



# Competition and cooperation between supply chains in multi-objective petroleum green supply chain: A game theoretic approach



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## ABSTRACT

Petroleum Supply Chain is one of the most important and sophisticated managing missions in both developing and developed countries. Nowadays, environmental pollution is another critical factor in designing the petroleum supply chain. This importance encourages the governments to minimize the amount of environmental pollution and maximize their obtained profit simultaneously, by enacting required legislations on the transportation modes and the refineries. Considering maximizing the job creation and each stakeholder's profit, and minimizing the emission of CO<sub>2</sub> and other greenhouse gases at the same time is called Sustainable Petroleum Supply Chain which has been paid little attention despite its significance. Therefore, the modelling of petroleum supply chain considering sustainability and pricing issues is investigated for the first time in this work and a sustainable competitive petroleum supply chain (SCPSC) model is developed to minimize pollution while maximizing the profits and job creation. This problem is a two level model. The first level in SCPSC is the competition between the supply chains of the government and the private sectors, which is modelled by the game theory approach including Nash and Stackelberg equilibria. The optimal price and demand for each supply chain determined in the first level are considered as the second level parameters. In the second level, the optimal values of the decisions in designing the petroleum supply chain will be obtained by solving Mixed Integer Linear Programming (MILP) under the mentioned three objective functions. Finally, the proposed model is applied to a real world case in the national Iranian oil company (NIOC). Based on the results of the Stackelberg equilibrium, the government profits increase by 11.12% while that of the private sector decreases by 25.4 and 28.11%. Increasing in the government profit is due to increased demand provided by government. The results show that the whole profit of the petroleum supply chain in Nash equilibrium is 9.8% more than that in the Stackelberg equilibrium.

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## 1. Introduction

Although, sustainable supply chain is currently of interest to many researchers, the sustainability has not been paid much attention in Petroleum Supply Chain (PSC). According to the Carter and Rogers' sustainability definition "the strategic, transparent integration and achievement of an organization's social, environmental, and economic goals is the systematic coordination of key inter-organizational business processes for improving the long term economic performance of the individual company and its

supply chain" (Carter and Rogers, 2008). Considering the following aims at the same time, sustainability is an inalienable part of the petroleum supply chain:

- Many environmental regulations have been enacted by the governments to reduce the environmental and pollution effects.
- Maximizing the profit of whole chain such as the other supply chains.
- Most of the governments enforce the stakeholders by enacting appropriate regulations to increase the created job in the chain.

Beside of the sustainability, the competition is another important factor in the PSC. Because, different parts of a petroleum supply chain are controlled by different stakeholders and stakeholders'

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attempt to maximize their profits emerges competition. This can be defined as a complete competition which is not enough to analyze the developing countries PSC, and we need to take into account the other ongoing factors.

In the developing countries, PSC is controlled by governments, which spend millions of dollars on refined petroleum products such as gasoline, diesel, etc. In these countries, large volume of subsidies causes artificial low prices and the economic and financial problems, this in turn causes fading out of optimization of economic consumption and market competitions (Cheon et al., 2013). Considering the economic and financial problems, countries take corrective measures such as gradual reduction of subsidies to ultimately complete elimination and inclusion of private sector in the petroleum supply chain. Therefore, like in the developed countries, there are various stakeholders in the petroleum supply chain and the main challenge is between the supply chains of the government and private sector to determine their price and demand balance. Hence, three other factors that play important role in optimizing of the PSC are: i) rate of the reduction in subsidies in different periods. ii) Government's subsidized price. iii) Government's unsubsidized price.

As mentioned before, the subsidies will be gradually diminished in each year with respect to government planning. Thus, the prices (subsidized and unsubsidized) are adjusted by the government and the private sectors for each of the products as well as their demand based on the amounts of subsidies i.e. determined and paid by the government every year. In other words, the government and private supply chains compete to determine the adjusted prices and demands which is referred as a competitive chain. This competition can be non-cooperative. Despite the significance of pricing in PSC, the pricing of petroleum products by considering competition between private sectors and government chains has hardly been dealt with in the literature. Just Moradi Nasab et al. (2016) proposes an integrated economic model (IEM) of fossil fuel energy planning for government and private sectors. In their study, there is competition between refineries within the refinery level, between distribution centers (DCs) within the distribution center level, and between refineries and DCs. Under these conditions, part of the refineries and DCs are under the control of government and the remainder are controlled by the private sectors. whereas, the non-cooperative competition between the private sectors and government chains have been considered in this study.

In addition to sustainability and competition, network design decisions including capacity expansion of facilities and pipelines routes, transportation modes, inventory and assignment for each sector, including private sectors chains and government chains are considered in PSC to achieve the global optimum solution (Shah et al., 2010; Hasani et al., 2013).

According to the literature, limited number of studies have been conducted in PSC, which are classified in Table 1. Each of these papers has focused on solving a sub-problem of the PSC network. However, local improvement at any sub-level does not necessarily lead to overall improvement (Shah et al., 2010).

As observed in the Table 1, most researches have focused on the midstream and downstream supply chains and only a few have dealt with all three levels of downstream, midstream and upstream. In addition, in spite of the importance of different stakeholder and competitions in PSC, the presence of different stakeholders in PSC has only been investigated by Fernandes et al. (2013). Although the competition and the presence of deferent stakeholders in the midstream petroleum supply chain including refineries which is important, only the competition in the downstream petroleum supply chain has been considered in Fernandes et al. (2013). Moreover, none of the works have considered pricing in PSC design (Table 1).

Regarding the decision making levels (Strategical, Tactical, Operational), only the tactical level in the supply chain was carried out in most researches and only a few works have simultaneously dealt with strategic and tactical levels. In the case of petroleum supply chain decisions, most works have covered inventory, procurement of raw materials and production rates by considering only one objective function, which is usually profit or costs. As it mentioned before, sustainability is an important aspect in PSC, but none of the studies have regarded it along with PSC network design.

According to Table 1, the only paper which investigates the environmental concerns in its constraints is (Ribas, 2011). Their concern is the amount of pollutants produced that should not exceed a minimum limit. Only (Khosrojerdi et al., 2012) has considered the two objective functions of profit and the level of customer service, simultaneously.

Altogether, none of these researches have studied the sustainability, competition, and PSC network design. Therefore, the necessity of proposing a multi-objective, multi-level, multi-stakeholder and multi-stage sustainable competitive petroleum supply chain (SCPSC) by considering the decisions in strategic and tactical levels is completely clear.

In this SCPSC, the network supply chain design (in strategic and tactical levels), pricing and demand decisions should be made at the same time. This SCPSC includes the government and private sector supply chains competing with each other to determine their petroleum price and demand. This competition is non-cooperative and one of the approaches for modeling such competition is the application of game theory. This study investigates the chain competition and sustainability in the supply chain simultaneously for the first time. Despite the importance of chains competition, none of the researches have studied it in the design phase of PSC. According to the literature, in the supply chains other than petroleum, only a few researches have studied the chain to chain competition.

McGuire and Staelin (1983) studied the effect of product substitutability on Nash Equilibrium distribution structures between two suppliers each selling through an independent retailer. Their results show that product distribution through a company store is preferable for low degrees of substitutability and the decentralized distribution system is preferable for a more highly substitutable one. Higher profits can be caused by supply chain decentralization, as explained by Moorthy (1988). This is linked to the concept of strategic interaction. Wu and Chen (2003) studied the quantity of competition between two chains in which a single manufacturer and two retailers facing a newsvendor demand are included. Baron et al. (2008) extended the seminal work of McGuire and Staelin (1983) and investigated the Nash Equilibrium of an industry with two supply chains. In their study, each chain includes a single manufacturer and a single retailer modeled by a Nash Bargaining on the Wholesale (BW) price. Their model is extended by Wu et al. (2009), who have considered demand uncertainty. Their results show that decentralization on the wholesale price may be Nash Equilibrium over infinitely many periods and integration in both chains is the unique Nash Equilibrium over one period decision. Other studies, which have considered chain to chain competition include Ai et al. (2012), Wu and Chen (2003), Nagurney et al. (2002), Rezapour et al. (2011a,b), Rezapour and Farahani (2010), Anderson and Bao (2010), Zhang (2006), Xiao and Yang (2008), Boyaci and Gallego (2004).

Sustainable supply chain is another aspect of SCPSC, and some works that have studied it are discussed briefly in the following. In this group, some researches have investigated the environmental dimension and greenhouse gas emission including Ubeda et al. (2011), Pan (2010), Wang et al. (2011) and Venkat (2007). Also, a

**Table 1**  
Characterizations of the supply chain.

Author (year)	Sector			Decision level			Supply chain decisions										
	Upstream	Midstream	Downstream	Strategic	Tactical	Operational	Location allocation	Capacity expansion	Capacity reduction	Inventory	Raw material Preparation	Production	Routing	Transportation modes	Multi-Stakeholders	Pricing	Multi-Objective
							Facility	Route	Facility								
Sear (1993)	✓	✓					✓						✓	✓			
Escudero et al. (1999)	✓	✓		✓							✓						
Dempster et al. (2000)	✓	✓		✓													
Pinto et al. (2000)	✓					✓				✓		✓					
Neiro and Pinto (2004)	✓	✓	✓	✓	✓					✓	✓	✓		✓			
Mendez et al. (2006)	✓					✓				✓		✓					
Pongsakdi et al. (2006)	✓	✓		✓						✓		✓					
Al-othman et al. (2008)	✓	✓	✓	✓						✓	✓	✓					
Kuo and Chang (2008)	✓	✓	✓			✓				✓	✓	✓					
Pitty et al. (2008)	✓	✓	✓	✓	✓					✓	✓	✓					
Kim et al. (2008)	✓	✓	✓	✓	✓		✓							✓			
Al-Qahtani and Elkamel (2008)	✓	✓		✓				✓			✓		✓				
Mirhassani (2008)		✓		✓						✓		✓					
Guyonnet et al. (2009)	✓	✓	✓							✓		✓					
Rocha and Grossmann, 2009	✓	✓				✓				✓		✓					
Al-Qahtani and Elkamel (2010)	✓	✓		✓							✓						
Ribas et al. (2010)	✓	✓		✓							✓						
Gill (2011)		✓		✓	✓		✓										
Khosrojerdi et al. (2012)	✓	✓		✓	✓		✓			✓		✓		✓			✓
Chen et al. (2012)	✓			✓						✓							
Fernandes et al. (2013)		✓		✓			✓	✓						✓		✓	
Guajardo et al. (2013)	✓	✓		✓						✓		✓					
Ghezavati et al. (2015)	✓	✓		✓			✓	✓				✓		✓			

number of researchers such as McGuire, Tao et al. (2010) and Bonney and Jaber (2011) have considered both dimensions of the classical economic order quantity (EOQ (and carbon emission simultaneously). Nevertheless, less attention has been paid to other dimensions including pollution and resource use in the literature (Luo et al. (2001) and Dotoli et al. (2006)). Furthermore, only few models considering other social criteria such as maximization of local employment (You et al. (2011)), maximization of the distribution of jobs to be created and maximization of supply chain corporate responsibility (Perez-Fortes et al. (2012)) have been identified. Du et al. (2011) studied the impact of emission 'cap-and-trade' mechanism in a two-echelon supply chain with the emission permit supplier and the emission dependent firm. They proposed game theory analytical model and achieved optimal decisions on permit pricing and production quantity by unique Nash Equilibrium. Swami and Shah (2013) considered the problem of coordination of a manufacturer and a retailer in which both supply chain members put in efforts for 'greening' their operations. They found that the ratio of the optimal greening efforts for the manufacturer and retailer is dependent on the ratio of their green sensitivity ratios and greening cost ratios. Zhang and Liu (2013) investigated the three level green supply chain system in which market demand correlates with product green degree. They used game theory to study four models including cooperative decision making, three level leader follower game, Stackelberg game I and Stackelberg game II. Their results showed that the supply chain system and participating members have the optimal level of profit under cooperative decision making. Dong et al. (2014) examined the order quantity of the retailer and sustainability investment of the manufacturer for the decentralized supply chain and determined the production quantity and sustainability investment for the centralized supply chain. Amin and Zhang (2014) studied a closed-loop supply chain network including multiple products, plants, recovery technologies, demand markets, and collection centers and proposed a mixed integer linear programming model. Boukherroub et al. (2015) considered all three aspects of sustainability (economic, environmental and social) simultaneously. They transposed the sustainable development principles to supply chain planning models by proposing an integrated approach. Their model is applied to a Canadian lumber industry case and is solved by using the weighted goal programming technique. Li and Li (2014) examined two sustainable supply chains under competition in product sustainability by the game model. They presented the equilibrium structures of the two chain system and generated the managerial insights. The more recent literature review of this research stream is presented in Alzaman (2014).

Literature survey shows that the most of researches in the chain to chain competition and sustainable supply chains are associated with the fields other than PSC. It is notable that, the investigation of the presence of different stakeholders and sustainability along with petroleum supply chain decisions have not been studied in any of the references.

In this study, a SCPSC model has been developed, which simultaneously optimizes the supply chain network design, price and demand of each of the products for government and private sectors, while considering the maximization of profits and employment, and the minimization of pollution. The supply chain network design of this model includes installation and capacity expansion of facilities and pipelines, transportation modes, inventory and assignment. The major contributions of this paper can be summarized as follows:

- 1 Several supply chains consisting of government SC and private sectors SCs in SCPSC have been considered, which have non-cooperative game characteristics to determine their refined

product prices. The government has two prices for refined products including subsidized and unsubsidized prices.

- 2 For the first time, all the three aspects of sustainability including economic, social and environmental have been considered in PSC as a multi-objective model and fuzzy theory used to solve it.
- 3 According to the literature, there are two approaches for increasing the capacity of facilities and pipeline routes, namely new facility installation and capacity expansion. These two approaches for facilities and pipeline routes have been not considered at the same time. In addition to determining the level of DC capacity, the number of tanks and their capacity allocation for each of the products must be determined.
- 4 Addressing the supply chain decisions consists of installation and capacity expansion of facilities and pipelines, transportation modes, inventory and assignment simultaneously. However, no studies have been performed on the supply chain competition in PSC by considering private sectors and subsidies paid by government.
- 5-In this proposed model, one DC may be under the ownership of more than one supply chains. In other words, one DC may be used by several supply chains. This leads to two features including the efficient utilization of the DC capacity and the reduction in costs.

The remaining contents of the paper are organized as follows. The problem description is briefly introduced in Section 2, followed by the assumptions of problem representation in Section 3. Section 4 presents the PSC network model, which is then followed by competition of supply chains in PSC in section 5. The fuzzy set theory is presented in section 6. The case study and the numerical study is analyzed in Section 7. Finally, Section 8 presents the summary of the research and recommendations for further works.

## 2. Problem description

Downstream SCPSC is divided into three major levels of refineries, DCs and customer zones, which are depicted in Fig. 1 (Moradi Nasab et al., 2016). In this SCPSC, consider the government SC and private sectors SCs, which compete with each other. In Fig. 1, three SC's are assumed including those of government and two private sectors competing with each other.

In this SCPSC, various decisions consisting of installation and capacity expansion of facilities and pipelines, transportation modes, inventory and assignments are considered by each supply chain (private and government sectors). These decisions must be optimal with respect to three objective functions including economic, social and environmental. Each of the supply chains (government and private sectors) transports the refined products such as gasoline or petrol, kerosene, jet fuel, and diesel oil from refineries to DC by pipelines and sells them based on their prices. Here, it is assumed that one DC may be under the ownership of more than one supply chain. For example DC 2 is under the ownership of government and private sector 1. The refined products can be transported to consumers through logistic network by road, water and rail. The government will release the refinery products with two unsubsidized and subsidized prices according to the amount of subsidies. The competition is one between the public and private sector supply chains. Only government prices (subsidized and unsubsidized) and private sector prices have been considered here as effective parameters on determining the demand.

Here, the linear demand function is used in which the demand for government and each of private sectors are given by Equations (1)–(3). Many economic and SCM studies (such as Wu (2013), Ai et al. (2012) and Anderson and Bao (2010)) have used linear price

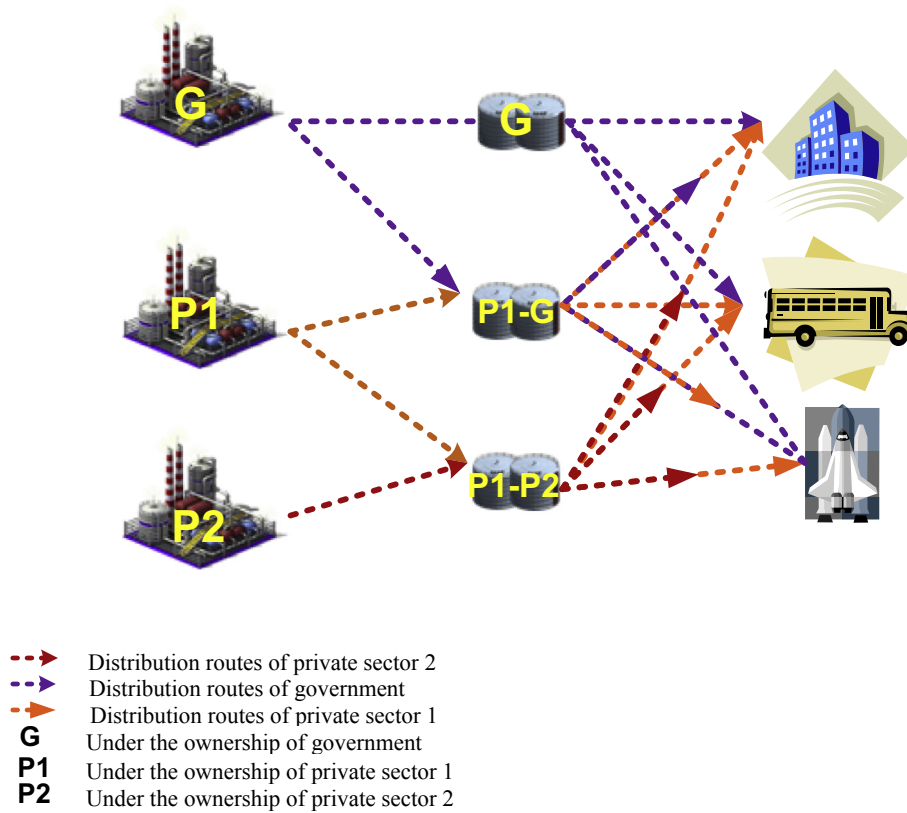


Fig. 1. Sustainable competitive petroleum supply chain (SCPSC).

dependent function (Azari Khojasteh et al. (2013)).

Based on Equations (1) and (2), the amount of the demand with unsubsidized and subsidized prices is a linear function of the subsidized price, unsubsidized price and the prices of the private sectors. Definitions of parameters, sets, and variables used in the proposed model are provided in Appendix A.1.

$$d_1^p = D_p - \alpha_1^p pr_1^p + \beta_1^p pr_1'^p + \sum_{e \in E_{ps}} \beta_2^e pr_2^e \quad \forall p \in P \quad (1)$$

$$d_1'^p = D_p - \alpha_1'^p pr_1'^p + \beta_1'^p pr_1^p + \sum_{e \in E_{ps}} \beta_2^e pr_2^e \quad \forall p \in P \quad (2)$$

Furthermore, the amount of each private sector demand is a linear function of its price, other private sector prices, subsidized and unsubsidized ones, as shown in Equation (3).

$$d_p^e = D_p - \alpha_2^e pr_2^e + \beta_1^p pr_1^p + \beta_1'^p pr_1'^p + \sum_{e', e' \neq e} \beta_2^{e'} pr_2^{e'} \quad \forall p \in P, e \in E_{ps} \quad (3)$$

Therefore, the problem is a SCPSC with multi-objective function, multi-echelon and multi-levels in which the amount of demands depends on competitive factors (prices of government and private sectors). Here, there is a two stage decision making problem, which is shown in Fig. 2.

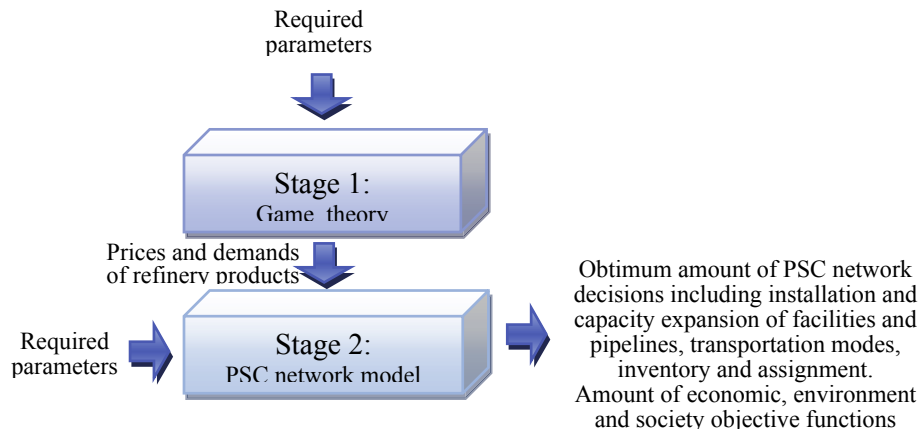


Fig. 2. Structure of the proposed model.



In the first stage, the prices and the demands of the government and private sectors will be determined by competition between them. The variables for government and two private sectors are shown in Fig. 3.

In the second stage, the value of the supply chain decisions will be obtained by solving PSC network design. These two stages of decision making are described in subsequent sections in detail.

### 3. Assumptions

Assumptions considered in the formulation of the problem are as follows:

1. As previously stated, the petroleum supply chain includes three sectors: upstream, midstream, and downstream. But the competition is considered in two sectors midstream and downstream. Being strategic, the government controls the upstream by itself and there is no competition in this sector. Therefore, there will be three levels in two mentioned sectors of proposed SCPSC model: refineries, DCs and customer zones.
2. For simplicity, it is assumed that there is no lost sale and associated cost in this model. Therefore, all the deterministic customer zone demands must be satisfied.
3. The transportation modes in the proposed model are pipelines, railway and roads. Based on PSC system in Iran, the stream between refineries and DCs is transported only by pipelines and the stream between DCs and customer zones is transported by other transportation modes; namely railway and roads.
4. For simplicity and according to the reality, the locations of existing refineries, DCs and pipeline routes are determined and may not be changed.
5. All the costs including the fixed and variable costs, installation and expansion capacity costs are known and constant per each facility during the modeling. In addition, all the possible

capacity and capacity expansion levels for the set of the candidate and existing refineries, DCs and pipeline routes are known.

6. Each facility holds the amount of inventory as safety stock and pays a fixed cost for holding inventory at each facility.

### 4. PSC network design

In this section, a PSC network design is presented for the sustainable multi-objective, multi-echelon and multi-product PSC problem as the second stage of decision making, which is a MILP model.

#### 4.1. Objective function

The model is formulated as a multi-objective linear programming model having three objectives simultaneously. The first objective of the model aims to maximizing the total profit while the second objective minimizes the created pollution by facilities and transportation modes. The third objective deals with maximization of the number of jobs created. The objectives are formulated as follows:

- maximization of the total profit

$$\text{Maximize } P_e = R_e - C_e \quad \forall e \in E \tag{4}$$

$$R_e (\forall e \in E_{ps}) = \sum_{p \in P} \sum_{v \in V} \sum_{l \in (L_e \cup L'_e)} \sum_{m \in M} q_{1m}^{pv} pr_2^{ep} \tag{5}$$

$$R_e (\forall e \in E_c) = pr_1^p SRPP\_Gp + pr_1^p NSRPP\_Gp \tag{6}$$

$$\begin{aligned} C_e (\forall e \in E) = & \sum_{e \in E} \sum_{k \in K'_e} \sum_{ek \in EK} x \cos t_k^{ek} x_k^{ek} e + \sum_{l \in L'_e} \sum_{el \in EL} x \cos t_l^{el} x_l^{el} e + \sum_{k \in K_e} \sum_{uk \in UK} u \cos t_k^{uk} \tau_k^{uk} e \\ & + \sum_{l \in L_e} \sum_{ul \in UL} \sum_{p \in P} u \cos t_l^{ul} p \tau_l^{ul} p e + \sum_{k \in (K_e \cup K'_e)} \sum_{l \in (L_e \cup L'_e)} \sum_{lv \in LV} \sum_{rv \in RV} r \cos t_{k1}^{lv} r v \tau_{k1}^{lv} r v e \\ & + \sum_{ev \in EV} \sum_{k \in K_e} \sum_{l \in L_e} y \cos t_k^{ev} y_k^{ev} l + \sum_{l \in L_e} \sum_{p \in P} F \cos t_l \left( ic_l^p + \sum_{ul \in UL} c_l^{ul} \tau_l^{ul} p e \right) \\ & + \sum_{k \in K_e} F \cos t_k \left( ic_k + \sum_{uk \in UK} c_k^{uk} \tau_k^{uk} e \right) + \sum_{k \in K'_e} \sum_{ek \in EK} F \cos t_k x_k^{ek} c_k^{ek} + \sum_{l \in L'_e} \sum_{ez \in EZ} \sum_{p \in P} F \cos t_l c_l^{ez} n_l^{p} e z e \\ & + \sum_{p \in P} \sum_{l \in (L_e \cup L'_e, e \in E_{ps})} pr_2^p h \cos t_l^p v_l^p e + \frac{1}{2} \sum_{p \in P} \sum_{l \in (L_e \cup L'_e, e \in E_c)} (pr_1^p + pr_1^p) h \cos t_l^p v_l^p e \\ & + \sum_{p \in P} \sum_{k \in (K_e \cup K'_e)} \sum_{l \in (L_e \cup L'_e)} q_{k1}^p q \cos t_{k1} + \sum_{l \in (L_e \cup L'_e)} \sum_{m \in M} \sum_{lc \in LCV} \sum_{v \in V} n \cos t_{1m}^{lc} v n_{1m}^{lc} v p e \\ & + \sum_{ez \in EZ} \sum_{l \in L_e} \sum_{p \in P} n \cos t_l^{ez} n_l^{p} e z e + \sum_{k \in (K_e \cup K'_e)} \sum_{k \in (L_e \cup L'_e)} \sum_{p \in P} q_{k1}^p p \cos t_k + \sum_{p \in P} \sum_{k \in (K_e \cup K'_e)} \sum_{k \in (L_e \cup L'_e)} q_{k1}^p p \cos t_l^p \\ & + \sum_{lev \in LEV} WCost_{lev} \left[ \sum_{k \in K'_e} \sum_{en \in EN} HE\_NK_{en}^{en' lev} + \sum_{k \in K_e} \sum_{en \in EN} \sum_{en' \in EN} HE\_EK_{en}^{en' lev} + \right. \\ & \left. \sum_{l \in L'} \sum_{en \in EN} \sum_{en' \in EN} HE\_N_{en}^{en' lev} + \sum_{l \in L_e} \sum_{en \in EN} \sum_{en' \in EN} HE\_EK_{en}^{en' lev} \right] \end{aligned} \tag{7}$$

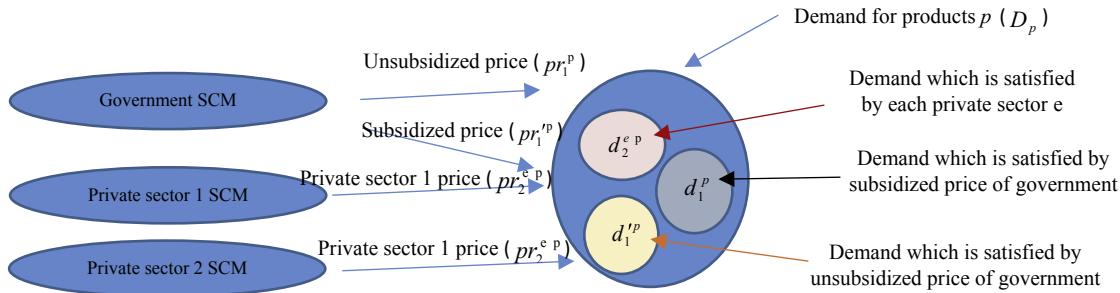


Fig. 3. The variables in stage 1 of proposed model.

Equation (4) presents the profit obtained by the difference between the revenue and costs. The revenues of each private sector and the government are calculated by Equations (5) and (6), respectively. In Equation (6), the income of public sector is from the sales by subsidized and unsubsidized prices shown by the first and second parts. The first and second terms in equation (7) show the installation costs of new refineries and DCs. The expansion costs of existing refineries and DCs are calculated by the third and fourth terms in equation (7). The installation costs of new and existing pipelines between refineries and DCs are obtained by the fifth and sixth terms in equation (7). The annualized costs, which consist of existing and new refineries and DCs are obtained using the seventh to tenth terms in equation (7). Terms 11 and 12 in equation (7) indicate the inventory costs of the existing and new refineries. The refined products are transposed between refineries and DCs by pipelines. The cost of these flow rates is achieved by term 13 in equation (7). In addition, the transportation of the refined products between DCs and customer zones is carried out by modes other than pipeline in this equation. This is obtained by term 14. The installation cost of storage tanks in DCs is calculated by term 15 in equation (7). The operating costs, which consist of existing and new refineries and DCs, are obtained using the terms 16–17 in equation (7). Finally, in equation (7), the hiring cost of refineries and DC employees are obtained by terms 18–21.

• **Minimization of created pollution by facilities and transportation modes**

$$\begin{aligned}
 \text{Minimize Pul}_e = & \sum_{k \in K'_e} \sum_{en \in EN} \sum_{ek \in EK} \lambda_{E_{en}} Nk_{en}^k c_k^{ek} x_k^{ek} e \text{ Pul}_k + \sum_{ez \in EZ} \\
 & \times \sum_{l \in L'} \sum_{p \in P} \sum_{en \in EN} \lambda_{E_{en}} Nl_{en}^l c_l^{ez} n_l^{p \text{ ez}} e \text{ Pul}_l + \sum_{k \in K_e} \\
 & \times \sum_{en \in EN} \sum_{uk \in UK} \lambda_{E_{en}} Nk_{en}^k c_k^{uk} \tau_k^{uk} e \text{ Pul}_k \text{ Per} \\
 & + \sum_{l \in L_e} \sum_{ul \in UL} \sum_{p \in P} \lambda_{E_{en}} Nl_{en}^l c_l^{ul} \tau_l^{ul} p e \text{ Pul}_l \text{ Per} \\
 & + \sum_{l \in (L_e \cup L')} \sum_{m \in M} \sum_{lcv \in LCV} \\
 & \times \sum_{v \in V} \text{pul}_{lcv}^v n_{lm}^{lcv} p e \text{ dis}_m^l \quad \forall e \in E
 \end{aligned} \tag{8}$$

The first and second terms of equation (8) show the amount of the pollution created by the candidate refineries and DCs, respectively.  $\lambda_{E_{en}}$  Indicates the sensitivity of a region to pollution.

The objective for the introduction of this parameter is the division of the range of planning to several areas based on the amount of pollution and the significance of its increase and allocation of  $\lambda_{E_{en}}$  to each area. Therefore,  $\lambda_{E_{en}}$  is a continuous

parameter smaller than 1. Higher and closer to one values of this parameter indicate the importance of pollution content in that area and increased pollution due to the installation or capacity expansion. Parameters  $Nk_{en}^k$ ,  $Nl_{en}^l$ ,  $Nek_{en}^k$  and  $Nel_{en}^l$  show which area the refineries and the new and existing DCs are located in.

The pollution caused by capacity expansion of refineries and DCs is calculated by the third and fourth terms. Finally, the pollution generated by transportation modes is obtained by the last term. The amount of pollution created is equal to the product of the distance travelled times the number of vehicles per unit distance.

• **Maximization of the number of jobs created**

$$\begin{aligned}
 \text{Maximize } S_e = & \sum_{lev \in LEV} \sum_{k \in K'_e} \sum_{en \in EN} \sum_{en' \in EN} Nk_{en}^k W_{en}^{en'} HE_{en}^{en'} lev \\
 & + \sum_{lev \in LEV} \sum_{k \in K_e} \sum_{en \in EN} \\
 & \times \sum_{en' \in EN} Nek_{en}^k W_{en}^{en'} HE_{en}^{en'} lev \sum_{lev \in LEV} \sum_{l \in L'} \sum_{en \in EN} \\
 & \times \sum_{en' \in EN} Nl_{en}^l W_{en}^{en'} HE_{en}^{en'} lev + \sum_{lev \in LEV} \sum_{l \in L_e} \sum_{en \in EN} \\
 & \times \sum_{en' \in EN} Nel_{en}^l W_{en}^{en'} HE_{en}^{en'} lev \quad \forall e \in E
 \end{aligned} \tag{9}$$

Equation (9) represents the number of jobs created by candidate refineries and DC installations obtained by the first two terms and the number of jobs created by the capacity expansion of refineries and DCs calculated by the last two terms. The work force is assumed to be relocated from other areas to the sites where new installations are made or the existing installations are expanded.  $W_{en}^{en'}$  shows the relocation of the work force from area  $en'$  to area  $en$ .

4.2. Model constraints

The model considers various constraints as follows:

$$\sum_{el \in EL} x_l^{el} e \leq 1 \quad \forall e \in E, l \in L'_e \tag{10}$$

$$\sum_{ek \in EK} x_k^{ek} e \leq 1 \quad \forall k \in K'_e, e \in E \tag{11}$$

$$z_1^{ez} e \leq 1 \quad \forall l \in L'_e, e \in E, ez \in EZ \tag{12}$$

$$M \times z_1^{ez} e \geq n_l^{p \text{ ez}} e \quad \forall l \in L'_e, p \in P, ez \in EZ, e \in E \tag{13}$$

$$\sum_{e \in E_i} \sum_{ul \in UL} \tau_1^{ulpe} \leq 1 \quad \forall l \in L_e, p \in P \tag{14}$$

$$\sum_{uk \in UK} \tau_k^{uke} \leq 1 \quad \forall k \in K_e, e \in E \tag{15}$$

$$\sum_{ev \in EV} y_{k1}^{ev} \leq 1 \quad \forall k \in K_e, l \in L_e, e \in E \tag{16}$$

$$\sum_{p \in P} \sum_{e \in E_i} \sum_{k \in (K_e \cup K'_e)} q_{k1}^p \leq ic_k + \sum_{uk \in UK} c_k^{uk} \tau_k^{uke} \quad \forall k \in K_e, e \in E_i \tag{17}$$

$$\sum_{e \in E_i} \sum_{k \in (K_e \cup K'_e)} q_{k1}^p + \sum_{e \in E_i} v_{01}^p \leq ic_1^p + \sum_{e \in E_i} \times \sum_{ul \in UL} c_1^{ul} \tau_1^{ulpe} \quad \forall l \in L_e, p \in P, e \in E_i \tag{18}$$

$$\sum_{ek \in EK} M_k^{ek} c_k^{ek} x_k^{ek} \leq \sum_{p \in P} \sum_{l \in (L_e \cup L'_e)} q_{k1}^p \leq \sum_{ek \in EK} c_k^{ek} x_k^{ek} \quad \forall k \in K'_e, e \in E \tag{19}$$

$$\sum_{ez \in EZ} M_1^{p,ez} c_1^{ez} n_1^{p,ez} \leq \sum_{k \in (K_e \cup K'_e)} q_{k1}^p \leq \sum_{ez \in EZ} c_1^{ez} n_1^{p,ez} \quad \forall l \in L'_e, e \in E, p \in P \tag{20}$$

$$\sum_{ez \in EZ} n_1^{p,ez} \leq M \sum_{el \in EL} x_1^{el} \quad \forall l \in L'_e, e \in E, p \in P \tag{21}$$

$$\sum_{p \in P} q_{k1}^p \leq \sum_{lv \in LV} \sum_{rv \in RV} cl_{lv} r_{k1}^{lv,rv} + \left( ic_{k1} + \sum_{ev \in EV} c_{k1}^{ev} y_{k1}^{ev} \right) \quad \forall e \in E, k \in K_e, l \in L_e \tag{22}$$

$$\sum_{lv \in LV} \sum_{rv \in RV} r_{k1}^{lv,rv} + R_{k1} \leq 1 \quad \forall e \in E, k \in K_e, l \in L_e \tag{23}$$

$$\sum_{ev \in EV} y_{k1}^{ev} \leq R_{k1} \quad \forall e \in E, k \in K_e, l \in L_e \tag{24}$$

$$\sum_{p \in P} q_{k1}^p \leq \sum_{lv \in LV} \sum_{rv \in RV} cl_{lv} r_{k1}^{lv,rv} \quad e \in E, k \in (K_e \cup K'_e), l \in L_e \text{ or } k \in K_e, l \in (L_e \cup L'_e) \tag{25}$$

$$\sum_{lv \in LV} \sum_{rv \in RV} r_{k1}^{lv,rv} \leq 1 \quad \forall k \in (K_e \cup K'_e), l \in (L_e \cup L'_e), p \in P, e \in E \tag{26}$$

$$q_{1m}^{pve} \leq \sum_{lcv \in LCV} tr_{cv}^{lcv} n_{1m}^{lcvpe} TPP \quad \forall l \in (L \cup L'), m \in M, p \in P, v \in V, e \in E \tag{27}$$

$$\sum_{e \in E} \sum_{l \in (L_e \cup L'_e)} \sum_{m \in M} \sum_{p \in P} n_{1m}^{lcvpe} \leq n \cdot \max_v^{lcv} \quad \forall lcv \in LCV, v \in V \tag{28}$$

$$\sum_{ez \in EZ} \sum_{p \in P} c_1^{ez} n_1^{p,ez} \leq \sum_{el \in EL} c_1^{el} x_1^{el} \quad \forall l \in L'_e, e \in E \tag{29}$$

$$\sum_{lv \in LV} \sum_{rv \in RV} r_{k1}^{lv,rv} \leq \sum_{ek \in EK} x_k^{ek} \quad \forall k \in K'_e, l \in (L_e \cup L'_e), e \in E \tag{30}$$

$$\sum_{lv \in LV} \sum_{rv \in RV} r_{k1}^{lv,rv} \leq \sum_{el \in EL} x_1^{el} \quad \forall k \in (K_e \cup K'_e), l \in L'_e, e \in E \tag{31}$$

$$\sum_{e \in E} \sum_{v \in V} \sum_{l \in (L_e \cup L'_e)} q_{1m}^{pve} = D_{p,m} \quad \forall m \in M, p \in P \tag{32}$$

$$\sum_{l \in (L_e \cup L'_e)} \sum_{m \in M} \sum_{v \in V} q_{1m}^{pve} = d_p^e \quad \forall e \in E, p \in P \tag{33}$$

$$\sum_{k \in (K_e \cup K'_e)} q_{k1}^p + v_{01}^p = \sum_{v \in V} \sum_{m \in M} q_{1m}^{pve} + v_1^p \quad \forall l \in (L'_e \cup L_e), p \in P, e \in E \tag{34}$$

$$\sum_{e \in E_i} v_1^p \leq ic_1^p + \sum_{e \in E_i} \sum_{ul \in UL} c_1^{ul} \tau_1^{ulpe} \quad \forall l \in L_e, p \in P, e \in E \tag{35}$$

$$v_1^p \leq \sum_{ez \in EZ} c_1^{ez} n_1^{p,ez} \quad \forall l \in L'_e, p \in P, e \in E \tag{36}$$

$$\sum_{e \in E_i} v_1^p \geq l_1 \left( ic_1^p + \sum_{e \in E_i} \sum_{ul \in UL} c_1^{ul} \tau_1^{ulpe} \right) \quad \forall l \in L_e, p \in P, e \in E \tag{37}$$

$$v_1^p \geq l_1 \left( \sum_{ez \in EZ} c_1^{ez} n_1^{p,ez} \right) \quad \forall l \in L'_e, p \in P, e \in E \tag{38}$$

$$\sum_{v \in V} \sum_{l \in (L_e \cup L'_e)} \sum_{m \in M} q_{1m}^{pve} = SRR_{Gp} + NSRR_{Gp} \quad e \in E_G, p \in P \tag{39}$$

• Employment constraints

$$\sum_{k \in K'_e} \sum_{en' \in EN} HE_{en'k}^{en'lev} + \sum_{k \in K_e} \sum_{en' \in EN} HE_{en'k}^{en'lev} + \sum_{l \in L'_e} \times \sum_{en' \in EN} HE_{en'l}^{en'lev} + \sum_{l \in L_e} \sum_{en' \in EN} HE_{en'l}^{en'lev} \leq Lab_{en}^{lev} \quad \forall en \in EN, lev \in LEV \tag{40}$$



$$\begin{aligned} & \sum_{ek \in EK} c_k^{ek} x_k^{ek} e W_{NK_{lev}} \\ & = \sum_{en \in EN} \sum_{en' \in EN} N_{en}^k HE_{en'k}^{en' lev} \quad \forall k \in K_e', e \in E, lev \in LEV \end{aligned} \quad (41)$$

$$\begin{aligned} & \sum_{uk \in UK} c_k^{uk} \tau_k^{uk} e W_{EK_{lev}} \\ & = \sum_{en \in EN} \sum_{en' \in EN} N_{en}^k HE_{EK_{en'k}}^{en' lev} \quad \forall k \in K_e, e \in E, lev \in LEV \end{aligned} \quad (42)$$

$$\begin{aligned} & \sum_{el \in EL} c_l^{el} x_l^{el} e W_{NL_{lev}} \\ & = \sum_{en \in EN} \sum_{en' \in EN} N_{en}^l W_{en'}^{en' lev} HE_{en'l}^{en' lev} \quad \forall l \in L_e', e \in E, lev \in LEV \end{aligned} \quad (43)$$

$$\begin{aligned} & \sum_{p \in P} \sum_{ul \in UL} c_l^{ul} \tau_l^{ul} p e W_{EK_{lev}} \\ & = \sum_{en \in EN} \sum_{en' \in EN} N_{en}^l HE_{EK_{en'l}}^{en' lev} \quad \forall l \in L_e', e \in E, lev \in LEV \end{aligned} \quad (44)$$

#### • Coverage constraint

$$\begin{aligned} & \sum_{ek \in EK} \sum_{e \in E} \sum_{k \in K_e'} x_k^{ek} e N_{en}^k + \sum_{uk \in UK} \sum_{e \in E} \sum_{k \in K_e} \tau_k^{uk} e N_{en}^k + \sum_{el \in EL} \sum_{e \in E} \\ & \times \sum_{l \in L_e'} x_l^{el} e N_{en}^l + \sum_{ul \in UL} \sum_{e \in E} \sum_{l \in L_e'} \tau_l^{ul} p e N_{en}^l NR_{en} + ND_{en} \\ & \leq Max\_num_{en} \quad \forall en \in EN \end{aligned} \quad (45)$$

The model selects at most one facility from the set of refineries and DCs to locate at sites  $k$  and  $l$ , respectively. This is shown in inequalities 10–11. The number of storage tanks for each product ( $p$ ) in each DC is determined only once in the model, which is satisfied by inequalities 12–13. Constraints 14–15 impose only one capacity expansion in capacity levels selected in the existing refineries and DCs, respectively. For each of the existing pipeline routes between refineries and DCs, only one expansion policy exists, as shown by constraint 16. The maximum capacities of existing refineries and DCs are limited by Equations (17) and (18).

Based on Equations (17) and (18), the amount of output stream from the existing DCs and refineries must be lower than that of the initial capacity plus the sum of capacity expansions carried out. According to Equations (19) and (20), the flow rate between refineries and DCs must be in the required range, minimum and maximum capacities. Therefore, if the flow rate from refineries and to DCs is at least greater than the minimum flow rate, the new refineries and DCs can be installed.

In addition, based on these two equations, the amount of output stream from the DCs and refineries must be lower than that of their capacity. If and only if a new DC is installed, the refined products are stored in it. This is satisfied by Equation (21). In other words, the number of storage tanks for each product in a new facility is determined when the new DC is installed.

The transportation of refined products is done between refineries and DCs by pipelines. Constraint 22 determines the flow

rates of the refined products between existing refineries and DCs. According to Equation (22), the stream of each specified refined product must be less than the sum of the pipeline capacity and all its capacity expansions in the existing pipelines and less than the capacity of the new pipeline. Constraint 23 shows that the new pipeline route may be built if and only if the pipeline route does not exist between a given existing refinery and DC. The expansion capacity is done in existing pipelines with respect to Equation (24). In the same way, Equations (25) and (26) show the constraints set for the refined product stream between two new refineries and DCs as well as two new and existing facilities. According to this equation, the stream via the pipeline exists when the pipeline is installed. Equation (26) also shows that the installation of pipelines between two facilities is carried out only once. The refined products are transported between DCs and customer zones by road and rail. The constraints in Equation (27) reflect the capacity of transportation modes except pipeline routes.

The amount of transportation stream via modes other than pipelines between two facilities should be smaller than the number of vehicles multiplied by their capacity.

The maximum number of the available vehicles is determined by Equation (28). In Equation (29), the sum of the capacities of the storage tanks in each DC must be less than its DC capacity.

Constraints 30–31 require that the new pipeline routes be built if and only if the new refineries and DCs are built. According to constraint 32, the customer zone demand must be satisfied. The amount of the given refinery product transported by each stakeholder to DCs is equal to the demand planned to satisfy by this stakeholder, based on Equation (33). Equation (34) shows that the sum of the input flow rates for each facility must be equal to the sum of the output, which flows from it. The inventory levels in each facility must be limited to its capacity, which is depicted by Equations (35) and (36).

It is assumed that in each of the new or existing facilities, some of the inventory is saved as confidence inventory, which is a percentage of the facility capacity.

Equations (37) and (38) show that the minimum inventory levels in each facility must be more than the minimum amount of inventory. Equation (39) shows that the amount of the product transported from DCs to customer zones by the government is divided into two groups including the products with subsidized and unsubsidized prices. The total number of employees in refineries and DCs must be less than the available laborers satisfied by constraint 40.

According to Equation (41)–(44), the number of the required laborers in each refinery and DC must be equal to the laborers hired for them. Based on Equation (41), the number of laborers employed in a new refinery ( $HE_{en'k}^{en' lev}$ ) is equal to the capacity of the new refinery ( $c_k^{ek}$ ) multiplied by the number of required work force ( $W_{NK_{lev}}$ ) per capacity unit.  $x_k^{ek}$  indicates installation or lack of it and is equal to zero when the installation does not take place. Equations (42)–(44) are also similar.

Equation (45) is called coverage equation. According to the coverage equation, the sum of the installation and capacity expansion and the existing facilities in each region must be less than the determined maximum number. The objective of this equation is uniform distribution of existing and new facilities in all areas.

## 5. Competition of supply chains in SCPSC

In this SCPSC, the government and private sector SCs have non-cooperative competition. As stated, the public and private sectors

compete with each other. Here, this competition has been studied by considering two different defaults. The petroleum supply chain consists of the three upstream, midstream and downstream sections. Competition exists only in downstream and midstream sections. In other words, the government does not attempt to privatize the upstream section due to its being strategic. Therefore, the upstream section is completely government controlled. The complete control of the upstream section by the government makes this section stronger and causes it to play a leadership role in the chain. Since private sector bases its decisions on those of the government, it has a follower role in the chain. Therefore, Stackelberg Equilibrium has been investigated by considering the leader and follower roles.

In Stackelberg Equilibrium, which was introduced by Stackelberg (1934), the leader player (government) chooses a strategy first and then the Stackelberg follower player (private sector) observes this decision and makes his own strategy choice.

On the other hand, this competition will also be investigated when the government does not play the role of leader player and both public and private sectors make their decisions simultaneously, which is referred to as Nash Equilibrium.

In Nash Equilibrium, the players (government and private sectors) choose strategies simultaneously; i.e., these are simultaneous move, one-shot games.

This game looks for a rational prediction of how the game will be played in practice.

Here the players (government and private sectors) determine their prices and competitive characteristics by one of the methods including Nash and Stackelberg Equilibria. This competition is formulated as the first stage of decision making, which is described in details in the next sections.

5.1. Nash competition of supply chains in SCPSC

The objective functions and the constraints of each supply chains are as follows:

• The Government SC:

Max

$$\pi_1(pr_1^p, pr_1^p) = \sum_{p \in P} pr_1^p d_1^p + \sum_{p \in P} pr_1^p d_1^p + C \max_{p \in P} [\tau_p(d_1^p + d_1^p)] \tag{46}$$

s.t.

$$d_1^p = D_p - \alpha_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e \in E_{PS}} \beta_2^e pr_2^e \quad \forall p \in P \tag{47}$$

$$d_1^p = D_p - \alpha_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e \in E_{PS}} \beta_2^e pr_2^e \quad \forall p \in P \tag{48}$$

$$(pr_1^p - pr_1^p)Z D_p \leq B_p \quad \forall p \in P \tag{49}$$

$$\sum_{e \in E_{PS}} d_2^e + (d_1^p + d_1^p) = D_p \quad \forall p \in P \tag{50}$$

• The Private sectors SC:

Max

$$\pi_2(pr_2^e) = \sum_p pr_2^e d_2^e - C \max_{p \in P} \left[ \frac{1}{\tau_p} d_2^e \right] \quad \forall p \in P, e \in E_{PS} \tag{51}$$

s.t.

$$d_2^e = D_p - \alpha_2^e pr_2^e + \beta_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e', e' \neq e} \beta_2^{e'} pr_2^{e'} \quad \forall p \in P, e \in E_{PS} \tag{52}$$

$$\sum_{e \in E_{PS}} d_2^e + (d_1^p + d_1^p) = D_p \quad \forall p \in P \tag{53}$$

According to government objective function, which is shown in relationship 46, the cost value is obtained based on the amount of the petroleum refined in the refinery. Relationship  $\left(\frac{1}{\tau_p}\right)$  shows the required amount of petroleum for each unit of the product. Assuming there are two products and 5 and 10 petroleum units are required to produce products one and two respectively, these two products can be produced if the maximum amount of the petroleum is available (10 units). Based on this explanation,  $\max_{p \in P} \left[\frac{1}{\tau_p} d_2^e\right]$  shows the required amount of the petroleum for the assumed products. Based on the demand (constraints 1 and 2), the profit function  $(\pi_1(pr_1^p, pr_1^p))$  can be formulated as:

$$\begin{aligned} \text{Max } \pi_1(pr_1^p, pr_1^p) &= \sum_p pr_1^p \left[ D_p - \alpha_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e \in E_{PS}} \beta_2^e pr_2^e \right] \\ &+ \sum_p pr_1^p \left[ D_p - \alpha_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e \in E_{PS}} \beta_2^e pr_2^e \right] \\ &- C \max_{p \in P} \left[ \frac{1}{\tau_p} \left( 2D_p - \alpha_1^p pr_1^p + \beta_1^p pr_1^p + 2 \right. \right. \\ &\left. \left. \times \sum_{e \in E_{PS}} \beta_2^e pr_2^e - \alpha_1^p pr_1^p + \beta_1^p pr_1^p \right) \right] \quad \forall p \in P \end{aligned} \tag{54}$$

Similarly, the profit function  $(\pi_2(pr_2^e))$  can be rewritten based on private sector demand (constraint 52) as follows:

$$\begin{aligned} \text{Max } \pi_2(pr_2^e) &= \sum_p pr_2^e \left[ D_p - \alpha_2^e pr_2^e + \beta_1^p pr_1^p + \beta_1^p pr_1^p \right. \\ &+ \left. \sum_{e', e' \neq e} \beta_2^{e'} pr_2^{e'} \right] - C \max_{p \in P} \left[ \frac{1}{\tau_p} \left( D_p \right. \right. \\ &- \left. \alpha_2^e pr_2^e + \beta_1^p pr_1^p + \beta_1^p pr_1^p \right. \\ &\left. \left. + \sum_{e', e' \neq e} \beta_2^{e'} pr_2^{e'} \right) \right] \quad \forall p \in P, e \in E_{PS} \end{aligned} \tag{55}$$

As observed, the variables  $pr_1^p, pr_1^p$  and  $pr_2^e$  are continuous. Therefore, the Kuhn-Tucker condition can be used to calculate the optimal solutions. Concavity is necessary if the Kuhn-Tucker condition is used to calculate the optimal solution. Therefore, it can be shown that  $\pi_1(pr_1^p, pr_1^p)$  and  $\pi_2(pr_2^e)$  are the jointly concave functions of  $pr_1^p, pr_1^p$  and  $pr_2^e$ , respectively.

**Proposition 1.** The function  $\pi_1(pr_1^p, pr_1^p)$  is concave of  $pr_1^p$  and  $pr_1^p$  when  $4\alpha_1^p\alpha_1^p \geq (\beta_1^p - \beta_1^p)^2$ .

**Proof.** The first order partial derivative of  $\pi_1(pr_1^p, pr_1^p)$  to  $pr_1^p$  and  $pr_1^p$  are:

$$\frac{\partial \pi_1}{\partial pr_1^p} = D_p - 2pr_1^p\alpha_1^p + \beta_1^p pr_1^p + \sum_{e \in E_{rs}} \beta_2^e pr_2^e + \beta_1^p pr_1^p + C \times \max_{p \in P} \left[ \frac{1}{\tau_p} (\alpha_1^p - \beta_1^p) \right] \tag{56}$$

$$\frac{\partial \pi_1}{\partial pr_1^p} = pr_1^p\beta_1^p + D_p + \beta_1^p pr_1^p - 2\alpha_1^p pr_1^p + \sum_{e \in E_{rs}} \beta_2^e pr_2^e + C \times \max_{p \in P} \left[ \frac{1}{\tau_p} \alpha_1^p \right] - C \max_{p \in P} \left[ \frac{1}{\tau_p} \beta_1^p \right] \tag{57}$$

The second order partial derivatives of  $pr_1^p$  and  $pr_1^p$  are:

$$\frac{\partial^2 \pi_1}{\partial pr_1^p \partial pr_1^p} = -2\alpha_1^p \quad \frac{\partial^2 \pi_1}{\partial pr_1^p \partial pr_1^p} = -2\alpha_1^p \tag{58}$$

$$\frac{\partial^2 \pi_1}{\partial pr_1^p \partial pr_1^p} = (\beta_1^p + \beta_1^p) \quad \frac{\partial^2 \pi_1}{\partial pr_1^p \partial pr_1^p} = (\beta_1^p + \beta_1^p)$$

The Hessian matrix is:  $H = \begin{bmatrix} -2\alpha_1^p & (\beta_1^p + \beta_1^p) \\ (\beta_1^p + \beta_1^p) & -2\alpha_1^p \end{bmatrix}$

To have a concave function the Hessian should be negative definite,  $\det(H) = [4\alpha_1^p\alpha_1^p - (\beta_1^p + \beta_1^p)^2] \geq 0$  which yields to  $4\alpha_1^p\alpha_1^p \geq (\beta_1^p + \beta_1^p)^2$ .

**Proposition 2.**  $\pi_2(pr_2^e, pr_2^e)$  is a concave function of  $pr_2^e$ .

**Proof.** The first order partial derivative of  $\pi_2(pr_2^e, pr_2^e)$  to  $pr_2^e$  is:

$$\frac{\partial \pi_2}{\partial pr_2^e} = D_p - 2pr_2^e\alpha_2^e + \beta_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e', e'' \neq e} \beta_2^{e'} pr_2^{e'} + C \max_{p \in P} \left[ \frac{1}{\tau_p} \alpha_2^e \right] \tag{59}$$

The second order partial derivatives of  $pr_2^e$ :

$$\frac{\partial^2 \pi_2}{\partial pr_2^e \partial pr_2^e} = -2pr_2^e\alpha_2^e \tag{60}$$

Since the  $\frac{\partial^2 \pi_2}{\partial pr_2^e \partial pr_2^e} \leq 0$ , then  $\pi_2(pr_2^e, pr_2^e)$  is a concave function.

With respect to concavity of  $\pi_1(pr_1^p, pr_1^p)$ , the Kuhn-Tucker condition can be used to calculate the optimal solution. Let  $\delta_p$  and  $\mu_p$  be the Lagrange multipliers for constraints 49 and 50, the Lagrange function  $L_{\pi_1}(pr_1^p, pr_1^p)$  can then be expressed as follows.

$$L_{\pi_1}(pr_1^p, pr_1^p) = \sum_p pr_1^p d_1^p + \sum_p pr_1^p d_1^p - C \max_{p \in P} \left[ \frac{1}{\tau_p} (d_1^p + d_1^p) \right] - \sum_p \delta_p [(pr_1^p - pr_1^p)Z_p D_p - B_p] - \sum_p \mu_p \left[ \sum_{e \in E_{rs}} d_2^e + (d_1^p + d_1^p) - D_p \right] \tag{61}$$

Then the Kuhn-Tucker condition for  $\pi_1(pr_1^p, pr_1^p)$  is:

$$\frac{\partial L_{\pi_1}}{\partial pr_1^p} = D_p - 2pr_1^p\alpha_1^p + \beta_1^p pr_1^p + \sum_{e \in E_{rs}} \beta_2^e pr_2^e + \beta_1^p pr_1^p + C \times \max_{p \in P} \left[ \frac{1}{\tau_p} (\alpha_1^p \lambda^p) \right] - \delta_p Z D_p - \mu_p I \beta_1^p + \mu_p \alpha_1^p - C \times \max_{p \in P} \left[ \frac{1}{\tau_p} (\beta_1^p) \right] - \mu_p \beta_1^p = 0$$

$$\frac{\partial L_{\pi_1}}{\partial pr_1^p} = pr_1^p\beta_1^p + D_p + \beta_1^p pr_1^p - 2\alpha_1^p pr_1^p + \sum_{e \in E_{rs}} \beta_2^e pr_2^e + C \times \max_{p \in P} \left[ \frac{1}{\tau_p} \alpha_1^p \right] + \delta_p Z D_p - \mu_p I \beta_1^p + \mu_p \alpha_1^p - \mu_p \beta_1^p - C \times \max_{p \in P} \left[ \frac{1}{\tau_p} \beta_1^p \right] = 0$$

$$\delta_p [(pr_1^p - pr_1^p)Z_p D_p - B_p] = 0 \quad (pr_1^p - pr_1^p)Z_p D_p \leq B_p \sum_e d_2^e + (d_1^p + d_1^p) = D_p \tag{62}$$

The Lagrange function,  $L_{\pi_2}(pr_2^e, pr_2^e)$ , can be expressed as follows.  $\mu_p$  is the Lagrange multiplier.

$$L_{\pi_2} = \sum_p pr_2^e d_2^e - C \max_{p \in P} \left[ \frac{1}{\tau_p} d_2^e \right] - \sum_p \mu_p \left[ \sum_{e \in E_{rs}} d_2^e + (d_1^p + d_1^p) - D_p \right] \tag{63}$$

The Kuhn-Tucker condition for the  $\pi_2(pr_2^e, pr_2^e)$  is:

$$\frac{\partial L_{\pi_2}}{\partial pr_2^e} = D_p - 2pr_2^e\alpha_2^e + \beta_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e', e'' \neq e} \beta_2^{e'} pr_2^{e'} + C \max_{p \in P} \left[ \frac{1}{\tau_p} \alpha_2^e \right] - \mu_p [-\alpha_2^e + (I + 1)\beta_2^e] = 0 \quad \forall p \in P, e \in E_{rs}$$

$$\sum_{e \in E_{rs}} d_2^e + (d_1^p + d_1^p) = D_p \tag{64}$$

By solving Equations (62) and (64) simultaneously, the optimal solution of Nash Equilibrium is obtained.

### 5.2. Stackelberg competition of supply chains in SCPSC

In this situation, the government and private sector supply

chains are considered the leader and follower players, respectively. The upstream and extraction in the petroleum supply chain play important roles in pricing. Therefore, since the upstream and extraction is under government control, the government supply chain is considered as the leader player. In other words, decisions of the private sectors are made based on the government decisions. This type of game is solved by using back induction technique. For this purpose, the optimal value of the  $pr_2^{e,p}$  is replaced in government objective function.

$$\begin{aligned}
 d_1^p &= D_p - \alpha_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e \in E_{PS}} \beta_2^{e,p} pr_2^{e,p} \quad \forall p \in P \\
 d_1^p &= D_p - \alpha_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e \in E_{PS}} \beta_2^{e,p} pr_2^{e,p} \quad \forall p \in P \\
 (pr_1^p - pr_1^p)ZD_p &\leq B_p \quad \forall p \in P \\
 \sum_{e \in E_{PS}} d_2^{e,p} + (d_1^p + d_1^p) &= D_p \quad \forall p \in P \\
 d_2^{e,p} &= D_p - \alpha_2^{e,p} pr_2^{e,p} + \beta_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e', e' \neq e} \beta_2^{e',p} pr_2^{e',p} \quad \forall p \in P, e \in E_{PS}
 \end{aligned}
 \tag{66}$$

$$\begin{aligned}
 \text{Max } \pi_1(pr_1^p, pr_1^p) &= \sum_p pr_1^p \left[ D_p - \alpha_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e \in E_{PS}} \frac{\beta_2^{e,p}}{2\alpha_2^{e,p}} \left[ D_p + \beta_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e', e' \neq e} \beta_2^{e',p} pr_2^{e',p} \right. \right. \\
 &\quad \left. \left. + C \max_{p \in P} \left[ \frac{1}{\tau_p} \alpha_2^{e,p} \right] - \mu_p [-\alpha_2^{e,p} + (I+1)\beta_2^{e,p}] \right] \right] + \\
 &\sum_p pr_1^p \left[ D_p - \alpha_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e \in E_{PS}} \frac{\beta_2^{e,p}}{2\alpha_2^{e,p}} \left[ D_p + \beta_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e', e' \neq e} \beta_2^{e',p} pr_2^{e',p} + C \max_{p \in P} \left[ \frac{1}{\tau_p} \alpha_2^{e,p} \right] \right. \right. \\
 &\quad \left. \left. - \mu_p [-\alpha_2^{e,p} + (I+1)\beta_2^{e,p}] \right] \right] \\
 &- C \max_{p \in P} \left[ \tau_p \left( 2D_p - \alpha_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e \in E_{PS}} \frac{\beta_2^{e,p}}{\alpha_2^{e,p}} \left[ D_p + \beta_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e', e' \neq e} \beta_2^{e',p} pr_2^{e',p} \right. \right. \right. \\
 &\quad \left. \left. + C \max_{p \in P} \left[ \frac{1}{\tau_p} \alpha_2^{e,p} \right] - \mu_p [-\alpha_2^{e,p} + (I+1)\beta_2^{e,p}] \right] - \alpha_1^p pr_1^p + \beta_1^p pr_1^p \right) \right] \\
 &\forall p \in P \\
 (pr_1^p - pr_1^p)ZD_p &\leq B_p \quad \forall p \in P \\
 \sum_{e \in E_{PS}} d_2^{e,p} + (d_1^p + d_1^p) &= D_p \quad \forall p \in P \\
 D_p - 2pr_2^{e,p} \alpha_2^{e,p} + \beta_1^p pr_1^p + \beta_1^p pr_1^p + \sum_{e', e' \neq e} \beta_2^{e',p} pr_2^{e',p} + C \max_{p \in P} \left[ \frac{1}{\tau_p} \alpha_2^{e,p} \right] - \mu_p [-\alpha_2^{e,p} + (I+1)\beta_2^{e,p}] &= 0 \\
 &\forall p \in P, e \in E_{PS}
 \end{aligned}
 \tag{65}$$

By solving model 65, the optimal solution of Stackelberg Equilibrium is obtained.

### 5.3. Cooperative competition of supply chains in SCPSC

The optimal solution of cooperative game is obtained by solving the following model.

$$\begin{aligned}
 \text{Max} \\
 \pi_1(pr_1^p, pr_1^p, pr_2^{e,p}) &= \sum_{p \in P} pr_1^p d_1^p + \sum_{p \in P} pr_1^p d_1^p + \sum_p pr_2^{e,p} d_2^{e,p} \\
 &- C \max_{p \in P} \left[ \frac{1}{\tau_p} d_2^{e,p} \right] - C \max_{p \in P} \left[ \frac{1}{\tau_p} (d_1^p + d_1^p) \right] \\
 &\times \left] \quad \forall p \in P, e \in E_{PS}
 \end{aligned}$$

s.t.

### 6. Fuzzy programming technique for multi-objective linear programming problems

The model proposed in section 4 is a three objective, deterministic MILP model including profit maximization, pollution minimization and maximization of job creation. There are many solution strategies like the Weighting Method, Constraint Method, Multi-objective Simplex Method, and Fuzzy Programming techniques to solve a multi-objective model (Alborzi et al., 2011).

Fuzzy programming technique approach is used here to solve the multi-objective MILP model. This approach converts the multi-objective model to a single objective one.  $\lambda$  variable is introduced here to indicate the overall satisfaction of the decision maker. The objective function indicates the maximization of this variable. 1 signifies 100% completion of each objective whereas 0 shows a lack of completion. Values between 0 and 1 show that the objectives

**Table 2**  
The required parameters.

Parameter	$\alpha_1^1$	$\alpha_1^2$	$\alpha_1^1$	$\alpha_1^2$	$\beta_1^1$	$\beta_1^2$
Corresponding random distribution	U (540,630)	U (180,190)	U (240,285)	U (90.8,105.8)	U (0.1,9.1)	U (0.1,3.1)
Parameter	$\beta_1^1$	$\beta_1^2$	$\beta_2^1$	$\beta_2^2$	$\beta_2^1$	$\beta_2^2$
Corresponding random distribution	U (0.5,3.5)	U (0.92,6.92)	U (0.524,15.524)	U (2.71,2.79)	U (1,16)	U (2.8,2.9)
Parameter	$\alpha_2^1$	$\alpha_2^2$	$\alpha_2^1$	$\alpha_2^2$		
Corresponding random distribution	U (262,263.5)	U (91.8,92)	U (250,260)	U (92.05,92.25)		

The obtained optimal parameters are shown in Table 3.

have been met at  $\lambda$  level.

Bellman and Zadeh (1970) presented the fuzzy set theory for the first time to manage the uncertainty coming from indirectness not being random. The Fuzzy optimization problems are also applied in multi-objective literature (Hwang et al., 1993). The concept of fuzzy theory with appropriate membership functions has been used to solve multi-objective linear programming problems by Zimmermann (1978). Fuzzy theory is capable of considering the minimization of some objectives besides some maximization objectives to make a fair balance between them. In other words, the fuzzy approach is used to solve the multi-objective problems. In order to obtain a unique optimal solution, the multi-objective linear programming problems is considered with two objectives as Equation (66a).

$$\begin{aligned} &\max f(x) \\ &\min g(x) \\ &\text{subject to:} \\ &h(x) \leq 0 \end{aligned} \tag{66a}$$

The steps of fuzzy approaches for solving a multi-objective problem are as follows:

Step 1: Each time, only one of the objectives with all of the constraints is maximized as follows:

$$\begin{aligned} &\max f(x) \quad \max g(x) \\ &\text{subject to:} \quad \text{subject to:} \\ &h(x) \leq 0 \quad h(x) \leq 0 \end{aligned} \tag{67}$$

The results of the models are called  $f^*$  and  $g^*$ .

Step 2: Each time, only one of the objectives with all of the constraints is minimized as follows:

$$\begin{aligned} &\min f(x) \quad \min g(x) \\ &\text{subject to:} \quad \text{subject to:} \\ &h(x) \leq 0 \quad h(x) \leq 0 \end{aligned} \tag{68}$$

The results of the models are called  $f^-$  and  $g^-$ .

Finally, the membership functions are found using Equations (69) and (70) based on the  $f(x)$  and  $g(x)$ , which are the maximized and minimized functions, respectively.

$$\mu(f) = \begin{cases} 0 & f(x) \leq f^- \\ \frac{f(x) - f^-}{f^* - f^-} & f^- \leq f(x) \leq f^* \\ 1 & f(x) \geq f^* \end{cases} \tag{69}$$

$$\mu(g) = \begin{cases} 0 & g(x) \leq g^- \\ \frac{g^* - g(x)}{g^* - g^-} & g^- \leq g(x) \leq g^* \\ 1 & g(x) \geq g^* \end{cases} \tag{70}$$

These membership functions must be bigger than  $\lambda$ . According to this, the changed multi-objective linear programming problem based on the fuzzy programming approach is as follows:

$$\begin{aligned} &\max \lambda \\ &\text{subject to:} \\ &h(x) \leq 0 \\ &f(x) \geq f^- + \lambda(f^* - f^-) \\ &g(x) \leq g^* - \lambda(g^* - g^-) \end{aligned} \tag{71}$$

The presented multi-objective, multi-echelon and multi-product PSC problem with fuzzy approach is solved by solving the above linear programming problem.

**7. Computational result**

In this section, the analytical results of competition between government and private sector SCs are compared with each other by using different methods. For this purpose, the proposed two stages decision making method was applied to a real world case in the National Iranian Oil country (NIOC), in which each supply chain includes refineries, DCs and customer zones with two refinery products (i.e. kerosene and gasoline). In this case, both the government and private sectors supply chains should decide on the prices and demands of the refinery products in addition to its network structure decisions, (i. e. installation and capacity expansion of facilities and pipelines, transportation modes, inventory and assignment).

Product prices and demands are determined for each of the public and private sector supply chains by competition between them in the first stage and the amounts of variables related to each

**Table 3**  
The obtained optimal parameters.

$\alpha_1^1$	620	$\beta_1^1$	0.1	$\beta_2^1$	3.524	$\alpha_2^1$	262
$\alpha_1^2$	190	$\beta_1^2$	2.1	$\beta_2^2$	2.715	$\alpha_2^2$	92
$\alpha_1^1$	255	$\beta_1^1$	0.5	$\beta_2^1$	3	$\alpha_2^1$	260
$\alpha_1^2$	92.8	$\beta_1^2$	2.92	$\beta_2^2$	2.812	$\alpha_2^2$	92.05
$D_1$	400,000	$D_2$	150,000	$B_1$	1,000e+7	$B_2$	1,000e+7





of the public and private sector supply chains are determined in the second stage. The numerical results of each of these stages will be discussed in details in the next sections.

7.1. First stage numerical results: determination of price and demand using game theory

In the first stage of the mentioned decision making method, the optimal prices of the refinery products and the amount of demands for the government and private sector chains in Nash Equilibrium competition are obtained by Equations (62)–(64) simultaneously.

By considering the value of zero for  $\delta_p$  variable, Nash Equilibrium is a mixed integer linear programming (MILP) model.

This model was solved by ILOG OPL Studio 3.6. Furthermore, to obtain the optimal prices of the refinery products and the amount of demands for the government and private sector chains in Stackelberg Equilibrium competitions, which is a mixed integer non-linear programming (MINLP) model, Equation (65) is solved by MATLAB. These computations have been done on a personal platform with 2.66 GHz CPU and 4GBRAM memory. The required parameters are generated randomly for chain competitions in a given SCPSC and are shown in Table 2.

Tables 4 and 5 represent the optimal prices of the refinery products and the amount of demands for government and private sector chains in Nash and Stackelberg Equilibria competitions. The prices are based on 1000 Rial per barrel. For example, for  $pr_2^1$  variable, the price is 1059.5 thousand Rial per barrel or 17658.33 Rial per liter.

Considering Tables 4 and 5, the total profit of the supply chain in Nash Equilibrium is greater than that in Stackelberg Equilibrium.

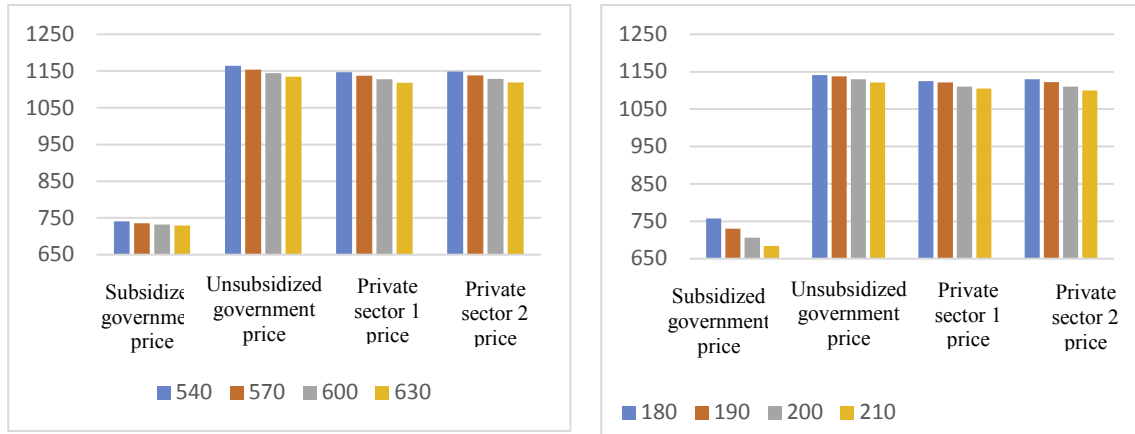
Comparison of public and private sector profits in Nash and Stackelberg Equilibria shows that the profits of the private and public sectors are maximum in Nash and Stackelberg Equilibria, respectively. The reason for this is the leader role of the government

in Stackelber Equilibrium. The demand estimated by the government in this equilibrium has increased compared with that by Nash Equilibrium. In addition, the government subsidized prices for both products has decreased while the corresponding unsubsidized price for both public and private sectors has increased. The increased demand met by the government in Nash Equilibrium is the major factor in increasing the government profit in this equilibrium. The sensitivity analysis on price elasticity coefficients of refined products for the public and private sectors for Nash and Stackelberg Equilibria are shown in Tables 6 through 7. Based on Table 6, the variation of elasticity coefficients for the subsidized prices ( $\alpha_1^1$  and  $\alpha_1^2$ ), has the greatest effect on the variations in the levels of private and public sector profits in Nash Equilibrium. In other words, profit functions of the public and private sectors have the greatest sensitivity towards the variations in elasticity coefficients of subsidized prices. As observed in Table 6, in Nash Equilibrium, by increasing the elasticity coefficients of government unsubsidized ( $\alpha_1^1$  and  $\alpha_1^2$ ) and subsidized prices ( $\alpha_1^1$  and  $\alpha_1^2$ ), which show the market competition, the public and private sector profits simultaneously increase. This is due to the simultaneous decrease of the public and private sectors subsidized and unsubsidized prices according to the diagrams in Fig. 4. The simultaneous decrease of the public and private sectors profits reduces the total profit of the supply chain by increasing the elasticity coefficients of unsubsidized ( $\alpha_1^1$  and  $\alpha_1^2$ ) and subsidized prices ( $\alpha_1^1$  and  $\alpha_1^2$ ) (Table 7).

With regards to elasticity coefficient variations of the process of each private sector, as observed in the Table 6, increasing the price coefficient of each sector ( $\alpha_2^1$ ,  $\alpha_2^2$ ,  $\alpha_2^1$ ,  $\alpha_2^2$ ) decreases the profit whereas the profits of other private or public sectors may decrease or increase. According to Table 7, increasing the price coefficient of each private sector ( $\alpha_2^1$ ,  $\alpha_2^2$ ,  $\alpha_2^1$ , and  $\alpha_2^2$ ) decreases the total profit of the supply chain. Increasing  $\beta_1^2$  and  $\beta_1^1$  elasticity coefficients increases the public sector profit whereas

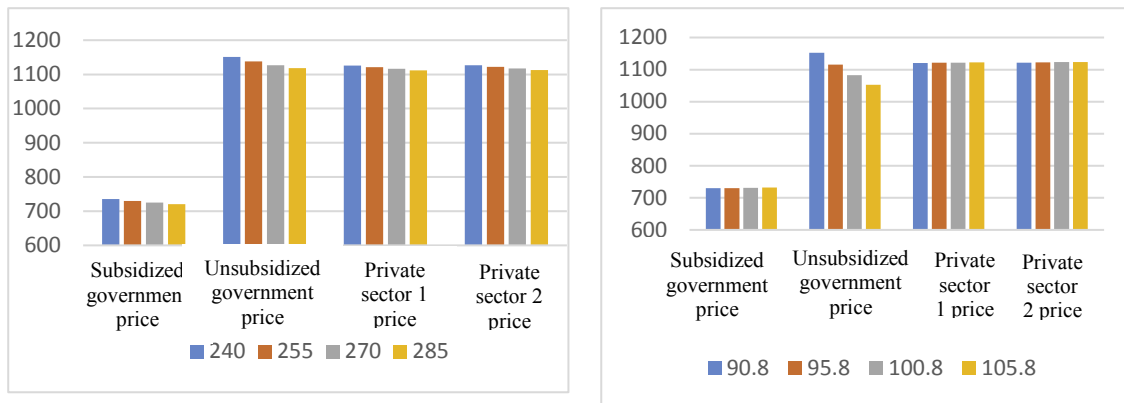
Table 7  
Optimal total values for supply chain profits from Nash Equilibrium.

Parameter	$\alpha_1^1$				$\alpha_1^2$				$\alpha_1^1$			
	540	570	600	630	180	190	200	210	240	255	270	285
$\Pi_S$ (1000 Rial)	4.41E+08	4.4E+08	3.8E+08	4.37E+08	4.39E+08	4.38E+08	4.38E+8	4.37E+08	4.46E+8	4.38E+8	4.3E+08	4.23E+8
Parameter	$\alpha_1^2$				$\beta_1^1$				$\beta_1^2$			
	90.8	95.8	100.8	105.8	0.1	3.1	6.1	9.1	0.1	1.1	2.1	3.1
$\Pi_S$ (1000 Rial)	4.39E+08	4.36E+08	4.32E+08	4.29E+08	4.38E+08	4.4E+08	4.43E+8	4.46E+08	4.35E+8	4.36E+8	3.79E+8	4.39E+8
Parameter	$\beta_1^1$				$\beta_1^2$				$\beta_1^1$			
	0.5	1.5	2.5	3.5	0.92	2.92	4.92	6.92	0.524	5.524	10.524	15.524
$\Pi_S$ (1000 Rial)	4.38E+08	4.39E+08	4.4E+08	4.41E+08	4.35E+08	4.38E+08	4.4E+08	4.43E+08	4.34E+8	4.4E+8	4.45E+8	4.5E+08
Parameter	$\beta_2^{12}$				$\beta_2^{21}$				$\beta_2^{22}$			
	2.71	2.74	2.77	2.79	1	6	11	16	2.8	2.813	2.816	2.9
$\Pi_S$ (1000 Rial)	4.38E+08	4.38E+08	4.38E+08	4.38E+08	4.35E+08	4.41E+08	4.46E+8	4.51E+08	4.38E+08	4.38E+8	4.38E+8	4.38E+8
Parameter	$\alpha_2^{11}$				$\alpha_2^{12}$				$\alpha_2^{21}$			
	262	262.5	263	263.5	91.8	91.85	91.9	92	250	253	256	260
$\Pi_S$ (1000 Rial)	4.38E+08	4.37E+08	4.37E+08	4.37E+08	4.38E+08	4.38E+08	4.38E+8	4.38E+08	4.43E+8	4.41E+8	4.4E+08	4.38E+8
Parameter	$\alpha_2^{22}$											
	92.05	92.25	92.3	92.3								
$\Pi_S$ (1000 Rial)	4.38E+08	4.37E+08	4.37E+08	4.37E+08								



Elasticity coefficient variations of government subsidized prices ( $\alpha_1^1$ ) for product 1

Elasticity coefficient variations of government subsidized prices ( $\alpha_1^2$ ) for product 2



Elasticity coefficient variations of government unsubsidized prices ( $\alpha_1^1$ ) for product 1

Elasticity coefficient variations of government unsubsidized prices ( $\alpha_1^2$ ) for product 2

Fig. 4. Variations of public and private sector profits with respect to elasticity coefficient variations of subsidized and unsubsidized prices in Nash Equilibrium.

increasing  $\beta_1^2$  and  $\beta_1^1$  elasticity coefficients decreases both public and private sector profits. With regards to  $\beta_2^1$ ,  $\beta_2^2$ ,  $\beta_2^1$ , and  $\beta_2^2$ , increasing the coefficient increases the government profit. As shown in Table 7, increasing  $\beta_2^1$ ,  $\beta_2^2$ ,  $\beta_2^1$ ,  $\beta_2^2$ ,  $\beta_1^2$ ,  $\beta_1^1$ ,  $\beta_1^2$  and  $\beta_1^1$  decreases the total profit of the supply chain.

According to Table 8, the greatest effects on the private and public sector profit function values is associated with the variations in the elasticity coefficients of government unsubsidized prices ( $\alpha_1^1$  and  $\alpha_1^2$ ) in Stackelberg Equilibrium. In other words, the public and private sector profits have the greatest sensitivity towards the variations in the elasticity coefficients of unsubsidized prices. As observed in Table 8, increasing the elasticity coefficients of the government unsubsidized prices ( $\alpha_1^1$  and  $\alpha_1^2$ ) decreases the government sector profits while increasing that of the private sector in Stackelberg Equilibrium. As shown in Fig. 5, increasing the elasticity coefficients decreases the government subsidized and

unsubsidized prices and the prices of private sectors although the price reductions in the private sectors are less than those in the public sector, which leads to increased public sector demand and decreased private sector demand and thus increased private sector profits, decreased public sector profits and increased total profit of the supply chain (Table 9).

According to Table 8, for increasing the elasticity coefficient of government subsidized prices ( $\alpha_1^1$  and  $\alpha_1^2$ ) despite reduction of the government subsidized and unsubsidized and private sector prices (Fig. 5), the government profit increases and that of the private sector decreases. This is due to the higher intensity of the reduction of government prices (subsidized and unsubsidized) compared with the private sector, which results in the increased demand in the government section and thus reduction of government profits. The total profit of the supply chain reduces as a result of increased elasticity coefficient of government subsidized prices ( $\alpha_1^1$  and  $\alpha_1^2$ ).

With regards to the variations in the elasticity coefficient of the

**Table 8**  
Optimal values of total profits of the supply chain from Stackelberg Equilibrium.

Parameter	$\alpha_1^1$				$\alpha_1^2$				$\alpha_1^1$			
	540	570	600	630	180	190	200	210	240	255	270	285
$\pi_1(pr_1^p, pr_1^p)$	1.8943E+8	1.7620E+8	1.7014E+8	1.6819E+8	1.7972E+8	1.6840E+8	1.6574E+8	1.6440E+8	1.8357E+8	1.8840E+8	1.9310E+8	1.9719E+8
$\pi_2(pr_2^p)$	1.1870E+8	1.2668E+8	1.3102E+8	1.3343E+8	1.2421E+8	1.3197E+8	1.3376E+8	1.3399E+8	1.3279E+8	1.3197E+8	1.2813E+8	1.2248E+8
$\pi_2(pr_2^{2p})$	1.2153E+8	1.2920E+8	1.3271E+8	1.3527E+8	1.2638E+8	1.3331E+8	1.3539E+8	1.3681E+8	1.3396E+8	1.3331E+8	1.3024E+8	1.2415E+8
Parameter	$\alpha_1^2$				$\beta_1^1$				$\beta_1^2$			
	90.8	95.8	100.8	105.8	0.1	3.1	6.1	9.1	0.1	1.1	2.1	3.1
$\pi_1(pr_1^p, pr_1^p)$	1.6179E+8	1.6219E+8	1.6326E+8	1.6441E+8	1.6440E+8	1.6129E+8	1.6108E+8	1.6082E+8	1.6396E+8	1.6036E+8	1.6440E+8	1.6168E+8
$\pi_2(pr_2^p)$	1.3717E+8	1.2488E+8	1.2154E+8	1.2066E+8	1.3399E+8	1.3734E+8	1.3876E+8	1.4029E+8	1.2776E+8	1.3249E+8	1.3399E+8	1.3590E+8
$\pi_2(pr_2^{2p})$	1.3836E+8	1.2805E+8	1.2438E+8	1.2387E+8	1.3681E+8	1.3900E+8	1.4091E+8	1.4163E+8	1.3069E+8	1.3496E+8	1.3681E+8	1.3776E+8
Parameter	$\beta_1^1$				$\beta_1^2$				$\beta_2^{11}$			
	0.5	1.5	2.5	3.5	0.92	2.92	4.92	6.92	0.524	5.524	10.524	15.524
$\pi_1(pr_1^p, pr_1^p)$	1.6440E+8	1.6724E+8	1.6924E+8	1.6896E+8	1.6404E+8	1.6440E+8	1.6510E+8	1.6856E+8	1.6006E+8	1.6483E+8	1.7594E+8	1.7792E+8
$\pi_2(pr_2^p)$	1.3399E+8	1.3408E+8	1.3418E+8	1.3424E+8	1.3361E+8	1.3399E+8	1.3715E+8	1.3568E+8	1.3694E+8	1.3367E+8	1.3173E+8	1.3063E+8
$\pi_2(pr_2^{2p})$	1.3681E+8	1.3748E+8	1.3798E+8	1.3581E+8	1.3538E+8	1.3681E+8	1.3826E+8	1.3837E+8	1.3670E+8	1.3651E+8	1.3416E+8	1.3954E+8
Parameter	$\beta_2^{12}$				$\beta_2^{21}$				$\beta_2^{22}$			
	2.71	2.74	2.77	2.79	1	6	11	16	2.8	2.813	2.816	2.9
$\pi_1(pr_1^p, pr_1^p)$	1.7658E+8	1.7493E+8	1.6795E+8	1.6686E+8	1.6436E+8	1.6904E+8	1.7911E+8	1.8371E+8	1.7263E+8	1.6440E+8	1.5881E+8	1.5666E+8
$\pi_2(pr_2^p)$	1.2537E+8	1.2859E+8	1.3253E+8	1.3333E+8	1.3173E+8	1.3560E+8	1.3195E+8	1.3197E+8	1.2740E+8	1.3399E+8	1.3786E+8	1.3878E+8
$\pi_2(pr_2^{2p})$	1.2809E+8	1.3093E+8	1.3451E+8	1.3686E+8	1.3593E+8	1.3583E+8	1.3000E+8	1.2771E+8	1.2947E+8	1.3681E+8	1.3857E+8	1.3990E+8
Parameter	$\alpha_2^{11}$				$\alpha_2^{12}$				$\alpha_2^{21}$			
	262	262.5	263	263.5	91.8	91.85	91.9	92	250	253	256	260
$\pi_1(pr_1^p, pr_1^p)$	1.6440E+8	1.7172E+8	1.7751E+8	1.7995E+8	1.6323E+8	1.6394E+8	1.6426E+8	1.6440E+8	1.6440E+8	1.6820E+8	1.7120E+8	1.7424E+8
$\pi_2(pr_2^p)$	1.3399E+8	1.2454E+8	1.2314E+8	1.2214E+8	1.3385E+8	1.3390E+8	1.3395E+8	1.3309E+8	1.3399E+8	1.3119E+8	1.2919E+8	1.2740E+8
$\pi_2(pr_2^{2p})$	1.3681E+8	1.2848E+8	1.2655E+8	1.2448E+8	1.3768E+8	1.3740E+8	1.3699E+8	1.3681E+8	1.3681E+8	1.3597E+8	1.3597E+8	1.3569E+8
Parameter	$\alpha_2^{22}$				Parameter $\alpha_2^{22}$							
	92.05	92.25	92.3	92.3	92.05	92.25	92.3	92.3				
$\pi_1(pr_1^p, pr_1^p)$	1.6440E+08	1.7774E+08	1.7518E+8	1.7167E+8	$\pi_2(pr_2^p)$	1.3681E+08	1.2308E+8	1.2241E+8	1.2041E+8			
$\pi_2(pr_2^{2p})$	1.3399E+08	1.1996E+08	1.1914E+8	1.1926E+8								

price of each private sector, increasing the amount of price coefficient of each private sector ( $\alpha_2^1$ ,  $\alpha_2^2$ ,  $\alpha_2^1$  and  $\alpha_2^2$ ), the public sector profit decreases while that of the government increases, as observed in Table 8. The total profit of the supply chain decreases as these coefficients increase (Table 9).

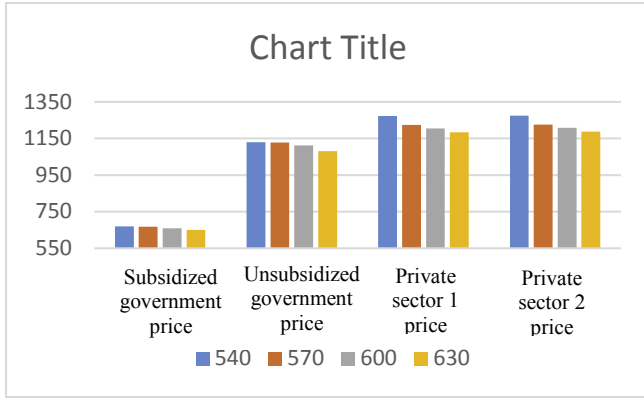
According to Table 8, increasing  $\beta_1^2$  and  $\beta_1^1$  elasticity coefficients increases public sector profit while reducing that of the public sector. However, increasing  $\beta_2^2$  and  $\beta_2^1$  elasticity coefficients increases private sector profit. Increasing  $\beta_1^2$ ,  $\beta_1^1$ ,  $\beta_2^2$  and  $\beta_2^1$  elasticity coefficients decreases the total profit of the supply chain (Table 9). With regards to  $\beta_2^1$ ,  $\beta_2^2$ ,  $\beta_2^2$  and  $\beta_2^2$  elasticity coefficients, the government profit increases similarly to the Nash Equilibrium (Table 8). As observed in Table 9, increasing  $\beta_2^1$ ,  $\beta_2^2$ ,  $\beta_2^2$ ,  $\beta_2^2$ ,  $\beta_2^1$ ,  $\beta_1^1$ ,  $\beta_1^2$  and  $\beta_1^1$  increases the total profit of the supply chain.

7.2. Second stage numerical values: determination of decision parameters of the public and private sector supply chains

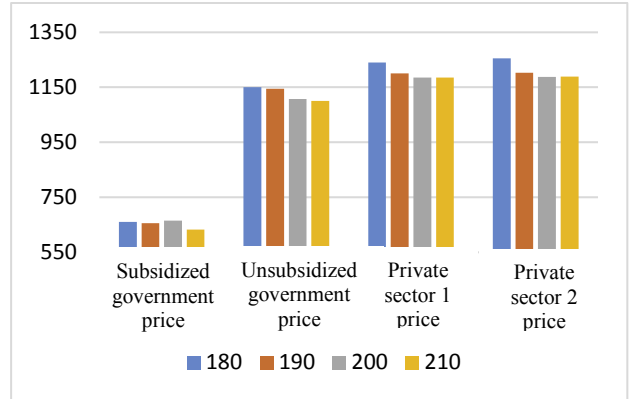
In the first stage, the numerical values of which have been presented in the previous section, the results of the competition between the supply chains of the private and public sectors (the first stage of the proposed model) including the prices and

demands of the public and private sectors were determined. In the second stage of SCPSC model, the prices and demands of the public and private sectors are the parameters of the model, which is a competitive supply chain (the second stage of the proposed model). The model proposed in the second stage is a MILP model whose outputs include stream rates, installation of new facilities, expansion of the existing facilities, installation of new pipelines, expansion of the existing pipelines, imports, exports, determination of the use of DCs for each of the stakeholders, the inventory and selection of transportation mode for each of the stakeholders including the government and each of the private sectors. Since the proposed model in the second stage is a multi-objective model, Fuzzy Programming Technique has been used to solve the model and convert it to a single objective model. Therefore, three objective functions are converted to one and three constraints are added to the existing constraints. Thus, the model is solved using GAMS 24.1.2 software. In order to use the Fuzzy Programming Technique, the minimum and maximum values of each function are first calculated. The minimum and maximum values are then placed in constraint 4–121. The optimal values are thus obtained by considering one objective function.

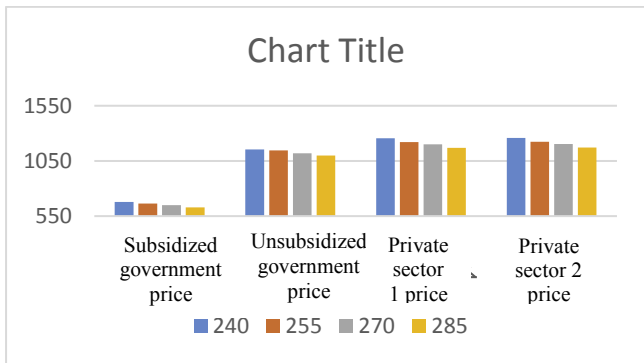
The values of the model objective function are given in Table 10. As observed in the table, the values of the objective function of the supply chain total profit are greater in Nash Equilibrium than they



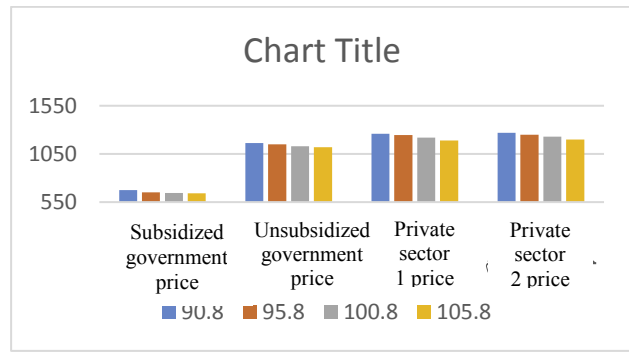
Elasticity coefficient variations of government unsubsidized prices ( $\alpha_1^1$ ) for product 1



Elasticity coefficient variations of government unsubsidized prices ( $\alpha_1^2$ ) for product 2



Elasticity coefficient variations of government subsidized prices ( $\alpha_1^1$ ) for product 1



Elasticity coefficient variations of government subsidized prices ( $\alpha_1^2$ ) for product 2

Fig. 5. Variations of public and private sector profits with respect to elasticity coefficient variations of subsidized and unsubsidized prices in Stackelberg Equilibrium.

are in Stackelberg Equilibrium. In addition, the pollution amount and job creation is maximum in Nash Equilibrium compared with Stackelberg Equilibrium.

As observed in Table 10, comparison of the values of profit objective functions, job creation and pollution for each individual stakeholder shows that the values of objective function are greater for the government than they are for the other two stakeholders in both equilibria (Fig. 6). The reason for this has to do with the high volume of the demands met by the government compared with the other two stakeholders. The government meets both demands for subsidized and unsubsidized prices leading to the high volume of the demands met by the government. The remarkable point in this model is that in Stackelberg Equilibrium in which the government plays the leader player role, the amount of profit has increased compared with Nash Equilibrium while the stakeholder profits have reduced.

The values of the costs associated with each of the stakeholders individually including costs of installation and pipeline expansion, facility installation, product transportation, inventory and fixed and variable costs in each of the equilibria are shown in Fig. 7. As observed, the costs corresponding to the installation and expansion

of facilities and pipelines, inventory costs and fixed and variable costs of government are greater compared with the first and second private sectors in both equilibria. According to Fig. 7, the comparison of Nash and Stackelberg Equilibria shows that the government costs are greater in Stackelberg equilibrium compared with those in Nash Equilibrium, which is due to the high number of demands met. However, according to Fig. 7, private sector costs (except for the inventory cost) are smaller in Stackelberg Equilibrium compared with those in Nash Equilibrium, which is due to the decreased demand met by the private sector.

With regards to the inventory costs, as observed in Fig. 7, the inventory costs for all stakeholders are greater in Stackelberg Equilibrium than those in Nash Equilibrium, which is due to the increased inventory volume in Stackelberg Equilibrium compared with Nash Equilibrium according to Fig. 8.

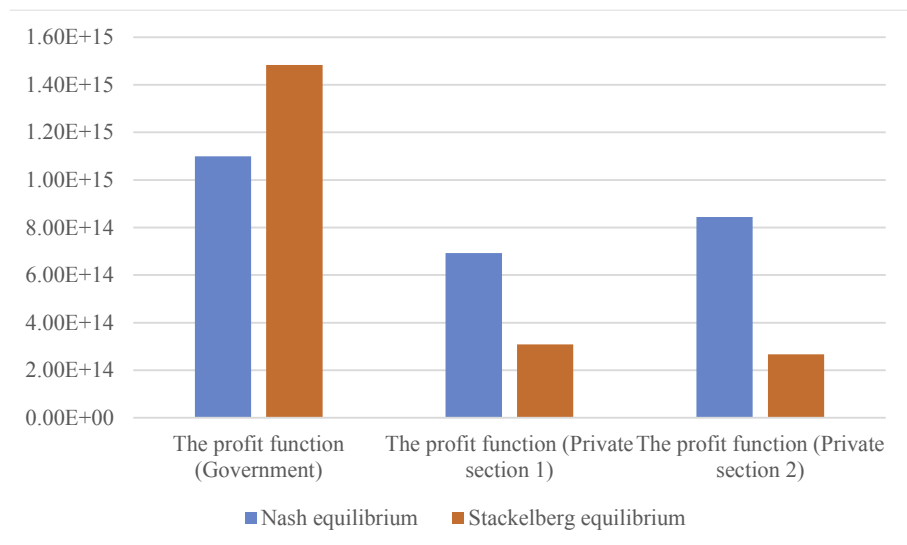
Fig. 9 shows that the stream of the products transported by the government is greater than that by the public sector. The amount of the stream of the products transported by the government is reduced in Stackelberg Equilibria compared with that in Nash Equilibria while this value is reduced in the private sector. This result verifies the difference in transportation costs between

**Table 9**  
Optimal values of total profits of the supply chain from Stackelberg Equilibrium.

Parameter	$\alpha_1^1$				$\alpha_1^2$				$\alpha_1^1$			
	540	570	600	630	180	190	200	210	240	255	270	285
$\Pi_S$ (1000 Rial)	4.3E+08	4.32E+08	4.34E+08	4.37E+08	4.3E+08	4.34E+08	4.35E+08	4.35E+08	4.5E+08	4.34E+08	4.21E+08	4.04E+08
Parameter	$\alpha_1^2$				$\beta_1^1$				$\beta_1^2$			
	90.8	95.8	100.8	105.8	0.1	3.1	6.1	9.1	0.1	1.1	2.1	3.1
$\Pi_S$ (1000 Rial)	4.37E+08	4.15E+08	4.09E+08	4.09E+08	4.35E+08	4.38E+08	4.41E+08	4.43E+08	4.22E+08	4.28E+08	4.35E+08	4.35E+08
Parameter	$\beta_1^1$				$\beta_1^2$				$\beta_1^1$			
	0.5	1.5	2.5	3.5	0.92	2.92	4.92	6.92	0.524	5.524	10.524	15.524
$\Pi_S$ (1000 Rial)	4.35E+08	4.39E+08	4.41E+08	4.39E+08	4.33E+08	4.35E+08	4.41E+08	4.43E+08	4.34E+08	4.35E+08	4.42E+08	4.48E+08
Parameter	$\beta_2^{12}$				$\beta_2^{21}$				$\beta_2^{22}$			
	2.71	2.74	2.77	2.79	1	6	11	16	2.8	2.813	2.816	2.9
$\Pi_S$ (1000 Rial)	4.3E+08	4.34E+08	4.35E+08	4.37E+08	4.32E+08	4.4E+08	4.41E+08	4.43E+08	4.3E+08	4.35E+08	4.35E+08	4.35E+08
Parameter	$\alpha_2^1$				$\alpha_2^2$				$\alpha_2^1$			
	262	262.5	263	263.5	91.8	91.85	91.9	92	250	253	256	260
$\Pi_S$ (1000 Rial)	4.35E+08	4.25E+08	4.27E+08	4.27E+08	4.35E+08	4.35E+08	4.35E+08	4.34E+08	4.37E+08	4.36E+08	4.35E+08	4.35E+08
Parameter	$\alpha_2^{22}$											
	92.05	92.13	92.22	92.3								
$\Pi_S$ (1000 Rial)	4.35E+08	4.21E+08	4.17E+08	4.11E+08								

**Table 10**  
Optimal values of the objective function for the whole supply chain and individual stakeholders.

Beneficiary	Objective function	Nash game value	Stackelberg game value
Government	$P_e$	1.08E+15	1.28E+15
	$P_{ul_e}$	5.19926E+12	6.29E+12
	$S_e$	21570	23279
Private sector 1	$P_e$	7.23E+14	5.39E+14
	$P_{ul_e}$	3.48061E+12	2.65E+12
	$S_e$	17989	16214
Private sector 2	$P_e$	8.82E+14	6.34E+14
	$P_{ul_e}$	4.24606E+12	3.12E+12
	$S_e$	19173	17188
Total profit of the supply chain	$P_e$	2.69E+15	2.45E+15
Total employment created by the supply chain	$P_{ul_e}$	1.30E+13	1.20E+13
Total pollution created by the supply chain	$S_e$	58732	56681
	$\lambda$	0.60105	0.5613



**Fig. 6.** Values of profit function for each of the stakeholders in Nash and Stackelberg Equilibria.

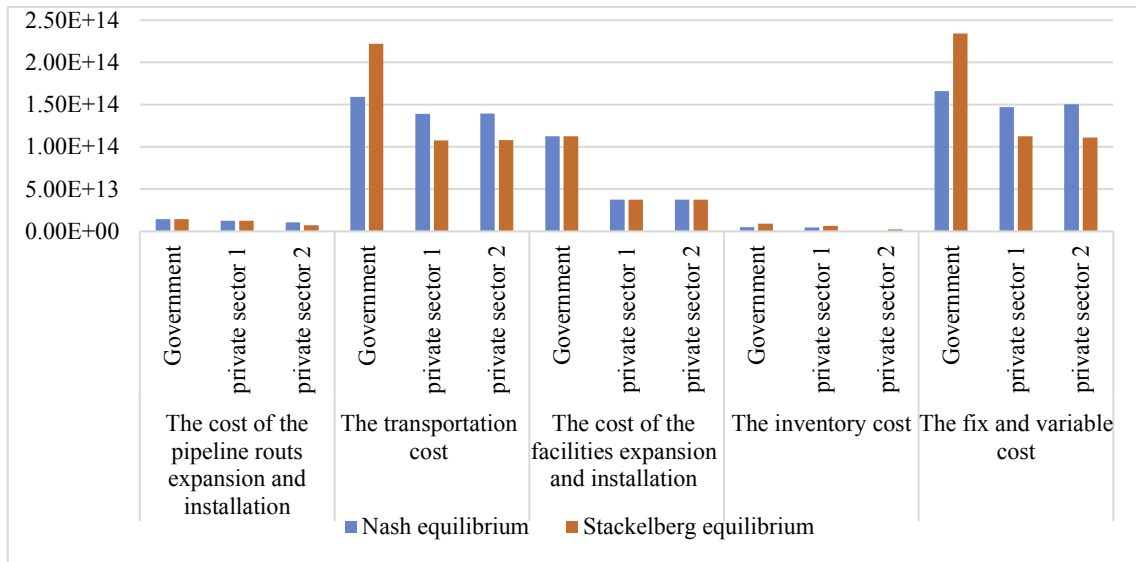


Fig. 7. The costs of stakeholders in Nash and Stackelberg Equilibria.

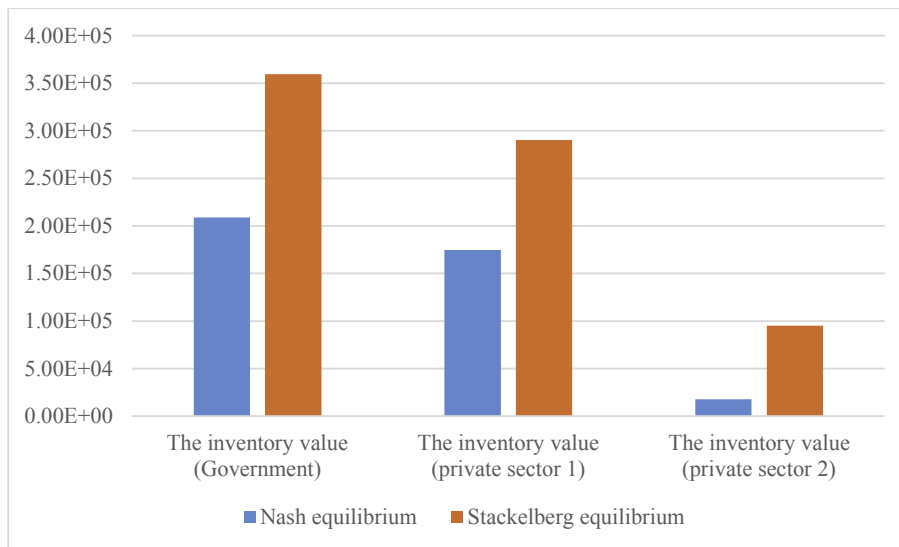


Fig. 8. Inventory of each of the stakeholders in Nash and Stackelberg Equilibria.

different stakeholders and between Nash and Stackelberg Equilibria (Fig. 8).

According to literature review, several valuable managerial insights for government and private sectors are as follows:

- When the government and each of the private sectors simultaneously compete with one another, they choose their prices based on Nash Equilibrium. The total profit of the supply chain is maximum.
- If the management policies decide that the private sectors have the highest possible profit share, of the two equilibria, Nash Equilibrium is chosen. However, Stackelberg Equilibrium is selected when the management policies decide that the government has the highest possible profit share
- If the management policies decide that the government have the highest possible demand share, Stackelberg Equilibrium is chosen.

- According to the sensitivity analysis, the Nash and Stackelberg Equilibria are sensitive to changes in elasticity coefficients of government subsidized and unsubsidized prices. Therefore, carefully determination of these coefficients is playing a major role in the amount of obtained profit.

### 8. Conclusions

Petroleum supply chain has been investigated with regards to sustainability for the first time in this paper. Few works have studied petroleum supply chain. Since there are different stakeholders in the petroleum supply chain in the developed countries and developing countries such as Iran are moving toward privatization of the supply chain and inclusion of various stakeholders, the issue of stakeholders in the petroleum supply chain is a challenging and important one. As it is known, the existence of stakeholders leads to competition among them to determine prices and demand balance.



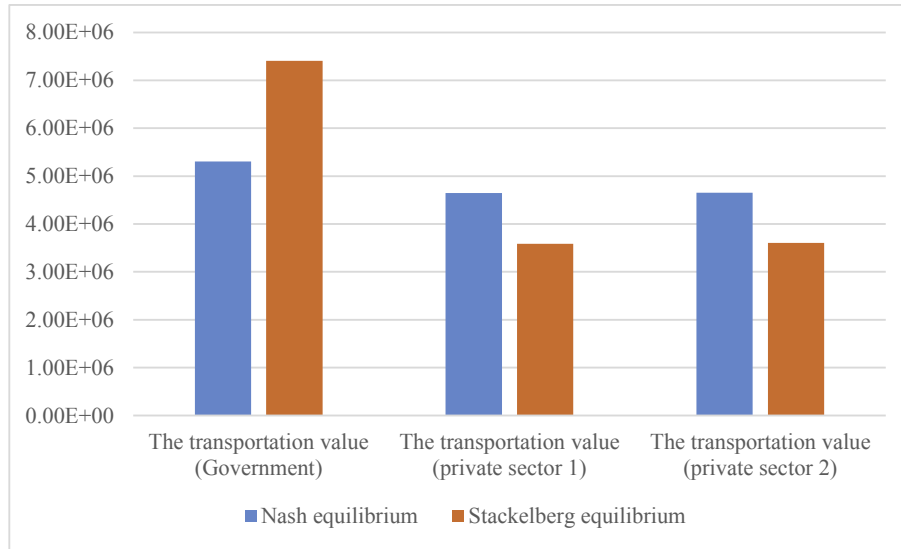


Fig. 9. Transportation values of different stakeholders in Nash and Stackelberg Equilibria.

Therefore, the multi-objective, multi-echelon and multi-product SCPSC model in three levels including refineries, DCs and customer zones have been proposed for the first time. The competition is considered between some SCs including government and private sector SCs based on refinery product prices and demands. The supply chain decisions and refinery product prices and demands are obtained by a two stages decision making method. In the first stage, the prices and the demands of the government and private sectors will be determined by competition between them. Here, two non-cooperative competition approaches; namely Stackelberg and Nash are considered to happen in the SCPSC with respect to the importance of the price and the demand values. Then, the results of the first stage are used as parameters in the second stage. In the second stage, the value of the supply chain decisions will be obtained by solving PSC network under the three objective functions including maximization of economic and social objective functions and minimization of environmental objective function, which is solved by fuzzy theory. One of the features considered in this study is related to DC ownership. One DC may be used by several supply chains. This leads to efficient utilization of the DC capacity and the reduction in costs. The proposed two stage decision making model was applied to a real world case in the NIOC.

Based on the obtained results, in Nash Equilibrium, in which the government and private sectors simultaneously determine their prices and demands, the total profit of the supply chain is 9.8% bigger than it is in Stackelberg Equilibrium, where the government is the leader player and the public sector is the follower player making its decisions according to the leader's. According to this study, Nash and Stackelberg competing models are sensitive to  $(\alpha_1^1$  and  $\alpha_1^2)$  and  $(\alpha_1^1$  and  $\alpha_1^2)$  coefficients, respectively. In Nash Equilibrium, increasing  $(\alpha_1^1$  and  $\alpha_1^2)$  decreases the total profit of the supply chain by 5% while in Stackelberg Equilibrium, increasing  $(\alpha_1^1$  and  $\alpha_1^2)$  increases the total profit of the supply chain by 1.6%. The important question is which equilibrium is the more appropriate one. Quite obviously, the selection depends on the government policies and economic situation. Nash Equilibrium is one of the close to reality equilibria and the stakeholders make their decisions

based on this equilibrium in most competitions. While considering that only downstream and midstream sectors have been privatized and the upstream sector is completely government controlled, state power in the chain is greater than that of the private sector. Thus, when the private sector first enters the game, the government preferable equilibrium is probably the Stackelberg Equilibrium. The government profit in this equilibrium is 11.12% greater compared with Nash Equilibrium. However, as time goes by, the government reduces its control over the chain and then Nash equilibrium becomes the preferable one. The amount of demand met by the government decreases and the public sector demand increases by in this equilibrium. The control measure of the government may be the total amount of subsidies paid.

A.1. :

Nomenclature

Indices	Sets		
Refinery	<i>k</i>	Refinery	<i>k</i>
DC	<i>l</i>	DC	<i>l</i>
Customer zone	<i>m</i>	Customer zone	<i>m</i>
Transportation mode	<i>v</i>	Transportation mode	<i>v</i>
Capacity of transportation mode	<i>lcv</i>	Capacity of transportation mode	<i>lcv</i>
Product	<i>p</i>	Product	<i>p</i>
Capacity of new refinery	<i>ek</i>	Capacity of new refinery	<i>ek</i>
Capacity of new DC	<i>el</i>	Capacity of new DC	<i>el</i>
Capacity expansion of existing refinery	<i>uk</i>	Capacity expansion of existing refinery	<i>uk</i>
Capacity expansion of existing DC	<i>ul</i>	Capacity expansion of existing DC	<i>ul</i>
Capacity expansion of existing pipeline	<i>ev</i>	Capacity expansion of existing pipeline	<i>ev</i>
Capacity of the storage tank in DC	<i>ez</i>	Capacity of the storage tank in DC	<i>ez</i>
Capacity of new pipeline	<i>lv</i>	Capacity of new pipeline	<i>lv</i>
New pipeline route	<i>rv</i>	New pipeline route	<i>rv</i>
Beneficiaries including government and private sectors	<i>e</i>	Beneficiaries including government and private sectors	<i>e</i>
regions	<i>en</i>	regions	<i>en</i>
Education levels	<i>lev</i>	Education levels	<i>lev</i>
Private sectors	<i>ps</i>	Private sectors	<i>ps</i>
Government	<i>G</i>	Government	<i>G</i>

**Parameters**

Demand of customer zone $m$ for products $p$ ( $m \in M, p \in P$ )	$D_{p,m}$
Demand for products $p$ ( $p \in P$ )	$D_p$
Self-price elasticity coefficient of private sector supply chain ( $e \in E, p \in P$ )	$\alpha_2^e$
Cross-unsubsidized price elasticity coefficient of government ( $p \in P$ )	$\beta_1^p$
Cross-subsidized price elasticity coefficient of government ( $p \in P$ )	$\beta_1^{p'}$
Cross-price elasticity coefficient of private sector supply chain ( $e \in E, p \in P$ )	$\beta_2^e$
Self-unsubsidized price elasticity coefficient of government ( $p \in P$ )	$\alpha_1^p$
Self-subsidized price elasticity coefficient of government ( $p \in P$ )	$\alpha_1^{p'}$
The minimum percentage of the demand, which are satisfied by subsidized price ( $p \in P$ )	$z_p$
The maximum available budget of government, which is determined as subsidies ( $p \in P$ )	$B_p$
The cost for each barrel of petroleum	$C$
Product $p$ ratio of one barrel ( $p \in P$ )	$\tau_p$
The subsidized amount of product $p$ ( $p \in P$ )	$SRR_G$
The capacity expansion amount of existing DC $l$ at level $ul$ ( $l \in L_e, ul \in UL$ )	$c_l^{ul}$
The capacity expansion of existing refinery $k$ at level $uk$ ( $k \in K_e, uk \in UK$ )	$c_k^{uk}$
The initial capacity of existing refinery $k$ ( $k \in K_e$ )	$i_k$
The initial capacity of existing DC $l$ for product $p$ ( $l \in L_e, p \in P$ )	$i_l^p$
The minimum capacity coefficients of new refinery $k$ at level $ek$ ( $k \in K', ek \in EK$ )	$M_k^{ek}$
The minimum capacity coefficients of new DC $l$ for product $p$ at level $el$ ( $l \in L', p \in P, el \in EL$ )	$M_l^{el}$
The initial inventory level in refinery $k$ for product $p$ ( $p \in P, k \in (K \cup K')$ )	$v_0^p$
The initial inventory level in DC $l$ for product $p$ ( $p \in P, l \in (L \cup L')$ )	$v_0^p$
The capacity of storage tank $ez$ in new DC $l$ ( $l \in L', e \in EL$ )	$c_l^{ez}$
The capacity of new DC $l$ at level $el$ ( $l \in L', el \in EL$ )	$c_l^{el}$
The capacity of new refinery $k$ at level $ek$ ( $k \in K', ek \in EK$ )	$c_k^{ek}$
The initial route pipeline capacity between refinery $k$ and DC $l$ ( $k \in K, l \in L$ )	$i_{k,l}$
The capacity of transportation mode $v$ at level $lcv$ ( $v \in V, lcv \in LCV$ )	$tr_{cv}^{lcv}$
The capacity expansion of existing route between existing refinery $k$ and existing DC $l$ at level $ev$ ( $k \in K, l \in L, ev \in EV$ )	$c_{k,l}^{ev}$
The amount of time periodic	$TPP$
The capacity of pipeline transportation mode at level $lv$ ( $lv \in LV$ )	$cl_{lv}$
Zero-one matrix, which represents the existing routes between existing refinery $k$ and existing DC $l$ ( $k \in K, l \in L$ )	$R_{k,l}$
The maximum available number of transportation mode $v$ at level $lcv$ at period $t$ ( $lcv \in LCV, v \in V, t \in T$ )	$n_{max_{cv}^{lcv}, t}$
The minimum inventory level of refinery $k$ ( $k \in (K \cup K')$ )	$l_k$
The minimum inventory level of DC $l$ ( $l \in (L \cup L')$ )	$l_l$
The big number	$M$
The installation cost of new refinery $k$ at level $ek$ ( $ek \in EK, k \in K'$ )	$x \cos t_k^{ek}$
The installation cost of new DC $l$ at level $el$ ( $l \in L'$ )	$x \cos t_l^{el}$
The expansion cost of existing refinery $k$ at level $uk$ ( $p \in P, k \in K_e, uk \in UK$ )	$u \cos t_k^{uk}$
The expansion cost of existing DC $l$ at level $ul$ for product $p$ ( $p \in P, l \in L_e, ul \in UL$ )	$u \cos t_l^{ul,p}$
The expansion cost of the existing pipeline route between existing refinery $k$ and existing DC $l$ at level $lv$ ( $ev \in EV, k \in K_e, l \in L_e$ )	$y \cos t_{k,l}^{lv}$
The holding cost of crude oil in refinery $k$ ( $k \in (K_e \cup K'_e)$ )	$h \cos t_k$
The holding cost of product $p$ in DC $l$ ( $p \in P, l \in (L_e \cup L'_e)$ )	$h \cos t_l^p$
The flow rate cost of crude oil between refinery $k$ and DC $l$ ( $k \in (K_e \cup K'_e), l \in (L_e \cup L'_e)$ )	$q \cos t_{k,l}$
The route installation cost of new route pipeline $rv$ between refinery $k$ and DC $l$ at level $lv$ in period $t$ ( $lv \in LV, rv \in RV, k \in (K_e \cup K'_e), l \in (L_e \cup L'_e)$ )	$r \cos t_{k,l}^{rv}$
Cost per unit of resource transport mode $v$ between DC $l$ and customer zone $m$ at level capacity $lcv$ ( $lcv \in LCV, v \in V, l \in (L_e \cup L'_e), m \in M$ )	$n \cos t_{lm}^{lcv}$

(continued)

The storage cost of storage policy $ez$ in new DC $l$ ( $ez \in EZ, p \in P, l \in L'_e$ )	$n \cos t_l^{ez}$
The operating cost of refinery $k$ ( $k \in (K_e \cup K'_e)$ )	$p \cos t_k$
The operating cost of product $p$ at DC $l$ ( $t \in T, p \in P, l \in (L_e \cup L'_e)$ )	$p \cos t_l^p$
The fixed cost of refinery $k$ ( $k \in (K_e \cup K'_e)$ )	$F \cos t_k$
The fixed cost of DC $l$ ( $l \in (L_e \cup L'_e)$ )	$F \cos t_l$
The cost per labor in each education level ( $lev \in LEV$ )	$WCos_{lev}$
The crude oil price	$OP$
The importance coefficient of pollution in region $en$ ( $en \in EN$ )	$\lambda_{en}$
Zero and one matrix showing the region $en$ in which the candidate refinery $k$ is located ( $en \in EN, k \in K'_e$ )	$Nk_{en}^k$
Zero and one matrix showing the region $en$ in which the existing refinery $k$ is located ( $en \in EN, k \in K_e$ )	$Ne_{en}^k$
Zero and one matrix showing the region $en$ in which the candidate DC $l$ is located ( $en \in EN, l \in L'_e$ )	$Nl_{en}^l$
Zero and one matrix showing the region $en$ in which the existing DC $l$ is located ( $en \in EN, l \in L_e$ )	$Ne_{en}^l$
The amount of pollution produced per barrel of crude oil refined in refinery $k$	$Pul_k$
The amount of pollution produced per barrel of crude oil stored in DC $l$	$Pul_l$
The amount of pollution produced per km by each transportation mode $v$ with capacity $lcv$ ( $lcv \in LCV, v \in V$ )	$Pul_{v^{lcv}}$
The percentage of pollution created by expansion of refinery or DC	$Per$
The distance between DC $l$ and customer zone $m$ ( $l \in (L_e \cup L'_e), m \in M$ )	$dis_m^l$
The importance of labor transportation from region $en$ to $en'$ ( $en \in EN, en' \in EN$ )	$W_{en}^{en'}$
The amount of availability labors for educational level $lev$ in region $en$ ( $en \in EN, lev \in LEV$ )	$lab_{en}^{lev}$
The required labor in educational level $lev$ for candidate refineries ( $lev \in LEV$ )	$W_{NK_{lev}}$
The required labor in educational level $lev$ for expansion of existing refineries ( $lev \in LEV$ )	$W_{EK_{lev}}$
The required labor in educational level $lev$ for candidate DC ( $lev \in LEV$ )	$W_{NL_{lev}}$
The required labor in educational level $lev$ for expansion of existing DC ( $lev \in LEV$ )	$W_{EL_{lev}}$
The maximum number of installation and expansion in each region $en$ ( $en \in EN$ )	$Max_{num_{en}}$

Model variables are divided into three categories including positive variables, binary variables and a positive integer as follows.

**Positive variables**

$q_{k,l}^{p,e}$	Flow rate of product $p$ from refinery $k$ to DC $l$ by transportation mode $v$ by stakeholder $e$ ( $p \in P, v \in V, e \in E, k \in (K_e \cup K'_e), l \in (L_e \cup L'_e)$ )
$q_{l,m}^{p,v,e}$	Flow rate of product $p$ from DC $l$ to customer zone $m$ by transportation mode $v$ by stakeholder $e$ ( $e \in E, p \in P, l \in (L_e \cup L'_e), m \in M, v \in V$ )
$v_1^{p,e}$	Volume of product $p$ inventory in DC $l$ by beneficiary $e$ ( $e \in E, p \in P, l \in (L_e \cup L'_e)$ )
$v_k^e$	Volume of crude oil inventory in refinery $k$ ( $e \in E, k \in (K_e \cup K'_e)$ )
$NSRR_G$	The unsubsidized amount of product $p$ ( $p \in P$ )
$pr_2^{p,p}$	The product $p$ price presented by private sector $e$ ( $e \in E, p \in P$ )
$pr_1^p$	The unsubsidized price of product $p$ presented by government ( $p \in P$ )
$pr_1^{p,p}$	The subsidized price of product $p$ presented by government ( $p \in P$ )
$d_2^p$	The amount of demand satisfied by stakeholder $e$ for product $p$ ( $e \in E, p \in P$ )
$d_1^p$	The amount of demand satisfied by unsubsidized price of government for product $p$ ( $p \in P$ )
$d_1^p$	The amount of demand satisfied by subsidized price of government for product $p$ ( $p \in P$ )
$\delta_p$	The Lagrange multiplier
$\mu_p$	The Lagrange multiplier

**Binary variables:**

$$\begin{aligned}
 x_1^{el} e &= \begin{cases} 1 & \text{if distribution center } l \text{ to be installed at level } el \text{ by beneficiary } e \\ 0 & \text{otherwise } (e \in E, el \in EL, l \in L'_e) \end{cases} \\
 x_k^{ek} e &= \begin{cases} 1 & \text{if refinery } k \text{ is installed at level } ek \text{ by beneficiary } e \\ 0 & \text{otherwise } (e \in E, ek \in EK, k \in K'_e) \end{cases} \\
 r_1^{ul} p e &= \begin{cases} 1 & \text{if distribution center } l \text{ is expanded at level } ul \text{ for product } p \text{ by beneficiary } e \\ 0 & \text{otherwise } (e \in E, p \in P, l \in L_e, ul \in UL) \end{cases} \\
 r_k^{uk} e &= \begin{cases} 1 & \text{if refinery } k \text{ is expanded at level } uk \text{ by beneficiary } e \\ 0 & \text{otherwise } (e \in E, k \in K_e, uk \in UK) \end{cases} \\
 y_{kl}^{ev} e &= \begin{cases} 1 & \text{if current rout between current refinery } k \text{ and distribution center } l \text{ is expanded} \\ & \text{at level } ev \text{ by beneficiary } e \\ 0 & \text{otherwise } (ev \in EV, e \in E, k \in K_e, l \in L_e) \end{cases} \\
 z_l^{ez} e &= \begin{cases} 1 & \text{if the number of storage tanks is determined at level } ez \text{ in distribution center } l \\ & \text{by beneficiary } e \\ 0 & \text{otherwise } (ez \in EZ, e \in E, l \in L') \end{cases} \\
 r_{kl}^{lv} e &= \begin{cases} 1 & \text{if rout } rv \text{ between refinery } k \text{ and distribution center } l \text{ is selected at level } lv \text{ by} \\ & \text{beneficiary } e \\ 0 & \text{otherwise } (ev \in EV, e \in E, k \in (K_e \cup K'_e), l \in (L_e \cup L'_e)) \end{cases}
 \end{aligned}$$

**Positive integer:**

$n_{lm}^{lv} p e$	Number of fleet of transportation mode $v$ at capacity level $lcv$ between DC $l$ and customer zone $m$ by beneficiary $e$ ( $lcv \in LCV, v \in V, e \in E, l \in (L_e \cup L'_e), m \in M$ )
$n_l^p ez e$	The number of storage tanks in capacity level in product $p$ in DC $l$ in by beneficiary $e$ ( $lcv \in LCV, v \in V, e \in E, l \in (L_e \cup L'_e), m \in M$ )
$HE\_NK_{en}^{en'} k^{lev}$	The number of labors in level $lev$ , which are worked in new refinery $k$ in region $en$ from region $en'$ ( $en \in EN, en' \in EN, lev \in LEV, k \in K'_e$ )
$HE\_EK_{en}^{en'} k^{lev}$	The number of labors in level $lev$ , which are worked in refinery $k$ in region $en$ from region $en'$ ( $en \in EN, en' \in EN, lev \in LEV, k \in K_e$ )
$HE\_N_{en}^{en'} l^{lev}$	The number of labors in level $lev$ , which are worked in DC $l$ in region $en$ from region $en'$ ( $en \in EN, en' \in EN, lev \in LEV, l \in L'_e$ )
$HE\_EK_{en}^{en'} l^{lev}$	The number of labors in level $lev$ , which are worked in DC $l$ in region $en$ from region $en'$ ( $en \in EN, en' \in EN, lev \in LEV, l \in L_e$ )

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