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Highlights

- We examine the impact of imprecise accounting information on optimal portfolio choicein the mean-variance sense.
- We provide a theoretical platform illustrating the exact way in which imprecise return errors alter the optimal vector of weights.
- An imprecise information set affects optimal weights through the mean of return and through the variance-covariance matrix.

Mean-variance Theory with Imprecise Accounting Information*

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Mean-variance Theory with Imprecise Accounting Information

Abstract

In this paper, we examine the impact of imprecise accounting information on optimal portfolio choice in the mean-variance sense. We provide a theoretical platform illustrating the exact way in which imprecise return errors affect portfolio choice and alter the optimal vector of weights. We demonstrate that the covariance between actual return and return error could partly offset the impact of low-quality information on variance-covariance matrix. This is in agreement with empirical evidence suggesting that optimal portfolio weights are highly sensitive to small estimation errors in expected returns, but they are less sensitive with respect to errors in return variance estimates.

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Keywords: Mean-variance theory; information quality; accrual quality; imprecise information.

1. Introduction

The seminal work of Markowitz (1952) on mean-variance theory has had a lasting impact on theory and practice in the areas of asset pricing and asset allocation. Markowitz laid the foundation for Modern Portfolio Theory and more specifically for the Capital Asset Pricing Model (see: Sharpe, 1964; Lintner 1965; and Mossin, 1966). He assumes that the joint distribution of asset returns can be described by vector of return means and variance-covariance matrix, which are known to investors. Markowitz makes an implicit assumption that return moments are measured by homogeneous investors with perfect precision.[1]

There is mounting evidence indicating noisy information releases (see, for example, Faust et al., 2000). In terms of accounting methods, while an accrual-based accounting system allows companies to measure their future liabilities and revenue receivable, it does make it hard for investors to estimate earnings (Kang and Yoo, 2007). To make things worse, empirical evidence reveals that managers have the power to manipulate earnings through accruals (for a comprehensive review on earnings management, see Healy and Wahlen, 1999). Cash flow is a primitive element in asset pricing and therefore accrual quality is a factor in the mapping accuracy between cash flow and returns. Thus, less than perfect accrual quality implies an imprecise information set. In this paper, we uncover how such imprecise information set leads to an inefficient portfolio choice.

[1] This assumption is also maintained in standard asset-pricing models (see, for example, Sharpe, 1964; Lintner 1965; Mossin, 1966; Merton, 1973), which rely on informationally efficient financial markets and well informed investors.

There is compelling evidence suggesting that mean-variance optimal portfolio weights in the Markowitz model are highly sensitive to seemingly trivial changes in expected returns (see, for example, Kallberg and Ziemba, 1984; Best and Grauer, 1991; and Chopra and Ziemba, 1993). Motivated by this evidence, we examine the theoretical impact of return errors generated by imprecise accounting information on the efficient frontier and asset allocation. Following Jacoby et al., (2014), we decompose the expected equity return into precise and imprecise part. As investors fail to obtain precise returns, we model the efficient frontier with an imprecise expected return vector. However, the decomposition offered by Jacoby et al. (2014) allows us to isolate the impact of the imprecise accounting information return error from the perfect-information solution. We further explore implications for the Capital Market Line (CML) and for asset pricing.

To the best of our knowledge, this is the first attempt to examine the role accounting information imprecision plays in mean-variance analysis. However, there are several papers looking at imprecise accounting information risk. Lambert et al. (2007) find that firms with more imprecisely estimated future cash flow have a higher cost of equity. Green and Hand (2011) estimate portfolio weights using a linear function of accruals, change in earnings, and asset growth. They find that this accounting information significantly improves the performance of the traditional mean-variance efficient portfolio. However, the potential direct effect of low-quality information on mean-variance efficient portfolio remains undiscussed.

Jacoby et al. (2014) derive an intertemporal asset-pricing model based on an imprecise information set. In the static version of their model, systematic imprecise information acts through three asset betas, distinct from the CAPM beta, and related to: (*i*) the covariance between the asset precise return and the aggregate (market) imprecise-information return error; (*ii*) the covariance between the asset and the market imprecise-information return errors (termed commonality in information quality); and (*iii*) the covariance between the asset imprecise-information return error and the precise market return.[2] Jacoby et al. (2014) present empirical evidence in support of their model. Our one-period portfolio-selection model is derived under the same information set assumed in the static version of the model of Jacoby et al. (2014).

The remainder of this paper is as follows. Section 2 provides a mathematical derivation of the imprecise accounting information adjusted efficient frontier. Section 3 discusses the impact of imprecise information on optimal portfolio weights. Conclusion is offered in Section 4.

2. Mathematical Derivation of the Imprecise Information Adjusted Efficient Frontier

In this section we derive the efficient frontier with imprecise accounting information in a single-period setting. We start by defining the imprecise information set. We then proceed to derive the Markowitz efficient frontier in an economy with imperfect quality information and N risky assets. Finally, we add a risk-free asset into the analysis and derive the modified Capital Market Line.

[2] Empirical work in this area includes Francis et al. (2005), who demonstrate that stocks loads positively on an aggregate (market) accounting information quality factor. Others test whether a market-wide accounting information quality factor is priced at the cross section of stock returns, with largely supportive results (see, Kravet and Shevlin, 2010; Kim and Qi, 2011; and Ogneva, 2012). A notable exception is the work of Core et al. (2008) that cannot support a priced accounting information quality factor.

2.1 The Information Set and Investor's Optimization Problem with Imprecise Accounting Information

We retain all conventional mean-variance theory assumptions with the exception of the perfect information set assumption. We assume a single-period economy with *N* risky assets, indexed by *i* (*i = 1,2,…,N*). Let $\tilde{r}_i$ designate the one-period return (one plus rate of return) on risky asset *i*, which is given by:

$$E(\tilde{r}_i) = \frac{E(D_i) + E(P_i)}{P_{0i}}, \tag{1}$$

where P_{0i} represents the current stock price of asset *i*; P_i and D_i represent asset *i*'s stock price and dividend per share at the end-of-period, respectively. We rewrite equation (1) as:

$$E(\tilde{r}_i) = \frac{E(FCF_i) - E(DCF_i) + E(P_i)}{P_{0i}},$$

where FCF_i and DCF_i represent the free cash flow per share and debt activities-related cash outflow per share by firm *i* during the underlying period, respectively.

We follow Jacoby et al. (2014) and define an error term, ν_i, as the random imprecise part of free cash flow per share for the single period. Thus, the expected equity return can be expressed as follows:

$$E(\tilde{r}_i) = \frac{E(FCF_i) - E(DCF_i) + E(P_i)}{P_{0i}} + \frac{E(\nu_i)}{P_{0i}}.$$

Let ψ_i denote the single-period imprecise information return error, where $\psi_i = \frac{v_i}{P_{0i}}$. Next, let $\tilde{r}_i$ and r_i represent the imprecise and precise one-period returns on asset *i*, respectively, such that $\tilde{r}_i = r_i + \psi_i$. In practice, investor solve the following optimization problem with imprecise return estimates:

$$\underset{B,\, V_i}{Max}\ E\left[u(W)\right]$$

such that:

$$1 = \frac{B}{W_0} + \sum_{i=1}^{N} \frac{V_i}{W_0} \text{ , and}$$

$$W = r_f B + \sum_{i=1}^{N} V_i \left(r_i + \psi_i\right) \text{ ,}$$

where W_0 is current wealth; W is random end-of-period wealth; B is the dollar amount invested in the risk-free asset; V_i is the dollar amount invested in the risky asset *i* (*i = 1, 2,...,N*).

Let x_f and x_i denote the investment weight of riskless asset and risky asset *i*, such that $x_f = \frac{B}{W_0}$ and $x_i = \frac{V_i}{W_0}$. Then we can rewrite the previous problem as:

$$\underset{x_f,\, x_i}{Max}\ E\left[u(W)\right]$$

such that:

$$1 = x_f + \sum_{i=1}^{N} x_i \text{ , and}$$

$$W = r_f W_0 x_f + \sum_{i=1}^{N} W_0 x_i \left(r_i + \psi_i\right) .$$

We assume investors' utility to be increasing and strictly concave: $u'(.) > 0$ and $u''(.) < 0$. We further assume that imprecise returns are jointly normal or that an investor's utility function is quadratic. This implies that investors are mean-variance optimizers. Next, we apply Taylor series expansion to expand the investor's utility function around the expected terminal wealth and the expected terminal utility is expressed as follows:

$$E[u(W)] = u[E(W)] + \frac{1}{2}u''[E(W)]\sigma^2$$

The portfolio variance under an imprecise information set is given by:

$$\begin{aligned} \sigma_{\tilde{r}_p}^2 &= \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j Cov\left(r_i + \psi_i, r_j + \psi_j\right) \\ &= Cov\left(\sum_{i=1}^{N} r_i + \sum_{i=1}^{N}\psi_i, \sum_{j=1}^{N} r_j + \sum_{j=1}^{N}\psi_j\right) \\ &= \sigma_{r_p}^2 + 2\sigma_{r_p,\psi_p} + \sigma_{\psi_p}^2 , \end{aligned} \tag{2}$$

where $\psi_p = \sum_{i=1}^{N} x_i\psi_i = \sum_{j=1}^{N} x_j\psi_j$ and $r_p = \sum_{i=1}^{N} x_i r_i = \sum_{j=1}^{N} x_j r_j$.

According to Markowitz (1952), portfolio *p* is mean-variance efficient if there exists no other portfolio *q*, such that: $\mu_q \geq \mu_p$ and $\sigma_{r_q}^2 \leq \sigma_{r_p}^2$. Thus, we construct the efficient frontier by solving the following optimization problem:

$$\underset{x_i}{Min} \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j Cov(r_i + \psi_i,\ r_j + \psi_j)$$

such that:

$\mu_{\tilde{r}_p} = x_f r_f + \sum_{i=1}^{N} x_i E(r_i + \psi_i)$, and

$1 = x_f + \sum_{i=1}^{N} x_i$.

Note that:

$$\begin{aligned}\mu_{\tilde{r}_p} &= x_f r_f + \sum_{i=1}^{N} x_i E(r_i + \psi_i) \\ &= x_f r_f + \sum_{i=1}^{N} x_i E(r_i) + \sum_{i=1}^{N} x_i E(\psi_i) \\ &= \mu_{r_p} + \mu_{\psi_p},\end{aligned} \tag{3}$$

where $\mu_{\tilde{r}_p} = E(\tilde{r}_p)$ and $\mu_{\psi_p} = E(\psi_p) = \sum_{i=1}^{N} x_i E(\psi_i)$. It is clear that our $\mu_{\tilde{r}_p}$ will be indifferent from the traditional mean of Markowitz model when $\mu_{\psi_p} = 0$. Equations (2) and (3) provide a theoretical platform for research examining the potential impact of mean and variance return errors on optimal portfolio choice. When the ex post return is lower than market expectation, managers have a strong incentive to meet the target by manipulate accruals and increase the imprecise information return error. Thus, the covariance between the precise return and the imprecise information return error is negative, which partially offsets the influence of the imprecise information set on the variance-covariance matrix.

2.2 Imprecise Accounting Information and the Efficient Frontier in a Market with only Risky Assets

In this subsection, we derive the efficient set in an imprecise accounting information economy with no riskless asset. Thus, investors will make a portfolio choice by solving the following optimization problem:

$$\underset{x_i}{Min} \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j Cov\left(r_i + \psi_i, r_j + \psi_j\right)$$

such that:

$$\mu_{\tilde{r}_p} = \sum_{i=1}^{N} x_i E\left(r_i + \psi_i\right) \text{, and}$$

$$1 = \sum_{i=1}^{N} x_i \ .$$

We use Lagrange multipliers to rewrite the investor's objective function as:

$$\underset{x_i,\gamma_1,\gamma_2}{Min} \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij}^{*} + \gamma_1 \left[\mu_{\tilde{r}_p} - \sum_{i=1}^{N} x_i E\left(r_i + \psi_i\right)\right] + \gamma_2 \left(1 - \sum_{i=1}^{N} x_i\right), \tag{4}$$

where $\sigma_{ij}^{*} = Cov\left(r_i + \psi_i, r_j + \psi_j\right)$.

The first-order conditions of optimization problem (4) are given by:

$$\sum_{j=1}^{N} x_j \sigma_{ij}^{*} - \gamma_1 E\left(r_i + \psi_i\right) - \gamma_2 = 0 \ , \tag{5.1}$$

$$\mu_{\tilde{r}_p} - \sum_{i=1}^{N} x_i E\left(r_i + \psi_i\right) = 0 \ , \tag{5.2}$$

$$1 - \sum_{i=1}^{N} x_i = 0 \ , \tag{5.3}$$

where x_i *(i = 1,2,…,N)*, are unique weights which satisfy first-order conditions (5.1) through (5.3) and generate the lowest variance for every given mean return level. According to Merton (1972), equation (5.1) suggests that:

$$x_k = \gamma_1 \sum_{j=1}^{N} M_{kj} E\left(r_j + \psi_j\right) + \gamma_2 \sum_{j=1}^{N} M_{kj} \ , \tag{6}$$

where M_{kj} *(k = 1,2,…,N)* are elements of the inverse of variance-covariance matrix of imprecise returns.

3. The Impact of Imprecise Information on Optimal Portfolio Weights

Equation (6) theoretically addresses the previous finding that, while optimal portfolio weights are highly sensitive to small errors in estimating expected returns, they are less sensitive with respect to errors in return variance estimates (see, for example, Kallberg and Ziemba, 1984; Best and Grauer, 1991; and Chopra and Ziemba, 1993). According to equation (6), the distortion of the optimal portfolio weight vector is produced by $E\left(\psi_j\right)$ and the impact of imprecise information return errors on the inverse of the variance-covariance matrix. Recall that equation (2) and (3) show that the covariance between the precise return and return error could partly balance out the impact of low-quality information on the variance-covariance matrix and its inverse (M_{kj}). Equation (6) reveals that the optimal weights are more sensitive to changes in mean of returns.

By solving the optimization problem, we obtain the variance of a mean-variance efficient portfolio as follow:

$$\sigma^2_{\tilde{r}_p} = \frac{\left(C\mu^2_{\tilde{r}_p} - 2A\mu_{\tilde{r}_p} + B\right)}{D},$$

where:

$$A = \sum_{k=1}^{N} \sum_{j=1}^{N} M_{kj} E\left(r_j + \psi_j\right),$$

$$B = \sum_{k=1}^{N} \sum_{j=1}^{N} M_{kj} E\left(r_j + \psi_j\right) E\left(r_k + \psi_k\right),$$

$$C = \sum_{k=1}^{N} \sum_{j=1}^{N} M_{kj},$$

$$D = BC - A^2.$$

Differentiating $\sigma^2_{\tilde{r}_p}$ with respect to $\mu_{\tilde{r}_p}$, and solving the first-order condition, we obtain the global minimum-variance portfolio[3] as follows:

$$\mu_{\tilde{r}_{p\min}} = \frac{A}{C} = \frac{\sum_{k=1}^{N} \sum_{j=1}^{N} M_{kj} E\left(r_j + \psi_j\right)}{\sum_{k=1}^{N} \sum_{j=1}^{N} M_{kj}}$$

$$\sigma^2_{\tilde{r}_{p\min}} = \frac{1}{C} = \frac{1}{\sum_{k=1}^{N} \sum_{j=1}^{N} M_{kj}}.$$

3.1 Imprecise Accounting Information and the Efficient Frontier in a Market with Risky Assets and a Riskless Asset

With a riskless asset, the investor's problem now becomes:

$$\underset{x_f, x_i}{Min} \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j Cov\left(r_i + \psi_i, r_j + \psi_j\right)$$

such that:

$$\mu_{\tilde{r}_p} = x_f r_f + \sum_{i=1}^{N} x_i E\left(r_i + \psi_i\right), \tag{7}$$

[3] A positive second derivative ensures that this is indeed a minimum.

$$1 = x_f + \sum_{i=1}^{N} x_i, \tag{8}$$

The solution to this optimization problem yields the following minimum-variance (efficient) frontier:

$$\sigma_{\tilde{r}_p} = \frac{\left(\mu_{\tilde{r}_p} - r_f\right)}{\sqrt{Cr_f^2 - 2Ar_f + B}}. \tag{9}$$

The imprecise information adjusted version of A, B and C demonstrates that the slope of the Capital Market Line has been altered. Thus, investors who construct portfolio based on the original mean-variance frontier will be in suboptimal portfolios.

4. Conclusion

We recast the Markowitz efficient frontier by allowing an imperfect information set. We demonstrate how imprecise information return errors affect optimal portfolio choice. As the influence of low-quality information rises, the distinction between the prefect information and the imprecise information (observed) efficient frontier turns larger, moving investors away from an optimal portfolio choice. This hurts the benefit of using the classical mean-variance theory in practice. Consistent with the estimation error literature, we find that with an imprecise information set expected return errors play a more significant role in affecting optimal weights than the role played by imprecise return variance estimates.

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