these data, however, only provide snapshot information [4,14–17] and reveal little about income volatility. To clarify the impact of mobility and volatility on income disparities, a more comprehensive model that contains both factors is needed.

Regarding income distribution models, Pareto's seminal work [18] has inspired an avalanche of related studies. For example, Champernowne has constructed a stochastic model to simulate Pareto's power law [19], and Mandelbrot has applied Lévy's theory to develop the Pareto-Lévy law [20]. A large portion of recent effort has been spent on constructing new models that are able to replicate the income's cumulative distributions. These include, among others, the κ-generalized distribution [21], kinetic exchange models [22,23], generalized Lotka–Volterra models [24], Boltzmann–Gibbs distribution and Yakovenko models [25–28]. These models are based on intuitive assumptions or phenomenological arguments such as multiplicative and additive randomness [26]. Although the models' prediction exactly matches the empirical data (such as data from the U.S. Internal Revenue Service [28]), the socioeconomic meanings of these models have not been fully explored. As a result, critical socioeconomic concepts such as mobility and volatility are not reflected in these models. This limits the opportunity to apply these models to social sciences and, in particular, makes it difficult for econo-physicists to communicate with social scientists [29]. In this paper, we examine the factors of mobility and volatility embedded in the simplest model—the Fokker–Planck equation-based model [27,28]. Our results show that this equation is more than a match for empirical data; it can provide insightful knowledge about the causal dynamics of economic inequality – especially how income mobility and volatility affect inequality – and can shed light on the decline of the American Dream.

2. Data

To overcome the limitations of cross-sectional data, we carried out a longitudinal study on household income variation using nationally representative data from the Panel Study of Income Dynamics (PSID) [30]. PSID is the world's longest-running household panel survey that has tracked thousands of American families since 1968. The PSID originally included two sub-datasets: a nationally representative sample designed by the Survey Research Center (SRC) at the University of Michigan, and a low-income family sample designed by the Bureau of Census. In this paper, we only used the SRC data since the other dataset was not nationally representative. The SRC data archived, among other things, the employment, income, wealth, and expenditures of members of 2930 households in 1968. As the members of SRC families grew up, moved out, and formed new families, they were interviewed separately as newly established households. These new families were referred to as the "split-offs". By 2013, a total number of 13,776 SRC households had been interviewed. The interviews were conducted annually from 1968 to 1997, and biennially thereafter.

In this paper, the group of families interviewed in 1968 was denoted as *G0*. These family heads' ages ranged from 20 to over 90. Many of the families were well established at the time of their first interviews. In this paper, we focused on split-offs established by the offspring or members of *G0*. We used *Cohort G1* to label those families, formed between 1969 and 1980 with a family head younger than 45 at the time of the first interview. The age was controlled so that the cohort had similar working experiences. Those formed between 1981 and 1990 with a head younger than 45 were *Cohort G2*, and those from 1991 to 2000 were *Cohort G3*. These three cohorts can be regarded as three generations. The average ages of the heads in *G1*, *G2,* and *G3* in 1980, 1990, and 2000 were 36, 37, and 39, respectively. Fig. 1 shows the median and mean incomes of *G1*–*G3* as well as the median of all U.S. household incomes (values not adjusted for inflation). The cohort medians increase at a rate of about \$3200/year and the mean value increments are about \$2400/year.

The total family income, $X(t)$, is the sum of all family members' taxable incomes, transferred money, and social security income. The PSID website has detailed descriptions of these variables. Because PSID did not collect data in the even years after 1997 (e.g., 1998, 2000, 2002, etc.), we chose not to analyze data from the even years before 1997 in order to keep the time step Δt constant, i.e., $\Delta t = 2$ in our computations. This allowed us to compute the income-rise, mobility, and volatility using the same time scale.

3. Definition, model, and method

3.1. Definition of mobility and volatility

Within each cohort, we classify its households into different social classes (i.e. income-based positions on the social ladder) according to their income-to-mean ratio $x(t) = X(t)/M(t)$, where $X(t)$ is the total family income in the year t and $M(t)$ is the mean income of the cohort. Since both $X(t)$ and $M(t)$ contain the same inflation factor, their ratio $x(t)$ is free from the influence of inflation. Due to unexpected events such as illness or unemployment, x (and X) can vary randomly over time. The variation $r(t) = x(t + \Delta t) - x(t)$ indicates the up-and-down of an individual family over Δt years. To investigate the relationship between r and inequality, we define the mobility of a social class as the mean value of r divided by Δt for all families in the class x in the year t. In other words, the mobility of a social class is the average change-of-status per year for all households within that class. The volatility of a social class is defined as the standard deviation of r divided by $2\Delta t$. The mathematical expression of mobility is $F(x) = \int_{-\infty}^{\infty} rA_x(r)dr/\Delta t$, and volatility is $S(x) = \int_{-\infty}^{\infty} [r - mean(r)]^2 A_x(r)dr/2\Delta t$, where $A_x(r)$ is the probability of having an r increment among the x-class families. As discussed later, $A_x(r)$ can also be a function of time t, but we omit the time argument in the above expressions for simplicity. Generally, $F(x) > 0$ (or < 0) means the entire social class tends to move upward (or downward) along the social ladder. Unlike current studies that quantify

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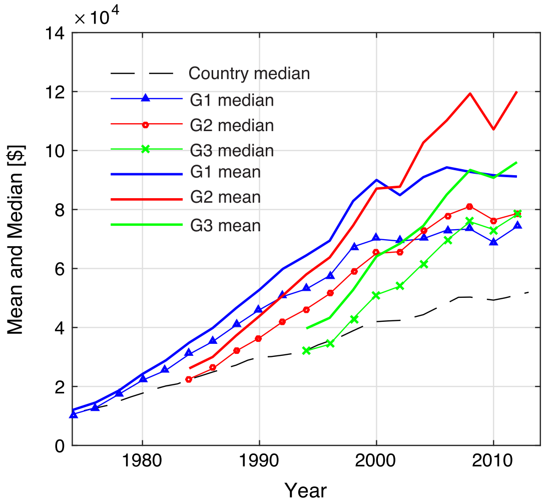

Fig. 1. The median and mean incomes of Cohorts G1, G2, and G3, and the median of U.S. income from 1975 to 2013. The cohort medians and means keep rising until the members retire, except for the 2001 and 2008 recessions.

mobility based on arbitrarily truncated social groups (e.g., classifying the households in the U.S. into five quintiles [31,32]), $F(x)$ precisely measures how much change occurs in each social class on a continuum. In an ideally stable environment, F and S remain stationary, unless significant changes occur in the labor market, industrial technology, public policy, and the like.

3.2. The Fokker–Planck equation

After defining mobility and volatility, we will subsequently examine their impacts on the income-based population distribution $p(x,t)$ using the Fokker–Planck equation. Here, the probability density $p(x,t)$ is defined as the x-class population normalized by the total number of a cohort. Much literature has described how to construct the Fokker–Planck equation from a master equation of p (e.g., [33]), but repeating the construction is necessary in order to uncover the factors of mobility and volatility embedded in the Fokker–Planck equation. The population of social class $x{-}r$ at year t is $p(x{-}r,t)dr$. The probability for this class of households to rise r is $A_{x-r}(r,t)$. Consequently, the probability $p(x,t)$ at x should increase $A_{x-r}(r,t)\,p(x{-}r,t)dr$ in the year $t+\Delta t$. Adding up all these contributions, $p(x,t)$ should increase $\int_{-\infty}^{\infty} A_{x-r}(r,t)\,p(x{-}r,t)dr$. Meanwhile, the x-class family can also move to other social classes, so the population at x will decrease $\int_{-\infty}^{\infty} A_x(r,t)\,p(x,t)dr$. The net increment of the x-class population is, therefore,

$$\Delta p = p(x,\ t+\Delta t) - p(x,\ t) = \int_{-\infty}^{\infty} A_{x-r}(r,t)\,p(x-r,\ t)dr - \int_{-\infty}^{\infty} A_x(r,t)\,p(x,\ t)dr. \tag{1}$$

We can now expand $p(x-r,\ t)$ to a Taylor series to the second order: $p(x-r,t) \approx p(x,t) - \frac{\partial p}{\partial x}r + \frac{1}{2}\frac{\partial^2 p}{\partial x^2}r^2$. Thus the net increment per year, $\Delta p/\ \Delta t$ can be approximated by $\frac{\Delta p}{\Delta t} \approx -\frac{1}{\Delta t}\frac{\partial}{\partial x}\left[p(x,t)\int A_x(r,t)\,rdr\right] + \frac{1}{\Delta t}\frac{\partial^2}{\partial x^2}\left[p(x,t)\int \frac{r^2}{2}A_x(r,t)\,dr\right]$. (If $A_x(r,t)$ is independent of time, it is a simple Fokker–Planck equation [27,33].) Obviously, $\int A_x(r,\ t)rdr/\Delta t$ is mobility F, and $\int r^2 A_x(r,\ t)dr/2\Delta t$ is close to volatility S defined in the previous section. Reorganizing the terms, we finally have the following:

$$\underbrace{\frac{\Delta p}{\Delta t}}_{\text{Change in distribution}} = \underbrace{-\frac{\partial}{\partial x}F(x,t)\,p(x,t)}_{\text{Impact of mobility}} + \underbrace{\frac{\partial^2}{\partial x^2}S(x,t)\,p(x,t)}_{\text{Impact of volatility}}. \tag{2}$$

In the last equation, $\int r^2 A_x(r,\ t)dr/2\Delta t$ is approximated by $S(x)$ because $mean\,(r^2)$ is much larger than $[mean\,(r)]^2$, as discussed later. This equation shows that changes in the income distribution are a result of both mobility and volatility, and their impacts are independent of each other. Together they keep driving p until their impacts balance each other, $\partial(Fp)/\partial x = \partial^2(Sp)/\partial x^2$, so that $\Delta p/\Delta t = 0$ and p reaches an equilibrium distribution p_e [26,28]. For time-independent F and S, the equilibrium is:

$$p_e(x) = \frac{C}{S(x)}\exp\left[-V(x)\right]. \tag{3}$$

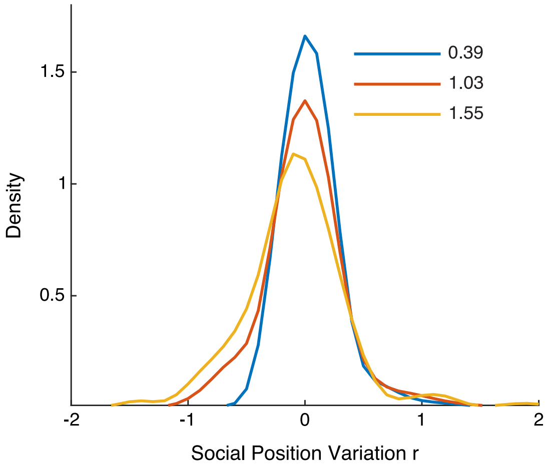

Fig. 2. The probability density function $A_x(r)$, based on the data of Cohort G3 in the 2000s. The selected x values are 0.39, 1.03 and 1.55.

Here $V(x) = -\int_0^x F(\eta)/S(\eta)\,d\eta$, and it diverges to $+\infty$ at the positive x limit (as shown later, $F(\eta)$ is negative for large η). The term C is a normalization factor and, following [26], we only consider $x > 0$. Few income types, except for those incurred through investment, can be negative. Therefore, in the PSID data, the probability of negative x is negligible (in the order of 0.1%). Note that if mobility or volatility changes, the balance will be broken and thus the distribution will shift until a new balance is achieved.

Eq. (1) connects the income distribution with changes of household income, and enables us to quantify inequality using the statistics of r. In this regard, we have followed the arguments in [26,28,34] and assumed that r is proportional to x for middle or higher income households (known as the Gibrat's law) and is nearly independent of x for lower classes. Therefore, as a simple model, mobility and volatility can be represented as $F = F_0 - ax$ and $S = S_0 + b(x - x_0)^2$, respectively. In addition, in the derivation of Eq. (2), $A_x(r)$ should vary fast with r but change little with x; $p(x, t)$ varies slowly with x so that a Taylor expansion up to the second order is accurate. Using *Cohort G3*'s data in the 2000s as a demonstration, we calculate the $A_x(r)$ of three x values to show its x-dependence (Fig. 2). A indeed varies rapidly with r and slowly with x. The maximum $\partial A/\partial x$ is in the order of 0.5, an order of magnitude smaller than the r-variation $\partial A/\partial r \approx 4$. Other cohort's data show the same pattern. The term $\partial p/\partial x$ is in the order of 0.4, which is also much smaller than $\partial A/\partial r$. Finally, the second moment $mean\,(r^2)$ ought to be finite and much larger than $[mean\,(r)]^2$. Indeed, $mean\,(r^2)$ is in the order of 0.1 and $[mean\,(r)]^2$ is less than 0.01. Hence, Eq. (2) is a good approximation to describe the impact of mobility and volatility on income inequality.

3.3. Numerical calculation of mobility, volatility, and probability distribution

The above model involves three distributions: $p(x,t)$, $F(x,t)$ and $S(x,t)$. They can be evaluated using the PSID data. We exclude missing values in the database in case that imputation affects our results. Regarding $F(x)$ and $S(x)$, they are essentially the mean value and standard deviation of r with a given x. In the computation, we first collect the r of all families located between $x - \varepsilon$ and $x + \varepsilon$, where $\varepsilon = 0.1$, and then divide the mean and standard deviation of r by Δt and $2\Delta t$ following the definitions. In this process, we combine the (x, r) data in a ten-year window to enlarge the sample size. Regarding the probability density distribution $p(x)$, it is evaluated using a diffusion-based density estimator [35], which is a state-of-the-art kernel-based estimation method [36].

4. Results and discussion

Fig. 3a shows the mobility of three cohorts *G1*–*G3* at corresponding ages. All the F-lines are close to a straight line for $x \le 2$. These lines cross 0 at $x \approx 1$ with a negative slope, $dF/dx < 0$. This indicates that, for all cohorts, the $x < 1$ families on average will improve in the coming year ($F > 0$), whereas the families at $1 < x < 2$ will move downward ($F < 0$). These trends prevent the distribution p from polarizing. The PSID data have a limited number of samples from the upper class, especially the "top 1%" [3], thus the $x \ge 2$ part of the F-line has large fluctuations due to the lack of enough samples. The mobility of *G1* in the 1980s is marked using a thick blue line. Inter- and intra-cohort comparisons in Fig. 3a indicate that the F-lines neither flatten nor steepen significantly or systematically. In other words, despite the growing inequality in the U.S., income mobility for the American majority has remained constant over the last few decades, which is consistent with previous findings [31,32].

By contrast, the U.S. household income volatility S has increased significantly since the 1990s. As shown in Fig. 3b, *G2*'s S-curves in the 1990s and *G3*'s in the 2000s are much higher than the base case of *G1* in 1980s (the thick blue line). Since

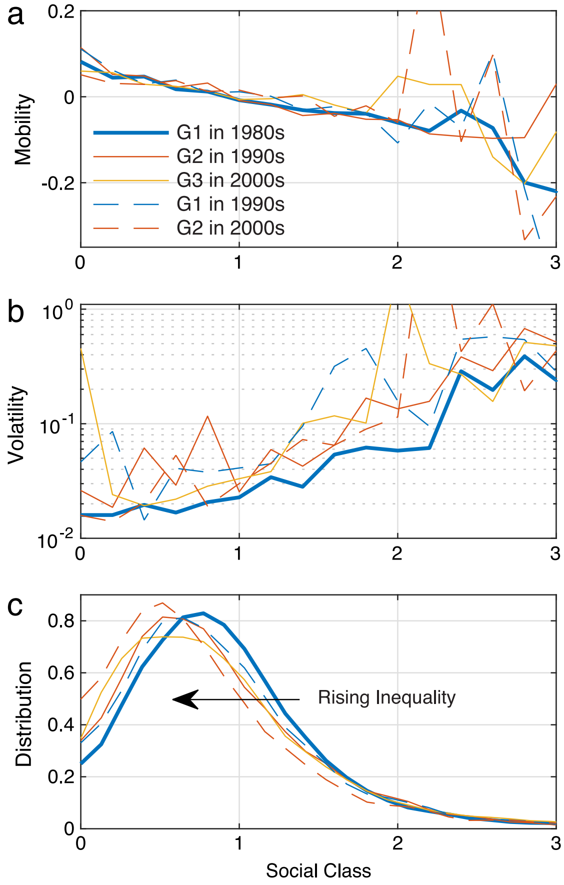

Fig. 3. The mobility, volatility and income-based population distribution. (a) Mobility of the Cohorts G1, G2, and G3, compared at corresponding ages. All F-lines are close to a straight line and have not changed significantly over the last three decades for $0 \leq x \leq 2$. The fluctuations at $x \geq 2$ are due to the lack of enough samples in the dataset. (b) Volatility curves in a log-linear plot. Volatility in the 1990s and 2000s is much larger than that in the 1980s (indicated by the thick blue line of G1). (c) The distribution p of Cohorts G1, G2, and G3 in the 1980s, 1990s, and 2000s. The peak shifts to the left, indicating higher inequality. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the cohorts in comparison are of the same age, the effect of age is minimized. The increasing volatility confirms the trend discussed in [7,10,12,13,37]. In order to demonstrate the effect of rising volatility on inequality, we used the simplified numerical expressions [28] $F = F_0 - ax$ and $S = S_0 + b\,(x - x_0)^2$. Fig. 3a indicates that $F_0 \approx 0.057$, and $a \approx 0.057$. Data in Fig. 3b suggest $x_0 \approx 0.2$. If mobility F is fixed, increasing S_0 causes the equilibrium income distribution p_e Eq. (3) to be flattened (Fig. 4a); increasing b causes the peak of p_e to drift to the left (Fig. 4b). Both changes result in higher disparity. Actually, the relationship among mobility, volatility and inequality can be intuitively understood through the profound fluctuation–dissipation theorem (FDT) [38], a crucial theorem in statistical physics. To take the classic Brownian motion in very viscous fluid as an example (i.e., the Ornstein–Uhlenbeck process), FDT boils down to Einstein's relation [38]: the diffusion coefficient is proportional to temperature times mobility coefficient. Diffusion is actually volatility. Drawing a parallel, if social mobility remains unchanged, volatility will be proportional to a "social temperature" [26] that characterizes the income distribution. Thus, increasing volatility will lead to higher social temperature, and hence, higher inequality. The positive correlation between volatility and inequality is demonstrated in Fig. 5, which shows the impacts of S_0 and b on the Gini coefficient (*GINI*) of the distribution p_e. Increments of S_0 from 0.02 to 0.04 and of b from 0.06 to 0.1 are enough to drive *GINI* to rise from 0.4 to 0.47, similar to the pattern of the U.S. household *GINI* increment (inset of Fig. 5). The peak of the PSID cohort income distribution $p\,(x,\ t)$ shifts to the left (Fig. 3c). This trend is similar to the one illustrated in Fig. 4b.

To conclude, not disputing that other factors may affect the growth of income inequality, e.g., [16,39], we find that rising volatility is a critical contributing factor. Moreover, the distribution p of *G1–G3* shifts to the left, illustrating that households in the middle sections decrease and those in the lower increase. This lends support to the view that the American middle class is in decline [40,41].

Rising volatility means that American households have witnessed larger up-and-down swings in incomes. This can be the result of a big salary raise or an expected job change. Without knowing these details, we cannot conclude whether the rise of volatility is desirable or not for households. However, these reasons are usually complex and difficult to trace in survey

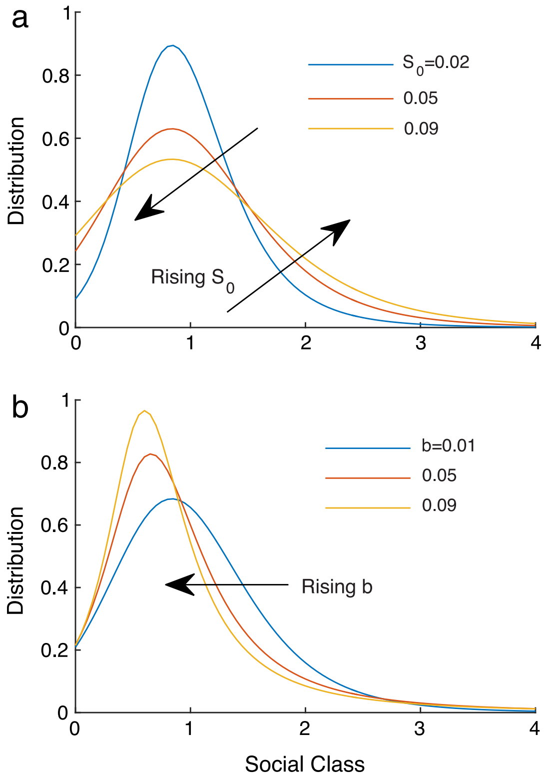

Fig. 4. The dependence of p_e on S, which is modeled as $S = S_0 + b(x - x_0)^2$. (a) The effect of S_0 with a constant $b = 0.02$. Rising S_0 can make the distribution flatter. (b) Variation of p_e due to b with a fixed $S_0 = 0.02$. The peak of p_e shifts to the left when b increases, i.e., the majorities move toward the lower-class end.

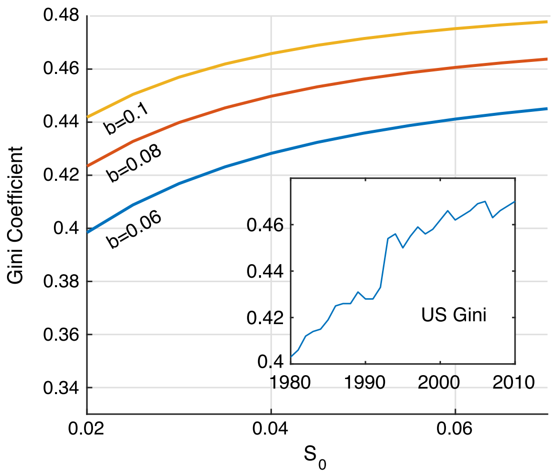

Fig. 5. The numerical relation between the Gini coefficient and volatility $S = S_0 + b(x - x_0)^2$, where $x_0 = 0.2$ is used in the computation. Inset: The Gini coefficient of U.S. households from 1980 to 2010.

data. To address this issue, we use the opportunity of salary-increase to measure the positive role of volatility-change:

$$\text{Opportunity} = \frac{\text{Number of families with increasing salary } X(t)}{\text{Total number of families in the cohort at year } t}. \quad (4)$$

Here we use the absolute salary X instead of x (=X/M, the normalized income) because X more accurately reflects how each individual family feels about their income. Fig. 6a shows the opportunities of *Cohorts G1–G3*. Evidently, these curves are on

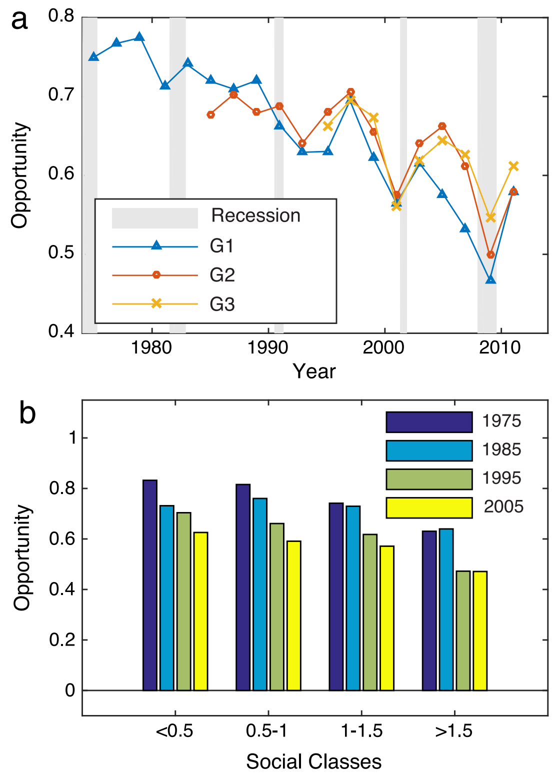

Fig. 6. Variation of income-growth opportunity from the 1970s to 2011. (a) The opportunity for all members in Cohorts G1, G2, and G3. It clearly shows a long-term declining trend. (b) The opportunities for four social classes in G1.

a long-term declining trend, from more than 75% in the late 1970s to less than 60% in recent years for all cohorts, regardless of their ages. To further investigate the decline, we take the data of *G1* in 1975, 1985, 1995, and 2005 and split the cohort into four social classes based on x. All four classes ($x < 0.5$, 0.5–1, 1–1.5, and >1.5) are affected by decreasing opportunity (Fig. 6b). That is, the probability of gaining more income has become lower and the chance of income reduction has become higher over the last few decades.

Therefore, in most cases, the rise in volatility is not due to salary increase. Instead, for the vast majority of families, the chance of earning more income and having a better life is decreasing. In other words, the younger generation is less likely to receive raises in income compared to the older generations at the same age. This generates the feeling that the American Dream has become more elusive. On the other hand, since mobility remains stable (the mean of r is constant), the surge of volatility (the standard deviation of r) suggests that a few households have enjoyed a large rise in salary, and this compensates for the income stagnation of the majority.

5. Conclusion

Econophysics is an exciting interdisciplinary research field, but it is not free from criticism, partially because the models and methods from statistical physics are not much acknowledged by mainstream social scientists (including economists, sociologists, and political scientists). In order to bridge this gap, we re-define the terms on the right-hand side of the Fokker–Planck equation (Eq. (2)) as income mobility and volatility, which are critical measures of income dynamics. This equation has two implications. (1) Both mobility and volatility affect income distribution, and their effects, which can be of the same significance, are independent and opposite, i.e., inequality is positively correlated with volatility but negatively with mobility. In this sense, the relationship among mobility, volatility, and inequality resembles the association among drift, diffusion, and temperature described by Einstein's relation. (2) Mobility and volatility influence income distribution through the aid of the distribution itself (i.e., the gradient of Fp or Sp causes the change in p, not F or S themselves). Thus, the Fokker–Planck equation is far more than a physical model that generates matching cumulative distributions. Instead, each building block of this equation has socioeconomic meaning and can provide illuminating insights into the dynamics of economic inequality. Our analyses show that increasing volatility, rather than mobility, is a major contributor to rising income inequality and the feeling about the fading American Dream in recent decades.

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References

[1] L. McCall, C. Percheski, Income inequality: New trends and research directions, Annu. Rev. Sociol. 36 (2010) 329–347. http://dx.doi.org/10.1146/annurev.soc.012809.102541.
[2] D.H. Autor, L.F. Katz, M.S. Kearney, Trends in U.S. wage inequality: Revising the revisionists, Rev. Econ. Stat. 90 (2008) 300–323.
[3] T. Piketty, Capital in the Twenty-First Century, Harvard University Press, Cambridge, 2014.
[4] T. Piketty, E. Saez, Inequality in the long run, Science 344 (2014) 838–843.
[5] J. Bivens, L. Mishel, The pay of corporate executives and financial professionals as evidence of rents in top 1% incomes, J. Econ. Perspect. 27 (2013) 57–77.
[6] L. Mishel, N. Sabadish, CEO pay in was extraordinarily high relative to typical workers and other high earners, Econ. Policy Inst. (Br. 367) (2013).
[7] P. Gottschalk, R. Moffitt, The growth of earnings instability in the U.S. labor market, Brook. Pap. Econ. Act. 2 (1994) 217–272.
[8] R. Breen, C. García-Peñalosa, Income inequality and macroeconomic volatility: An empirical investigation, Rev. Dev. Econ. 9 (2005) 380–398. http://dx.doi.org/10.1111/j.1467-9361.2005.00283.x.
[9] M. Gittleman, M. Joyce, Have family income mobility patterns changed? Demography 36 (1999) 299–314.
[10] J.S. Hacker, The Great Risk Shift: The New Economic Insecurity and the Decline of the American Dream, Oxford University Press, 2006.
[11] P. Gosselin, High Wire: The Precarious Financial Lives of American Families, Basic Book, 2008.
[12] D. Shin, G. Solon, Trends in men's earnings volatility: What does the panel study of income dynamics show? J. Public Econ. 95 (2011) 973–982. http://dx.doi.org/10.1016/j.jpubeco.2011.02.007.
[13] K.E. Dynan, D.W. Elmendorf, D.E. Sichel, The evolution of household income volatility, in: Division of Research & Statistics and Monetary Affairs, Federal Reserve Board, 2007.
[14] T. Piketty, E. Saez, Income inequality in the United States, 1913-1998, Quart. J. Econ. CXVIII (2003) 1–39. http://dx.doi.org/10.1162/00335530360535135.
[15] R.V. Burkhauser, J. Larrimore, K.I. Simon, A "second opinion" on the economic health of the American middle class, Natl. Tax J. 65 (2012) 7–32.
[16] P. Armour, R.V. Burkhauser, J. Larrimore, Deconstructing income and income inequality measures: A Crosswalk from market income to comprehensive income, Amer. Econ. Rev. 103 (2013) 173–177. http://dx.doi.org/10.1257/aer.103.3.173.
[17] Y. Berman, E. Ben-Jacob, Y. Shapira, The dynamics of wealth inequality and the effect of income distribution, PLoS One 11 (2016) 8–10. http://dx.doi.org/10.1371/journal.pone.0154196.
[18] B.C. Arnold, Pareto Distribution, John Wiley & Sons, Ltd, 2015.
[19] D.G. Champernowne, A model of income distribution, Econom. J. 63 (1953) 318–351.
[20] B. Mandelbrot, The Pareto-Levy law and the distribution of income, Internat. Econom. Rev. 1 (1960) 79–106.
[21] F. Clementi, T. Di Matteo, M. Gallegati, G. Kaniadakis, The κ-generalized distribution: A new descriptive model for the size distribution of incomes, Physica A 387 (2008) 3201–3208.
[22] P. Bhattacharyya, A. Chatterjee, B.K. Chakrabarti, A common mode of origin of power laws in models of market and earthquake, Physica A 381 (2007) 377–382.
[23] A. Chatterjee, B.K. Chakrabarti, Kinetic exchange models for income and wealth distributions, Eur. Phys. J. B 60 (2007) 135–149.
[24] S. Solomon, P. Richmond, Stable power laws in variable economies; Lotka–Volterra implies Pareto-Zipf, Eur. Phys. J. B-Condens. Matter Comp. Syst. 27 (2002) 257–261.
[25] A. Drăgulescu, V.M. Yakovenko, Exponential and power-law probability distributions of wealth and income in the United Kingdom and the United States, Physica A 299 (2001) 213–221.
[26] A.C. Silva, V.M. Yakovenko, Temporal evolution of the thermal and superthermal income classes in the USA during 1983–2001, Europhys. Lett. 69 (2005) 304–310. http://dx.doi.org/10.1209/epl/i2004-10330-3.
[27] V.M. Yakovenko, J.B. Rosser, Colloquium: Statistical mechanics of money, wealth, and income, Rev. Modern Phys. 81 (2009) 1703–1725. http://dx.doi.org/10.1103/RevModPhys.81.1703.
[28] A. Banerjee, V.M. Yakovenko, Universal patterns of inequality, New J. Phys. 12 (2010) 75032.
[29] K.P.Y. Thébault, S. Bradley, A. Reutlinger, Modelling inequality, British J. Philos. Sci. (2017). http://dx.doi.org/10.1093/bjps/axw028. axw028.
[30] PSID, https://psidonline.isr.umich.edu/.
[31] Income Mobility in the U.S. from 1996 to 2005, Department of the treasury, 2007.
[32] R. Chetty, N. Hendren, R. Kline, E. Saez, N. Turner, Is the United States still a land of opportunity? Recent trends in intergenerational mobility, Amer. Econ. Rev. 104 (2014) 141–147.
[33] N.G. van Kampen, Stochastic Processes in Physics and Chemistry, North-Holland Publishing Company, 1981.
[34] M. Jagielski, R. Kutner, Modelling of income distribution in the European Union with the Fokker–Planck equation, Physica A 392 (2013) 2130–2138. http://dx.doi.org/10.1016/j.physa.2013.01.028.
[35] Z.I. Botev, J.F. Grotowski, D.P. Kroese, Kernel density estimation via diffusion, Ann. Statist. 38 (2010) 2916–2957. http://dx.doi.org/10.1214/10-AOS799.
[36] B.W. Silverman, Density Estimation for Statistics and Data Analysis, CRC press, 1986.
[37] P. Gottschalk, R. Moffitt, The rising instability of U.S. earnings, J. Econ. Perspect. 23 (2009) 3–24. http://dx.doi.org/10.1257/jep.23.4.3.
[38] R. Kubo, The fluctuation–dissipation theorem, Rep. Progr. Phys. 29 (1966) 255–284. http://dx.doi.org/10.1088/0034-4885/29/1/306.
[39] C. Goldin, L. Katz, The Race Between Education and Technology, Havard Univ. Press, Cambridge, 2008.
[40] The American middle class is losing ground, Pew Research Center, 2015.
[41] J.E. Foster, M.C. Wolfson, Polarization and the decline of the middle class: Canada and the U.S., J. Econ. Inequal. 8 (2010) 247–273. http://dx.doi.org/10.1007/s10888-009-9122-7.

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