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An extended VIKOR method based on entropy measure for the failure modes risk assessment – A case study of the geothermal power plant (GPP)

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ABSTRACT

Process equipment failures (PEFs) are recognized as one of the leading causes of process accidents. Failure modes and effect analysis (FMEA) as a risk assessment technique, has widely been used in a variety of process industries. The conventional form of FMEA uses three parameters of severity (S), occurrence (O), and detection (D) as risk factors to calculate a risk priority number (R.P.N) and rank the failure modes based on this number. But several shortcomings associated with the FMEA have limited its applicability. This study aims at the development of an extension of FMEA that could efficiently handle the vagueness and uncertainty exists in the experts' judgments in process of failure modes ranking in conventional FMEA. In this paper we used the concept of the Z number to capture the inherent uncertainty exists in the experts' judgments. In addition, we used Shannon entropy concept to deploy objective weights to adjust subjective weights assigned by experts. Furthermore, the fuzzy VIKOR technique applied to rank and prioritize the failure modes based on the minimum individual regret and the maxi group utility. A numerical example is presented to illustrate an application of the proposed method in a geothermal power plant (GPP). Results are also compared with the conventional FMEA. A sensitivity analysis was conducted to validate the obtained results. Findings indicate that the application of the proposed approach (subjective-objective ranking) in fuzzy environment can improve the applicability of the conventional FMEA method.

1. Introduction

Because of some problems such as global warming and lack of fossil resources, renewable energy, specifically geothermal energy has received more attention. According to the available evidences, the use of geothermal energy is more cost effective than the use of conventional fossil fuel resources (Lund et al., 2005; Yari, 2010).

Geothermal Power Plants (GPPs) are type of power plants that supply the electricity by means of geothermal energy. GPPs have much equipment in common with conventional power-generating stations. They utilize many of identical components, including generators, transformers, turbines, and other regular power generating equipment (Feili et al., 2013).

Recently, the majority of process accidents has happened due to failures in process equipments which might be due to deviations from intended design objectives or departures from desired operating conditions (Mohammadfam et al., 2013). Some reports revealed that about 60% of process accidents are originated by equipment failures (Prem et al., 2010). According to the NRC report, about 67% of all accidents are occurred as a result of equipment failures (Meel et al., 2007). Although many efforts have been made to improve the safety of processes, equipment related accidents are still happening (Stricoff, 2012). As a result, a high level of reliability and safety is a critical prerequisite for the continuous operation of process industries. Nevertheless, few studies have specially focused on the estimation of the reliability of GPPs (Feili et al., 2013).

The FMEA is an efficient technique used for accident prevention and risk analysis and is applied to discover and eliminate recognized or potential failures to improve the reliability and safety of complex systems (Hu-Chen et al., 2013). FMEA is intended to present information required for risk management decision making (Stamatis, 2003). While the usefulness of the FMEA has been confirmed, the traditional RPN model is vulnerable to a number of limitations (Liu et al., 2013; Tay and Lim, 2006). The most important

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limitation of traditional FMEA is the application of crisp numbers to extract the evaluator's judgment about risk factors (i.e. Occurrence (O), Severity (S), and Detection (D)). Unfortunately, the crisp numbers used in developing the RPN, demonstrates several considerable limitations when the FMEA is applied in the real-world situations (Liu et al., 2012b). Furthermore, the other limitations of FMEA that could be found in literature are as follows:

- Different combinations of O, S and D may produce exactly the same value of RPN, but their hidden risk implications may be totally different (Mohamed and Aminah Robinson, 2010).
- The mathematical formula for calculating RPN is questionable and debatable (Geum et al., 2011).
- RPNs are not continuous with many holes (Liu et al., 2011a,b).
- The mathematical form adopted for calculating the RPN is strongly sensitive to variations in risk factor evaluations (Gargama and Chaturvedi, 2011; Liu et al., 2011a,b, 2012a).
- The RPN elements have many duplicate numbers without any difference in interpretation (Chang and Cheng, 2009, 2010).

Regarding to these limitations, numerous approaches have been recommended in the literature to improve the FMEA methodology. In brief, these methods can be divided into four main categories including, multi-criteria decision making (MCDM), mathematical programming (MP), artificial intelligence (AI), and hybrid approaches. A comprehensive list of these methods could be found in (Liu et al., 2013). Some important methods include analytic hierarchy process (AHP) (Aslani et al., 2014), data envelopment analysis (DEA) (Chin et al., 2009; Garcia et al., 2013), technique for ordering preference by similarity to ideal solution (TOPSIS) (Fu et al., 2014; Vahdani et al., 2014), decision making trial and evaluation laboratory (DEMATEL) (Chang et al., 2014), grey theory (Chang et al., 2013; Razi et al., 2013), evidential reasoning approach (Du et al., 2016; Hu-Chen et al., 2013), expert system (Tay and Lim, 2006; Yang et al., 2008), hybrid approaches (Chang, 2009; Chang and Cheng, 2011; Gargama and Chaturved, 2011; Kutlu and Ekmekçioğlu, 2012; Liu et al., 2011a,b; Wang et al., 2009; Zhang and Chu, 2011), and so forth.

As stated above, to overcome the aforementioned shortcomings, some papers have treated the risk prioritization of the failure modes as a multiple criteria decision-making (MCDM) problem; accordingly, an extensive array of mathematical methods has been applied to provide the problems with adequate and more accurate solutions. The MCDM, which uses decision matrices, can offer support techniques for the comparison and ranking of alternatives (Chang, 2016).

Among the MCDM methods, the VIKOR method has fascinated special application for ranking of the alternatives. The VIKOR method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria, and on proposing compromise solution (Opricovic and Tzeng, 2007).

Liu et al. (2012a) used an extended VIKOR to determine risk priorities of the failure modes. They used the concepts of the "minimum individual regret" and "maximum group utility" to rank the failure modes. Liu et al. (2015) developed an integrated methodology that incorporate the fuzzy AHP technique for weighting the risk factors and entropy weighting method to determine the importance of the failure modes with respect to the risk factors. They also used the VIKOR method to rank the failure modes. Mandal et al. (2015) developed a methodology that utilized the VIKOR method for ranking the human errors. Liu et al. (2014) used the grey relational projection method and D numbers to develop a risk priority model for the risk evaluation in FMEA. Mandal and Maiti (2014) used the Fuzzy similarity value to rank and prioritize the failure modes in FMEA. They developed a new approach that integrates the concepts of similarity value measure of fuzzy numbers and possibility theory.

Although these efforts have eliminated the shortcomings of the conventional FMEA to some extent, a problem still exist. In the majority of the MCDM methods including those that are used to improve the applicability of FMEA, the weight of criteria are determined only based on the subjective judgments of decision makers. While it is convenient for the experts to represent their judgments in terms of the linguistic variables, though there are some limitations for expressing their opinions in this manner. Subjective judgments often are presented by linguistic variables in terms of the fuzzy numbers. Although the fuzzy numbers could handle the fuzziness of the experts' information, they could not reflect the partial reliability exists in the experts' judgments (Kang et al., 2012). As an example, one may express his opinion about the severity of a failure mode as "very high" or "catastrophic" but he may hesitate about his opinion. In other words, it may be better to represents it as "catastrophic, with high degree of reliability". To deal with this limitation we used the Z-number. The concept of the Z-number was proposed by L. A. Zadeh (Zadeh, 2011). A Z-number is an ordered pair $Z = (A, R)$; where A is an imprecise restriction on values of X and R is an imprecise estimation of reliability of A (Kang et al., 2012). Furthermore, the subjective fixed weight methods could deviate the indexes' weights because of subjective factors. While subjective methods specify weights only based on the preference or opinions of decision makers, objective methods employ mathematical models (i.e. entropy method or multiple objective programming) to automatically avoid the use of decision makers' preferences. Objective weighting approach is particularly appropriate for situations where consistent subjective weights could not be acquired (Deng et al., 2000).

Regarding to the abovementioned limitations, this study aims at the development of a framework for evaluation of the equipment failure modes by the FMEA method applying the two-facet approach (subjective-objective ranking) in fuzzy environment. In this work, in one hand we are using the Z-numbers to obtain the expert's opinions (subjective weights) about the importance of the risk factors, and on the other hands we are using the entropy concept to consider the objective weighting of the failure modes. The contribution of this work is the application of the Z-numbers to handle the partial reliability associated with the expert's opinions when they want to express their judgments about the FMEA risk factors.

To the best of our knowledge, this work is the first study that uses the objective-subjective weighting method by Z-number combined with the VIKOR approach for ranking of the failure modes in a geothermal power plant (GPP).

2. Material and methods

2.1. Failure mode and effects analysis (FMEA)

The FMEA technique initially was developed as a formal design method in the 1960s by the aerospace industry (Bowles and Peláez, 1995). It has confirmed to be a practical and powerful means in evaluating potential failures and putting them off from occurring (Sankar and Prabhu, 2001). Currently FMEA has widely been used in different industries including chemical, mechanical, aerospace, nuclear, automotive, electronics, and medical technology industries (Chang and Cheng, 2011; Chin et al., 2009; Sharma et al., 2005).

As the main characteristic of the FMEA that makes it different from other risk assessment tools, the key concern of FMEA is to put emphasis on the prevention of failures, rather than to present a solution following the occurrence of a failure. This feature can help safety professionals to regulate the current programs, utilize the recommended measures to decrease the probability of failures, decline the failure rates, and keep away from hazardous accidents (Liu et al., 2012b). Standards that are generally referred to, when applying an FMEA include IEC 60812 (International Electrotechnical Commission, 2006), BS 5760-5 (British Standards Institute, 1991); Replaced by BS EN 60812:2006 (EN, 2006), and MIL-STD-1629A (Department of Defense, 1980).

The traditional FMEA uses three variables as its risk factors including occurrence (O), severity (S), and detection (D). To obtain the RPN value of a PFM, the three risk factors are evaluated using the 10-point scale presented in Tables 1–3.

As one major limitation of the FMEA, the risk factors in the conventional FMEA are expressed by crisp numbers. This makes the decision process more difficult and the evaluation results more unreliable. Generally, it is difficult and often misleading to give a crisp numerical assessment of the risk factors (O, S and D), in the FMEA (Braglia et al., 2003a, 2003b). It is more convenient for decision makers to express their judgments by linguistics variable such as “likely”, “important”, or “very low” (Xu et al., 2002). In this regards, we used the Z-numbers to deal with the subjective judgments of the experts (we referred as DMs herein) about the risk factors as well as the importance of failure modes with respect to the risk factors. On the other hand the entropy weighting method was used to regulate the subjective weights. Finally, the fuzzy VIKOR method was applied to rank the failure modes.

2.2. Z-number

In the process of failure modes evaluation, the experts often tend to use linguistic variables to express their judgments about the failure modes. A linguistic variable can be defined as a variable whose values are presented in linguistic terms. Linguistic variables can be used in situations when it is hard to describe those conditions in traditional quantitative expressions (Zadeh, 1965). These linguistic values can also be characterized by fuzzy numbers. Trapezoid fuzzy numbers (TrFNs) and triangular fuzzy numbers (TFNs) are the two most commonly used types of fuzzy numbers. TFNs and TrFNs are characterized by A = (l, m, n) and B = (l, m, n, s) respectively. The corresponding membership function for TFNs and TrFNs are illustrated as Eqs. (1) and (2), respectively (Liu et al., 2012b).

$$\mu_{\tilde{A}}(x) = \left\{ \begin{matrix} 0, x < l \\ \left(\frac{x-l}{m-l}\right), l \leqslant x \leqslant m \\ \left(\frac{n-x}{n-m}\right), m \leqslant x \leqslant n \\ 0, x > n \end{matrix} \right\} \tag{1}$$

Table 1
Traditional FMEA scale for occurrence (O) (Liu et al., 2014).

Rating	Probability of failure	Possible failure rates
10	Extremely high: failure almost inevitable	⩾ in 2
9	Very high	1 in 3
8	Repeated failures	1 in 8
7	High	1 in 20
6	Moderately high	1 in 80
5	Moderate	1 in 400
4	Relatively low	1 in 2000
3	Low	1 in 15,000
2	Remote	1 in 150,000
1	Nearly impossible	⩽1 in 1,500,000

Table 2
Traditional FMEA scale for severity (S) (Liu et al., 2014).

Rating	Effect	Severity of effect
10	Hazardous without warning	Highest severity ranking of a failure mode, occurring without warning and consequence is hazardous
9	Hazardous with warning	Higher severity ranking of a failure mode occurring with warning, consequence is hazardous
8	Extreme	Operation of system or product is broken down without compromising safe
7	Major	Operation of system or product may be continued but performance of system or product is affected
6	Significant	Operation of system or product is continued and performance of system or product is degraded
5	Moderate	Performance of system or product is affected seriously and the maintenance is needed
4	Low	Performance of system or product is small affected and the maintenance may not be needed
3	Minor	System performance and satisfaction with minor effect
2	Very minor	System performance and satisfaction with slight effect
1	None	No effect

Table 3
Traditional FMEA scale for detection (D) (Liu et al., 2014).

Rating	Detection	Likelihood of detection by design control
10	Absolute uncertainty	Potential occurring of failure mode cannot be detected in concept, design and process FMEA/mechanism and subsequent failure mode
9	Very remote	The possibility of detecting the potential occurring of failure mode is very remote/mechanism and subsequent failure mode
8	Remote	The possibility of detecting the potential occurring of failure mode is remote/mechanism and subsequent failure mode
7	Very low	The possibility of detecting the potential occurring of failure mode is very low/mechanism and subsequent failure mode
6	Low	The possibility of detecting the potential occurring of failure mode is low/mechanism and subsequent failure mode
5	Moderate	The possibility of detecting the potential occurring of failure mode is moderate/mechanism and subsequent failure mode
4	Moderately high	The possibility of detecting the potential occurring of failure mode is moderately high/mechanism and subsequent failure mode
3	High	The possibility of detecting the potential occurring of failure mode is high/mechanism and subsequent failure mode
2	Very high	The possibility of detecting the potential occurring of failure mode is very high/mechanism and subsequent failure mode
1	Almost certain	The potential occurring of failure mode will be detect/ mechanism and subsequent failure mode

$$\mu_{\tilde{A}}(x) = \left\{ \begin{matrix} 0 \quad x < l \\ \left(\frac{x-l}{m-l}\right) \; l \leqslant x \leqslant m \\ 1 \quad m \leqslant x \leqslant n \\ \left(\frac{s-x}{s-n}\right) \; n \leqslant x \leqslant s \\ 0 \quad x > s \end{matrix} \right\} \tag{2}$$

where l, m, n, and s are the vectors of the fuzzy number. Let $\tilde{A}$ and $\tilde{B}$ be two positive TrFNs parameterized by $\tilde{A} = (a1, a2, a3, a4)$ and $\tilde{B} = (b1, b2, b3, b4)$, then the algebraic operations for the TrFNs are as follows (Liu et al., 2012b):

$$\text{Addition operation}: \tilde{A} \oplus \tilde{B} = [a1 + b1, a2 + b2, a3 + b3, a4 + b4] \quad (3)$$

$$\text{Subtraction operation}: \tilde{A} \ominus \tilde{B} = [a1 - b1, a2 - b2, a3 - b3, a4 - b4] \quad (4)$$

$$\text{Multiplication operation}: \tilde{A} \otimes \tilde{B} = [a1.b1, a2.b2, a3.b3, a4.b4] \quad (5)$$

$$\text{Division operation}: \frac{\tilde{A}}{\tilde{B}} = \left[\frac{a1}{b1}, \frac{a2}{b2}, \frac{a3}{b3}, \frac{a4}{b4}\right] \quad (6)$$

$$\text{Distance between two TpFNs}: d_v(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{4}[(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2 + (d_1 - d_2)^2]} \quad (7)$$

Linguistic variables are often described by fuzziness. This means that we often exert soft constraints on values of variables of interest. But, It must be noted that it is not adequate to take into consideration only fuzziness when dealing with real-world imperfect information. The other critical property of information is its deficient reliability. Indeed, in fuzzy numbers, uncertainty is expressed by a numerical membership function. This implies that they do not consider inferred uncertainty interval. Undeniably, any assessment of values of interest, be it exact or soft, are dependent on the confidence in sources of information that cannot completely cover the entire complexity of real-world phenomenon. As a consequence, fuzziness from the one side and partial reliability form the other side are robustly interconnected each other. So as to take into consideration this fact, Zadeh (2011) proposed the notion of a Z-number as a more adequate solution for explanation of real-world information. in fact, a Z-number is an ordered pair Z = (A, R); of fuzzy numbers (usually triangular and trapezoidal), used to describe a value of a variable X, where the part "A", is an imprecise constraint on values of X and part "R" is an rough estimation of reliability of "A" and is regarded as a value of probability measure of A. In this paper we used the concept of Z-number to deal with partial reliability of the experts' judgments about the importance of risk factors as well as the evaluation of failure modes. As there are different preferences and individual backgrounds in the decision making team, the DMs in the team may implement linguistic term sets with dissimilar granularities and membership functions to declare their judgments. Choice of linguistic variables is absolutely established by DMs themselves (Büyüközkan et al., 2008; Herrera et al., 2000). Consequently, we adopted seven-point scale triangular fuzzy numbers (TFNs) and trapezoidal fuzzy numbers (TpFNs) to describe the importance of risk factors and the rating of alternatives. While the importance of risk factors and the rating of alternatives were expressed by TrFNs and TFNs, their associated reliabilities (probability measures) were presented by TFNs shown in Tables 4 and 5.

2.3. Fuzzy analytical hierarchy process (FAHP)

Analytical hierarchy process (AHP) technique is one of the most widely used multi criteria decision making (MCDM) methods (Tavana et al., 2016). Even though the conventional AHP takes account of experts' opinions and performs a multiple-criteria assessment, it is not capable of revealing human's fuzzy opinions (Seçme et al., 2009). The fuzzy set theory, puts together the comparison process more flexibly and potently in order to clarify experts' preferences (Kahraman et al., 2003). In this study, we adopted the FAHP method to determine the weight of the risk factors. The geometric mean method (Zheng et al., 2012) is one of the extensions of the AHP that used to determine the weights of the FMEA risk factors. The procedure of calculating subjective weights based on fuzzy AHP is explained in Section 2.6.

2.4. VIKOR technique

Compromise ranking method (VIKOR- a Serbian abbreviation for Vlsekriterijumska Optimizacija I Kompromisno Resenje) first was proposed by Opricovic (Opricovic and Tzeng, 2007). It is one of the MCDM methods that was developed for ranking and selection of the optimum choice among a set of alternatives when there are conflicts between the criteria in complex systems. VIKOR establishes a multi-criteria ranking index with regard to the particular measure of "closeness" to the "ideal" solution (Liou et al., 2011; Opricovic and Tzeng, 2002, 2004, 2007).

The basis for the development of the VIKOR method is the following Lp metric:

$$L_k^p = \left\{ \sum_{j=1}^{n} \left[\frac{w_j(|f_j^* - f_{kj}|)}{(|f_j^* - f_j^-|)} \right] \right\}^{1/p} \quad (8)$$

In Eq. (8), $1 \leqslant p \leqslant \infty$; k = 1, 2, ..., m, and w_j is the influential weight that in our study was extracted from subjective judgments of experts. Let the possible alternatives and criteria presented as V_1, V_2, ..., V_k and $j_1, j_2, \ldots, j_k$ respectively, then fkj can be reflected as performance scores of alternative Vk and the jth criterion.

The procedure of ranking and prioritizing PFMs based on VIKOR approach is explained in Section 2.6.

2.5. Entropy method

The entropy weight method was first transferred from the field of thermodynamics to information domain (Shannon, 2001). In the information domain the uncertainty of signals in communication processes are known as "information entropy" (Ji et al., 2015). While subjective methods (e.g. Delphi and AHP) are used to determine subjective weights of criteria, objective methods such as entropy weight method are utilized to eliminate man-made instabilities and yield more realistic results. In information theory, the Shannon entropy can be used to establish the extent of disorder and its effectiveness in system information. The smaller the entropy value, the smaller is the degree of disorder in the system and the higher is the weight (LI et al., 2011). Considering the mentioned attributes, Shannon's concept is competent to be used as a weighting calculation method (Lihong et al., 2008; Wang and Lee, 2009). The procedure of calculating objective weights based on Shannon's entropy is explained in Section 2.6.

2.6. The proposed method

Considering the Z-numbers, Entropy, AHP, and VIKOR principles, the proposed framework for risk evaluation by the FMEA method based on the abovementioned methods can be shown as Fig. 1.

As mentioned previously, the problem of risk assessment of equipment failures with the FMEA method can be considered as a group multiple criteria decision-making (GMCDM) issue. Let a GMCDM with K decision makers DM_k (k = 1, 2,... ,K), n decision criteria C_j (j = 1, 2,..., n) and m alternatives Ai (i = 1, 2,..., m); then it is possible to assess m alternative (herein the PFMs) with respect to the n criteria (herein the risk factors) applying VIKOR method. The main steps of the proposed algorithm continue as the following:

1. Identification of the objectives of risk assessment process and determination of the analysis level.

Fig. 1. The proposed framework for evaluation of the risks by the FMEA method based on the VIKOR-AHP-Entropy methods in fuzzy environment.

2. Arrangement of the decision making team.
3. Determining the process equipment and their failure modes.
4. Selection of appropriate linguistic variables (Z-numbers) and corresponding membership functions for evaluation of risk factors.
5. Determination of the subjective weights of risk factors (O, S, and D) by using fuzzy AHP method.
   5.1. Obtaining the importance of risk factors from the experts by using Z-numbers shown in Table 4
   5.2. Converting the Z-numbers to regular fuzzy numbers as follows (Kang et al., 2012):
      5.2.1. Convert the second part of the Z-number to a crisp number using the Eq. (9)

$$\alpha = \frac{\int x\mu_{\tilde{R}}(x)dx}{\int \mu_{\tilde{R}}(x)dx} = \frac{1}{3}[(a_3 - a_1) + (a_2 - a_1)] + a_1 \tag{9}$$

In which α is a triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$

      5.2.2. Add the weight of the second part of the Z-number (reliability part) to the first part (restriction part). So, the weighted Z-number can be represented as:

$$\tilde{Z}_\alpha = \left\{\langle x, \mu_{\tilde{A}_\alpha}(x)\rangle | \mu_{\tilde{A}_\alpha}(x) = \alpha\mu_{\tilde{A}}(x), x \in [0,1]\right\} \tag{10}$$

      5.2.3. Convert the irregular fuzzy number from previous step (i.e. weighted restriction) to regular fuzzy number. The regular fuzzy set can be indicated as:

$$\tilde{Z}' = \tilde{a}_{ij}^K = \left\{\langle x, \mu_{\tilde{Z}'}(x)\rangle | \mu_{\tilde{Z}'}(x) = \mu_{\tilde{A}}\sqrt{\left(\frac{x}{\alpha}\right)}, x \in [0,1]\right\} \tag{11}$$

   5.3. Calculation of the aggregated fuzzy relative importance ($\tilde{a}_{ij}$)

Let $\tilde{a}_{ij}^K = \left(a_{ij1}^k, a_{ij2}^k, a_{ij3}^k, a_{ij4}^k\right)$, be the fuzzy relative importance of criterion i with criterion j provided by the kth DM. Consequently, the aggregated fuzzy relative importance ($\tilde{a}_{ij}$) can be determined as:

$$(\tilde{a}_{ij}) = (\tilde{a}_{ij1}, \tilde{a}_{ij2}, \tilde{a}_{ij3}, \tilde{a}_{ij4}); \begin{cases} \tilde{a}_{ij1} = min_k\{a^k_{ij1}\} \\ \tilde{a}_{ij2} = \frac{1}{k}\sum_{k=1}^{K} a^k_{ij2} \\ \tilde{a}_{ij3} = \frac{1}{k}\sum_{k=1}^{K} a^k_{ij3} \\ \tilde{a}_{ij4} = max_k\{a^k_{ij4}\} \end{cases} \quad (12)$$

5.4. Establishing the pair wise comparison matrixes among all risk factors

The results of the DMs' comparisons is constructed as a fuzzy pair wise comparison matrix ($\tilde{\mathrm{A}}$), such that

$$\tilde{\mathrm{A}} = [\tilde{\mathrm{a}}_{ij}]\begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \dots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \dots & \tilde{a}_{nn} \end{bmatrix} \quad (13)$$

5.5. Calculation of the fuzzy weights of each criterion

Using the comparison matrix ($\tilde{\mathrm{A}}$), criteria weights can be determined as follows:

$$\alpha_j = \left[\prod_{j=1}^{n} l_{ij}\right]^{1/n}; \beta_j = \left[\prod_{j=1}^{n} m_{ij}\right]^{1/n}; \gamma_j = \left[\prod_{j=1}^{n} n_{ij}\right]^{1/n}; \delta_j = \left[\prod_{j=1}^{n} s_{ij}\right]^{1/n} \quad (14)$$

$$\alpha = \sum_{j=1}^{n}\alpha_j; \beta = \sum_{j=1}^{n}\beta_j; \gamma = \sum_{j=1}^{n}\gamma_j; \delta = \sum_{j=1}^{n}\delta_j \quad (15)$$

where l, m, n, and s are the vectors of the TrFN $\tilde{A}$, that $\tilde{A} = (l, m, n, s)$.

The fuzzy weights of risk factors can be acquired as:

$$\tilde{w}^s_j = (w^s_{j1}, w^s_{j2}, w^s_{j3}, w^s_{j4}) = (\alpha_j\delta^{-1}, \beta_j\gamma^{-1}, \gamma_j\beta^{-1}, \delta_j\alpha^{-1})j \in \{1, 2, \dots, n\} \quad (16)$$

5.6. Defuzzification of the values of $\tilde{w}^s_j$ applying Eq. (17)

$$\bar{x}_o(\tilde{w}^s_j) = \bar{w}^s_j = \frac{w^s_{j1}w^s_{j2} + w^s_{j3}w^s_{j4} + \frac{1}{3}(w^s_{j4} - w^s_{j3})^2 - \frac{1}{3}(w^s_{j2} - w^s_{j1})^2}{-w^s_{j1} - w^s_{j2} + w^s_{j3} + w^s_{j4}} \quad (17)$$

5.7. Normalization of the criteria weights applying Eq. (18)

$$w^s_j = \frac{\bar{w}^s_j}{\sum_{j=1}^{n}\bar{w}^s_j} \quad (18)$$

where $\bar{w}^s_j$ is referred to as the crisp number of fuzzy weight determined by Eq. (17) and w^s_j is the subjective weight of criterion j.

6. Obtaining the objective weights using the entropy method
   6.1. Normalization of the evaluation index using Eq. (19) which yields the projection value of criteria.

$$P_{ij} = \frac{x_{ij}}{\sum_j x_{ij}} \quad (19)$$

6.2. Calculation of the entropy value of every index applying Eq. (20).

$$e_j = -k\sum_{j=1}^{n} P_{ij}\ln_{(P_{ij})}; k = ln(m)^{-1} \quad (20)$$

6.3. Definition of the divergence by means of Eq. (21)

$$div_j = 1 - e_j \quad (21)$$

6.4. Calculation of the normalized (objective) weights of indexes applying Eq. (22)

$$w^o_j = \frac{div_j}{\sum_{j=1}^{n} div_j} \quad (22)$$

7. Calculation of the S, R and Q values by using the fuzzy VIKOR method
   7.1. Aggregation of the DMs' linguistic evaluations of each failure mode with respect to risk factors

Suppose the fuzzy rating of ith alternative with respect to jth criterion of kth DM be presented as $\tilde{x}_{ijk} = (x_{ijk1}, x_{ijk2}, x_{ijk3}, x_{ijk4})$. Consequently, the aggregated fuzzy rating $\tilde{x}_{ijk}$ with regard to criterion C_j could be determined as:

$$(\tilde{x}_{ijk} = (x_{ijk1}, x_{ijk2}, x_{ijk3}, x_{ijk4}) | i = 1, 2, \dots, m; j = 1, 2, \dots, n; k = 1, 2, \dots, K)$$

$$\begin{cases} x_{ij1} = \min_k\{x_{ijk1}\} \\ x_{ij2} = \frac{1}{k}\sum_{k=1}^{K} x_{ijk2} \\ x_{ij3} = \frac{1}{k}\sum_{k=1}^{K} x_{ijk3} \\ x_{ij4} = \max_k\{x_{ijk4}\} \end{cases} \quad (23)$$

7.2. Establishing the decision matrix $\tilde{D}$

The decision matrix $\tilde{D}$ could be briefly shown as:

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{bmatrix} \quad (24)$$

7.3. Determination of aspired and tolerable levels

Aspired and tolerable levels correspond to the best f_j^* and the worst f_j^- values of all criterion ratings respectively (j = 1, 2, ..., n) that can be obtained as follows:

$$f^*_j = \begin{Bmatrix} max_i x_{ij}, & \text{for benefit criteria} \\ \min_i x_{ij}, & \text{for cost criteria} \end{Bmatrix} i = 1, 2, \dots, m \quad (25)$$

$$f^-_j = \begin{Bmatrix} min_i x_{ij}, & \text{for benefit criteria} \\ max_i x_{ij}, & \text{for cost criteria} \end{Bmatrix} i = 1, 2, \dots, m \quad (26)$$

7.4. Defuzzification of the decision matrix $\tilde{D}$

The following equation could be used to obtain the crisp values of decision matrix.

$$\text{defuzz}(xij) = \frac{-x_{ij1}x_{ij2} + x_{ij3}x_{ij4} + \frac{1}{3}(x_{ij4} - x_{ij3})^2 - \frac{1}{3}(x_{ij2} - x_{ij1})^2}{-x_{ij1} - x_{ij2} + x_{ij3} + x_{ij4}} \quad (27)$$

7.5. Calculation of the normalized fuzzy distance (NFD) d_{ij}, i = 1, 2, ..., m, j = 1, 2, ..., n,

$$d_{ij} = \frac{d(\tilde{f}^*_j, \tilde{x}_{ij})}{d(\tilde{f}^*_j, \tilde{f}^-_j)} \quad (28)$$

7.6. Calculation of the values of S_i and R_i using Eqs. (29) and (30) respectively:

$$S_i = \sum_{j=1}^{n} w^c_j d_{ij} \quad (29)$$

$$R_i = max_j(w^c_j d_{ij}) \quad (30)$$

where $w^c_j = \varphi w^s_j + (1 - \varphi)w^o_j$ is the combination weights of criteria, and $\phi \in [0, 1]$, 7.7. Computation of the index value (Q_i, i = 1, 2, ..., m) using Eq. (31)

$$Q_i = \upsilon \frac{S_i - S^*}{S^- - S^*} + (1-\upsilon)\frac{R_i - R^*}{R^- - R^*} \quad (31)$$

In Eq. (31) values of S^*, S^-, R^*, R^- can be determined as follows:

$$S^* = min_i\{S_i\},\ S^- = max_i\{S_i\},\ R^* = min_i\{R_i\},\ R^- = max_i\{R_i\} \quad (32)$$

In Eq. (31) the "ν" parameter introduces the weight of the strategy of the maximum group utility. On the other hand, (1 – ν) is the weight of individual regret.

7.8. Ranking or improving the alternatives

Arrange alternatives decreasingly with respect to the values of S_i, R_i and Q_i. The results create three ranking lists with regard to values of S_i, R_i, and Q_i.

7.9. Proposition of compromise solution

If the following two conditions are verified, the alternative $A^{(1)}$ (which is the best ranked by the measure Q (minimum)) is proposed as the compromise solution (Opricovic and Tzeng, 2007):

C1. Acceptable advantage: Fulfils with $Q(A^{(2)}) - Q(A^{(1)}) \geqslant DQ$, which $A^{(2)}$ is the alternative with second score in the grading list by Q; DQ = 1/(m – 1).
C2. Acceptable stability: The alternative $A^{(1)}$ should also be the top ranked by S or/and R. this defined as "voting by majority rule" for v > 0.5, "voting by consensus" for v = 0.5, or "with veto" for (v < 0.5).

If one of the above conditions is not fulfilled, subsequently a set of compromise solutions is proposed, as the following:

- Alternatives A(1) and A(2) as compromise solution, if only the condition C2 is not satisfied or alternatives A(1), A(2), ..., A (M) if the condition C1 is not fulfilled; A(M) is established by the Q(A(M)) Q(A(1)) < DQ for maximum M.

3. Illustrative example

To demonstrate the applicability of the proposed method, a Geothermal Power Plant (GPP) was selected as the case of the study. The aforementioned GPP has been established 15 years ago with about 600 personnel. It has about 15 geothermal wells. The output power of the plant is approximated about 100 MW with the maximum power of 300 MW.

The GPP desires to identify and rank the most critical failure modes of process. With respect to the identified failure modes, appropriate preventive measures could be applied.

In the current study, five DMs were selected from different fields including mechanical, power, chemical, HSE and operational workers. The characteristics of DMs are presented in Table 6.

One of the critical steps in the application of the FMEA is to decompose a system to its individual components. In this paper, we used equipment block diagram (EBD) of the GPP which contained different systems including generator and electrical, turbine and auxiliaries, production and transmission, cooling system, and gas extraction divisions (Fig. 2).

Subsequently, the DMs were requested to represent their judgments about 25 potential failure modes (PFMs) of the GPP (Feili et al., 2013). Table 7 shows the PFMs and their related causes and effects.

The decision group decided to apply two types of linguistic variables for evaluation, one for the assessment of the importance weight of the risk factors (Table 4) and another for the evaluation of the failure modes with respect to the risk factors (Table 5). To compare the results of the proposed method with those of the conventional FMEA, the DMs were also asked to declare their judgments about each risk factor based on traditional RPN rating (Table 8).

Table 4
Z-numbers for the importance weight of risk factors.

$\tilde{A}$ (restriction component)		$\tilde{R}$ (reliability component)	
Linguistic variable	TFNs and TPFNs	Linguistic variable	TFNs
Equally important (EI)	(0, 0, 0.1, 0.2)	Very low (VL)	(0, 0, 0.1)
Very weakly important (VWI)	(0.1, 0.2, 0.2, 0.3)	Low (L)	(0, 0.1, 0.3)
Weakly important (WI)	(0.2, 0.3, 0.4, 0.5)	Medium low (ML)	(0.1, 0.3, 0.5)
Medium important (MI)	(0.4, 0.5, 0.5, 0.6)	Medium (M)	(0.3, 0.5, 0.7)
Strong important (SI)	(0.5, 0.6, 0.7, 0.8)	Medium high (MH)	(0.5, 0.7, 0.9)
Very strongly important (VSI)	(0.7, 0.8, 0.8, 0.9)	High (H)	(0.7, 0.9, 1)
Absolutely important (AI)	(0.8, 0.9, 1, 1)	Very high (VH)	(0.9, 1, 1)

Table 5
Z-numbers for the fuzzy rates of failure modes (PFMs).

$\tilde{A}$ (restriction component)		$\tilde{R}$ (reliability component)	
Linguistic variable	TFNs and TPFNs	Linguistic variable	TFNs
Very poor (VP)	(0, 0, 1, 2)	Very low (VL)	(0, 0, 0.1)
Poor (P)	(1, 2, 2, 3)	Low (L)	(0, 0.1, 0.3)
Medium poor (MP)	(2, 3, 4, 5)	Medium low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(4, 5, 5, 6)	Medium (M)	(0.3, 0.5, 0.7)
Medium good (MG)	(5, 6, 7, 8)	Medium high (MH)	(0.5, 0.7, 0.9)
Good (G)	(7, 8, 8, 9)	High (H)	(0.7, 0.9, 1)
Very good (VG)	(8, 9, 10, 10)	Very high (VH)	(0.9, 1, 1)

Table 6
Characteristics of the DMs used to evaluate risk factors importance and failure modes precedence.

DMs	Title	Experience	Education	Age
1	Mechanical Engineer	23	PHD	48
2	Power Engineer	18	M.S.c	42
3	Chemical Engineer	24	M.S.c	45
4	Safety Supervisor	15	B.S.	39
5	Operator	28	High school	52

Subsequently, the DMs were asked to declare their judgments about the importance of O, S, and D parameters using Z-numbers. Aggregated and normalized weights of the risk factors are presented in Table 9.

To make decision about the priority of the PFMs, it is necessary to examine each PFM with respect to risk factors. To do this, the DMs were requested to state their opinions about the PFMs with reference to O, S, and D, respectively. The results of these comparisons are shown in Table 10.

After the DMs deployed linguistic variables (in terms of Z-number) to evaluate each PFM (Table 10), linguistic values were converted to fuzzy numbers applying Eqs. (9)–(11). Afterward, the fuzzy numbers were aggregated using Eq. (23). Consequently the aggregated fuzzy values of the PFMs rates were defuzzified using Eq. (27) the results are shown in Table 11.

Results of the entropy calculations are shown in Table 12. Projection value of each risk factor was calculated using Eq. (19). Then, the entropy value (ej) and degree of divergence (div_j) were calculated with reference to w_j^o using Eqs. (20) and (21).

The best fj^* and the worst fj^- values of all risk factors were determined using Eqs. (25) and (26) as shown in Table 13.

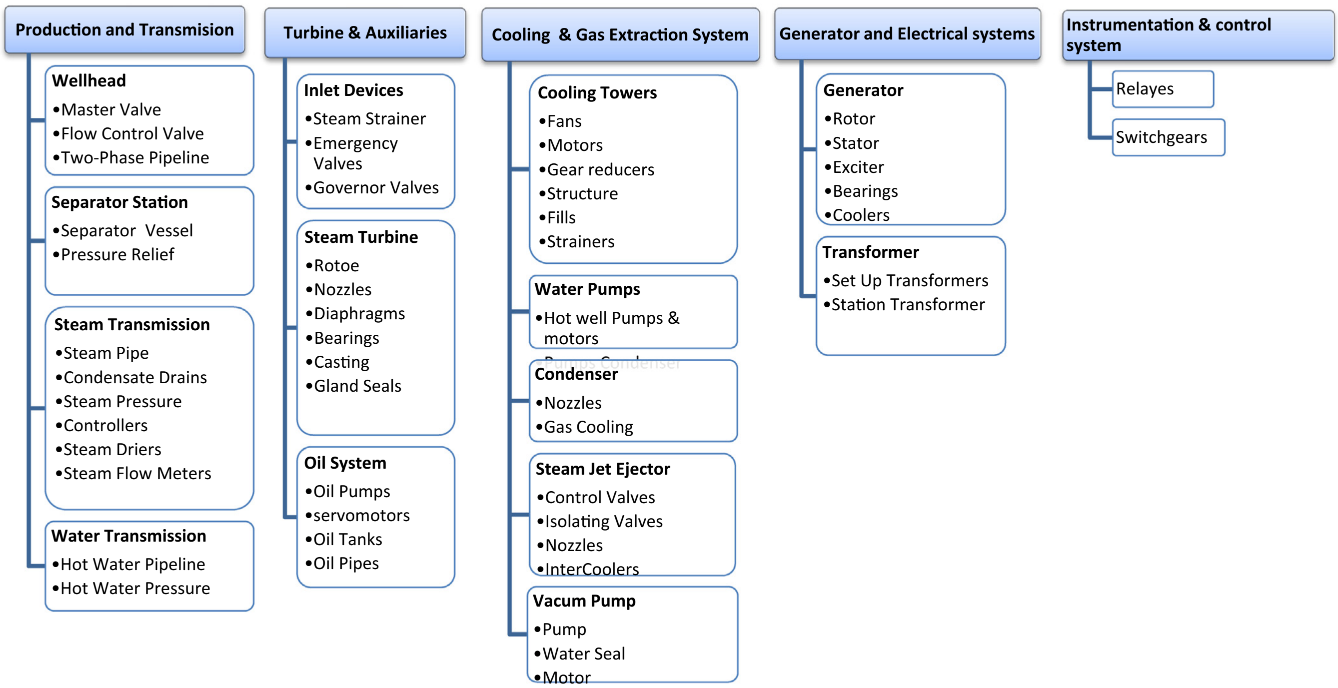

Fig. 2. Equipment block diagram (EBD) of GPP.

Table 7
Failure modes of the GPP.

	Failure	Failure mode	Cause	Effect
Production and Transmission	PFM1	Sticking valves	Environmental effect	Valves lost disk, scaling
	PFM2	Leaking glands	Separator, wrong quality	Split, crack
	PFM3	Blocked pipes	Deformation, pipeline burst	Deformation
	PFM4	Worn valve disks	Leakage, rupture	Loss of well
	PFM5	Failed traps	Pressure devices	Wrong specification
	PFM6	Dislodged pipes	Wrong operation	Wet steam, downtime
	PFM7	Steam quality degradation	Turbine damage, damage of blades	Reduced turbine efficiency
	PFM8	Scaling problems (calcium, silica, sulfide compounds, etc.)	The plugging and deposit problems in brine handling system, well pipe, injection lines, etc.	Production losses, reduced efficiency
	PFM9	Corrosion problems (carbon dioxide, iron sulfide, oxygen, etc.)	Stress corrosion cracking (SCC) in steam turbines, failure of pipe, production lines, well injections, and equipment	Reduced safety efficiency and power transmission lines. Production losses
Turbine and auxiliaries	PFM10	Scaling on rotor and diaphragms blades	Turbine worn blades, vibration	Reduced efficiency, vibration of rotor, loss of control
	PFM11	Wear and corrosion	Blocked blades	Reduced safety
	PFM12	Sticking of valves	Sticking, leaking	Reduced efficiency
	PFM13	Rotor vibration	Inadequate flow, low pressure	Loss of control
Cooling and NCG extraction system	PFM14	Fouling of condenser tubes	Corrosion on tubes	Poor cooling, loss of efficiency
	PFM15	Blocking of nozzles	Scaling, corrosion	Poor cooling, loss of efficiency
	PFM16	Fouled cooling tower fins	Fan blade failure	Poor cooling, loss of efficiency
	PFM17	Vacuum pump water seal breaking	Water seal break	Loss of vacuum
Generator and electrical systems	PFM18	Rotor vibration	Poor lubrication of bearing	Misalignment
	PFM19	Loose stator coils	Wrong operation	Cost of repair, downtime
	PFM20	Arcing of switch gears	Wrong operation	Poor cooling, corona effect
	PFM21	Failure of motors	Excitation under voltage	Downtime
	PFM22	Failure of transformers	Excitation under voltage	Downtime
Instrumentation and control system	PFM23	H2S damage of copper	Faulty instrument	Safety risk
	PFM24	Wrong control signal	Damage cables	Inefficiency, downtime
	PFM25	Failure of protective relay	Wrong calibration	Inefficiency, downtime

The values of S, R and Q were calculated for all the PFMs as shown in Table 14. Table 15 presents the ranking of the PFMs by S, R, and Q in a descending order.

In VIKOR method, the ν parameter was introduced as the weight of the strategy of the "majority of attributes" (Rao, 2008). Generally, the value of ν is considered as 0.5. It is worth mentioning that ν plays an essential role in the ranking of the PFMs (Rezaie et al., 2014). Considering Eq. (31) it can clearly seen that ν is an effective parameter in establishing the importance of the index rank. Consequently, to validate the obtained results, a sensitivity analysis of ν in the interval [0, 1] was conducted. The results of the sensitivity analysis are shown in Fig. 3.

Table 8
Conventional evaluation of failure modes of the GPP using RPN.

Failure mode	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Occurrence	3	7	5	4	8	9	9	9	8	7	7	3	4	3	7	8	3	3	3	5	9	8	4	8	3
Severity	8	3	4	5	5	3	5	9	8	4	3	7	7	4	4	5	5	8	7	5	4	3	4	6	5
Delectability	3	3	7	5	8	3	2	3	2	5	5	4	3	6	7	7	7	3	3	2	6	6	2	8	5
RPN	72	63	140	100	320	81	90	243	128	140	105	84	84	72	196	280	105	72	63	50	216	144	32	384	75

Table 9
Aggregated and normalized weights of the risk factors.

	Severity				Occurrence				Detection			
Aggregated fuzzy weights	(0.141	0.277	0.363	0.612)	(0.224	0.340	0.428	0.975)	(0.115	0.262	0.346	0.496)
Defuzzified weight	0.355				0.520				0.305			
Normalized weight	0.301				0.441				0.258			

Table 10
Evaluation of the PFMs with regard to the risk factors using Z-numbers.

	O										S										D									
	DM1		DM2		DM3		DM4		DM5		DM1		DM2		DM3		DM4		DM5		DM1		DM2		DM3		DM4		DM5	
	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R	A	R
FM1	P	VH	P	H	MP	VH	MP	VH	MP	VH	G	H	VG	VH	G	VH	G	H	G	VH	P	H	MP	VH	P	H	M	VH	MP	VH
FM2	G	H	MG	VH	VG	VH	G	H	G	VH	P	H	P	VH	MP	M	M	M	MP	VH	P	VH	MP	VH	MP	VH	M	VH	P	H
FM3	MG	H	MP	VH	M	VH	M	VH	G	H	P	VH	MP	VH	MG	VH	MP	VH	P	VH	MG	VH	G	H	G	VH	VG	VH	G	VH
FM4	P	VH	MP	VH	MG	VH	MP	VH	P	H	MP	H	MP	H	M	VH	M	VH	G	VH	M	VH	M	M	M	M	MG	VH	M	M
FM5	VG	VH	G	VH	G	H	G	VH	G	H	M	VH	M	VH	MG	H	M	VH	M	VH	VG	VH	VG	VH	VG	VH	VG	H	VG	MH
FM6	VG	H	VG	H	VG	H	VG	VH	VG	VH	P	VH	P	VH	MP	VH	MP	H	MP	VH	P	VH	P	H	MP	H	MP	VH	MP	VH
FM7	VG	VH	VG	VH	G	VH	VG	VH	VG	VH	M	VH	M	VH	MG	H	M	H	M	H	P	VH	P	H	P	H	P	VH	P	VH
FM8	VG	H	VG	VH	VG	VH	VG	VH	VG	H	VG	VH	VG	H	VG	H	VG	VH	VG	VH	P	VH	P	VH	MP	VH	P	H	M	MH
FM9	G	VH	G	VH	G	VH	VG	VH	VG	VH	VG	H	VG	VH	VG	VH	G	VH	VG	VH	P	VH	P	VH	P	H	P	H	P	VH
FM10	G	VH	G	H	VG	H	MG	VH	G	VH	P	H	M	H	MG	VH	M	VH	MP	VH	MP	VH	MP	VH	MG	VH	MG	VH	G	H
FM11	G	VH	VG	VH	VG	VH	G	H	MG	VH	P	H	P	VH	MP	VH	M	VH	M	VH	MG	VH	M	VH	M	VH	MP	H	M	VH
FM12	P	H	P	H	P	VH	MP	VH	MP	H	MG	VH	G	VH	G	VH	G	VH	MG	H	P	VH	MP	VH	MP	H	G	VH	P	VH
FM13	P	H	MP	MH	G	VH	MP	VH	P	VH	MG	VH	VG	VH	VG	VH	VG	H	G	MH	P	MH	M	VH	MP	VH	P	H	MP	VH
FM14	P	VH	MP	VH	M	H	P	H	P	VH	MP	VH	MP	VH	MG	VH	M	VH	MP	H	M	H	MG	H	MG	VH	M	VH	G	VH
FM15	MG	VH	MG	H	G	VH	G	VH	G	VH	P	H	MP	H	M	VH	MG	VH	MP	VH	MG	H	G	VH	MG	H	VG	H	G	VH
FM16	VG	VH	VG	VH	G	VH	G	H	VG	VH	G	H	M	VH	MP	VH	M	VH	MP	VH	MG	VH	MG	VH	G	VH	G	H	VG	VH
FM17	P	H	MP	VH	P	VH	P	VH	P	VH	MP	VH	MP	VH	MP	VH	G	H	M	VH	G	VH	G	VH	MG	H	MG	VH	MG	VH
FM18	P	H	MP	VH	MP	H	MP	VH	M	VH	G	H	G	VH	VG	VH	G	VH	G	VH	MP	VH	M	H	MP	VH	P	VH	P	VH
FM19	MP	VH	P	H	P	H	P	VH	P	VH	G	VH	MG	MH	VG	H	G	VH	MG	VH	MP	VH	MP	VH	P	VH	P	VH	MP	VH
FM20	M	VH	M	H	MG	VH	M	MH	M	VH	MG	H	P	VH	G	VH	M	VH	M	VH	P	MH	P	VH	P	VH	P	VH	MP	H
FM21	VG	VH	VG	VH	VG	VH	VG	VH	VG	VH	MP	VH	G	H	MP	VH	MP	VH	P	MH	G	MH	MG	VH	MG	VH	G	H	G	VH
FM22	VG	H	VG	VH	G	VH	VG	MH	VG	VH	MP	H	P	H	P	VH	MP	VH	P	VH	VG	VH	G	MH	G	MH	M	VH	MP	VH
FM23	M	VH	M	VH	MP	VH	M	MH	M	VH	MP	VH	MP	VH	M	H	MP	H	M	VH	P	VH	MP	VH	P	VH	P	VH	P	MH
FM24	VG	VH	VG	VH	VG	VH	G	H	VG	H	MG	VH	MG	H	G	VH	G	VH	MG	MH	G	VH	G	VH	VG	H	VG	H	VG	H
FM25	P	VH	P	H	MP	VH	M	VH	MP	VH	M	VH	MG	MH	MP	VH	G	H	M	VH	MG	VH	MG	VH	MP	VH	MP	H	M	VH

Table 11
Defuzzified fuzzy values of PFMs.

Failure mode	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Occurrence	2.77	7.14	5.14	3.87	7.83	8.52	8.41	8.57	7.98	7.14	7.29	2.64	4.06	3.06	6.68	8.09	2.58	3.25	2.56	5.11	8.83	7.79	3.79	8.16	3.17
Severity	7.83	2.87	3.96	4.95	5.38	2.77	5.32	8.57	8.16	4.15	3.27	6.68	7.02	4.46	4.05	4.98	4.87	7.88	6.72	4.77	4.15	2.65	3.75	6.24	4.91
Detectability	3.15	3.17	7.18	4.79	8.18	2.75	1.86	2.92	1.92	5.17	4.67	4.28	3.01	5.94	7.00	7.08	6.58	3.17	2.84	2.44	6.34	5.53	2.46	8.00	4.67

Table 12
Entropy measure, divergence and objective weights of risk factors.

	Severity	Occurrence	Detection
ej	0.974	0.984	0.972
div_j	0.026	0.016	0.028
w_j^o	0.370	0.229	0.401

Table 13
Calculated values of the f^* and f^-.

	Severity				Occurrence				Detection			
	l	m	n	s	l	m	n	s	l	m	n	s
fj^*	0.887	2.044	2.230	4.619	0.845	2.230	2.602	4.646	0.887	1.858	1.858	2.787
fj^-	7.659	8.617	9.574	9.574	7.433	8.362	9.292	9.345	7.096	7.982	8.869	9.345

Table 14
Calculated values of S_i, R_i and Q_i for PFMs.

Failure mode	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
S_i	0.392	0.347	0.520	0.371	0.751	0.389	0.461	0.726	0.605	0.539	0.460	0.395	0.426	0.350	0.585	0.707	0.378	0.422	0.293	0.319	0.672	0.500	0.191	0.820	0.337
R_i	0.288	0.249	0.279	0.148	0.324	0.320	0.315	0.334	0.313	0.249	0.259	0.229	0.254	0.212	0.268	0.296	0.244	0.291	0.230	0.143	0.335	0.283	0.081	0.315	0.159
Q_i	0.580	0.477	0.666	0.325	0.926	0.630	0.680	0.923	0.790	0.629	0.584	0.482	0.548	0.417	0.698	0.844	0.492	0.608	0.401	0.275	0.882	0.656	0.067	0.966	0.316

Table 15
Ranking of the PFMs by the proposed and conventional methods.

Failure mode	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
S_i	16	21	9	19	2	17	11	3	6	8	12	15	13	20	7	4	18	14	24	23	5	10	25	1	22
R_i	10	16	12	22	3	4	6	2	7	16	14	19	15	20	13	8	17	9	18	23	1	11	24	5	21
Q_i	15	19	9	22	2	11	8	3	6	12	14	18	16	20	7	5	17	13	21	24	4	10	25	1	23
RPN	16	17	8	11	2	14	12	4	9	8	10	13	13	16	6	3	10	16	17	18	5	7	19	1	15

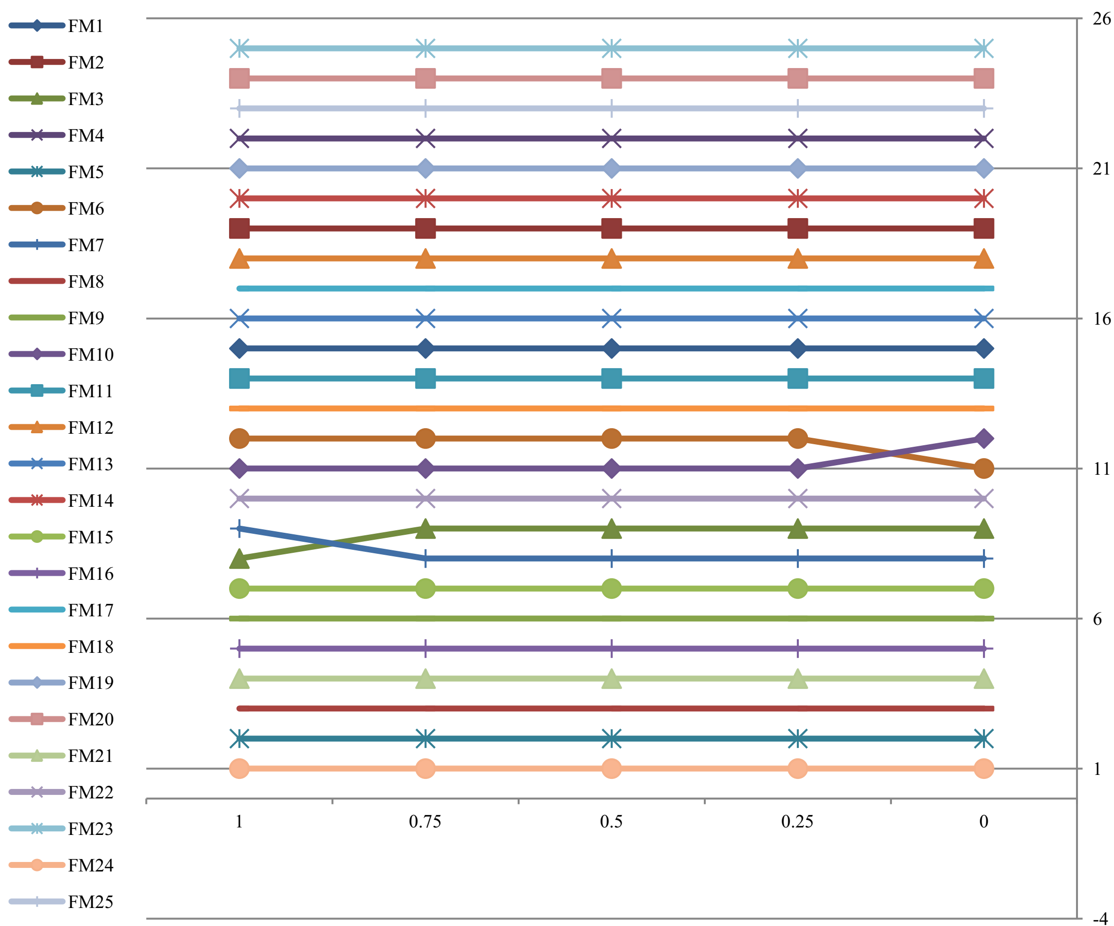

Fig. 3. Sensitivity analysis of the potential failure modes (PFMs).

Table 16
Ranking of the PFMs by different values of Φ.

	Failure mode	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Q_i	$\Phi = 0$	22	15	7	19	2	11	10	5	9	12	13	18	20	16	6	4	14	17	24	23	3	8	25	1	21
	$\Phi = 0.5$	15	19	9	22	2	11	8	3	6	12	14	18	16	20	7	5	17	13	21	24	4	10	25	1	23
	$\Phi = 1$	8	21	16	20	4	17	10	1	2	13	18	11	9	24	12	7	19	6	15	22	5	14	25	3	23

4. Discussion

In this study, we used linguistic variables in terms of Z-number to capture the opinions of the experts about the importance of the FMEA risk factors as well as for comparison of the PFMs with regard to the risk factors. The weights of risk factors were determined by using the Fuzzy AHP and the ranking of the PFMs was done by the VIKOR method. Although some works applied the VIKOR method (Liu et al., 2015) to rank the failure modes, the contribution of this work is the application of the Z-numbers to handle the partial reliability associated with the judgments of the experts. Besides, we used the trapezoidal fuzzy numbers to determine the weight of risk factors. Trapezoidal fuzzy numbers (TpFN) have many advantages over triangular fuzzy numbers as they have more generalized form in relation to triangular ones (Gajivaradhan and Parthiban, 2015).

According to Liu et al. (2013) the most frequently applied method to FMEA was found to be Artificial intelligence (AI) with a proportion of 40% (within that the most popular approach is fuzzy rule-base system (FRBs)), followed by MCDM, integrated approaches, and mathematical programming. Also, the MCDM approaches were the next most applied methods with 22.5% of all methods. The most used method for FMEA risk analysis is FRBs that suffer from limitations such as: (i) Difficulty to define appropriate membership functions for the risk factors and risk priority levels; (ii) Rule explosion problem in defining fuzzy RPN model; (iii) difficulty of the construction of a fuzzy if-then rule base and highly costly and time-consuming of the method; (iv) Inability of the method to distinguish between different fuzzy if-then rules with the same consequence but different antecedents. Also the MCDM methods such as AHP, though deal with the inequality of the risk factor weights, they didn't consider the uncertainty of the data. Furthermore, mathematical programming approaches such as fuzzy DEA (data envelopment analysis) suffer from the problems such as complexity of calculations and lack of a full ranking for prioritization of the failure modes. While in the distance-based methods such as TOPSIS, the failures are prioritized based on the measurement of the Euclidean distance of an alternative from an ideal goal but, it does not consider the relative importance of these distances (Zhang and Wei, 2013) while the VIKOR method deal with this limitation.

To demonstrate the effectiveness of the proposed method, a real-world study was conducted in a geothermal power plant and the obtained results of the proposed method were compared with the traditional FMEA. Table 15 shows the ranking order of the integrated risks of the 25 PFMs. As it can be seen from Table 15 the ranking of the PFMs in the proposed method and conventional is not exactly the same (except that for PFM1), although the trend is similar with a slight difference. One reason for this difference is the PFMs with the similar RPNs in the conventional ranking. While the conventional FMEA considers a similar rank for the PFMs with the equal RPNs (whereas they have inherently different risks), the proposed method take into account this inherent difference and assigns different ranking for them. To demonstrate how the proposed method can benefit the analysis, it is necessary to have a close look at the results shown in Table 15. While PFM16 (RPN = 280) obtained the 3rd rank in conventional ranking, it achieved the 5th place in the proposed method. On the other hand FM8 (RPN = 243) acquired the 3rd place in the proposed method. At first glance it seems reasonable that the PFM which has the more value of RPN (herein PFM16) to gets the lower rank (higher priority), but the proposed method has assigned the lower priority to what that obtained the lower RPN (i.e. PFM8). One reason for this difference is the weighing factor that has been used for the risk factors in the proposed method. While PFM16 has the risk factors (O = 8, S = 5, and D = 7), the risk factors for the PFM 8 are (O = 9, S = 9, and D = 3). Since the weights of the severity (S) and occurrence (O) is determined as WS = 0.441 and WO = 0.301, by the Fuzzy AHP, it is expected that those PFMs have higher values of O and S to have the more priority.

On the other hand the PFM1 (O = 3, S = 8, D = 3, RPN = 72), PFM14 (O = 3, S = 4, D = 6, RPN = 72), and PFM18 (O = 3, S = 8, D = 3, RPN = 72) got the 16th place in the ranking queue of the conventional method as the corresponding RPN calculated as 72. The ranking rationale for the PFM14 is as same as for the PFM8 and PFM16 as stated in the previous case. A close look at the values of the PFM (1) and PFM (18) risk factors reveals that the values of the three risk factors for both PFM1 and PFM18 are exactly the same, nevertheless they obtained different ranking in the proposed method. The question is how the proposed method can do this? Although the traditional method can't handle this situation and differentiate between two failure modes, the proposed method efficiently handled this problem and assigned different ranking for the PFMs (1) and (18). The answer is originated from the rationale of the VIKOR for ranking the alternatives. The VIKOR is based on an aggregating function representing "closeness to the ideal", which based on the compromise programming method. The VIKOR method determines a compromise solution, providing a maximum "group utility" for the "majority" and a minimum of an individual regret for the "opponent". As the aggregated fuzzy values of failure modes for the PFM1 and PFM18 are different (2.768, 7.832, 3.152 for the PFM1 and 3.252, 7.877, and 3.17 for the PFM18), then the VIKOR method after calculation of the closeness to the idea distances, offers different rank for these failure modes while considering their weights. Again, this can be explained by the ability of the proposed method that was raised from the novelty of the method in the shed of subjective-objective weighing of the method.

Another interesting case is the PFM12 (O = 3, S = 7, D = 4, RPN = 84) and PFM13 (O = 4, S = 7, D = 3, RPN = 84) which both cases got the same rank (i.e. 13th) in the conventional approach. Although they are inherently two different cases of PFMs (different values of O and D), they were ranked the same in conventional FMEA. Again, the proposed method differentiates these PFMs according to the corresponding weights and ranks the PFM12 as 18th (because of the higher value of D and lower value of O) and PFM13 as 16th (because of the higher value of O and lower value of D). The abovementioned examples could present the adequacy of the proposed method for the PFMs with the same RPNs.

It is worth mentioning that before making any decision, the compromise ranking was conducted in terms of the risk factors value. The ideal solution for the risk factors will take place when the occurrence probability and severity of the PFMs are low and delectability is high. On the other hand, the worst case will happen when occurrence probability and severity of the PFMs are very high and their delectability is very low. Consequently, in this paper the compromised solution corresponds to the failure mode closest to the ideal solution, which indicates that it has the lowest risk compared with the other cases. Bearing this in mind, the compromise solution for the failure modes is the PFM23 as it satisfies two conditions of the compromise solution. As the ranking of the three parameters of S and Q are the same for the PFM23, then the condition "C1" is satisfied. On the other hand as $Q(A(2)) - Q\,(A(1)) \geqslant DQ$, then $0.224{-}0 > 1/(25{-}1)$ then condition "C2" also is satisfied. Therefore, the PFM23 can be proposed as the compromise solution.

To further demonstrate the effect of the application of combination weights (subjective and objective weights) in the obtained results, we performed the procedure of the proposed approach with taking into account only the subjective ($\Phi = 1$) or objective weights ($\Phi = 0$) of risk factors. As it can be seen from Table 16 the ranking of the PFMs is completely different, for different values of the weight restriction (Φ) parameters (i.e. $\Phi = 0$, $\Phi = 0.5$, and $\Phi = 1$). The findings revealed that when considering only subjective or objective weights, the results of the failure modes ranking are affected heavily so that the only constant ranking was obtained for the PFM23, with all other PFMs were achieved different rank with different Φ values. This bias must be considered carefully when changing the weight restriction (Φ) value with respect to the real situation of the analysis and experts' opinions.

Furthermore, the results obtained from the sensitivity analysis (for the ν value) are illustrated in Fig. 3. As it can be seen, the ranking of the most PFMs is not affected by considering different values of ν. In the other words, one can say that these PFMs are similar in terms of both maximum group utility (MGU) and minimum individual regret (MIR) except that for PFMs 1, 2, 5, 6, 8, 10, 11 and

12. As it can be observed from Fig. 3 the ranking of PFMs 5, 10, 11, and 12 were aggravated (i.e. obtained higher risk priority or lower rank in Q) when the ν value increased. On the other hand the ranking of PFMs 1, 2, 6, and 8 were improved (i.e. obtained lower risk priority or higher rank in Q) when the ν value was increased. However, it must be noted that this trend is not the same for these PFMs. While one focuses on MIR the PFM11 has lower rank than PFM1, but this will be reversed when one focuses in MGU. This trend is the same for the PFMs (2 and 12), (5 and 8), (6 and 10). On the other hand, when emphasizing on MIR, PFM11 gets lower rank than PFM1 while PFM1 obtains higher rank than PFM11 when focus is on MGU. This analysis confirms the reliability of the results obtained from the proposed methodology.

As final words, it is worth mentioning that, the provided example has demonstrated the applicability of the proposed method in the real-world applications. Besides, the sensitivity analysis of the results has validated the reliability of the suggested model. In addition to the sensitivity analysis results, the opinions of the domain' experts has confirmed the efficacy of the method for the risk analysis in the real environments. Moreover, the obtained result has verified the superiority of the method with regard to the conventional FMEA.

5. Conclusion

Despite of developments in risk assessment domain (especially quantitative risk assessment methods), methods like the FMEA has obtained extended applicability because of its simplicity and consuming less time. On the other hand, because of its intrinsic ambiguity, the FMEA method has been criticized and its application has become limited. In this paper a framework based on AHP-Entropy was proposed that, in one hand can capture the subjective judgments of decision makers in term of the Z numbers and on the other hand can apply objective weights based on Shannon entropy to regulate the subjective weights. In addition, the Z numbers can handle the inherent reliability associated with the judgments of the decision makers. Furthermore, application of the fuzzy VIKOR method for ranking the PFMs can suggest a compromise solution to decision makers with the "maximum group utility" for the "majority" and the "minimum of the individual regret" for the "opponent". A sensitivity analysis confirmed the reliability of the data obtained from the proposed framework in a real-world application. The main limitation of the proposed method is the interrelations between risk factors (O, S, and D) that are not considered in the failure modes assessment process, which may cause biased ranking of the results. One solution for this problem is to applying methods such as ANP (analytical network process) instead of the AHP method for determining the weights of risk factors. As a recommendation for future research, it is suggested that the proposed method will be tested in other working area to further validate the reliability of the method. Also the comparison of the proposed method with other developed methods for criticality analysis of the FMEA is recommended.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.ssci.2016.10.006.

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