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Modified vertical bearing capacity for circular foundations in sand using reduced friction angle

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ABSTRACT

Bucket foundations, which are large cylindrical structures that are open at the base and closed at the top, recently have attracted much attention in offshore projects. To demonstrate the relationship between the vertical bearing capacity of a bucket foundation relative to the corresponding capacity of the circular plate, several loading tests on small-scale bucket and surface foundations were performed at Aalborg University. In the current research, the modified vertical bearing capacity of circular surface footings was investigated with the reduced friction angle. A linear relationship with reasonable accuracy was found between the relative density and the reduced friction angle.

1. Introduction

Energy production from offshore wind turbines is expected to increase substantially in the near future, because offshore wind turbines are expected to become more profitable. However, the foundations connected with offshore wind turbines are a large economic expense. Suction buckets are tubular steel foundations that are installed by sealing the top and applying suction inside the bucket. The hydrostatic pressure difference and the deadweight cause the bucket to penetrate the soil. This installation procedure allows the bucket to be connected to the rest of the structure before installation, thereby reducing the number of installation steps. A bucket foundation (also denoted as a skirted foundation or suction caisson) is large cylindrical structure that is open at the base and closed at the top. The cylindrical part is called the “bucket skirt,” and the upper plate that closes the bucket is called the “bucket lid” or “top plate”. The bucket foundation can be used as a single foundation (monopod) or as a multiple foundation system (e.g., tripod).

In November 2002, the first bucket foundation for a fully operational wind turbine was installed at an offshore test facility in Frederikshavn, in the northern part of Jutland, Denmark (Fig. 1). In the case of vertical loading, a bucket foundation is assumed to behave similarly to an embedded circular foundation. Thus, soil trapped within the bucket is expected to behave the same or nearly the same as a rigid cluster. The bucket foundation, which is constrained laterally by the skirts, prevents the soil within the bucket from experiencing large deformations (Achmus and Abdel-Rahman, 2005; Abdel-Rahman and Achmus, 2005; and Ibsen, 2008).

Terzaghi (1943) presented a formula for the general bearing capacity of circular foundations located in saturated sand. This formula is based on the principle of superposition, which results in a conservative estimate of the bearing capacity (Hansen, 1975). Several authors have performed extensive studies to determine the values of the bearing capacity factors for the relationship proposed by Terzaghi (1943). Although some factors have been determined to be exact, others are still being discussed.

Since 2006, a series of experimental investigations have been carried out at Aalborg University to modify the expression presented by Byrne and Houlsby (1999) (Eq. (4)) that relates the vertical bearing capacities of bucket and surface foundations founded on sand. The current research seeks to determine corrected values for the bearing capacity of circular surface footings located on sand, by using the reduced friction angle (Hansen, 1979). The results given in the present work were utilized to obtain a corrected form of the relationship proposed by Byrne and Houlsby (1999), which will be presented in a later publication.

2. Bearing capacity factors

Prandtl (1920) presented an exact solution for the bearing capacity factor N_q:

$$N_q = e^{\pi \tan\varphi} \tan^2(45+(\varphi/2)) \quad (1)$$

where φ is the friction angle. Although this equation calculates N_q exactly, it has not yet been possible to determine the exact value

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Nomenclature

γ	effective unit weight of the soil
D	diameter of the foundation
q	effective overburden pressure, $q=d\cdot\gamma$
d	skirt length
k	coefficient of lateral earth pressure
δ	friction angle between the skirt and the surrounding soil
ψ	dilation angle

Fig. 1. Bucket foundation after installation in Nov., 2002.

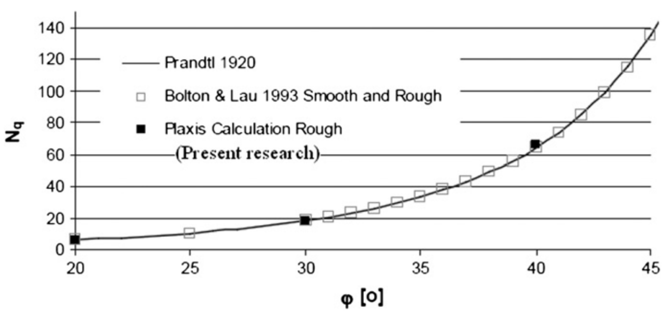

Fig. 2. Plain strain values of N_q suggested by different authors.

for the bearing capacity factor N_γ. Many attempts have been made to calculate N_q (Fig. 2).

One of the first expressions for N_γ was proposed by Hansen (1961), as follows:

$$N_\gamma = 1.8(N_q - 1)\tan\varphi \tag{2}$$

This expression was developed based on the results from the lower-bound solution given by Lundgren and Mortensen (1953) and the upper-bound solution given by Meyerhof (1951). Only the value at $\varphi=30^\circ$ was given by Lundgren and Mortensen (1953). The corresponding values at $\varphi=20^\circ$ and $\varphi=40^\circ$ were calculated with the kinematically admissible rupture shape proposed by Lundgren and Mortensen (after Hansen, 1961). Fig. 3 shows the values of N_γ suggested by different authors, compared to the values by Lundgren and Mortensen (1953) and Meyerhof (1951). The lower-bound results by Lundgren and Mortensen fit the results from Martin (2004) and the relationship given in the Danish Code of Practice (DS 415, 1998) for foundation engineering. The results from Martin (2004) are the converged solutions obtained from the Analysis of Bearing Capacity (ABC) program, version 1.0. These results are argued to be exact values (Martin, 2005).

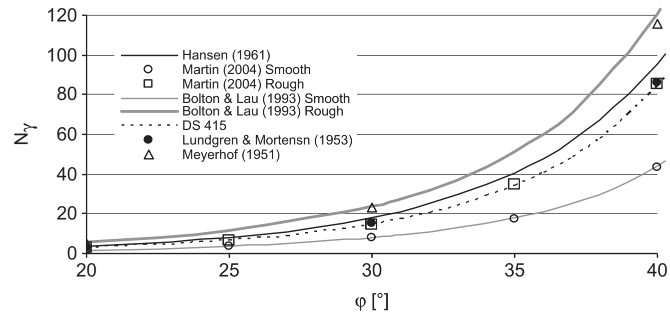

Fig. 3. Values of N_γ suggested by different authors compared with the lower and upper bound values by Lundgren and Mortensen (1953) and Meyerhof (1951), respectively.

The value of the bearing capacity factor N_γ according to DS 415 (1998) is given by:

$$N_\gamma = (1/4)((N_q - 1)\cos\varphi')^{3/2} \tag{3}$$

Fig. 3 also shows that, in the case of rough foundations, the results of Bolton and Lau (1993) are higher than the corresponding values obtained by Meyerhof (1951). Martin (2004) claims that the rupture figure used in the calculations by Bolton and Lau is incorrect for the case of a rough base. For a smooth strip footing, the values of the bearing capacity N_γ given by Martin (2004) and Bolton and Lau (1993) are identical. The value of N_γ given by Martin (2004) and Bolton and Lau (1993) for a friction angle of 20° was verified for a smooth strip foundation from the corresponding FE calculation by Clausen et al. (2007).

3. Expressions for vertical bearing capacity

Byrne and Houlsby (1999) performed a test series that included 17 tests on circular surface footings with a diameter of 50 mm on very dense and dry sand. Based on these tests, Byrne and Houlsby suggested the use of the general bearing capacity formula given in Eq. (4). This formula utilizes the bearing capacity factors from Bolton and Lau (1993) for estimating the vertical bearing capacity of a bucket foundation:

$$\frac{V_{peak}}{V_0} = 1 + 0.89\frac{d}{D} \tag{4}$$

In Eq. (4), V_{peak} and V_0 are the vertical bearing capacities of bucket foundations and the corresponding surface foundations, respectively. The friction between the soil and skirt is ignored, and a friction angle of approximately 46° is used. The friction angle can be determined from the measured bearing capacities of the surface foundations.

The normalized bearing capacity of a bucket foundation is given by the following relationship that includes all of the terms in Eq. (4):

$$\frac{V_{peak}}{V_0} = 1 + \frac{d}{D}\frac{2}{N_\gamma}\left(N_q + \frac{d}{D}2K\tan\delta\right) \tag{5}$$

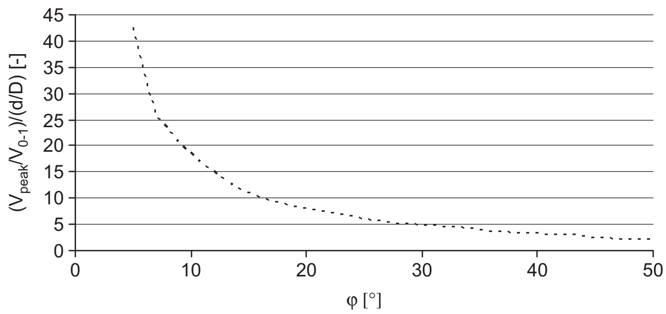

Fig. 4. Variation of the constant value in the analytical relation of V_{peak}/V_0, ignoring the skirt friction.

Table 1
Fitted values for constants in Eq. (7) for the bearing capacity factors.

	Circular foundation		Strip foundation	
	Smooth	Rough	Smooth	Rough
c_1	0.1	0.16	0.12	0.25
c_2	1.33	1.33	1.51	1.5
c_3	0.715	0.8	1	1
c_4	1.42	1.5	1	1

Because the values of the bearing capacity factors given by Bolton and Lau (1993) are incorrect for foundations with a rough base, Eq. (4) can be rewritten with the recommended expression of N_γ given by Eq. (3):

$$\frac{V_{peak}}{V_0} = 1+2.1\frac{d}{D} \quad (6)$$

Eq. (6) was derived with a friction angle of 48°, which was found by back-calculation of the experiments from Byrne and Houlsby (1999). The relationship in Eq. (6) is only valid for the small-scale tests performed by Byrne and Houlsby (1999). The constant in Eq. (6) is a function of the friction angle, as shown in Fig. 4. The result from rewriting the relationship in Eq. (6) shows that the vertical bearing capacity of a skirted foundation relative to the corresponding capacity of a circular plate is significantly larger than that assumed by Byrne and Houlsby (1999).

In a plane strain situation, the bearing capacity factors N_q and N_γ can be calculated with Eqs. (1) and (2), respectively. In the case of a smooth base, N_γ must be determined from Martin (2004) or Bolton and Lau (1993).The values of the bearing capacity factors are also fitted to the following expressions:

$$N_\gamma = c_1 \cdot ((N_q-1)\cos\varphi)^{c_2}$$
$$N_q = c_3 \cdot e^{c_4.\pi.\tan\varphi}\tan^2(45+\varphi/2) \quad (7)$$

where the constants c_i are given in Table 1.

4. Experimental setup

The test box used to investigate the behavior of circular surface footings was improved in connection with this work. The structure of the test box is illustrated in Figs. (5) and (6). By redesigning the drainage system in the bottom of the test box, the depth of the sand sample was increased by approximately 100 mm, to 530 mm.

The test box consists of a rigid steel construction, with inner horizontal dimensions of 1600 × 1600 mm and an inner total depth of 650 mm (Fig. 5). In the bottom of the test box, a drainage system provides the sand with water through a drainage layer.

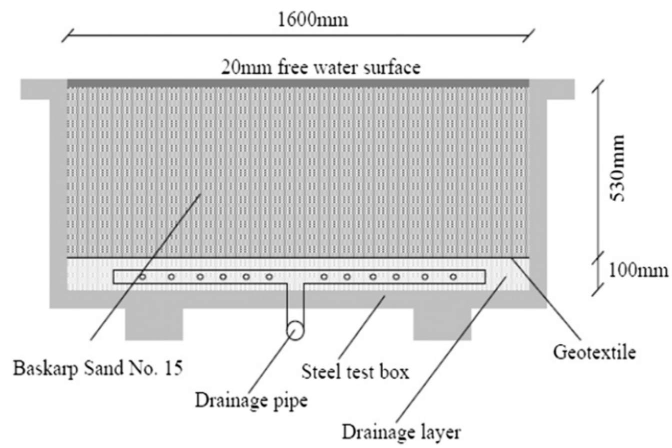

Fig. 5. Structure of the test box used for the small-scale loading tests.

Fig. 6. Test box used for loading tests on bucket foundations.

The drainage system consists of a set of perforated pipes, which leads the water in and out of the test box. Through jets in the pipes, the water in the drainage layer is distributed across the entire area of the test box before entering the sand above. The drainage layer consists of stones around the drainage pipes with diameters of 2 to 5 mm. A sheet of Geotextile was placed between the sand layer and the drainage layer to prevent the sand from penetrating the drainage layer.

The sand used in the test box is Aalborg University Sand No. 1. The sand was saturated with water during the experiments. Aalborg University Sand No. 1 is introduced briefly in Section 4.1, but the full details regarding its characteristic behavior can also be found in Ibsen et al., (1995).

4.1. Soil properties

Aalborg University Sand No. 1 is graded sand from Sweden. The largest grains are round in shape, whereas the small grains have sharp edges. The main component of Aalborg University Sand No. 1 is quartz, but it also contains feldspar and biotite. The properties of Aalborg University Sand No. 1 are well described, due to an extensive testing program performed at Aalborg University. Triaxial and other tests, including classification analyses, have been performed, and the details are described in Ibsen et al. (1995). Some of the grading results are as follows:

- Mean grain size, d_{50}=0.14 mm.
- Coefficient of uniformity, $U=(d_{60}/d_{10})=1.78$
- Grain density, d_s=2.64

- Maximum void ratio, $e_{max}=0.858$
- Minimum void ratio, $e_{min}=0.549$

5. Results and discussion

Results from vertical load tests on circular surface footings with rough bases are presented in the following figures. Different diameters (i.e., 50, 100, and 200 mm) were experimentally modeled and tested. Failure was defined as the peak value of the vertical load; if no peak was obtained during further vertical deformation, then failure was defined as the residual value. The vertical load at failure was denoted as V_0 for circular surface footings (i.e., bucket foundations with an embedment ratio of zero).

Figs. 7–9 show that there is large scatter in the relationship between the relative density and the vertical bearing capacity. The vertical bearing capacity of the surface footings V_0 was usually calculated while ignoring the influence of the overburden pressure, which was caused by the deformations that developed during the loading procedure. The measured capacities in Figs. 7–9 include the highly variable contributions from the deformations.

To exclude the contribution of the deformations to the bearing capacity, the friction angle must be determined. The bearing capacity formula in Eq. (8) (Terzaghi, 1943) was used to determine the friction angle with the bearing capacity factors in Eq. (7) for circular foundations. Because the sand was glued onto the base of the circular plates, the bearing capacity factors were determined under the assumption of a rough base, with the following formula:

$$V=\gamma'\frac{D}{2}N_\gamma\left(\frac{\pi D^2}{4}\right)+q'N_q\left(\frac{\pi D^2}{4}\right) \tag{8}$$

where $q=w\cdot\gamma$ and w is the vertical settlement at failure. The unit soil weight γ is known for each test from the void ratio determined with the CPT experiments. After determining the friction angle, we determined the corresponding V_0-capacity by using only the first term in Eq. (8). This capacity is denoted as the corrected V_0-capacity,$V_{0_{corr}}$.

The friction angle was estimated from the bearing capacity formula under the assumption of soil behavior with associated flow. This assumption means that the calculated friction angle does not correspond to the triaxial one. Hansen (1979) suggested the use of a reduced friction angle in the bearing capacity formula to account for the degree of nonassociation arising from the difference between the friction and dilation angles. The value of this reduced friction angle is given by Hansen (1979):

$$\tan\varphi_d=\frac{\sin\varphi\cos\psi}{1-\sin\varphi\sin\psi} \tag{9}$$

Within the current research, the reduced friction angle ϕ_d was calculated from the bearing capacity tests with surface footings by using Eq. (8). The results are shown in Figs. 10 and 11.

In Fig. 10, the calculated friction angle was differentiated with respect to the different data series from which the experiments originated. To investigate the reason for the scatter, the results in Fig. 11 were differentiated with respect to the diameter of the tested footings. The results shown in Figs. 10 and 11 reveal that there was no systematic scatter that could be assigned to the size of the foundation or the test series. Instead, the scatter is assumed to be due to the evaluation of the relative density from the laboratory-CPT. Whereas the relative density of the tested sand was determined as a mean value over depth, a small variation with depth was observed in some tests. The scatter also is ascribed to experimental errors, such as skewed settlements of the plate and loads that were applied at an inclined angle from the vertical.

The results from the laboratory tests were fitted to the following linear relation, which is also shown in Fig. 11:

$$\varphi_d=0.214D_r+22.86 \tag{10}$$

The linear relationship between the relative density and the reduced friction angle was described with reasonable accuracy by Eq. (10). It must be noted that Eq. (10) is only valid for stress levels under which the experiments were performed and for the sand tested.

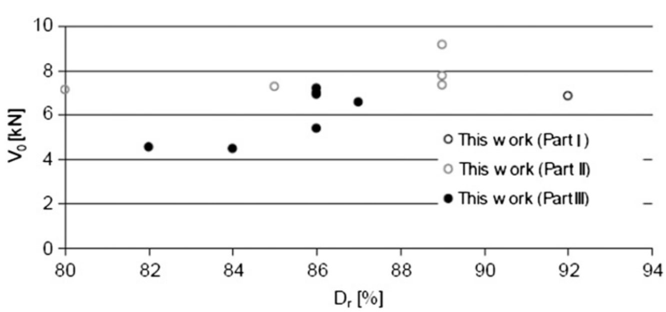

Fig. 7. Results from the vertical loading of circular footings with $D=200$ mm.

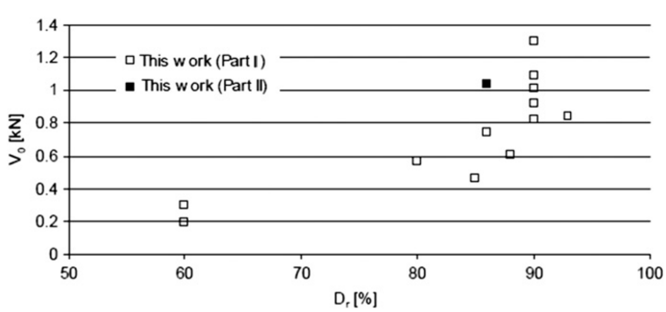

Fig. 8. Results from the vertical loading of circular footings with $D=100$ mm.

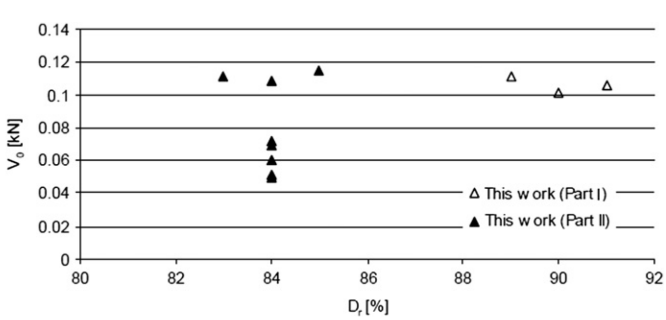

Fig. 9. Results from the vertical loading of circular footings with $D=50$ mm.

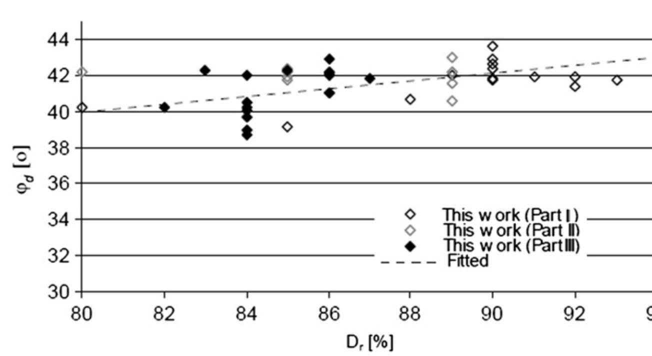

Fig. 10. Reduced friction angle calculated from V_0 experiments.

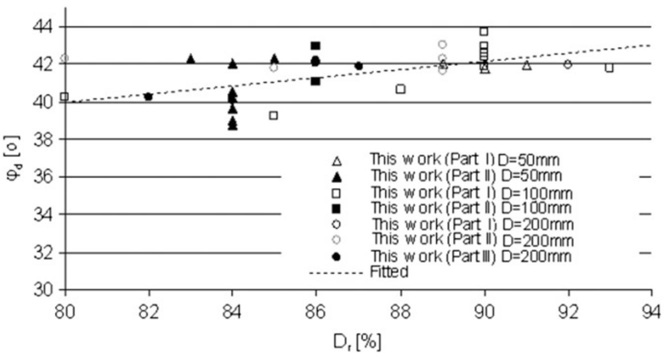

Fig. 11. Reduced friction angle calculated from V_0 experiments.

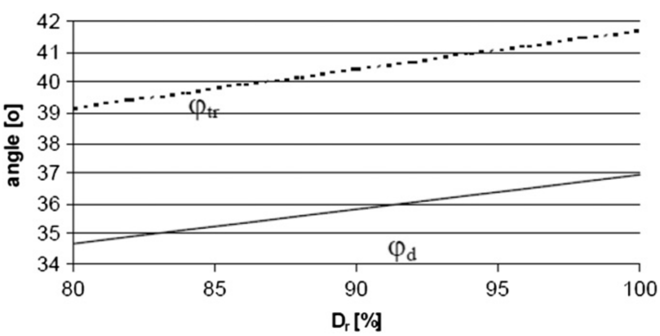

Fig. 12. Theoretical variation of the friction angle for sand with $U=3$ and $\varphi_{cl}=30^{\circ}$ (Ibsen and Lade, 1998), according to Eqs. (9) and (11).

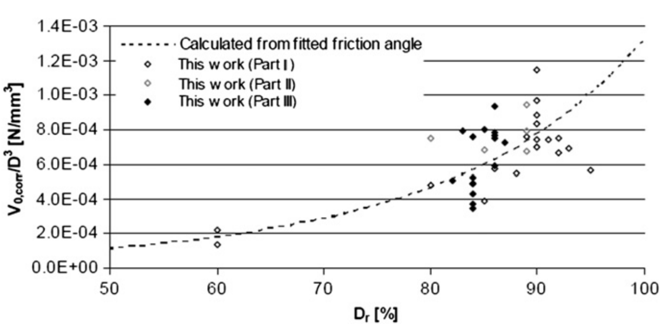

Fig. 13. Corrected values of the measured V_0 bearing capacity versus the relative density of the sand measured with the CPT tests.

The linear fit was also examined for nonassociated flow based on the following example. The friction and dilation angle of sand are assumed to be linear, and are frequently estimated as follows:

$$\varphi_{tr}=30^{\circ}-\frac{3}{U}+\left(14-\frac{4}{U}\right)D_r \tag{11}$$

where U is the coefficient of uniformity. The dilation angle, ψ, is assumed to follow$\psi=\varphi_{tr}-\varphi_{cl}$, where φ_{cl} is the characteristic friction angle that is assumed to be constant for a given sand (as shown in Ibsen and Lade, 1998). Combining Eqs. (9) and (11) results in a linear relationship between the relative density and the reduced friction angle. This linear relationship is shown in Fig. 12 for a sand with $U=3$ and $\varphi_{cl}=30^{\circ}$.

The values of $V_{0_{corr}}$ are shown in Fig. 13. To compare the results, the bearing capacity $V_{0_{corr}}$ was normalized by the diameter to the third power. The corrected values of the vertical bearing capacity in Fig. 13 show, in spite of the scatter, good agreement with the variation of the theoretical line. The theoretical line in the figure was calculated with the fitted relationship between the reduced friction angle and the relative density given by Eq. (10).

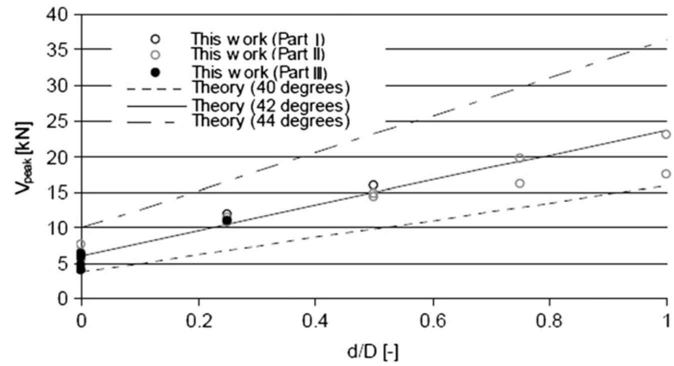

Fig. 14. Results from bearing capacity tests on buckets with $D=200$ mm, corrected for deformations.

A similar procedure was followed for bucket foundations with different embedment ratios, which will be discussed in detail in a later work. Results from the performed bearing capacity tests on buckets with $D=200$ mm as a function of the embedment ratio d/D are shown in Fig. 14. The values in this figure were corrected for deformation by using the reduced friction angle and bearing capacity formula, as explained earlier. The modified theoretical bearing capacity is shown for friction angles of 40°, 42°, and 44°, and the reduced triaxial friction angle of 42° captures the measured capacities very well.

6. Conclusions

Bucket foundations represent a new concept in offshore wind turbines. To obtain a new general formula for the bearing capacity of bucket foundations, a series of experimental studies were performed on circular surface footings (i.e., bucket foundations with an embedment ratio of zero) placed on Aalborg University Sand No. 1. For the experimental modeling, the test box used to investigate the behavior of the circular surface footings consisted of a rigid steel construction with inner horizontal dimensions of 1600×1600 mm and an inner total depth of 650 mm. The circular footings were placed on Aalborg University Sand No. 1, which is primarily quartz, but also contains feldspar and biotite. Different diameters of footings (e.g., 50, 100, and 200 mm) were tested.

The results of the vertical bearing capacity versus the relative density demonstrated large scatter. The measured bearing capacities of surface footings had to be corrected, due to the influence of deformations that developed during the loading procedure. To remove the contribution of the deformations from the bearing capacity, the friction angle was determined from the experiments and analytical solutions. A new expression for the reduced friction angle was developed with the data from the V_0 experiments. Finally, corrected values of V_0 were tabulated, which showed good agreement with the variation of the theoretical line.

References

Abdel-Rahman, K., Achmus, M., 2005. Finite element modelling of horizontally loaded monopile foundations for offshore wind energy converters in Germany. International Symposium on Frontiers in Offshore Geotechnics (ISFOG), Perth, Australia.

Achmus, M., Abdel-Rahman, K., 2005. Design of monopile foundations for offshore wind energy plants. 11th International Coloquium on Structural and Geotechnical Engineering, Cairo, Egypt.

Byrne, B.W., Houlsby, G.T., 1999. Drained behaviour of suction caisson foundations on very dense sand. Offshore Technology Conference, May.

Bolton, M.D., Lau, C.K., 1993. Vertical bearing capacity factors for circular and strip footings on Mohr–Coulomb soil. Can. Geotech. J. 30, 1024–1033.

Clausen, J., Damkilde, L., Krabbenhoft, K., 2007. Efficient finite element calculation of Ngamma. Proceedings of the 11th International Conference on Civil,

Structural and Environmental Computing. Barry H.V Topping (Ed). Civil Comp Press.
Ds 415, 1998. Norm for Fundering (code of practice for foundation engineering. 4th. edn., Danish Standard Copenhagen, In Danish.
Hansen, J.B., 1961. Bulletin No.11, a general formula for bearing capacity. The Danish Geotechnical Institute, Copenhagen.
Hansen, B., 1975. The bearing capacity of footings. First Baltic Conference on Soil Mechanics and Foundations Engineering, Gdansk, Poland, pp. 22–25.
Hansen, B., 1979. Definition and use of frictional angles, Proc. Int. Conf. VII, ECSMFE Brighton, UK.
Ibsen, L.B., Lade, P.V., 1998. The role of the charactristic line in static soil behavior. Localiozation and Bifurcation Theory for Soils and Rocks, Adachi, Oka & Yashima (Eds.), Balkema, Rotterdam, pp. 221–230.
Ibsen, L.B., Broup, M., Hedegaard, J., 1995. Data report 9501: Triaxial Tests on Baskarp Sand No.15. Geotechnical Engineering Group, Aalborg Univeresity.
Ibsen, L.B. 2008. Implementation of a new foundation concept for offshore wind farms. Int. Proc 15th Nordic Geotechnical Meeting.Norway, pp. 19–33.
Lundgren, H., Mortensen, K., 1953. Determination by the theory of plasticity of the bearing capacity of continuous footings on Sand. Proc. 3rd Int. Conf. Soil. Mech. Vol. 1, Zurich.
Martin, C.M., 2004. User guide for ABC-analysis of bearing capacity, Version 1, Department of Engineering Science University of Oxford, October 2004, OUEL Report No.226/03.
Martin, C.M., 2005. Exact bearing capacity calculations using the method of characteristics. Issues lecture, 11th Int.Conf. of IACMAG, Turin.
Meyerhof, G., 1951. The ultimate bearing capacity of foundations. Geotechnique 2, 301–332.
Prandtl, L., 1920. Uber die harte plastischer Korper, Nachr.D.Ges.D.Wiss, Gottingen.
Terzaghi, K., 1943. Theoretical Soil Mechanics. John Wiley & Sons, New York.