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Pricing estimation of a barrier option in an IoT scenario

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HIGHLIGHTS

- We propose a mathematical model to estimate pricing barrier option.
- We extend a previous approach to a more complex framework.
- We use data and information in real-time within the IoT scenario.
- We show a real case study to assess the proposed approach.

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ABSTRACT

IoT systems are able to manage very great amount of different types of data. In our paper we propose a mobile app which uses data processed by an IoT framework to estimate the price of a European barrier price. This software is based on an algorithm: in input it receives the values of maturity, strike price, interest rate, barrier level and in output it gives the value of the price. The algorithm implements a mathematical procedure involving numerical and statistical issues, as quadrature formulas and statistical tests. The validity of our methodology is verified by applying it to a real case.

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1. Introduction

In this paper we extend the methodology of [1] to the case of European barrier options. More precisely, we consider an IoT scenario¹ extracting and managing very huge data and describe a mobile app which makes use of this information to estimate the no-arbitrage price of this kind of option. In IoT scenarios very sophisticated and efficient tools are able to infer, classify and store data coming from different types of sources (an example of a IoT financial data flow is presented in [3]). The advantages of this kind of approach is investors are updated about the conditions of the financial world and of new markets (for IoT we refer to [4,5]).

IoT frameworks already find applications in different sectors: (i) monitoring of drivers' performance in insurance (see [6]); (ii) creation of apps to improve trades, as *Mobile Location Confirmation*, *Alfa-Bank Sense*, *Groceries by MasterCard*; (iii) management of data in cultural heritage (see [7–9]).

European barrier options are a very old and simple example of exotic options, and they can be defined as a vanilla (or basic)

option to which some mathematical constraints, named *barriers*, are added. More precisely, as basic options, an investor is provided with the right to buy (Call) or to sell (Put) a quantity of a good (underlying) in a future date (maturity) by paying a further amount (strike price): the great difference consists in the presence of the barriers, which determines the beginning of the option (knock-in) or the end (knock-out). From a classification point of view, we distinguish different kinds of barrier options: (i) *up* (the initial value of the underlying is below the barriers) or *down* (the initial value of the underlying is above the barriers); (ii) *discrete* (the barriers are represented by a numerable set of values) or *continuous* (the barriers are a real regular function); (iii) *single* (the barrier is a real half-line) or *double* (the barrier is a delimited real interval). In this paper we focus our attention on single discrete down knock-out barrier options.

Pricing a barrier option is a very complex, involving many sophisticated mathematical tool. In simple cases it is possible to achieve a closed for barrier option price: in the case of barrier options with continuous monitoring of the barrier (see [10]), or in the case of discrete barrier options by using the continuous barrier formulas with a correction (see [11]). In general, this topic can only be analyzed by using numerical techniques, in particular binomial and trinomial lattices [12], finite differences schemes [13] and

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¹ The term IoT first appears in [2].

integration [14], or statistical methodologies, in particular Monte Carlo methods [15,16].

The paper is organized in the following way. In Section 3 we introduce the analytical model we focus our attention on numerical issues and we present a numerical example; in Section 4 we draw the conclusions.

2. Collecting financial data from an IoT framework

The Internet of Things has been on the mind of financial services industry for a while now. There are several ways IoT frameworks can help and support financial services, which tend to deal with intangible assets, especially in the world of on-line finance. IoT is able to sense and gather data in the physical world converting the data into digital and actionable output. In this scenario, financial institutes can retrieve a more clear picture of the consumer, understanding how they move and where they spend money, allowing them to make accurate lending decisions. Customers will be able to make transactions and inquiries using different devices. Financial institutes will also be able to collect information and monitor behavior patterns to provide better and sound financial advice and services.

Recent studies have highlighted that the quantity of available data has increased because of not only the development of internet technologies, but also of interactions among different markets and the growth of financial transitions of banks. At the moment IT systems are partially able to take advantage from these data, but, as stated by the authors in [17], thanks to the power of IoT, this situation will improve: banking and financial institutes will offer their customers a truly bespoke experience, with insight, advice and offers that reflect real-time events and situations taking place in their customers' lives.

Application of IoT in financial services is no longer limited to human imagination. In fact, its application opportunities are as limitless as human imagination. IoT powered customer information used in insurance sector can unlock a world of possibilities for financial institutes.

The rise of the pervasive IoT Technology has revolutionized the way how the financial advisor, stock brokers, and other professionals embrace the IoT adoption for better decision-making capabilities.²

Adopting IoT brings opportunities in parallel with some challenges to Finance Services, for example:

1. Customer Experience: The Personalization, which means offering potential solutions by collecting and analyzing data from customer behaviors,
2. Risk Management: The opportunities mostly lie in the insurance sector.
3. Investment: Investment banks can benefit with the help of the data aggregated by insurance companies.

We remark that IoT cannot be considered as a simple extension of traditional IT world, but it introduces a novel technology that connects objects and people to a network in order to give information about the objects state, movement, position and so on. Several financial transactions and services are based on information from intangible sources while the IoT is fundamentally about gathering, processing and creating value from data about tangible objects.

From a financial point of view, it is essential to understand how IoT-generated data create value for companies and/or consumers. In this paper we discuss an IoT framework (illustrated in Fig. 1), which relies on a smart mobile application able to retrieve and collect data about a particular feature (for example the volatility of the underlying) from the whole IoT environment.

The proposed IoT framework is composed by three layers:

² <https://www.linkedin.com/pulse/internet-things-iot-financial-services-industry-fsi-introduction-rao>.

- **Data Collection layer:** this layer is responsible of the collection of data coming from multiple sources (e.g. database, web-site, sensors, actuators, people, mobile devices, etc.).
- **Computing layer:** This layer is characterized by a software engine responsible for computing the volatility estimation of a financial option. It receives all necessary data coming from the Data Collection layer as input. The mathematical model described in the next section is implemented here.
- **Visualization layer:** This layer is responsible for the visualization of alerts, queries and results coming from the Computing Layer. In details, a mobile application has been developed to show the computed option price estimation and the related volatility.

We can image the following scenario. A trader is going to invest its money in an option, of which he knows the price established by its financial institute. After he has selected the time interval, the interest rate, the strike price and the underlying asset, he runs our mobile application, which takes all the necessary information in real-time and computes the price; he decides to compare this value with the one received by its institute.

3. The model

In this section we illustrate our analytical and numerical model. We assume that the classical assumptions of the Black-Scholes market (see [18]): (a) completeness; (b) absence of arbitrages; (c) possibility of short selling; (d) absence of any frictions; (e) presence of two assets, a risky asset (with time-dependent volatility) and a risk-free bond (with a constant interest rate). In the following we adopt the following notations: (i) $[0; T]$ is the time interval; (ii) $(\Omega; F, F_t; \mathbb{P})$ indicates a filtered probability space; (iii) W_t denotes a standard Brownian motion; (iv) S_t is the risky asset; (v) r indicates the interest rate; (vi) σ_t denotes the volatility of S_t ; (vii) $N(\mu; v)$ is a Normal with mean μ and variance v ; (viii) $\Phi(\cdot)$ indicates the standard Normal cumulative distribution function; (ix) $cov(X; Y)$ is the covariance of the random variables X and Y .

The risky asset evolves according to the following risky-neutral dynamics:

$$dS_t = rS_t dt + \sigma_t S_t dW_t. \tag{1}$$

Assigned an initial value S_0 , the (1) states that S_t is a log-normal process, given by³:

$$S_t = S_0 e^{\int_0^t (r - 0.5\sigma_s^2) ds + \int_0^t \sigma_s dW_s}$$

$$\ln S_t \sim N \left(\ln S_0 + \int_0^t (r - 0.5\sigma_s^2) ds; \int_0^t \sigma_s^2 ds \right).$$

3.1. Barrier option price

In our context we consider a particular kind of discrete barrier option, *knock-out down barrier option*. We suppose that the underlying can be monitored in a sequence of N instants $\{t_n\}_{n=1}^N$; in symbols, for every $n = 1, \dots, N$, in t_n the value of the underlying and the barrier are respectively S_n and B_n ; the price of a down

³ We apply the Ito's lemma to the process $Y_t = f(t, S_t)$, where $f(t, x) = \ln x$:

$$dY_t = \left(\frac{\partial f(t, S_t)}{\partial t} + \frac{1}{2} \sigma_t^2 S_t^2 \frac{\partial^2 f(t, S_t)}{\partial x^2} \right) dt + \frac{\partial f(t, S_t)}{\partial x} dS_t =$$

$$= \left(r - \frac{1}{2} \sigma_t^2 \right) dt + \sigma_t dW_t.$$

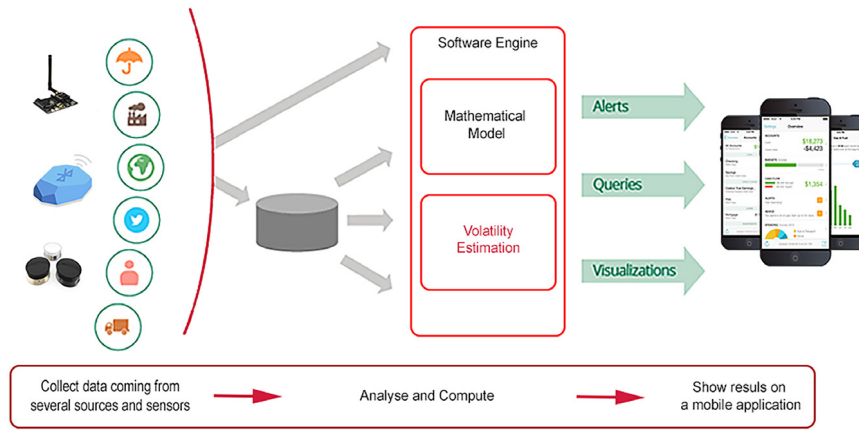


Fig. 1. An Internet of Things framework for financial data collecting and computation.

knock-out barrier option can be expressed in terms of expectation respect to the risk-adjusted probability measure \mathbb{Q} :

$$P = e^{-rT} E^{\mathbb{Q}} \left[\prod_{n=1}^N \mathbb{1}_{\ln B_n; +\infty}(\ln S_n) H(S_T) \right], \quad (2)$$

where $H(S_T)$ is the pay-off function of a vanilla option with underlying S_t , maturity T and strike price K

$$H(S_T) = \begin{cases} \max\{S_T - K, 0\} & \text{if Call} \\ \max\{K - S_T, 0\} & \text{if Put} \end{cases} \quad (3)$$

In the case in which the sequence S_n tends to 0 for the input N , the random variables $\mathbb{1}_{\ln B_n; +\infty}(\ln S_n)$ and $H(S_T)$ are not correlated: the formula (2) can be rearranged as⁴

$$P = e^{-rT} \prod_{n=1}^N E^{\mathbb{Q}} \left[\mathbb{1}_{\ln B_n; +\infty}(\ln S_n) \right] E^{\mathbb{Q}} [H(S_T)]. \quad (4)$$

We conclude this subsection by deriving a numerical computation of (4). For every $n = 1, \dots, N$, let $\tilde{\sigma}_n$ be the value of the volatility observed in t_n ; we set $\sigma_n = \tilde{\sigma}_n \sqrt{t_n}$ and $\mu_n = \ln S_0 + rt_n - (1/2)\sigma_n^2$. It results:

$$\begin{aligned} P_c &= e^{-rT} \left(\prod_{n=1}^N \frac{1}{\sqrt{2\pi}\sigma_n} \int_{\ln B}^{+\infty} e^{-\frac{(x-\mu_n)^2}{2\sigma_n^2}} dx \right) E^{\mathbb{Q}} [H(S_T)] = \\ &= \frac{e^{-rT}}{(2\pi)^N / 2} \left(\prod_{n=1}^N \int_{\frac{\ln B - \mu_n}{\sigma_n}}^{+\infty} e^{-\frac{y^2}{2}} dy \right) E^{\mathbb{Q}} [H(S_T)] = \\ &= \left[\prod_{n=1}^N \Phi \left(-\frac{\mu_n - \ln B}{\sigma_n} \right) \right] (e^{-rT} E^{\mathbb{Q}} [H(S_T)]). \end{aligned}$$

The value of the term $R = e^{-rT} E^{\mathbb{Q}} [(S_T - K)^+]$ is computed by using the Black-Scholes formula, and it depends on the kind of option. In the case of a call, we have:

$$\begin{aligned} R &= S_0 \Phi(d1) - Ke^{-rT} \Phi(d2) \\ d1 &= [\ln(S_0/K) + (r + 0.5\sigma_N^2)T] / \sigma_N \sqrt{T} \\ d2 &= d1 - \sigma_N \sqrt{T} \end{aligned}$$

In the previous computations we have made use of the log-normality of S_t . In conclusion, for a Call discrete knock-out down

barrier option P_c we have determined the following numerical estimator:

$$P_c = \left[\prod_{n=1}^N \Phi \left(\frac{\ln B - \mu_n}{\sigma_n} \right) \right] [S_0 \Phi(d1) - Ke^{-rT} \Phi(d2)]. \quad (5)$$

The price of a Put P_p can be obtained by applying the Put-Call parity:

$$P_p = P_c + Ke^{-rT} - S_0. \quad (6)$$

In the next subsection we deal with the numerical issues concerning the formula (5).

3.2. Numerical and statistical issues

As it has been remarked in the previous subsection, the step described in (5) rely on the log-normality of the underlying (for numerical methods we refer to [19]). We test this property by applying the *Jarque-Berra test* [20] to a Z -dimensional sample of observations of underlying $\{S_z\}_{z=1}^Z$, available in 0.

The function $\Phi()$ is evaluated by means of a *Laguerre quadrature*.⁵

The value of the cumulative distribution function of the standard Normal in a generic real number a can be rewritten, by setting $y + a$, as:

$$\begin{aligned} \Phi(a) &= \frac{1}{\sqrt{2\pi}} \int_{-a}^{+\infty} e^{-\frac{x^2}{2}} dx = \\ &= \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} e^{-y} e^y e^{-\frac{(y-a)^2}{2}} dy = \\ &= \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} \omega(y) f(y; a) dy \quad \forall a \in \mathbb{R}. \\ \omega(y) &= e^{-y} \\ f(y; a) &= e^y e^{-\frac{(y-a)^2}{2}}. \end{aligned}$$

⁵ The choice of a Gaussian formula is due to a classical theorem of numerical calculus, which ensures that this class of numerical methods have the highest precision degree. The main Gaussian formulas in unbounded intervals are *Laguerre* and *Hermite*: we have preferred the first one because, as it will be evident, is more suitable for our situation.

⁴ We remark that for a Brownian motion we have:

$cov(\ln S_n; \ln S_m) = \min\{t_n; t_m\} \quad n, m = 1, \dots, N \quad n \neq m.$

We obtain the following approximation for (4):

$$P_c = \left[\prod_{n=1}^N \left(\sum_{m=1}^M A_m(a_n) \right) \right] \times \left[\sum_{m=1}^M (S_0 A_m(d1) - Ke^{-rT} A_m(d2)) \right] \tag{7}$$

$$A_m(a) = \frac{1}{\sqrt{2\pi}} w_m f(y_m; a); \quad a_n = (\mu_n - \ln B) / \sigma_n.$$

$$m = 1, \dots, M \quad n = 1, \dots, N.$$

The terms y_m and w_m are respectively the nodes and the coefficients of Laguerre formula. Since they depend on the observations of the volatility, which is generally an endogenous variable: this implies that a procedure to achieve them is necessary. In our context we simplify this point by assuming that the volatility is constant and exogenous.

We summarize our numerical and statistical procedure. It is composed of the following steps: (i) **testing the log-normality of the underlying**; (ii) **determination of coefficients and parameters of Laguerre polynomials**; (iii) **calculation of weights and nodes of Laguerre–Gauss formula**; (iv) **approximation of standard normals evaluations by mean of a Gaussian quadrature formula**. The input variables are: (i) the maturity T ; (ii) the interest rate r ; (iii) the volatility σ ; (iv) the value of the underlying in $0 S_0$; (v) the set of barrier B_n ; (vi) the strike price K ; (vii) the number of time subintervals N ; (ix) the dimension of the sample Z ; (x) the number of the nodes M ; (xi) observations of the underlying S_m . The output variable are the estimation of the barrier call price c_0 and put price p_0 .

Algorithm 1 Estimation of barrier option prices

Require: $T, r, \sigma, S_0, N, B_n, K, N, Z, M, S_m$
Ensure: c_0, p_0 .

- 1: **Initialization**
 $\omega(y) = e^{-y}; \quad f(y; a) = e^y e^{\frac{(y-a)^2}{2}}$
 $\sigma_n = \sigma \sqrt{t_n}; \quad \mu_n = \ln S_0 + r t_n - 0.5 \sigma_n^2.$
 $n = 1, \dots, N.$
- 2: **Jarque–Berra Statistic**
 $JB = \frac{Z}{6} \left(S^2 + \frac{1}{4}(C - 3)^2 \right).$
 $S = [(1/Z) \sum_{z=1}^Z (\ln S_z - \mu)^3] / v^{3/2}; \quad C = [(1/Z) \sum_{z=1}^Z (\ln S_z - \mu)^4] / v^2$
 $\mu = (1/Z) \sum_{z=1}^Z \ln S_z; \quad v = (1/Z) \sum_{z=1}^Z (\ln S_z - \mu)^2.$
- 3: **Computation of the coefficients b_m and c_m of Laguerre polynomials L_m and parameters $a_n, d1$ and $d2$**
 $b_m = 2(m - 1) + 1; \quad c_m = (m - 1)^2 \quad b_1 = c_1 = 1$
 $d1 = [\ln(S_0/K) + (r + 0.5\sigma_N^2)T] / \sigma_N \sqrt{T} \quad d2 = d1 - \sigma_N \sqrt{T}$
 $a_n = (\mu_n - \ln B) / \sigma_n; \quad n = 1, \dots, N \quad m = 1, \dots, M.$
- 4: **Calculation of weights w_m and nodes y_m of Laguerre–Gauss formula**
 $L_m(y_m) = 0; \quad w_m = (M!) y_m / [L_{M+1}(y_m)]^2.$
- 5: **Evaluation of barrier option price**

$$P_c = \left[\prod_{n=1}^N \left(\sum_{m=1}^M A_m(a_n) \right) \right] \left[\sum_{m=1}^M S_0 A_m(d1) - Ke^{-rT} \sum_{m=1}^M A_m(d2) \right].$$

Table 1
Historical values of the underlying.

Date	Value1	Value2	Value3	Value4	Value5
01/01/2018	79.22	76.15	73.27	70.57	68.07
02/01/2018	87.14	83.77	80.59	77.63	74.88
03/01/2018	95.0608	91.3853	87.9205	84.6837	81.6860
05/01/2018	87.14	83.77	80.59	77.63	74.88

The real value of the option P_{real} has been computed by means of a Monte Carlo method (in the following we sketch the codex):

Monte Carlo estimation for barrier option

- (1) **Draw** $i \times T$ numbers $u_{it} \sim N(0, 1), i = 1, \dots, I, t = 1, \dots, T.$
- (2) **Generate** $S_{it} = S_0 \exp \left[\left(r - \frac{\sigma^2}{2} \right) t_v + \sigma \sqrt{t_i} u_{it} \right]$
- (3) **Finding** the minimums $m_t, t = 1, \dots, T$ of $S_i^{(t)}, i = 1, \dots, I.$
- (4) **Computing** the following quantity

$$\tilde{P} = \frac{1}{T} \sum_{t=1}^T \mathbb{1}_{[B; +\infty)}(m_t) H(S_{it}).$$

3.3. Numerical example

In this subsection we present a numerical example of our framework. We have calculated the price of a European barrier call with constant barrier $B = 9$, stock price $S_0 = 10$, strike price $K = 10$, starting date $t_0 = 05/01/2018$, maturity $05/01/2022$, interest rate $r = 0.05$, the annual volatility $\sigma = 0.1$ (all the amounts are expressed in Euro). The value $P_{real} = 1.43$ has been found with a number of simulations $I \times T = 100 \times 1000$. The instants t_n in which the underlying is monitored are equidistant and time subintervals have length equal to $h = T/N$ or, equivalently, $t_n = nh, n = 1, \dots, N.$

The next figure illustrates the pop-up of our mobile app, implementing our software in the IoT scenario (see Fig. 2).

We first have verified that the underlying satisfies the log-normality hypothesis by means of Jarque–Berra test: we have considered observations relative to the period from 01/01/2018 to 05/01/2018 and we have reported some of these values in Table 1: the first column is relative to the date of observations, the others are relative to the available value. The result of the test is 0: this means that our hypothesis cannot be rejected.

The Table 2 shows estimations of the call price. Our goal consists in analyzing how the number of the instants in which the underlying is monitored impacts our We have fixed the number of nodes $M = 20$, while the value of the time subintervals N varies. The table is structured in the following way: (i) the column **N** contains the different values of subintervals; (ii) the column **Price** lists the approximations of the call price relative to our procedure; (iii) in the columns **Err abs** and **Err rel** the values of the absolute and relative error have respectively been inserted. We observe that an increase of N does not improve the accuracy (in fact both absolute and relative errors are very low when $N = 1$): this is verified especially when the order of magnitude of data is not very high.

4. Conclusions

In this paper we have presented a numerical and statistical framework for the evaluation of European barrier option price in a Black–Scholes model, characterized by the following assumptions: (i) completeness of the market; (b) absence of arbitrages;

Fig. 2. Barrier option calculator.

Table 2
Call price estimations.

N	Price	Err abs	Err rel
1	1.81	0.39	0.28
5	1.32	0,1	0.07
10	0.92	0.5	0.35
15	0.65	0.77	0.55
20	0.45	0.97	0.68
50	0.05	1.37	0.96

(c) possibility of short selling; (d) absence of any frictions; (e) risky asset described by a log-normal process. This algorithm has been implemented in a mobile app in an IoT scenario: this approach can communicate and store large amounts of data and gives traders updated information of different nature. Our procedure involves mathematical tools, as quadrature formulas and statistical testing, and it has been used for the resolution of a real case.

Our procedure has been applied to discuss the problem in a very simple case, in which financial parameters are constant and exogenous. In the future our goals are the extension of our methodology to the more complex options with stochastic and time dependent volatility.

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