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# Modeling of decision-making process for moving straight using inverse Bayesian inference

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ABSTRACT

Humans sometimes make unreasonable decisions when viewed in objective terms. Even in the real world, we may lose sense of direction by turning around the corner several times or mistaking the estimation of travel distance. We experimented in virtual space how we lose sense of direction under what circumstances. In the experiment, subjects viewed a three-dimensional space displayed on a computer display in the first person's perspective and were instructed to go straight from the start to the goal position. Results showed that unreasonable selections that strayed from the centerline connecting the start and goal positions were frequently made. The change in the direction is more influential than the change in the distance, and the angle of turning also affects. Furthermore, the subject's decision – making process was modeled using Bayesian inference and inverse Bayesian inference. Comparing the two models, when the decision-making pattern suddenly changed, the model by inverse Bayesian inference could follow up faster than the Bayesian inference.



## 1. Introduction

Human beings perceive and behave based on uncertain information and sometimes make unreasonable judgments when viewed in objective terms. Modeling of the process used for instantly making some kind of decision—even if unreasonable—from uncertain information for which conditions can suddenly change is important not only for understanding the human cognitive process but also for constructing intelligent systems that can adapt to actual environments.

Mach (Mach, 1980) and Helmholtz (Helmholtz, 1925) (Helmholts, 1962) suggested that *stochastic inference* provides a basis for the human cognitive process based on uncertain information. Additionally, advances in artificial intelligence research have driven research of cognition and behavior based on such stochastic inference (Knill, 1998) (Shimozaki, 2003) (Murray, 2003) (Saunders and Knill, 2004). Given that a causal relation between two events, such as "if $p$ then $q$" is true, the contraposition "if not $q$ then not $p$" is true, but the converse "if $q$ then $p$" or the converse of the contrapositive "if not $p$ then not $q$" is not necessarily true. It is known, however, that humans will sometimes perceive the converse to be true and make unreasonable judgments as a result. This lies in the tendency to make any asymmetry in cause-and-effect relations symmetric and is therefore called "symmetry bias," which has come to be studied in detail through standardized experiments (Hattori, 2003) (Wasserman et al., 1990) (Anderson and Sheu, 1995) (Hattori and Oaksford, 2007). In these experiments, subjects were asked to evaluate the extent to which the casual relation of presented events could be relied upon and attempts were made to model the processes involved. For example, the contingency model of Jenkins (Jenkins and Ward, 1965) and the probabilistic contrast model of Cheng (Cheng and Novick, 1992) have been proposed. In addition, Hattori discovered that the causal inference parameter $(P(p|q)P(q|p))^{1/2}$ that deals symmetrically with "if $p$ then $q$" and "if $q$ then $p$" had a high correlation with this sense of reliability (Hattori and Oaksford, 2007).

Gigerenzer showed by experiment that human thinking is performed along the lines of *Bayesian inference*, and, that it can be influenced by converting expressions for rate of event occurrence into "frequency" or "probability" (Gigerenzer and Hoffrage, 1995). Meanwhile, Knill described how humans and other living organisms perform stochastic inference based on uncertain information obtained from the real world and reported that many experimentally observed examples could be explained by a Bayesian

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perceptual system (Knill and Pouget, 2004). In contrast to reacting suddenly to particular input, this system integrates information that propagates over space and time. He explained that this property holds not only for inputted information but also for uncertainty in the results of behavior. In addition, Manktelow (Manktelow, 2012) classified and analyzed how humans perceive a variety of uncertain events such as coin-toss games, probability that a weather report is accurate, existence of the Loch Ness Monster, etc. It was described here that Bayesian inference, which is used to update a person's hypothesis after observing an event as posterior probability, is the basis for human intuitive cognition. There are also many studies that children's developmental process is in line with Bayesian principles (Bonawits et al., 2012), (Goodman et al., 2011), (Gopnik and Wellman, 2012), (Griffiths et al., 2012). Pellicano, meanwhile, described how the perceptual process of autistic patients follows Bayesian inference and that autistic people tend to dislike unconventional stimuli that do not agree with experience (Pellicano and Burr, 2012). This result suggests the ability and limitations of the Bayesian inference.

As mentioned above, the Bayesian inference has become widely used to model various cognitive and decision-making processes and hundreds of studies have been done (Bowers and Davis, 2012) (Jones and Love, 2011) (Lee and Sarnecka, 2011) (Perfors et al., 2011) (Dawson and Gerken, 2011). However, Knill pointed out that Bayesian inference does not respond sensitively to unconventional input (Knill and Pouget, 2004). As described later in the next section (Fig. 1), simple Bayesian inference does not immediately follow a sudden change in data such that the model itself changes. Meanwhile, Gallistel et al. have proposed a method “Bayesian change point analysis” which detects sudden change of data (Gallistel et al., 2004), (Gallistel, 2009), (Baker et al., 2016), (Papachristos and Gallistel, 2006). First, for the null hypothesis that data does not change abruptly and alternative hypotheses, calculate the ratio of likelihood, that is, the Bayes factor. Using the sign of the logarithm of the Bayes factor (i.e., weight of the evidence), it is determined whether or not there is a change in the data. When it is evaluated that there is a change, the time at which the distance between the line connecting the start point and the end point of accumulated data and the cumulative value becomes maximum is set as the change point. This method has been applied to the analysis of learning curves of infants and animals. But their Bayesian approach is not an attempt to model the cognitive process itself.

In response to the above, Arecchi has proposed “inverse Bayesian inference” which refers to inverse transposition of terms in Bayes formula (Arecchi, 2011). Independent of Arecchi, Gunji et al. also proposed inverse Bayesian inference which refers to replacing conditional probability by marginal probability (Gunji et al., 2016a,b) (Gunji et al., 2016a,b) (Gunji et al., 2017). In this paper, we refer to “inverse Bayesian inference” in Gunji's sense. While Bayesian inference predicts beforehand the probability that a hypothesis will be selected to directly derive an optimal solution, inverse Bayesian inference determines the likelihood of a hypothesis afterwards to alleviate previously prescribed rules. In this paper, we test by experiment whether inverse Bayesian inference is applicable to modeling of the human decision-making process.

When moving or changing direction, humans continue to update their own local coordinate system to the world coordinate system. Liberman rotated the goldfish in the direction of the two axes orthogonal to each other, and then observed the nerve cells of the goldfish which became impossible to swim. Before rotation, the microfilaments of the cytoskeleton parallel to the dendrites collapsed after rotation, indicating that decision-making to select the coordinate system is also done at the cellular level (Liberman et al., 2008) (Igamberdiev and Shklovskiy-Kordi, 2017).

We designed this experimental system to make visual information presented to the subject and the results of the subject's button operations both incomplete and uncertain so that decision-making might change suddenly. A total of 418 trials were performed in the experiment, and it was often found that decision-making would change abruptly. We model these decision processes using Bayesian inference and inverse Bayesian inference and compare them.

In this experiment, the information presented to the subjects is uncertain, and the pattern of decision of the subjects is designed to change suddenly. The uncertainty of the information obtained in the actual decision making depends on the situation at that time and the knowledge of the subject. In addition, even if it is possible to make a reasonable judgment with sufficient time and necessary knowledge, in actual circumstances, there are many cases where judgment must be made immediately. Therefore, we designed an experiment that (i) does not rely on past knowledge and experience, (ii) reproduces the situation to be judged one after another, and (iii) the degree of uncertainty can be controlled.

## 2. Bayesian inference and inverse bayesian inference

### 2.1. Bayesian inference

First, we show a simple example of Bayesian inference. Time series data $d$:{$d^{0}.d^{T-1}$($T$ is data length)} obtained by experiment takes on binary values, that is, −1 or 1. Given $N$ hypotheses, we denote the prior probability of hypothesis $h_i$ ($i = 1.N$) at time step $t$ as $P^{t}(h_i)$ and the probability (likelihood) that the data at time step $t$ is 1 for a given hypothesis as $P^{t}(d|h_i)$. Initial values of these probabilities are $P^{0}(h_i) = 1/N$ and $P^{0}(d|h_i) = i/(N+1)$. Now, the posterior probability $P^{t}(h_i|d)$ of each hypothesis $h_i$ according to $d^{t}$ at each step in this time series of data is successively calculated by Eq. (1).

$$ifd^{t} = 1P^{t}(h_i|d) = P^{t}(h_i)P^{t}(d|h_i)/\sum_{k=1.N}(P^{t}(h_k)P^{t}(d|h_k)) \quad (1)$$

$$ifd^{t} = -1P^{t}(h_i|d) = P^{t}(h_i)(1-P^{t}(d|h_i))/\sum_{k=1.N}(P^{t}(h_k)(1-P^{t}(d|h_k)))$$

Next, posterior probability $P^{t}(h_i|d)$ obtained in this way is substituted in the prior probability of the hypothesis, $P^{t+1}(h_i)$ in the next step.

$$P^{t+1}(h_i) = P^{t}(h_i|d) \quad (2)$$

The above process is Bayesian inference. Here, likelihood $P^{t}(d|h_i)$ of hypothesis $h_i$ for which prior probability $P^{t}(h_i)$ is maximum shows the change in event probability as inferred from the data.

### 2.2. Inverse bayesian inference

Next, in inverse Bayesian inference (Gunji et al., 2016a,b), the process described above continues with the following process. First, using random number $r$ equal to or greater than 0 and less than $\sum_{k=1\cdot N}(1\text{-}P^{t}(h_k))$, we select hypothesis $h_i$ with the smallest possible prior probability $P^{t}(h_i)$ satisfying equation (3).

$$Q_{i-1} \le r \prec Q_i \text{where} Q_i = \sum_{k=1.i}(1-P^{t}(h_k)), Q_0 = 0 \quad (3)$$

Likelihood $P^{t+1}(d|h_i)$ of the above hypothesis is now substituted by the most recent moving average $P^{t}{}_{mov}(d)$ of the frequency of $d^{t} = 1$. In other words, this process updates the hypothesis itself. $s$ indicates the interval of moving average.

$$P^{t+1}(d|h_i) = P^{t}{}_{\text{mov}}(d) = \sum_{k=t\text{-}s.t}(d^{k}/(s+1)) \quad (4)$$

As described above, the Eqs. (1) to (4) are combinations of Bayesian inference and inverse Bayesian inference, and therefore are called BIB (both Bayesian and Inverse Bayesian) inference (Gunji

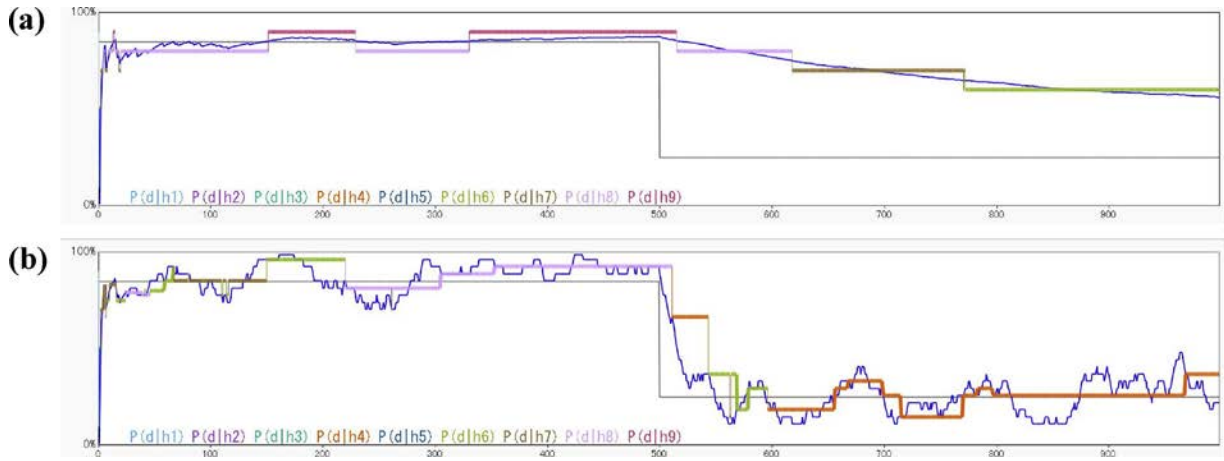


**Fig. 1.** (a) Change in cumulative average of the data from the start $P^{t}_{ac}(d)$ (blue), and likelihood $P^{t}(d|h_i)$ for maximum prior probability $P^{t}(h_i)$ by Bayesian inference. (b) Moving average of the data $P^{t}_{mov}(d)$ whose interval is 25 (blue), and likelihood $P^{t}(d|h_i)$ by BIB inference. Both data are randomly generated and the probability that 1 will occur is 0.85 when $T \leq 500$, and 0.25 when $500 < T$ (gray). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

et al., 2016a,b). Fig. 1 shows a simple example of Bayesian inference (a) and BIB inference (b). The binary time series data (1 or −1) used here is randomly generated, and the probability that 1 will occur is 0.85 before 500 steps and 0.25 thereafter (gray). The staircase lines with color indicate the likelihoods with the maximum prior probabilities, the BIB inference follows the sudden change in probability, while Bayesian inference follows asymptotically. In addition, the blue lines show (a) the cumulative average from the start and (b) the moving average with the interval of 25 steps, which change with the same tendency as the result of each inference.

Here, joint probability $P(d, h)$ satisfies Eq. (5).

$$P(d, h) = P(h|d)P(d) \tag{5}$$

Eq. (6) can therefore be obtained from Eq. (2) of Bayesian inference.

$$P^{t}(d, h) = P^{t+1}(h)P^{t}(d) \tag{6}$$

In addition, Eq. (7) can be obtained from Eqs. (4) and (5) of inverse Bayesian inference.

$$P^{t}(d,h) = P^{t}(h)P^{t-1}(d) \tag{7}$$

In Eqs. (6) and (7), the prior probability and the data probability become the temporal converse of each other. In Eq. (6), the prior probability $P^{t+1}(h)$ is predicted beforehand to directly derive an optimal solution. On the other hand, in Eq. (7), data probability $P^{t-1}(d)$ is determined by *post-diction* to alleviate previously prescribed rules. That's the reason why the name of this method is called *inverse* Bayesian inference (Gunji et al., 2016a,b).

The problem of perception and decision making can be thought of in the framework of *endophysics* such as understanding the nature of the world by the internal observer (Rossler, 1996). *Retrocausality* is a way of thinking about opposite causality and time (Matsuno, 2017). Bayesian inference (Eq. (1)) in BIB inference corresponds to the forward direction of time, while the inverse Bayesian inference (Eq. (4)) corresponds to the reverse flow of time. When considering human reasoning ability and decision-making, the logic which is normally thought not to depend on time has deep connection with time (Gunji et al., 2006) (Sasai and Gunji, 2008) (Sawa and Gunji, 2014). Sawa proposed a *double Homunculus model* in which two differential equations influenced each other and expressed the recursive structure of the observer. By combining this model, various kinds of logic gates can be constructed, and the reverse logic gate allows us to consider of the reverse flow and uncertainty of time (Sawa and Igamberdiev, 2016) (Sawa and Igamberdiev, 2017). In order to realize this, the *double Homunculus model* adopts hidden and dummy variables. In BIB inference, these variables correspond to likelihood and prior probability.

In the following sections, we first describe our experimental method for having subjects perform successive decision-making based on incomplete and uncertain information. We then use Bayesian inference and BIB inference to model the decision-making process obtained by this perceptual experiment including unreasonable judgments. Finally, we present the results of comparing these models with experimental data.

## 3. Materials and methods

### 3.1. Design

In this experiment, the decision-making process for moving to a certain destination was reproduced in a virtual space. Even in the real world, by misunderstanding the target direction by repeating turning left or right several times, or misunderstanding the estimation of the distance traveled, we may lose a sense of direction. In order to reproduce such a situation, we developed software that proceeded in the virtual space by right and left selection of subjects. In this experiment, (i) subjects do not need any special prior knowledge, and (ii) they need to continue deciding which way to proceed. Also, (iii) the degree of uncertainty is controlled by the forced selection of the software. The left and right data series selected by the subject are analyzed using Bayesian and BIB inference shown in Section 2 with binary values of 1 and −1, respectively.

### 3.2. Procedure and tasks

As shown in Fig. 2 (left) and Table 1, there are four types of three-dimensional (3D) objects: (a) *rectangles*, (b) *hexagons*, (c) *walls created in ad hoc*, and (d) *pillars*. These objects are arranged within a prescribed area. The 3D objects (a) to (c) were assumed to test the sense of direction, and the 3D object (d) was assumed to test the sense of migration. As shown in Fig. 2 (right), the space viewed from the first person's perspective consisting of 3D objects is displayed on a computer screen. The viewpoint position in 3D space moves according to the subject's left/right key selection. In the case of (a) to (c), the view direction is also changed. In the case of (c), walls are automatically created on the left and right in the next dead end (impossible in reality). In the case of (d), the pillars-like objects are randomly arranged, and the viewpoint advances

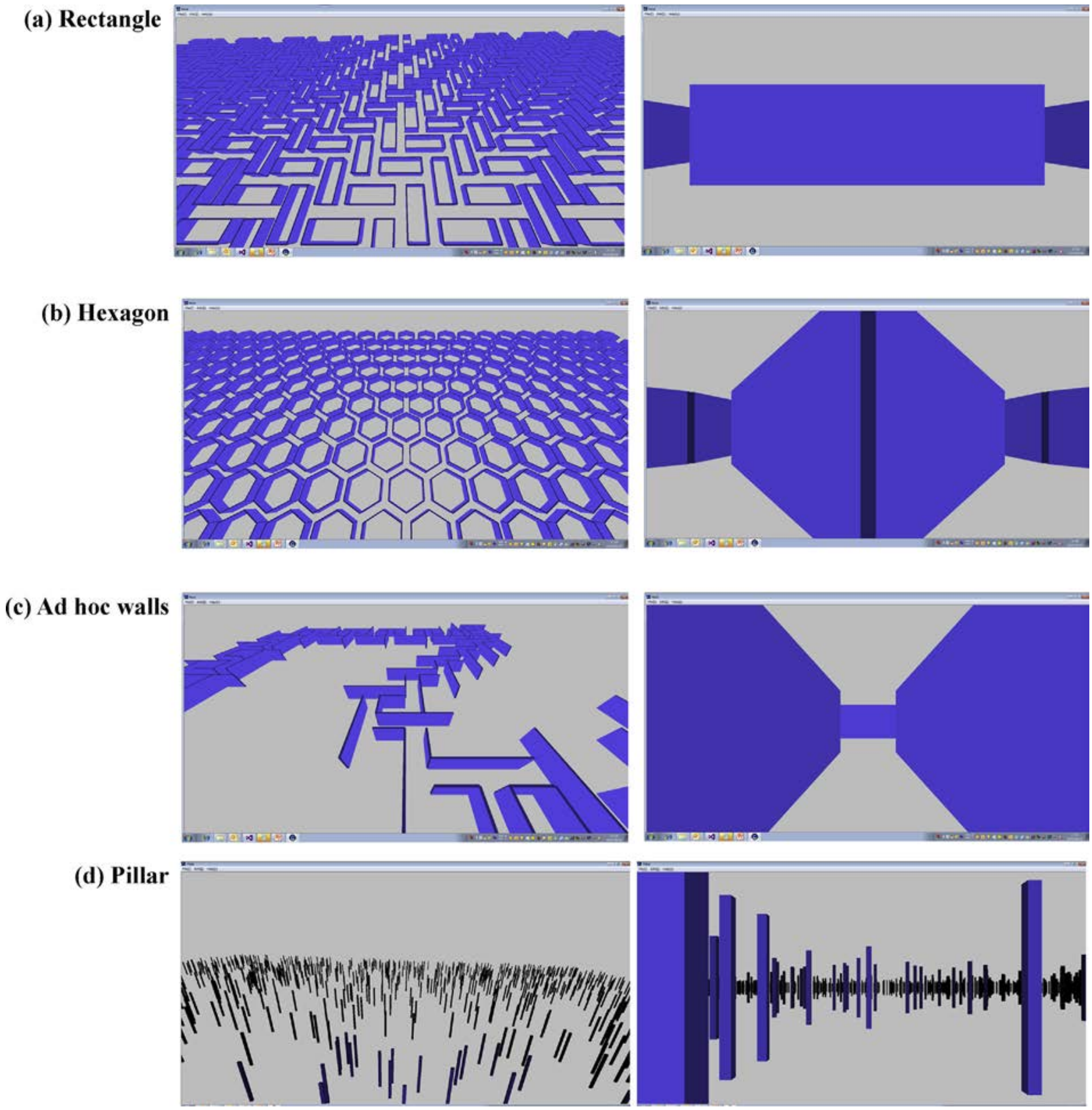


**Fig. 2.** Screenshot of experiment on moving through a virtual three-dimensional space arranged with objects. The viewpoint in this three-dimensional space moves according to the subject's selection of a left or right key. The subjects were instructed to move in the forward direction from the start position. Key-operations and the elapsed time were recorded.

**Table 1**
Task list of this experiment. For tasks 1–4, only the subject selects direction of movement, while for tasks 5–8, the software randomly selects left and right, once in three times. The subjects perform all tasks 1–3 times.

| Task | 3D model | selection by |
|---|---|---|
| 1 | (a) Rectangles | subject only |
| 2 | (b) Hexagons | |
| 3 | (c) Ad hoc walls | |
| 4 | (d) Pillars | |
| 5 | (a) Rectangles | subject-> subject-> software |
| 6 | (b) Hexagons | |
| 7 | (c) Ad hoc walls | |
| 8 | (d) Pillars | |

in a zigzag manner diagonally forward by selection of the left and right of the subject. The migration length in the left or right (lateral) direction is set randomly within a certain range while the migration length in the forward direction is fixed. In all cases, the time to move to the point where the next selection is required is 1000 milliseconds. In tasks 1–4, only the subject selects the direction of movement. To increase the degree of uncertainty, in tasks 5–8, the software automatically selects the left or right key once for each of the three key selections.

If the same experiment were to be performed in the third person's perspective such as in Fig. 2 (left), the subject would likely arrive at the correct goal position with ease. This is because such a view would enable the subject to determine the relationship between the start and goal positions and his or her current location at all times. However, in the first persons' perspective of this experiment as shown in Fig. 2 (right), the subject cannot look backwards and can only obtain information in the forward direction within a certain field of view angle (40° in this experiment). So, the information presented to the subject is uncertain in this experiment.

### 3.3. Variables and research questions

Independent and dependent variables of this experiment are as follows:

Independent variables

- 3D models (*rectangles*, *hexagons*, *ad hoc walls*, and *pillars*)
- Selection left or right by subjects and software (subject only and both subject and software)

Dependent variables

- Left or right selection sequence
- Time required for judgment
- Number of unreasonable judgements (straying from the centerline connecting the start and goal positions)
- Goal position

The research questions of this experiment are as follows:

| | |
|---|---|
| **RQ1** | Does the subject's goal position change depending on the 3D model? |
| **RQ2** | Does time of subject's judgment change depending on 3D model? |
| **RQ3** | Does the ratio of subject's unreasonable judgment change depending on 3D model? |
| **RQ4** | Do the subject's goal position, the ratio of irrational judgment, and the judgment time change with the degree of uncertainty? |
| **RQ5** | Can Bayesian inference and BIB inference model actual data? |
| **RQ6** | Does the accuracy of Bayesian inference and BIB inference change with the degree of uncertainty? |
| **RQ7** | Does BIB inference provide higher accuracy than Bayesian inference? |
| **RQ8** | Does the difference in error between Bayesian inference and BIB inference change as the degree of uncertainty increases? |

### 3.4. Participants and instruction

The subjects were naïve college students numbering 30 in total (18 men and 12 women) without any detailed knowledge of this field. The subjects were instructed that "the goal lies straight ahead—select either the left or right arrow key to advance toward the goal" and were encouraged to move straight ahead as much as possible. In each trial, the subjects performed these key operations until 3D objects could no longer be seen, which took about 50 steps. In tasks 3 and 7 (*ad hoc walls*), each experiment was continued up to 50 steps. Each task was completed in about 1 min and a break of about 30 s was set up until the next task execution. The order of the tasks was 4, 8, 1, 5, 2, 6, 3, and 7, and the subjects were conducted 1–3 times. There was no notable difference between the same tasks of the same subjects, and no influence of fatigue and accustomed was observed. Due to ethical issues, all the participants signed a form in which they accepted to participate in the experiment and let their data to be used in anonymized and aggregated form for research purposes.

## 4. Results

### 4.1. Result of experiments

Examples of experimental results are shown in Fig. 3. The line graph in the figure represents the locus of viewpoint motion when observing the 3D space from above. The vertical direction in this graph corresponds to forward direction in the experiment. The points on this locus indicate locations where left or right was selected, and the numerical values indicate elapsed time (second) since the start of the experiment.

The distribution of the distance between the goal position and the actual arrival point for each task is shown in Fig. 4. The results of the experiment using *pillars* (tasks 4 and 8) are significantly smaller than that of the other 3D objects (RQ1). Fig. 5 shows the distribution of time until the subject selects left and right keys. In the tasks of the *pillars*, the time required for the decision is significantly shorter (RQ2).

The result in straying from the centerline connecting the start and goal positions can be seen. Fig. 6 shows the percentage of such an unreasonable judgment among left and right selection by subjects in about 50 steps in each trial. This percentage does not include selection by software. The proportion of unreasonable judgment when the 3D object is *rectangles* and *hexagons* is higher than that of *ad hoc* and *pillars* (RQ3).

As shown in Fig. 4 to Fig. 6, these three results (goal position, judgment time, and percentage of unreasonable judgment) were unrelated to the presence or absence of forced selection by the program except task 4 & 8 in unreasonable decision (RQ4). In tasks of *pillars*, the subject reaches a position not far from the correct goal, the unreasonable judgment rate is low, and the judgment time is short. Therefore, it can be said that these are the easiest tasks for the subject. That is, the influence of the *change of the distance* on the task is smaller than the *change of the direction*. In tasks of *rectangles* and *hexagons*, the distance from the correct goal was large, the ratio of unreasonable judgment was high, and it took time to judge, so these may be difficult tasks for the subject. A possible reason is that in the case of *rectangles,* there are two types of distances traveled by left or right selection. Also, in the case of *hexagons*, turning 60$^\circ$ is rarely experienced in reality. For these reasons, it may be difficult in these two cases.

### 4.2. Evaluation of results using bayesian inference and BIB inference

In this section, we calculate Bayesian inference and BIB inference described in section 2 setting $d^t = 1$ when left is selected and $d^t = -1$ when right is selected in the experiment. The number of hypotheses $N$ is set to 9. Fig. 7(a) shows the prior probability $P^t(h_i)$ of hypothesis $h_i$ against time obtained only by Eqs. (1) and (2) (i.e. only Bayesian inference), and Fig. 7(b) shows those obtained by Eqs. (1) to (4) (i.e. BIB inference). The probability where $d^t = 1$, that is, likelihood $P^t(d|h_i)$ of each hypothesis by Bayesian inference is invariable but likelihood by BIB inference is updated sequentially as shown in Fig. 8.

Fig. 9(a) shows average from start time $P^t_{ac}(d)$ for percentage occurrence of $d^t = 1$ and the likelihood of Bayesian inference $P^t(d|h_i)$ for the hypothesis having maximum prior probability $P^t(h_i)$, and Fig. 9(b) shows moving average $P^t_{mov}(d)$ and likelihood of BIB inference. The results of Bayesian inference follow average $P^t_{ac}(d)$ from start time while the results of BIB inference follow moving average $P^t_{mov}(d)$. It can therefore be said that Bayesian inference is effective for a steady data-appearance pattern and that BIB inference is effective for a suddenly changing data-appearance pattern.

Next, in order to further investigate whether actual behavior can be modeled, comparison was made based on the cumulative value of the likelihood with the highest prior probability. Here, we compared cumulative change of experimental data $c_{raw}{}^t = \sum_{k=0.t} d^k$ with cumulative change of the results obtained by Bayesian inference and BIB inference expressed as $c_{inf}{}^t = \sum_{k=0.t}(2P^k(d|h_i) - 1)$. Here, since $P^t(d|h_i)$ takes a value from 0 to 1, it is converted from $-1$ to 1. As shown by the example in Fig. 10, Bayesian inference cannot track the sudden change in behavior occurring near the middle of the experiment while BIB inference can.

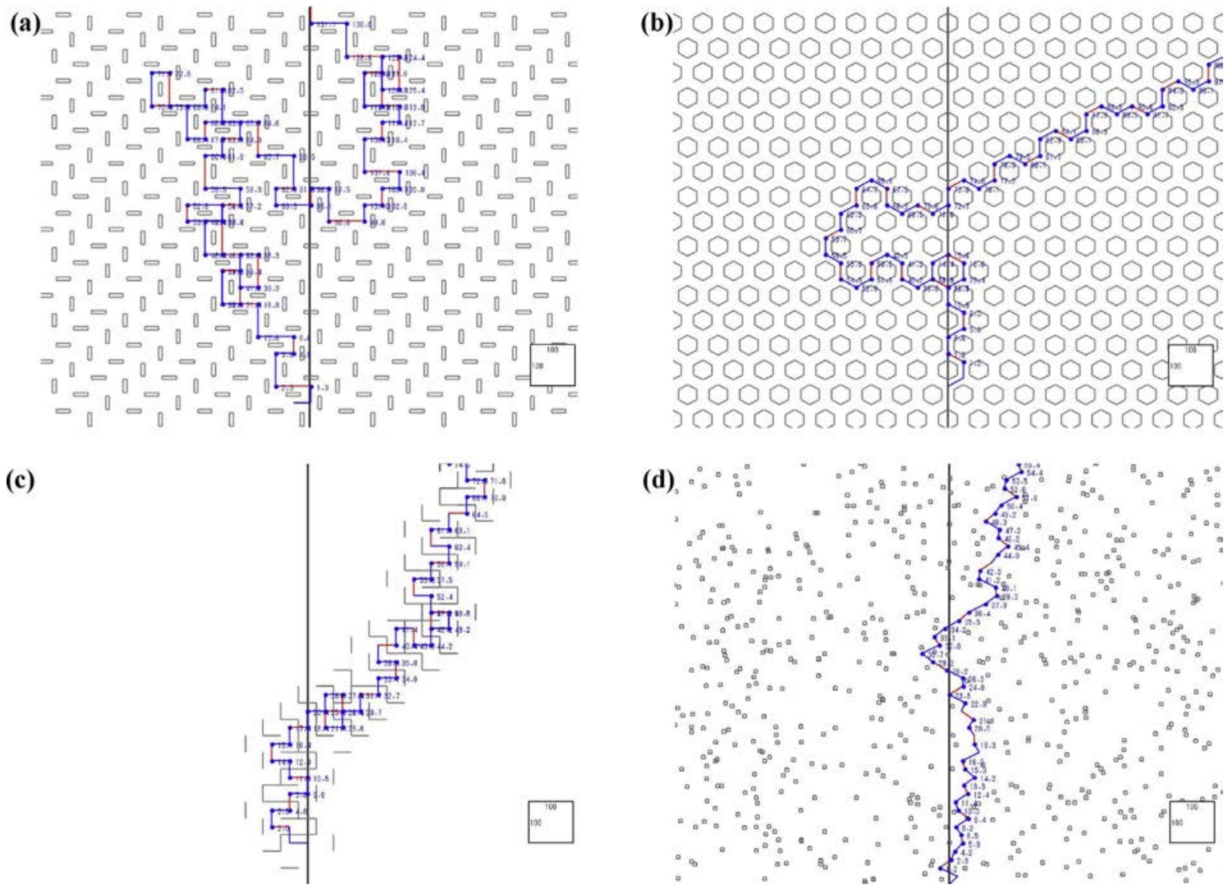


**Fig. 3.** Bird's-eye view of experimental results showing movement history. (a) task 5, (b) task 6, (c) task 7, and (d) task 8. The line graph is the locus of viewpoint motion when observing the three-dimensional space from above and the vertical direction corresponds to forward direction. Points on the locus indicate locations where left or right was selected, and the numerical values indicate elapsed time (second) since the start of the experiment. The red line shows the trajectory selected by the software. The lower right square shows the scale of the space, and the length in the longitudinal direction (goal direction) and the length in the lateral direction are both 100. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

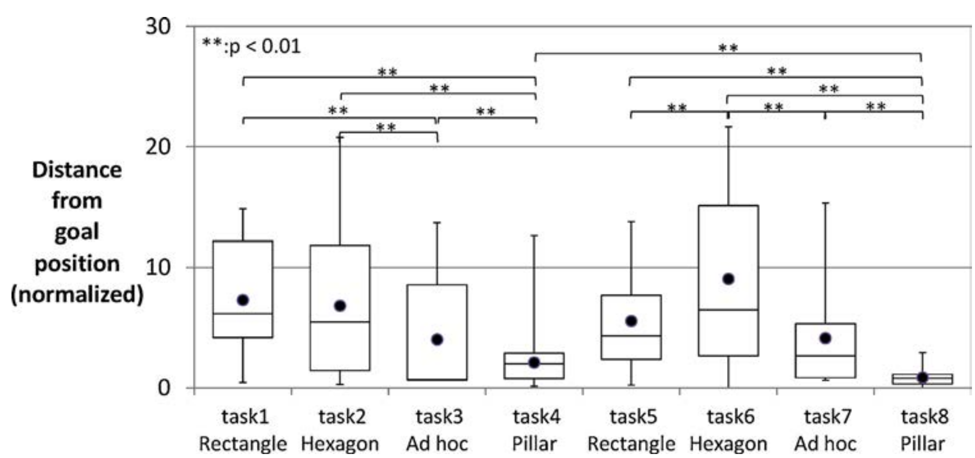


**Fig. 4.** Distributions of displacement from the correct goal position at the end of the test. The value of the vertical axis is normalized by the average value of the moving distance of each step. Variances of tasks 4 and 8 ((d) *pillars*) are significantly smaller. In this figure (also in Fig. 5, Fig. 11, and Fig. 14), the upper and lower ends of the box are 75% and 25% of the data distribution, and the middle line shows the median value. The black circle indicates the average value, and the upper end and the lower end of the whisker indicate the maximum and minimum values. The number of trials for tasks 1–8 are 49, 51, 50, 58, 51, 52, 50, and 57, (total 418 trials) respectively.

Table 2 and Fig. 11 show the mean square error $E = \sum_{t=0.T\text{-}1}(c_{raw}{}^{t} \sum_{t=0\cdot T\text{-}1}(c_{raw}{}^{t} - c_{inf}{}^{t})^{2}/T$ *between the actual data and the inference results.* $E^{1/2}$ *in*

Table 2 means *times* where the inference result is different from the actual selection. That is, Bayesian inference can be estimated within the error range of 0.80–5.52 times, and in the BIB inference it can be estimated within the error range of 0.80–5.65 times (RQ5). In tasks 5–8 where there is forcible selection by software as compared with tasks 1–4, the mean square error *E* becomes large. That is, as the degree of uncertainty of information increases, the estimation error increases for both Bayesian inference and BIB inference (RQ6). The average value of the mean squared error excluding tasks

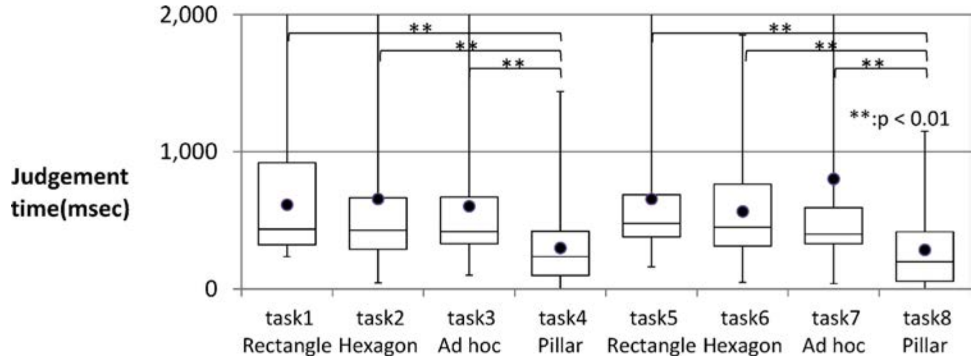


**Fig. 5.** Time taken to select left and right of the subject (millisecond). Time for tasks 4 & 8 ((d) *pillars*) was significantly shorter.

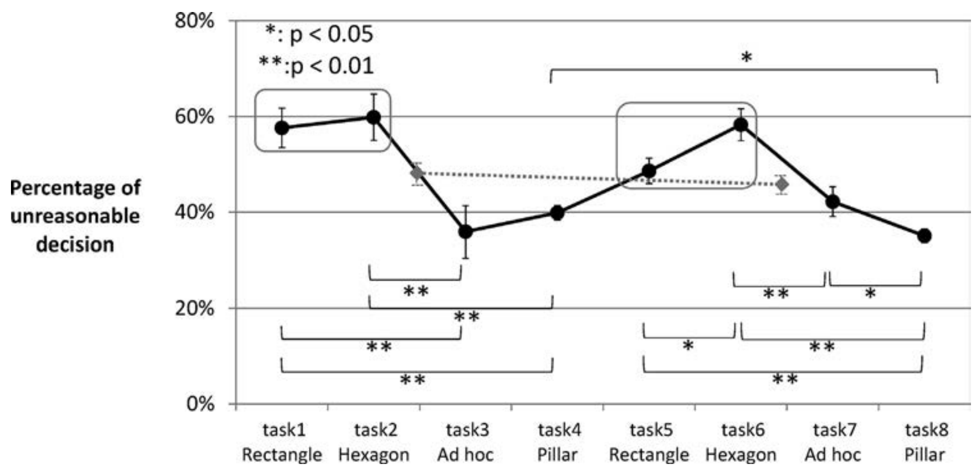


**Fig. 6.** Percentage of unreasonable judgment among subjects' decisions in each test. Error bars indicate standard error. Tasks 1, 2, 5, and 6 ((a) *rectangles* & (b) *hexagons*) are higher than tasks 3, 4, 7, and 8 ((c) *ad hoc* & (d) *pillars*) in terms of unreasonable judgment. The dotted line shows the average value of task 1–4 and task 5–8.

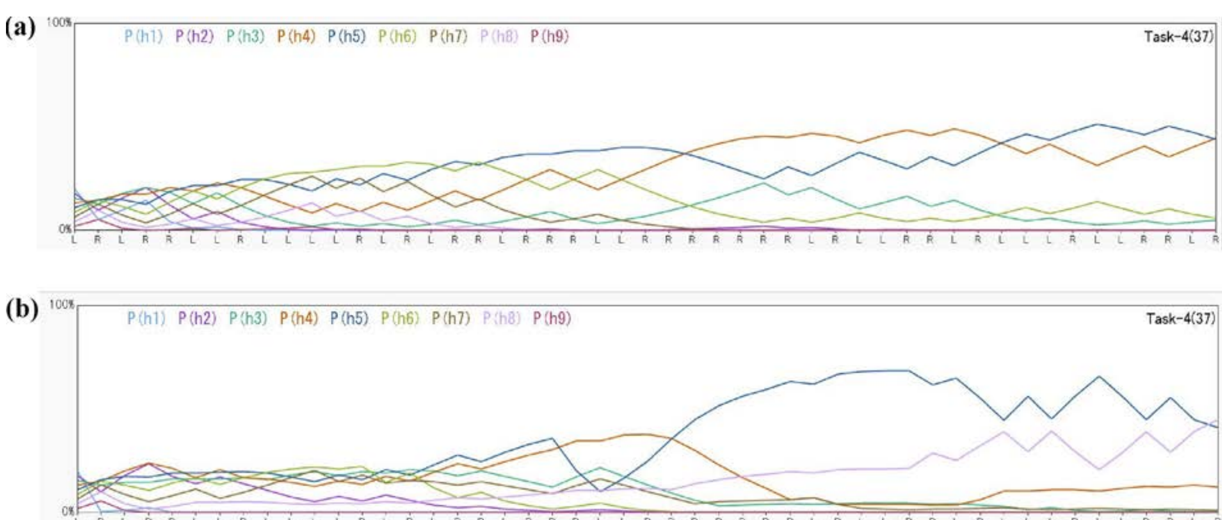


**Fig. 7.** Change in prior probability $P^t(h_i)$ by Bayesian inference (a) and BIB inference (b). In this graph, one of the results of task 4 is used. The moving-average interval used by inverse Bayesian inference was set to 7. (Also in Fig. 8, Fig. 10, and Fig. 13).

1 and 5 and the median of all tasks were smaller in BIB inference than in Bayesian inference. However, these significant differences (T test $p < 0.05$) were not observed except task 8 (RQ7). Fig. 12 shows the difference in error between Bayesian estimation and BIB estimation. There is no significant difference, but as the degree of uncertainty is higher, the result of BIB inference tends to be slightly advantageous (RQ8).

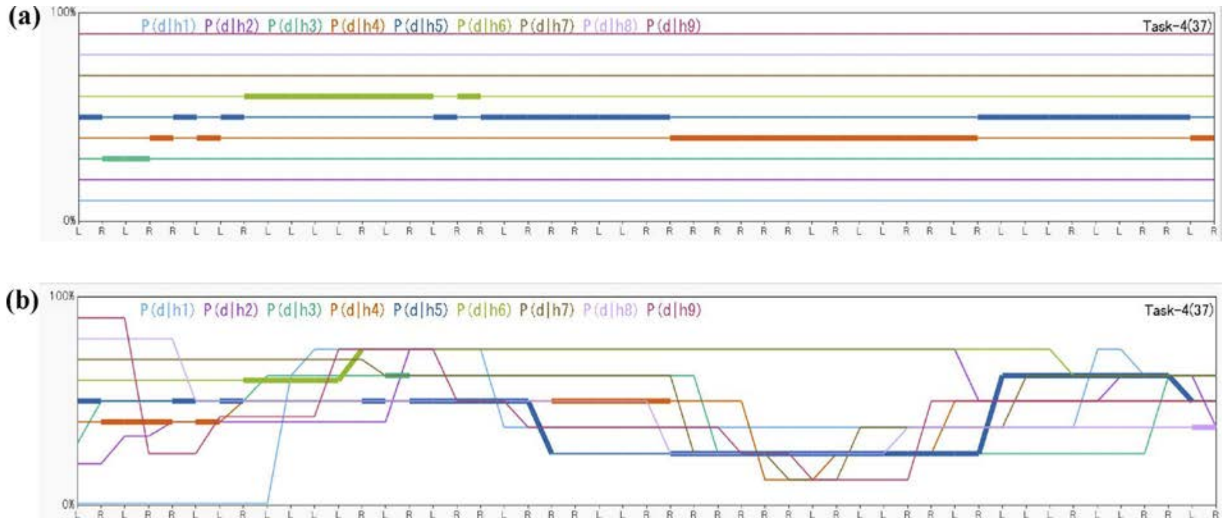


**Fig. 8.** Change in likelihood $P^t(d|h_i)$ of each hypothesis of Bayesian inference (a) and BIB inference (b). The thick segments on these plots indicate the hypothesis having the maximum prior probability $P^t(h_i)$. Likelihood is constant in Bayesian inference, but the likelihood of BIB inference is updated according to the moving average $P^t{}_{mov}(d)$.

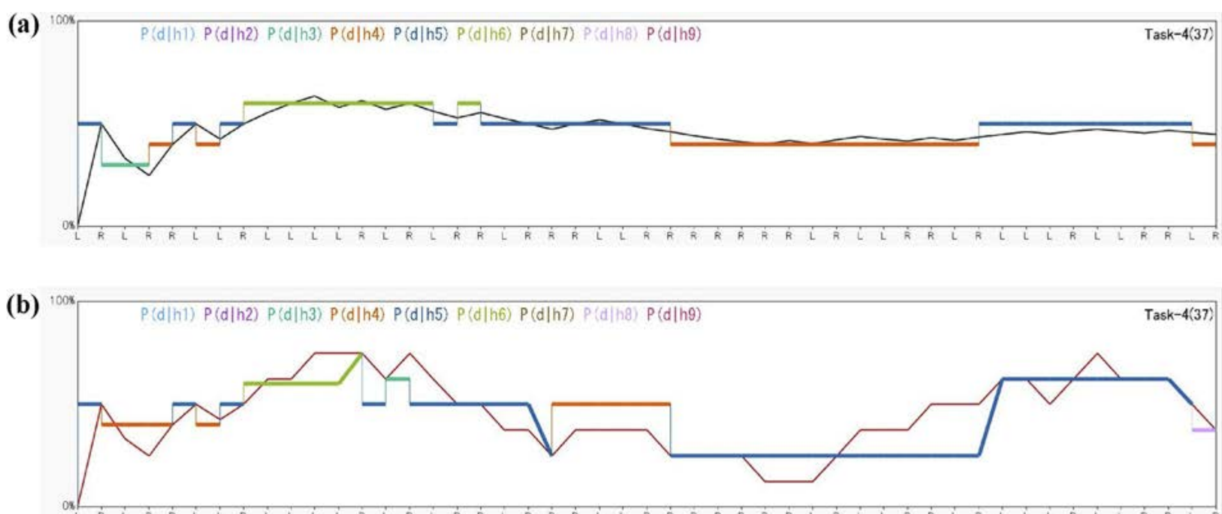


**Fig. 9.** (a) Change in cumulative average from the start position $P^t{}_{ac}(d)$ (black) and likelihood $P^t(d|h_i)$ for maximum prior probability $P^t(h_i)$ by Bayesian inference. (b) Moving average $P^t{}_{mov}(d)$ (red) and likelihood by BIB inference. The likelihood by Bayesian inference change in line with the average from start time while those by BIB inference change in line with the moving average. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

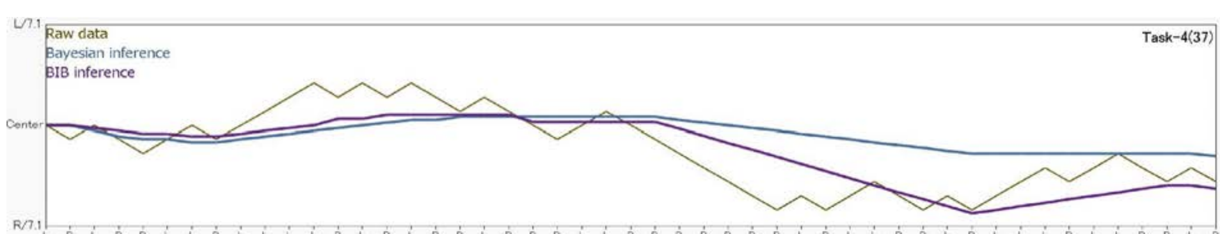


**Fig. 10.** Comparison of actual measurements and inferred values at each step. The yellow plot shows cumulative change of experimental data $c_{raw}{}^t$ and the turquoise and purple plots show cumulative change $c_{inf}{}^t$ of the results obtained by Bayesian and BIB inference, respectively. Bayesian inference cannot keep up with sudden changes, but BIB inference can. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

## 5. Discussion

As shown in Section 4, the goal position, the judgment time, and percentages of the unreasonable decisions did not change depending on the presence or absence of software intervention (RQ4), but a large change was observed in the inference error (RQ6). Also, when there was software intervention, the error of BIB inference was somewhat lower than the Bayesian inference (RQ8). In the event

**Table 2**
Mean square error of inferred values $c_{inf}{}^{t}$ relative to experimental data $c_{raw}{}^{t}$ for each task.

| | task1 | | task2 | | task3 | | task4 | | task5 | | task6 | | task7 | | task8 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Bayes | BIB | Bayes | BIB | Bayes | BIB | Bayes | BIB | Bayes | BIB | Bayes | BIB | Bayes | BIB | Bayes | BIB |
| mean (E) | 0.72 | 0.95 | 0.76 | 0.67 | 0.64 | 0.64 | 8.13 | 6.73 | 30.52 | 31.91 | 23.25 | 18.57 | 19.03 | 15.75 | 18.76 | 14.76 |
| $E^{1/2}$ | 0.85 | 0.98 | 0.87 | 0.82 | 0.80 | 0.80 | 2.85 | 2.60 | 5.52 | 5.65 | 4.82 | 4.31 | 4.36 | 3.97 | 4.33 | 3.84 |
| SD | 0.54 | 2.26 | 0.45 | 0.37 | 0.46 | 0.60 | 6.76 | 4.68 | 22.66 | 35.93 | 25.89 | 25.16 | 20.05 | 15.10 | 18.57 | 14.79 |
| median | 0.54 | 0.52 | 0.57 | 0.55 | 0.53 | 0.51 | 5.64 | 5.54 | 26.96 | 18.42 | 16.51 | 13.10 | 13.51 | 13.34 | 12.07 | 10.67 |
| T test p | 4.4E-01 | | 8.1E-02 | | 9.9E-01 | | 9.7E-02 | | 6.8E-01 | | 1.9E-01 | | 5.9E-02 | | 2.8E-03 | |

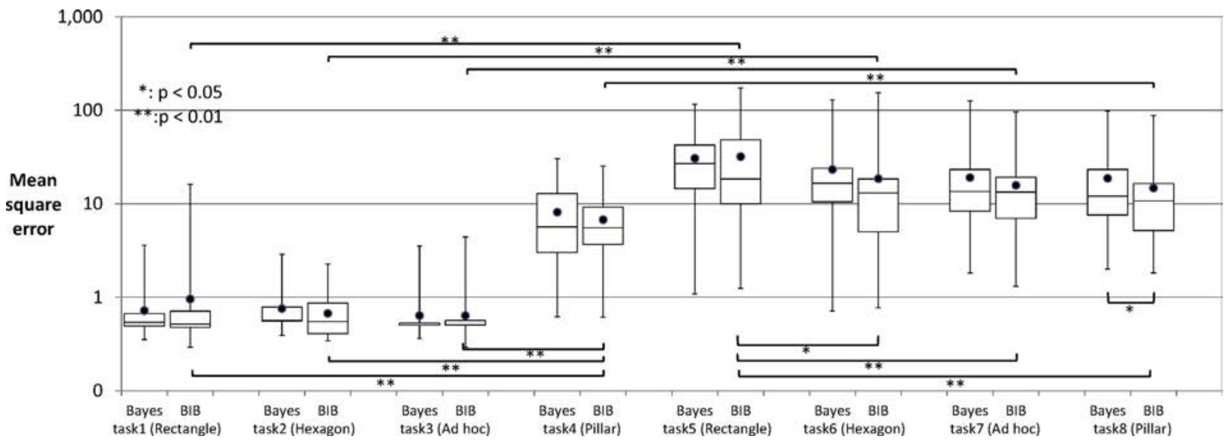


**Fig. 11.** Mean square error of inferred values $c_{inf}{}^{t}$ relative to experimental data $c_{raw}{}^{t}$ for each task.

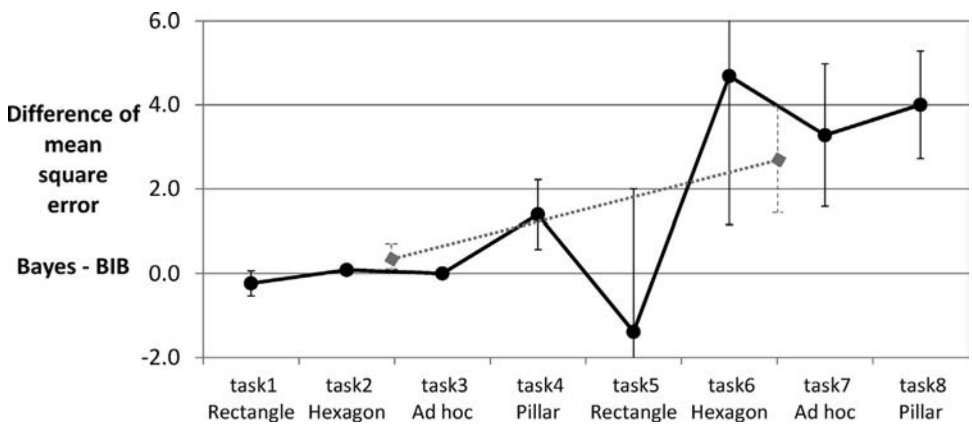


**Fig. 12.** Difference in mean squared error between Bayesian inference and BIB inference. There is no significant difference, but as the degree of uncertainty is higher, the result of BIB inference tends to be slightly advantageous. The dotted line shows the average value of task 1–4 and task 5–8.

that data behavior suddenly changes, a divergence arises between the cumulative average $P^{t}{}_{ac}(d)$ and the moving average $P^{t}{}_{mov}(d)$. For example, when suddenly selecting the same direction in succession in contrast to past behavior as shown by the arrow in Fig. 13(a), a large divergence arises between the cumulative average $P^{t}{}_{ac}(d)$ and the moving average $P^{t}{}_{mov}(d)$ as shown in Fig. 13(b) in black and colored lines, respectively. As shown in Fig. 14, such a divergence increases as the degree of uncertainty increases, and shows the same tendency as the inference error for each task. In Bayesian inference, the prior probability $P^{t}(h_i)$ of the hypothesis with the likelihood $P^{t}(d|h_i)$ near the cumulative average $P^{t}{}_{ac}(d)$ is high, but in BIB inference, the likelihood $P^{t}(d|h_i)$ is updated by the value of moving average $P^{t}{}_{mov}(d)$, and as a result, the prior probability $P^{t}(h_i)$ of that hypothesis becomes high.

The results of an experiment involving movement through the first person's perspective revealed many judgments that could be considered unreasonable when viewed in objective terms. In addition, judgments of this type would suddenly appear. This is thought to be because the subject would suddenly feel that "I went far" and think "I must return." We modeled such a decision-making process by both Bayesian inference and BIB inference and showed that the latter could deal with such sudden changes.

As Gigerenzer (Gigerenzer and Hoffrage, 1995), Knill (Knill, 1998), and Manktelow (Manktelow, 2012) pointed out, the results of this experiment also suggest that human decision making process can be modeled by Bayesian inference. However, as the degree of uncertainty of information increased, accuracy of inference tended to decrease. BIB inference can also be estimated with the same precision as Bayesian inference, and there is a possibility of

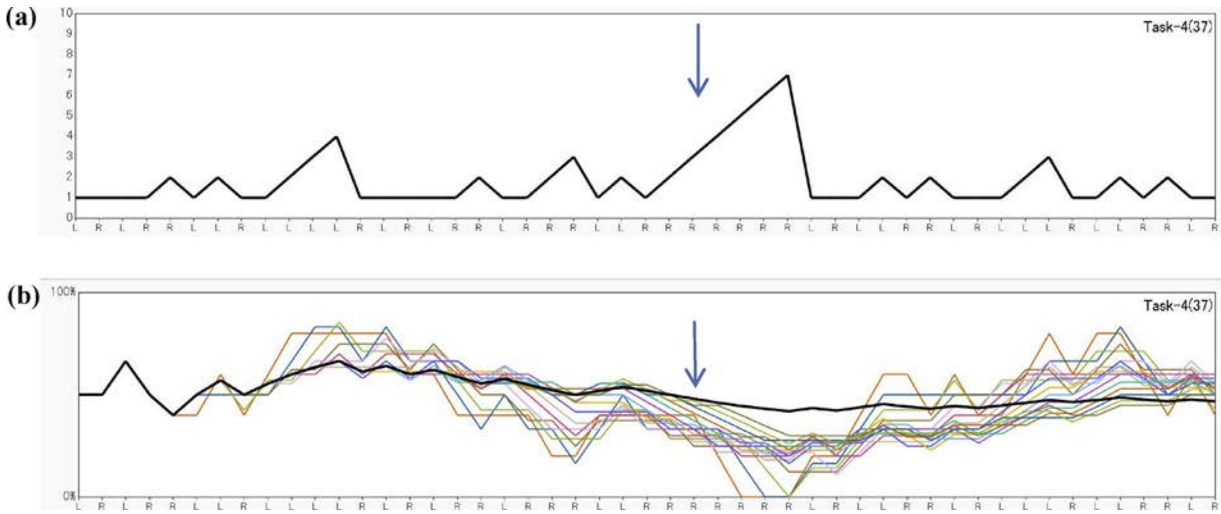


Fig. 13. Change in number of times same direction was consecutively selected (a) and change in cumulative average $P^t{}_{ac}(d)$ (black) and (b) moving average $P^t{}_{mov}(d)$ (colored, interval: 4–20). As indicated by the arrow, consecutive selection of the right direction results in a divergence between cumulative average and moving average. A difference consequently arises between the results of Bayesian inference and BIB inference with the latter performing sensitive tracking of immediately previous changes.

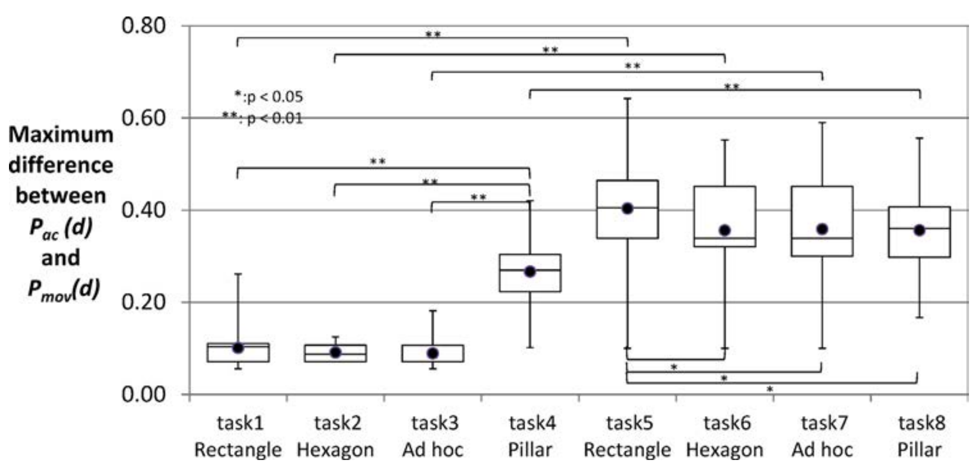


Fig. 14. Maximum difference between the cumulative average $P^t{}_{ac}(d)$ and the moving average $P^t{}_{mov}(d)$.

being able to estimate with higher accuracy in situations where the behavior changes suddenly.

In the BIB inference mentioned in Section 2, moving average was applied to the likelihood in the process of inverse Bayesian inference. In addition to this, it is also possible to use weighted averages, exponential moving averages and others. The interval of moving average corresponds to how much information of the past is taken as an internal state. This value was about 7 s in the experiment described in Section 4 and about one month in the economic indices mentioned in Section 5. We will consider the optimal method to capture from the past information into the internal state.

Apart from this experiment, we consider that BIB inference can be used to model the various types of decision making processes of organisms including humans. Also, BIB inference can easily deal with multiple values and continuous values in addition to binary data as presented here. For extending to the multiple values, let $P^t(d_m|h_i)$ be the likelihood that hypothesis $h_i$ takes values in each interval $m$ ($m = 0, 1, \ldots, M-1$), where $M$ is the total number of discrete values. For Bayesian and inverse Bayesian inference, Eqs. (8) and (9) are calculated, respectively.

$$P^{t+1}(h_i) = P^t(h_i|d_m) = P^t(d_m|h_i)P^t(h_i) / \sum\nolimits_{k=1.N} P^t(d_m|h_k)P^t(h_k) \quad (8)$$

$$P^t(d_m|h_i) = P^t{}_{\mathrm{mov}}(d_m) = \sum\nolimits_{u=t-s.t} P^u(d_m)/(s+1) \quad (9)$$

The inference examples of economic indicators, such as stock price, exchange rate, and oil price are shown in Fig. 15. These indicators represent the *culmination* of many human decision-making, and sometimes they show unexpected behavior. As shown in the figures, when the number of data is large, the model using BIB inference can better represent the actual data.

## 6. Conclusion

In the experiments of moving in a virtual three-dimensional space, many unreasonable judgments were observed. Depending on the three-dimensional model, the goal position, judgment time,

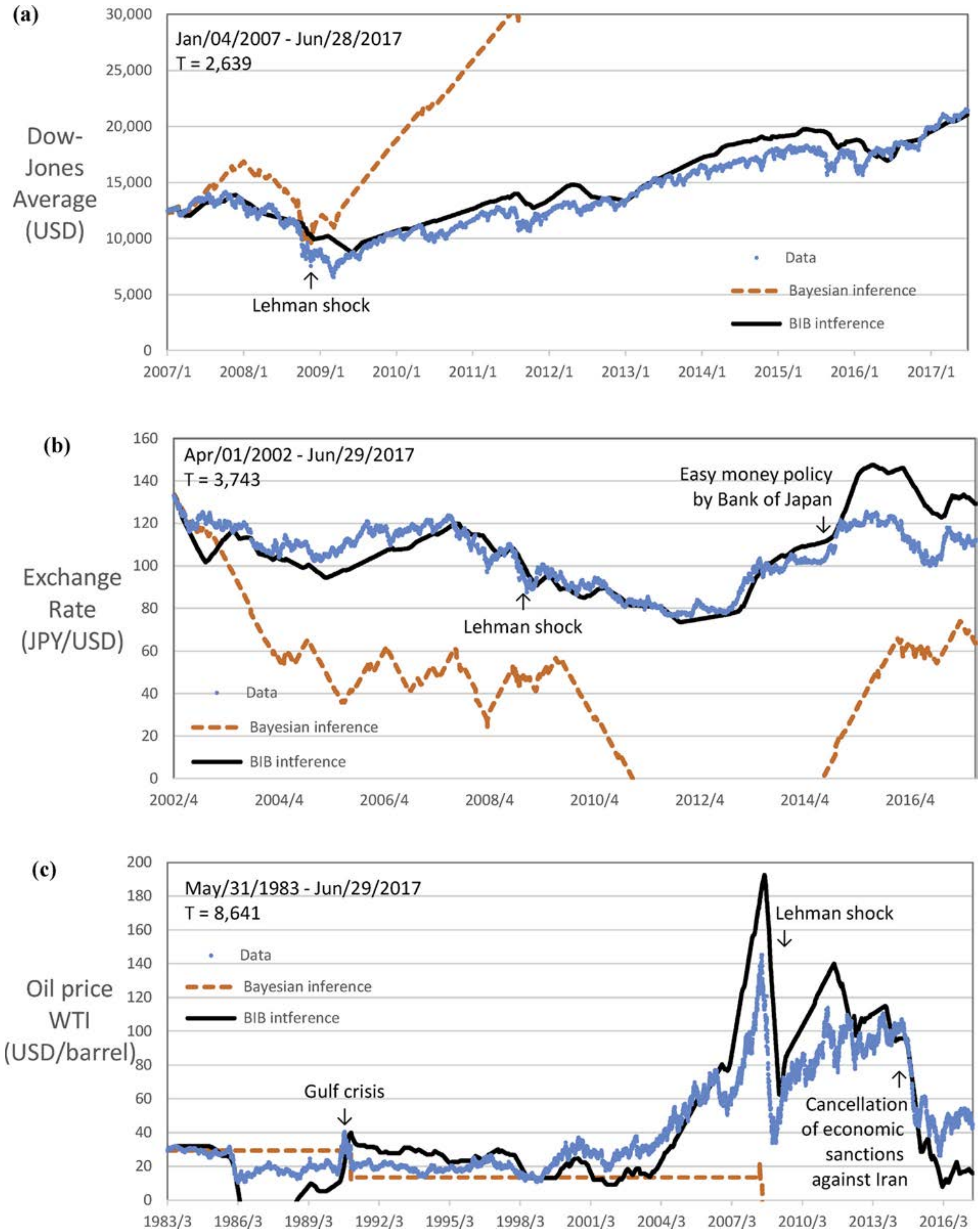


**Fig. 15.** Inference examples of economic indicators. (a) Stock price (Dow-Jones Average, 2017), (b) Exchange rate of Japanese Yen vs US Dollar (Exchange rate, 2017), and (c) Oil price of West Texas Intermediate (Oil price, 2017). The number of hypotheses *N* is 28, the number of sections of values *M* is 28, and the interval of moving average is 32. Even in unconventional situations such as the Lehman shock, BIB inference can follow the actual data.

and unreasonable judgment ratio changed. However, differences due to changes in the degree of uncertainty were not observed. The change in the direction is more influential than the change in the distance, and the angle of turning also affects. As a result of applying the time series data of the judgment of the subjects to the Bayesian and BIB inference, both results are roughly consistent with the actual data. As the degree of uncertainty increases, the inference error also increases. The result of BIB inference tends to match the actual data rather than that of the Bayesian inference. It is thought that the decision-making process in which the strategy suddenly changes can be modeled by BIB inference. In addition, when the number of data is large, the effectiveness of BIB inference may become remarkable. In the future, we are planning experiments to verify whether we can walk straight in real city blocks.

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## Conflict of interest

We have no competing interests.

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YH and PGY designed an experimental setup and the method to analyze data. AY and YN conducted cognitive experiments. YH wrote the main manuscript text and prepared all figures. All authors reviewed the manuscript.

## References

Anderson, J., Sheu, C., 1995. Causal inferences as perceptual judgements. Meomory Cognition 23, 510–524.
Arecchi, F., 2011. Phenomenology of Consciousness: from Apprehension to Judgment. Nonlinear Dynamics. Psychol. Life Sci. 15 (3), 359–375.
Baker, S., Leslie, A., Gallistel, C., Hood, B., 2016. Bayesian change-point analysis reveals developmental change in a classic theory of mind task. Cognit. Psychol. 91, 124–149, http://dx.doi.org/10.1016/j.cogpsych.2016.08.001.
Bonawits, E., Van Schijndel, T., Friel, D., Schulz, L., 2012. Children balance theories and evidence in exploration, explanation and learning. Cognit. Psychol. 64, 215–234.
Bowers, J., Davis, C., 2012. Bayesian just-so stories in psychology and neuroscience. Psychol. Bull. 138 (3), 389–414, http://dx.doi.org/10.1037/a0026450.
Cheng, P., Novick, L., 1992. Covariation in natural causal induction. Psychol. Rev. 99, 365–382.
Dawson, C., Gerken, L., 2011. When global structure "Explains Away" local grammar: a Bayesian account of rule-induction in tone sequences. Cognition 120 (2011), 350–359, http://dx.doi.org/10.1016/j.cognition.2010.11.016.
Dow-Jones Average, 2017. Retrieved from https://jp.investing.com/indices/us-30-historical-data.
Exchange rate, 2017. Retrieved from https://www.mizuhobank.co.jp/rate/market/historical.html.
Gallistel, C., Fairhurst, S., Balsam, P., 2004. The learning curve: implications of a quantitative analysis. Proc. Natl. Acad. Sci. 101, 13124–13131.
Gallistel, C., 2009. The importance of proving the null. Psychol. Rev. 116, 439–453.
Gigerenzer, G., Hoffrage, U., 1995. How to improve Bayesian reasoning without instruction: frequency formats. Psychol. Rev. 102, 684–704.
Goodman, N., Ullman, T., Tenenbaum, J., 2011. Learning a theory of causality. Psychol. Rev. 118, 110–119.
Gopnik, A., Wellman, H., 2012. Reconstructing constructivism: causal models, Bayesian learning mechanisms, and the theory. Psychol. Bull. 138 (6), 1085.
Griffiths, T., Vul, E., Sanborn, A., 2012. Bridging levels of analysis for probabilistic models of cognition. Curr. Directions Psychol. Sci. 21, 263–268.
Gunji, Y.P., Haruna, T., Sawa, K., 2006. Principles of biological organization: local–global negotiation based on material cause. Physica D 219 (2), 152–167, http://dx.doi.org/10.1016/j.physd.2006.06.001, 15 July.
Gunji, P.-Y., Shinohara, S., Haruna, T., Basios, V., 2016a. Inverse Bayesian inference as a key of consciousness featuring a macroscopic quantum logical structure. Biosystems 152, 44–65.
Gunji, P.-Y., Sonoda, K., Basios, V., 2016b. Quantum cognition based on an ambiguous representation derived from a rough set approximation. Biosystems 141, 55–66.
Gunji, Y., Minoura, M., Kojima, K., Horry, Y., 2017. Free will in Bayesian and inverse Bayesian inference-driven endo-consciousness. Prog. Biophys. Mol. Biol., http://dx.doi.org/10.1016/j.pbiomolbio.2017.06.018.
Hattori, M., Oaksford, M., 2007. Adaptive non-interventional heuristics for covariation detection in causal induction: model comparison and rational analysis. Cognit. Sci. 31 (5), 765–814.
Hattori, M., 2003. Adaptive heuristics of covariation detection: a model of causal induction. Proceedings of the 4th International Conference on Cognitive Science and the 7th Australasian Society for Cognitive Science Joint Conference (ICCS/ASCS 2003) Vol. 1, 163–168.
Helmholts, H., 1962. In: Southall, J.P.C. (Ed.), In Helmholtz's Treatise on Physiological Optics. Dover Publications, Translated from the 3d German edn.
Helmholtz, H., 1925. The perceptions of vision. In: Southall, J.P. (Ed.), Physiological Optics, Vol. III (Trans.).
Igamberdiev, A.U., Shklovskiy-Kordi, N.E., 2017. The quantum basis of spatiotemporality in perception and consciousness. Prog. Biophys. Mol. Biol. 130, 15–25, http://dx.doi.org/10.1016/j.pbiomolbio.2017.02.008.
Jenkins, H., Ward, W., 1965. Judgment of contingency between responses and outcomes. Psychol. Monogr. General Appl. 79, 1–17.
Jones, M., Love, B., 2011. Bayesian fundamentalism or enlightenment? On the explanatory status and theoretical contributions of Bayesian models of cognition. Behav. Brain Sci. 34, 169–231, http://dx.doi.org/10.1017/S0140525X10003134.
Knill, D., Pouget, A., 2004. The Bayesian Brain: the Role of Uncertainty in Neural Coding and Computation. Trends in Neurosciences, vol. 27. Psychology Press, Making. London, pp. 712–719.
Knill, D., 1998. Discrimination of planar surface slant from texture: human and ideal observers compared. Vision Res. 38, 1683–1711.
Lee, M., Sarnecka, B., 2011. Number-knower levels in young children: insights from Bayesian modeling. Cognition 120 (2011), 391–402, http://dx.doi.org/10.1016/j.cognition.2010.10.003.
Liberman, E.A., Minina, S.V., Moshkov, D.A., Santalova, I.M., Chistopolskiy, I.A., Shklovski-Kordi, N.E., 2008. Experimental testing of the role of cytoskeleton in the solution by neurons of problems facing the brain. Biochemistry (Moscow) 73 (4), 479–482, http://dx.doi.org/10.1134/S0006297908040147.
Mach, E., 1980. Contributions to the Analysis of the Sensations. Open Court Publishing Co., Chicago, IL USA.
Manktelow, 2012. Thinking and reasoning an introduction to the psychology of reason. Judgment Decis.
Matsuno, K., 2017. From quantum measurement to biology via retrocausality. Prog. Biophys. Mol. Biol., http://dx.doi.org/10.1016/j.pbiomolbio.2017.06.012.
Murray, R., 2003. Saccadic and perceptual. performance in visual search tasks. II. Letter discrimination. J. Opt. Soc. Am. A Opt. Image Sci. Vis. 20, 1356–1370.
Oil price, 2017. Retrieved from https://jp.investing.com/commodities/crude-oil-historical-data.
Papachristos, S., Gallistel, C., 2006. Autoshaped head poking in the mouse: a quantitative analysis of the learning curve. J. Exp. Anal. Behav. 85, 293–308.
Pellicano, E., Burr, D., 2012. When the world becomes 'too real': a Bayesian explanation of autistic perception. Trends Cogn. Sci. 16 (10), 504–510.
Perfors, A., Tenenbaum, J., Griffiths, T., Xu, F., 2011. A tutorial introduction to Bayesian models of cognitive development. Cognition 120 (2011), 302–321, http://dx.doi.org/10.1016/j.cognition.2010.11.015.
Rossler, O.E., 1996. 1996). ultraperspective and endophysics. Biosystems 38 (2–3), 211–219, http://dx.doi.org/10.1016/0303-2647(95)01593-0.
Sasai, K., Gunji, Y.P., 2008. A logic-based dynamical model of abstract biological network derived from time-state-scale re-entrant form. Biosystems 2, 182–188, http://dx.doi.org/10.1016/j.biosystems.2008.02.004.
Saunders, J., Knill, D., 2004. Visual feedback control of hand movements. J. Neurosci. 24, 3223–3234.
Sawa, K., Gunji, Y.P., 2014. An emergence of formal logic induced by an internal agent. Biosystems 124, 39–45, http://dx.doi.org/10.1016/j.biosystems.2014.08.005.
Sawa, K., Igamberdiev, A.U., 2016. The Double Homunculus model of self-reflective systems. Biosystems 144, 1–7, http://dx.doi.org/10.1016/j.biosystems.2016.03.007.
Sawa, K., Igamberdiev, A.U., 2017. The dynamics of interaction of reflexive subjects operating with the two-valued versus many-valued logic. Prog. Biophys. Mol. Biol., http://dx.doi.org/10.1016/j.pbiomolbio.2017.08.007.
Shimozaki, S., 2003. An ideal observer with channels versus feature-independent processing of spatial frequency and orientation in visual search performance. J. Opt. Soc. Am. A Opt. Image Sci. Vis. 20, 2197–2215.
Wasserman, E., Dorner, W., Kao, S., 1990. Contribution of specific cell information to judgments of interevent contingency. J. Exp. Psychol. Learn. Memory Cognition 16 (3), 509–521.