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Economic Accounting in the Simple Hotelling Model☆

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ABSTRACT

Because of its simplicity, the simple Hotelling model of exhaustible-resource extraction is a useful vehicle to address two themes in economic accounting: (1) Foundational equalities in the calculation of depreciation and in double-entry bookkeeping, or circular flow, apply in both accounting and economics. (2) Incentives, decisions and outcomes are crucial in economics. It is argued that financial accounting satisfies the demands of the first theme but not the second and that green accounting falls short on both. Both can be satisfied if capital gains are included in net income and product. Comparisons are made to studies in general economics.

1. Introduction: resource accounting on a two-way street

A finding of the analysis of green accounting for an exhaustible resource in the simple Hotelling model is that, if marginal extraction cost is constant, the depreciation of the reserve at any time is equal to the net value of output. Contrary to financial (corporate) accounting, which makes no provision for depreciation of the reserve, green accounting holds that in this central case the resource makes no contribution to the net income of a firm, sector or nation, but that the value of sales is the realization of value embodied in the resource.

The Hotelling model holds that resources are a form of capital. Returns to manufactured capital contribute to its owners' and to national net income. Like manufactured capital, resource capital is used up in production and is valued according to its contribution to social well-being. In green accounting, however, resource capital is subject to different accounting rules. Asheim and Hartwick (2011: 2303) consider it an "anomaly that no entry for natural capital appears on the income side of the accounts".

Accounting organizes raw data for use in empirical analysis and further theoretic development. Its aim is to provide a conceptual and numeric rendering of decisions and their consequences. For centuries financial accounting has been a highly successful economic institution for recording and interpreting commercial transactions. Yet green accountants find that it goes wrong in a model with no market failure and optimal decisions.

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The present paper examines the economics of accounting *in the simple Hotelling model*. In the model, a known, homogeneous reserve is exploited in continuous time by symmetric, competitive sellers under conditions of constant returns to scale, stationary demand and perfect knowledge. There is no exploration or development; at an initial time $t = 0$ a stock $S(0) = S_0$ is simply held by the owners. Extraction cost is zero. The stock of the resource earns capital gains consistently with Hotelling's rule that price rises at the rate of interest.

Gaudet (2007) identifies several ways in which the model is not an accurate portrayal of resource exploitation. It abstracts from exploration, development, the locations of different deposits, the physical and chemical properties of the resource, realistic technology and geology, and uncertainty. Furthermore, the use of sunk capital in producing the resource leads to a fundamental problem of accounting (Cairns, 2009, 2013; see also El Serafy, 1989 for early insights). The conclusion that net income is zero does not survive many of these extensions. In most, however, the *Hotelling rent* is considered to measure depreciation.

There are good reasons for studying this unrepresentative model. (1) Widely recognized as the foundation of resource economics, it is being applied in green accounting for various non-renewable resources. An economically accurate perception of income is essential to assess the performance of resource-producing firms and economies, as well as to formulate taxes and other policies. (2) With no sunk capital, the fundamental problem does not arise. There are no complications from other assets or from nonlinearity. Moreover, in the model, the distinction between capital-stock and commodity-flow approaches (Wei 2013) is moot. (3) A scooter on what Heal (2007: 7) calls a *two-way street* between resource economics and broader economics, the model is of particular interest because Hotelling (1925) also wrote on accounting for depreciation. While it is oversimplified, its specific analytic results render an intricate discussion of the following issues more manageable.

A Accounting can be for wealth, defined as the present value of consumption, or welfare, frequently defined as the present value of the utility of consumption.
B Hicks (1946) enunciated three concepts of income. Income no. 1 refers to maintaining present value in the short term, nos. 2 and 3 to sustaining current value in the long term.
C The contributions of a given resource sector and of its firms must be addressed.
D The relationship of accounting for the resource and for other assets has to be clarified.
E Accounting can be in terms of real or present-value prices.
F Double-entry bookkeeping plays a lead role in accounting and, in the guise of circular flow, in economics. Resulting accounting identities are central.

The following accounting equalities apply to the present value of net benefits, denoted by V.

E1, Adding Up. *The undiscounted sum of depreciation is equal to value.*

Depreciation in period $t+1$ is $V_t - V_{t+1}$. If $\lim_{t\to\infty} V_t = 0$ then $V_0 = \sum_{t=0}^{\infty} (V_t - V_{t+1})$.

E2, Balance. *Net income and product are equal.*

In economics, E2 expresses the circular flow; in accounting, it is a double-entry condition.

A main message of the model is that capital gains have allocational significance. From a study of issues A through F, in the context of equalities E1 and E2, the present paper concludes that the resource should be considered to earn an income in the form of capital gains. It explains that not including capital gains in income can be consistent with double-entry bookkeeping (with the circular flow) but that it neglects aspects of decisions that are critical to an economic assessment. Divergences from financial and green accounting are the imputation of (a) a strictly positive net income in the form of capital gains as the resource price evolves to support equilibrium and (b) the recording of these gains on the product side. It is argued that these two entries complete the accounting consistently with incentives and decisions and with findings elsewhere in economics.

2. Up the Two-Way Street: Resource Accounting

2.1. Wealth vs. welfare accounting

Wealth is the present value of consumption at the market rate, r_t. When there is a single consumption good, C, wealth can be defined as $V(t) = \int_t^{\infty} C_s exp\left[-\int_t^s r_\tau d\tau\right] ds$, with C as numeraire. When there are many consumption goods they are aggregated using prices.

Wealth accounting is closely linked to the aim of economic accounting, namely, to evaluate decisions and their outcomes. Its conception of income is tied to capital theory: "If income equals consumption plus the change in total wealth, then we have a neatly self-contained and intuitive theory of income and wealth – income equals the return on total wealth. . ." (Hamilton and Ruta 2009: 56). The market values of wealth and of output are reported. In a distorted economy, shadow values are used.

The underlying context of the model is the pursuit of welfare, frequently defined as the integral of the utility of consumption, $U(C)$, discounted at the social rate of time preference, ρ: $W(t) = \int_t^\infty U(C_s)\, e^{-\rho(s-t)} ds$ (cf. Weitzman 1976, 2003; Sefton and Weale, 1996, 2006; Asheim and Wei 2009; Arrow et al., 2012). Welfare can be maximized by decentralizing to a competitive industry the optimal net price of the resource and force of interest. A natural behavioral assumption is that the owners' objectives pertain to their wealth. The assumption conforms with Irving Fisher's separation theorem, by which a consumer's decision problem can be broken down into a maximization of present value followed by an allocation of that value among consumption goods over time (cf. Becker 2008).

An interpretation of Sefton and Weale's (1996, 2006) important analyses is that they admit of a link between wealth accounting and welfare accounting. Their point of departure (1996: 32) is to *define* net product Y as the sum of consumption C and a vector of net investments $\dot{K}$ having the price vector P in terms of C as numeraire: $Y = C + P \cdot \dot{K}$. Let the market discount factor from s to $t < s$ be represented by $\delta(t,s) = exp\left[-\int_t^s r_\tau d\tau\right]$. Sefton and Weale derive expressions for *savings* in wealth terms and in welfare terms and show that they are equal to net investment:

$$\frac{1}{U'(C_t)} \int_t^\infty U'(C_s)\dot{C}_s e^{-\rho(s-t)} ds = P \cdot \dot{K} = \int_t^\infty \dot{C}_s \delta(t,s) ds. \tag{1}$$

Their Propositions 3 and 5 (2006) show that net investment $P \cdot \dot{K}$ is proportional to the rate of change of welfare yet is accounted in linear prices. Asheim and Wei (2009) also interpret an expression comparable to $\int_t^\infty \dot{C}_s \delta(t,s)\, ds$ as net investment. In a distorted economy, their interpretation applies to shadow prices for the appropriate social-welfare functional.

This thread is taken up in Section 3, which discusses general approaches. For the moment, it is observed that Eq. (1) provides a parallel between welfare and wealth accounting.

2.2. The Hamiltonian

In the Hotelling model, let the value of the stock at time $t = 0$ be represented by

$$V(0) = \max \int_0^\infty p(t)q(t)\delta(0,t)dt \quad \text{s.t.} \int_0^\infty q(t)dt \le S_0. \tag{2}$$

The current-value Hamiltonian of this problem is

$$H = pq - \lambda q. \tag{3}$$

The first-order condition is that $\frac{\partial H}{\partial q} = p - \lambda = 0$.

Green accountants have adopted the Hamiltonian as the definition of net product. Since the Hamiltonian is equal to zero at all times, the resource is considered to produce no net product or income. Depreciation is *defined* to be $-\lambda \dot{S} = pq$, the Hotelling rent.

However, the integral of the Hotelling rent pq does not add up to value:

$$\int_0^\infty p(t)q(t)dt > \int_0^\infty p(t)q(t)\delta(0,t)dt = V(0).$$

According to E1, the Hotelling rent cannot be the depreciation. Therefore, the Hamiltonian cannot be the net product or income.

"What gets measured gets managed" (Heal 2012: 147). A task of economic accounting is to elucidate how and why an asset acquires as well as loses value (appreciates as well as depreciates). The adjoint condition is that

$$\dot{\lambda} = r\lambda - \frac{\partial H}{\partial S} = r\lambda > 0, \text{ or } \dot{p} = rp > 0. \tag{4}$$

Since its (relative) price rises through time, the resource acquires value through capital gains, $\dot{p}S$, as well as loses value through depletion, q.

The value of the stock (the wealth) remaining in the sector at time $t > 0$ is

$$V(t) = \int_t^\infty p(s)\, q(s)\, \delta(t,s)\, ds = p(t) \int_t^\infty q(s)\, ds = p(t) S(t). \tag{5}$$

Hotelling (1925) and Samuelson (1937) define depreciation as $-\frac{dV}{dt} = -\dot{V}$. Differentiating the sides of the first equality of Eq. (5) yields that

$$-\dot{V}(t) = p(t)q(t) - \int_t^{\infty} r(t)p(s)q(s)\delta(t,s)ds = p(t)q(t) - r(t)V(t) < p(t)q(t). \tag{6}$$

The capital gain $\dot{p}S = rpS$ is a consequence of the decrease in the stock through deliberate management. In Eq. (6) it is incorporated into depreciation. Depreciation is less than the net value of output at t, the value used in green accounting based on the Hamiltonian, which can be called the value of the depletion of the reserve. To first order,

$$V(t) = p(t)q(t)dt + V(t+dt)\exp(-r(t)dt) = p(t)q(t)dt + V(t+dt)(1 - r(t)dt),$$

so that

$$V(t) - V(t+dt) = [p(t)q(t) - r(t)V(t+dt)]dt.$$

On the LHS, depreciation is a difference in current terms; on the RHS, after the passage of dt, $V(t+dt)$ is discounted by $r(t)dt$ less than its contribution to $V(t)$.

The net cash flow, pq, is equal to the rental, interest on plus depreciation of the stock's value:

$$pq = rV + \left(-\dot{V}\right) = rV - \left(p\dot{S} + \dot{p}S\right) = rV - d(pS)/dt. \tag{7}$$

Eq. (7) shows that depreciation is equal to the value of the change in the stock (the value of depletion, pq) *plus* the change in the value of the stock. Given the transversality condition, $\lim_{t\to\infty} p(t)S(t) = 0$, it is confirmed that E1 holds:

$$-\int_t^{\infty}\left(p\dot{S} + \dot{p}S\right)ds = -\int_t^{\infty}\left(\frac{d(pS)}{dt}\right)ds = p(t)S(t) = V(t).$$

Since Hotelling (1925) also wrote the seminal paper on economic depreciation, it is of more than historical interest to note that Hotelling (1931: 170) disagreed with recording the rent λq as the depreciation and thereby rendering income equal to zero:

"It has been said that if the value of ore removed from the ground could be claimed as a deduction from income, then a mining company having no income except from the sale of ore could escape payment of income tax entirely. The fallacy of this contention may be examined by considering the value of the mine at time t. . .any particular production schedule fixes the value of the mine at such a figure that the income at any time, after allowing for depletion, is exactly equal to the interest on the value of the investment at that time."

To maintain wealth intact, depreciation $-\dot{V} = pq - rV$ (not resource rent or depletion $\lambda q = pq$) must be invested elsewhere from cash flow, pq. In the Hotelling model with V as objective, it is optimal that $pq > rV$ be consumed and that value V decrease. In the definition of Hicksian income no. 1 (Hicks 1946), the condition that $\dot{V} = 0$ is a purely formal, instantaneous property of the return to wealth: Since $\dot{V} = rV - pq$, $\dot{V} = 0 \Leftrightarrow pq = rV$. Similarly, $\dot{W} = \rho W - U(C)$ and $\dot{W} = 0 \Leftrightarrow U(C) = \rho W$. Since Hicks (1946: 177) "eschews utility", he likely intended that $\dot{V} = 0$ rather than that$\dot{W} = 0$.

Writing of incomes nos. 2 and 3 as what a *prudent* decision-maker sustains "in each ensuing week" (pp. 172, 174), Hicks implicitly switches to maximin as objective (Solow 1974; Hartwick 1977; Cairns and Long 2006; Cairns and Martinet 2014; Fleurbaey 2015). This notion does not apply if value is given by V (or W): prudence consists of maximizing value, not of maintaining it. By yielding benefits to holding the stock, the capital gain $\dot{p}S$ provides the incentive to manage it well. Income no. 1, which is appropriate to the model, is

$$rV = pq + p\dot{S} + \dot{p}S = H + \dot{p}S = \dot{p}S, \tag{8}$$

i.e., the required return on the remaining reserve. Eq. (8) is the fundamental capital-market-equilibrium (no-arbitrage) condition: interest on the value of the stock, rV, is equal to the dividend, $pq + p\dot{S}$, plus the capital gain,$\dot{p}S$. The total return rV is equated across all assets in efficient capital markets and is what is relevant to managers' incentives and decisions.

Consistently with E2, in the equation $rV = \dot{p}S$ the income rV is balanced by an equal value $\dot{p}S$ on the product side. The gain is not realized in the capital market and consumed but remains with the resource. Let the term *accretion* be coined for it. Eqs. (7) and (8) indicate that, though it involves a change in price rather than quantity, an accretion is akin to a net investment. Any investment involves a choice of a "strike" time. Reflection suggests that a realized capital gain is a reward to the choice of a strike time: optimal timing gives the decision maker more wealth than sub-optimal timing. Since accounting aims to evaluate the management of the resource, the choice ought to affect economic accounting.

If the assumption of stationarity is relaxed, demand may drop to zero for a period and then pick up. Hotelling's rule holds throughout. As Hotelling (1931: 171) notes, "a year in which the mine failed to operate would still be set down as yielding an income equal to the rate of interest on the investment value." Similarly, if conditions are changing there is an optimal time to begin production; capital gains accrue at rate $r(t)$ from 0 to $p(0)S_0$ on (-∞,0] (Cairns and Davis 2007).

Green accounting takes the Hamiltonian, $H = pq + \lambda \dot{S} = 0$, as net product and by a rearrangement of Eq. (6) accounts for net income as interest on the value net of the capital gain:

$$H = rV - \dot{p}S. \tag{9}$$

Often, the value $V(t)$ is written as a function of the stock $S(t)$. However, the decisions in the model are made by individual, competitive firms that take the (optimal) price path as given.[1] The firm's optimal-value function is equal to $p(t)S(t)$. As viewed by a firm, then, the problem is non-autonomous, and time t is another state variable; value should be written $V(t) = f(S(t), t)$. Then,

$$\dot{f}(S,t) = \frac{\partial f}{\partial S}\dot{S} + \frac{\partial f}{\partial t} = -\lambda q + \dot{p}S, \text{ and}$$

$$rV = pq + \dot{V} = pq + \dot{f} = (p - \lambda)q + \dot{p}S.$$

Where there are capital gains, the Hamiltonian is not equal to net income. A method of accounting for income that is consistent with Hotelling and Samuelson's approach to depreciation and with E1 and E2 records as net product,

$$H + \dot{p}S = rV. \tag{10}$$

Both Eqs. (9) and (10) equate a measure of net product (H or $H + \dot{p}S$) to a measure of net income ($rV - \dot{p}S$ or rV). Both satisfy double-entry equality E2. However, green accounting does not satisfy adding-up equality E1. Financial accounting satisfies both equalities. By neglecting capital gains, financial and green accounting neglect the incentives provided by capital gains. The definitions of net product and income in Eq. (10) and of depreciation in Eq. (6) satisfy both accounting and economic requirements.

2.3. Interest and income

The standard practice in economics, of accounting in *real* prices, is followed herein. In the context of the broader economy, the capital gain, $\frac{\dot{p}}{p} = r$, is not related to general inflation but is a change in a *relative* price. As has been stressed in resource economics since Hotelling's original analysis, it has allocational significance.

A different notion is current in some papers on green accounting. The *present-value* price for any $s > t$ at time t is $\pi(s) = p(s)\delta(t,s)$. The use of present-value prices allows for a more compact notation but does not affect any decision. Beyond notation, it has implications for the view of income. Since $\delta(t,s) \in (0,1)$ and $\frac{d\delta(t,s)}{ds} < 0$, the discount factor is tantamount to a price deflator. Consider any price index $\gamma(t,s) \in (0,1)$ and let $v(s) = p(s)\gamma(t,s)$. The index induces an interest rate which is less than r by the value $i = \frac{1}{1/\gamma}\frac{d(1/\gamma)}{dt} = -\frac{\dot{\gamma}}{\gamma}$, and a revised discount factor of

$$\beta(t,s) = \exp\left\{-\int_t^s (r(\tau) - i(\tau))\,d\tau\right\} = \delta(t,s)\exp\left\{\int_t^s i(\tau)\,d\tau\right\} = \frac{\delta(t,s)}{\gamma(t,s)}.$$

The present value is

$$\int_t^\infty p(s)q(s)\delta(t,s)\,ds = \int_t^\infty [p(s)\gamma(t,s)]\,q(s)\,\frac{\delta(t,s)}{\gamma(t,s)}\,ds = \int_t^\infty v(s)q(s)\beta(t,s)\,ds.$$

Present value is thus invariant to the use of any continuous price index, be it exact, approximate or inaccurate (and even to a discrete approximation of a Divisia index as is used in practice). Income is not invariant but depends on the index. The return on present value, rV, can be considered real or nominal depending on whether r is specified to be a real or nominal interest rate.

If the index chosen is $\gamma(t,s) = \delta(t,s)$, so that present-value prices prevail (e.g. a gold standard for a Hotelling gold reserve), then $i(\tau) = r(\tau)$ and the accounting interest rate is nil. Furthermore, income is $(r(\tau) - i(\tau))V(\tau)$, for the invariant value $V(\tau)$, so that what may be called *present-value income* for a Hotelling exhaustible resource is nil.

Only in present-value accounting is the net cash flow the realization of value embodied in the resource. Contrary to green accounting, if $r > 0$, depreciation cannot be equal to the net cash flow. The *real* income of the resource, measured as rV for real interest rate r, is positive.

[1] Thanks to Julien Daubanes for raising this point.

Ramsey's formula, $r = \rho + \frac{\dot{u}'}{u'} = \rho + \frac{\dot{p}}{p}$, gives another perspective on real income. Accounting in welfare terms, with utility as numeraire, is at rate ρ. The change in *welfare* is $p\dot{S} = -pq$, net investment excluding the capital gain. Accounting in wealth terms, with consumption as numeraire, is at rate r and includes the capital gain, $\frac{\dot{p}}{p}$, in income.

2.4. The social-accounting matrix

The aggregate *social-accounting matrix* is an analytic tool used by Hartwick (2000, 2011) to obtain many insights to national accounting. The matrix ensures that E2 holds at each point in time. The last entry in a column or row is equal to sum of the preceding entries. The total levels of net income and product are recorded in the bottom, right-hand entry.

Income from the firms of a sector is added to form the sectorial income, and from the sectors to form the national income. The analysis herein is confined to the sectorial (resource) rather than the national matrix. The contributions to the entries of such matrices can help to clarify the differences in the accounting for a nonrenewable resource among financial accounting, green accounting and the method proposed in the present paper. In these matrices, several entries have equilibrium values pq. Having a common value renders the calculations easy but requires attention to the meanings in the various placements.

The first matrix depicts a standard financial account. Depreciation of the resource is not recognized. Neither is a capital gain or accretion to the resource. Net income and product are pq.

	Consumption	= Net Product
Net Revenue	pq	pq
= Net Income	pq	pq

Green accounting addresses the extension to non-commercial but still economic decisions. A green account is represented in a second matrix. Positive depreciation (negative net investment) of pq offsets net revenue from sales. Net income and net product are recorded as being nil.

	Consumption	+ Net Investment	= Net Product
Net Revenue	pq	0	pq
+ Change in Stock	0	$-pq$	$-pq$
= Net Income	pq	$-pq$	0

For the circular flow, there are many possible "circles within circles". The one in green accounting does not "add up" (E1) and gives spurious results. In financial accounts depreciation adds up and satisfies the double-entry condition (E2). But it does not capture all of the economic features of the problem. A third matrix has an additional line and column for the capital gains (of a mine or the sector), equal to $\dot{p}S = rpS = rV$. The double-entry condition is satisfied.

	Consumption	+ Net Investment	+ Accretion	= Net Product
Net Revenue	pq	0	0	pq
+ Change in Stock	0	$-pq$	0	$-pq$
+ Capital Gain	0	0	$\dot{p}S = rpS$	rpS
= Net Income	pq	$-pq$	rpS	$rpS = rV$

Some authors reason that, because the terms of trade move against consumers over time, the capital gain to owners is offset by a capital loss to consumers. The offset is argued to be why capital gains should not "count" in macro accounts and why net income is nil. For example, Arrow et al. (2012: 333) argue, "[I]n a closed economy there is no need to adjust for capital gains or losses, since the future gains to owners will be exactly offset by the losses to future consumers." This reasoning is different from that based on the Hamiltonian, which applies to aggregate outcomes for society as a whole without distinguishing among the interests of groups within society.

The third matrix offers a contrary interpretation. Consumption is on the product side, not the income side. The capital gain appears on the income side and is equal to an accretion on the product side. Consumption and capital gain do not *offset* each other; rather, accretion and capital gain *balance* each other on the two sides of the accounts. This balance provides an interpretation of Hartwick's (2000: 115) principles that, for an exhaustible resource, the flow value of the stock rpS should be included in net product and the capital gain in net income. Below, similar principles are discussed for all capital assets.

In the Hotelling model all future outcomes are determined as of $t = 0$. The terms of trade are set before extraction begins, for the life of the reserve. Since the objective (2) is the present value of consumption as well as profit, decisions maximize consumers' benefits as well as owners'. Consumers and owners are parties to the terms of trade through the market. The availability of the resource is a benefit to *both*, and to society as a whole.

There is no externality in the Hotelling model; the resource is exploited optimally. The interpretation concerning losses to consumers may arise from the emphasis in resource economics that the resource rent is an opportunity or user cost. The user cost is an expression of the fact that the greatest benefit is obtained from optimal timing. The greatest obtainable benefit is still a benefit. Diewert (2009) observes that a user *cost* is also a user *benefit*. His observation illustrates the analytic power of double-entry bookkeeping. The capital gains are essential to the decisions.

3. Down the two-way street: general accounting

The neglect of capital gains is contrary to capital theory (Diewert and Lawrence 2000: 273):

> "The fundamentals of capital measurement. . .were laid out by Dale Jorgenson (1963). . .This theory, which lays out the relationships between asset prices, rental prices, depreciation and the relative efficiencies of vintages of durable inputs, has been refined and extended by a large number of authors. . .Unfortunately, the United Nations' (1993) System of National Accounts has not yet incorporated this well established theory into its production accounts, partly because the SNA regards interest as an income transfer rather than being a productive reward for postponing consumption and partly because capital gains are also regarded as being unproductive."

The SNA shares many precepts with financial accounting, including those related to capital gains. An important question, only partly examined herein, is how financial and national accounting have made substantial practical contributions in spite of this theoretic shortcoming.

Hill and Hill (2003: 607) note that "Even in a perfect foresight setting, the concept of income is not straightforward. . .The general consensus that emerges from the literature is that, with perfect foresight, all capital gains are included in Hicksian income no. 1." The latter "is equal to consumption C_t plus $V_t - V_{t+1}$, the actual change in the value of the stock of assets existing at the beginning of the period. . .This change can be decomposed into the capital gain. . .and net investment during the period. . ." (Hill and Hill 2003: 610).

Hill and Hill's observations are illustrated succinctly in Diewert's (2006) general model of depreciation with changes in the price of an asset. Over the time interval [0,1], the real price of an asset K_0 changes from P_0 to P_1. Physical deterioration of the asset is given by $K_0 - K_1$; it has value $P_1 (K_0 - K_1)$. Total user cost U_0 (the gross value of capital services) is the counterpart in discrete time of the user cost proposed by Jorgenson (1963). It has three components:

$$U_0 = (1 + r) P_0 K_0 - P_1 K_1 = r P_0 K_0 + P_1 (K_0 - K_1) - (P_1 - P_0) K_0.$$

The first term is the reward to waiting. The second is the value of deterioration, of the "using up" or "consumption" of capital in the period. The third is the capital gain or loss.

The net value of capital's services is its contribution to net product and net income. Which of two expressions should be considered to be the net income from the asset turns on two notions of Hicksian income no. 1, the maximum amount that can be consumed while maintaining capital intact (Hicks 1946: 173). One notion is appropriate if the analyst is interested in its implications for the maintenance of (a pecuniary measure of) *physical* capital:

$$Y_P = r P_0 K_0 - (P_1 - P_0) K_0.$$

The term $(P_1 - P_0) K_0$ is a price effect that is removed from the definition. The second notion is

$$Y_F = r P_0 K_0.$$

The price effect (the capital gain) is not subtracted from income. Consuming income Y_F leaves real *financial* capital intact. Diewert derives both. In a footnote, he expresses a preference for the second.

A further justification of the second is that, if there is obsolescence of the asset, then $P_0 > P_1$ and $Y_P > Y_F$. Obsolescence is comparable to deterioration but appears as a price effect. Moreover, a capital gain or loss motivates decisions.

In the notation of Hotelling's model, $p_1 = P_1 = (1 + r) P_0 = (1 + r) p_0$, $K_0 = S_0$ and

$$Y_P = r P_0 S_0 - [(1 + r) p_0 - p_0] S_0 = 0,$$

as in green accounting. The green accounts focus implicitly on the depletion of environmental capital. An exhaustible resource can only be depleted. Maintaining environmental capital intact entails an augmentation of another resource; the two can be compared only in pecuniary (not physical) terms. A strict focus on physical capital cannot be maintained. Diewert's preferred measure, $Y_F = r P_0 S_0$, includes both interest on the initial stock and the capital gain. Financial capital or wealth is the focus. In the case that $q_0 = 0$, income is also $Y_F = r P_0 S_0$.

With these notions in mind one can begin to interpret Sefton and Weale's (2006) intricate and thorough study of the national accounts. Among their contributions, they clearly indicate micro foundations of national accounting, expressing the national accounts as sums over accounts for households, firms and sectors. Even though there may be equal capital losses in other sectors, income in the resource sector includes its capital gains. They show the strong relationship of welfare accounting to wealth accounting for the dominant, discounted-utilitarian concept of welfare. Furthermore, they show that prices matter: Capital is not aggregated physically but using prices.

As in their earlier paper, Sefton and Weale begin by assuming, in their Eq. (6), that net national product is given by the sum of the values of consumption and investment, $Y = C + I$. The derivations of further expressions for income depend on that definition. Real income is equal to the sum of the return to wealth rV and the effects of changing factor prices expressed as discounted values of changes in interest and wage rates (Sefton and Weale, 2006: Eq. (37)). Their Eqs. (40) and (50) show that, in closed and open economies, the sum of these discounted values is the negative of the real capital gains, denoted herein by Π. For individual sectors and for the macro aggregate, the unnumbered equation on p. 234 can be written

$$C + I = Y = rV - \Pi. \tag{11}$$

Eq. (11) indicates that the definition, $Y = C + I$, presupposes that capital gains are to be excluded from net product and thus from net income. It is also consistent, however, to rearrange Eq. (11) to include capital gains on both sides of the accounts:

$$rV = C + I + \Pi. \tag{12}$$

Since Π is a linear form, both methods yield linear forms. The difference in method lies not in the formal modeling but in the initial definition of net product, $Y = C + I$. Consistency with this definition is what leads to netting the capital gains out of income.[2]

Sefton and Weale (2006) show that $I \propto dW/dt$ (the proportionality factor being the marginal utility of money). As an instantaneous and not a long-term property – not an indicator of sustainability – the economic significance of the proportionality is attenuated: Welfare is maintained, "in each ensuing week" in Hicks's phraseology, only if investment is valued at maximin shadow prices and not the shadow values for a discounted-utilitarian or other concept of welfare (Cairns and Martinet 2014). For these other concepts, there is no imperative to maintain welfare, even instantaneously. Moreover, in the wealth accounts, $rV - C - \Pi = I \propto dW/dt$: To find the value that is proportional to the instantaneous change in welfare one nets capital gains as well as consumption out of the interest on wealth.

In an application of Sefton and Weale's and Asheim and Wei's analyses of welfare accounting, Wei (2012) obtains an intermediate result. Wei decomposes capital gains into those related to changes in prices and those related to changes in interest rates (denoted herein by E_t); only the gains related to changes in prices are included. The reason, Wei argues, is that the capital gains related to changes in interest rates have to be re-invested in order to maintain welfare. Wei proposes a second welfare-accounting method that is consistent with double-entry bookkeeping. Again, the capital gains related to changes in interest rates can be added to both income and product in Wei's study to produce the income and product of the present study: since $Y_t = r_t V_t - E_t$ it follows that $r_t V_t = Y_t + E_t$.

Asheim and Hartwick (2011) examine a general economy but stress accounting for natural resources. In a competitive economy with a vector of *present-value* prices π for capital stocks K and π_C for a scalar consumption good C, they note that $\pi_C C + \pi \cdot \dot{K} + \dot{\pi} \cdot K$ is maximized and is equal to zero. In *current* prices for capital, P, with consumption as numeraire,

$$C + P \cdot \dot{K} = \left(rP - \dot{P}\right) \cdot K. \tag{13}$$

The same re-arrangement as above yields that

$$C + P \cdot \dot{K} + \dot{P} \cdot K = rP \cdot K.$$

National output (with consumption as numeraire) is traditionally considered to be $C + P \cdot \dot{K}$. Asheim and Hartwick's social-accounting matrix (their Table 1) displays capital gains as being netted out of interest on the value of the capital stocks, on the income and product sides as in Eq. (13). Adding the gains to both sides gives a matrix comparable to the third matrix in Section 2.4 above. Since the view of net product and net income is not limited to the value of the marginal product, the re-arrangement also resolves a second anomaly that Asheim and Hartwick uncover in their analysis.

It remains to reconcile with the analysis Sefton and Weale's (1996) striking finding that domestically used resources do not contribute to income but that exported ones do. To translate their notation to that of the present paper, let investment in manufactured capital be denoted by I, imports of the consumption good by M, exports of the resource by X and domestic resource use by Q. A re-arrangement of their Eq. (46) using the result of Eq. (47) is:

$$\int_0^\infty r\delta C dt = [C + I + pX - M - (pX + pQ)]_{t=0} + \int_0^\infty r\delta M dt, \text{ or that}$$

$$\int_0^\infty r\delta (C - M)\, dt = [C - M + (I - pQ) + (pX - pX)]_{t=0} \tag{14}$$

Only domestically produced consumption $(C - M)$ is reported as part of net product or of income in Eq. (14). The RHS is in the textbook form of net product but for receipts of foreign exchange for the exported resource: The value $(pX - pX)$ reports a transfer of resource for foreign exchange, netting to zero. There is no special addition to product (or income) arising from exports. Foreign exchange is used to purchase imports, M, over time. An alternative way to express Eq. (14) following Sefton and Weale's Eqs. (46) and (47) splits income and product into domestic and foreign using the account for holdings of foreign exchange, H, where $\dot{H} = rH + pX - M$:

$$\int_0^\infty r\delta (C - M)\, dt + [rH]_{t=0} = \{(C - M) + (I - pQ) + (pX - pX) + [M + \left(\dot{H} - pX\right)]\}_{t=0}.$$

[2] Compare the remarks on Ramsey's rule in subsection 2.3 above.

The terms in square brackets are foreign income and product.

The capital gain is not reported in Eq. (14). As is discussed above, it can be added to both sides to account for decisions about extraction, $X+Q$, and to show an income and a corresponding product as an accretion.

The upshot is that, in an optimization model, welfare and wealth accounting use the same shadow prices and net investments. The correspondence between wealth and welfare is analogous to that between the budget and utility in the separating-hyperplane theorem. The change in welfare is proportional to $P \cdot \dot{K}$, with the same values of P and$\dot{K}$ as in wealth accounting. This point is clear in the section on "practical wealth accounting" of Hamilton and Ruta (2009: 60–63). They simply net out capital gains to find the change in welfare. Indeed, if the welfare integral is equal to a stationary function $W(K)$, not necessarily discounted utilitarian, then

$$\dot{W}(K) = \sum_i \frac{\partial W}{\partial K_i} \dot{K}_i = P \cdot \cdot K.$$

Therefore, other reasoning than formally economic is required to justify financial accountancy's use of a system based on Eq. (11). Diewert (2006) notes that the measurement of capital is far from simple. Price and real effects are mingled in the choice of a strike time. Malinvaud (2008) writes of a controversy in economics about anticipated capital gains but favors their inclusion in income. As Hill and Hill (2003) and Hill (2004) stress, in statistics that include capital gains, income can be volatile for some resource-extracting economies. *Unanticipated* capital gains do not motivate decisions. Hill and Hill argue that unanticipated gains should not be included in income. They should be used to revalue assets, as in financial accounting: Income from the revalued assets should be incorporated into income, but the revaluation should not be depreciated. There is, however, reason to include anticipated capital gains in income: Anticipated capital gains do motivate decisions. The difficulty lies in estimating how much of capital gains are anticipated. Under uncertainty, a reasonable estimate, even if hard to make, is better than an estimate of zero. The capital gain is interest net of dividends. In the case at hand, $rV = pq + \dot{V} = pq + p\dot{S} + \dot{p}S$ by Eq. (10). The dividend is $pq + p\dot{S}$ and the anticipated capital gain is $\dot{p}S = rV - \left(pq + p\dot{S}\right)$.

The view of the present paper is that the main issues for economic accounting are (a) objectives, incentives and decisions and (b) the *provenance* of income. These contrast with aims sometimes expressed, namely, (1) stability and sustainability and (2) the *use* of income. Income is a benefit to the deployment of capital. Income is independent of preferences, is objective to the extent possible. How income is used is, in keeping with Fisher's separation theorem, a distinct choice, dependent on (subjective) individual preferences

The following considerations support the inclusion of capital gains.

- Prices, as well as material flows, are vital to decisions and economic equilibrium.
- Prices record obsolescence (or amelioration as for a fine violin) but quantities do not.
- Interest and capital gains are productive.
- The return that is equalized through arbitrage includes capital gains. The gains appear in Jorgenson's model of capital measurement.
- The expression of comprehensive income as the return on comprehensive capital formalizes their reciprocal relationship.

4. Mid-rush traffic report

Even if they lack realism it is important that accounting method apply in simple, transparent models. The Hotelling model is a salient theoretic example of the role of timing and of the capital gains that are the rewards to those decisions, arguing for recognizing capital gains in economic accounting.

The present paper is not a call to change accounting practice so much as an effort to understand accounting statistics and their implications for measures of income. There is no question of the usefulness of financial accounting for recording and interpreting commercial outcomes: It satisfies the market test. The difference between financial and economic accountancy lies in their scope, not their validity. Financial accountancy aims for an accurate recording of transactions. Economics is the study of decisions. The aim of economic accounting is broader.

As Diewert (2006) argues, the productivity of waiting is comparable to that of transport. The reward to waiting is interest, a component of which is the capital gain. There must be an income from a reserve to justify holding it as an asset. The Hotelling model is an unadorned vehicle for conceptualizing income and the role of capital gains. Since capital gains are neglected in both green and financial accounting (other than via revaluations in asset accounts), it is worthwhile to emphasize what the Hotelling model implies for a system based on shadow accounting prices.

- Financial accounting can be extended from transactions to all decisions in the model in a natural way that is consistent with accounting method and economic principles.
- Prices in the model (p, λ and r) are "the right" prices from a welfare perspective: they are competitive, with no market failure.
- The price p in the model is a real, relative price.
- Green accounting "doesn't add up", deceptively so since it satisfies a double-entry condition and fits the social-accounting matrix.

- Capital gains are the motivation for decisions. They are components of the returns to assets, which are equalized through arbitrage in the capital market.
- Capital gains are not mere transfers within society that offset each other. They balance with a positive entry on the other side of the accounts.
- Accounting for capital gains can be done in a way that is consistent with fundamental accounting principles in the social-accounting matrix.
- Hicksian income no. 1, what can be consumed while leaving capital value *V intact*, is *rV*. It includes the capital gain.
- Hicksian income is not what is optimally consumed. The Hotelling model is a prime example of how sustainability or even maintaining capital intact over the next short time interval does not motivate Hicksian income no. 1.
- The accounting method can be used for welfare accounting: the sign of the change in welfare is proportional to net investment at the appropriate shadow prices.

The resource's income and productivity stem from the fact that society can wait to deploy it in its most highly valued use. Changes in price are important because prices are central inputs to decisions in decentralized decision making. Accordingly, the accounting method proposed has introduced two unconventional, balancing entries to the green accounts, namely, capital gains and accretions of those gains. Even with its oversimplifications, Hotelling's model does not give a justification to treat all of the net cash flow from an exhaustible resource as depreciation, nor to conclude that there is no net income from an exhaustible resource.

References

Arrow, K., Dasgupta, P., Goulder, L., Mumford, K., Oleson, K., 2012. Sustainability and the measurement of wealth. Environ. Dev. Econ. 17 (3), 317–353.
Asheim, G., Hartwick, J., 2011. Anomalies in green national accounting. Ecol. Econ. 70, 2303–2307.
Asheim, G., Wei, T., 2009. Sectoral income. Environ. Resour. Econ. 42, 65–87.
Becker, R., 2008. Capital theory. In: Durlauf, S., Blum, L., MacMillan, Palgrave (Eds.), The New Palgrave Dictionary of Economics Online Palgrave MacMillan 18 December 2010. , second ed.
Cairns, R., Davis, G., 2007. Strike when the force is with you: optimal stopping with application to resource equilibria. Am. J. Agric. Econ. 89, 461–472.
Cairns, R., Long, N.V., 2006. Maximin: a direct approach to sustainability. Environ. Dev. Econ. 11, 275–300.
Cairns, R., Martinet, V., 2014. An environmental-Economic measure of sustainable development. Euro. Econ. Rev. 69, 4–17.
Cairns, R., 2009. Green accounting for black gold. Energy J. 30 (4), 133–159.
Cairns, R., 2013. The fundamental problem of accounting. Can. J. Econ. 46 (2), 634–655.
Diewert, W.E., Lawrence, D.A., 2000. Progress in Measuring the Price and Quantity of Capital. In: Jorgenson, Dale W., Lau, L.J. (Eds.), Econometrics and the Cost of Capital: Essays in Honor. MIT Press, Cambridge MA, pp. 32–273.
Diewert, W.E., 2006. The Measurement of Income, Ch. 7 of The Measurement of Business Capital, Income and Performance. A tutorial presented at the University Autonoma of Barcelona, Spain (September 21–22; revised June 2006, available at http://www.economics.ubc.ca/faculty-and-staff/w-erwin-diewert/).
Diewert, W.E., 2009. The aggregation of capital over vintages in a model of embodied technical progress. J. Prod. Anal. 32, 1–19.
El Serafy, S., 1989. The proper calculation of income from depletable natural resources. In: Ahmad, J. Yusuf, Salah, El Serafy, Lutz, Enrst (Eds.), Environmental Accounting for Sustainable Development. World Bank, Washington DC.
Fleurbaey, M., 2015. Sustainability and social welfare. J. Environ. Econ. Manage. 71 (2), 34–53.
Gaudet, G., 2007. Natural resource economics under the rule of Hotelling. Can. J. Econ. 40 (4), 1033–1059.
Hamilton, K., Ruta, G., 2009. Wealth accounting, exhaustible resources and social welfare. Environ. Resour. Econ. 42, 53–64.
Hartwick, J., 1977. Intergenerational equity and the investing of rents from exhaustible resources. Am. Econ. Rev. 66 (5), 972–974.
Hartwick, J., 2000. National Accounting and Capital. Edward Elgar, Cheltenham UK.
Hartwick, J., 2011. Green national income and green national product. Ann. Rev. Resour. Econ. 3, 21–35.
Heal, G., 2007. A celebration of environmental and resource economics. Rev. Environ. Econ. Policy 1, 7–25.
Heal, G., 2012. Reflections –- defining and measuring sustainability. Rev. Environ. Econ. Policy 6 (1), 147–163.
Hicks, J., 1946. Value and Capital. Oxford, Clarendon.
Hill, R., Hill, T., 2003. Expectations, capital gains, and income. Econ. Inq. 41 (4), 607–619.
Hill, R., 2004. Accounting for unexpected capital gains on national assets in net national product. Empirical Econ. 29, 803–824.
Hotelling, H., 1925. A general mathematical theory of depreciation. J. Am. Stat. Assoc. 20, 340–353.
Hotelling, H., 1931. The economics of exhaustible resources. J. Pol. Econ. 39 (2), 137–175.
Jorgenson, D., 1963. Capital theory and investment behavior. Am. Econ. Rev. 53 (2), 247–259.
Malinvaud, E., 2008. Capital gains and losses. In: Durlauf, S.N., Blume, L.E. (Eds.), The New Palgrave Dictionary of Economics. , second edition. Palgrave Macmillan, p. e2008, http://dx.doi.org/10.1057/9780230226203.0192 (The New Palgrave Dictionary of Economics Online, accessed 05 March 2013 http://www.dictionaryofeconomics.com/article?id=pde2008_C000031\TEXTsymbol.
Samuelson, P., 1937. Some aspects of the pure theory of capital. Q. J. Econ., 469–496.
Sefton, J., Weale, M., 1996. The net national product and exhaustible resources: the effects of foreign trade. J. Public Econ. 61, 21–47.
Sefton, J., Weale, M., 2006. The concept of income in a general equilibrium. Rev. Econ. Stud. 73, 219–249.
Solow, R., 1974. Intergenerational equity and exhaustible resources. Rev. Econ. Stud. 41, 29–45 (Symposium Issue).
Wei, T., 2012. Capital gains and income arising from nonrenewable resources. Environ. Resour. Econ. 52, 293–300.
Wei, T., 2013. Comparing approaches to valuing sectoral net investments. Resour. Energy Econ. 35, 316–328.
Weitzman, M., 1976. On the welfare significance of net national product. Q. J. Econ. 90, 156–162.
Weitzman, M., 2003. Income, Wealth and the Maximum Principle. Harvard University Press, Cambridge MA.