RCS Uncertainty Quantification Using the Feature Selective Validation Method

Min Su, Dijun Liu, Ning Fang, and Baofa Wang

Abstract—Uncertainty quantification is an important issue in the field of radar cross section (RCS) research. To quantify the impact of specific uncertainty factor on RCS, a novel approach based on the feature selective validation (FSV) method combined with Monte Carlo (MC) method is proposed in this paper. MC method is applied as the basic framework for uncertainty analysis, and FSV is initially employed to compare the results derived from sufficient uncertainty simulations. To facilitate and enhance the massive data assessment, a novel single and direct indicator of FSV is proposed as a quantitative descriptor of data uncertainty. The feasibility of the proposed method in RCS uncertainty quantification is benchmarked through many RCS evaluation examples. The impact of attitude uncertainty on the target RCS, including the scene of dynamic flight, is also studied by the proposed method.

Index Terms—Data similarity, feature selective validation (FSV) method, Monte Carlo (MC) method, radar cross section (RCS), uncertainty quantification (UQ).

I. INTRODUCTION

R ECENTLY, many practical problems with the uncertain feature referred as uncertain feature, referred as uncertainty quantification (UQ), have received unprecedented attentions [1]. Due to the existence of uncertainty, the radar cross section (RCS) data obtained from dynamic/static measurement or electromagnetic simulation always have some mutual difference, which is even obvious sometimes. This leads to the question about the reliability of the dynamic targets' RCS, which has also become a bottleneck in RCS research. Therefore, the uncertainty issue is not only an inevitable problem but also a valuable study topic in this field. Extensive efforts have been devoted to analyze and evaluate RCS measurement uncertainty [2], [3]. However, in the measurement process, results are often interfered by a variety of factors simultaneously, such as attitude disturbance of dynamic targets, system noise, frequency drift, environmental clutter, and so on. It is difficult to extract the impact of one of the abovementioned uncertainties from the measurement data. Therefore,

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there is a great necessity to study the impact of the specific uncertainty on RCS data by means of simulation.

The RCS simulation of complex target is a complicated computational process which needs to take many practical factors into account, such as sheltering, multiple scattering [4], and so on. Such simulation systems are difficult to describe with all-inclusive mathematical models. Hence, the reliability of the dynamic RCS simulation has to be considered. A more feasible way of RCS UQ is the sampling-based statistical methods. Monte Carlo (MC) method is considered to be one of the most popular UQ methods [5], [6]. Its simplicity and nonintrusive characters simplify the implementation by repeating uncertain experiments and sample statistics with sufficient amounts. Although the convergence efficiency of such method is relatively low, it is widely used in the analysis of various complex electromagnetic problems because of its strong adaptability and less constraint on conditions. These advantages of MC method make it a good choice in UQ study, since it is a classical samplingbased statistical approach, the comparison and evaluation of numerical simulation data are critical for UQ.

Several data comparison methods are available in nowadays, such as correlation and *R*-factor methods [7]. Generally, these methods intend to quantify the difference of two datasets, and provide an objective tool for engineers to validate data. However, they have intrinsic limitations and neither of them is flexible enough to adjust to the practical requirement. Beyond these two, feature selective validation (FSV), as an innovative data evaluation method developed recently, is first applied in electromagnetic compatibility area [8]. This method was originally proposed by Martin [9] in 1999, and its fundamental was presented successively in the subsequent literatures [10]-[12]. FSV provides a prototype of data validation, which has been adopted by IEEE standard [13] accompanied with the relevant practical guidance [14]. The descriptions and applications of this method were also published in many literatures [15]-[18]. FSV method analyzes the data in two aspects, including amplitude trend and variation feature. The evaluation results could describe the detailed data difference, and also provide the conclusion in natural language rank form.

Although FSV method could offer abundant indicators and information, huge assessment results will significantly increase the processing burden especially for the RCS uncertainty study under MC simulations. A scalar descriptor as the quantization of validation result would be a better option for deeper analyses in this situation, which also eases the implementation of massive data assessments.

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To quantify the impact of specific uncertainty factor on target RCS character, this paper presents a novel approach by combining MC and FSV methods. Data similarity, served as the validation output quantitatively representing the uncertainty, is proposed to meet massive data assessments requirements. Meanwhile, the RCS of complex target under different degrees of attitude disturbance is applied to demonstrate the feasibility and efficiency of the method.

The rest of this paper is organized as follows. Section II introduces the methodology of RCS UQ, including FSV method, the data similarity assessment based on FSV, and the UQ framework combining MC and FSV. In Section III, the effectiveness and feasibility of the proposed method are verified by numerical examples of RCS, and the influence of attitude disturbance is further investigated. Section IV draws the conclusion.

II. METHODOLOGY

A. Feature Selective Validation Method

The preliminary stage of FSV evaluation is the data decomposition into respective frequency bands. The point-by-point comparisons between datasets are followed by using three indicators including amplitude difference measure (ADM), feature difference measure (FDM), and global difference measure (GDM). These indicators can be represented in point-by-point type XDM(*i*) (XDM could be ADM, FDM, or GDM), or characterized in confidence histogram form XDMc.

ADM indicates the difference on data amplitude and trend, which could be calculated by the low-frequency component. Since the overall magnitude of data is mainly considered in the process of RCS assessment, the direct current (dc) component cannot be ignored to avoid the bias of the assessment result. In [15], offset difference measure, as the supplement of ADM, is utilized to describe the difference on dc component. The equation used to calculate the point-by-point ADM is as follows:

$$ADM(i) = \frac{||\text{Lo}_{1}(i)| - |\text{Lo}_{2}(i)||}{\frac{1}{N} \sum_{j=1}^{N} (|\text{Lo}_{1}(j)| + |\text{Lo}_{2}(j)|)} + \frac{||\text{dc}_{1}(i)| - |\text{dc}_{2}(i)||}{\frac{1}{N} \sum_{j=1}^{N} (|\text{dc}_{1}(j)| + |\text{dc}_{2}(j)|)} \cdot e^{\frac{||\text{dc}_{1}(i)| - |\text{dc}_{2}(i)||}{\frac{1}{N} \sum_{j=1}^{N} (|\text{dc}_{1}(j)| + |\text{dc}_{2}(j)|)}}$$
(1)

where N is the length of compared data, and Lo denotes the low-frequency component of data, and dc is the direct current component.

FDM, reflecting the difference of variation feature for data, is computed by adopting the derivatives of low-frequency and high-frequency components:

$$FDM(i) = 2 |FDM_1(i) + FDM_2(i) + FDM_3(i)|$$
 (2)

$$FDM_{1}(i) = \frac{|Lo'_{1}(i)| - |Lo'_{2}(i)|}{\frac{2}{N}\sum_{j=1}^{N} (|Lo'_{1}(j)| + |Lo'_{2}(j)|)}$$
(3)

$$FDM_{2}(i) = \frac{|Hi'_{1}(i)| - |Hi'_{2}(i)|}{\frac{6}{N} \sum_{j=1}^{N} (|Hi'_{1}(j)| + |Hi'_{2}(j)|)}$$
(4)

TABLE I FSV INTERPRETATION SCALE

FSV value XDM(<i>i</i>)	FSV Interpretation
Less than 0.1	Excellent
Between 0.1 and 0.2	Very Good
Between 0.2 and 0.4	Good
Between 0.4 and 0.8	Fair
Between 0.8 and 1.6	Poor
Greater than 1.6	Very Poor

$$FDM_{3}(i) = \frac{|Hi_{1}''(i)| - |Hi_{2}''(i)|}{\frac{7.2}{N} \sum_{j=1}^{N} (|Hi_{1}''(j)| + |Hi_{2}''(j)|)}$$
(5)

where Hi is the high-frequency component of data, Hi' and Hi'' denote the first and second derivatives of Hi, and Lo' is the first derivative of the Lo component.

Combining ADM and FDM, the GDM term with weights can be defined as

$$\text{GDM}(i) = \sqrt{W_A \cdot \text{ADM}(i)^2 + W_F \cdot \text{FDM}(i)^2} \qquad (6)$$

where W_A and W_F are weighting factors of ADM and FDM.

Owing to the electromagnetic scattering physics in optical region, the dynamic radar data are sensitive to the attitudes. So the high attitude sensitivity leads to the dynamic of RCS data change rapidly. The minor attitude disturbances may result in apparent difference of data distribution. Thus, in the process of RCS data comparison, FDM tends to get worse result than ADM, and GDM merely reflects FDM results while ignoring the ADM.

According to (6), the original GDM formula mainly emphasizes on the largest value, which may overestimate the GDM term. For RCS data, a good ADM (small value) is easily overwhelmed by the poor FDM (large value), which makes ADM contribute little in GDM term. Only using FDM in assessment evaluation is not a prior choice, since the overall tendency of data also cannot be neglected.

One possible solution to the aforementioned problem is the modification of weighing factor, which is achieved by limiting the sum of the weights to one in case of ADM or FDM being neglected. Thus, (6) can be expressed as

$$\text{GDM}(i) = \sqrt{\frac{W_A \cdot \text{ADM}(i)^2 + W_F \cdot \text{FDM}(i)^2}{W_A + W_F}}.$$
 (7)

Utilizing the calculated XDM(i), we can derive the average value as

$$XDM_{avg} = \frac{1}{N} \sum_{i=1}^{N} XDM(i).$$
(8)

According to the above-mentioned point-by-point result [XDM(i)], a more intuitive hierarchical description XDMc can be obtained by using histogram statistics under certain mapping rules. As listed in Table I [11], the evaluated results are ranked into six levels according to the FSV mapping rule, which corresponds to the natural language description as "excellent," "very good," "good," "fair," "poor," and "very poor."

In brief, FSV provides an objective and comprehensive tool for data comparison. It gives not only the point-by-point results [XDM(i)] describing the partial difference, but also the natural language ranked results (XDMc) as the overall descriptor. This might extend the application scenario of FSV. In the following sections, FSV is applied to compare the RCS data with intensive fluctuations.

B. Data Similarity

To further investigate the RCS uncertainty, multiple measurements or simulations are needed with corresponding statistical study of FSV evaluation results. For MC simulation, statistics of multiple comparisons with respect to a reference should be taken to analyze the performance of specific model or parameters. The mean value XDM_{avg} is a single scalar which is relatively abstract and incomprehensible. Though the confidence histogram XDMc could be readily understood, but it may not be suitable for comparison in many situations such as the dispersive distribution of the histogram. For this reason, a quantified indicator, similarity, is introduced to describe the data validation, which should take advantages from both XDM_{avg} and XDMc. In other words, the similarity should be able to easily interpret the performance of the results.

According to confidence histogram XDMc, data similarity XDM_{sim} can be derived as

$$XDM_{sim} = \sum_{r=1}^{6} P(r) S(r)$$
 (9)

where r is validation rank, and P(r) is the probability of each rank representing the histogram values. S(r) is the similarity of each rank.

IEEE standard describes the validation rating scale (VRS) method, which allows visual comparisons between any two datasets [13]. It explains each rank in visual comparisons. Through a large number of RCS data evaluations and the conclusions from visual assessments, further definitions of these interpretations are developed for the RCS validation situation. For example, "excellent" means "perfect or almost perfect match," which could be explained as "100% area of data matches." "Very good" means "minor variations allowable," and visually "nearly 90% area has agreement." "Good" means "generally good agreement across the data," which means "nearly 75% area has agreement" for RCS data. "Fair" corresponds to "reasonable agreement over many portions of the data," and visually "nearly 60% area has agreement" for RCS data. "Poor" could be explained as "minor agreement" in VRS, and ought to be "nearly 30% area agreement." Finally, the "very poor" could be considered as "virtually no discernable agreement," and it also means "nearly 0% area has agreement."

According to the above-mentioned discussions, the mapping scale of nature language rank and its corresponding similarity are introduced in Table II. For special RCS data evaluation, the similarity means "how many areas with agreement." It is reasonable that the similarity of "excellent" is defined as 100%, while "very poor" corresponds to 0%. "Fair" has the meaning of

TABLE II Similarity Mapping Relation

Rank r	FSV Interpretation	Similarity $S(r)$
1	Excellent	100%
2	Very Good	90%
3	Good	75%
4	Fair	60%
5	Poor	30%
6	Very Poor	0%



Fig. 1. Framework for uncertainty quantification using the MC method.

qualified, and its similarity is 60%. The similarities of remaining ranks are also defined in Table II.

It should be noted that the similarity mapping S(r) in Table II is completely determined by the operating personnel based on the actual situation. It has a strong operability, and can be flexibly adjusted according to practical application process.

C. Uncertainty Quantification Based on the Monte Carlo Method

As the prerequisite to quantify the impact of the specific uncertainty on RCS data, MC simulation method is adopted as the basic framework of RCS UQ. It models the input parameters of the system as random (or pseudo-random) variables, and obtains the deterministic description of the system response through a large number of simulations. Such method is applicable to black box systems which are difficult to be described with deterministic mathematical models and uncertain input parameters. The target electromagnetic scattering characteristics under uncertainty interference could be considered as this kind of black box. In this paper, we establish a stochastic model to describe the uncertainty and calculate the macroscopic statistics (mean and variance) through the MC simulation method. The influence of uncertain factor on target RCS could be analyzed. The main framework for UQ based on MC is shown in Fig. 1.

As described in Fig. 1, reference data are acquired using simulation model without the interference of random parameters. Uncertain factors such as attitude disturbance of the target, system noise, frequency drift, environmental clutter, etc., are selected and modeled according to the research requirements. Based



Fig. 2. Geometric model of generic missile.

on these preliminary works, massive electromagnetic simulations are carried out and many experimental data affected by uncertainty are obtained. Since the FSV is a one-to-one data comparison method, these data are also compared with reference data one by one. Thus, the similarities of each data to the reference are derived. Finally, the data similarity is obtained through the statistical analyses (usually the mean value), which characterizes the influence of the current uncertainty factor on target RCS.

Based on the above-mentioned idea, the impact of various uncertainty interferences on RCS is analyzed by changing the parameters of random variables, such as amplitude of the target attitude disturbance. Meanwhile, we change the fixed variables to investigate the effects of same uncertainty factor such as the frequency of incident waves under different conditions.

III. EXAMPLES AND APPLICATIONS

A. RCS Assessment Using FSV

In order to illustrate the performance of FSV in RCS evaluation, datasets from a generic missile model are utilized in this part, which can be seen as a representative of the complex target. The missile model is similar to the one in [19], with a length of 0.9906 m and a wingspan of 0.635 m, as illustrated in Fig. 2. Assume that a vertical polarized wave with radar frequency 10 GHz is incident to target. The azimuth $\theta = 0^{\circ}$ means that the radar wave incidents on the nose of the missile, while $\theta = 180^{\circ}$ denotes the wave incidence to the tail. In this paper, backscattering RCS was calculated by applying the theory of physical optics and the physical theory of diffraction. In Fig. 3, solid line shows backscattering RCS of missile with azimuth from 0° to 180°, the sampling interval is 0.5°, while pitch and roll angles of target are all equal to 0°.

In practical measurements and engineering applications, target attitude uncertainty is ubiquitous and un-neglectable. Since the attitude uncertainty leads to different results in subsequent procedures, learning its influence on RCS is of great significance. Generally, an effective way is to analyze similarity between the RCS data with and without the attitude uncertainty. Namely, we can compare the data containing attitude disturbance with the reference data without disturbance.

To simplify the attitude uncertainty model, the Gaussian random model is added to the original attitude sequence as disturbance interference. Then, the corresponding RCS data can be



Fig. 3. RCS data of the generic missile model.



Fig. 4. ADM results of generic missile RCS evaluation from FSV.

calculated. One of the simulation results with the mean attitude disturbance of 0° and standard deviation of 0.5° is shown as the dashed line in Fig. 3.

As depicted in Fig. 3, the RCS difference is not very obvious from subjective judgment. Such difference is mainly caused by the small quantity of attitude disturbance. Therefore, when engineers performing visual assessment on such data, they tend to conclude the high data similarity, and the assessment result is probably "excellent" or "very good." Visually, the majority (nearly 90%) of data area reflect good matches in amplitudes. From the variation feature view point, some points within 20° and 80° indicate disagreement. Therefore, it is considered that almost 75% of the regions are similar.

The quantitative evaluation of data is performed by using the FSV method. The confident histogram ADMc is shown in Fig. 4, with little difference in data magnitude and trend. The assessment result is mainly concentrated within "excellent," which is basically consistent with visual comparison. The ADM similarity is 94.71% with good consistency compared with the visual result of 90%.

Fig. 5 shows the FDM results. FSV could discover the subjective imperceptible differences on data variation feature. Under certain attitudes, particularly in the region where the data changes larger and faster (between 30° and 90° in this case), two datasets have more obvious differences. This phenomenon is caused by the higher RCS attitude sensitivity on the broadside of the missile model. In this region, small changes of attitude



Fig. 5. FDM results of generic missile RCS evaluation from FSV.



Fig. 6. GDM results of generic missile RCS evaluation from FSV according to (6).

would cause large RCS variations, thus leading to FDM "worse" than ADM. This indicates that FDM results are in line with the law of RCS, and it is beneficial for RCS data assessment. The FDM_{sim} result of 74.72% has a good agreement with visual evaluation of 75%. This also proves the effectiveness of the definition of S(r) in Table II for RCS data assessment.

According to (6) with weighting factors all equal to one, GDM results are obtained as shown in Fig. 6. For comprehensive consideration, GDM is taken as the ultimate result of RCS data assessment. In this instance, the same weight factor leads to that the GDM values are preciously showing no difference with FDM, and the effect of ADM is seemingly neglected. So the current weight setting makes the RCS data validation not agree with the true feature.

Fig. 7 shows the GDM results of Fig. 3 from (7) with all weighting factors equal to one. Compared with the results in Fig. 6, the probabilities of "excellent" and "very good" have increased significantly, while the rest are reduced. The GDM result is optimized and is between the result of FDM and ADM. This illustrates that the GDM from (7) ensures a balance between ADM and FDM without ignoring any of them.

Therefore, according to (9), the similarity of example in Fig. 3 could be obtained and listed in Table III. The high similarity of 94.71% on amplitude trends is observed, with a low similarity of 74.72% on variation feature. The GDM_{sim} by (6) is 73.19% which similar to the FDM, and it seems to neglect the ADM part. By (7), the total similarity is 80.24% with all weight equal to one. The similarity results also demonstrate the effectiveness of the weight adjustment from (7).



Fig. 7. GDM results of generic missile RCS evaluation from FSV according to (7).

TABLE III Data Similarity of Fig. 3

XDM	XDM _{avg}	XDM _{sim}
ADM	0.098	94.71%
FDM	0.357	74.72%
GDM by (6)	0.378	73.19%
GDM by (7)	0.267	80.24%



Fig. 8. RCS correlation coefficients in different attitude disturbances in 10 GHz.

B. Influence of Attitude Disturbance on RCS

In this part, the influences of attitude uncertainty on the missile RCS are analyzed by evaluating massive simulation data. We add random attitude disturbance to target's attitude sequence for simulate attitude uncertainty in the above-mentioned missile target. MC simulation is performed under each disturbance amplitude with the increase of 0.1°. RCS data from the attitude disturbance are compared with the reference one without disturbance.

To verify the accuracy of proposed approach, correlation coefficient is applied for the data comparison first. The RCS correlation coefficients with varying attitude disturbances ranging from 0.1° to 2° are shown in Fig. 8. Obviously, the data correlation coefficient decreases with the increment of attitude disturbance amplitude.

For the proposed method, data similarities under each disturbance amplitude are obtained from the FSV method. The mean



Fig. 9. XDM_{avg} in different attitude disturbances in 10 GHz.



Fig. 10. RCS similarities in different attitude disturbances in 10 GHz.

value of data similarities in each disturbance amplitude is chosen to describe the influence of attitude disturbance on RCS. Figs. 9 and 10 show the XDM_{avg} and data similarities with varying attitude disturbances ranging from 0.1° to 2° correspondingly.

As can be seen, the data similarity decreases with the increase of attitude disturbance magnitude. Apparently, the result reflects the good consistency with the correlation coefficient, which proves the reasonability of the proposed method.

In addition, more information can be mined from the FSV results. The reduction rate of ADM_{sim} approximates to a constant when the disturbance amplitude is within specific scope. The similarity reduces about 2% by each 0.1° more attitude disturbance. The FDM similarities are lower than those of ADM, and the global similarity GDM_{sim} is closer to the FDM similarity. It means that the RCS variation feature is more affected by the attitude disturbance, and it plays a dominant role in the assessment.

Figs. 9 and 10 show that both XDM_{avg} and XDM_{sim} reveal the same principle of RCS uncertainty. As described in Section II-B, the mean value XDM_{avg} is a single scalar which is relatively abstract and incomprehensible, especially for massive data comparison. The XDM_{sim} combines the advantages of XDM_{avg} and XDMc, which is easy for both statistical analysis



Fig. 11. RCS similarities over attitude disturbances.



Fig. 12. RCS similarities over different frequencies.

and understanding. Thus, it would be more suitable for massive RCS data processing.

Fig. 11 shows the trends of RCS similarities (take the GDM similarity as the ultimate result) with different attitude disturbances. Obviously, the degree of data similarity decreases with raising attitude disturbance. Higher uncertainty of the RCS data has been explored in all listed frequencies with similarity reduction. It can also be seen that the similarity curve reflects the negative correlation relationship with the frequency.

Fig. 12 presents the trends of RCS similarities over different frequencies under specific disturbance amplitude. The global similarity of RCS becomes lower with the increment of frequency. This comes from the higher attitude sensitivity of radar target RCS under higher frequency. Hence, for the missile target introduced in Fig. 2, the effect of attitude disturbance is essentially proportional to the electrical size of the target.

C. RCS Uncertainty Quantification of Dynamic Simulation

To demonstrate the practicality of the method, RCS datasets from dynamic flight simulation are utilized. As shown in Fig. 13, the missile moves along the straight line from the starting point (10 000, 10 000, 0) to the point (-12 000, 13 000, 5000) with the unit of meter. The radar is located at the origin (0, 0, 0).



Fig. 13. Flight track of dynamic simulation.



Fig. 14. RCS of dynamic simulation scene.



Fig. 15. RCS similarities over attitude disturbances in dynamic simulation.

There are 1024 even sampling points within a flight simulation. The RCS simulation results are shown in Fig. 14. The solid line shows backscattering RCS of missile in a flight process. We add random attitude disturbance model in the original dynamic target attitude series to simulate attitude uncertainty in the flight process. Simulation results with mean attitude disturbance of 0° and standard deviation of 0.5° are shown as the dashed line.

MC simulation with attitude disturbance is performed massively for this dynamic flight scene. RCS similarities are evaluated from the FSV method. The mean value of data similarities in each disturbance is chosen to describe the influence of attitude disturbance on RCS. Fig. 15 shows the data similarities with varying attitude disturbances ranging from 0.1° to 2° . As can be seen, the data similarity decreases with the increase of attitude disturbance magnitude, which is also consistent with the previous results.

The above-mentioned results also indicate that XDM_{sim} is more intuitive and informative in massive data evaluation, and the corresponding conclusions have more practical significance. The uncertainty analysis results from the proposed method also reflect the high consistency with the expectation according to the principal of electromagnetic theory. Meanwhile, the example of dynamic simulation again illustrates the practical significance of the proposed method.

IV. CONCLUSION

This paper proposes a novel UQ approach for RCS data evaluation based on MC and FSV methods. The performance of the proposed method has been demonstrated by assessing the target RCS with different attitude disturbances. The data similarity XDM_{sim} is developed as a new indicator of FSV to assess the massive data more conveniently and efficiently. The impact of attitude uncertainty on target RCS is also studied to further verify the effectiveness of proposed method.

As presented in this paper, the current approach still has some limitations to be considered as generally applicable. Many critical parameters are empirically obtained from massive RCS assessments and comparisons with the visual evaluation. The adaptive parameter adjustment achieved by machine learning might be a potential solution to extend the application area of the proposed method.

This paper aims at proposing a new perspective in RCS study considering the uncertainty interference. As a starting point of RCS UQ under the complicated electromagnetic environment, the topic and approach introduced in this paper are expected to attract more attention on the study about the radar target characteristics within more practical backgrounds.

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