A Cardiovascular Model for Blood Pressure Control Systems

P. C. A. Ang, B. W. Ang and K. Y. Zhu

Department of Electronic and Computer Engineering
Ngee Ann Polytechnic, Singapore
Email: zku2@np.edu.sg

***Abstract* - A cardiovascular model for blood pressure control system is developed in this paper. This model is used to simulate hypertensive patients in order to design control systems for regulation of blood pressure. The stability of the model is also investigated. The model can accurately represent human arterial blood pressure and therefore, the control system designed and simulated based on the model can be considered very near to clinical trial.**

I. INTRODUCTION

The cardiovascular system consists of the heart which is a muscular pumping device and a closed-loop system of vessels called arteries, veins, and capillaries. Human heart has four chambers, the right atrium, right ventricle, left atrium and left ventricle [1]. The right atrium will receive deoxygenated blood from the body. Then the blood will flow into the right ventricle through the tricuspid valve. During the contraction of ventricles, the right ventricle will pump the blood into the lung through the pulmonary valve. The left atrium will receive the oxygenated blood from the lung. The blood in the left atrium will flow through the mitral valve into the left ventricle. After that, the blood in the left ventricle will be pumped through the aortic valve into aorta. The blood received by the heart from vena cava is the deoxygenated blood. However, the blood pumped from the heart into the body is oxygenated. The oxygenated blood will contain more oxygen while the deoxygenated blood will contain more carbon dioxide. The main purpose of the blood flowing through the lung is for gaseous exchange such as removing the carbon dioxide from the blood and it to take in oxygen. The oxygenated blood will be pumped back into the body again.

Due to pumping motion, the human blood pressure fluctuates between the systole and diastole periods. During the systole period, the heart beats to pump the blood out of the heart. During the diastole period, the heart relaxes before the next beat to allow blood to fill back up into the heart. The systolic and diastolic values of the arterial blood pressure for the healthy adult should be around 120/80 mmHg respectively.

Research in human cardiovascular system has attracted many researchers and technologists over the past few decades. It is mainly researched for the understanding of the mechanism and characteristics of the cardiovascular system. In recent years, development of artificial human heart becomes one of the hot and interesting areas. Another important research in biomedical engineering is about hypertension control. The main purpose of this paper is to develop the human cardiovascular system model as it can become an useful tool in the design of blood pressure control systems. This paper is organized as follows: Section II represents the cardiovascular system model, where the mathematical equations describing the system are given. Simulation is carried out in section III. In section IV, stability of the system model is investigated. Applications for cardiovascular system model are given in section V and section VI represents the conclusion.

II. CARDIOVASCULAR SYSTEM MODEL

A. Applications of the system models

The cardiovascular system model developed in this section can be used as an assistant tool in the analysis and understanding of characteristics of human cardiovascular system, especially in the development of artificial human heart. However, the motivation of the model is in the design

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and control of human hypertension blood pressure control systems.

The major advantage of using a model for the testing of a controller is that a much wider range of patients can be simulated by a model than can be accomplished in experiments on a few different dogs [2]-[4]. In the cardiovascular model used for these simulations, it is possible to change, independently or in any combination, the resistances of the arteries and veins, their compliances, the strength of the heat's contraction, heart rate and many other parameters. The effect of different values of the infusion delay, sensitivity to drugs, amount of recirculation, and speed of action of drug can be studied.

Fig. 1 shows the block diagram of the hypertension control using a drug Sodium Nitroprusside (SNP). The patient model includes three sections as follows:

- SNP pharmacokinetic model,
- SNP pharmacodynamic model,
- Cardiovascular model.

For the SNP pharmacokinetic model, the input is SNP infusion rate, the outputs are the concentration of SNP in the arteries and the concentration of SNP in the veins, which are used as the inputs of the following model. For SNP pharmacodynamic model, the outputs are arterial resistance change and venous unstressed volume change, which are used as the inputs of the following model. Finally, for cardiovascular model, the output is arterial pressure, which is used to compute mean arterial pressure.

Another interesting application of the cardiovascular system model is in the artificial heart design. The artificial heart is a medical device which consists of the ventricles that connected to aorta, left atrium, right atrium and pulmonary artery. We just need to focus on these few parts in the cardiovascular system model. We will need to know the amount pressure require the both ventricles to pump the sufficient amount of blood out to the lung and other part of the body, the amount of bloods that is stored in each ventricle before pump to out from the ventricles and the opening and closing of four heart's valves. In the cardiovascular system model, it creates a feedback controller of the cardiovascular system which will show the pressure, flow rate of the blood and volume of blood in various part of the human's body.

With this information, we are able to improve the function of the artificial heart which can work as similar as the real heart. Then, the patients are able to live longer until they can receive a donor heart. Beside this, we also must use those materials which are biocompatible and the efficiency of the motor must be optimized in order to ensure heat generated by the motor is minimal.

B. Mathematical model

The cardiovascular system model is presented here. Fig. 2 shows the block-diagram of the cardiovascular system. Before presenting the model, the following notation, which is used throughout the paper, is given.

1. Notation:

P = Pressure, F = Flow rate, Q = Volume
C = Compliance, R = Resistance,

2. Heart

First, the heart model is given here. It is divided into the right atrium, right ventricle, left atrium and left ventricle. Due to the presences of the tricuspid valve, pulmonary valve, mitral valve and aortic valve, the heart is considered as a nonlinear system.

Right Atrium

$$F_{V2} - F_{RA} = C_{RA}\frac{dP_{RA}}{dt},$$

$$L_{RA}\frac{dF_{RA}}{dt} + R_{RA}F_{RA} = \begin{cases} P_{RA} - P_{RV}, & \text{for } P_{RA} - P_{RV} \geq 0 \\ 0, & \text{for } P_{RA} - P_{RV} < 0 \end{cases},$$

$$Q_{RA} = C_{RA}P_{RA},$$

Right Ventricle

$$F_{RA} - F_{RV} = C_{RV}\frac{dP_{RV}}{dt},$$

$$L_{RV}\frac{dF_{RV}}{dt} + R_{RV}F_{RV} = \begin{cases} P_{RV} - P_{P1}, & \text{if } P_{RV} - P_{P1} \geq 0 \\ 0, & \text{if } P_{RV} - P_{P1} < 0 \end{cases},$$

$$Q_{RA} = C_{RA}P_{RA},$$

Left Atrium

$$F_{L2} - F_{LA} = C_{LA}\frac{dP_{LA}}{dt},$$

$$L_{LA}\frac{dF_{LA}}{dt} + R_{LA}F_{LA} = \begin{cases} P_{LA} - P_{LV}, & \text{for } P_{LA} - P_{LV} \geq 0 \\ 0, & \text{for } P_{LA} - P_{LV} < 0 \end{cases},$$

$$Q_{LA} = C_{LA}P_{LA},$$

Left ventricle

$$F_{LA} - F_{LV} = C_{LV}\frac{dP_{LV}}{dt},$$

$$L_{LV}\frac{dF_{LV}}{dt}+R_{LV}F_{LV}=\begin{cases}P_{LV}-P_{A1}, & \text{if} \quad P_{RV}-P_{A1}\geq 0\\ 0, & \text{if} \quad P_{LV}-P_{A1}<0\end{cases},$$

$$Q_{LV}=C_{LV}P_{LV}$$

3. Pulmonary Circulation

The pulmonary circulation mainly consists of blood vessels and therefore, without loss of generality we divide it into 6 segments. Then it can be described using a series of first-order differential equations below.

$$P_{P1}-P_{P2}=F_{P1}R_{P1}+L_{P1}\frac{dF_{P1}}{dt},$$

$$F_{RV}-F_{P1}+R_{PW1}C_{P1}(\frac{d(F_{RV}-F_{P1})}{dt})=C_{P1}\frac{dP_{P1}}{dt},$$

$$P_{P2}-P_{P3}=F_{P2}R_{P2}, \qquad F_{P1}-F_{P2}=C_{P2}\frac{dP_{P2}}{dt},$$

$$F_{P2}-F_{P3}=C_{P3}\frac{dP_{P3}}{dt}, \qquad P_{P3}-P_{L1}=F_{P3}R_{P3},$$

$$P_{L1}-P_{L2}=F_{L1}R_{L1}, \qquad F_{P3}-F_{L1}=C_{L1}\frac{dP_{L1}}{dt},$$

$$F_{L1}-F_{L2}=C_{L2}\frac{dP_{L2}}{dt}, \qquad P_{L2}-P_{LA}=F_{L2}R_{L2}+L_{L2}\frac{dF_{L2}}{dt},$$

where the indices P and L represent the arterial and vein, respectively.

4. Systemic circulation

Similarly, for the systemic circulation it is also divided into 6 segments and also combined as one loop. This is because with one loop the arterial blood pressure can be accurately denoted. The following are the differential equations about the systemic circulation.

$$P_{A1}-P_{A2}=F_{A1}R_{A1}+L_{A1}\frac{dF_{A1}}{dt},$$

$$F_{LV}-F_{A1}+R_{AW1}C_{A1}(\frac{d(F_{LV}-F_{A1})}{dt})=C_{A1}\frac{dP_{A1}}{dt},$$

$$P_{A2}-P_{A3}=F_{A2}R_{A2}, \qquad F_{A1}-F_{A2}=C_{A2}\frac{dP_{A2}}{dt},$$

$$P_{A3}-P_{V1}=F_{A3}R_{A3}, \qquad F_{A2}-F_{A3}=C_{A3}\frac{dP_{A3}}{dt},$$

$$P_{V1}-P_{V2}=F_{V1}R_{V1}, \qquad F_{A3}-F_{V1}=C_{V1}\frac{dP_{V1}}{dt},$$

$$F_{V1}-F_{V2}=C_{V2}\frac{dP_{V2}}{dt}, \quad P_{V2}-P_{RA}=F_{V2}R_{V2}+L_{V2}\frac{dF_{V2}}{dt},$$

where the indices A and V represent the arterial and vein, respectively.

III. SIMULATION

We have written matlab codes to display all three hemodynamic signals together which are the blood volume, flow rate and pressure in a particular part of the heart. This will allow the user to understand the relationship of the blood volume, pressure and flow rate in that particular part of the heart easier. The graphs in the display are also arranged in the way of how blood flow through the body which will allow the user to identify the differences easily.

Fig. 3 shows the blood pressures in the left ventricle, aorta and vein. In the blood pressure control through injection of SNP, the mean arterial pressure (MAP) will be regulated. Fig. 4 shows the volume, blood flow and pressure signals of the various part of the heart. The highest pressure is in the left ventricle as it is pumping blood to the other parts of the body. When the blood flows along the arteries and veins back to the right atrium through vena cava, the pressure will decrease as indicated in the display. At the right ventricle, the pressure increased again as the right ventricle is pumping deoxygenated blood to the lungs through the pulmonary artery to be oxygenated again. The oxygenated blood is then pumped back to the left atrium through the pulmonary veins. The pressure will also decrease as the blood flows from the pulmonary artery back to the left atrium. The pressures at the left and right atrium are not high although they contract like the ventricles as they are pumping blood to the ventricles only.

The blood volume in the ventricles and atrium change a lot due to the contraction and relaxation of the heart. The blood volume in the aorta, vena cava, pulmonary artery and vein do not change a lot as they do not contract like the heart.

The highest flow rate is in the left ventricle as it is pumping blood to the other parts of the body. When the blood flows along the arteries and veins back to the right atrium through the vena cava, the flow rate will decrease as indicated in the display. At the right ventricle, the flow rate increased again as the right ventricle is pumping deoxygenated blood to the lungs through the pulmonary artery to be oxygenated again. The oxygenated blood is then pumped back to the left atrium through the pulmonary veins. The flow rate will also decrease as the blood flows from the pulmonary artery back to the left atrium. The flow rates at the left and right atrium are not high although they contract like the ventricles as blood is flowing from the atria into the ventricles only.

Fig. 5 shows the blood pressures, volumes and flow rates of the pulmonary artery and abdominal aorta. It can be seen that the waveforms at both parts are almost in phase as both ventricles of heart contract simultaneously.

IV. STABILITY ANALYSIS

Stability of the cardiovascular model developed in the previous section is investigated here. Since the blood pumped out from the left ventricle is bounded in terms of its pressure, volume and flow rate, the system stability can be analyzed separately, i.e. we can analyze the stability for the systemic circulation and pulmonary circulation. For this, we first rewrite the pulmonary circulation system in the matrix form as follows:

$$\frac{dX_p}{dt} = A_p X_p + B_p \tag{7}$$

where

$$X_p = \begin{bmatrix} P_{P1} & F_{P1} & P_{P2} & P_{P3} & P_{L1} & P_{L2} & F_{L2} \end{bmatrix}^T$$

and A_p and B_p are the system matrices.

For the pulmonary circulation, we will investigate the stability from Nyquist stability criterion when varying the parameters of the system.

Similarly, for the systemic circulation system, we write it in the matrix form:

$$\frac{dX_s}{dt} = A_s X_s + B_s \tag{8}$$

where:

$$X_s = \begin{bmatrix} P_{A1} & F_{A1} & P_{A2} & P_{A3} & P_{V1} & P_{V2} & F_{V2} \end{bmatrix}^T$$

and A_s and B_s are the system matrices.

Fig. 6 shows the Nyquist plots representing the systemic circulation system for the variations of the compilances, inertances and resistances in the system. It can be seen that the system is stable under all cases.

V. APPLICATIONS FOR CARDIOVASCULAR SYSTEM MODEL

Applications for cardiovascular system model can be seen in fig 7 and 8.

Fig 7 shows a graphic user interface with a case study of different conditions of blood pressure. This is to demonstrate that our model can be used as a case study for different scenarios of the cardiovascular system such as different types of blood pressure conditions (hypotension and hypertension), stenosis in blood vessels or mitral valve incompetence. In fig 7, it displays the blood pressures in the left heart for 3 different conditions. It is useful in allowing researchers who are developing an artificial heart to understand the possible blood pressures that can occur in different people. For example, if the researcher develops an artificial heart to withstand a pressure of less than120mmHg and this heart is given to a patient who has hypertension which causes his left ventricular pressure to reach as high as 140mmHg, the medical device may crack and fail as it cannot withstand the pressure of that patient.

Fig 8 shows a flash movie of animated heart and 3 types of hemodynamic signals which are blood pressure, flow rate and volume. The heart is animated in accordance to the signals. This is to allow the user to have a clearer understanding of the relationship of the pressure, volume and flow in any part of the cardiovascular system. In fig 8, it shows the left ventricle's hemodynamic signals with respect to the animated heart. As the volume signal decreases, there is lesser number of arrows in the left ventricle as there is 1 arrow shown in the animated heart when we used 3 arrows to show that the blood volume signal is at its peak. When the pressure and flow signal increases, the size of the left ventricle for the animated heart decreases too. Therefore, the hemodynamic signals can give a clearer understanding to the user why the heart is animated in this manner which is similar to how our heart contracts and relaxes.

VI. CONCLUSION

A cardiovascular model for design of blood pressure control system is developed in this paper. This model faithfully represents the human arterial blood pressure along the circulation system. The most important hemodynamic parameters can also be provided from the model, especially all the waveforms are coordinately simulated, which should be very useful in the design. Although the model is developed for the blood pressure control system, it would be useful in the design of artificial heart. It can also be used in teaching and other applications such as anesthesia control and other drug injection control.

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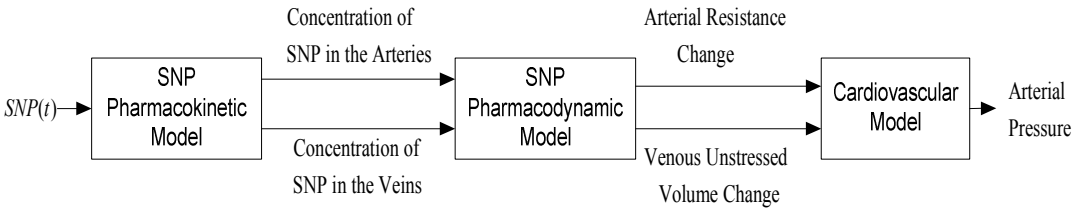


Fig. 1. SNP drug system model for MAP blood pressure control

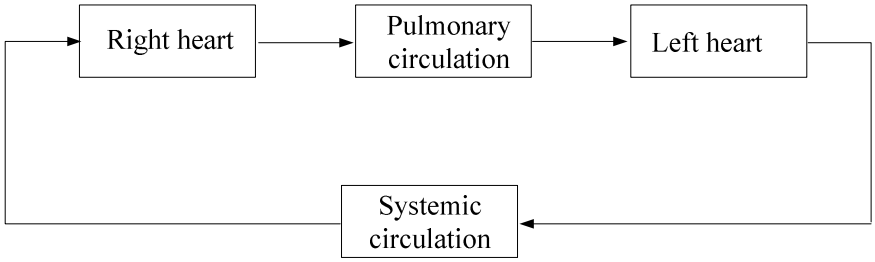


Fig. 2. Block-diagram of the cardiovascular system

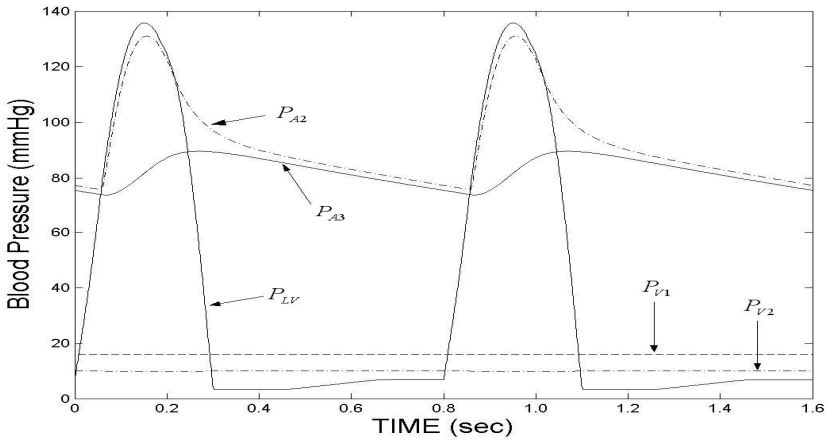


Fig. 3. P_{A2} - arterial blood pressure; P_{LV} - left ventricular blood pressure; P_{A3} -arterial blood pressure far away from heart, P_{V1} and P_{V2} are vein blood pressure

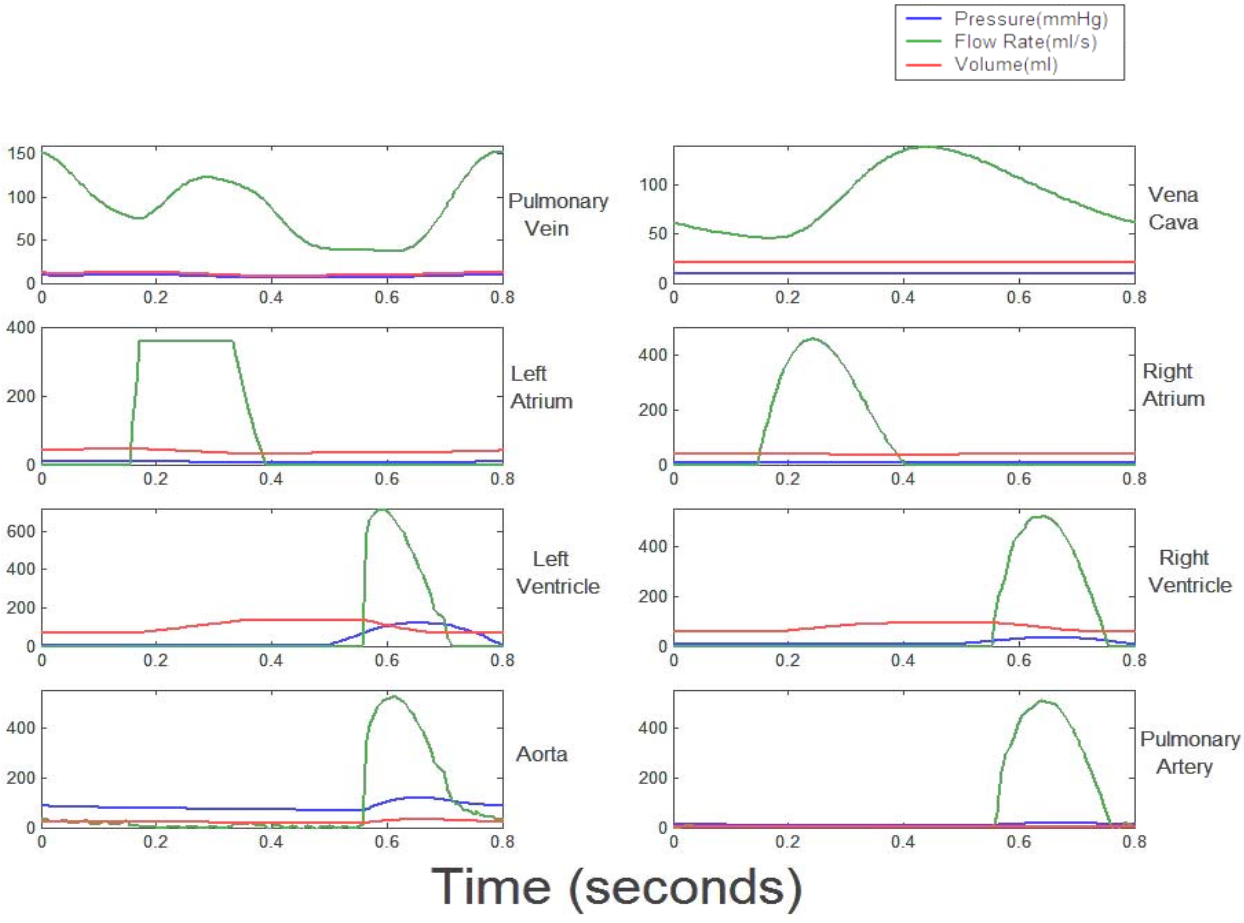


Fig. 4. Blood pressures, volumes and flow rates at different parts of the cardiovascular system

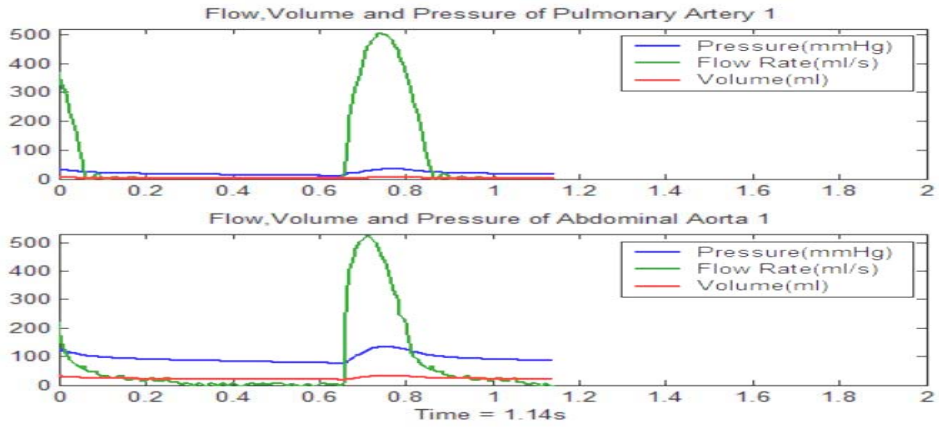
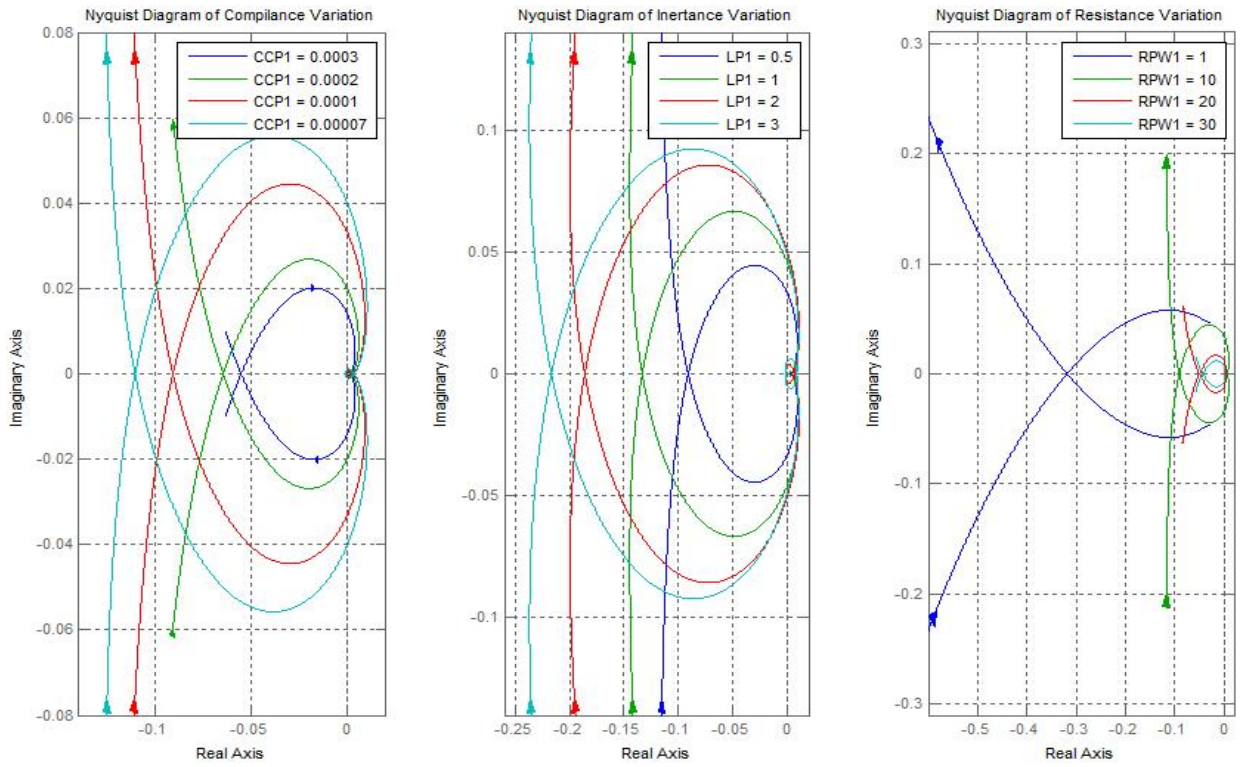


Fig. 5. Relationship between the pulmonary artery and aorta



(a) (b) (c)

Fig. 6: Nyquist plots for variation of (a) compilances, (b) inertances, (c) resistances

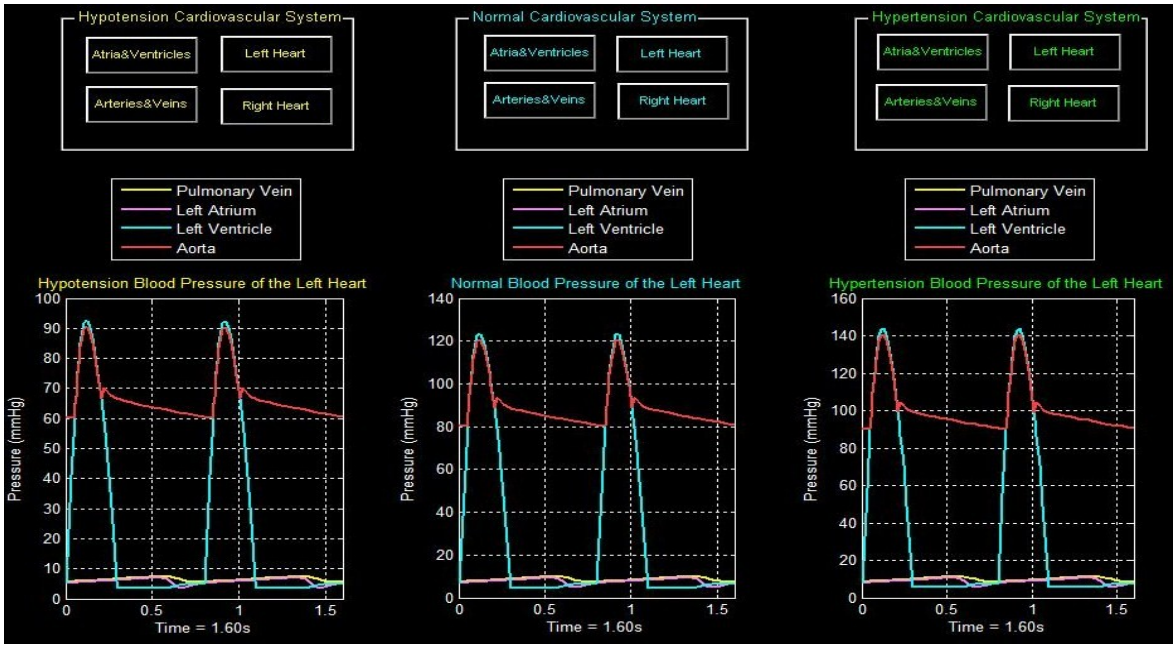


Fig. 7: Graphic User Interface for case study of different conditions of blood pressures

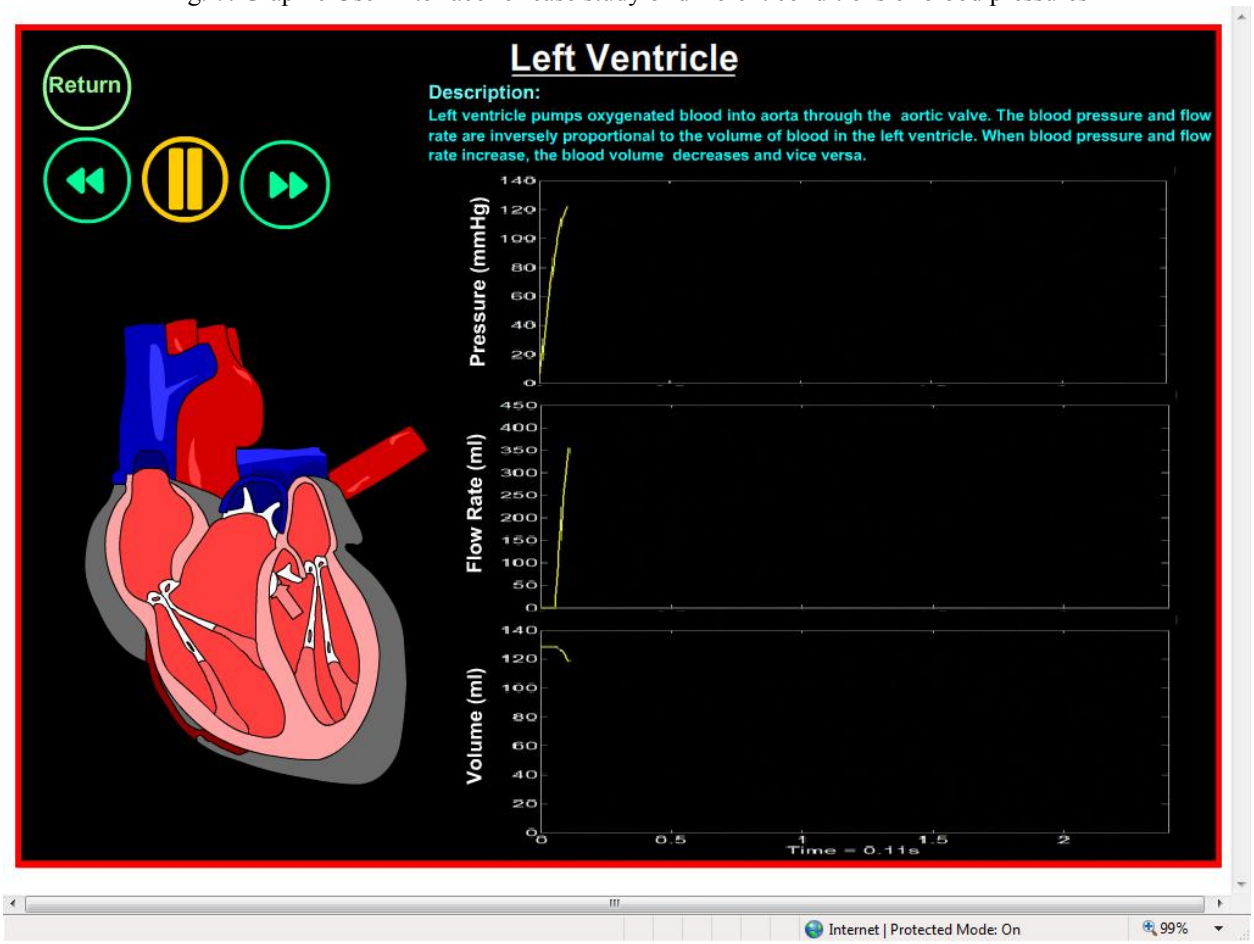


Fig. 8: Flash movie to display hemodynamic signals with accordance to an animated heart