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International Journal of Production Research

ISSN: 0020-7543 (Print) 1366-588X (Online) Journal homepage: http://www.tandfonline.com/loi/tprs20

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To cite this article: Ying Wu, Haiyan Li, Qinglong Gou & Jibao Gu (2017): Supply chain models with corporate social responsibility, International Journal of Production Research, DOI: 10.1080/00207543.2017.1346833

To link to this article: http://dx.doi.org/10.1080/00207543.2017.1346833

Published online: 06 Jul 2017.

International Journal of Production Research, 2017
https://doi.org/10.1080/00207543.2017.1346833

Supply chain models with corporate social responsibility

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(*Received 13 October 2016; accepted 16 June 2017*)

Currently, corporate social responsibility (CSR) has become a critical issue because more than 88% of consumers think companies should try to achieve their business goals while improving society and the environment. This paper focuses on a CSR supply chain where an Original Equipment Manufacturer's (OEM's) sales can be significantly reduced because of its oversea supplier's social misconduct. Specifically, as in a conventional wholesale price contract the supplier determines its wholesale price and the OEM decides its order quantity, but in a CSR supply chain, the supplier can autonomously change its CSR cost once a minimum requirement is satisfied. A higher CSR cost means that the supplier invests more in its corporate social responsibility and the OEM's sales will be less likely to be influenced by negative CSR events. The equilibrium solutions show an important dilemma – although the supplier's profit increases in the basic CSR requirement, the supplier will always use the minimum CSR cost under the conventional wholesale price contract, which eventually leads to a low supply chain profit. Thus, we introduce two different contracts to handle this problem: the flexible quantity contract and the wholesale price incentive contract, which are, respectively, a 'tough' way and a 'beneficent' way for the OEM to solve the problem. Although the two ways cannot (always) coordinate the supply chain, we show that they both will significantly improve the supply chain performance. Our results also show that in some conditions, one strategy will dominate, whereas in different conditions the other strategy dominates.

Keywords: supply chain management; supply chain disruption; pricing model; corporate social responsibility; game theory

1. Introduction

With the increasing social consciousness of consumers, corporate social responsibility (CSR) has become a critical issue for most firms. For instance, polls by Epstein-Reeves[1] in 2010 showed that more than 88% of consumers thought companies should try to achieve their business goals while improving society and the environment, and 83% of consumers thought companies should support charities and nonprofits with financial donations.

Under such a circumstance, a company's reputations and sales could be significantly damaged by the misconduct of its suppliers, a case in point being Mattel, a toy company. Because its independent oversea vendor in the upstream supply chain outside of their direct hierarchical control manufactured the toys using lead paint, millions of toys had to be recalled, and sales also declined (Hoyt, Lee, and Tseng 2008; Tang 2008). Another case was Sodexo. An *Escherichia coli* outbreak among nearly 11,000 German pupils happened (FAZ 2012) because of contaminated frozen strawberries from its supplier. In both cases, one member which did not realise the CSR's importance eventually destroyed the CSR implementation and hurt the entire supply chain.

While a downstream firm's CSR strategy could be thoroughly damaged by its suppliers, the following factors compound the difficulty when the firm is a multinational one. First, owing to the globalisation tendency most suppliers of the multinational firms are located in developing countries overseas (Laudal 2011). While the long distance makes it hard to supervise the supplier's production activities, cultural and other differences will make the problem even more complex – an acceptable CSR practice in one country may be strongly disapproved in another country. Second, unlike other clauses in a contract which are derived from legal principles and the law, CSR clauses are usually based on frameworks that provide companies with scorecards and guidelines to measure the effectiveness of their proposed CSR policies and evaluate the CSR ideas propounded. A CSR clause is more likely to be a simple lip service because it has no actual legal implications and therefore is not a contract clause in the strictest sense. Third, how to measure CSR is another problem. Different from any other corporate activities, CSR deals with issues like environmental pollution, child labour and product safety. Those issues are often outside of the traditional boundaries of a business and cannot be measured by using traditional indicators such as return on investment (Mellahi 2013).

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Some interesting questions arise within such a situation. First, how will the supply chain members act when facing ambiguous CSR clauses? Second, to what degree will a firm's profit be hurt because it cannot control its supplier's CSR activities? Third, can a firm avoid or mitigate its oversea supplier's misconduct through a proper supply chain contract?

To answer those questions, we focus on a CSR supply chain consisting of an OEM and its oversea supplier where the OEM resells the supplier's product under its own name and branding. The OEM is the leader, the supplier is the follower. Under the conventional wholesale price contract where the supplier determines its wholesale price and the OEM decides its order quantity, we suppose that the supplier can autonomously adjust its CSR cost. A higher CSR cost means that the supplier pays more attention to its CSR, and there is a lower probability that misconduct will happen and be exposed. We also assume that: (i) a minimum CSR cost requirement should be met, and (ii) the OEM's sales will be reduced when one or more acts of misconduct are exposed. The sequence of events is as follows. The supplier determines its wholesale price firstly and then the OEM decides its order quantity. After that, the supplier produces the product with a certain CSR cost, where the CSR cost is its decision variable. Misconduct may happen during the production process with a probability affected by the supplier's CSR investment. The market condition eventually is realised and the OEM sets its retail price to consumers.

Using the Stackelberg game we derive the equilibrium solutions and find a dilemma – the supplier's profit increases with the minimum CSR requirement, yet the supplier always uses the minimum CSR cost under the conventional wholesale price contract. This eventually leads to a low supply chain performance. To handle this dilemma, we introduce two different CSR supply chain contracts, i.e. the flexible quantity contract and the wholesale price incentive contract. The former contract refers to a tough OEM who can change its order quantity without any cost, and the latter corresponds to a beneficent OEM who will reward the supplier if no misconduct occurs. Although the two contracts cannot (always) coordinate the supply chain, we show that they both can improve the supply chain performance significantly. Also, comparison between the two contracts shows that neither strategy can always dominate the other one – the supply chain should choose the two alternative contracts conditionally.

This paper contributes to the literature as follows. First, while most previous CSR literature comes from empirical studies (Luo and Bhattacharya 2006; Gupta, Briscoe, and Hambrick 2017), our work is among the few papers using operations management models to investigate the role of supply chain contracts in improving the CSR supply chain performance. Second, our work also provides a new perspective on supply chain disruption management. Most previous literature incorporates the disruption as an exogenous factor affecting the supply chain decisions (Hendricks and Singhal 2005; Tomlin 2006; Sawik 2013), our model differs by assuming the probability of the market disruption is endogenously affected by the supplier's CSR investment. Third, this study enriches the supply chain contract literature about the CSR environment. Different from the conventional supply chain that can be coordinated by various contracts (Wang and Webster 2009; Gou et al. 2016), in most cases we cannot find a contract always able to coordinate the CSR supply chain – the CSR supply chain can only be coordinated under some specific conditions.

The remainder of this paper is organised as follows. Section 2 provides a general overview of the pertinent literature. Section 3 develops the basic models, including the decentralised case and the centralised case. In Section 4, we consider the flexible quantity contract under which the OEM can change its order quantity to any level lower than its original order without paying any additional cost. Section 5 incorporates the case that the OEM encourages the supplier to invest more in CSR activities with a wholesale price incentive contract in a beneficent way. Comparison between the two contracts is provided in Section 6. Conclusions and future directions are in Section 7. We provide all proofs in the Appendix 1.

2. Literature review

Literature related to this paper comes mainly from three streams, i.e. (i) corporate social responsibility, (ii) supply chain contracts and (iii) supply chain disruption management.

2.1 *Corporate social responsibility literature*

Most previous literature on corporate social responsibility involves case study and empirical research, trying to indicate that the firm's investment in social responsibility can create value from various aspects (Porter and Kramer 2006). For instance, Moir (2001) proved that although it is expensive, consumers are willing to buy an environmentally friendly product; Fombrun, Gardberg, and Barnett (2000) proposed that CSR activities can manage firm's risk, especially, reputation risk from stakeholder groups. As a consequence, most previous literature advocated that companies should carry out CSR activities as a means to build customer loyalty (Sen and Bhattacharya 2001), to improve their business reputation (Lu, Wang, and Lee 2013), and even just to respond to a government's call (McWilliams, Siegel, and Wright 2006).

Besides the empirical studies, there are also a few papers using OM models to solve various CSR problems. Ni, Li, and Tang (2010) considered a particular wholesale price contract under which the wholesale price is linearly related to the supplier's CSR performance, i.e. $w = w_0 + ky$ where y is the supplier's CSR performance, and $k \geq 0$ is the downstream firm's marginal pay for the supplier's CSR performance. Specifically, the authors considered six scenarios within two different dimensions. Comparison among the six scenarios demonstrated that it is optimal to let the supplier determine the k and y as the Stackelberg leader. With a similar setting, Hsueh (2015) considered a three-echelon supply chain consisting of multiple suppliers, a manufacturer and multiple retailers, trying to determine the CSR performance levels of all the supply chain members and the transaction quantity between them with a bilevel programming model. Another important work is Hsueh (2014). Within a newsvendor setting, the author proved that the CSR supply chain could be coordinated by a new revenue sharing contract.

It should be noted that all those three papers assume that the sales are linear in the supplier's CSR performance, either in a deterministic and a stochastic form. Our paper differs from them by incorporating the fact that a scandal comes unexpectedly and its negative effect on sales is often terrible. Also, the contracts proposed by our work are more enforceable because they can avoid various difficulties in measuring the supplier's CSR performance.

2.2 *Supply chain contract literature*

Supply chain coordination is among the most important issues of supply chain management. Researchers have introduced various contracts such as buyback contracts (Taylor 2002; Pasternack 2008), revenue sharing contracts (Pasternack 2002; Cachon and Lariviere 2005), quantity-flexibility contracts (Tsay 1999; Tsay and Lovejoy 1999), sales-rebate contracts (Krishnan, Kapuscinski, and Butz 2004) and quantity discount contracts (Moorthy 1987) to coordinate the pricing and inventory decisions of a supply chain, usually within a newsvendor model setting, other research tried to investigate whether those contracts can still coordinate supply chains under various conditions or circumstances.

For instance, the behavioural OM literature supposed that the decision-makers will be affected by certain behavioural factors and then analysed whether a specific contract can coordinate the supply chain (Cui, Raju and Zhang 2007; Wu and Niederhoff 2014; Li, Sethi, and Zhang 2016; Zhang, Li, and Gou 2016), OM-Marketing interface literature usually included the firm's advertising or promotion decisions, trying to find a proper contract coordinating these decisions in a supply chain setting (Yue et al. 2006; Krishnan, Kapuscinski, and Butz 2004; Zhang et al. 2013; Gou et al. 2014; Zhao et al. 2014; Li, Petruzzi, and Zhang 2016). Other literature explored similar problems under various channel structures such as dual channel (Dumrongsiri et al. 2008), online-to-offline (Zhao et al. 2016; He et al. 2017) or express supply chain (Meuffels et al. 2010; Liu et al. 2016).

Our work is in line with the latter stream by considering contracts for the CSR supply chain where the sales may be seriously reduced by the supplier's CSR misconduct. Specifically, we propose two different contracts, i.e. the flexible quantity contract and the wholesale price incentive contract, to improve the CSR supply chain performance, and we also specify the conditions under which one dominates the other.

2.3 *Supply chain disruption management literature*

Supply chain disruption refers to disturbances caused by natural disasters and man-made disasters, including earthquakes, hurricanes, floods, terrorist incidents and other unexpected occurrences (Yu, Zeng, and Zhao 2009). By treating those disruptions as exogenous events, previous literature has investigated various strategies, either operational strategy or finance strategy, to mitigate or manage the disruption risk. Tomlin (2006) and Schmitt, Snyder, and Shen (2010) focused on inventory policies. Chopra, Reinhardt, and Mohan (2007) and Yang et al. (2009) considered the emergency sourcing strategy. Dada, Petruzzi, and Schwarz (2007) suggested the dual sourcing strategy. Ang, Iancu, and Swinney (2016) found that a firm with dual sourcing or multiple sourcing strategy may still face the same tier 2 supplier, and thus they suggested firms should optimise their sourcing strategy in the multitier supply chain network. Lodree and Taskin (2008) and Dong and Tomlin (2012) investigated the role of 'insurance' in managing the disruption risk.

Our work is highly related to literature using contracts to coordinate the supply chain with disruptions. Xiao, Qi, and Yu (2007) and Zhang et al. (2012) considered a supply chain with a supplier and two retailers where the two retailers face a certain market disruption, and proposed mechanisms to coordinate the supply chain. Hou, Zeng, and Zhao (2010) studied a buy-back contract between a buyer and a backup supplier when the buyer's main supplier experiences disruptions, and found that the two different types of disruptions, i.e. the supply disruption and the demand disruption, lead to different optimal strategies in terms of order quantity for the buyer and the return price for the backup supplier.

Our work differs from the above literature by assuming the probability of the possible market disruption is endogenously determined by the supplier's CSR investment. To the best of our knowledge, this is the first paper that investigates how a downstream firm should use contracts encouraging its supplier's CSR investment to reduce its market disruption. Our results also demonstrate why famous companies would more likely to outsource its products from suppliers in underdeveloped countries – the low CSR requirement there plays an important role.

3. Model development

Consider a supply chain consisting of an OEM M and its supplier S. The firm M, as the brand owner, controls the design, development, and distribution of the products, but purchases/outsources its products from/to the supplier S. Apple and Foxconn is an example of such a relationship.

Due to the increasing consciousness of CSR issues among consumers, a firm's goodwill is also influenced by the performance of its suppliers. A supplier's misconduct will reduce the OEM's sales significantly. The social misconduct could be irresponsible practices, including extended working hours, low payment, using child labour, and so on. To model this phenomenon, we assume that the OEM M's demand is $D_1 = a_1 - bp$ if no exposure of the supplier S's negative social misconduct occurs and is $D_2 = a_2 - bp$ on the contrary (Anand and Goyal 2009; Wang et al. 2017). Here, $a_1 > a_2 > 0$ are the maximum potential market under the two situations, $p > 0$ is the retail price and $b > 0$ is the price sensitivity. In particular, $a_1 - a_2$ will take a larger value when consumers are more conscious of a firm's CSR, and will take a smaller one on the contrary.

Following Xia, Zu, and Shi (2015), which developed an initial analytic framework that links a firm's social performance with its economic performance, we suppose that the occurrence of the supplier S's negative social events follows a Poisson distribution and the probability that one or more negative events happen is

$$\xi = 1 - e^{-\lambda}, \tag{1}$$

where λ is the average number of breakouts related to the supplier's misconduct. A smaller λ implies a higher level of the supplier's social responsibility performance. Consequently, the probability that no misconduct happens is $\tau = 1 - \xi = e^{-\lambda}$.

As for the supplier's CSR cost, we also follow Xia, Zu, and Shi (2015) and assume that for each unit of product, the CSR cost is quadratic in τ, i.e.

$$c = K\tau^2. \tag{2}$$

While the quadratic form of Equation (2) reflects the diminishing return of the CSR investment, $\tau = e^{-\lambda} \in [0, 1]$ implies that K is the firm's maximum CSR investment. Combining Equations (1) and (2) we obtain the relationship between ξ and c as shown in Equation (3),

$$\xi = 1 - (c/K)^{1/2}, \tag{3}$$

where $0 < c_0 \le c \le K$. Here, c_0 is the supplier's minimum CSR cost and K is the maximum one. Usually, the minimum cost c_0 is to meet the basic requirement from either the government or other external social forces. The maximum cost K indicates that a firm can avoid its misconduct completely with an extremely high CSR cost. Thus, one cannot expect to produce higher profits with a CSR cost c higher than K. We should note that once the supplier's CSR cost is lower than K, negative social events will happen with a certain probability. Another note is that a large K, as well as a large c_0, often occurs in developed countries with high social responsibility awareness. Equation (3) indicates that a firm can reduce its exposure probability of social misconduct by undertaking more social responsibility, under which a higher CSR cost will occur. Specifically, when it just meets the basic requirement, its CSR cost is c_0 and the exposure probability reaches its maximum value, i.e. $\xi_0 = 1 - \sqrt{c_0/K}$.

To simplify the mathematics, we assume that the condition $a_2 - bK \ge 0$ is held throughout the paper. This constraint means that even when the supplier utilises the maximum CSR cost K but unfortunately faces the minimum potential market base, the entire supply chain still has a positive profit margin.

For easy reference, we list all notation in Table 1.

3.1 *The decentralised case*

In this subsection, we consider that the two channel members use the traditional push wholesale price contract as a benchmark. The supplier's decision variables include its wholesale price w and its CSR cost c, whereas the OEM's decision variables are its retail price p and its order quantity q. The decision sequence is shown in Figure 1. Before the selling season, the supplier S firstly declares its wholesale price w and then the OEM M determines its order quantity q. Then, the supplier manufactures the products at a unit CSR cost c, under which harmful CSR events may happen at a probability $\xi = 1 - \sqrt{c/K}$. At the beginning of the selling season, the demand uncertainty $D = D_1$ or D_2 is resolved, and the firm M receives its order, i.e. the q units of product, and determines its retail price p.

Table 1. Notation.

Notation	Description
M	A symbol to indicate the OEM
S	A symbol to indicate the supplier
i	An index to indicate whether there are exposures of the supplier's negative social misconduct – $i = 1$ means no exposure and $i = 2$ means at least one
D_i	The market demand under the above two situations
ξ	The probability that one or more negative exposures occur
λ	The average number of breakouts related to supplier's social misconduct
c_0	The supplier's minimum CSR cost per unit product to meet the public requirement
K	The supplier's maximum CSR cost per unit product
c	The supplier's cost related to its social responsibility – a supplier's decision variable
w	The supplier's wholesale price – a supplier's decision variable
q	The OEM's order quantity – a OEM's decision variable
p	The retail price – a OEM's decision variable
π_S	The supplier's profit
π_M	The OEM's profit

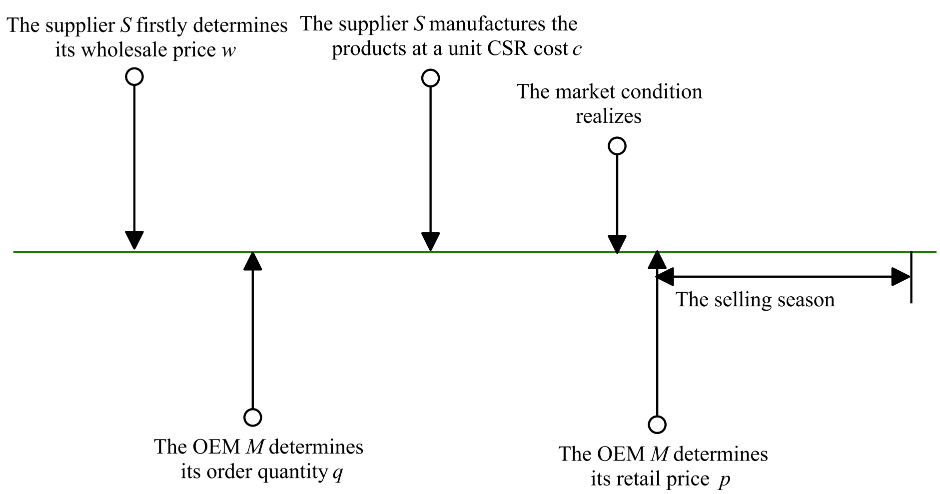

Figure 1. The sequence of events.

To avoid messing in mathematics, we normalise all other costs and the product salvage value to zeroes (Guo 2009). The supplier's profit is then

$$\pi_S = (w - c)q, \tag{4}$$

and the OEM's profit is

$$\pi_M^{(i)} = p\min(a_i - bp, q) - wq, \tag{5}$$

where $i = 1$ or 2. Here, $i = 1$ means that no exposure of social misconduct happens during the production season and $i = 2$ means the contrary. Specifically, when the ordered products arrive the OEM and the market condition is realised at the beginning of the selling season, the OEM's objective is to determine its retail price p maximising its profit as given in Equation (5). But when the OEM decides its order quantity, whether $D = D_1$ or D_2, is not known yet. As a consequence, its objective at this time point is to maximise its expected profit given by Equation (6), i.e.

$$E(\pi_M) = (1-\xi)\max_p\,[p\min(a_1 - bp, q) - wq] + \xi\max_p\,[p\min(a_2 - bp, q) - wq], \tag{6}$$

where ξ is given by Equation (3). In the following we utilise a backward induction approach to derive the equilibrium decisions of the two channel members. Lemma 1 provides the OEM's optimal retail price when the potential market is realised.

LEMMA 1 *Given the market demand function $D = a - bp$ and the order quantity q, the OEM's optimal retail price is*

$$p(q) = \begin{cases} \frac{a-q}{b} & \text{if} \quad 0 < q \le \frac{a}{2} \\ \frac{a}{2b} & \text{if} \quad q > \frac{a}{2}, \end{cases} \tag{7}$$

under which the OEM's profit is

$$\pi_M = \begin{cases} \frac{aq-q^2}{b} - wq & \text{if} \quad 0 < q \le \frac{a}{2} \\ \frac{a^2}{4b} - wq & \text{if} \quad q > \frac{a}{2}. \end{cases} \tag{8}$$

The proof of Lemma 1 is provided in Appendix 1.

Lemma 1 illustrates that the OEM's retail price is determined by both its order quantity q and the market condition. Specifically, we have $\partial p(q)/\partial a \ge 0$ and $\partial p(q)/\partial q < 0$, implying a large potential market a will lead to a high retail price whereas a high order quantity q will result in a low retail price. Since $a_1 > a_2 > 0$, the exposure of the supplier's social misconduct will usually reduce the retail price.

As for the supplier, once its wholesale price w and the OEM's order quantity q are determined, it tries to determine its optimal CSR cost c to maximise its profit given by Equation (4). Noting that $\partial\pi_S/\partial c = -q < 0$, we know that under the push wholesale price contract, the supplier would always like to invest less in its CSR as shown in Proposition 1.

PROPOSITION 1 *Under the push wholesale price contract, given the wholesale price w and the OEM's order quantity q, the supplier S's optimal decision on its CSR cost is to use its minimum value, i.e. $c = c_0$, under which the probability of negative CSR exposure is $\xi = \xi_0 = 1 - \sqrt{c_0/K}$.*

The proof of Proposition 1 is provided in Appendix 1.

Proposition 1 demonstrates that despite the fact that the wholesale price w and the order quantity q are determined before the supplier determines its CSR cost c, they do not influence the supplier's CSR cost decision under the conventional wholesale price contract. The contract gives the supplier no incentive to improve its social responsibility and the exposure probability of its negative social misconduct reaches the maximum value, i.e. $\xi = \xi_0 = 1 - \sqrt{c_0/K}$.

With Lemma 1 and Proposition 1 we obtain the OEM's problem when it decides its order quantity q, i.e.

$$\max_q E(\pi_M) = \begin{cases} (1-\xi_0)\frac{a_1q-q^2}{b} + \xi_0\frac{a_2q-q^2}{b} - wq & \text{if } 0 \le q \le \frac{a_2}{2} \\ (1-\xi_0)\frac{a_1q-q^2}{b} + \xi_0\frac{a_2^2}{4b} - wq & \text{if } \frac{a_2}{2} < q \le \frac{a_1}{2} \\ (1-\xi_0)\frac{a_1^2}{4b} + \xi_0\frac{a_2^2}{4b} - wq & \text{if } \quad q > \frac{a_1}{2}, \end{cases} \tag{9}$$

and the supplier's problem when it determines its wholesale price w, i.e.

$$\max_w \pi_S(w) = (w - c_0)q. \tag{10}$$

Solving the game backward, we obtain the equilibrium solutions for the decentralised scenario as in Proposition 2.

PROPOSITION 2 *Let $\xi_0 = 1 - \sqrt{c_0/K}$. Suppose the supplier and OEM make their decisions independently with the wholesale price contract, the equilibrium solutions of the CSR supply chain are as follows:*

(i) *when $a_2 \le a_1 \le 2a_2$ holds, the supplier's equilibrium wholesale price is*

$$w^* = \frac{(1-\xi_0)a_1 + \xi_0 a_2}{2b} + \frac{c_0}{2}, \tag{11}$$

the OEM's equilibrium order quantity is

$$q^* = \frac{(1-\xi_0)a_1 + \xi_0 a_2}{4} - \frac{bc_0}{4}, \tag{12}$$

under which the (expected) profit of the two channel members are

$$\pi_S^* = \frac{((1-\xi_0)a_1 + \xi_0 a_2 - bc_0)^2}{8b}, \tag{13}$$

and

$$E\pi_M^* = \frac{((1-\xi_0)a_1 + \xi_0 a_2 - bc_0)^2}{16b}; \tag{14}$$

(ii) *when $a_1 > 2a_2$ holds, the supplier's equilibrium wholesale price is*

$$w^* = \begin{cases} \frac{(1-\xi_0)a_1}{2b} + \frac{c_0}{2} & \text{if} \quad 0 \le c_0 \le \frac{(a_1-2a_2)^2}{b^2K} \\ \frac{(1-\xi_0)a_1+\xi_0 a_2}{2b} + \frac{c_0}{2} & \text{if} \quad \frac{(a_1-2a_2)^2}{b^2K} < c_0 \le K, \end{cases} \tag{15}$$

the OEM's equilibrium order quantity is

$$q^* = \begin{cases} \frac{a_1}{4} - \frac{bc_0}{4(1-\xi_0)} & \text{if} \quad 0 \le c_0 \le \frac{(a_1-2a_2)^2}{b^2K} \\ \frac{(1-\xi_0)a_1+\xi_0 a_2}{4} - \frac{bc_0}{4} & \text{if} \quad \frac{(a_1-2a_2)^2}{b^2K} < c_0 \le K, \end{cases} \tag{16}$$

under which the (expected) profit of the two channel members are

$$\pi_S^* = \begin{cases} \frac{((1-\xi_0)a_1-bc_0)^2}{8b(1-\xi_0)} & \text{if} \quad 0 \le c_0 \le \frac{(a_1-2a_2)^2}{b^2K} \\ \frac{((1-\xi_0)a_1+\xi_0 a_2-bc_0)^2}{8b} & \text{if} \quad \frac{(a_1-2a_2)^2}{b^2K} < c_0 \le K, \end{cases} \tag{17}$$

and

$$E\pi_M^* = \begin{cases} \frac{\xi_0 a_2^2}{4b} + \frac{((1-\xi_0)a_1-bc_0)^2}{16b(1-\xi_0)} & \text{if} \quad 0 \le c_0 \le \frac{(a_1-2a_2)^2}{b^2K} \\ \frac{((1-\xi_0)a_1+\xi_0 a_2-bc_0)^2}{16b} & \text{if} \quad \frac{(a_1-2a_2)^2}{b^2K} < c_0 \le K. \end{cases} \tag{18}$$

The proof of Proposition 2 is provided in Appendix 1.

Proposition 2 demonstrates that the CSR consciousness of consumers and the basic CSR requirement of a society will interactively influence the supply chain decisions. Since ξ_0 is the probability that the potential market a equals a_2 and $1-\xi_0$ is the probability that the potential market a equals a_1, the term $(1-\xi_0)a_1 + \xi_0 a_2$ is the expected potential market. Then, the expression of Equation (11) illustrates that when the CSR consciousness of consumers is not high, i.e. $a_2 \le a_1 \le 2a_2$, the equilibrium wholesale price is mainly determined by the expected potential market and the supplier's CSR cost c_0. However, Equation (15) demonstrates this is not always the case. Specifically, when consumers pay more attention to CSR so that $a_1 > 2a_2$, a low wholesale price will result when the basic requirement c_0 is low, i.e. $0 \le c_0 \le (a_1 - 2a_2)^2/b^2K$. Compared with the equilibrium wholesale price under the condition of $c_0 > (a_1 - 2a_2)^2/b^2K$, a term $\xi_0 a_2/2b$ has been deleted, implying that the equilibrium wholesale price will be significantly lower if $0 \le c_0 \le (a_1 - 2a_2)^2/b^2K$. To make this conclusion clear, we vary c_0 from 0 to 10 while keeping other parameters fixed, i.e. $a_1 = 1850$, $a_2 = 900$, $b = 10$ and $K = 10$, and draw the equilibrium solutions in Figure 2. From Figure 2, we see that the supplier is more likely to use a low price strategy to attract a large order quantity when c_0 is small, implying that the local government of the supplier has a low basic CSR requirement.

The left subfigure of Figure 2 shows the trend of the wholesale price and the OEM's order quantity as c_0 changes when $c_0 > (a_1 - 2a_2)^2/b^2K$, and the right subfigure reflects the tendency of the profits. We also plot the trends of the equilibrium solutions for the case of $a_2 \le a_1 \le 2a_2$ in Figure 3, behind which the parameters are $a_1 = 1500$, $a_2 = 900$, $b = 10$, $K = 10$. In Figures 2 and 3, π_S^* is the supplier's profit at equilibrium, and $E\pi_M^*$ and $E\pi^*$ are the expected profit of the OEM and the entire supply chain, respectively.

Figures 2 and 3 illustrate the following dilemmas: (i) although the supplier's profit is an increasing function in the basic CSR requirement c_0, the supplier will always use the minimum c_0; (ii) although the supplier can benefit from a higher requirement c_0, the host country of the supplier has little incentive in increasing its basic CSR requirement.

We will investigate solutions to the first dilemma using supply chain contracts in Sections 4 and 5, but first the OEM's profit trend in Figures 2 and 3 will help us explain the second dilemma. In Figure 2, as c_0 increases, the OEM's profit will drop suddenly at the break-point $c_0 = (a_1 - 2a_2)^2/b^2K$ and then increase gradually. Note the OEM's expected profit is significantly higher at the left of the break-point, when there are many suppliers competing for the same order, the OEM

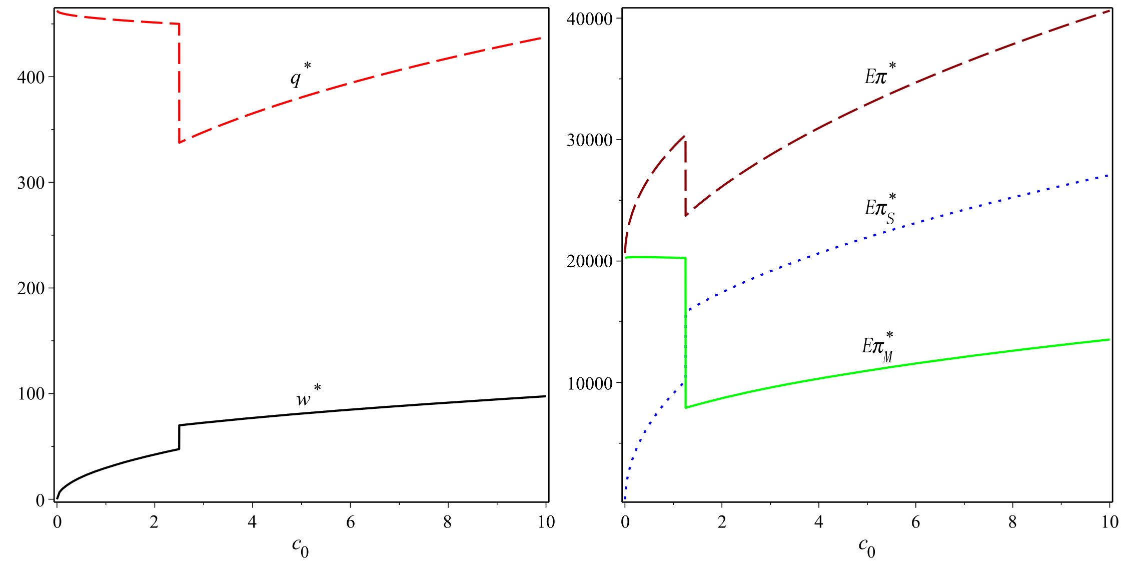

Figure 2. The equilibrium solutions with different c_0 when $a_1 > 2a_2$.

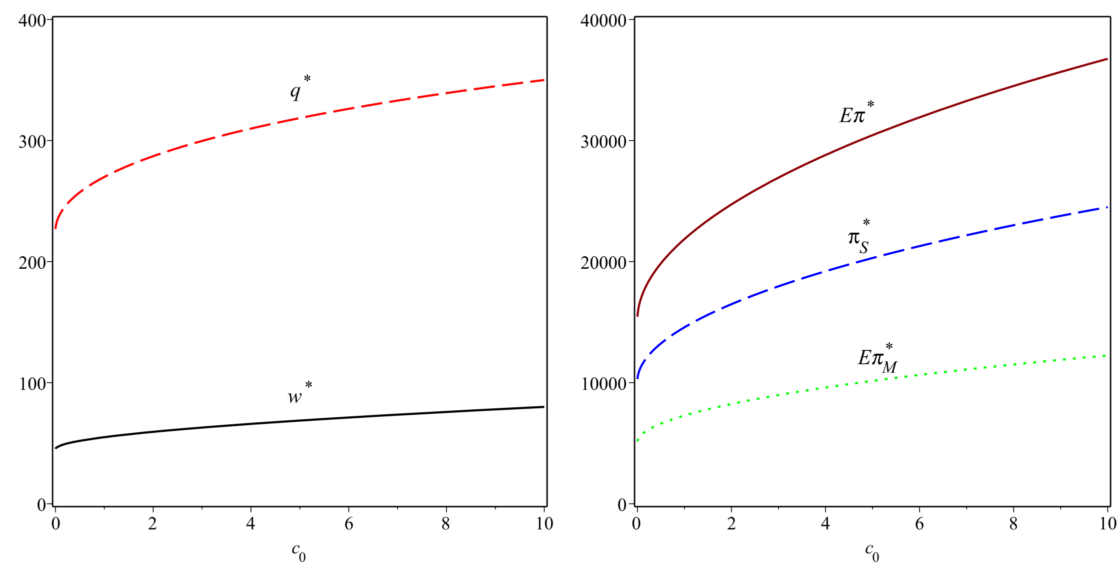

Figure 3. The equilibrium solutions with different c_0 when $a_2 < a_1 \leq 2a_2$.

will more likely choose a supplier with a lower CSR requirement. Although Figure 3 indicates that the OEM would like a high CSR requirement, the condition of Figure 3 indicates that such an intention exists only when consumers' consciousness of the OEM's social responsibility is not so high, i.e. $a_2 \leq a_1 \leq 2a_2$. Therefore, the comparison between Figures 2 and 3 explains why most famous brands such as Apple and Dell prefer to outsource their production to oversea suppliers with a low CSR requirement. When consumers in developed countries are more conscious of the social responsibility of their firms, OEMs are more likely to outsource their production to oversea suppliers with lower CSR requirements.

Furthermore, Figures 2 and 3 illustrate that the supplier's profit will be much higher than the OEM's profit either when the CSR consciousness of consumers is not so high (i.e. $a_2 \leq a_1 \leq 2a_2$) or when both the CSR consciousness of consumers and the society's basic CSR requirement are high (i.e. $a_1 > 2a_2$ and $c_0 > (a_1 - 2a_2)^2/b^2K$). On the contrary, when the CSR consciousness of consumers is high and the society's basic CSR requirement is low, the OEM's profit will be much higher than the supplier's profit. This demonstrates that the CSR consciousness in a famous brand's country or area, plays an important role in the brand's quest to earn a high profit margin in the current CSR supply chain.

3.2 *Centralised decisions: a benchmark*

While Section 3.1 shows the equilibrium solutions of the two channel members when they make independent decisions, in this subsection we establish a performance benchmark by considering the case that the two members act as an integrated firm.

When the demand is realised, we denote it as $D = a - bp$ where a could either be a_1 or a_2, so the integrated system's profit function for any given production quantity q is

$$\pi = p\min(a - bp, q) - cq. \tag{19}$$

Under such a condition, the system's optimal retail price is exactly as given in Equation (7), under which the optimal profit is

$$\pi = \begin{cases} \frac{aq-q^2}{b} - cq & \text{if} \quad 0 \le q \le \frac{a}{2} \\ \frac{a^2}{4b} - cq & \text{if} \quad q > \frac{a}{2}. \end{cases} \tag{20}$$

Here, Equation (20) has the same form as Equation (8) but w is replaced with c.

Consequently, when the integrated system determines its production quantity and its CSR cost, its objective function is

$$E\pi = (1-\xi)\max_p\left[p\min(a_1 - bp, q) - cq\right] + \xi\max_p\left[p\min(a_2 - bp, q) - cq\right], \tag{21}$$

where $\xi = 1 - \sqrt{c/K}$ is given by Equation (3).

Maximising Equation (21) with constraint of $c \ge c_0$ and $q \ge 0$, we have the following proposition.

Proposition 3 *Let* $A = 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2Kc_0}$ *and* $B = 2bK + \sqrt{a_2^2 + b^2K^2}$. *When the OEM and the supplier integrate as a single firm, their optimal production quantity is*

$$\tilde{q} = \begin{cases} \frac{(a_1-a_2)\sqrt{c_0/K}}{2} + \frac{a_2-bc_0}{2} & \text{if} \quad a_2 < a_1 \le a_2 + b\sqrt{Kc_0} \\ \frac{a_1}{2} - \frac{b\sqrt{Kc_0}}{2} & \text{if} \quad a_2 + b\sqrt{Kc_0} < a_1 \le A \\ \frac{a_1}{6} + \frac{\sqrt{a_1^2+3a_2^2}}{6} & \text{if} \quad A < a_1 \le B \\ \frac{a_1}{2} - \frac{bK}{2} & \text{if} \quad a_1 > B, \end{cases} \tag{22}$$

and their optimal decision on the CSR cost is

$$\tilde{c} = \begin{cases} c_0 & \text{if} \quad a_2 < a_1 \le A \\ \frac{1}{K}\left(\frac{2a_1-\sqrt{a_1^2+3a_2^2}}{3b}\right)^2 & \text{if} \quad A < a_1 \le B \\ K & \text{if} \quad a_1 > B, \end{cases} \tag{23}$$

under which the system's expected profit is

$$E\tilde{\pi} = \begin{cases} \frac{((a_1-a_2)\sqrt{c_0/K}+a_2-bc_0)^2}{4b} & \text{if} \quad a_2 < a_1 \le a_2 + b\sqrt{Kc_0} \\ \frac{(a_1^2-a_2^2)\sqrt{c_0/K}}{4b} + \frac{bc_0\sqrt{Kc_0}}{4} - \frac{a_1c_0}{2} + \frac{a_2^2}{4b} & \text{if} \quad a_2 + b\sqrt{Kc_0} < a_1 \le A \\ \frac{(a_1^2+3a_2^2)^{\frac{3}{2}}+a_1(a_1^2-9a_2^2)}{54b^2K} + \frac{a_2^2}{4b} & \text{if} \quad A < a_1 \le B \\ \frac{a_1^2-a_2^2}{4b} + \frac{bK^2}{4} - \frac{a_1K}{2} + \frac{a_2^2}{4b} & \text{if} \quad a_1 > B. \end{cases} \tag{24}$$

The proof of Proposition 3 is provided in Appendix 1.

4. The flexible quantity contract: the role of a tough OEM

In Section 3.1, a dilemma was observed between the supplier S and the OEM M under the push wholesale price contract – although the two members could both benefit from a high CSR, the supplier will always manufacture the products with the minimum CSR cost c_0 rather than a higher CSR cost, under which both channel members could obtain less profit. In this section, we show that a tough OEM may resolve such a dilemma.

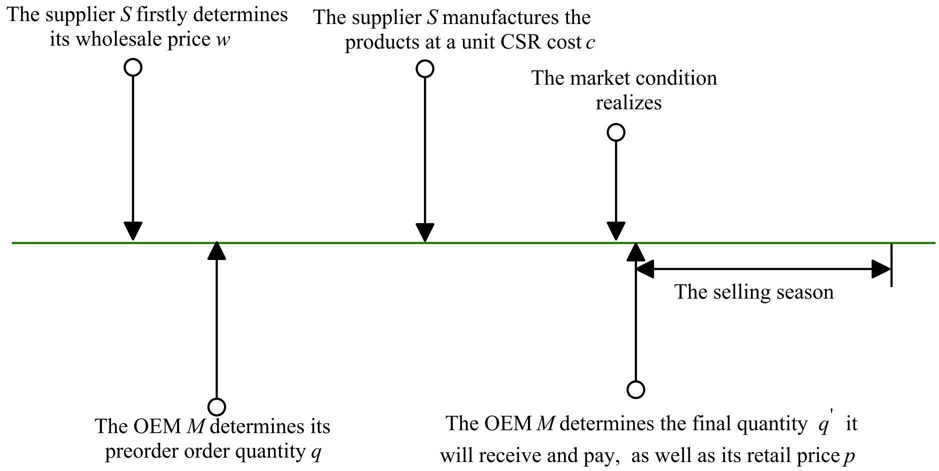

Figure 4. The sequence of events under the flexible quantity contract.

Specifically, we consider the case that the OEM is so tough that it can change its order quantity from q to any q' which is less than q when the demand is realised. Such a case commonly exists between the famous brands and their suppliers in developing countries. To cover such written and unwritten arrangements, we say that the supplier provides the OEM with a flexible quantity contract so that the OEM can change the order quantity without any cost. The supplier S, owing to its inferior status, has to produce a quantity of products sufficient to meet all the contractually possible requirements from the OEM. That is to say, despite the fact that supplier's production quantity is determined by the OEM, the supplier has to undertake the entire market risk due to possible production misconduct.

The sequence of events is shown in Figure 4. The supplier S firstly announces its wholesale price w. Then the OEM M determines its order quantity q. The supplier has to manufacture enough products to meet the OEM's possible requirement in future, with a unit CSR cost c. The demand is realised at the beginning of the selling season. The OEM eventually determines the quantity q' for sales and pays the supplier wq' for its products. Here, the only difference to that in the decentralised case shown in Figure 1 is that the OEM can change its order quantity from q to any $q' \leq q$ just before the selling season.

Let q_1 (q_2) be the OEM's optimal sales when no negative misconduct occurs (one or more harmful events happen) conditioning that the OEM can purchase the products after the demand is realised. It is obvious that $0 < q_2 \leq q_1$. Denoting w as the wholesale price, the OEM's objective function is

$$\pi_M = (p - w)(a_i - bp), \quad i = 1 \text{ or } 2. \tag{25}$$

By maximising Equation (25) we obtain the OEM's optimal retail price as $\hat{p}_i = (a_i + bw)/2b$ $(i = 1, 2)$, under which $q_1 = (a_1 - bw)/2$ and $q_2 = (a_2 - bw)/2$.

Under the flexible quantity contract, although the OEM could theoretically obligate the supplier to any q large enough so that the OEM will not take any demand risk, we should note that the OEM's choice q will equal q_1. In fact, since the supplier also knows the maximum potential market a_1, it can anticipate the OEM's optimal and maximum order quantity q_1. Thus, any choice of q larger than q_1 is unbelievable –in such a case the supplier would prepare q_1 unit of products at most. Also, given the order q_1, the OEM will not change the delivery amount when no negative CSR events happen, but when negative events do occur, it will change its order from q_1 to q_2. Here, $q_1 = (a_1 - bw)/2$ and $q_2 = (a_2 - bw)/2$.

With the OEM's decisions of q_1 and q_2, when the supplier determines its CSR cost c, its objective function becomes

$$E\pi_S = [(1 - \xi)q_1 + \xi q_2]w - cq_1, \tag{26}$$

where $\xi = 1 - \sqrt{c/K}$ given by Equation (3).

Maximising Equation (26) we obtain the supplier's optimal decision on its CSR cost c for any wholesale price w, which is shown in Proposition 4.

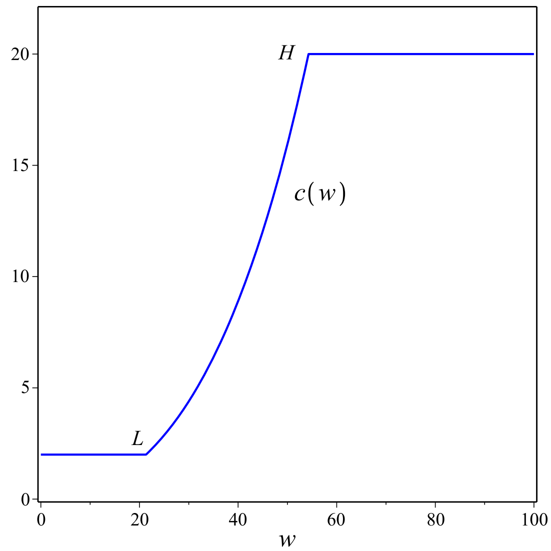

Figure 5. The equilibrium solutions with different w.

Proposition 4 *Under the flexible quantity contract, for any wholesale price w, the supplier's optimal decision on its CSR cost c is*

$$c^T(w) = \begin{cases} c_0 & \text{if} \quad 0 \le w \le \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} \\ \frac{w^2(a_1-a_2)^2}{4K(a_1-bw)^2} & \text{if} \quad \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} < w \le \frac{2a_1K}{2bK+a_1-a_2} \\ K & \text{if} \quad w > \frac{2a_1K}{2bK+a_1-a_2}, \end{cases} \tag{27}$$

under which the supplier's profit is

$$E\pi_S^T(w) = \begin{cases} \frac{w(a_1-a_2)\sqrt{c_0/K}}{2} + \frac{(a_2+bc_0)w}{2} - \frac{bw^2}{2} + \frac{a_1c_0}{2} & \text{if} \quad 0 \le w \le \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} \\ \frac{w^2(a_1-a_2)^2}{(a_1-bw)8K} + \frac{(a_2-bw)w}{2} & \text{if} \quad \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} < w \le \frac{2a_1K}{2bK+a_1-a_2} \\ \frac{(a_1-bw)(w-K)}{2} & \text{if} \quad w > \frac{2a_1K}{2bK+a_1-a_2}, \end{cases} \tag{28}$$

the OEM's expected profit is

$$E\pi_M^T(w) = \begin{cases} \frac{(a_1+a_2-2bw)(a_1-a_2)\sqrt{c_0/K}+(a_2-bw)^2}{4b} & \text{if} \quad 0 \le w \le \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} \\ \frac{(a_1+a_2-2bw)(a_1-a_2)^2w}{8bK(a_1-bw)} + \frac{(a_2-bw)^2}{4b} & \text{if} \quad \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} < w \le \frac{2a_1K}{2bK+a_1-a_2} \\ \frac{(a_1-bw)^2}{4b} & \text{if} \quad w > \frac{2a_1K}{2bK+a_1-a_2}, \end{cases} \tag{29}$$

and the expected profit of the entire supply chain is

$$E\pi^T(w) = \begin{cases} \frac{(a_1^2-a_2^2)\sqrt{c_0/K}+a_2^2}{4b} + \frac{2(bw-a_1)c_0-bw^2}{4} & \text{if} \quad 0 \le w \le \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} \\ \frac{(a_2-bw-a_1)(a_1-a_2)^2w+2Kb^2w^3-2Kb^2w^2}{8bK(a_1-bw)} + \frac{a_2^2}{4b} & \text{if} \quad \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} < w \le \frac{2a_1K}{2bK+a_1-a_2} \\ \frac{(a_1-bw)(a_1+bw-2Kb)}{4b} & \text{if} \quad w > \frac{2a_1K}{2bK+a_1-a_2}. \end{cases} \tag{30}$$

The proof of Proposition 4 is provided in Appendix 1.

While Proposition 1 indicates the supplier S will always utilise the minimum CSR cost c_0 under the push wholesale price, Equation (27) demonstrates that under the flexible quantity contract the supplier's CSR cost c is mainly determined by its wholesale price w. As shown in Figure 5, when the wholesale price is low, i.e. $0 \le w \le \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2}$, the supplier will still use a minimum CSR cost c_0. However, such a case will change as the wholesale price increases. Specifically, when the wholesale price is extremely high, i.e. $w > \frac{2a_1K}{2bK+a_1-a_2}$, it will set its CSR cost $c = K$; and for a moderate wholesale price, i.e. $\frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} < w \le \frac{2a_1K}{2bK+a_1-a_2}$, the supplier's CSR cost c increases in w.

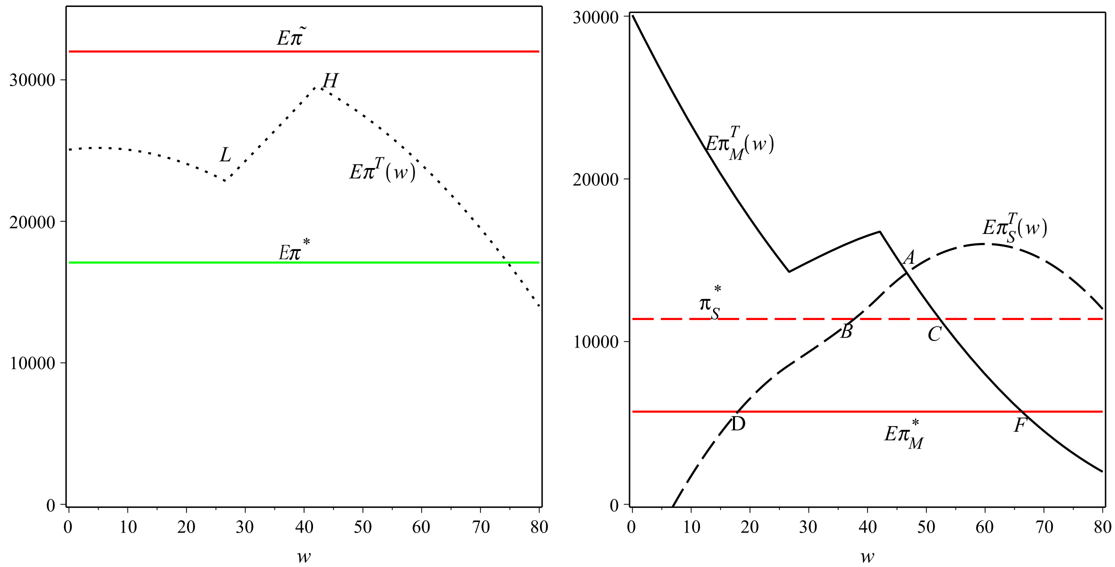

Figure 6. The equilibrium solutions with different w.

Since the supplier's CSR cost depends on the wholesale price now, the flexible quantity contract should solve the dilemma mentioned in Section 3.1, and the supply chain performance should be improved. To show the performance of the flexible quantity contract, we vary w from 0 to 80 while keeping other parameters fixed ($a_1 = 2000$, $a_2 = 900$, $b = 20$, $K = 20$ and $c_0 = 5$) and draw the profits of the supplier, the OEM and the entire supply chain in Figure 6. In Figure 6, π_S^*, $E\pi_M^*$ and $E\pi^*$ are the equilibrium profit of the supplier, the equilibrium expected profit of the OEM and that of the entire supply chain for the decentralised case, $E\pi_S^T(w)$, $E\pi_M^T(w)$ and $E\pi^T(w)$ are those under the flexible quantity contract, and $E\tilde{\pi}$ is the optimal supply chain profit under the centralised case.

From Figure 6 we can find that: (i) although the supply chain performance is lower than that under the centralised case, in a large range of the wholesale price w the supply chain's total profit is much higher than that under the decentralised case – the flexible quantity contract can improve the performance of the CSR supply chain significantly; and (ii) when the wholesale price w is between the points B and F, both the supplier and OEM can benefit from the flexible quantity contract (compared with the decentralised case). It should be noted that although we use specific values here, the tendency in Figure 6 is a typical one – we experimented with changing all the parameters using a large variation but found the trends are all similar.

In fact, the flexible quantity contract is effective because of the following two aspects. First, owing to the flexible quantity contract, the OEM can be free from the demand risk and increase its order quantity, which eventually increases the expected sales and the supply chain profit. This explains what we see in Figure 5 and the left part of Figure 6 – when the wholesale price is low, the supplier's CSR cost stays the same as that in the decentralised case, but the profit of the entire supply chain increases. Second, since the supplier has to undertake all the possible misfortune resulted from its misconduct, it has to increase its investment in CSR cost. The rule 'each one should carry his own load' benefits both the supplier and the OEM.

5. The wholesale price incentive contract: the role of a beneficent OEM

While Section 4 indicates that the flexible quantity contract can solve the dilemma that under the wholesale price contract the supplier will always manufacture the products with a minimum CSR cost, we also find the profit of the entire supply chain cannot reach the optimal under the centralised case. In this section, we try to investigate whether a beneficent OEM with an incentive program can do a better job.

In particular, we consider the case that besides the conventional wholesale contract, the OEM M offers the supplier S the following reward program – the OEM will give the supplier an additional reward s if no exposure of negative CSR events occurs during the production period. We call this a wholesale price incentive contract.

The sequence of events is shown in Figure 7. Before the supplier manufactures the products at a unit CSR cost c, the supplier should determine its wholesale price w, and the OEM should decide its unit additional reward s and its order quantity. Then, just as in the push wholesale price contract, the market will be realised at the beginning of the selling season and then

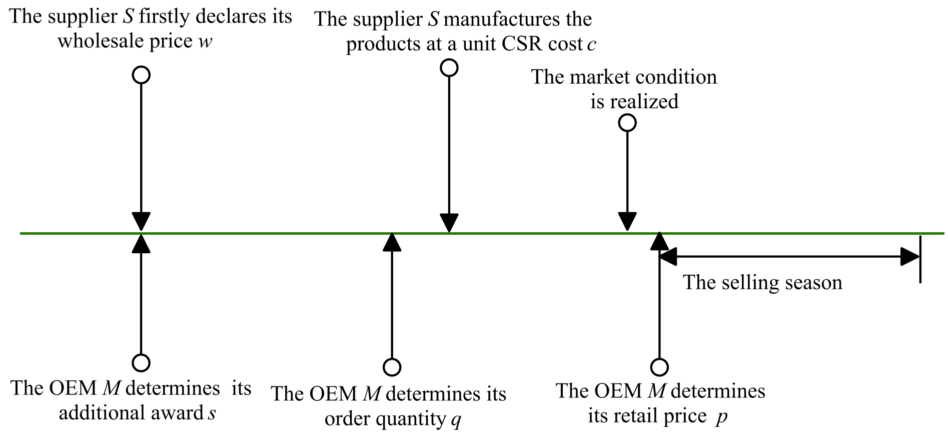

Figure 7. The sequence of events under the wholesale price contract.

the OEM will try to find its optimal retail price to maximise its profit. Here, a difference from the decentralised case is that an additional reward s should also be determined when the supplier declares its wholesale price w. In particular, because the supplier's wholesale price w will be influenced by the reward s significantly, it is more likely that the supplier and the OEM will collaboratively determine (w, s) in a cooperative manner.

While the sequence of decisions on w, s and q may differ, once those three variables are given, the supplier's decision on c, as well as the OEM's pricing strategy on p, will be the same. Specifically, if no negative CSR events occur, the OEM aims to maximise

$$\pi_M = p\min(a_1 - bp, q) - (w + s)q; \tag{31}$$

otherwise, it maximises

$$\pi_M = p\min(a_2 - bp, q) - wq. \tag{32}$$

From Equations (31) and (32) we see that the value of w and s do not influence the OEM's decision on p, the OEM's optimal p will be the same as that given by Equation (7). When the OEM decides its order quantity, its objective function is

$$E\pi_M = \begin{cases} (1-\xi)(\frac{a_1 q - q^2}{b} - sq) + \xi\frac{a_2 q - q^2}{b} - wq & \text{if} \quad 0 \le q \le \frac{a_2}{2} \\ (1-\xi)(\frac{a_1 q - q^2}{b} - sq) + \xi\frac{a_2^2}{4b} - wq & \text{if} \quad \frac{a_2}{2} < q \le \frac{a_1}{2} \\ (1-\xi)(\frac{a_1^2}{4b} - sq) + \xi\frac{a_2^2}{4b} - wq & \text{if} \quad q > \frac{a_1}{2}. \end{cases} \tag{33}$$

When the supplier S decides its CSR cost c, it tries to maximise its expected profit given by the following equation, i.e.

$$E\pi_S = (w - c)q + (1 - \xi)sq, \tag{34}$$

where $\xi = 1 - \sqrt{c/K}$ is the probability that one or more negative CSR events occur. Maximising Equation (34) with the constraint of $c_0 \le c \le K$, we have the following proposition.

Proposition 5 *Under the wholesale price incentive contract, given the wholesale price w, the order quantity q and the additional unit reward s, the supplier S's optimal decision in CSR cost is*

$$c = \begin{cases} c_0 & \text{if} \quad 0 \le s \le 2\sqrt{Kc_0} \\ \frac{s^2}{4K} & \text{if} \quad 2\sqrt{Kc_0} < s \le 2K \\ K & \text{if} \quad s > 2K. \end{cases} \tag{35}$$

The proof of Proposition 5 is provided in Appendix 1.

Proposition 5 demonstrates that the dilemma mentioned in Section 3.1 may also be solved by an incentive program. While under the flexible quantity contract the supplier's CSR cost c depends only on its wholesale price w (see Equation (27)), for this type of contract Equation (35) illustrates that the supplier's CSR cost c depends only on the OEM's unit additional reward s – the OEM's order quantity q and the supplier's wholesale price w do not influence the supplier's decision about its CSR cost c.

While Equation (35) shows that the OEM's reward can affect the supplier's decision on its CSR cost, the following observations indicate that the reward strategy may work at low efficiency. First, the supplier's CSR cost retains at $c = c_0$ when $0 \le s \le 2\sqrt{Kc_0}$, implying that the OEM's reward strategy is entirely in vain under such a condition – its incentive reward s does not affect the supplier's decision about CSR cost c. Second, let $\phi = c/s$, the ratio between the supplier's investment in improving its social responsibility and the OEM's reward. It can be easily proved that $0 < \phi < 1/2$ when $s > 2\sqrt{Kc_0}$. This means that the supplier will use no more than a half of the OEM's reward to improve its social responsibility performance. As a consequence, a doubt arises here about whether the OEM's wholesale reward strategy can work in improving the supply chain performance with social responsibility concerned consumers. To answer this question, we first present the optimal (w, s) contract that maximises the supply chain profit in Proposition 6.

PROPOSITION 6 *Under the wholesale price incentive contract, for any w and s satisfying*

$$w = c_0 - s\sqrt{c_0/K} \text{ and } w \ge 0, \tag{36}$$

the supply chain will obtain its maximum profit, i.e.

$$E\pi^B = \begin{cases} \frac{((a_1-a_2)\sqrt{c_0/K}+a_2-bc_0)^2}{4b} & \text{if } a_2 < a_1 \le a_2 + b\sqrt{Kc_0} \\ \frac{(a_1^2-a_2^2)\sqrt{c_0/K}}{4b} + \frac{bc_0\sqrt{Kc_0}}{4} - \frac{a_1c_0}{2} + \frac{a_2^2}{4b} & \text{if } a_1 > a_2 + b\sqrt{Kc_0}. \end{cases} \tag{37}$$

The proof of Proposition 6 is provided in Appendix 1.

Comparing Equation (37) with Equation (24), we find that when $a_2 < a_1 < 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2Kc_0}$ the supply chain's maximum profit under the wholesale price incentive strategy equals the supply chain's maximum profit in the centralised case. Furthermore, Proposition 6 indicates any (w, s) satisfying Equation (36) can make the supply chain achieve its maximum profit under the wholesale price incentive contract. It may seem that the wholesale price incentive contract can coordinate the CSR supply chain and allocate the supply chain profit between the two members, and do so with high flexibility. Unfortunately, this appearance is false for the following reasons.

First, for any w and s satisfying Equation (36), we have $0 \le s \le \sqrt{Kc_0}$. Under such a condition, Equation (35) indicates that the supplier's optimal decision on its CSR cost is c_0. Substituting $c = c_0$ and $w = c_0 - s\sqrt{c_0/K}$ into Equation (34), we always have $E\pi_S = 0$. This means that even though the wholesale price incentive contract can coordinate the supply chain under certain conditions, a transfer payment from the OEM to the supplier is necessary, otherwise the supplier will never accept such an arrangement because its profit will be zero.

Second, since the supplier's optimal CSR cost is always c_0, just as in the decentralised case, the wholesale price incentive contract cannot solve the dilemma raised in Section 3. In fact, although $E\pi_B = E\tilde{\pi}$ when $a_2 < a_1 < 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2Kc_0}$ holds, we can see under such a condition the supply chain's optimal decision is also $c = c_0$. The reason that the wholesale price incentive contract can achieve the supply chain's maximum profit here is because it transfers the demand risk from the OEM to the supplier, which eventually enlarges the OEM's order quantity.

For clarity we summarise the above conclusion in the corollary below.

COROLLARY 1

(i) *When $a_2 < a_1 < 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2Kc_0}$ is satisfied, with a proper transfer payment, any wholesale price incentive contract satisfying*

$$w = c_0 - s\sqrt{c_0/K} \text{ and } w \ge 0,$$

can coordinate the CSR supply chain;

(ii) *The wholesale price incentive contract cannot solve the CSR supply chain dilemma.*

6. Tough or beneficent: a comparison

In Sections 4 and 5, we introduce two different contracts to improve the CSR supply chain performance, i.e. the flexible quantity contract having the OEM act in a tough way, and the wholesale price incentive contract in which the OEM has a beneficent attitude. In this section, we will compare the two contracts to investigate under what condition one of these will perform better than the other. Since the wholesale price incentive contract can coordinate the supply chain when $a_2 < a_1 < 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2Kc_0}$ but the flexible quantity contract cannot, we have the following corollary.

COROLLARY 2 *When $a_2 < a_1 < 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2Kc_0}$ is satisfied, combined with a proper transfer payment, for any wholesale price w, there always exists a wholesale price incentive contract performing better than any flexible quantity contract.*

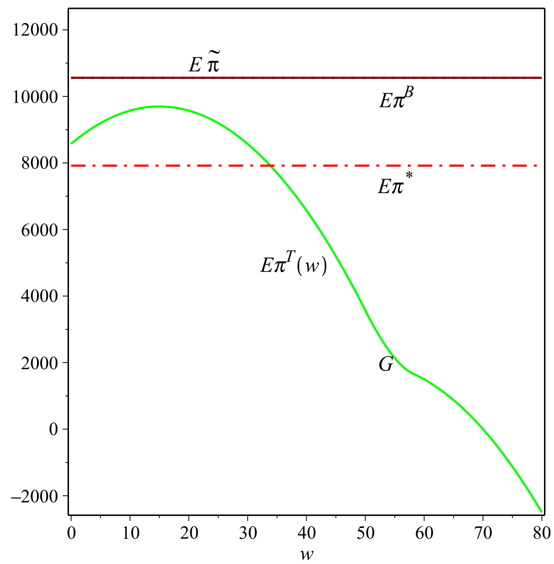

Figure 8. Comparison of the two contracts when $a_2 < a_1 < 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2Kc_0}$.

To illustrate Corollary 2 clearly, we vary w from 0 to 80 and plot the optimal supply chain profits under the two contracts in Figure 8 (other parameters are fixed as $a_1 = 1400$, $a_2 = 1000$, $b = 20$, $K = 50$ and $c_0 = 15$). Figure 8 shows that the supply chain's optimal expected profit under the flexible quantity contract is always lower than that under the wholesale incentive contract.

In fact, under the condition of $a_2 < a_1 < 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2Kc_0}$, the supply chain's optimal CSR cost under the centralised case is $c = c_0$, implying that the supply chain has no incentive to improve its CSR cost and the CSR dilemma does not exist. Thus, the flexible quantity contract looks useless to solve the CSR dilemma under such a condition. On the contrary, the wholesale incentive contract can flexibly change the risk allocation between the two channel members[2] and eventually coordinate the supply chain. Thus, the wholesale incentive contract dominates the flexible quantity contract in this case.

However, when $a_1 \geq 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2Kc_0}$, the situation changes. The flexible quantity contract can solve the CSR dilemma but has limitation in allocating the risk, whereas the wholesale incentive contract can allocate the risk flexibly but cannot solve the CSR dilemma. Which contract will perform better? The answer is no longer apparent.

Because of the mathematical complexity, we cannot provide an analytical answer for the above question. As an alternative, we vary the parameters in a large range, plot the trends of the supply chain profits along with the changing of w under the two contracts, and eventually obtain three patterns as shown in Figure 9. For the three subfigures of Figure 9, all other parameters except a_1 are the same as those in Figure 8, i.e. $a_2 = 1000$, $b = 20$, $K = 50$ and $c_0 = 15$. Figure 9(a) with $a_1 = 2300$ represents the scenario that a_1 is slightly larger than $2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2Kc_0}$, Figure 9(b) with $a_1 = 2900$ is for the scenario that a_1 is moderately higher, and Figure 9(c) with $a_1 = 3500$ shows the trend when a_1 is extremely large.

From Figure 9 we find the following trends. First, as w increases, the supply chain expected profit under the flexible quantity contract, i.e. $E\pi^T$, has two local maximum points, J and H. Second, while $E\pi^T(J)$ is always larger than $E\pi_B$ for all $a_1 \geq 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2Kc_0}$, $E\pi^T(H)$ has an increasing trend when a_1 increases. While the former demonstrates that when $a_1 \geq 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2Kc_0}$ holds, there always exists a w making the flexible quantity contract perform better than the wholesale incentive contract, the latter indicates that the flexible quantity contract will do a better job when a_1 is larger. In fact, under the flexible quantity contract, from Figure 5, we can see the following results. First, the supplier's equilibrium decision in CSR cost is always c_0 when w is small (the left of point L). Although c_0 is smaller than in the centralised case, because the supplier undertakes all the demand risk, the OEM's order quantity will be large enough. This eventually makes the flexible quantity perform better than the wholesale incentive contract. However, we should also note that the OEM's order quantity decreases in w, which is why we see that the supply chain profit $E\pi^T$ decreases in w to the right of point J. Second, when w is between points L and H, the supplier's CSR cost will increase sharply as w increases. Because the CSR dilemma can be solved under such a condition, we see that $E\pi^T$ also increases in w. Figure 9 shows that the flexible

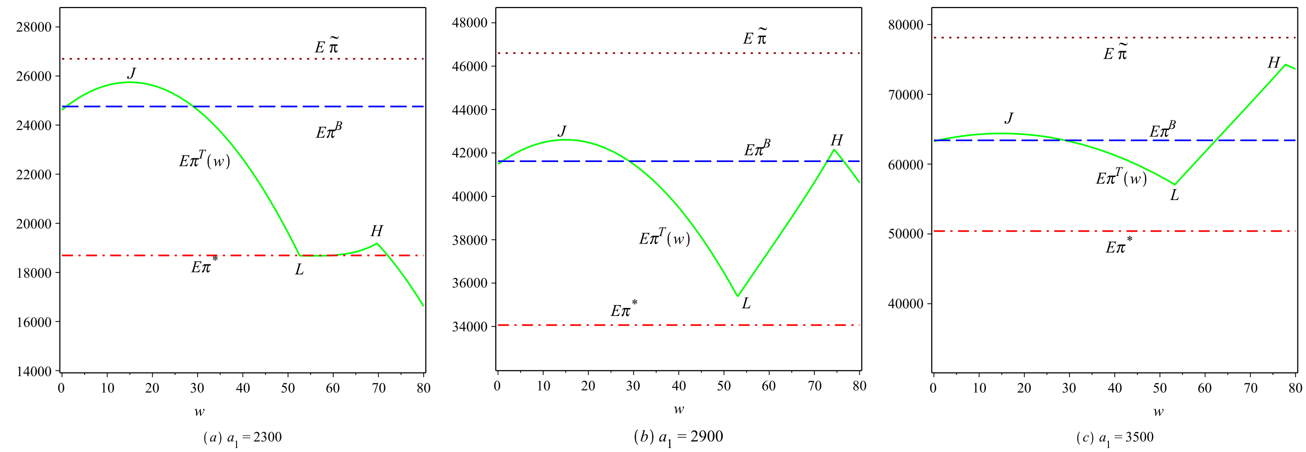

Figure 9. Comparison of the two contracts when $a_1 \geq 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2 K c_0}$.

quantity contract's ability to resolve the CSR dilemma will improve the supply chain profit significantly when a_1 is large. Third, when w is large enough (to the right of point H), the supplier's CSR cost stays the same at K. In such a case, the increasing of w will just exacerbate the double marginalisation effect – $E\pi^T$ is then decreasing in w.

Corollary 3 summarises the above discussions.

Corollary 3 *When $a_1 \geq 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2 K c_0}$ satisfies, there always exists a flexible quantity contract performing better than any wholesale price incentive contract.*

7. Conclusions

Corporate social responsibility (CSR) has become an increasingly vital issue for multinational enterprises. In particular, within the supply chain, a firm's sales and reputation can be deeply damaged by the misconduct of its suppliers.

In this paper, we investigate how a firm should use supply chain contracts to avoid or mitigate a supplier's possible misconduct, focusing on a CSR supply chain consisting of an OEM and an overseas supplier, and introducing a decision model for the CSR supply chain. Our model incorporates these two facts: (i) the possibility of the market disruption can be reduced through a higher CSR investment, and (ii) the society has a minimum requirement for the supplier's CSR. The supplier has to determine its wholesale price and CSR cost, while the OEM has to decide its order quantity and retail price.

A dilemma of the CSR supply chain appears from its equilibrium solutions – the supplier always utilises the minimum CSR cost, which eventually leads to a low supply chain efficiency. This paper considers two contracts, the flexible quantity contract and the wholesale price incentive contract, to solve this problem. Our results show that while both types of contract can improve the supply chain performance, neither contract always dominates the other. A contract dominating in some conditions will be dominated in other conditions. Besides the above dilemma, our results can also explain why, in most cases, the OEM prefers to select a supplier whose host country has a relatively low CSR requirement.

A practical value of our paper is that although most CSR clauses have no actual legal implications and are difficult to apply, our model proves the usefulness of some simple CSR contracts just considering the market results. Specifically, the model indicates that a proper supply chain contract will help the OEM induce its supplier to invest more in its CSR, reduce the possible market disruption, and, moreover, benefit both the OEM and the supplier profits. The CSR contracts which we have proposed are especially suitable for a multinational OEM and its oversea supplier.

Limitations and possible future research of this study include the following. First, we consider that the sales total is only affected by the supplier's CSR cost in the current setting, which implicitly assume that the OEM itself has no CSR misconduct. This assumption could be relaxed if one needs to incorporate the synergy effect of the OEM and the supplier's CSR investment. Second, we use deterministic models in the current paper to avoid the mathematical complexity – an extension of the current work to a newsvendor setting could be an interesting work. Third, because of the difficulties in mathematics, we cannot derive an explicit expression of all the decision variables for the proposed contracts. A simple mathematical model that does not lose the properties of the CSR supply chain should be investigated.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported by the National Natural Science Foundation of China [grant number 71671170], the Youth Fund Project of National Social Science of China [grant number 13CGL156], and the Scientific and Technological Research Program of Chongqing Municipal Education Commission [grant number KJ1400906].

Notes

1. James Epstein-Reeves, 'Consumers Overwhelmingly Want CSR' 15 December 2010, Forbes (online) http://www.forbes.com/sites/csr/2010/12/15/new-study-consumers-demand-companies-implement-csrprograms/.
2. The flexible quantity contract also changes the risk allocation between the two supply chain members, but it lets the supplier undertake all the risk – it cannot allocate the risk with flexibility.

References

Anand, K. S., and M. Goyal. 2009. "Strategic Information Management under Leakage in a Supply Chain." *Management Science* 55 (3): 438–452.

Ang, E., D. Iancu, and R. Swinney. 2016. "Disruption Risk and Optimal Sourcing in Multitier Supply Networks." *Management Science*. doi:10.1287/mnsc.2016.2471.

Cachon, G. P., and M. A. Lariviere. 2005. "Supply Chain Coordination with Revenue-sharing Contracts: Strengths and Limitations." *Management Science* 51 (1): 30–44.

Chopra, S., G. Reinhardt, and U. Mohan. 2007. "The Importance of Decoupling Recurrent and Disruption Risks in a Supply Chain." *Naval Research Logistics (NRL)* 54 (5): 544–555.

Cui, H. T., J. S. Raju, and J. Zhang. 2007. "Fairness and Channel Coordination." *Management Science* 5 (8): 1303–1314.

Dada, M., N. C. Petruzzi, and L. B. Schwarz. 2007. "A Newsvendor's Procurement Problem When Suppliers are Unreliable." *Manufacturing & Service Operations Management* 9 (1): 9–32.

Dong, L., and B. Tomlin. 2012. "Managing Disruption Risk: The Interplay between Operations and Insurance." *Management Science* 58 (10): 1898–1915.

Dumrongsiri, A., M. Fan, A. Jain, and K. Moinzadeh. 2008. "A Supply Chain Model with Direct and Retail Channels." *European Journal of Operational Research* 187 (3): 691–718.

FAZ, Frankfurter Allgemeine Zeitung. 2012. "Verseuchte Erdbee-renlösten Brechdurchfall aus." Accessed October 3. http://www.faz.net/aktuell/gesellschaft/gesundheit/norovirus-erkrankungen-verseuchte-erdbeerenloesten-brechdurchfall-aus-11919056.html

Fombrun, C. J., N. A. Gardberg, and M. L. Barnett. 2000. "Opportunity Platforms and Safety Nets: Corporate Citizenship and Reputational Risk." *Business and Society Review* 105 (1): 85–106.

Guo, L. 2009. "The Benefits of Downstream Information Acquisition." *Marketing Science* 28 (3): 457–471.

Gou, Q., S. Sethi, J. Yue, and J. Zhang. 2016. "Push and Pull Contracts in a Local Supply Chain with an Outside Market." *Decision Sciences* 47 (6): 1150–1177.

Gou, Q., J. Zhang, L. Liang, Z. Huang, and A. Ashley. 2014. "Horizontal Cooperative Programmes and Cooperative Advertising." *International Journal of Production Research* 52 (3): 691–712.

Gupta, A., F. Briscoe, and D. C. Hambrick. 2017. "Red, Blue, and Purple Firms: Organizational Political Ideology and Corporate Social Responsibility." *Strategic Management Journal* 38 (5): 1018–1040.

He, Y., J. Zhang, Q. Gou, and G. Bi. 2017. "Supply Chain Decisions with Reference Quality Effect under the O2O Environment." *Annals of Operations Research*. doi:10.1007/s10479-016-2224-2.

Hendricks, K. B., and V. R. Singhal. 2005. "An Empirical Analysis of the Effect of Supply Chain Disruptions on Long-run Stock Price Performance and Equity Risk of the Firm." *Production and Operations Management* 14 (1): 35–52.

Hou, J., A. Z. Zeng, and L. Zhao. 2010. "Coordination with a Backup Supplier through Buy-back Contract under Supply Disruption." *Transportation Research Part E: Logistics and Transportation Review* 46 (6): 881–895.

Hoyt, D., H. L. Lee, and M. M. Tseng. 2008. *Unsafe for Children: Mattel's Toy Recalls and Supply Chain Management*. Stanford University, Graduate School of Business, Case No. GS63.

Hsueh, C. F. 2014. "Improving Corporate Social Responsibility in a Supply Chain through a New Revenue Sharing Contract." *International Journal of Production Economics* 151: 214–222.

Hsueh, C. F. 2015. "A Bilevel Programming Model for Corporate Social Responsibility Collaboration in Sustainable Supply Chain Management." *Transportation Research Part E: Logistics and Transportation Review* 73: 84–95.

Krishnan, H., R. Kapuscinski, and D. A. Butz. 2004. "Coordinating Contracts for Decentralized Supply Chains with Retailer Promotional Effort." *Management Science* 50 (1): 48–63.

Laudal, T. 2011. "Drivers and Barriers of CSR and the Size and Internationalization of Firms." *Social Responsibility Journal* 7 (2): 234–256.

Li, M., N. Petruzzi, and J. Zhang. 2016. "Overconfident Competing Newsvendors." *Management Science*. doi:10.1287/mnsc.2016.2469.

Li, M., S. P. Sethi, and J. Zhang. 2016. "Competing with Bandit Supply Chains." *Annals of Operations Research* 240 (2): 617–640.

Liu, X., Q. Gou, L. Alwan, and L. Liang. 2016. "Option Contracts: A Solution for Overloading Problems in the Delivery Service Supply Chain." *Journal of the Operational Research Society* 67 (2): 187–197.

Lodree Jr, E. J., and S. Taskin. 2008. "An Insurance Risk Management Framework for Disaster Relief and Supply Chain Disruption Inventory Planning." *Journal of the Operational Research Society* 59 (5): 674–684.

Lu, W. M., W. K. Wang, and H. L. Lee. 2013. "The Relationship between Corporate Social Responsibility and Corporate Performance: Evidence from the US Semiconductor Industry." *International Journal of Production Research* 51 (19): 5683–5695.

Luo, X., and C. B. Bhattacharya. 2006. "Corporate Social Responsibility, Customer Satisfaction, and Market Value." *Journal of Marketing* 70 (4): 1–18.

McWilliams, A., D. S. Siegel, and P. M. Wright. 2006. "Corporate Social Responsibility: Strategic Implications." *Journal of Management Studies* 43 (1): 1–18.

Mellahi, K. 2013. *Measuring CSR is Difficult and is not Always Desirable*. London: University Business School of Warwick University Pres. Erifrim tarihi June 30, 2014. http://www.wbs.ac.uk/news/measuring-csr-comes-with-problems-and-is-not-always-desirable/

Meuffels, W. J. M., H. A. Fleuren, F. C. A. M. Cruijssen, and E. R. Van Dam. 2010. "Enriching the Tactical Network Design of Express Service Carriers with Fleet Scheduling Characteristics." *Flexible Services and Manufacturing Journal* 22 (1–2): 3–35.

Moir, L. 2001. "What Do We Mean by Corporate Social Responsibility?" *Corporate Governance: The International Journal of Business in Society* 1 (2): 16–22.

Moorthy, K. S. 1987. "Comment Managing Channel Profits: Comment." *Marketing Science* 6 (4): 375–379.

Ni, D., K. W. Li, and X. Tang. 2010. "Social Responsibility Allocation in Two-echelon Supply Chains: Insights from Wholesale Price Contracts." *European Journal of Operational Research* 207 (3): 1269–1279.

Pasternack, B. A. 2002. "Using Revenue Sharing to Achieve Channel Coordination for a Newsboy Type Inventory Model." In *Supply Chain Management: Models, Applications, and Research Directions*, edited by J. Geunes, P. M. Pardalos, and H. Edwin Romeijn, 117–136. London: Springer US.

Pasternack, B. A. 2008. "Optimal Pricing and Return Policies for Perishable Commodities." *Marketing Science* 27 (1): 133–140.

Porter, M. E., and M. R. Kramer. 2006. "The Link between Competitive Advantage and Corporate Social Responsibility." *Harvard Business Review* 84 (12): 78–92.

Sawik, T. 2013. "Integrated Selection of Suppliers and Scheduling of Customer Orders in the Presence of Supply Chain Disruption Risks." *International Journal of Production Research* 51 (23–24): 7006–7022.

Schmitt, A. J., L. V. Snyder, and Z. J. M. Shen. 2010. "Inventory Systems with Stochastic Demand and Supply: Properties and Approximations." *European Journal of Operational Research* 206 (2): 313–328.

Sen, S., and C. B. Bhattacharya. 2001. "Does Doing Good Always Lead to Doing Better? Consumer Reactions to Corporate Social Responsibility." *Journal of Marketing Research* 38 (2): 225–243.

Tang, Christopher S. 2008. "Making Products Safe: Process and Challenges." *International Commerce Review* 8 (1): 48–55.

Taylor, T. A. 2002. "Supply Chain Coordination under Channel Rebates with Sales Effort Effects." *Management Science* 48 (8): 992–1007.

Tomlin, B. 2006. "On the Value of Mitigation and Contingency Strategies for Managing Supply Chain Disruption Risks." *Management Science* 52 (5): 639–657.

Tsay, A. A. 1999. "The Quantity Flexibility Contract and Supplier-customer Incentives." *Management Science* 45 (10): 1339–1358.

Tsay, A. A., and W. S. Lovejoy. 1999. "Quantity Flexibility Contracts and Supply Chain Performance." *Manufacturing & Service Operations Management* 1 (2): 89–111.

Wang, N., Q. Gou, X. Fang, and L. Liang. 2017. "Supply Chain Performance and Coordination under Two Wholesale Price Contracts with the Market Disruption." *International Transactions in Operational Research* 24 (4): 713–736.

Wang, C. X., and S. Webster. 2009. "The Loss-averse Newsvendor Problem." *Omega: The International Journal of Management Science* 37 (1): 93–105.

Wu, X., and J. A. Niederhoff. 2014. "Fairness in Selling to the Newsvendor." *Production and Operations Management* 23 (11): 2002–2022.

Xia, Y., X. Zu, and C. Shi. 2015. "A Profit-driven Approach to Building a "People-responsible" Supply Chain." *European Journal of Operational Research* 241 (2): 348–360.

Xiao, T., X. Qi, and G. Yu. 2007. "Coordination of Supply Chain after Demand Disruptions When Retailers Compete." *International Journal of Production Economics* 109 (1): 162–179.

Yang, Z., G. Aydin, V. Babich, and D. R. Beil. 2009. "Supply Disruptions, Asymmetric Information, and a Backup Production Option." *Management Science* 55 (2): 192–209.

Yu, H., A. Z. Zeng, and L. Zhao. 2009. "Single or Dual Sourcing: Decision-making in the Presence of Supply Chain Disruption Risks." *Omega: The International Journal of Management Science* 37 (4): 788–800.

Yue, J., J. Austin, M. C. Wang, and Z. Huang. 2006. "Coordination of Cooperative Advertising in a Two-level Supply Chain When Manufacturer Offers Discount." *European Journal of Operational Research* 168 (1): 65–85.

Zhang, J., Q. Gou, L. Liang, and Z. Huang. 2013. "Supply Chain Coordination through Cooperative Advertising with Reference Price Effect." *Omega: The International Journal of Management Science* 41 (2): 345–353.

Zhang, J., Q. Gou, J. Zhang, and L. Liang. 2014. "Supply Chain Pricing Decisions with Price Reduction during the Selling Season." *International Journal of Production Research* 52 (1): 165–187.
Zhang, W. G., J. Fu, H. Li, and W. Xu. 2012. "Coordination of Supply Chain with a Revenue-sharing Contract under Demand Disruptions When Retailers Compete." *International Journal of Production Economics* 138 (1): 68–75.
Zhang, Y., J. Li, and Q. Gou. 2016. "An Allocation Game Model with Reciprocal Behavior and its Applications in Supply Chain Pricing Decisions." *Annals of Operations Research*. doi:10.1007/s10479-016-2165-9.
Zhao, F., D. Wu, L. Liang, and A. Dolgui. 2016. "Lateral Inventory Transshipment Problem in Online-to-Offline Supply Chain." *International Journal of Production Research* 54 (7): 1951–1963.

Appendix 1

Proof of Lemma 1

Proof. When the market demand is realised, the OEM has to find an optimal p to maximise the following objective function, i.e.

$$\pi_M(p) = p\min(D, q) - wq, \tag{A1}$$

where $D = a - bp$ and $a = a_1$ or a_2.

The OEM could set either a price low enough, i.e. $0 \leq p \leq (a-q)/b$, leading to the situation that the demand is larger than its order quantity q $(D \geq q)$, or a price high enough to lead to $D < q$. Thus, Equation (A1) turns into the following,

$$\pi_M(p) = \begin{cases} pq - wq & \text{if } 0 \leq p \leq \frac{a-q}{b} \\ (a-bp)p - wq & \text{if } \quad p > \frac{a-q}{b}. \end{cases} \tag{A2}$$

It is easy to examine that the function $\pi_M(p)$ in Equation (A2) is a continuous function. Also, we have

$$\frac{\partial \pi_M}{\partial p} = \begin{cases} q & \text{if } 0 \leq p \leq \frac{a-q}{b} \\ a - 2bp & \text{if } \quad p > \frac{a-q}{b}, \end{cases} \tag{A3}$$

and

$$\frac{\partial^2 \pi_M}{\partial p^2} = \begin{cases} 0 & \text{if } 0 \leq p \leq \frac{a-q}{b} \\ -2b & \text{if } \quad p > \frac{a-q}{b}. \end{cases} \tag{A4}$$

Equations (A3) and (A4) indicate $\pi_M(p)$ increases linearly in p when $0 \leq p \leq (a-q)/b$ and is a concave function when $p > (a-q)/b$. Letting $a - 2bp = 0$ we obtain $\bar{p} = a/2b$, which is the point leading to the maximum value for the function $(a-bp)p - wq$ without any constraints.

Thus, we have the following: (i) when $\bar{p} \leq (a-q)/b$, i.e. $q \leq a/2$, $\pi_M(p)$ obtains its maximum value at $p = (a-q)/b$, under which $\pi_M = aq - q^2/b - wq$; (ii) when $\bar{p} > (a-q)/b$, i.e. $q > a/2$, $\pi_M(p)$ obtains its maximum value at $p = a/2b$, under which $\pi_M = a^2/4b - wq$. □

Proof of Proposition 1

Proof. Given the value of w and q, the supplier determines the optimal c to maximise $\pi_S = (w-c)q$ with constraint $c \geq c_0$. Since $\partial\pi_S/\partial c = -q < 0$, the supplier would always prefer a smaller c. As a consequence, its optimal decision in CSR cost is $c = c_0$. Substituting $c = c_0$ into Equation (3), we have $\xi_0 = 1 - \sqrt{c_0/K}$. □

Proof of Proposition 2

Proof. When the OEM determines its order quantity, its objective function is given by Equation (9), and the supplier aims to maximise its profit given by Equation (10) when it decides its wholesale price w. Here, the supplier acts as a leader and the OEM as a follower.

We use backward induction to solve the problem. We first derive the OEM's optimal decision on q given the supplier's wholesale price w.

First, differentiating the objective function of Equation (9) with respective to q, we have

$$\frac{\partial E\pi_M}{\partial q} = \begin{cases} \frac{(1-\xi_0)a_1+\xi_0 a_2}{b} - w - \frac{2q}{b} & \text{if } \quad 0 \leq q \leq \frac{a_2}{2} \\ \frac{(1-\xi_0)a_1}{b} - w - \frac{2(1-\xi_0)q}{b} & \text{if } \quad \frac{a_2}{2} < q \leq \frac{a_1}{2} \\ -w & \text{if } \quad q > \frac{a_1}{2}, \end{cases} \tag{A5}$$

and

$$\frac{\partial^2 E\pi_M}{\partial q^2} = \begin{cases} -\frac{2}{b} & \text{if } \quad 0 \leq q \leq \frac{a_2}{2} \\ -\frac{2(1-\xi_0)}{b} & \text{if } \frac{a_2}{2} < q \leq \frac{a_1}{2} \\ 0 & \text{if } \quad q > \frac{a_1}{2}. \end{cases} \tag{A6}$$

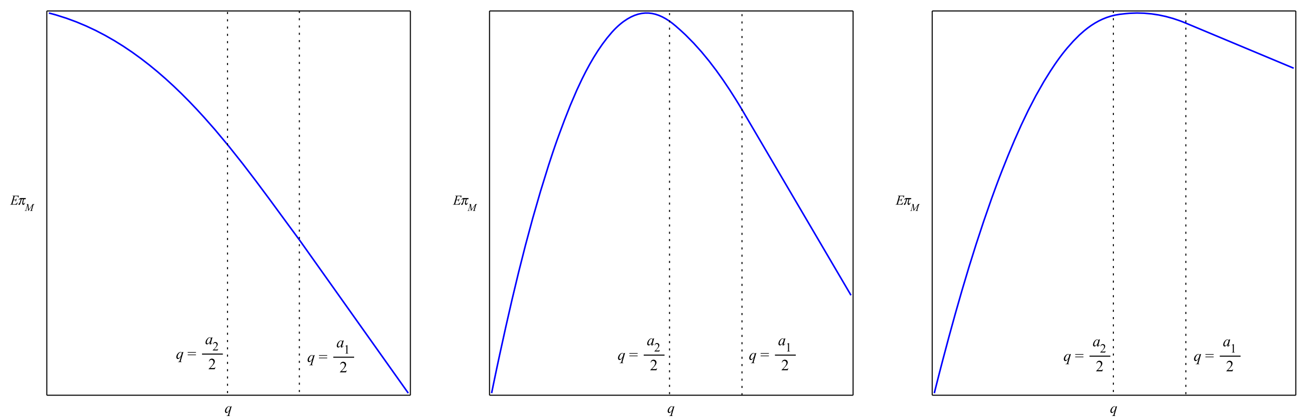

Figure A1. The equilibrium solutions with different q.

Second, we note that the objective function in Equation (9) and $\partial E\pi_M/\partial q$ in Equation (A5) are both continuous functions of q. Also, Equation (A6) indicates that $\partial E\pi_M/\partial q$ is a nonincreasing function of q. With the above properties of $E\pi_M$, and noting $a < b$, we can conclude that:

If $\frac{\partial E\pi_M}{\partial q}|_{q=a} \geq 0$ *and* $\frac{\partial E\pi_M}{\partial q}|_{q=b} < 0$, *then the point leading to the maximum value of* $E\pi_M$ *must be located in the internal* $[a, b]$.

Consequently, the OEM's optimal q can belong to the following three cases (see Figure A1).

Case 1: When $\frac{\partial E\pi_M}{\partial q}|_{q=0} < 0$, (i.e. $w > \frac{(1-\xi_0)a_1+\xi_0 a_2}{b}$), the optimal order quantity is $q = 0$.

Case 2: When $\frac{\partial E\pi_M}{\partial q}|_{q=0} > 0$ and $\frac{\partial E\pi_M}{\partial q}|_{q=a_2/2} < 0$ (i.e. $\frac{(1-\xi_0)a_1+\xi_0 a_2}{b} < w \leq \frac{(1-\xi_0)(a_1-a_2)}{b}$), the optimal order quantity q falls in $[0, a_2/2]$, and $q = \frac{(1-\xi_0)a_1+\xi_0 a_2}{2} - \frac{wb}{2}$.

Case 3: When $\frac{\partial E\pi_M}{\partial q}|_{q=a_2/2} \geq 0$ and $\frac{\partial E\pi_M}{\partial q}|_{q=a_1/2} < 0$, (i.e. $0 \leq w \leq \frac{(1-\xi_0)(a_1-a_2)}{b}$), the optimal order quantity $q = \frac{a_1}{2} - \frac{bw}{2(1-\xi_0)}$.

Summarising the above three cases, we obtain the OEM's optimal order quantity q in response to the supplier's wholesale price w, i.e.

$$q(w) = \begin{cases} \frac{a_1}{2} - \frac{bw}{2(1-\xi_0)} & \text{if} \quad 0 \leq w \leq \frac{(1-\xi_0)(a_1-a_2)}{b} \\ \frac{(1-\xi_0)a_1+\xi_0 a_2}{2} - \frac{wb}{2} & \text{if} \quad \frac{(1-\xi_0)(a_1-a_2)}{b} < w \leq \frac{(1-\xi_0)a_1+\xi_0 a_2}{b} \\ 0 & \text{if} \quad w > \frac{(1-\xi_0)a_1+\xi_0 a_2}{b}. \end{cases} \tag{A7}$$

Substituting Equation (A7) into Equation (10), we obtain the supplier's objective function when it determines its wholesale price, given that it can anticipate the OEM's best response, i.e.

$$\pi_S(w) = \begin{cases} \frac{(w-c_0)[(1-\xi_0)a_1 - wb]}{2(1-\xi_0)} & \text{if} \quad 0 \leq w \leq \frac{(1-\xi_0)(a_1-a_2)}{b} \\ \frac{(w-c_0)[(1-\xi_0)a_1+\xi_0 a_2 - wb]}{2} & \text{if} \quad \frac{(1-\xi_0)(a_1-a_2)}{b} < w \leq \frac{(1-\xi_0)a_1+\xi_0 a_2}{b} \\ 0 & \text{if} \quad w > \frac{(1-\xi_0)a_1+\xi_0 a_2}{b}. \end{cases} \tag{A8}$$

For convenience, we denote $w_0 = \frac{(1-\xi_0)(a_1-a_2)}{b}$ and $w_1 = \frac{(1-\xi_0)a_1+\xi_0 a_2}{b}$ for temporary use here. We can see that π_S is continuous at points $w = w_0$ and $w = w_1$. Differentiating Equation (A8) with respect to w we have

$$\frac{\partial \pi_S}{\partial w} = \begin{cases} \frac{a_1}{2} + \frac{(c_0-2w)b}{2(1-\xi_0)} & \text{if} \quad 0 \leq w \leq w_0 \\ \frac{(1-\xi_0)a_1+\xi_0 a_2}{2} + \frac{(c_0-2w)b}{2} & \text{if} \quad w_0 < w \leq w_1 \\ 0 & \text{if} \quad w > w_1, \end{cases} \tag{A9}$$

and

$$\frac{\partial^2 \pi_S}{\partial w^2} = \begin{cases} \frac{-b}{1-\xi_0} & \text{if} \quad 0 \leq w \leq w_0 \\ -b & \text{if} \quad w_0 < w \leq w_1 \\ 0 & \text{if} \quad w > w_1. \end{cases} \tag{A10}$$

From Equations (A9) and (A10), we have $\frac{\partial \pi_S}{\partial w}|_{w=0} > 0$ and $\partial^2 \pi_S/\partial w^2 < 0$ when $0 \leq w \leq w_1$. Furthermore, we can verify $\frac{\partial \pi_S}{\partial w}|_{w=w_0^-} > \frac{\partial \pi_S}{\partial w}|_{w=w_0^+}$. These two facts illustrate that $\partial \pi_S/\partial w$ is a monotone decreasing function of w. In addition, we can also see that $\frac{\partial \pi_S}{\partial w}|_{w=0} = \frac{a_1}{2} + \frac{c_0 b}{2(1-\xi_0)} > 0$ and $\frac{\partial \pi_S}{\partial w}|_{w=w_1^-} = -\frac{(a_1-a_2)\sqrt{c_0/K}}{2} - \frac{a_2 - bc_0}{2} < 0$.

Thus, similar to the way seen in the proof for deriving the OEM's optimal order quantity q, the supplier's optimal wholesale price will belong to the following three cases (see Figure A2).

Case 1: When $\frac{\partial \pi_S}{\partial w}|_{w=w_0^-} \leq 0$ (i.e. $a_1 > 2a_2 + b\sqrt{c_0 K}$), the optimal wholesale price $w^* = \frac{(1-\xi_0)a_1+bc_0}{2b}$ which satisfies the first-order condition must be located in $[0, w_0]$ and be the maximum point.

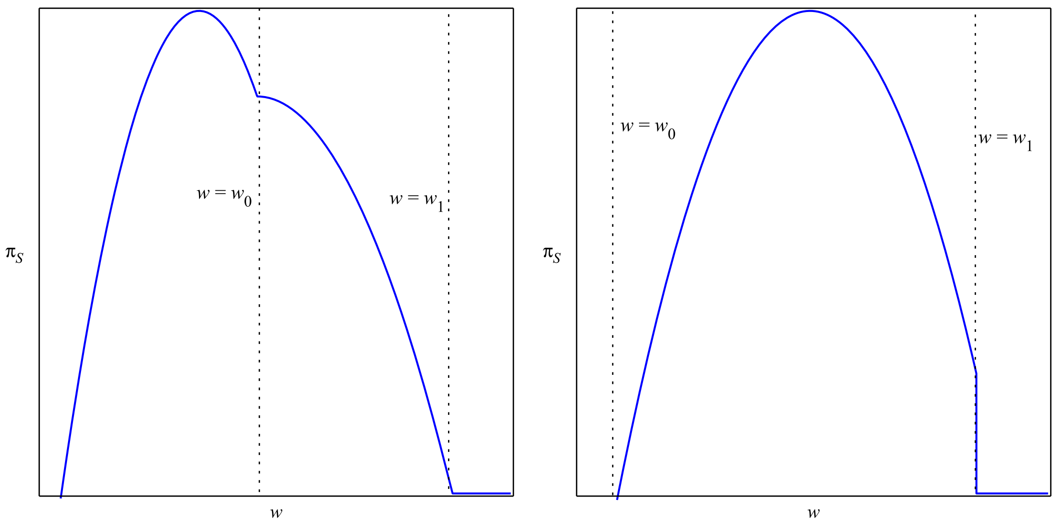

Figure A2. The equilibrium solutions with different w.

Case 2: If $\frac{\partial \pi_S}{\partial w}|_{w=w_0^-} > 0$ and $\frac{\partial \pi_S}{\partial w}|_{w=w_0^+} \leq 0$ hold simultaneously, the optimal wholesale price must be $w^* = w_0$. However, the former condition is equivalent to $a_1 \leq 2a_2 + b\sqrt{Kc_0}$ and the latter is equivalent to $a_1 > a_2 + \frac{bc_0+a_2}{\sqrt{c_0/K}}$. Since $c_0 \leq K$, the above two conditions result in an empty set, implying such a case does not exist.

Case 3: If $\frac{\partial \pi_S}{\partial w}|_{w=w_0^-} > 0$ (i.e. $a_1 \leq 2a_2 + b\sqrt{Kc_0}$), the optimal wholesale price must be located in the interval $(w_0, w_1]$, and $w^* = \frac{(1-\xi_0)a_1+\xi_0 a_2+bc_0}{2b}$. Here, $w^* = \frac{(1-\xi_0)a_1+\xi_0 a_2+bc_0}{2b}$ is the wholesale price which leads to $\frac{\partial \pi_S}{\partial w} = 0$.

Synthesising the above cases we obtain the optimal wholesale price in Equation (A11).

$$w^* = \begin{cases} \frac{(1-\xi_0)a_1+\xi_0 a_2}{2b} + \frac{bc_0}{2b} & \text{if } a_2 \leq a_1 \leq 2a_2 + b\sqrt{Kc_0} \\ \frac{(1-\xi_0)a_1}{2b} + \frac{bc_0}{2b} & \text{if } \quad a_1 > 2a_2 + b\sqrt{Kc_0}. \end{cases} \tag{A11}$$

Substituting Equation (A11) into (A7), we have

$$q^* = \begin{cases} \frac{(1-\xi_0)a_1+\xi_0 a_2}{4} - \frac{bc_0}{4} & \text{if } a_2 \leq a_1 \leq 2a_2 + b\sqrt{Kc_0} \\ \frac{(1-\xi_0)a_1}{4(1-\xi_0)} - \frac{bc_0}{4(1-\xi_0)} & \text{if } \quad a_1 > 2a_2 + b\sqrt{Kc_0}. \end{cases} \tag{A12}$$

Rewriting the conditions of Equations (A11) and (A12) into these of c_0, we obtain the results in Proposition 2. □

Proof of Proposition 3

Proof. When the integrated firm determines its CSR cost c and its production quantity q, it aims to maximise its expected profit given by Equation (21) where $\max_p[p \min(a_i - bp, q) - cq](i = 1, 2)$ can be obtained by substituting $a = a_i$ in Equation (20), i.e.

$$E\pi = \begin{cases} (1-\xi)\frac{a_1 q - q^2}{b} + \xi\frac{a_2 q - q^2}{b} - cq & \text{if } \quad 0 \leq q \leq \frac{a_2}{2} \\ (1-\xi)\frac{a_1 q - q^2}{b} + \xi\frac{a_2^2}{4b} - cq & \text{if } \frac{a_2}{2} < q \leq \frac{a_1}{2} \\ (1-\xi)\frac{a_1^2}{4b} + \xi\frac{a_2^2}{4b} - cq & \text{if } \quad q > \frac{a_1}{2}, \end{cases} \tag{A13}$$

where $\xi = 1 - \sqrt{c/K}$.

Here, we firstly derive the firm's optimal production quantity q for any possible CSR cost c.

The first derivative and second derivative of $E\pi$ on production quantity q are

$$\frac{\partial E\pi}{\partial q} = \begin{cases} \frac{(a_1-a_2)\sqrt{c/K}+a_2-2q}{b} - c & \text{if } \ 0 \leq q \leq \frac{a_2}{2} \\ \frac{(a_1-2q)\sqrt{c/K}}{b} - c & \text{if } \frac{a_2}{2} < q \leq \frac{a_1}{2} \\ -c & \text{if } \quad q > \frac{a_1}{2}, \end{cases} \tag{A14}$$

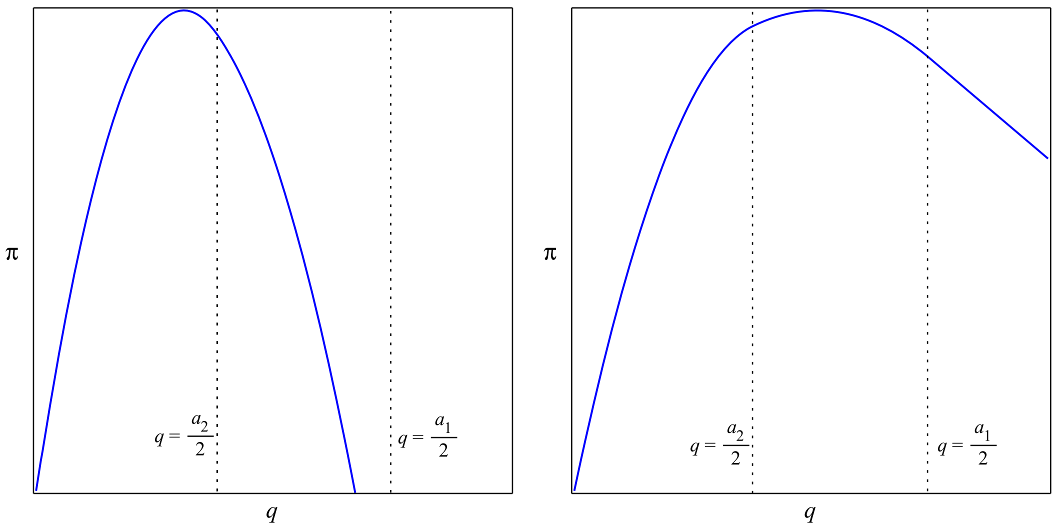

Figure A3. The equilibrium solutions with different q.

and

$$\frac{\partial^2 E\pi}{\partial q^2} = \begin{cases} -\frac{2}{b} & \text{if } 0 \le q \le \frac{a_2}{2} \\ -\frac{2\sqrt{c/K}}{b} & \text{if } \frac{a_2}{2} < q \le \frac{a_1}{2} \\ 0 & \text{if } \quad q > \frac{a_1}{2}. \end{cases} \tag{A15}$$

It is easy to prove that both $E\pi$ in Equation (A13) and $\partial E\pi/\partial q$ in Equation (A14) are continuous by examining their values at points $q = \frac{a_2}{2}$ and $q = \frac{a_1}{2}$. When $a_2 - bK \ge 0$ holds, we can prove that $\frac{\partial E\pi}{\partial q}|_{q=0} > 0$. Also, we have $\frac{\partial E\pi}{\partial q}|_{q=a_1/2} = -c < 0$. Consequently, the firm's optimal q can belong to the following two cases (see Figure A3).

Case 1: When $\frac{\partial E\pi}{\partial q}|_{q=a_2/2} < 0$ (i.e. $c > \frac{(a_1-a_2)^2}{b^2K}$), the optimal quantity q must be in $[0, a_2/2]$ and $q = \frac{(a_1-a_2)\sqrt{c/K}}{2} + \frac{a_2-bc}{2}$.

Case 2: When $\frac{\partial E\pi}{\partial q}|_{q=a_2/2} \ge 0$ (i.e. $0 \le c \le \frac{(a_1-a_2)^2}{b^2K}$), the optimal order quantity must be in $(a_2/2, a_1/2]$ and $q = \frac{a_1}{2} - \frac{b\sqrt{Kc}}{2}$.

Summarising the above two cases we obtain the optimal production q for any given CSR cost c, i.e.

$$q = \begin{cases} \frac{a_1}{2} - \frac{b\sqrt{Kc}}{2} & \text{if } 0 \le c \le \frac{(a_1-a_2)^2}{b^2K} \\ \frac{(a_1-a_2)\sqrt{c/K}}{2} + \frac{a_2-bc}{2} & \text{if } \quad c > \frac{(a_1-a_2)^2}{b^2K}, \end{cases} \tag{A16}$$

under which the integrated firm's expected profit turns into

$$E\pi(c) = \begin{cases} \frac{(a_1^2-a_2^2)\sqrt{c/K}}{4b} + \frac{bc\sqrt{Kc}}{4} - \frac{a_1c}{2} + \frac{a_2^2}{4b} & \text{if } 0 \le c \le \frac{(a_1-a_2)^2}{b^2K} \\ \frac{((a_1-a_2)\sqrt{c/K}+a_2-bc)^2}{4b} & \text{if } \quad c > \frac{(a_1-a_2)^2}{b^2K}. \end{cases} \tag{A17}$$

Now, we should derive the optimal c that maximises the expected profit given by Equation (A17). Letting $c_1 = \frac{(a_1-a_2)^2}{b^2K}$, we have $E\pi|_{c=c_1^-} = E\pi|_{c=c_1^+} = \frac{a_2^2}{4}$, implying that $E\pi(c)$ in Equation (A17) is a continuous function of c.

The first and second derivative of Equation (A17) are

$$\frac{\partial E\pi}{\partial c} = \begin{cases} \frac{3b\sqrt{Kc}}{8} + \frac{a_1^2-a_2^2}{8b\sqrt{Kc}} - \frac{a_1}{2} & \text{if } 0 \le c \le \frac{(a_1-a_2)^2}{b^2K} \\ \frac{\{(a_1-a_2)\sqrt{c/K}+(a_2-bc)\}(a_1-a_2-2b\sqrt{Kc})}{4b\sqrt{Kc}} & \text{if } \quad c > \frac{(a_1-a_2)^2}{b^2K}, \end{cases} \tag{A18}$$

and

$$\frac{\partial^2 E\pi}{\partial c^2} = \begin{cases} \frac{3b^2cK-a_1^2+a_2^2}{16bc\sqrt{Kc}} & \text{if } 0 \le c \le \frac{(a_1-a_2)^2}{b^2K} \\ \frac{(3bc+a_2)(a_2-a_1)}{8bc\sqrt{Kc}} + \frac{b}{2} & \text{if } \quad c > \frac{(a_1-a_2)^2}{b^2K}. \end{cases} \tag{A19}$$

For convenience, we let $f_1 = \frac{3b\sqrt{Kc}}{8} + \frac{a_1^2-a_2^2}{8b\sqrt{Kc}} - \frac{a_1}{2}$ and $f_2 = \frac{((a_1-a_2)\sqrt{c/K}+a_2-bc)(a_1-a_2-2b\sqrt{Kc})}{4b\sqrt{Kc}}$.

For the first derivative in Equation (A18) we have $\frac{\partial E\pi}{\partial c}|_{c=c_1^-} = \frac{\partial E\pi}{\partial c}|_{c=c_1^+} = -\frac{a_2}{4}$, indicating that $\frac{\partial E\pi}{\partial c}$ is also a continuous function of c and $\frac{\partial E\pi}{\partial c}|_{c=c_1} < 0$. In combination with the relationship between c_1 and K, we have the following three cases.

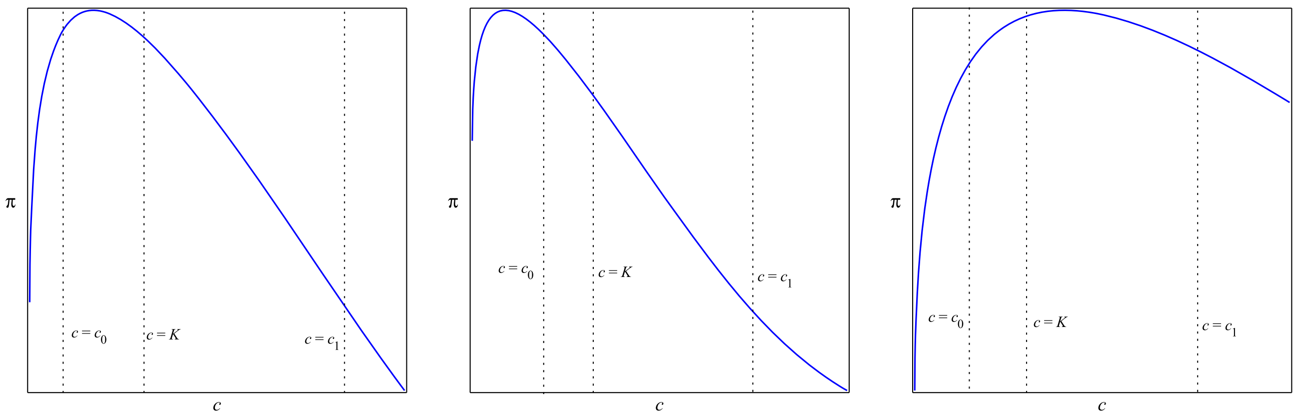

Figure A4. The equilibrium solutions with different c.

Case 1: If $c_1 > K$, i.e. $a_1 > a_2 + bK$, we discuss the optimal solutions under the case $c_0 \le c \le K$. We examine $\frac{\partial^2 E\pi}{\partial c^2} < 0$, the objective function is a concave function with respect to c.

Let $f_1 = 0$, the first-order condition gives the CSR cost $c_2 = \frac{1}{K}(\frac{2a_1 - \sqrt{a_1^2 + 3a_2^2}}{3b})^2$ and $c_3 = \frac{1}{K}(\frac{2a_1 + \sqrt{a_1^2 + 3a_2^2}}{3b})^2$. Comparing c_1 with c_2 and c_3, we have $c_2 < c_1$ and $c_3 > c_1$. Thus, we have one solution to discuss, i.e. c_2 (see Figure A4).

(i) If $2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2 K c_0} \le a_1 \le 2bK + \sqrt{a_2^2 + b^2 K^2}$, i.e. $c_0 \le c_2 \le K$, the optimal CSR cost $\tilde{c} = c_2$.

(ii) If $a_1 \le 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2 K c_0}$, i.e. $c_2 \le c_0$, the optimal CSR cost $\tilde{c} = c_0$.

(iii) If $a_1 > 2bK + \sqrt{a_2^2 + b^2 K^2}$, i.e. $c_2 > K$, the optimal CSR cost $\tilde{c} = K$.

Case 2: In this case, $c_0 \le c_1 \le K$, i.e. $a_2 + b\sqrt{Kc_0} \le a_1 \le a_2 + bK$. Now, since the objective function is a piecewise function, we discuss the optimal solutions under $c_0 \le c \le c_1$ and $c_1 \le c \le K$.

When $c_1 \le c \le K$, we have $\frac{\partial E\pi}{\partial c} < 0$, that is to say, the optimal CSR cost is always c_1. Due to the fact that $\frac{\partial E\pi}{\partial c}|_{c=c_1} < 0$, the firm's profit function decreases in c at the point c_1, thus, the optimal CSR cost cannot be c_1. That is to say, the optimal CSR cost is in $[c_0, c_1)$.

When $c_0 \le c \le c_1$, let $f_1 = 0$, referring to case 1, the CSR cost $c_2 = \frac{1}{K}(\frac{2a_1 - \sqrt{a_1^2 + 3a_2^2}}{3b})^2$, we derive $c_2 < c_1$. Thus, we have following analysis (see Figure A5).

(i) If $a_1 \le 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2 K c_0}$, i.e. $c_2 \le c_0$, we derive $\frac{\partial^2 E\pi}{\partial c^2} < 0$ in the interval $[0, c_0]$. Combining the facts that $c_2 \le c_0$, $\frac{\partial E\pi}{\partial c}|_{c=c_0} < 0$, $\frac{\partial E\pi}{\partial c}|_{c=c_1} < 0$, and the integrated firm's profit decreases in c_0, the optimal CSR cost is $\tilde{c} = c_0$.

(ii) If $a_1 > 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2 K c_0}$, i.e. $c_0 \le c_2 \le c_1 \le K$, we derive $\frac{\partial^2 E\pi}{\partial c^2} < 0$, the optimal CSR cost turns into $\tilde{c} = c_2$.

Case 3: If $c_1 \le c_0$, i.e. $a_1 \le a_2 + b\sqrt{Kc_0}$, we discuss the optimal CSR cost under $c_0 \le c \le K$. In this case, we have the first derivative of $E\pi$ with respect to c, $\frac{\partial E\pi}{\partial c} < 0$, thus, $\tilde{c} = c_0$ (see Figure A6).

Summarising the above analysis and using notation $A = 2b\sqrt{Kc_0} + \sqrt{a_2^2 + b^2 K c_0}$ and $B = 2bK + \sqrt{a_2^2 + b^2 K^2}$, the optimal CSR cost is

$$\tilde{c} = \begin{cases} c_0 & \text{if} \quad a_2 < a_1 \le A \\ \frac{1}{K}(\frac{2a_1 - \sqrt{a_1^2 + 3a_2^2}}{3b})^2 & \text{if} \quad A < a_1 \le B \\ K & \text{if} \quad a_1 > B. \end{cases} \tag{A20}$$

Substituting Equation (A20) into Equation (A16), the optimal order quantity expression turns into

$$\tilde{q} = \begin{cases} \frac{(a_1 - a_2)\sqrt{c_0/K}}{2} + \frac{a_2 - bc_0}{2} & \text{if } a_2 < a_1 \le a_2 + b\sqrt{Kc_0} \\ \frac{a_1}{2} - \frac{b\sqrt{Kc_0}}{2} & \text{if } a_2 + b\sqrt{Kc_0} < a_1 \le A \\ \frac{a_1}{6} + \frac{\sqrt{a_1^2 + 3a_2^2}}{6} & \text{if} \quad A < a_1 \le B \\ \frac{a_1}{2} - \frac{bK}{2} & \text{if} \quad a_1 > B. \end{cases} \tag{A21}$$

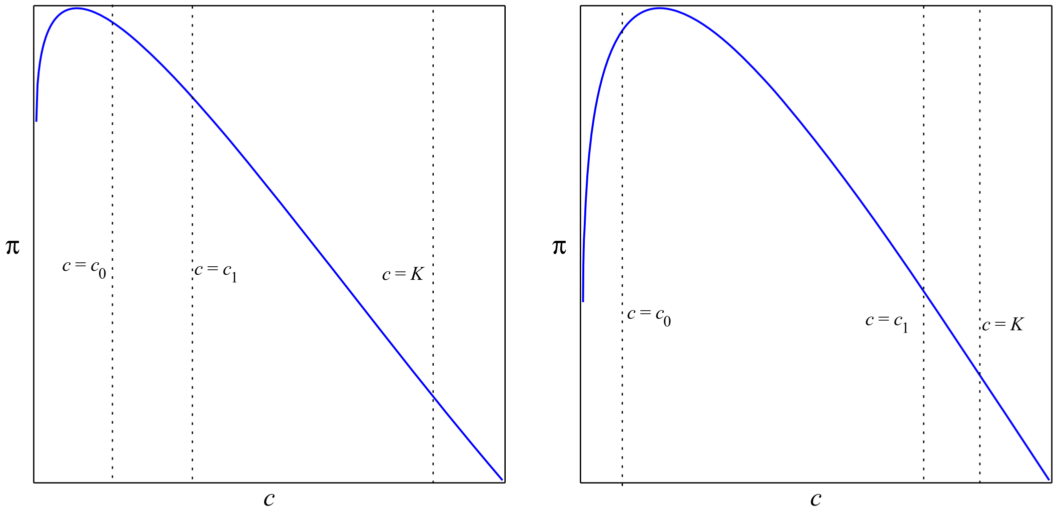

Figure A5. The equilibrium solutions with different c.

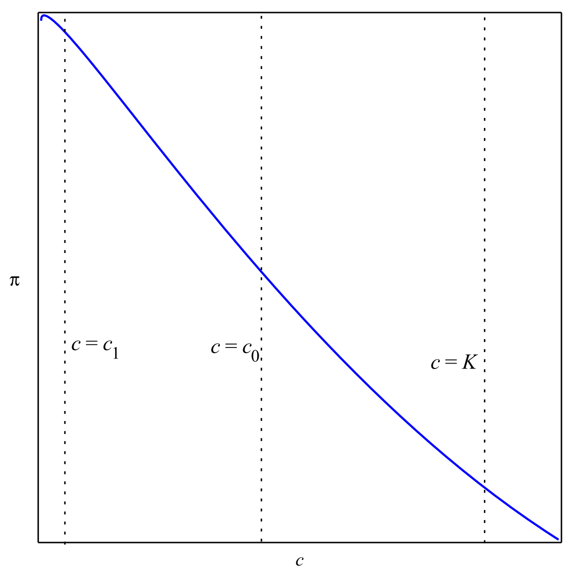

Figure A6. The equilibrium solutions with different c.

Substituting Equation (A20) and Equation (A21) into Equation (A13), the integrated firm's profit turns into

$$E\tilde{\pi}(c) = \begin{cases} \frac{(a_1^2-a_2^2)\sqrt{c_0/K}}{4b} + \frac{bc_0\sqrt{Kc_0}}{4} - \frac{a_1c_0}{2} + \frac{a_2^2}{4b} & \text{if } a_2 + b\sqrt{Kc_0} < a_1 \leq A \\ \frac{((a_1-a_2)\sqrt{c_0/K}+a_2-bc_0)^2}{4b} & \text{if } a_1 \leq a_2 + b\sqrt{Kc_0} \\ \frac{2(a_1^2+3a_2^2)^{\frac{3}{2}}+2a_1(a_1^2-9a_2^2)+27a_2^2bK}{108b^2K} & \text{if } A < a_1 \leq B \\ \frac{a_1^2-a_2^2}{4b} + \frac{bK^2}{4} - \frac{a_1K}{2} + \frac{a_2^2}{4b} & \text{if } a_1 > B. \end{cases} \tag{A22}$$

□

Proof of Proposition 4

Proof. When using the push wholesale price contract with flexible quantity, the OEM decides its retail price p, and the order quantity q can be determined accordingly, whereas the supplier decides its wholesale price w and the CSR cost c. Knowing from Equation (25), we

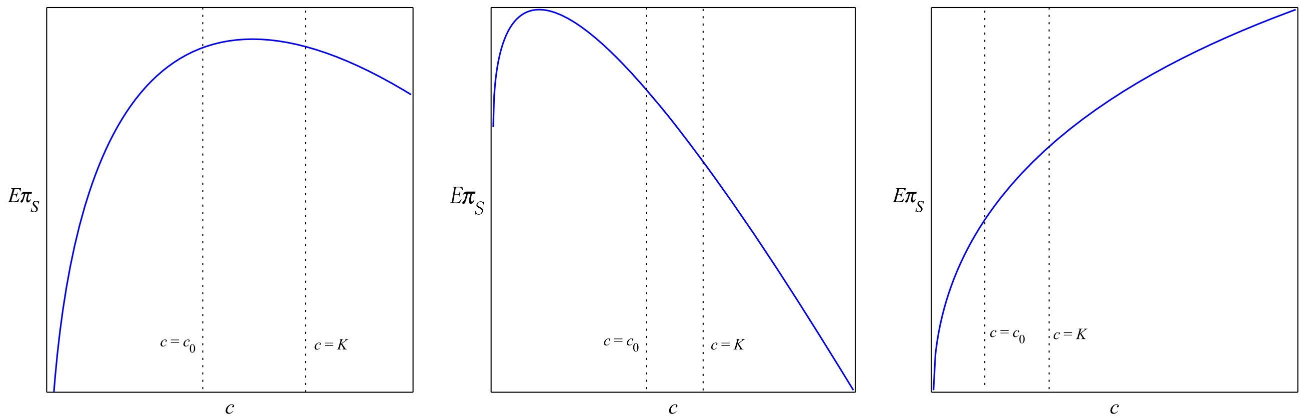

Figure A7. The equilibrium solutions with different c.

have the retail prices $p_1 = \frac{a_1+bw}{2b}$, $p_2 = \frac{a_2+bw}{2b}$ and the order quantities $q_1 = \frac{a_1-bw}{2}$, $q_2 = \frac{a_2-bw}{2}$. Thus, the supplier's profit function and the OEM's profit function can be described as follows, i.e.

$$E\pi_S = \left(\sqrt{\frac{c}{K}}q_1 + \left(1-\sqrt{\frac{c}{K}}\right)q_2\right)w - cq_1, \tag{A23}$$

and

$$E\pi_M = \sqrt{\frac{c}{K}}(p_1-w)q_1 + \left(1-\sqrt{\frac{c}{K}}\right)(p_2-w)q_2. \tag{A24}$$

We use backward induction to solve the problem. We first derive the supplier's optimal decision on c given the supplier's decisions on wholesale price w.

Differentiating $E\pi_S$ with respect to c , we have the first derivative and second derivative,

$$\frac{\partial E\pi_S}{\partial c} = \frac{bw-a_1}{2} + \frac{(a_1-a_2)w}{4\sqrt{cK}}, \tag{A25}$$

and

$$\frac{\partial^2 E\pi_S}{\partial c^2} = -\frac{(a_1-a_2)w}{8c\sqrt{cK}}. \tag{A26}$$

Thus, letting $\frac{\partial E\pi_S}{\partial c} = 0$, we have the supplier's CSR cost

$$c = \frac{w^2(a_1-a_2)^2}{4K(a_1-bw)^2}. \tag{A27}$$

From Equations (A25) and (A26), we have $\frac{\partial^2 E\pi_S}{\partial c^2} < 0$, and $\frac{\partial E\pi_S}{\partial c}|_{c=0} > 0$. These two facts illustrate that the objective function is concave function. Consequently, the supplier's optimal w can belong to the following three cases (see Figure A7).

Case 1: If $\frac{\partial E\pi_S}{\partial c}|_{c=c_0} > 0$ and $\frac{\partial E\pi_S}{\partial c}|_{c=K} < 0$, i.e. $\frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} < w \le \frac{2a_1K}{2bK+a_1-a_2}$, the optimal CSR cost must be located in the interval $(c_0, K]$, and $c^t = \frac{w^2(a_1-a_2)^2}{4K(a_1-bw)^2}$.

Case 2: If $\frac{\partial E\pi_S}{\partial c}|_{c=c_0} < 0$ and $\frac{\partial E\pi_S}{\partial c}|_{c=K} < 0$, i.e. $0 \le w \le \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2}$, the supplier's optimal CSR cost is $c^t = c_0$.

Case 3: If $\frac{\partial E\pi_S}{\partial c}|_{c=c_0} > 0$ and $\frac{\partial E\pi_S}{\partial c}|_{c=K} > 0$, i.e. $w > \frac{2a_1K}{2bK+a_1-a_2}$, the supplier's optimal CSR cost is $c^t = K$.

Here, for convenience, let $w_2 = \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2}$ and $w_3 = \frac{2a_1K}{2bK+a_1-a_2}$.

Summarising the above cases, we obtain that the supplier's optimal CSR cost is

$$c^T = \begin{cases} c_0 & \text{if} \quad 0 \le w \le \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} \\ \frac{w^2(a_1-a_2)^2}{4K(a_1-bw)^2} & \text{if} \quad \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} < w \le \frac{2a_1K}{2bK+a_1-a_2} \\ K & \text{if} \quad w > \frac{2a_1K}{2bK+a_1-a_2}. \end{cases} \tag{A28}$$

Substituting Equation (A28) into Equation (A23), we have

$$E\pi_S^T(w) = \begin{cases} \frac{w(a_1-a_2)\sqrt{c_0/K}}{2} + \frac{(a_2+bc_0)w}{2} - \frac{bw^2}{2} + \frac{a_1c_0}{2} & \text{if} \quad 0 \le w \le \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} \\ \frac{w^2(a_1-a_2)^2}{(a_1-bw)8K} + \frac{(a_2-bw)w}{2} & \text{if} \quad \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} < w \le \frac{2a_1K}{2bK+a_1-a_2} \\ \frac{(a_1-bw)(w-K)}{2} & \text{if} \quad w > \frac{2a_1K}{2bK+a_1-a_2}. \end{cases} \tag{A29}$$

Substituting Equation (A28) into Equation (A24), we have

$$E\pi_M^T(w) = \begin{cases} \frac{(a_1+a_2-2bw)(a_1-a_2)\sqrt{c_0/K}+(a_2-bw)^2}{4b} & \text{if} \quad 0 \le w \le \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} \\ \frac{(a_1+a_2-2bw)(a_1-a_2)^2w}{8bK(a_1-bw)} + \frac{(a_2-bw)^2}{4b} & \text{if} \quad \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} < w \le \frac{2a_1K}{2bK+a_1-a_2} \\ \frac{(a_1-bw)^2}{4b} & \text{if} \quad w > \frac{2a_1K}{2bK+a_1-a_2}. \end{cases} \tag{A30}$$

Summarising Equation (A29) and Equation (A30), we have

$$E\pi^T(w) = \begin{cases} \frac{(a_1^2-a_2^2)\sqrt{c_0/K}+a_2^2}{4b} + \frac{2(bw-a_1)c_0-bw^2}{4} & \text{if} \quad 0 \le w \le \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} \\ \frac{(a_2-bw-a_1)(a_1-a_2)^2w+2Kb^2w^3-2Kb^2w^2}{8bK(a_1-bw)} + \frac{a_2^2}{4b} & \text{if} \quad \frac{2a_1\sqrt{Kc_0}}{2b\sqrt{Kc_0}+a_1-a_2} < w \le \frac{2a_1K}{2bK+a_1-a_2} \\ \frac{(a_1-bw)(a_1+bw-2Kb)}{4b} & \text{if} \quad w > \frac{2a_1K}{2bK+a_1-a_2}. \end{cases} \tag{A31}$$

□

Proof of Proposition 5

Proof. We use backward induction to solve the problem. We first derive the supplier's optimal decision on c given the OEM's decisions and supplier's wholesale price w.

First, differentiating the objective function of Equation (34) with respect to c, we have

$$\frac{\partial \pi_S}{\partial c} = -q + \frac{sq}{2\sqrt{cK}}, \tag{A32}$$

and

$$\frac{\partial^2 \pi_S}{\partial c^2} = -\frac{sq}{4c\sqrt{cK}}. \tag{A33}$$

Thus, let $\frac{\partial \pi_S}{\partial c} = 0$, the CSR cost is

$$c = \frac{s^2}{4K}. \tag{A34}$$

From Equation (A33), we have $\frac{\partial^2 \pi_S}{\partial q^2} < 0$, so the objective function is a concave function. Besides, $\frac{\partial \pi_S}{\partial c}|_{c=0}$ indicates that the objective function increases at the point $c = 0$. Consequently, the supplier's optimal c can belong to the following three cases.

Case 1: If $\frac{\partial \pi_S}{\partial c}|_{c=c_0} > 0$ and $\frac{\partial \pi_S}{\partial c}|_{c=K} < 0$, i.e. $2\sqrt{Kc_0} \le s \le 2K$, the optimal CSR cost must be in the interval $[c_0, K]$, and $c^T = \frac{s^2}{4K}$.

Case 2: If $\frac{\partial \pi_S}{\partial c}|_{c=c_0} < 0$ and $\frac{\partial \pi_S}{\partial c}|_{c=K} < 0$, i.e. $0 \le s \le 2\sqrt{Kc_0}$, the optimal CSR cost is $c^T = 0$.

Case 3: If $\frac{\partial \pi_S}{\partial c}|_{c=c_0} > 0$ and $\frac{\partial \pi_S}{\partial c}|_{c=K} > 0$, i.e. $s > 2K$, the optional CSR cost is $c^T = K$.

Summarising the above three cases, we obtain the CSR cost c for any given additional reward s, i.e.

$$c = \begin{cases} c_0 & \text{if} \quad 0 \le s \le 2\sqrt{Kc_0} \\ \frac{s^2}{4K} & \text{if} \quad 2\sqrt{Kc_0} < s \le 2K \\ K & \text{if} \quad s > 2K. \end{cases} \tag{A35}$$

□

Proof of Proposition 6

Proof. From the expression of the optimal CSR cost we know that it is just related to the additional reward s, so we can calculate the optimal w and q without taking optimal CSR cost c into the objective function.

We derive the OEM's optimal decision on q given the supplier's wholesale price w and the additional reward s.

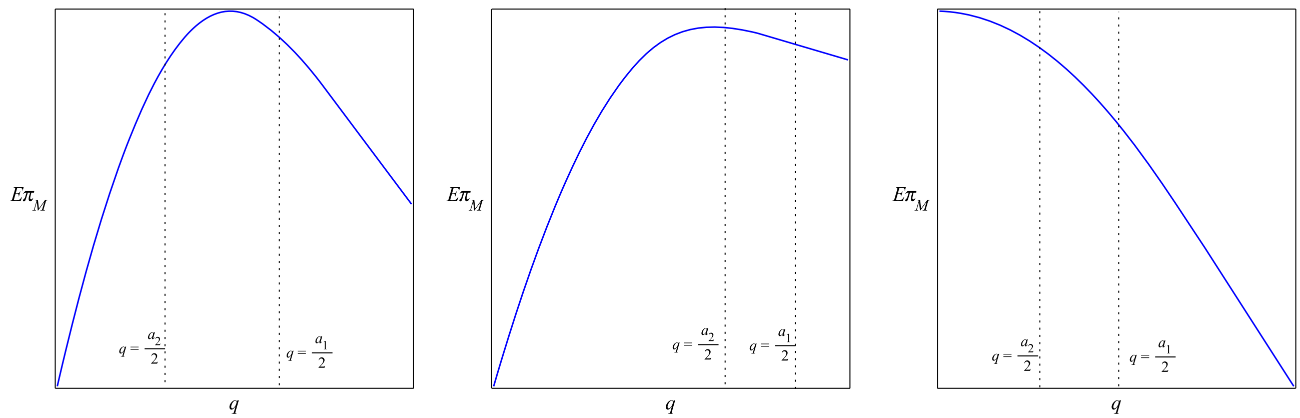

Figure A8. The equilibrium solutions with different q.

First, differentiating the objective function of Equation (33) with respective to q, we have

$$\frac{\partial E\pi_M}{\partial q} = \begin{cases} \frac{(a_1-a_2-bs)\sqrt{c/K}+a_2-2q}{b} - w & \text{if } 0 \le q \le \frac{a_2}{2} \\ \frac{(a_1-2q-bs)\sqrt{c/K}}{b} - w & \text{if } \frac{a_2}{2} < q \le \frac{a_1}{2} \\ -\sqrt{\frac{c}{K}}s - w & \text{if } \quad q > \frac{a_1}{2}, \end{cases} \tag{A36}$$

and

$$\frac{\partial^2 E\pi_M}{\partial q^2} = \begin{cases} -\frac{2}{b} & \text{if } 0 \le q \le \frac{a_2}{2} \\ -\frac{2}{b}\sqrt{\frac{c}{K}} & \text{if } \frac{a_2}{2} < q \le \frac{a_1}{2} \\ 0 & \text{if } \quad q > \frac{a_1}{2}. \end{cases} \tag{A37}$$

Second, we can observe that the objective function $E\pi_M$ and $\frac{\partial E\pi_M}{\partial q}$ are both continuous functions of q. Furthermore, we can verify $\frac{\partial E\pi_M}{\partial q}|_{q=a_1/2} < 0$.

The OEM's optimal order quantity q will belong to the following three cases (see Figure A8).

Case 1: If $\frac{\partial E\pi_M}{\partial q}|_{q=0} > 0$ and $\frac{\partial E\pi_M}{\partial q}|_{q=a_2/2} > 0$, i.e. $0 \le w \le \frac{(a_1-a_2-bs)\sqrt{c/K}}{b}$, the optimal order quantity will be located in $[a_2/2, a_1/2]$, the optimal order quantity will be $q = \frac{(a_1-bs)\sqrt{c/K}-bw}{2\sqrt{c/K}}$.

Case 2: If $\frac{\partial E\pi_M}{\partial q}|_{q=0} > 0$ and $\frac{\partial E\pi_M}{\partial q}|_{q=a_2/2} < 0$, i.e. $\frac{(a_1-a_2-bs)\sqrt{c/K}}{b} < w \le \frac{(a_1-a_2-bs)\sqrt{c/K}+a_2}{b}$, the optimal order quantity will be in $[0, a_2/2]$, and the optimal order quantity will be $q = \frac{(a_1-a_2-bs)\sqrt{c/K}}{2} + \frac{a_2-bw}{2}$.

Case 3: If $\frac{\partial E\pi_M}{\partial q}|_{q=0} < 0$ and $\frac{\partial E\pi_M}{\partial q}|_{q=a_2/2} < 0$, i.e. $w > \frac{(a_1-a_2-bs)\sqrt{c/K}+a_2}{b}$, the optimal order quantity will be $q = 0$.

Therefore, synthesising the above cases we obtain the optimal order quantity as

$$q^B = \begin{cases} \frac{a_1-bs}{2} - \frac{bw}{2\sqrt{c/K}} & \text{if} \quad 0 \le w \le \frac{(a_1-a_2-bs)\sqrt{c/K}}{b} \\ \frac{(a_1-a_2-bs)\sqrt{c/K}}{2} + \frac{a_2-bw}{2} & \text{if} \quad \frac{(a_1-a_2-bs)\sqrt{c/K}}{b} < w \le \frac{(a_1-a_2-bs)\sqrt{c/K}+a_2}{b} \\ 0 & \text{if} \quad w > \frac{(a_1-a_2-bs)\sqrt{c/K}+a_2}{b}. \end{cases} \tag{A38}$$

In addition, the OEM makes the same decisions on order quantity as in centralised system. We have calculated the optimal order quantity of integrated firm in the proof of Proposition 3 as follows, i.e.

$$\tilde{q} = \begin{cases} \frac{a_1}{2} - \frac{b\sqrt{Kc}}{2} & \text{if } 0 \le c \le \frac{(a_1-a_2)^2}{b^2K} \\ \frac{(a_1-a_2)\sqrt{c/K}}{2} + \frac{a_2-bc}{2} & \text{if } \quad c > \frac{(a_1-a_2)^2}{b^2K}. \end{cases} \tag{A39}$$

We now discuss the OEM's order quantity in different cases.

Case 1: The optimal CSR cost is c_0 and the additional reward s is in $[0, 2\sqrt{Kc_0}]$. Therefore, we have following analysis.

(i) If $\frac{a_1}{2} - \frac{b\sqrt{Kc_0}}{2} = \frac{a_1-bs}{2} - \frac{bw}{2\sqrt{c_0/K}}$, we derive $w = c_0 - s\sqrt{c_0/K}$, and that requires conditions that $0 \le c_0 \le \frac{(a_1-a_2)^2}{b^2K}$.

(ii) If $\frac{a_1}{2} - \frac{b\sqrt{Kc_0}}{2} = \frac{(a_1-a_2-bs)\sqrt{c_0/K}}{2} + \frac{a_2-bw}{2}$, we derive $w = \frac{(a_2-a_1)\xi_0}{b} + (K-s)\sqrt{c_0/K}$, requiring conditions $0 \le c_0 \le \frac{(a_1-a_2)^2}{b^2K}$ and $\frac{(a_1-a_2)^2}{b^2K} < c_0 \le \frac{a_1^2}{b^2K}$. These last two conditions imply the empty set, so such a case does not exist.

(iii) If $\frac{(a_1-a_2)\sqrt{c_0/K}}{2}+\frac{a_2-bc_0}{2}=\frac{a_1-bs}{2}-\frac{bw}{2\sqrt{c_0/K}}$, we derive $w=\frac{(a_1-a_2)\xi_0\sqrt{c_0/K}}{b}+(c_0-s)\sqrt{c_0/K}$, which requires conditions $c_0>\frac{(a_1-a_2)^2}{b^2K}$ and $c_0\le\frac{(a_1-a_2)^2}{b^2K}$. Since these two conditions result in an empty set, we see that this case also does not exist.

(iv) If $\frac{(a_1-a_2)\sqrt{c_0/K}}{2}+\frac{a_2-bc_0}{2}=\frac{(a_1-a_2-bs)\sqrt{c_0/K}}{2}+\frac{a_2-bw}{2}$, we derive $w=c_0-s\sqrt{c_0/K}$, which requires conditions that $\frac{(a_1-a_2)^2}{b^2K}<c_0\le\frac{\{a_1-a_2+\sqrt{(a_1-a_2)^2+4a_2bK}\}^2}{4b^2K}$.

We obtain the supplier's optimal wholesale price w in response to the OEM's additional reward s, i.e. $w=c_0-s\sqrt{c_0/K}$. This expression requires the conditions $0\le c_0\le\frac{\{a_1-a_2+\sqrt{(a_1-a_2)^2+4a_2bK}\}^2}{4b^2K}$, that is, $0\le c_0\le K$.

Case 2: The optimal CSR cost $c=\frac{s^2}{4K^2}$ and the additional reward s are located in $(2\sqrt{Kc_0},2K]$.

(i) If $\frac{2a_1-bs}{4}=\frac{a_1-bs}{2}-\frac{bwK}{s}$, we derive $w=-\frac{s^2}{4K}$, and it requires conditions that $0\le s\le\frac{2(a_1-a_2)}{b}$.

(ii) If $\frac{2a_1-bs}{4}=\frac{(a_1-a_2-bs)s}{4K}+\frac{a_2-bw}{2}$, we derive $w=\frac{(a_1-a_2)s+2a_2K+bsK-bs^2-2a_1K}{2Kb}$. It requires conditions that $0\le s\le\frac{2(a_1-a_2)}{b}$ and $\frac{2(a_1-a_2)}{b}<c_0\le\frac{2a_1}{b}$, which imply the empty set, so such a case does not exist.

(iii) If $\frac{(a_1-a_2)s}{4K}+\frac{4a_2K-bs^2}{8K}=\frac{a_1-bs}{2}-\frac{bwK}{s}$, we derive $w=(\frac{a_1-bs}{2}-\frac{2(a_1-a_2)s+4a_2K-bs^2}{8K})\frac{s}{bK}$. It requires conditions that $s>\frac{2(a_1-a_2)}{b}$ and $s\le\frac{2(a_1-a_2)}{b}$, which imply the empty set, so such a case does not exist.

(iv) If $\frac{(a_1-a_2)s}{4K}+\frac{4a_2K-bs^2}{8K}=\frac{(a_1-a_2-bs)s}{4K}+\frac{a_2-bw}{2}$, we derive $w=-\frac{s^2}{4K}$, which requires conditions that $\frac{2(a_1-a_2)}{b}<s\le\frac{a_1-a_2+\sqrt{(a_1-a_2)^2+4a_2bK}}{b}$.

Case 3: The optimal CSR cost $c=K$ and the additional reward s is in $(2K,+\infty)$.

(i) If $\frac{a_1-bK}{2}=\frac{a_1-bs-bw}{2}$, we derive $w=K-s$, requiring the conditions $a_1>a_2+bK$.

(ii) If $\frac{a_1-bK}{4}=\frac{a_1-a_2-bs}{2}+\frac{a_2-bw}{2}$, we derive $w=K-s$, implying condition that $a_1>a_2+bK$ and $bK<a_1\le a_2+bK$, which imply the empty set, so such a case does not exist.

(iii) If $\frac{a_1-a_2}{2}+\frac{a_2-bK}{2}=\frac{a_1-a_2-bs}{2}$, we derive $w=K-s$, requiring conditions $a_1\le a_2+bK$ and $a_1>a_2+bK$, implying the empty set, so such a case does not exist.

(iv) If $\frac{a_1-a_2}{2}+\frac{a_2-bK}{2}=\frac{a_1-a_2-bs}{2}+\frac{a_2-bw}{2}$, we derive $w=K-s$, requiring the conditions $bK<a_1\le a_2+bK$.

Summarising the above cases, we have the

$$w=\begin{cases} c_0-s\sqrt{c_0/K} & \text{if } 0\le s\le 2\sqrt{Kc_0}\\ -\frac{s^2}{4K} & \text{if } 2\sqrt{Kc_0}<s\le 2K\\ K-s & \text{if } s>2K.\end{cases}\tag{A40}$$

From Equation (A40), it is easy to know that the supplier's wholesale price w cannot be $-\frac{s^2}{4K}$ and $K-s$ on account of $w>0$. Therefore, when $0\le s\le\sqrt{Kc_0}$ holds, the supplier's wholesale price is

$$w=c_0-s\sqrt{c_0/K}.\tag{A41}$$

From the above analysis, we can derive the profit of total supply chain as follows.

If $0\le s\le 2\sqrt{Kc_0}$, we have $c=c_0$ and $w=c_0-s\sqrt{c_0/K}$. According to the centralised case, we obtain the profit of the entire supply chain, i.e.

$$E\pi^B=\begin{cases}\frac{(a_1^2-a_2^2)\sqrt{c_0/K}}{4b}+\frac{bc_0\sqrt{Kc_0}}{4}-\frac{a_1c_0}{2}+\frac{a_2^2}{4b} & \text{if } 0\le c_0\le\frac{(a_1-a_2)^2}{b^2K}\\ \frac{\{(a_1-a_2)\sqrt{c_0/K}+a_2-bc_0\}^2}{4b} & \text{if } \frac{(a_1-a_2)^2}{b^2K}<c_0\le K.\end{cases}\tag{A42}$$

□