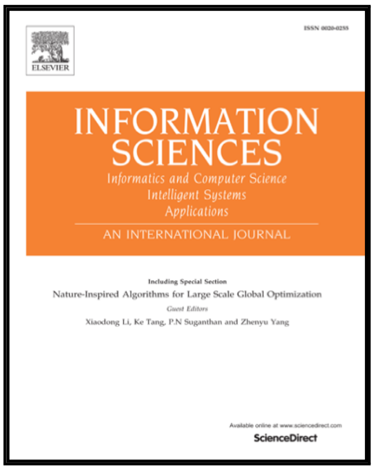
Accepted Manuscript



Understanding influence power of opinion leaders in e-commerce networks: An opinion dynamics theory perspective

Yiyi Zhao , Gang Kou , Yi Peng , Yang Chen

PII: S0020-0255(17)31026-5
DOI: 10.1016/j.ins.2017.10.031
Reference: INS 13206

To appear in: *Information Sciences*

Received date: 16 March 2017
Revised date: 12 October 2017
Accepted date: 13 October 2017

Please cite this article as: Yiyi Zhao , Gang Kou , Yi Peng , Yang Chen , Understanding influence power of opinion leaders in e-commerce networks: An opinion dynamics theory perspective , *Information Sciences* (2017), doi: 10.1016/j.ins.2017.10.031

Understanding influence power of opinion leaders in e-commerce networks: An opinion dynamics theory perspective

Yiyi Zhao[a], Gang Kou[a], Yi Peng[b], Yang Chen[a]

[a]School of Business Administration

Southwestern University of Finance and Economics, Chengdu, China

No.555, Liutai Ave, Wenjiang Zone

Chengdu, 611130, China

kougang@swufe.edu.cn

[b]School of Management and Economics

University of Electronic Science and Technology of China, Chengdu, China

No.2006, Xiyuan Ave, West Hi-Tech Zone

Chengdu, 611731, China

Corresponding author: Yi Peng, pengyi@uestc.edu.cn

Abstract

In this paper, from the perspective of opinion dynamics theory, we investigate the interaction mechanism of a group of autonomous agents in an e-commerce community (or social network), and the influence power of opinion leaders during the formation of group opinion. According to the opinion's update manner and influence, this paper divides social agents within a social network into two subgroups: opinion leaders and opinion followers. Then, we establish a new bounded confidence-based dynamic model for opinion leaders and followers to simulate the opinion evolution of the group of agents. Through numerical simulations, we further investigate the evolution mechanism of group opinion, and the relationship between the influence power of opinion leaders and three factors: the proportion of the opinion leader subgroups, the confidence levels of opinion followers, and the degrees of trust toward opinion leaders. The simulation results show that, in order to maximize the influence power in e-commerce, enhancing opinion leaders' credibility is crucial.

Keywords: E-commerce network, opinion dynamics, bounded confidence rule, opinion leader subgroup, herd behavior.

1. Introduction

The rapid development of Internet technology and Web 2.0 has stimulated the growth of customer-centered e-commerce, which has recently received increased attention in the fields of business applications, business strategies, and user behavior [34]. Within the e-commerce environment, agents access social knowledge and share experiences peer-to-peer (P2P) or through word of mouth (WOM), and then make their own decisions. In such a collective decision-making process, opinions play a fundamental role since they can deeply interact with each other [3].

During the public opinion dissemination process, we find some agents who can exert influence on the opinions, decisions, and actions of the majority of other agents. These agents are referred to by scholars as opinion leaders; they can be found in various social situations and have various definitions in the fields of social, management, and information sciences [9, 25, 10].

In the propagation process of public opinion, opinion leaders have a profound impact on the opinion formation of ordinary agents. In the field of business and marketing, Rogers [23] showed that a small group of influential opinion leaders determines the utility ratio of a given innovation. Compared to the spread of public opinion in a social network without opinion leaders, opinions tend to propagate faster in a social network with opinion leaders [17]. In addition, a number of zealot opinion leaders with definite objectives were employed in [22] to generate momentum and influence voters' decision-making behaviors, while Amblard and Deffuant [2] and Deffuant *et al.* [6] applied bounded confidence theory to construct opinion dynamics models to analyze the influence of opinion leaders in social networks. The results revealed that, as long as the confidence levels of ordinary agents in a social group are sufficiently high, even if the initial opinions of the ordinary agents are dissimilar to those of the opinion leaders, the opinion leaders are eventually able to guide the ordinary agents to accept their desired opinions. Considering that, in some cases, opinion leaders cannot always help spread the desired opinion, Afshar and Asadpour [1] extended the traditional Deffuant-Weisbuch model and built an informed agents model. According to this model, informed agents are common agents possessing desired information. They initially pretend to have opinions similar to those of others, and gradually change their opinions toward the desired information through intentional interactions.

In e-commerce networks, the role of opinion leaders is mainly reflected in two aspects: influencing consumers' decision outcomes and dispersal of opinions by word of mouth (WOM). According to Chaudhry and Irshad [4], average consumers will often consider the opinions of opinion leaders in their purchase decision-making processes. Moreover, it was pointed out in [28] that the degree of discrepancy in opinion leaders' impact on the purchase decisions of average consumers is mainly caused by differences between the cultural background and product focus of both opinion leaders and average consumers. Villanueva *et al*. [30] believed that opinion leaders could provide product information and advice for purchase decisions to other consumers through frequent WOM communication, and thereby affect the attitudes, beliefs, and behaviors of other consumers. Samson [24] pointed out that opinion leaders with a higher confidence level are more willing to become WOM communicators, and can enhance consumers' purchase intentions through positive WOM communication. Further, WOM communication focuses on the process of "opinion leader → WOM communication → consumer behavior"; however, most studies tend to divide the process into two stages (opinion leaders and WOM communication, and WOM communication and consumer behaviors) and analyze the relationship between the two variables involved in each stage, respectively. Liu et al. [17] used a time-varying hypergraph to model online social networks, and a domain-aware approach to identify effective opinion leaders. As far as we know, there is no quantitative research that focuses on the evolution of opinion interaction (consumer behavior) as a direct relationship between opinion leaders and opinion followers (consumers). In order to understand the impact of opinion leaders on opinion followers, it is necessary to consider the mechanism of opinion interaction between opinion leaders and opinion followers.

The dissemination process of public opinion is a complex system of co-evolution of opinions and networks, and involves many variables, such as network structure, the number of agents involved, and description of opinions. Besides, it is difficult for probability- or statistics-based mathematical models to describe the dynamic evolution of collective opinions. Opinion dynamics models focus on the interaction mechanism between opinions, and assume that agents will decide their own opinions based on those of their opinion neighbors in the network. On that account, an opinion dynamics model is more suitable for the study of the opinion dissemination mechanism on user relationship-based social media platforms.

The main purposes of this paper are (1) to explore both the influence of opinion leaders on the decision-making process and opinion formation of ordinary agents during the dissemination of public opinion in social networks; and (2) to provide a theoretical basis, as well as suggest feasible measures, for enterprises, undertakings, and government departments to design appropriate measures to guide and control public opinion. In this paper, opinion leaders have definite target opinions and are interested in influencing the updating process of opinion followers' opinions. In addition, the opinion leaders in this paper are not necessarily leaders based on social class, nor from official organizations, and may not even possess observable leadership qualities. In fact, opinion leaders on social media platforms could be ordinary agents that have the ability to exert significant influence on others' opinions due to their professional background or familiarity with specific events.

The paper contributes to literature by building a new opinion dynamics model for a social group with two opinion leader subgroups with opposite target opinions, based on the bounded confidence principle. It analyzes the relationships between the influence power of the opinion leaders and some factors, such as the proportion of the opinion leaders, the confidence levels of the followers, and the trust degrees of the followers toward the opinion leaders.

The rest of this paper is organized as follows. Section 2 provides a background on bounded confidence opinion dynamics and social network theory. Section 3 first gives a motivation example for opinion leaders, and then builds novel bounded confidence opinion dynamics models for opinion followers, positive and negative opinion leaders, respectively. Section 4 presents some quantitative results through computer simulations to study the influence power of opinion leaders and the evolution of the group opinion. Section 5 concludes the paper.

2. Background

This section introduces the basic knowledge regarding opinion dynamics and social network modeling.

2.1 Bounded confidence opinion dynamics

Opinion dynamics is an important class of social dynamics that studies the formation and dissemination of public opinion on social networks. Based on the methods by which opinions are manifested, opinion dynamics can be classified as either discrete or continuous. Since the 1960s, researchers have proposed a series of models with the continuous updating of opinions in order to study the sufficient conditions for a group of experts to reach a consensus [26]. These were pioneer studies that modeled and analyzed the evolution of continuous opinions. In 2002, Krause and Hegselmann [8] and Deffuant and Weisbuch [32] proposed, respectively, two bounded confidence-based opinion dynamics models—the Hegselmann-Krause (HK) and Deffuant-Weisbuch (DW) models. Specifically, the original HK model is described by

$$x_i(t+1) = \frac{\sum_{j:|x_i(t)-x_j(t)|\le\varepsilon} a_{ij}x_j(t)}{\sum_{j:|x_i(t)-x_j(t)|\le\varepsilon} a_{ij}}, \tag{2-1}$$

while the original DW model is described by

$$\begin{cases} x_i(t+1) = x_i(t) + \mu[x_j(t) - x_i(t)] \\ x_j(t+1) = x_j(t) + \mu[x_i(t) - x_j(t)] \end{cases} \tag{2-2}$$

for the case $|x_i(t) - x_j(t)| \le \varepsilon$, where $x_i(t)$ is the opinion of agent i, ε is the confidence level, a_{ij} is the interaction weight of agent j on agent i, and μ is the convergence parameter. The main difference between these two models is that the DW model adopts an asynchronous opinion updating process, while the HK model adopts a synchronous updating process. The bounded confidence rule suggests that each agent has his/her/its own confidence range when trusting others; only when the differences between the opinions of other agents and his/her/its own are not greater than a specific threshold or confidence level, will he/she/it share and exchange opinion with those agents.

In a bounded confidence model, the confidence level and initial opinion of an agent usually determine the opinion neighbors that he/she/it is likely to communicate with at different time instants. There are three possible final states of collective opinions simulated by a bounded confidence model—consensus, opinion polarization, and opinion fragmentation—that are closely related to the confidence levels, initial opinions, and some convergence parameters. On the one hand, researchers commit themselves to consensus measures or optimal consensus under some specific conditions. For example, some consensus measure algorithms were proposed in [7] and [35], for online-offline social networks and large-scale group decision-making, respectively. For group decision-making, [13, 14] built some interesting optimal consensus models based on minimum cost and maximal return. On the other hand, many researchers devote themselves to studying the relationships between the final opinion pattern (not just consensus) and influence factors. For example, for bounded confidence models, Lorenz [18, 19] categorized agents from a social group into high- and low-confidence subgroups, and proposed respectively heterogeneous HK and DW models. In the modified models, agents within the same subgroup have the same confidence levels, while those in different subgroups have different confidence levels. In addition to considering the heterogeneity of agents, Lorenz [20] also examined the evolutionary mechanism of public opinions in a dynamic social network, where the inter-agent influence changes according to a Markov chain. Mirtabatabaei and Bullo [21] also considered a heterogeneous HK model with a time-varying communication network, which assumes an equilibrium exists in the collective opinion dynamics.They then applied the nonlinear system theory to analyze the convergence of the collective opinions. For the Erdos-Renyi social network topology, based on the traditional HK model, Su and Liu [27] examined the coevolution of opinions and the interconnection network, and obtained some results on consensus or fragmentation for group opinions. The original HK and DW models were applied in [31] to investigate online consumer reviews in e-commerce networks, and to analyze some influence factors in the opinion evolution. Very recently, for a heterogeneous social network, a horizontal and vertical division principle for agents has been proposed for agents, and multi-level heterogeneous HK models and leader-follower opinion dynamics models have been constructed, to systematically analyze the impact of different factors on the spread of public opinion [11-16, 37-39]. Motivated by [16], a time-varying confidence level update rule was proposed in [36], based on in-degrees and out-degrees of agents to extend the original HK model.

2.2 Social network topology

The dissemination process of public opinion on social networks is essentially a coevolution of opinions and the associated network topology. In this subsection, we give a network description of the relationships among agents.

In this study, we assume that opinion leaders have definite target opinions and, thus, are not affected by the opinions of opinion followers. The opinion leaders only exchange opinions with other opinion leaders in the same subgroup. Moreover, opinion leaders play dominant roles in the formation of collective opinions, and are concerned with guiding opinion followers.

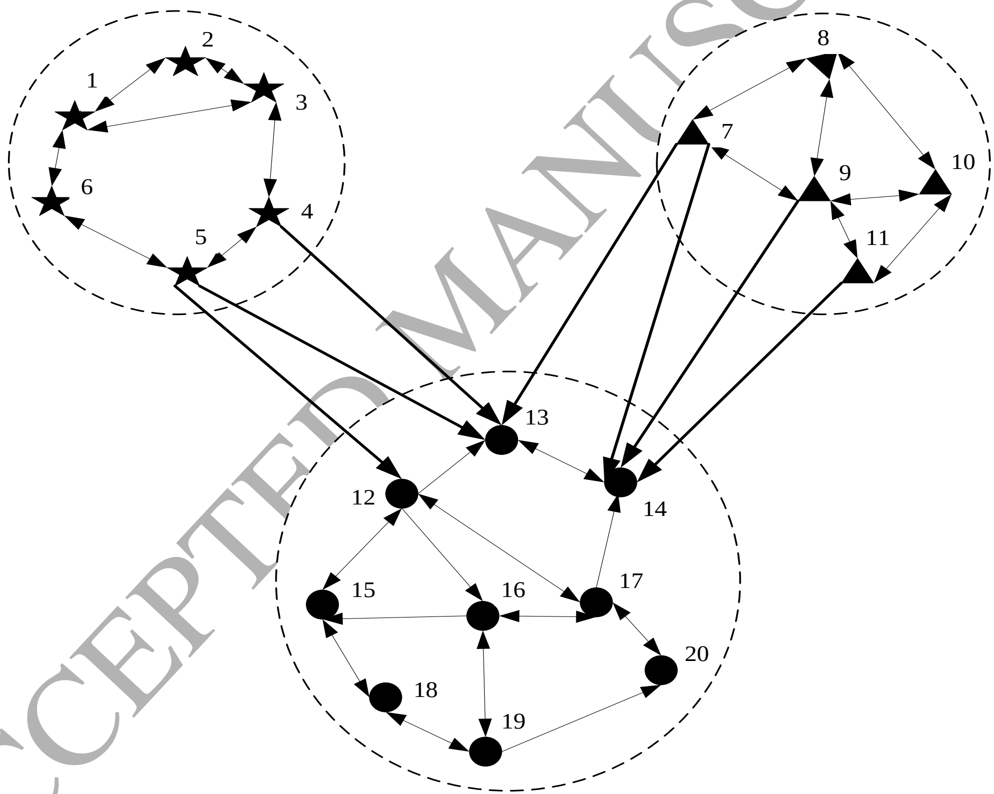


Figure 1: Opinion exchange in a network with two opinion leader subgroups at a fixed time

Figure 1 shows the interactions between opinion followers, as well as opinion followers and opinion leaders, at a certain time instant. As shown in Figure 1, the 20 agents are divided into three subgroups (described with dotted lines) according to their different target opinions. Agents marked with a star shape (labeled 1-6) are opinion leaders with positive target opinions, those in the upper right corner (marked with a triangle shape and labeled 7-11) are opinion leaders with negative target opinions, and those remaining (marked with a circle shape and labeled 12-20) are opinion followers. Opinion leaders from the two subgroups have their own explicit target opinions; opinion followers, however, do not have a specific target opinion, and only exchange their opinions with agents within their confidence ranges, including opinion leaders. If the opinion leaders in the same subgroup have the same target opinions and confidence levels, then the interactions among leaders within the same leader subgroups are symmetrical. Considering the diversity and universality of opinion followers, we assume that their confidence levels are heterogeneous and satisfy a uniform distribution within the interval $[0,1]$. Based on these assumptions, the opinion exchange between the opinion followers may not respect the same bi-directional symmetrical mode as that of the opinion leaders. When the opinion difference of two opinion followers is less than the confidence level of one of the agents, an exchange of opinions may be asymmetrical, such as that between agents 12 and 16, 15 and 16, and 14 and 17. At a given time t, only when the opinion difference between any two given opinion followers is not greater than the confidence levels of both agents, can there be a symmetrical exchange—such as that between agents 13 and 14, 12 and 15, and 15 and 18. Since this paper assumes that the opinions of the opinion followers do not influence those of the opinion leaders, and the opinion leaders from different leader subgroups do not communicate due to their dissimilar target opinions, there are no edges between the two opinion leader subgroups. The edges between the opinion leader group and opinion follower subgroup are directed toward the opinion follower group. Specifically, opinion leaders 1 and 7, as well as 3 and 9, belong to two different subgroups; hence, they do not have an opinion exchange. However, the opinion leaders 4 and 5 (from the positive opinion group) have an influence on the opinions of the opinion followers 12 and 13, respectively, and the opinion leaders 7, 9, and 11 have an influence on the opinions of the opinion followers 13 and 14; yet, the aforementioned opinion followers cannot affect the opinion updating of the corresponding opinion leaders.

3. Modeling the Influence Power of Opinion Leaders

By utilizing social network service platforms as a channel to communicate and exchange opinions, users or agents have formed large-scale social networks that connect "acquaintances of acquaintances." Although social network platforms are applications of technological architecture, they can eventually lead to the formation of various types of real-life relationships. Generally, opinion leaders are believed to have an important influence on consciousness, decision-making, and the behavior of ordinary agents in the dissemination process of public opinion. For example, [36] proposed a trust evaluation algorithm for opinion leaders in the cloud networks by using the opinion leaders' recommendations and removal of the troll entities. A new graph-based comprehensive reputation model and a hybrid trust-based recommender system were built in [33] and [40], respectively, to improve the role of opinion leaders in social commerce. Additionally, an online survey experiment was conducted in [29] to show that opinion leaders' recommendations could increase the trust of ordinal agents on some particular media.

3.1 A motivation example

In this paper, the opinion leaders are defined as the agents that have definite, unwavering target opinions The agents except opinion leaders in a social network are called opinion followers. The leaders are not affected by the opinions of the opinion followers during the opinion update processes. They have the intention to influence the opinions of others. Additionally, the opinion leaders in this paper are not required to be important agents in any official organization or institution, and can be ordinary agents who have a significant amount of information about the object or event in the public opinion. The agents, except opinion leaders in a social network, are called opinion followers. For example, peer-to-peer (P2P) lending is a classical social network service in the field of finance. It is well known that, with the quick development of internet and e-commerce, many P2P lending companies—such as Zopa, Prosper, Lending Club Funding Circle, and RateSetter—provide online investment platforms to attract lenders and investors to identify and purchase loans. All of the countries in the world have high expectations for P2P lending in financial innovation; however, in China, a P2P lending company called Ezubao launched in July 2014 and was subsequently shut down in February 2016 because it was accused of a Ponzi scheme. Consequently, about 900,000 customers and their 50 billion Renminbi were involved in the Ezubao case. Two classes of opinion leaders essentially influenced the opinions of agents: (1) Ezubao, as well as some associated media and experts; and (2) the third-party P2P rating organizations. In order to persuade more agents to use Ezubao, the company made significant investments into advertisements in certain well-known media, such as China Central Television, local television stations, metros in big cities, and experts' popularization. Moreover, the third-party P2P rating organizations often published some advanced-risk warning suggestions and reported that Ezubao was rated as Level C. Thus, when opinion leaders hold polarized or even fragmental opinions, it is interesting to investigate how the opinions evolve for opinion leader subgroups as well as for the whole group. The questions surrounding the key factors associated with the opinion leaders' influnece power remain to be further explored.

3.2 Bounded confidence-based opinion dynamics

We now construct a new model to analyze the influence power of opinion leaders, based on the framework of the bounded confidence theory. We further reveal the evolutionary mechanism of group opinions under the influence of multiple opinion leader subgroups. In reality, there are more than two opinion leader subgroups in a given social network group. The differences in the target opinions held by these subgroups are not necessarily substantially large. In order to simplify the analysis process, without loss of generality, this paper assumes a situation where only two subgroups of opinion leaders exist in a given social network. Each leader subgroup has its own target opinion. Moreover, the leader subgroups are referred to as positive and negative if the target opinions are completely positive and negative, respectively. By establishing a model of influence power for opinion leaders, this paper systematically investigates the relationships between the influence power of opinion leaders and some associated factors, such as the proportion of opinion leader subgroups, the confidence levels of opinion followers, and their degrees of trust toward the opinion leaders.

Suppose there is a social network with N agents, among whom N_1 is opinion followers, N_2 is opinion leaders with a positive target opinion, N_3 is opinion leaders with a negative target opinion, and $N_1+N_2+N_3=N$. Then, when an event occurs, agents' initial opinions of the event may be diverse; specifically, agents can hold an opinion somewhere between the completely positive and completely negative opinions of the event. When the opinions at time t of all agents are denoted by $x_i(t)$, without loss of generality, the completely positive and completely negative opinions of the event are, respectively, defined by $x_i(t)=1$ or 0.5 and $x_i(t)=-1$ or -0.5, for $i=1,\cdots,N$. We assume that the initial opinions of all agents $x_i(0)$ obey a uniform distribution within the interval $[-1,1]$ or $[-0.5,0.5]$. At any time t, the opinion of agent i satisfies $x_i(t)\in[-1,1]$ or $[-0.5,0.5]$. For convenience of description, we denote $X(t)=col(x_1(t),x_2(t),\cdots,x_N(t))\in\mathbf{R^N}$ as the vector of the collective opinions, and $X(0)$ as the initial opinion profile at time $t = 0$.

An opinion-updating model is proposed for the opinion followers as follows:

$$\begin{aligned} x_i^F(t+1) = {} & (1-\alpha_i-\beta_i)\frac{1}{N_i^F(t)}\sum_{j=1}^{N_1} a_{ij}(t)x_j(t) + \alpha_i \frac{1}{N_i^P(t)}\sum_{j=N_1+1}^{N_1+N_2} a_{ij}(t)x_j(t) \\ & + \beta_i \frac{1}{N_i^N(t)}\sum_{j=N_1+N_2+1}^{N} a_{ij}(t)x_j(t), \end{aligned} \qquad (3\text{-}1)$$

where the updating weight $a_{ij}(t) = \begin{cases} 1, & \|x_i(t)-x_j(t)\| \le \varepsilon_i^F \\ 0, & otherwise \end{cases}$, $i = 1,\cdots,N_1$, $j = 1,\cdots,N$, and ε_i^F represents the confidence level of the opinion follower i. $N_i^F(t) = \sum_{j=1}^{N_1} a_{ij}(t)$ is the total number of opinion neighbors in the opinion follower subgroup of opinion follower i at time t, and $N_i^P(t) = \sum_{j=N_1+1}^{N_2} a_{ij}(t)$ and $N_i^N(t) = \sum_{j=N_1+N_2+1}^{N_3} a_{ij}(t)$ are the numbers of opinion leaders who are the opinion neighbors of agent i from the positive and negative leader subgroups, respectively. α_i, β_i, and $1-\alpha_i-\beta_i$ are the degrees of trust assigned to the positive opinion leader subgroup, negative opinion leader subgroup, and opinion follower subgroup, respectively, and $\alpha_i, \beta_i, 1-\alpha_i\text{-}\beta_i \in [0,1]$. When $\alpha_i = 0$ or $\beta_i = 0$, opinion follower i completely distrusts the positive or negative opinion leaders, respectively. Contrarily, if $\alpha_i = 1$ or $\beta_i = 1$, then opinion follower i is completely influenced by the opinion leaders from the positive or negative opinion subgroups, respectively.

According to their definition and characteristics, opinion leaders tend to have a relatively comprehensive range of information on the same event, when compared to the opinion followers. In addition, their target opinions are very specific, and they only exchange opinions with the opinion leaders that meet their confidence levels in the same subgroup. In order to achieve a common goal, the confidence levels between opinion leaders in the same subgroup are relatively high. Thus, a relatively moderate value $\varepsilon_i^P = \varepsilon_i^N = 0.25$ is assigned as the confidence level for the opinion leaders. Then, the opinion-updating model of opinion leaders with the positive target opinion can be described as follows:

$$x_i^P(t+1) = (1-w_i)\frac{1}{N_i^P(t)}\sum_{j=N_1+1}^{N_2} a_{ij}(t)x_j(t) + w_i d, \quad i = N_1+1,\cdots\cdots,N_1+N_2, \qquad (3\text{-}2)$$

where $a_{ij}(t)=\begin{cases}1, & \|x_i(t)-x_j(t)\|\le\varepsilon_i^P \\ 0, & otherwise\end{cases}$, ε_i^P is the confidence level, and $N_i^P(t)=\sum_{j=N_1+1}^{N_2}a_{ij}(t)$ is the number of neighbors of positive opinion leader i. The variable d is the value of the target opinion of the positive opinion leader subgroup, which is a constant that falls in the interval $[0,1]$; w_i and $1-w_i$ are the influence weights of the target opinion d and of other positive opinion leaders that satisfy the condition $\|x_i(t)-x_j(t)\|\le\varepsilon_i^P$ on the positive leader i, respectively. Further, for simplicity, the values of w_i are assumed to be the same, that is, $w_i=w_j$.

Similarly, the opinion-updating model of the opinion leaders with the negative target opinion can be described as follows:

$$x_i^N(t+1)=(1-z_i)\frac{1}{N_i^P(t)}\sum_{j=N_1+N_2+1}^{N}a_{ij}(t)x_j(t)+z_ig,\quad i=N_1+N_2+1,\cdots\cdots,N,\quad (3\text{-}3)$$

where $a_{ij}(t)=\begin{cases}1, & \|x_i(t)-x_j(t)\|\le\varepsilon_i^N \\ 0, & otherwise\end{cases}$; $N_i^N(t)=\sum_{j=N_1+N_2+1}^{N}a_{ij}(t)$ represents the number of neighbors of negative leader i; g is a value of the negative target opinion, which is a constant between $[-1,0]$; and z_i and $1-z_i$ are respectively the influence weights of the target opinions and other negative opinion leaders on the negative leader $\boldsymbol{i}$. In addition, the values of z_i are assumed to be the same.

By establishing the opinion dynamics models of the opinion followers and leaders, we define the influence power of the opinion leaders as the ratio between the number of the opinion followers eventually led by the opinion leaders and that of opinion followers at the initial stage, which can be described as follows:

$$\eta=\frac{N_f}{N_F},\quad (3\text{-}4)$$

where N_f is the number of the opinion followers that have similar opinions as those of the opinion leaders. The next section is devoted to the analysis of the relationship between the influence power of the leaders and the fraction of the opinion leaders, the opinion followers' confidence levels, and the degrees of trust toward the opinion leaders.

4. Simulation Results and Quantitative Analysis

4.1 Data preparation

Based on Models (3-1), (3-2), and (3-3), a computer simulation method is adopted to investigate the influence power of the opinion leaders and the evolution of the collective opinions.

As shown in [38], the evolution of the collective opinions tends to become relatively stable when the network size reaches 2,000 nodes. This indicates that a further increase in the network size will have no significant impact on the influence power of the opinion leaders and the opinion evolution process of the whole group. Therefore, this paper assumes that the size of the considered network is $N = 2000$. For all of the computer experiments, the Monte Carlo simulation is conducted 1000 times. Unless otherwise specified, the following assumptions are applied to all of the experiments:

(1) the size of the social network is $N = 2000$;

(2) the initial opinions of the opinion followers and positive and negative opinion leaders all obey a uniform distribution;

(3) the confidence levels of both the positive and negative opinion leader groups are, respectively, $\varepsilon_i^P = \varepsilon_i^N = 0.25$; the confidence levels of the opinion followers satisfy a uniform distribution;

(4) the degrees of trust of the opinion followers toward the positive and negative opinion leaders satisfy $\alpha_i + \beta_i = 0.8$, while those between opinion followers satisfy $1 - \alpha_i - \beta_i = 0.2$;

(5) the target opinion of the positive opinion leaders is $d = 0.5$, and that of the negative opinion leaders is $g = -0.5$; and

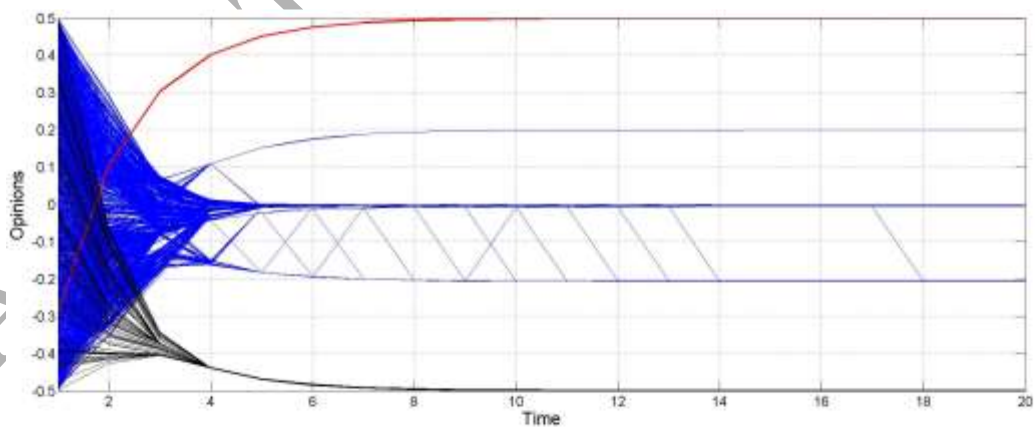
(6) the influence weights of the target opinions on both the positive and negative opinion leader subgroups are 0.5, that is, $w = z = 0.5$.

4.2 Proportion of Opinion Leaders

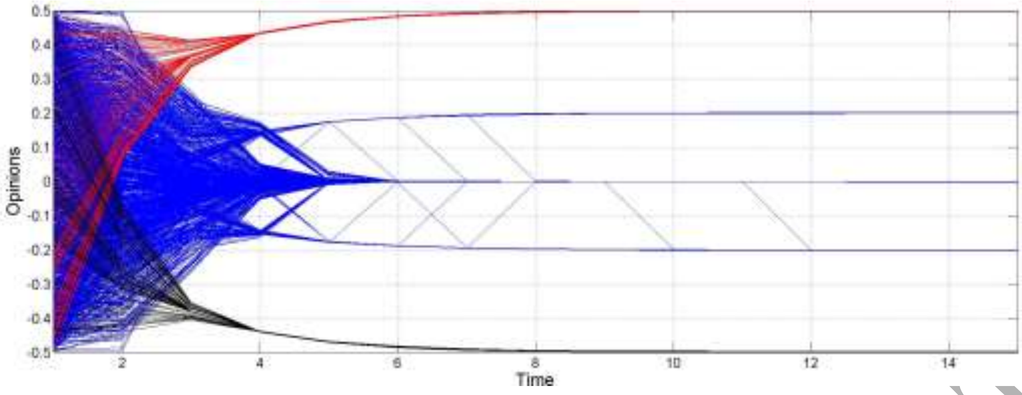
In the following simulation experiment, the proportion of negative opinion leaders is set as $P_N = 5\%$. Thus, in a social network with 2000 nodes, the number of negative opinion leaders is constantly 100. The degrees of trust of the opinion followers toward the positive and negative opinion leaders are respectively set as $\alpha_i = \beta_i = 0.4$, and those between the opinion followers are 0.2. Without loss of generality, we alter the proportion of positive opinion leaders to observe the impact of such changes on the evolution of the collective opinions and influence power of the opinion leaders.

In this experiment, the proportion P_P of positive opinion leaders is initially set as 0 and, accordingly, that of opinion followers is set as $P_F = 0.9500$. As the proportion of positive opinion leaders P_P increases to 0.0005, 0.001, 0.005, 0.0100, and a maximum of 0.9400, the proportion of opinion followers in the same social network is reduced to 0.9495, 0.9490, 0.9450, 0.9400, and eventually 0.0100, respectively.

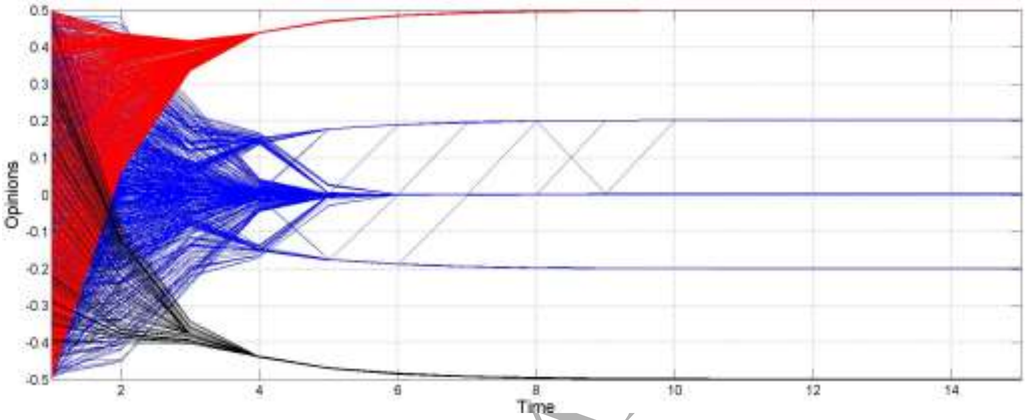
Figure 2 illustrates the evolution of the collective opinions of the three subgroups when the proportion of positive opinion leaders changes from 0.0005 to 0.94. The red, blue, and black solid lines represent the evolution trajectories of the opinions of the positive opinion leaders, opinion followers, and negative opinion leaders, respectively, over time. Regardless of the proportion of the positive opinion leaders (0.0005, 0.5000, 0.8000, or even 0.9400) or the opinion leaders' initial opinions, both the positive and negative opinion leaders can swiftly (in less than six time steps) converge to the target opinion of the corresponding subgroup. However, the opinion evolution of the opinion followers is relatively more complex and requires more time to reach a stable state. Regardless of which subgroup (positive or negative) is at an advantage in size, the final opinions of followers are divided into three clusters at the values of -0.2, 0, and 0.2. Furthermore, no follower's opinion converges to the target opinions (0.5 and -0.5), suggesting that, in a social network with two opposing opinion leader groups, none of the opinion leaders appears to have absolute influence power on the opinions of the followers.



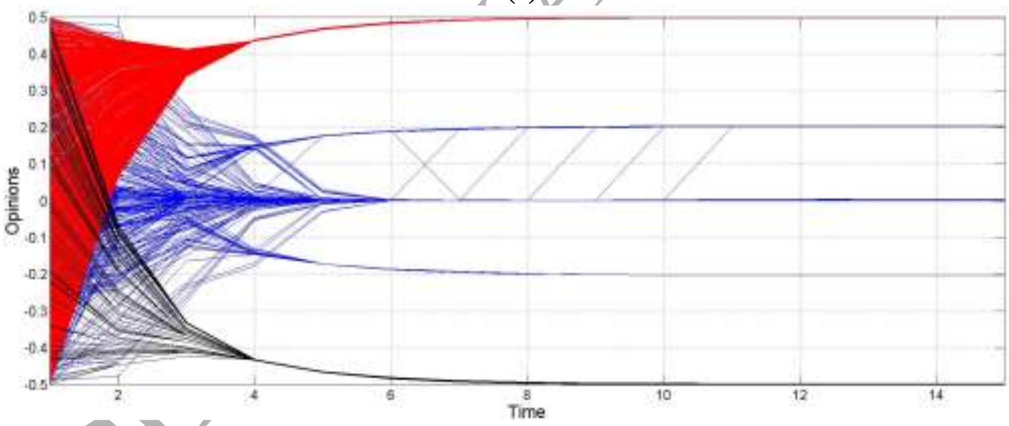
(a)



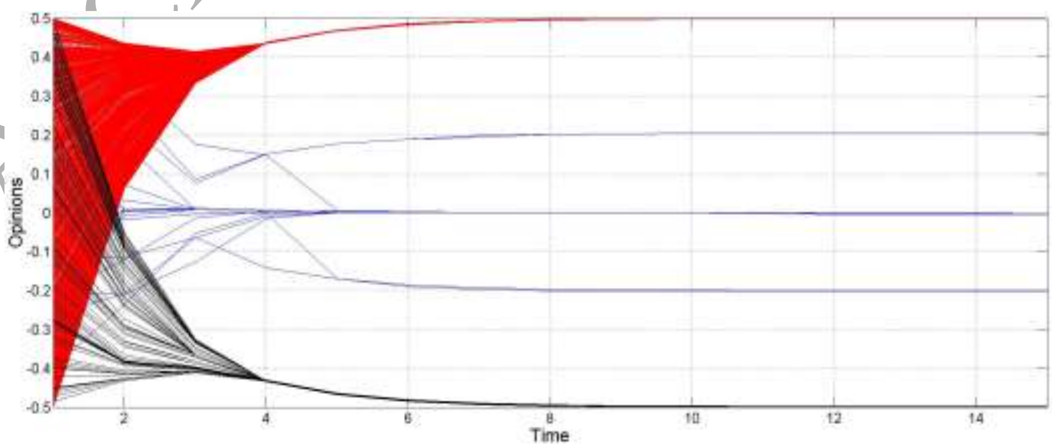
(b)



(c)



(d)



(e)

Figure 2: Evolution of collective opinions with different fractions of positive opinion leaders (a) $P_P = 0.0005$; (b) $P_P = 0.0500$; (c) $P_P = 0.5000$; (d) $P_P = 0.8000$; (e) $P_P = 0.9400$

In order to more clearly and systematically demonstrate the relationship between the influence power and the proportion of positive opinion leaders, we utilize Table 1 to present the number of opinion followers influenced by the positive opinion leaders. The first row of Table 1 is the sub-interval of opinions, while the first column represents the proportion of the positive opinion leaders in the entire social network. Other elements in the table are the numbers of the influenced opinion followers. According to this table, when there are two opposing opinion leader subgroups, regardless of there being only one or as many as 1880 positive opinion leaders in the entire social network, the final opinions of the opinion followers remain to be distributed in any of the three positions in the opinion interval, -0.2, 0, and 0.2. In addition, the opinion followers never follow the opinion leaders, as none of their opinions falls in the target opinion interval of either $[0.4, 0.5]$ or $[-0.5, -0.4]$. Unlike the results in [38], where there is only one opinion leader group in the entire social network, the final opinions of the opinion followers tend to lie in two opinion-intervals: the target opinion and completely opposing opinion of opinion leaders.

Table 1: Number distribution of the influenced opinion followers versus the proportion of the positive opinion leaders

Distribution / P_P	[-0.5,-0.4]	[-0.4,-0.3]	[-0.3,-0.2]	[-0.2,-0.1]	[-0.1,0]	[0,0.1]	[0.1,0.2]	[0.2,0.3]	[0.3,0.4]	[0.4,0.5]
0.0005	0	0	291	0	1582	0	26	0	0	0
0.0010	0	0	204	0	1615	0	79	0	0	0
0.0050	0	0	172	0	1659	0	59	0	0	0
0.0100	0	0	103	0	1694	0	83	0	0	0
0.0500	0	0	89	0	1632	0	78	0	0	0
0.1000	0	0	91	0	1516	0	93	0	0	0
0.1500	0	0	71	0	1436	0	92	0	0	0
0.2000	0	0	62	0	1346	0	92	0	0	0
0.2500	0	0	77	0	1240	0	83	0	0	0
0.3000	0	0	53	0	1154	0	92	0	0	0
0.3500	0	0	38	0	1065	0	97	0	0	0
0.4000	0	0	39	0	972	0	88	0	0	0

0.4500	0	0	36	0	888	0	76	0	0	0
0.5000	0	0	38	0	788	0	74	0	0	0
0.5500	0	0	26	0	712	0	61	0	0	0
0.6000	0	0	25	0	618	0	57	0	0	0
0.6500	0	0	30	0	525	0	45	0	0	0
0.7000	0	0	18	0	431	0	51	0	0	0
0.7500	0	0	20	0	348	0	32	0	0	0
0.8000	0	0	19	0	250	0	30	0	0	0
0.8500	0	0	8	0	176	0	16	0	0	0
0.9000	0	0	5	0	84	0	10	0	0	0
0.9400	0	0	2	0	16	0	2	0	0	0

Figure 3 directly demonstrates that, with increases in the proportion of positive opinion leaders, the proportion of influenced opinion followers in the three opinion subintervals $[-0.3,-0.2]$, $[-0.1,0]$, and $[0.1,0.2]$ does not represent a monotonic increasing/decreasing or linear correlation, but manifests a certain degree of fluctuation at a certain level. In addition, when the proportion of positive opinion leaders reaches the minimum or maximum values, the fluctuation of the influence power in the three subintervals becomes relatively obvious.

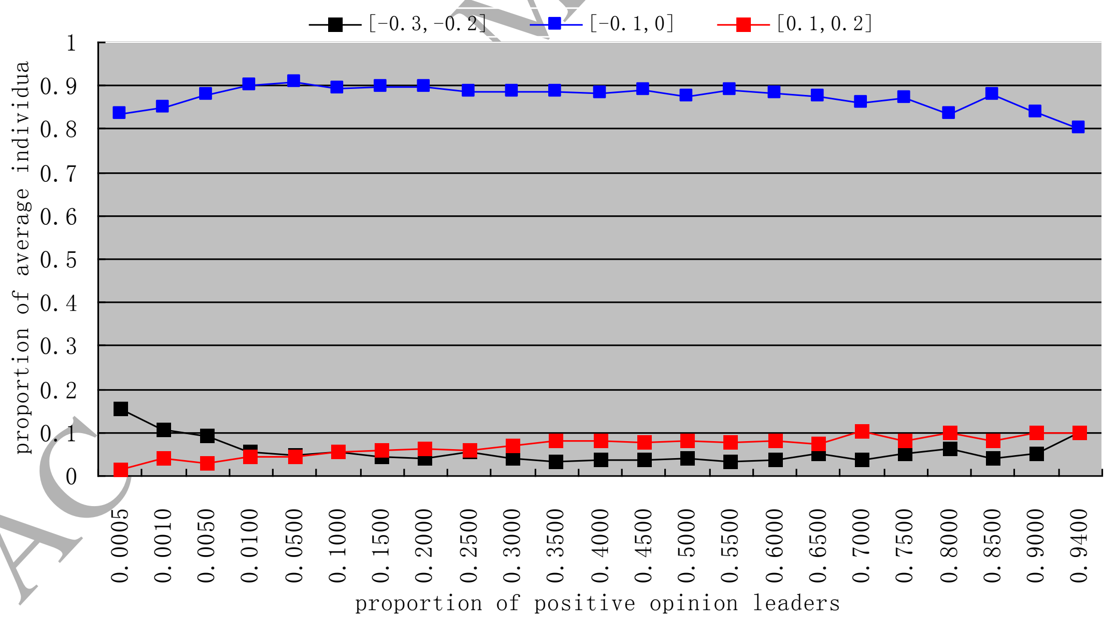


Figure 3: Relationship between influence power and the proportion of positive opinion leaders

We observe from the simulation experiment that the final opinions of the followers tend to form three clusters. The distribution of those influenced by the opinion leaders has the following features. First, the largest cluster is located in the middle of the opinion interval. When the opinion interval is $[-0.5, 0.5]$, the largest opinion cluster is located in the subinterval $[-0.1, 0]$. Second, except for the largest opinion cluster in the middle opinion subinterval, the final opinions of the remaining opinion followers form the two subintervals $[-0.3, -0.2]$ and $[0.1, 0.2]$. Third, as the proportion of the positive leaders increases, that of the influenced opinion followers in the subinterval $[0.1, 0.2]$ tends to increase at the beginning. However, the proportion in the subinterval $[0.1, 0.2]$ shows a fluctuation at the final stage. The proportion of the influenced opinion followers in the subinterval $[-0.3, -0.2]$ presents a drop followed by a rising trend. The changes in the proportion of opinion followers in the middle opinion cluster $[-0.1, 0]$ show a similar pattern as that in the subinterval $[0.1, 0.2]$.

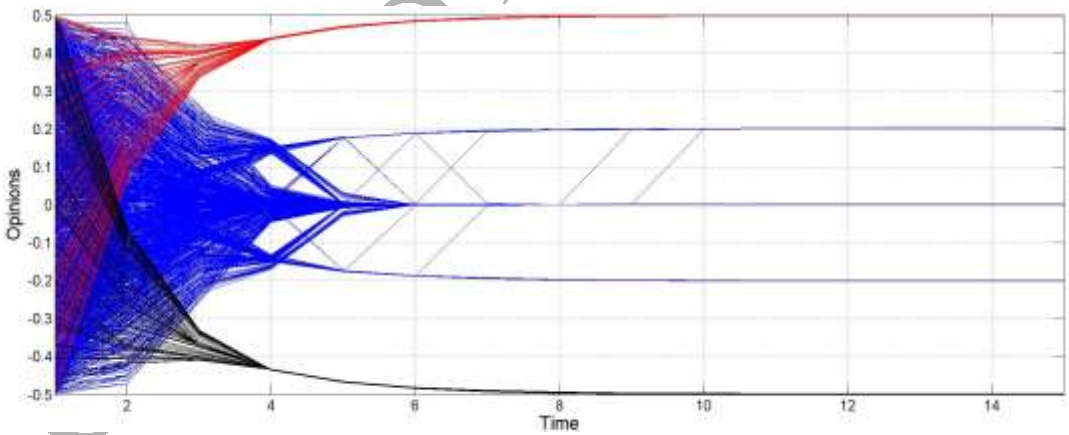
The simulation experiment reveals that, when there are two opinion leader subgroups in a social network, the influence power of either opinion leader subgroup is restricted, even if the fraction of the opinion leaders is large enough. Thus, it is necessary to investigate the reasons for the restriction in the exertion of the opinion leaders' influence power on the opinion followers. The next subsection further examines the impact of the opinion followers' confidence levels on the opinion leaders' influence power in order to investigate whether heterogeneity and low confidence levels are the main constraints of the opinion leaders' influence power.

4.3 Confidence Levels of Opinion Followers

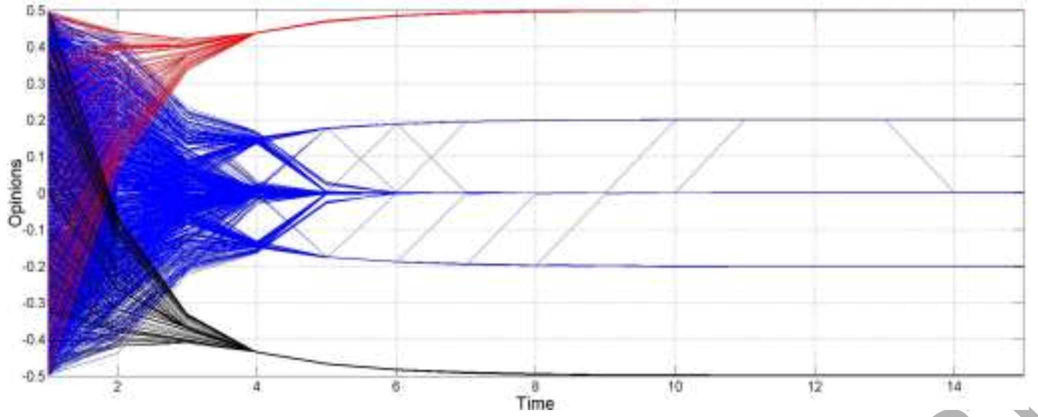
For more accurate analysis of the impact of the confidence levels of the opinion followers on the influence power of the opinion leaders, two situations are analyzed separately, namely that of the positive and negative opinion leaders having a similar influence on opinion followers, and that of the positive opinion leaders having a greater influence. The following assumptions are adopted in the subsequent analyses. The initial opinions of all agents obey a uniform distribution within the interval $[-0.5, 0.5]$, and remain fixed in the experiment. The proportions of the positive and negative opinion leaders are fixed at $P_P = P_N = 0.05$. Thus, in a social network that consists of 2000 agents, the numbers of positive opinion leaders, negative opinion leaders, and opinion followers are 100, 100, and 1800, respectively. The degrees of trust of the opinion followers toward both of the opinion leader subgroups are given by $\alpha_i = \beta_i = 0.4$. The opinion followers are heterogeneous; hence, their confidence levels can be dissimilar. However, all confidence levels obey a uniform distribution. The target opinions of the positive and negative opinion leaders are defined as 0.5 and

-0.5, respectively. In this simulation experiment, we continuously reduce the range of the confidence levels of the opinion followers. At the same time, we increase their confidence levels in order to examine the impact of the opinion followers' confidence levels on the influence power of the opinion leaders, as well as explore the evolution of the collective opinions.

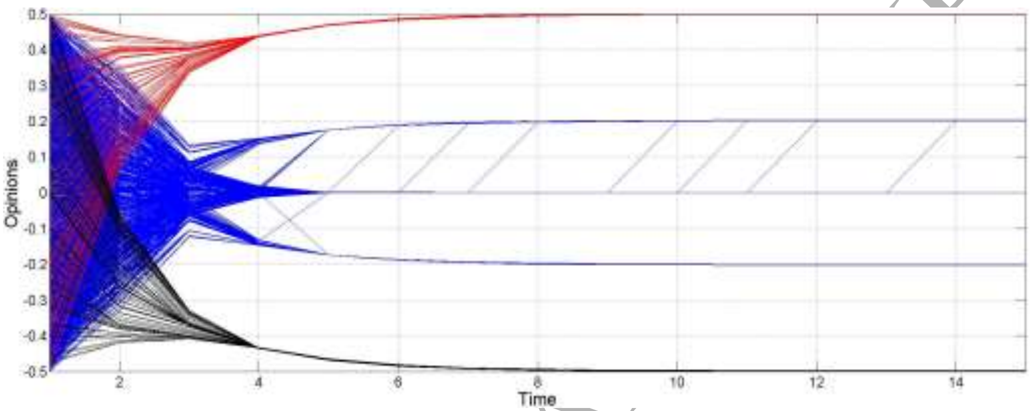
In Figure 4 and Table 2, the numbers of the positive and negative opinion leaders are identical, and the opinion followers have equal degrees of trust toward the positive and negative opinion leaders. As the range of the confidence levels of the opinion followers shrinks and confidence levels simultaneously increase, both the positive and negative opinion leaders reach their target opinions quickly. Furthermore, from Table 2, the opinion evolution of the opinion followers presents the following characteristics. First, with the increase in confidence levels, the number of the final opinion clusters of the opinion followers decreases, and their final opinions tend to become more aggregated. Second, the convergence speed of the opinion followers' opinions is accelerated with the increase of their confidence levels. Third, regardless of the final opinions of the opinion followers being divided into one or three clusters, these clusters are symmetrically distributed within the opinion interval; no obvious bias toward either opinion leader subgroup is observed. Fourth, the increase in confidence levels cannot distinctly improve the influence power.



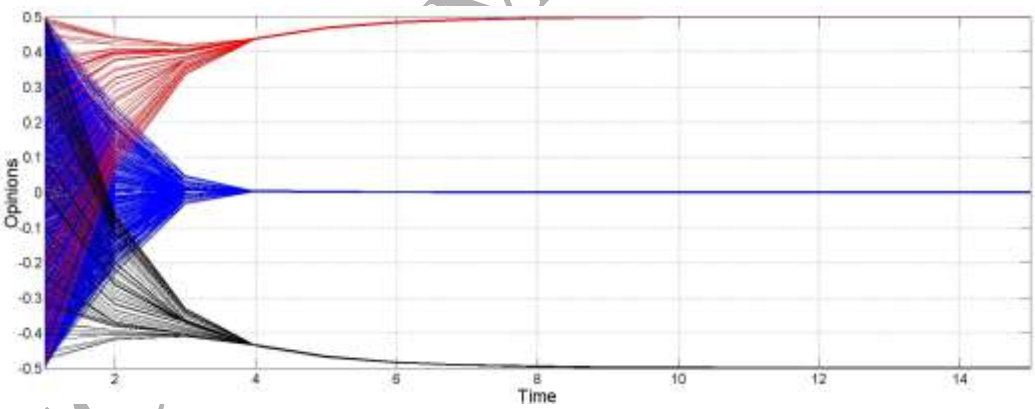
(a)



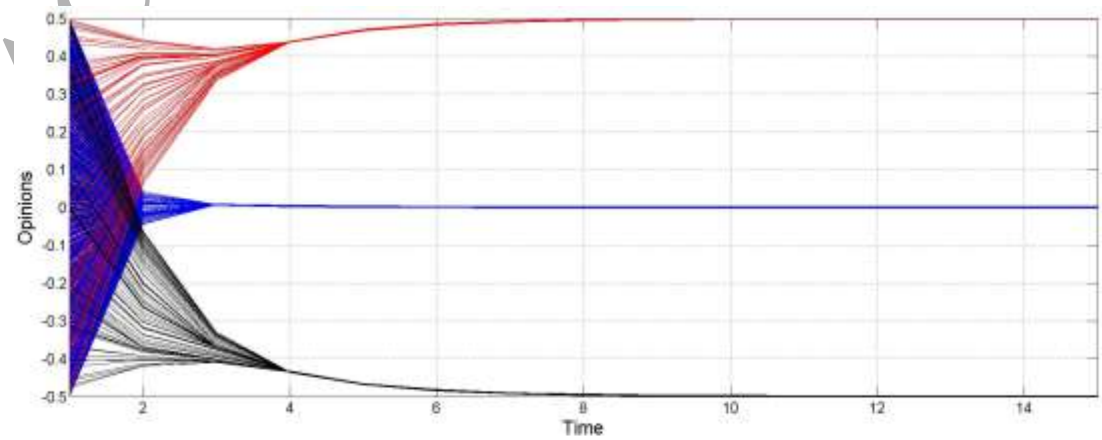
(b)



(c)



(d)



(e)

Figure 4: Evolution of the collective opinions with heterogeneous confidence levels (a) $\varepsilon_i^F \in [0,1]$; (b) $\varepsilon_i^F \in [0.1,1]$; (c) $\varepsilon_i^F \in [0.3,1]$; (d) $\varepsilon_i^F \in [0.5,1]$; (e) $\varepsilon_i^F \in [0.9,1]$

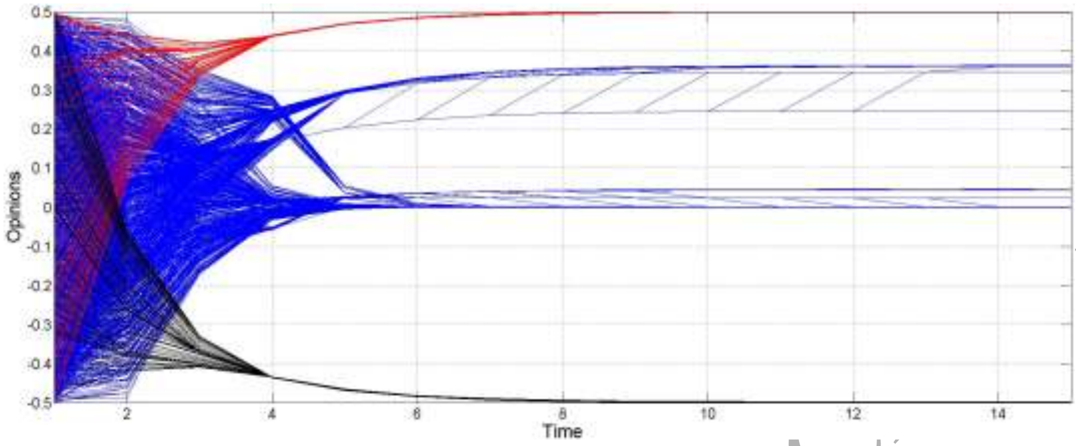
Table 2: Number distribution of the influenced opinion followers with heterogeneous confidence levels

Distribution / ε_i^F	[-0.5,-0.4]	[-0.4,-0.3]	[-0.3,-0.2]	[-0.2,-0.1]	[-0.1,0]	[0,0.1]	[0.1,0.2]	[0.2,0.3]	[0.3,0.4]	[0.4,0.5]
[0,1]	0	0	89	0	1632	0	78	0	0	0
[0.1,1]	0	0	95	0	1598	0	106	0	0	0
[0.2,1]	0	0	114	0	1577	0	108	0	0	0
[0.3,1]	0	0	117	0	1550	0	132	0	0	0
[0.4,1]	0	0	5	0	1733	0	61	0	0	0
[0.5,1]	0	0	0	0	1799	0	0	0	0	0
[0.6,1]	0	0	0	0	1799	0	0	0	0	0
[0.7,1]	0	0	0	0	1799	0	0	0	0	0
[0.8,1]	0	0	0	0	1799	0	0	0	0	0
[0.9,1]	0	0	0	0	1799	0	0	0	0	0

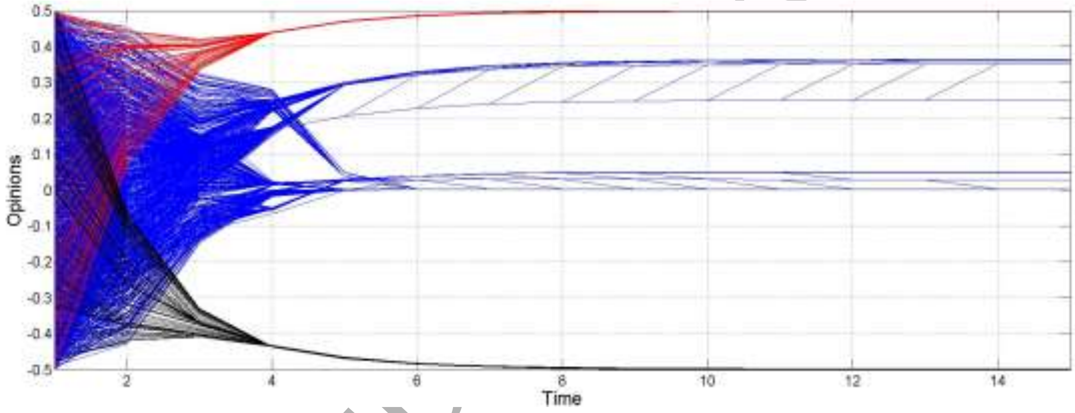
In order to explicitly demonstrate the relationship between the confidence levels of the opinion followers and the opinion leaders' influence power, we reduce the length of the confidence level range of the opinion followers in each experiment—for example, $[0.1,1]$, $[0.2,1]$, $[0.3,1]$, ... $[0.9,1]$—to analyze the changes in the number distribution of the influenced opinion followers. When the confidence level range is reduced from $[0,1]$ to $[0.1,1]$, $[0.2,1]$, and $[0.3,1]$, the number of the influenced agents in the opinion subinterval $[-0.1,0]$ gradually decreases, while those in the opinion subintervals $[-0.3,-0.2]$ and $[0.1,0.2]$ show an increasing trend. As the confidence level range is further reduced to $[0.4,1]$, drastic changes in the number of the influenced opinion followers are observed among the three opinion clusters. The agents, whose final opinions belong to $[-0.3,-0.2]$ and $[0.1,0.2]$ for longer confidence level ranges, tend to converge to the middle opinion subinterval. When the range reaches $[0.5,1]$, all opinion followers reach consensus in the middle opinion subinterval $[-0.1,0]$. This result shows that, after the confidence levels of the opinion followers have increased to $[0.5,1]$, the collective opinions of the opinion followers will reach a consensus at the compromise opinion 0. The results also indicate that, in a social network with equal degrees of trust toward the positive and negative opinion leader subgroups, increasing the confidence levels of the opinion followers to a certain degree is conducive to the influence power of the opinion leaders. However, when the confidence levels of the opinion followers surpass a certain threshold, the opinion leaders' influence power will not increase. Hence, the situation in which the opinion followers' opinions are completely dominated by one of the opinion leaders' groups is unlikely to occur. When the opinion followers' confidence levels are sufficiently large, they appear to have more difficulties in making a decision between the opinions of the positive and negative opinion leaders. As a result, the influence power of the positive and negative opinion leaders offset one another. Moreover, due to herd mentality, more agents tend to abandon the opinion leaders they initially followed and join the largest opinion group.

In the above experiment, the positive and negative opinion leader subgroups have the same propagation. Moreover, the opinion followers have equal degrees of trust in the opinion leaders. This well-matched case may be the reason for restricting the exertion of the influence power of the opinion leaders. In order to investigate the impact of the opinion followers' confidence levels on the influence power of the opinion leaders, under the condition of two leader subgroups not being well matched, we alter the opinion followers' degrees of trust toward the positive opinion leaders α_i from 0.4 to 0.6. This is so that the positive opinion leaders may have a stronger influence on the agents. Correspondingly, their degrees of trust toward the negative opinion leaders are reduced to $\beta_i = 0.2$. We then repeat the process of the previous simulation, with the other parameters unchanged.

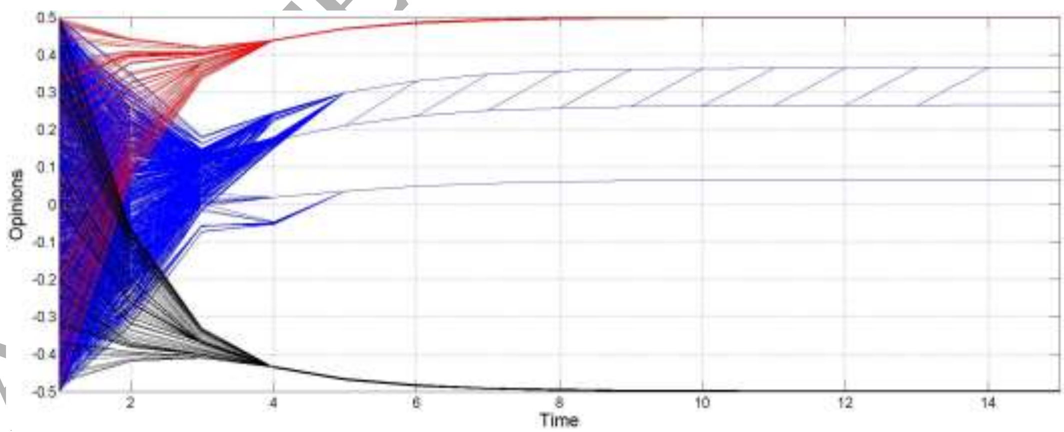
Figure 5 shows an evolution of the collective opinions with the reducing range of confidence levels when the opinion followers have different degrees of trust toward the positive and negative opinion leaders. Compared to Figure 4, the final opinions of the opinion followers in Figure 5 tend to shift toward the target opinion of the positive opinion leader subgroup. Moreover, Figure 5 illustrates that, as confidence levels increase, the opinion followers start to shift (to different degrees) toward the target opinion of the positive opinion leader subgroup, and eventually cause a situation in which the number of opinion followers in the opinion interval $[-0.5, 0]$ becomes zero. In addition, as the degrees of trust increase, although more opinion followers begin to shift toward the target opinion of the positive opinion leaders, none of them actually reaches the target opinion of 0.5; rather, their opinions become consistent within the opinion subinterval $[0.1, 0.2]$. These results show that the increase in the opinion followers' degrees of trust toward the positive opinion leaders can strengthen the influence power of the positive opinion leaders. However, when the degrees of trust are assigned as $\alpha_i = 0.6$, the influence power of the positive opinion leaders still cannot be fully exploited.



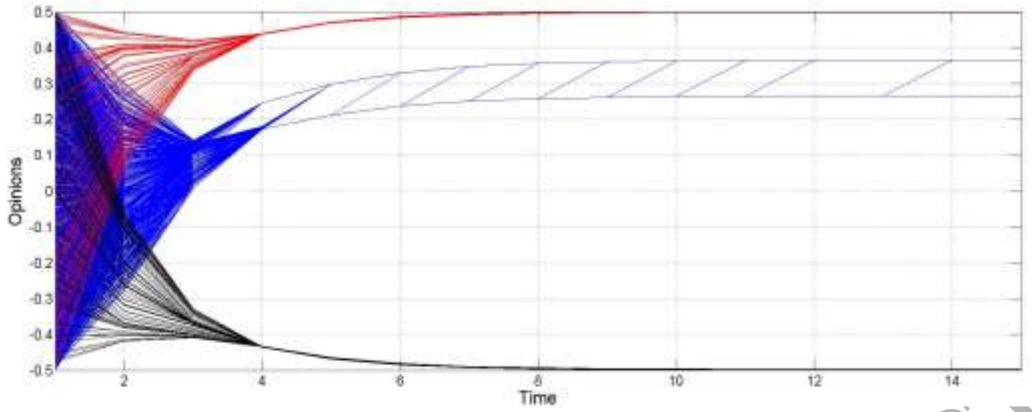
(a)



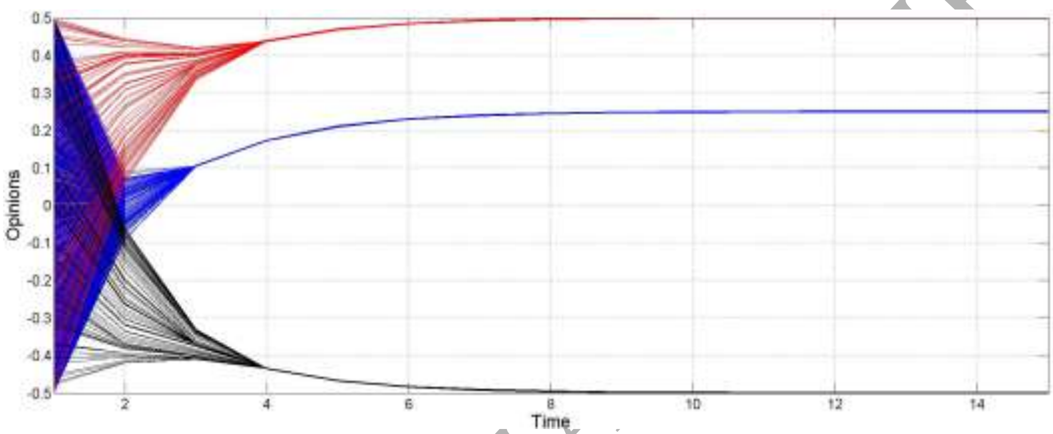
(b)



(c)



(d)



(e)

Figure 5: Evolution of the collective opinions with heterogeneous confidence levels (a) $\varepsilon_i^F \in [0,1]$; (b) $\varepsilon_i^F \in [0.1,1]$; (c) $\varepsilon_i^F \in [0.3,1]$; (d) $\varepsilon_i^F \in [0.4,1]$; (e) $\varepsilon_i^F \in [0.8,1]$

Table 3 presents the number distribution of the opinion followers when reducing the confidence level range, under the condition that the opinion followers have different degrees of trust toward the positive and negative opinion leaders. Compared to Table 2, when the opinion followers have larger degrees of trust toward the positive opinion leaders and these leaders become more influential, the number distribution of the influenced agents presents some substantial changes. Firstly, the pattern of their final opinions being distributed in the three subintervals ($[-0.1,0]$, $[-0.3,-0.2]$, $[0.1,0.2]$) disappears. The final opinions of the followers skew right toward 0.5. Specifically, in addition to $[-0.1,0]$, the opinions of the followers are distributed in the subintervals $[0,0.1]$, $[0.1,0.2]$, $[0.2,0.3]$, and $[0.3,0.4]$, to the right of the middle opinion 0. Secondly, with the narrowing of the confidence level range, some of the opinion followers, whose opinions are distributed in the center interval, begin to move toward the opinion intervals $[0.2,0.3]$ and $[0.3,0.4]$; these are closer to the target opinion of the positive opinion leaders. When the confidence level range is reduced to $[0.4,1]$, all opinion followers are aggregated in the subintervals $[0.2,0.3]$

and $[0.3, 0.4]$. It is worth noting that after the confidence levels ε_i^F have reached $[0.3,1]$, the number of the influenced opinion followers in the subinterval $[0.2, 0.3]$ begins to increase, while that of the influenced opinion followers in the subinterval $[0.3, 0.4]$ correspondingly decreases. After the confidence level range is narrowed to the subinterval $[0.7,1]$, all followers reach a consensus in the subinterval $[0.1, 0.2]$. Compared to when the positive and negative opinion leaders are well matched, the final collective opinions in this experiment are closer to the target opinion of the positive opinion leaders. Lastly, compared to when the positive and negative opinion leaders are well matched, the opinion followers need a greater overall confidence level in order to reach consensus. Specifically, when the opinion followers have equal degrees of trust toward the opinion leaders and the confidence levels satisfy $\varepsilon_i^F \in [0.5,1]$, the opinion followers reach consensus. However, when the opinion followers have unequal degrees of trust, the confidence levels should reach $\varepsilon_i^F \in [0.8,1]$, in order for the followers to come to a final consensus.

Table 3: Number distribution of the influenced opinion followers with heterogeneous confidence levels

ε_i^F \ Distribution	[-0.5,-0.4]	[-0.4,-0.3]	[-0.3,-0.2]	[-0.2,-0.1]	[-0.1,0]	[0,0.1]	[0.1,0.2]	[0.2,0.3]	[0.3,0.4]	[0.4,0.5]
[0,1]	0	0	0	0	497	0	420	746	136	0
[0.1,1]	0	0	0	0	352	21	0	474	952	0
[0.2,1]	0	0	0	0	123	29	0	567	1080	0
[0.3,1]	0	0	0	0	0	32	0	618	1149	0
[0.4,1]	0	0	0	0	0	0	0	721	1078	0
[0.5,1]	0	0	0	0	0	0	0	832	967	0
[0.6,1]	0	0	0	0	0	0	0	1102	697	0
[0.7,1]	0	0	0	0	0	0	0	1460	339	0
[0.8,1]	0	0	0	0	0	0	1799	0	0	0
[0.9,1]	0	0	0	0	0	0	1799	0	0	0

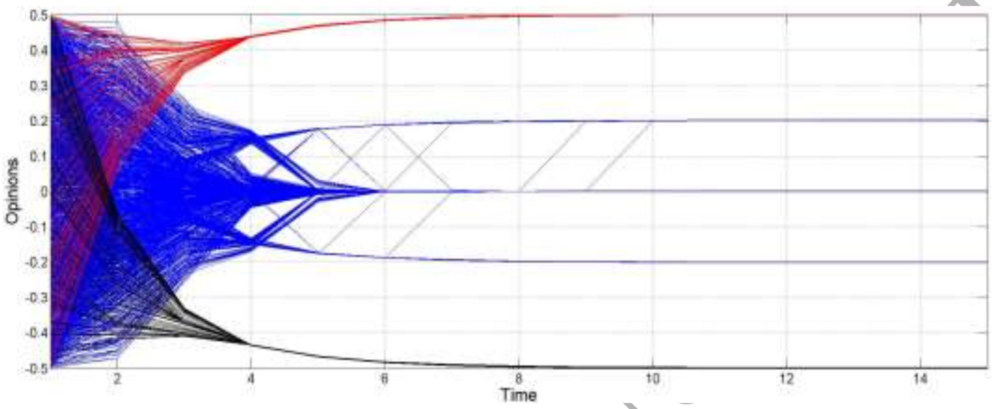
On that account, in a social network that consists of more than one opinion leader group, the confidence levels of the opinion followers are no longer the decisive factor that affects the influence power of the opinion leaders. However, regardless of whether the positive and negative opinion leader subgroups are matched or unmatched, increasing the confidence levels of the followers is helpful in reaching a consensus. Furthermore, although the increase in the degrees of trust toward positive opinion leaders could enhance their influence on the opinion followers, the influence is still not at its maximum potential. These results show that in a social network with multiple opinion leader subgroups, the influence power of either opinion leader subgroup tends to be restricted by the other subgroups—particularly when these subgroups hold opposing target opinions. In addition, it becomes increasingly more difficult for the opinion followers to make decisions and determine their opinion neighbors when multiple leader subgroups exist in the social network.

4.4 Trust Degrees Toward Opinion Leaders

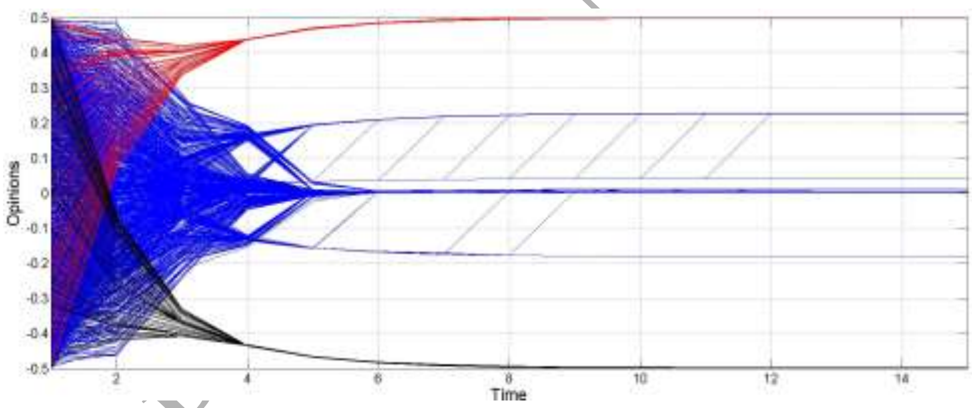
Although the previous section revealed that enhancing the degrees of trust of the opinion followers toward the positive opinion leaders is conducive to the influence power of these leaders, the relationship between the opinion followers' degrees of trust toward the leaders and the influence power of these leaders remains to some extent unclear. In order to check whether the relationship is positively correlated, we again conduct a series of simulation experiments.

Assume that the proportion of opinion followers is $P_F = 0.9$, and that the proportions of positive and negative opinion leaders are both $P_P = P_N = 0.05$. We will investigate the impact of the opinion followers' degrees of trust toward one opinion leader subgroup on the influence power of the opinion leaders as well as the evolution of the collective opinions. It is assumed that the degrees of trust satisfy the condition $\alpha_i + \beta_i = 0.8$. We then gradually increase the opinion followers' degrees of trust toward the positive opinion leader subgroup α_i, and reduce those toward the negative opinion leader subgroup, to investigate the relationship between the trust degree α_i and the influence power of the positive opinion leaders.

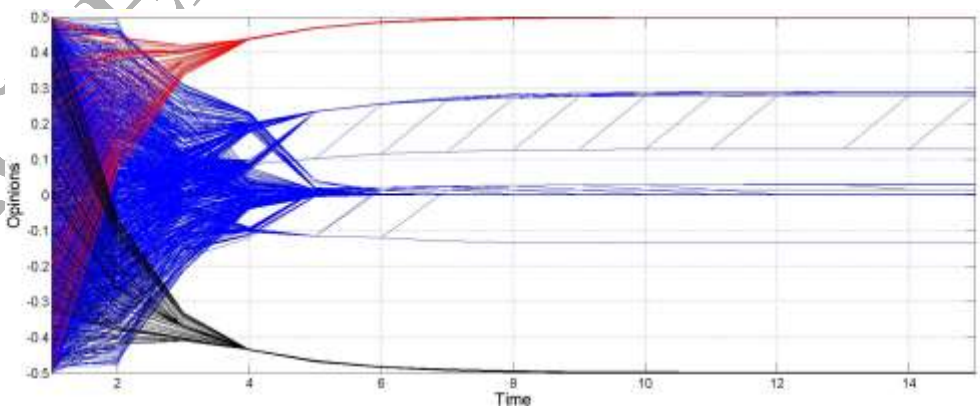
Figure 6 illustrates the evolution of the collective opinions when the degrees of trust of the opinion followers toward the positive opinion leaders take different values. We find that both the positive and negative opinion leaders' opinions relatively swiftly (approximately six time steps) converge to the target opinions 0.5 and -0.5, respectively, while the final opinions of the followers are relatively fragmental and take longer to reach the steady states. When the degrees of trust of the opinion followers toward the positive and negative opinion leaders are the same ($\alpha_i = \beta_i = 0.4$), the opinions of the opinion followers symmetrically form three opinion clusters centered on the intermediate opinion 0 [Figure 6(a)]. When the degrees of trust of the opinion followers toward the positive opinion leaders α_i increase from 0.4 to 0.43, 0.5, 0.6, 0.7, and 0.8, the three opinion clusters begin to shift toward the target opinion 0.5 of the positive opinion leaders. This indicates that the positive opinion leaders influence more opinion followers. Further, when these degrees of trust toward the leaders reach the maximum value α_i=0.8, some of the opinion followers reach consensus on the target opinion of the positive opinion leaders, whereas the opinions of those remaining are concentrated at the center opinion 0 [Figure 6(f)]. Note that none of the opinions of any follower converges to the target opinion -0.5 of the negative subgroup.



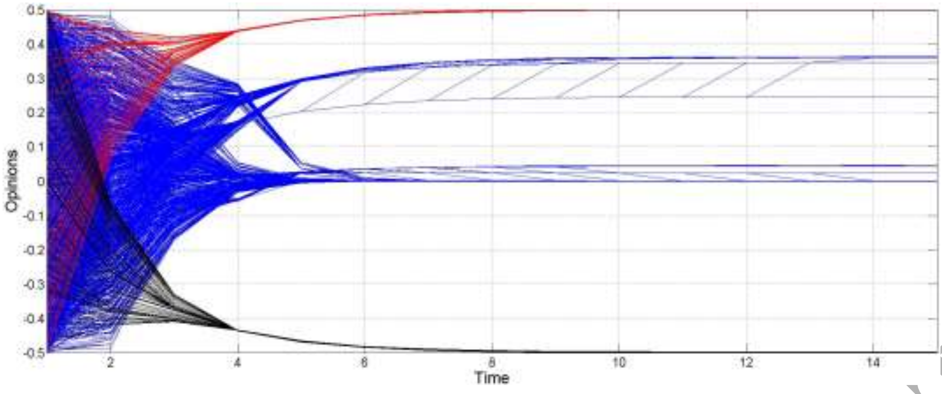
（a）



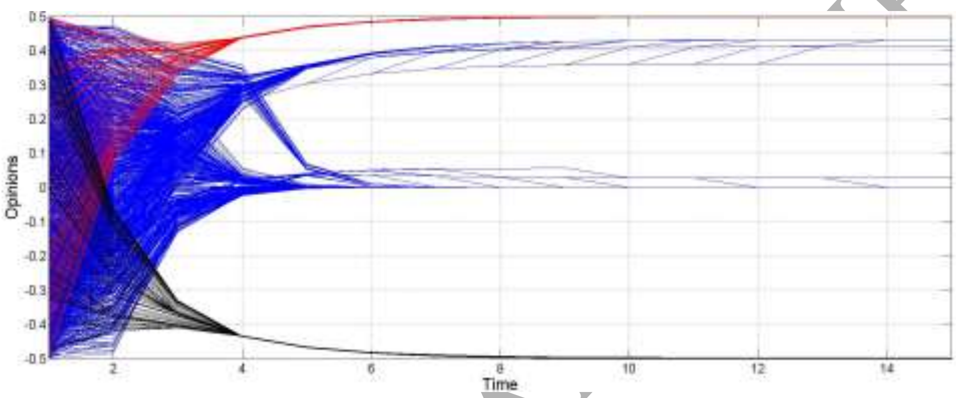
（b）



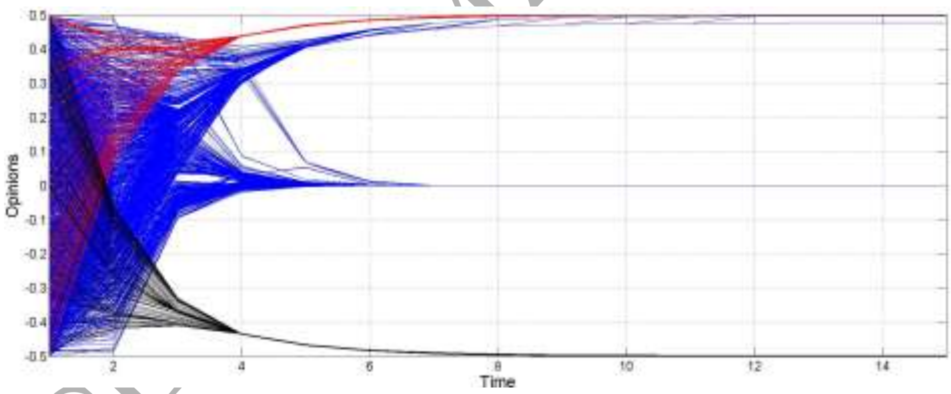
（c）



（d）



（e）



（f）

Figure 6: Evolution of the collective opinions with different degrees of trust
(a) $\alpha_i = 0.40$; (b) $\alpha_i = 0.43$; (c) $\alpha_i = 0.50$; (d) $\alpha_i = 0.60$; (e) $\alpha_i = 0.70$; (f) $\alpha_i = 0.80$

Table 4 presents the detailed relationship between the degrees of trust toward the positive opinion leaders and the number distribution of the opinion followers in the 10 opinion subintervals.

Table 4: Number distribution of the influenced opinion followers with different degrees of trust

Distribution / α_i	[-0.5,-0.4]	[-0.4,-0.3]	[-0.3,-0.2]	[-0.2,-0.1]	[-0.1,0]	[0,0.1]	[0.1,0.2]	[0.2,0.3]	[0.3,0.4]	[0.4,0.5]
0.40	0	0	83	0	1609	0	107	0	0	0
0.41	0	0	63	0	1518	0	218	0	0	0
0.42	0	0	65	0	1420	0	314	0	0	0
0.43	0	0	54	0	1421	0	324	0	0	0
0.44	0	0	39	0	584	800	376	0	0	0
0.45	0	0	36	0	547	768	448	0	0	0
0.50	0	0	0	3	516	681	0	599	0	0
0.60	0	0	0	0	497	0	420	746	136	0
0.70	0	0	0	0	430	0	0	0	1369	0
0.80	0	0	0	0	367	0	0	0	0	1432

The following characteristics can be observed from Table 4 above. First, the numbers distributed in the three subintervals $[-0.3,-0.2]$, $[-0.1,0]$, and $[0.1,0.2]$ have changed. With the increase in the opinion followers' degrees of trust toward the positive leaders, a larger proportion of opinion followers leave the opinion subintervals $[-0.3,-0.2]$ and $[-0.1,0]$, and join in $[0.1,0.2]$, which is closer to the target opinion of the positive opinion leaders. Even if there is a subtle increase, for example, 0.01 of α_i from 0.4 to 0.41, 0.42, 0.43, and 0.45, there is a substantial increase in the number of opinion followers in subinterval $[0.1,0.2]$, and the numbers in the other two intervals tend to correspondingly decrease. When the degrees of trust reach $\alpha_i \geq 0.5$, the number of opinion followers in the interval $[-0.3,-0.2]$ is reduced to zero. Second, there are changes in the distribution of the number of opinion followers across the 10 opinion subintervals. With the enhancement of the opinion followers' degrees of trust toward the positive opinion leaders, their opinions show a tendency to move toward the opinion subinterval that is close to the target opinion of the positive opinion leaders (0.5). When $\alpha_i > 0.5$, apart from some opinion followers that remain in the interval $[-0.1,0]$, the remaining agents all converge to the subinterval that is closer to the target opinion of the positive opinion leaders. When $\alpha_i = 0.6$, the opinion followers are distributed between $[-0.1,0]$, $[0.1,0.2]$, $[0.2,0.3]$, and $[0.3,0.4]$. Third, compared to the situation in which there is only one group of opinion leaders in the network, the influence power of opinion leaders seems to be weakened in a network with multiple opinion leader subgroups when the opinion followers' degrees of trust toward the opinion leaders are the same. For example, when the degrees of trust of the opinion followers toward the positive opinion leaders are $\alpha_i = 0.6$ and $\alpha_i = 0.7$, no opinion follower falls into the range of the target opinion of the positive opinion leader $[0.4, 0.5]$. Fourth, when the opinion followers' degrees of trust toward the positive opinion leaders are $\alpha_i = 0.8$, although there are still negative opinion leaders in the social network, the degrees of the trust toward the negative opinion leaders become $\beta_i = 0$. Thus, the model (3-1) can be revised as follows:

$$x_i^F(t+1) = (1-\alpha_i)\frac{1}{N_i^F(t)}\sum_{j=1}^{N_1} a_{ij}(t)x_j(t) + \alpha_i \frac{1}{N_i^P(t)}\sum_{j=N_1+1}^{N_1+N_2} a_{ij}(t)x_j(t). \tag{4-1}$$

In this case, the opinion updating of the opinion followers under the situation of the model (4-1) is the same as that of the situation in which there is a single opinion leader group in the network. In other words, the negative opinion leaders have no influence on the opinion evolution of the opinion followers.

4.5 Lessons learned

From the above analysis, we note that: (1) The proposed models (3-1)-(3-3) are very general bounded confidence models, which can be reduced to the model (4-1) with one subgroup of opinion leaders, or the HK model without opinion leaders under some conditions. (2) The above simulation experiment fully demonstrates that, when other conditions remain unchanged, and the degrees of the trust of opinion followers toward the positive opinion leaders increase, the influence of positive opinion leaders is likely to grow. On that account, in a social network with multiple opinion leader subgroups, improving opinion followers' degrees of trust toward opinion leaders is an essential approach to enhance the influence power of these opinion leaders. (3) In future, real opinion data could be collected from social networks to validate the proposed models (3-1)-(3-3). In recent years, with the rapid development of Internet technology, people can easily use tools, including Scribe, Chukwa, Kafka, and Flume, to acquire data on social media platforms through techniques such as web crawlers and application programming interfaces.

5. Conclusions

This paper considered a social network with multiple opinion leader subgroups. It established a very general bounded confidence-based opinion dynamics model for opinion leaders and followers, when the opinion leader subgroups possessed different target opinions. We then utilized a computer simulation technique to investigate the relationship between the proportion of opinion leaders, confidence levels of opinion followers, and degrees of trust of opinion followers toward the opinion leaders. The results provided a quantitative analysis for the collective decision-making of a social group in e-commerce networks. In summary, through the comparative analysis of the three factors, the degrees of trust of opinion followers toward opinion leaders have a more important effect on the influence power of opinion leaders. Thus, in order to maximize the propagation effect in e-commerce, enhancing opinion leaders' credibility is a crucial precondition.

We noted that opinion dynamics research generally uses computer simulation methods to investigate the opinion evolution mechanism for different influence factors. When group opinions evolve in an e-commerce environment, we may use some tools, including Scribe, Chukwa, Kafka, and Flume, to acquire opinion data on social media platforms. Future research lies in using the acquired data to test the degree of approximation between mathematical models and the actual processes of opinion dissemination on social media platforms. Thus, it would help to continuously improve the mathematical model, as well as deepen the understanding of the principle of evolution of public opinion.

Acknowledgements

This research has been partially supported by grants from the National Natural Science Foundation of China (#71503206, #71471149 and #71433001), and the Major project of the National Social Science Foundation of China (# 15ZDB153).

References

[1] M. Afshar, M. Asadpour (2010), Opinion formation by informed agents, Journal of Artificial Societies and Social Simulation 13 (4), 1-16.

[2] F. Amblard, G. Deffuant (2004), The role of network topology on extremism propagation with the relative agreement opinion dynamics, Physica A 343, 725-738.

[3] J. A. Balazs, J. D. Velasquez (2016), Opinion mining and information fusion: a survey, Information Fusion, 27, 95-110.

[4] S. Chaudhry, W. Irshad (2013), Opinion leadership and its role in buyer decision making, Academy of Contemporary Research Journal 7(1), 7-14.

[5] M. Chiregi, N. Jafari Navimipour (2017), A new method for trust and reputation evaluation in the cloud environments using the recommendations of opinion leaders' entities and removing the effect of troll entities, Computers in Human Behavior, 60:280-292.

[6] G. Deffuant, F. Amblard, G. Weisbuch (2004), Modelling group opinion shift to extreme: the smooth bounded confidence model, Arxiv:cond-mat/0410199, 1-12.
[7] Y. Dong, Z. Ding, F. Chiclana, E. Herrera-Viedma (2017), Dynamics of public opinions in an online and offline social network. IEEE Transactions on Big Data，DOI:10.1109/TBDATA.2017.2676810.
[8] R. Hegselmann, U. Krause (2002), Opinion dynamics and bounded confidence, models, analysis and simulation, Journal of Artificial Societies and Social Simulation 5(3), 1-33.
[9] E. Katz (1957), The two-step flow of communication: An up-to-date report on an hypothesis, The Public Opinion Quarterly 21(1), 61-78.
[10] P. Kotler, G. Armstrong(2011), Principles of marketing, Prentice Hall.
[11] G. Kou, D. Ergu, Y. Chen (2016), Pairwise comparison matrix in multiple criteria decision making, Technological and Economic Development of Economy, DOI: http://dx.doi.org/10.3846/20294913.2016.1210694, 22(5): 738–765.
[12] G. Kou, D. Ergu, J. Shang (2014), Enhancing data consistency in decision matrix: adapting hadamard model to mitigate judgment contradiction, European Journal of Operational Research, DOI: HTTP://DX.DOI.ORG/10.1016/j.ejor.2013.11.035, 236 (1): 261-271.
[13] G. Kou, C. Lin (2014), A cosine maximization method for the priority vector derivation in AHP, European Journal of Operational Research, DOI: HTTP://DX.DOI.ORG/10.1016/j.ejor.2013.10.019, 235 (2014) 225–23.
[14] G. Kou, Y. Lu, Y. Peng, Y. Shi (2012), Evaluation of classification algorithms using mcdm and rank correlation, International Journal of Information Technology & Decision Making, DOI: HTTP://DX.DOI.ORG/10.1142/S0219622012500095, Vol. 11, Issue: 1, 197-225.
[15] G. Kou, Y. Peng, G. Wang (2014), Evaluation of clustering algorithms for financial risk analysis using mcdm methods, Information Science, DOI: HTTP://DX.DOI.ORG/10.1016/j.ins.2014.02.137s, 275:1-12.
[16] G. Kou, Y. Zhao, Y. Peng, Y. Shi (2012), Multi-level opinion dynamics under bounded confidence, PLoS One 7(9), e43507.
[17] S. Liu, C. Jiang, Z. Lin, Y. Ding, et al. (2015), Identifying effective influencers based on trust for electronic word-of-mouth marketing: A domain-aware approach, Information Sciences, 306, 34–52.
[18] J. Lorenz (2006), Consensus strikes back in the Hegselmann-Krause model of continuous opinion dynamics under bounded confidence, Journal of Artificial Societies and Social Simulation 9, 1- 8.
[19] J. Lorenz (2007), Continuous opinion dynamics under bounded confidence: a survey, International Journal of Modern Physics C 18, 1819-1838.
[20] J. Lorenz (2010), Heterogeneous bounds of confidence, meet, discuss and find consensus, Complexity 4(15), 43-52.
[21] A. Mirtabatabaei, F. Bullo (2012), Opinion dynamics in heterogeneous networks: convergence conjectures and theorems, SIAM Journal on Control and Optimization 50(5), 2763-2785.

[22] M. Mobilia (2003), Does a single zealot affect an infinite group of voters, Physical Review Letters 91 (2), 028701.
[23] E. M. Rogers (1995), Diffusion of Innovations, New York Free Press.
[24] A. Samson (2010), Product usage and firm-generated word of mouth, International Journal of Market Research 52(4), 459-482.
[25] M. R. Solomon(2014), Consumer behavior: buying, having, and being, Prentice Hall.
[26] M. Stone (1961), The opinion pool, The Annals of Mathematical Statistics, 32, 1339-1342.
[27] J. Su, B. Liu, et al. (2014), Co-evolution of opinions and directed adaptive networks in a social group, Journal of Artificial Societies and Social Simulation, 17 (2) 4.
[28] P. Tejavibulya, S. Eiamkanchanalai (2011), The impacts of opinion leaders towards purchase decision engineering under different types of product involvement, Complexity System and Engineering Management 2, 12-22.
[29] J. Turcotte, C. York, J. Irving, et al. (2015), News recommendations from social media opinion leaders: Effects on media trust and information seeking, Journal of Computer-mediated Communication, 20(5):520-535.
[30] J. Villanueva, S. Yoo, et al. (2008), The Impact of marketing-induced versus word-of-mouth customer acquisition on customer equity growth, Journal of Marketing Research 45(1), 48-59.
[31] Y. Wan, B. Ma, Y. Pan (2017), Opinion evolution of online consumer reviews in the e-commerce environment, Electronic Commerce Research, doi:10.1007/s10660-017-9258-7.
[32] G. Weisbuch, G. Deffuant, F. Amblard, et al. (2002), Meet, discuss and segregate, Complexity 7(3), 55-63.
[33] S. Yan, X. Zheng, Y. Wang, et al. (2015), A graph-based comprehensive reputation model: Exploiting the social context of opinions to enhance trust in social commerce, Information Sciences, 318:51-72
[34] S. Yu (2012), The dynamic competitive recommendation algorithm in social network services, Information Sciences, 187, 1–14.
[35] H. Zhang, Y. Dong, E. Herrera-Viedma(2017), Consensus building for the heterogeneous large-scale GDM with the individual concerns and satisfactions. IEEE Transactions on Fuzzy Systems, DOI: 10.1109/TFUZZ.2017.2697403.
[36] Y. Zhang, Q. Liu, S. Zhang (2017), Opinion formation with time-varying bounded confidence. PLoS ONE 12(3): e0172982. https://doi.org/10.1371/journal.pone.0172982.
[37] Y. Zhao, G. Kou, Y. Peng, S. Li (2012), On modeling and analysis of opinion formation with heterogeneous confidence levels for emergencies, Systems Engineering -Theory & Practice, 32(5), 971-976.
[38] Y. Zhao, G. Kou (2014), Bounded confidence based opinion formation for opinion leaders and opinion followers, Studies in Informatics and Control 23(2), 153-162.
[39] Y. Zhao, L. Zhang, M. Tang, G. Kou(2016), Bounded confidence opinion

dynamics with opinion leaders and environmental noises, Computers & Operations Research, vol. 74, pp. 205-213,2016.
[40] X. Zheng, C. Chen, J. Hung, et al. (2015), A hybrid trust-based recommender system for online communities of practice, IEEE Transactions On Learning Technologies, 8(4): 345-356.