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Mixed-lubrication analysis of misaligned bearing considering turbulence

Fangrui Lv [a], Chunxiao Jiao [a], Na Ta [a], Zhushi Rao [a,b,*]

[a] Institute of Vibration, Shock and Noise, State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University, Shanghai 200240, PR China
[b] Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration, Shanghai 200240, PR China

A R T I C L E I N F O

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A B S T R A C T

Some bearings operate in mixed-lubrication regime due to the low rotational speed and the heavy load and misalign due to shaft's gravity. For large bearings, the maximum film thickness is much larger than minimum nominal film thickness and turbulence may locally develop. This paper proposes an approach to analyze misaligned mixed-lubrication bearing considering turbulence. Based on the average flow model proposed by Patir and Cheng and the Ng-Pan turbulence model, a generalized average Reynolds equation is derived. The calculation procedure is established by finite difference method. The results show that turbulence remarkably increases friction coefficient, slightly increases the minimum nominal film thickness, and decreases the transition speed from mixed-lubrication regime to hydrodynamic lubrication regime.

1. Introduction

Bearings are important parts of rotor systems, and their lubrication behavior affects the reliability of the whole system. Some bearings operate under low rotational speed and heavy load conditions, and the minimum liquid film thickness is on the same order of magnitude as roughness. The asperities contact occurs locally as well, and the bearing operates in mixed lubrication regime. Meanwhile, the bearings supporting rotors and the rotor's gravity may misalign the journal bearing. In recent years, many research concerns bearings operating in mixed lubrication regime [1–6] and bearings with journal misalignment [6–9]. Considering bearing deformation, Kraker et al. [10] proposed a mixed elastohydrodynamic-lubrication model based on the average Reynolds equation [11]. To analyze the mixed lubrication bearings with herringbone-groove, Han et al. [12] presented a new shape with herringbone mesh used for finite difference method. Some research proved the influence of journal misalignment on bearing behavior is non-negligible. For example, Litwin et al. [13] analyzed the influence of journal misalignment on water-lubricated bearings and indicated that the load carrying capacity decreases when journal misalignment angle increases. He et al. [6] investigated the effect of journal deflection on a mixed lubrication bearing and concluded journal deflection increases the rotational speed at which mixed lubrication transits to hydrodynamic lubrication. The research above assumed the lubricant flow is laminar. However, for large bearing with high eccentricity ratio, the maximum film thickness is great and turbulence may locally occur.

Several turbulence models, such as those by Constantinescu [14,15], Ng-Pan [16] and Elrod-Ng [17], have been widely applied to turbulent lubrication analysis. Frene [18] used Constantinescu model with local transition concept to investigate a trust bearing in both laminar and turbulent regimes. Bou-Said and Nicolas [19] studied the effect of misalignment on hybrid bearing performance in the laminar and turbulent regimes. Bouard et al. [20] used the Constantinescu, Ng-Pan, and Elrod-Ng models to study the influence of turbulence on the performance of tilting-pad bearing and concluded all the models give similar results. Based on Elrod-Ng model, Braunetiere [21] presented a modified model for flows with low Reynolds number. Shenoy and Pai [22] investigated the effect of misalignment and turbulence on an externally adjustable bearing. Zhang et al. [23] proposed an approximate solution of carrying capacity of sliding bearings with turbulent flow that applies to bearings with high eccentricity ratio and heavy load. Susilowati et al. [24] compared bearing performance in turbulent and laminar regimes by three-dimensional CFD and concluded the pressure has the same trend in both the flow regimes. Mallya et al. [25] investigated tribological performance of a water-lubricated bearing with journal misalignment operating in turbulent regime and concluded that the turbulent lubricant and journal misalignment increase the bearing carrying capacity. The research above involved the analysis of the influence of turbulence on hydrodynamic lubrication bearing. However, the influence analysis of turbulence on mixed-lubrication bearings has not been reported.

So, to analyze the influence of turbulence on misaligned bearing, an approach for misaligned mixed-lubrication analysis considering

* Corresponding author. Institute of Vibration, Shock and Noise, State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University, Shanghai 200240, PR China.
E-mail address: zsrao@sjtu.edu.cn (Z. Rao).

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Nomenclature

A	area of bearing extended surface
c	radial clearance $\psi = c/r$
f	friction coefficient
f_{asp}	surfaces contact friction coefficient
$F_{asp\xi}$	asperity contact force in horizontal direction
$F_{asp\eta}$	asperity contact force in vertical direction
F_f	total friction force
F_{foil}	friction force arising from lubricant shearing
F_{fasp}	asperity contact friction force
$F_{oil\xi}$	hydrodynamic force in horizontal direction
$F_{oil\eta}$	hydrodynamic force in vertical direction
h	(nominal) film thickness
h'	film thickness ratio
h_T	local film thickness
$\bar{h}_T$	average film thickness
$h_{\min}$	minimum film thickness
$h_{\max}$	maximum film thickness
k_x, k_z, k_τ	turbulence coefficients
L	bearing length
$p(\bar{p})$	(mean) hydrodynamic pressure
p_{asp}	equivalent asperities contact pressure
r	journal radius
R_L	local Reynolds number
R_c	critical Reynolds number
z	axial coordinate
δ_1	surface roughness of surface 1
δ_2	surface roughness of surface 2
ε_0	eccentricity ratio at bearing axial midplane (referred as to eccentricity ratio)
ε_z	eccentricity ratio at each axial section
θ_0	attitude angle at the bearing axial midplane (referred as to attitude angle)
θ_z	attitude angle at each axial section
μ	dynamic viscosity of lubricant
$\bar{\tau}_{yx}$	mean shear stress in x-direction
φ	bearing circumferential angle
$\phi_x, \phi_z, \phi_{sx}$	flow factors
$\phi_f, \phi_{fsx}, \phi_{fpx}$	shear stress factors
γ	misalignment angle

turbulence is proposed, and the numerical procedure is established. The influence of turbulence on a misaligned mixed-lubrication bearing is analyzed.

2. Modeling

2.1. Geometry of misaligned bearing

Gravity acting on the shaft often bends bearing's journal along the vertical direction. Fig. 1 shows the geometry of a bearing with journal misalignment in the vertical direction.

The misalignment angle γ is usually small, so $\tan\gamma \approx \gamma$. According to Fig. 1, eccentricity ratio at each axial section can be written as:

$$\varepsilon_z = \sqrt{\frac{z^2}{c^2}\gamma^2 + \frac{2z}{c}\varepsilon_0\gamma\cos\theta_0 + \varepsilon_0^2} \tag{1}$$

where z is axial coordinate; c is radial clearance; ε_0 is eccentricity ratio at the bearing axial midplane. θ_0 is attitude angle at the bearing axial midplane. In this paper, the eccentricity ratio at bearing axial midplane is hereinafter referred to as eccentricity ratio, and the attitude angle at bearing axial midplane is referred to as attitude angle.

The attitude angle at each axial section is:

$$\begin{cases} \theta_z = \arctan\left(\dfrac{\varepsilon_0\sin\theta_0}{\frac{z}{c}\gamma + \varepsilon_0\cos\theta_0}\right) & \left(\frac{z}{c}\gamma + \varepsilon_0\cos\theta_0\right) > 0 \\ \theta_z = \arctan\left(\dfrac{\varepsilon_0\sin\theta_0}{\frac{z}{c}\gamma + \varepsilon_0\cos\theta_0}\right) + \pi & \left(\frac{z}{c}\gamma + \varepsilon_0\cos\theta_0\right) < 0 \end{cases} \tag{2}$$

Asperities may contact at the area around the minimum film thickness. So, as Fig. 1 shows, the distance between the mean levels of the two surfaces is referred as to nominal film thickness h, and the local film thickness h_T is defined as:

$$h_T = h + \delta_1 + \delta_2 \tag{3}$$

The maximum film thickness $h_{\max}$ and minimum nominal film thickness $h_{\min}$ are shown in Fig. 1.

The nominal film thickness of a vertical misaligned bearing is:

$$h = c[1 + \varepsilon_z\cos(\varphi - \theta_z)] \tag{4}$$

where φ is circumferential angle.

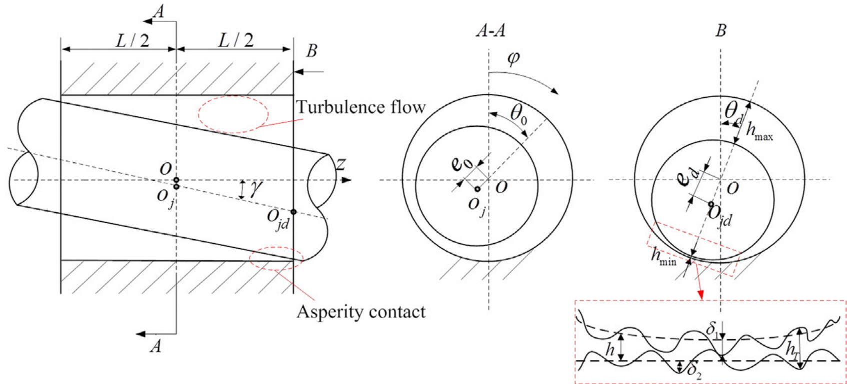

Fig. 1. Geometry model of the misaligned journal bearing.

Table 1
Turbulence coefficients of Ng-Pan model.

Model	TBA	ETA	TBB	ETB	TBC	ETC
Ng-Pan	0.00113	0.9	0.000358	0.96	0.0012	0.94

2.2. Numerical model

For mixed lubrication analysis not considering turbulence, the liquid in the whole bearing can be described by the average Reynolds equation proposed in Ref. [11]. However, for mixed lubrication analysis of bearing considering turbulence, the average Reynolds equation cannot describe the turbulent flow. Therefore, this paper derives a generalized average Reynolds equation to calculate the mixed lubrication bearing considering turbulence.

According to Ng-Pan turbulence model, the time-averaged flow in x- and z-direction can be described by:

$$q_{xt} = \int_0^h v_{xt} dy = \frac{Uh}{2} - \frac{h^3}{\mu k_x}\frac{\partial p}{\partial x} \tag{5}$$

$$q_{zt} = \int_0^h v_{zt} dy = -\frac{h^3}{\mu k_y}\frac{\partial p}{\partial z} \tag{6}$$

where k_x and k_z are turbulence factors, $\begin{cases} k_x = 12(1 + TBA \cdot R_L^{ETA}) \\ k_z = 12(1 + TBB \cdot R_L^{ETB}) \\ k_x = k_z = 12 \end{cases}$ $\begin{matrix} \textit{Turbulent flow} \\ \textit{laminar flow} \end{matrix}$ .R_L is local Reynolds number. If $R_L > R_c$, the fluid flow is turbulent, while it is laminar if $R_L \le R_c$, being R_c critical Reynolds number and $R_c = {}^{41.1}/\sqrt{\psi}$; the turbulence coefficients are listed in Table 1.

Considering the rough surfaces of bearing and journal, the average flow in x- and z-direction can be described by:

$$\overline{q_{xt}} = \frac{1}{\Delta z}\int_z^{z+\Delta z}\left(\frac{Uh}{2} - \frac{h^3}{k_x\mu}\frac{\partial p}{\partial x}\right)dz \tag{7}$$

$$\overline{q_{zt}} = \frac{1}{\Delta x}\int_x^{x+\Delta x}\left(-\frac{h^3}{k_z\mu}\frac{\partial p}{\partial z}\right)dx \tag{8}$$

According to [11], Eq. (7) and Eq. (8) can be expressed as:

$$\overline{q_{xt}} = -\frac{\phi_x h^3}{k_x\mu}\frac{\partial \overline{p}}{\partial x} + \frac{U\overline{h_T}}{2} + \frac{U}{2}\sigma\phi_s \tag{9}$$

$$\overline{q_{zt}} = -\frac{\phi_z h^3}{k_z\mu}\frac{\partial \overline{p}}{\partial z} \tag{10}$$

where ϕ_x, ϕ_z, and ϕ_{sx} are flow factors defined in Ref. [11]; h and h_T are nominal and local liquid film thickness, respectively.

Assuming steady-state conditions, the average flow balances on the control volume. Therefore:

$$\frac{\partial \overline{q_{xt}}}{\partial x} + \frac{\partial \overline{q_{zt}}}{\partial z} = 0 \tag{11}$$

By substituting Eqs. (9) and (10) into Eq. (11), the average Reynolds equation considering turbulence can be described as:

$$\frac{\partial}{\partial x}\left(\frac{\phi_x h^3}{k_x\mu}\frac{\partial \overline{p}}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\phi_z h^3}{k_z\mu}\frac{\partial \overline{p}}{\partial z}\right) = \frac{U}{2}\frac{\partial \overline{h_T}}{\partial x} + \frac{U\sigma}{2}\frac{\partial \phi_s}{\partial x} \tag{12}$$

When roughness converges to zero, it can be demonstrated that Eq. (12) is degraded into:

$$\frac{\partial}{\partial x}\left(\frac{h^3}{k_x\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{k_z\mu}\frac{\partial p}{\partial z}\right) = \frac{U}{2}\frac{\partial h}{\partial x} \tag{13}$$

When the fluid flow is laminar, it can be demonstrated that Eq. (12) is degraded into:

$$\frac{\partial}{\partial x}\left(\phi_x\frac{h^3}{12\mu}\frac{\partial \overline{p}}{\partial x}\right) + \frac{\partial}{\partial z}\left(\phi_z\frac{h^3}{12\mu}\frac{\partial \overline{p}}{\partial z}\right) = \frac{U_1}{2}\frac{\partial \overline{h}_T}{\partial x} + \frac{U_1}{2}\sigma\frac{\partial \phi_{sx}}{\partial x} \tag{14}$$

The mean hydrodynamic pressure distribution is obtained by solving Eq. (12) with the Reynolds boundary conditions. The carrying capacity of the liquid film is given by:

$$F_{oil\xi} = -\iint_A \overline{p}\sin\varphi dA \tag{15}$$

$$F_{oil\eta} = -\iint_A \overline{p}\cos\varphi dA \tag{16}$$

where $F_{oil\xi}$, $F_{oil\xi}$ are hydrodynamic force in the horizontal and vertical directions, respectively; A is area of the bearing extended surface.

For bearings with high eccentricity ratio, the maximum film thickness $h_{\max}$ is much greater than the minimum (nominal) film thickness $h_{\min}$. Asperities may contact in the region surrounding the minimum (nominal) film thickness such that in this area mixed lubrication regime develops. Meanwhile, in the bearing, especially in the area of thick liquid film, the local Reynolds number may be larger than the critical Reynolds number leading to the development of local turbulent flow. If turbulence takes place only in the hydrodynamic lubrication region (as shown in Fig. 2),

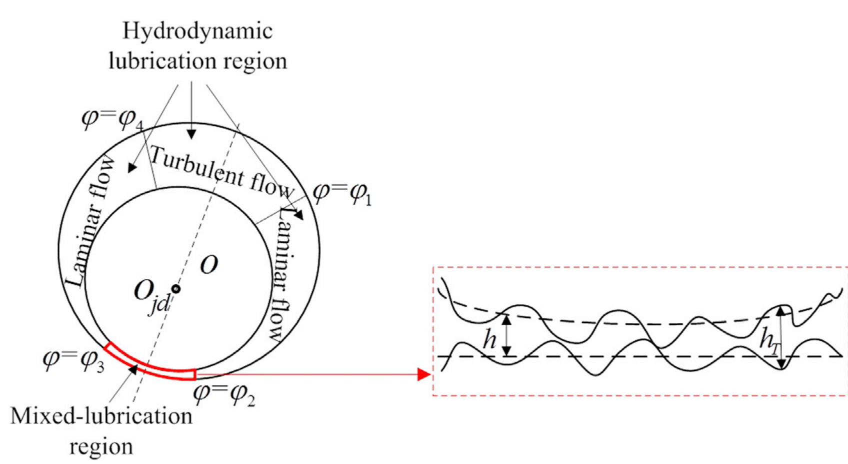

Fig. 2. Geometry model of bearing including asperity contact and turbulence.

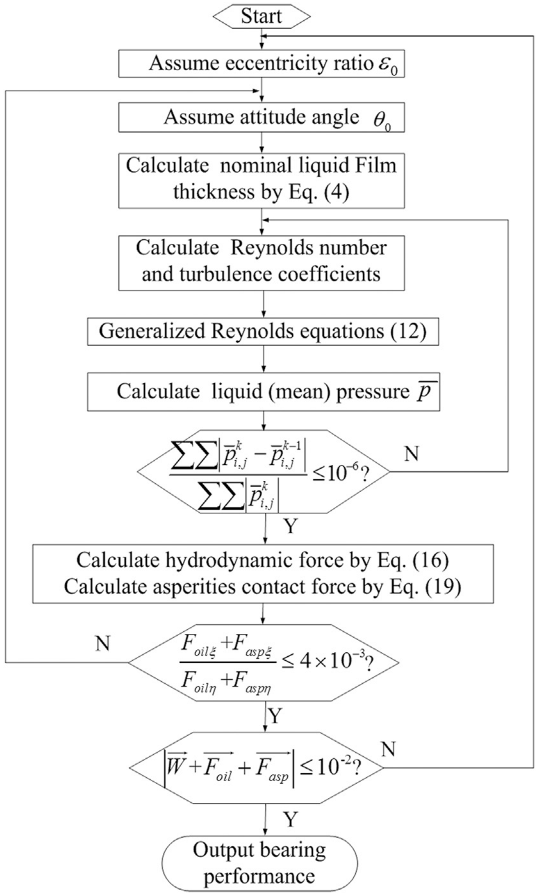

Fig. 3. The flow chart of numerical calculation.

the bearing can be analyzed through combining Eq. (13) and Eq. (14), but the calculation process is complicated. Analyzing the same bearing through Eq. (12), the calculation process simplifies.

2.3. Asperity contact force

The minimum nominal film thicknesses of some bearings are of the same order of magnitude as the roughness of surfaces. The equivalent asperities contact pressure is calculated in Ref. [26]:

$$p_{asp} = \frac{16\sqrt{2}}{15}\pi(\lambda\beta\sigma)^2 E'\sqrt{\frac{\sigma}{\beta}}\int F_{5/2}\left(\frac{h}{\sigma}\right) \tag{17}$$

where λ is density of asperity, ζ is curvature radius of asperity, E' is composite elastic modulus of the bearing bush and journal, $F_{5/2}(t) = \int_t^{\infty}(s-t)^{5/2}\phi^*(s)ds$.

Asperity contact forces can be written as:

$$F_{asp\xi} = -\iint_A p_{asp}\sin\varphi dA \tag{18}$$

Table 2
Bearing's parameters by Ref. [28].

Journal diameter (m)	0.06
Length-diameter ratio	1.1
Roughness of bearing bush (μm)	0
Roughness of journal (μm)	0
Radial clearance (μm)	30
Lubricant viscosity (Pa·s)	9×10^{-3}
Rotational speed (rpm)	3000

$$F_{asp\eta} = -\iint_A p_{asp}\cos\varphi dA \tag{19}$$

2.4. Friction coefficient

The shear stress of liquid in a turbulent flow can be calculated by:

$$\tau_{yx} = -\frac{k_\tau \mu U_0}{h} - \frac{h}{2}\frac{\partial p}{\partial x} \tag{20}$$

where $\begin{cases} k_\tau = 1 + TBC\cdot R_L^{ETC} & \textit{Turbulent flow} \\ k_\tau = 1 & \textit{laminar flow} \end{cases}$, the values of TBC and ETC are listed in Table 1.

According to [27], the mean shear stress of liquid in mixed-lubrication flow region can be calculated by:

$$\overline{\tau}_{yx} = -\frac{k_\tau \mu U_0}{h}\left(\phi_f - \phi_{fsx}\right) - \phi_{fpx}\frac{h}{2}\frac{\partial \overline{p}}{\partial x} \tag{21}$$

The friction force arising from liquid shearing is

$$F_{foil} = \iint_A \overline{\tau}_{yx}dA \tag{22}$$

Asperity contact friction force is

$$F_{fasp} = \iint_A f_{asp}p_{asp}dA \tag{23}$$

where f_{asp} is coefficient of friction of surfaces in contact.

The total friction force is supposed to be the sum of the liquid friction force term caused by liquid shearing and friction force term caused by asperity contact:

$$F_f = F_{foil} + F_{fasp} \tag{24}$$

where $F_{foil} = \iint_A \overline{\tau}_{yx}dA$, $F_{fasp} = \iint_A f_{asp}p_{asp}dA$, f_{asp} is coefficient of friction when surfaces contact.

The coefficient of friction is

$$f = F_f / W \tag{25}$$

3. Numerical procedure and validation

3.1. Numerical procedure

To analyze mixed lubrication bearing considering the journal misalignment and turbulence, the numerical procedure shown in Fig. 3 is established. Equation (4) is used to calculate the nominal thickness of liquid film. According to the film thickness of grid nodes, dynamic viscosity and density of the lubricant, and the linear velocity of the journal, the local Reynolds number can be determined, and the corresponding turbulence coefficients can be calculated. Reynolds boundary conditions are taken as the boundary conditions. The mean pressure distribution can be got by solving the generalized average Reynolds Equation (12). Finite difference method and over-relaxation iterative method are used for solving the generalized average Reynolds equation. Considering the journal misalignment, grids cover the whole width of bearing. The static equilibrium position of the journal center is got when the resultant of hydrodynamic force and asperity contact force balances the external load.

Table 3
The calculated maximum hydrodynamic pressures versus misalignment angles.

Misalignment angle (deg)	Reference (MPa)	Present study (MPa)
0	33.1	33.0
0.004	39.6	39.4
0.007	63.6	64.4
0.01	415.4	414.8

Table 4
The bearing parameters used for numerical analysis.

Journal diameter (mm)	800	Bearing width (mm)	1600
Misalignment angle (mrad)	0.19	Bearing's radial clearance (mm)	1.52
Roughness of journal (μm)	1.6	Roughness of bearing bush (μm)	2.6
Journal to bearing contact friction coefficient	0.2	Fluid dynamic viscosity (Pa·s)	0.011
Journal speed (rpm)	0–200	External applied load (kN)	320

3.2. Validation

The calculation program is validated by comparison with the bearing calculated by Sun [28], whose main parameters are listed in Table 2. Table 3 compares the maximum pressures of liquid film calculated by Sun [28] and by present study and shows the results are close to each other. Note that the calculation does not include mixed-lubrication regime thus a further validation of the program in mixed-lubrication is needed and will be addressed in the future work.

4. Results and discussion

The influence of turbulence on misaligned mixed-lubrication bearing is analyzed using the bearing whose parameters are listed in Table 4.

Liquid film pressure distribution is the key indicator of bearing performances. This paper discusses the six following cases: Case 1, considering turbulence and journal misalignment at rotational speed of 160 r/min; Case 2, considering journal misalignment without turbulence at rotational speed of 160 r/min; Case 3, not considering turbulence and journal misalignment at rotational speed of 160 r/min; Case 4, considering turbulence and journal misalignment at rotational speed of 190 r/min; Case 5, considering journal misalignment without turbulence at rotational speed of 190 r/min; Case 6, not considering turbulence and journal misalignment at rotational speed of 190 r/min. As Fig. 4a) and b) show, under the same external load, the maximum mean film pressure slightly decreases when considering turbulence, and a similar conclusion can be drawn from Fig. 5a) and b). However, the change value of maximum mean film pressure is larger when rotational speed is 190 r/min because at higher rotational speed the turbulence is more intense. The comparison of Fig. 4 b) and c) shows the maximum mean film pressure remarkably increases and the location of pressure peak moves away from the bearing axial midplane when considering journal misalignment.

Following Fig. 6 shows the nominal liquid film thickness distribution at down-warping end at the rotational speed of 160 r/min. When considering turbulence, the minimum nominal film thickness slightly increases because turbulence increases carrying capacity, and therefore, the bearing with a lower eccentricity ratio can balance the external load. Moreover, the turbulence is more intense at higher rotational speed (see Fig. 7), therefore the difference between the minimum nominal film thicknesses of bearing with and without considering the effect of turbulence is larger at the rotational speed of 190 r/min. Figs. 6 and 7 also show the minimum nominal film thickness remarkably decreases when considering journal misalignment.

The frictional behavior of bearings remarkably affects noise and the risk of seizure of rotor systems. Stribeck curves in Fig. 8 show that friction coefficient of mixed-lubrication bearing increases when turbulence is

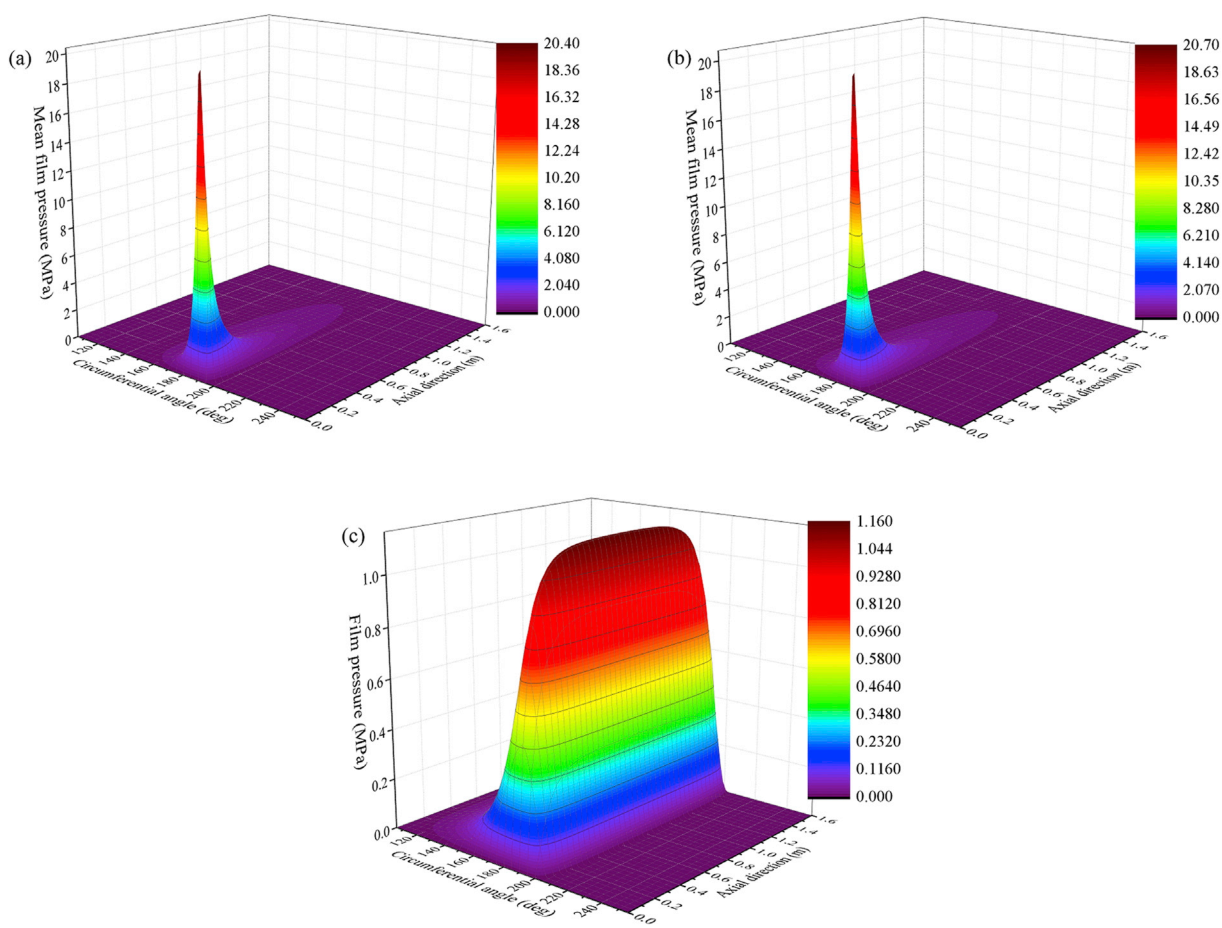

Fig. 4. Mean hydrodynamic pressure distribution at rotational speed of 160 r/min: (a) Considering turbulence and journal misalignment; (b) Considering journal misalignment without turbulence; (c) Not considering journal misalignment and turbulence.

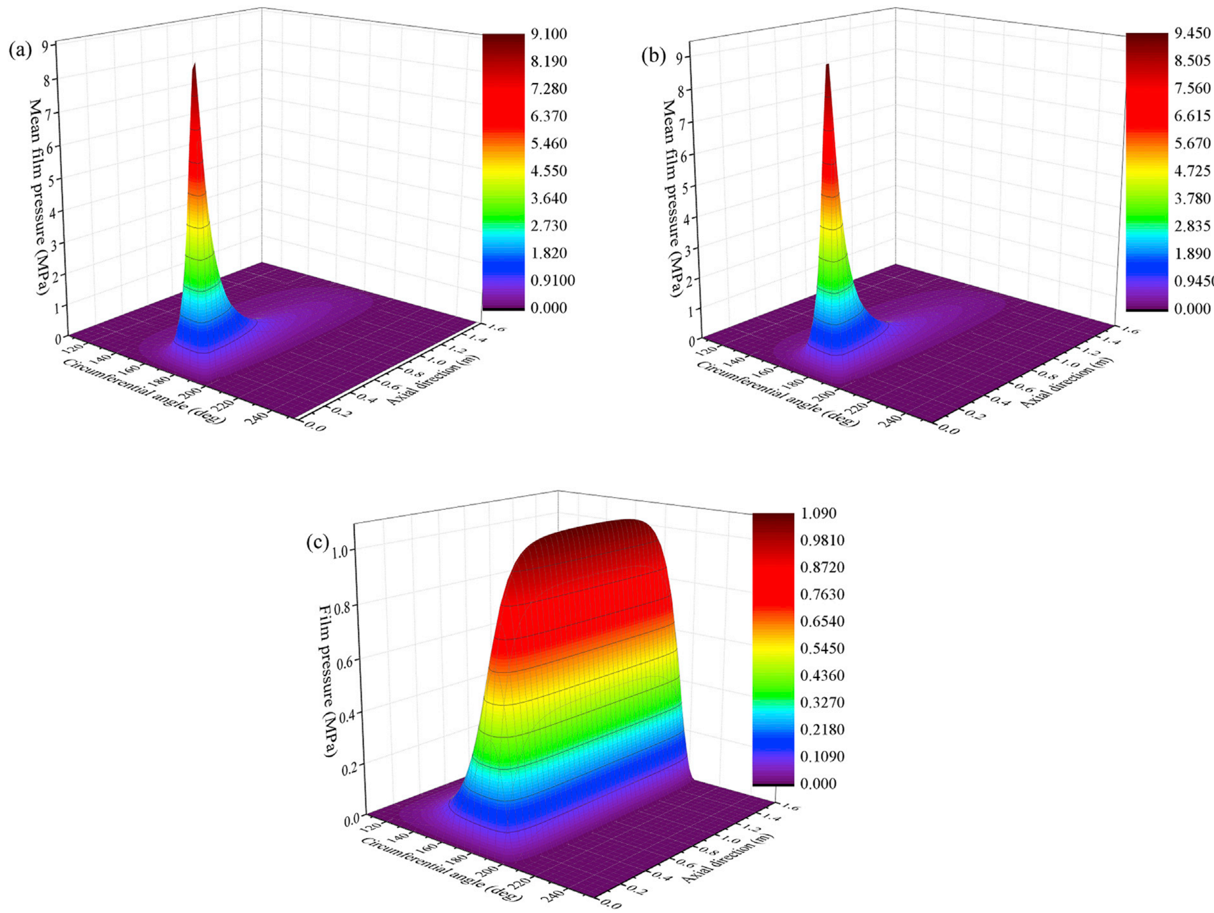

Fig. 5. Mean hydrodynamic pressure distribution at rotational speed of 190 r/min: (a) Considering turbulence and journal misalignment; (b) Considering journal misalignment without turbulence; (c) Not considering journal misalignment and turbulence.

considered. In engineering, the point of minimum coefficient of friction is approximately treated as the transition point from mixed-lubrication (ML) regime to hydrodynamic lubrication (HL) regime. According to this criterion, the locations of the points with minimum friction coefficient in curve A and curve B indicate that turbulence decreases the journal speed at which the transition from ML to HL occurs. Fig. 9 shows the asperity friction forces versus rotational speed. The force drops to zero at the rotational speed of 190 r/min when considering turbulence, while the force drops to zero at the rotational speed of 195 r/min without considering turbulence. Therefore, the turbulence indeed decreases the journal speed at which the transition from ML to HL occurs because turbulence increases the minimum nominal film thickness and leads to less asperities contact. Curve C in Fig. 9 shows that from 150 r/min to 220 r/min the bearing operates in the HL regime when journal misalignment is not considered. Accordingly, journal misalignment increases the journal speed at which the transition from ML to HL occurs.

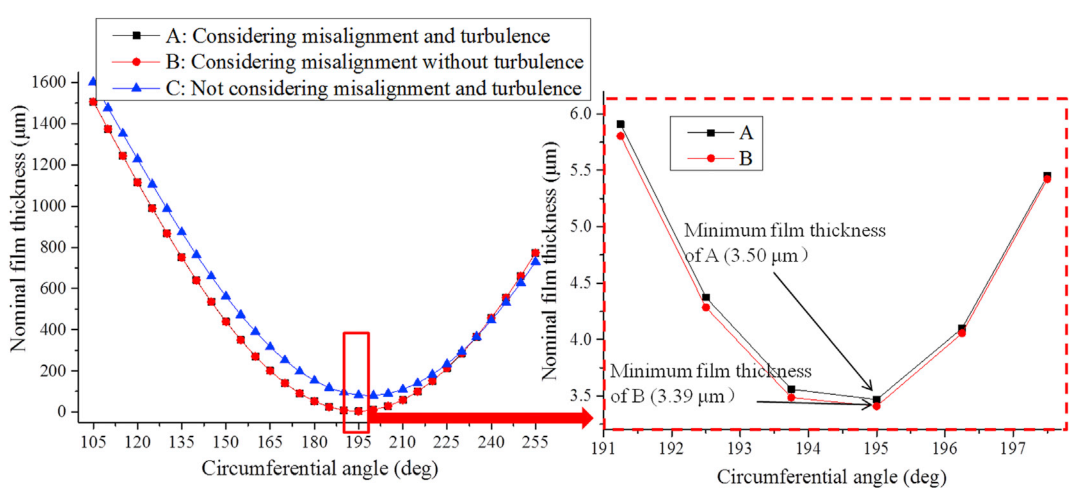

Fig. 6. Nominal film thickness distribution at the rotational speed of 160 r/min.

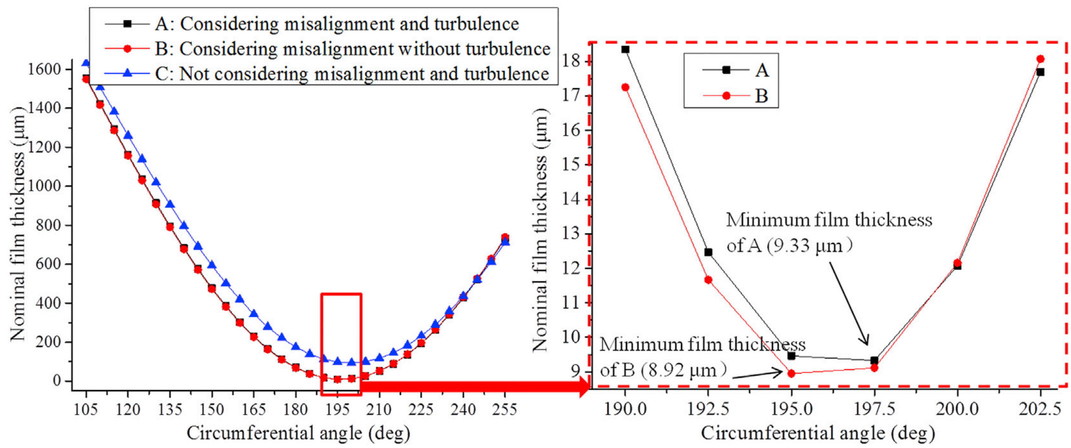

Fig. 7. Nominal film thickness distribution at the rotational speed of 190 r/min.

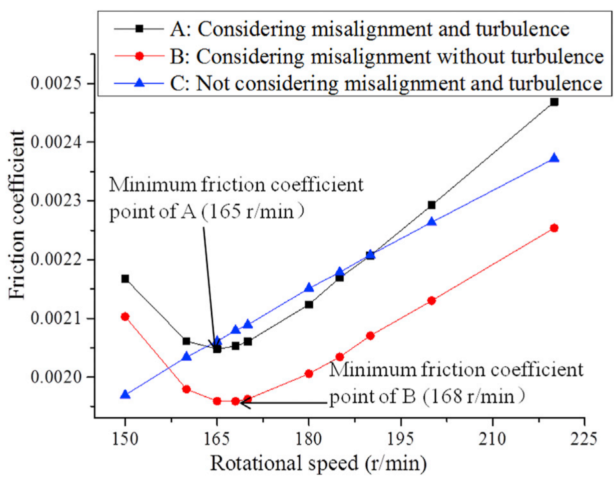

Fig. 8. Stribeck curve for the different models.

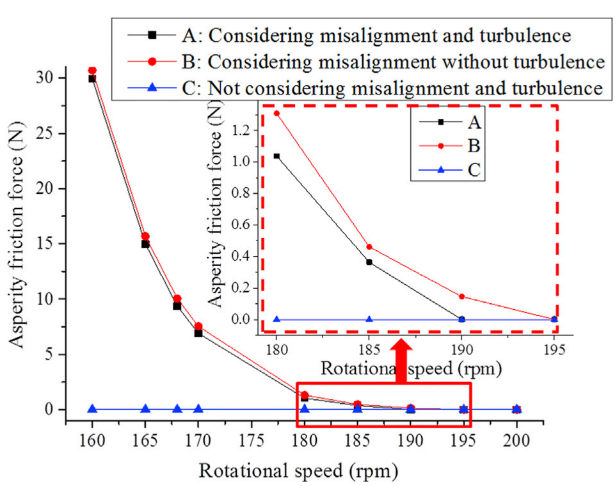

Fig. 9. Asperity friction force versus speed for the different models.

5. Conclusions

(1) To analyze large bearing with high eccentricity ratio, an approach for mixed-lubrication analysis of bearing considering journal misalignment and turbulence is proposed. Based on the average flow model proposed by Patir and Cheng, and the Ng-Pan turbulence model, a generalized average Reynolds equation is derived and the numerical calculation procedure performed by finite difference method and over-relaxation iterative method is established.

(2) Under the same external applied load, the turbulence remarkably increases the coefficient of friction, slightly increases the minimum nominal film thickness, and decreases the journal speed at which ML-HL transition occurs. The influence of turbulence on bearing performance is more remarkable when the rotational speed increases.

(3) Under the same external applied load, when considering journal misalignment, the minimum nominal film thickness remarkably decreases, the maximum mean film pressure remarkably increases, and the journal speed at which ML-HL transition occurs increases.

Future work will conclude experimental validation of the numerical program.

Acknowledgments

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