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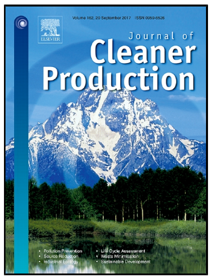

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Ellen Kenia Fraga Coelho, Geraldo Robson Mateus

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A Capacitated Plant Location Model for Reverse Logistics Activities

Ellen Kenia Fraga Coelho[1], Geraldo Robson Mateus[1]

[1]UFMG – Universidade Federal de Minas Gerais.
Avenida Presidente Antônio Carlos, 6627 - Pampulha, Belo Horizonte - MG 31270-901

Email address: ellenkfc@gmail.com, mateus@dcc.ufmg.br.

ABSTRACT

Product remanufacturing is one of the most profitable activities in reverse logistics. Running a business plan, in which companies take responsibility for the waste generated at their end-of-life products, involves making important strategic decisions. One of the challenges in planning the reverse flow of products is decide where installing the reprocessing facilities. This decision influences directly the transport variables costs and the facilities installation fixed costs. This paper proposes a model for the Capacitated Plant Location Problem in Reverse Logistics (CPL-RL), in which we assume that offered material in each collection center is aimed at a single facility for reprocessing. This restriction includes specific cases where there is no logistic availability in the network to send the collected material to different locations. The Mixed Integer Problem (MILP) is solved using an algorithm in two steps. In the first step, reduction tests are performed, which ones determine a priori which facilities are opened /closed. If all facilities are fixed opened or closed then the solution is optimal. Although not all facilities can have their status defined that way, the resultant problem has a less number of variables and it is solved using Benders method. The dataset was randomly generated and the results showed that the applied techniques are appropriate, achieving the optimal solution for all test problems.

Keywords: Reverse Logistics, Reduction Tests, Benders Decomposition, Algorithms.

1. INTRODUCTION

An annual report of the StEP (Solving the E-waste Problem) initiative, in association with Massachusetts Institute of Technology (MIT), estimated that 49 million tons of electronic waste were produced worldwide in 2013, with a predicted increase of 33% over the next five years (Huabo Duan, 2013). The government is greatly concerned with the residues produced, and some measures are being undertaken. The European Union (EU) was a pioneer in this field; in the early 2000s, the EU established three important directives that became laws in February 2003: the European Waste Electrical and Electronic Equipment Directive (WEEE), Restriction of Hazardous Substances Directive (RoHS) and End-of-Life Vehicles Directive (ELV).

The WEEE directive establishes goals for the collection, recovery and recycling of electronic waste per inhabitant per year. The RoHS directive defines manufacturers' restrictions on the components of their new electronic devices. Unlike the WEEE directive, the ELV directive introduced (in 2000) the concept of holding automotive vehicle manufacturers responsible for the end of life of their products. This directive aims to decrease mineral exploration, mainly by reusing heavy metals and several components of used vehicles.

All of these laws have been revised and updated since their implementation. Following a global trend, in December 2010, the Brazilian government approved law 12.305/10, which regulates the National Policy for Solid Waste. In addition to legal requirements, the reasons for companies to engage in sustainable projects include financial gains and Eco-friendly advertising (Fleischmann et al., 2000). The efficacy of reverse logistics may be decreased due to difficulties in controlling the return of products and materials; thus, companies tend to consider the reverse flow of products an expense and consequently their business plans and strategic decisions assign lower priority to reverse flows (Saavedra et al., 2013). Conversely, companies that propose to search for alternatives to the reverse flow of their products may receive economic benefits if their strategy is well considered and implemented. Managers should give as much attention to the reverse flow of products as to the direct flow because when poorly conducted, reverse flows may significantly decrease profits (Blackburn (2004)). Reverse logistics is an important part of sustainable management, which involves a series of activities to ensure that the returned products are reprocessed properly or simply eliminated responsibly (Rogers et al., 1999). Different approaches can be used for the end of life (EOL) of a product; the main ones include reconditioning, recycling, reuse, remanufacturing and repair (Rommert and A., 1999; Saavedra et al., 2013). Remanufacturing stands out among the possibilities for reprocessing used products because it is a process that returns the product with the same characteristics as a new product. Initially, products to be remanufactured undergo a process of disassembly, cleaning, repair, replacement of damaged parts and reassembly. Subsequently, they are subjected to the same tests as new products. Remanufacturing is responsible for 0.4% of the economy of the United States and is an industry that earns approximately US$ 53 billion per year (Steeneck and Sarin, 2013). When we compare the production of computers and peripherals, which earns approximately US$ 56 billion per year (Lund, 1996; Lund, 2010), we notice a strong appeal for companies to focus their efforts on remanufacturing. According to Giutini and Gaudette (2003), the production cost of remanufactured products is approximately 45-60% of the production cost of a new product. These remanufactured products are commercialized in specific markets for 50-70% of the price of a new product. Kerr and Ryan (2001) argue that remanufacturing is the most Eco-efficient way to reuse used products.

However, there is some difficulty in identifying a remanufactured product because some reconditioned products are described as remanufactured. The inaccurate use of terminology affects the understanding of consumers about remanufactured products; thus, consumers do not clearly realize the benefits of purchasing this type of product (Saavedra et al., 2013). The literature contains different types of studies of remanufacturing. A study by Lee et al. (2010), for example, presents the partial disassembly of returned materials as an alternative, thus simplifying the reassembly of remanufactured products. In a recent study, Zeng et al. (2015) proposed an economic alternative of replacing the fixed locations of reprocessing centers with containers. This proposal is interesting regarding mobility but would be suitable only when demands are lower.

One of the major strategic decisions that are important for remanufacturing companies is deciding where to establish these reprocessing locations. In the present study, we propose a Capacitated Plant Location Problem for Reverse Logistics Activities (CPL-RL) and it is modeled based on the Reverse Supply Chain Design Problem (RSCP), (Li, 2011; Santibanez-Gonzalez and Diabat, 2013). We address specific cases in which the flows from collection centers are indivisible. The objective is to minimize the costs of transportation and management as well as the fixed installation cost of reprocessing facilities.

Regarding operational research, several approaches to assist in strategic decision-making are possible. Studies have explored the supply chain design problem with different objectives, from redesigning the logistic network to reducing the transportation of materials (Jayaraman et al., 2003) to minimizing the CO^2 emissions (Kannan et al., 2012; Zhang et al., 2014). Some mathematical models that aim to solve reverse flow problems (and/or to integrate them with direct flow) are addressed in closed-loop supply chain studies. According to Dekker (2004), three areas are considered in reverse logistics: storage, distribution and production. Decisions regarding distribution involve planning the network facilities location to distribute products in the direct flow, reverse flow, storage, collection and reprocessing of returned products.

Several authors have proposed mixed-integer programming models to address reverse logistics problems. The Mixed Integer Programming (MIP) models have been highly used in combinatorial optimization problems, allowing strong formulations (Glover et al., 2011). In general, commercial packages have been used to solve these models (Alumur et al., 2012; Jayaraman et al., 2003; Salema et al., 2010; Zhou and Wang, 2008). In particular, Alumur et al., (2012), solved a multi-period model for a time horizon of five years. Supply chains with closed circuits can be found in the following studies: Amin and Zhang, (2012) proposed a mixed-integer linear programming model to configure the network and extend the model for the condition that the remanufactured products are sent to the secondary market. Barbosa-Povoa et al., (2010) worked with a Closed Loop Supply Chain (CLSC) model extended by adding a separate demand for remanufactured products. Lu and Bostel, (2007) presented a two-level location problem with three types of facility to be located in a specific reverse logistics system. The MIP considers “forward” and “reverse” flows and their mutual interactions. An algorithm based on Lagrangian heuristics was developed. Mutha and Pokharel, 2009 introduced a mathematical model for the design of a reverse logistic network where the returned products need to be consolidated in the warehouse before they are sent to reprocessing centers. Pishvaee et al. (2010) performed a stochastic study that combined multiple objective functions.

Location problems are common in operational research and address important aspects of designing a reverse logistics network (Daskin et al., 2005; Santibanez-Gonzalez and Luna, 2012); for a review, see Eiselt and Marianov, (2011).

The incorporation of more than one level into the Capacitated Plant Location Problem (CPLP) can be found in the following studies: Jayaraman et al. (2003), Li (2011), Santibanez-Gonzalez and Diabat (2013), Tragantalerngsak et al. (2000).

In the latter two studies, the authors chose exact methods. Tragantalerngsak et al. (2000) combined Lagrangian relaxation with a branch-and-bound algorithm, whereas Santibanez-Gonzalez and Diabat (2013) used the classic Benders' decomposition method with pareto-optimal cuts. The two first authors (Jayaraman et al., 2003, Li, 2011) chose to solve their problems using a particle swarm optimization (PSO) heuristic and an iterative concentrator heuristic (CH) in two phases, respectively.

Other studies have used heuristic techniques to solve the CPLP at three levels: Santibanez-Gonzalez and Diabat (2013) developed a two-phase tabu search that divided the logistic network project into a location problem and a reverse flow problem; Min et al. (2006) chose to develop a genetic algorithm for a model to return products from online sales; Sun (2012) generated a hybrid formulation using ADD/DROP procedure with a tabu search heuristic; and Bornstein and Azlan (1998) performed reduction tests and a simulated annealing heuristic. ADD/DROP heuristics (Akinc and Khumawala, 1977) are derived from reduction tests in which decision-making is based on a comparison of the fixed and variable costs for CPLP. A summary of these methods is described by Jacobsen (1983).

Domschke and Drexl (1985) added some priority rules to ensure the viability required to execute the DROP procedure. Conversely, Campêlo and Bornstein (2001), Mateus and Thizy (1999), Mateus and Bornstein (1991), based on the dominance criteria between fixed and variable costs and the submodularity property, defined exact and approximate tests for CPLP.

An economic interpretation of these exact tests, related to the international market, can be found in Mateus and Luna (1992).
The present study provides models and methods to help public, private and mixed organizations plan their reverse logistics networks. Although using classic models and algorithms from the literature, they are extended and improved to solve realistic problems. The main contributions are the indivisibility of the flows from collection centers to reprocessing facilities, extensions of reduction tests for three level plant location problems and integration between reduction tests and Benders decomposition.
In the next sections, the following will be presented: the mathematical model in section 2, techniques used to solve the proposed problem (3, 4), the results (6) and the conclusions in section 7.

2. MATHEMATICAL MODEL

The mixed-integer linear programming (MILP) model for the CPL−RL problem can be described as a two-level CPLP with known demand. The problem approached in the present study considers the model as a single product and the supply of materials available in collection centers as indivisible. The first level includes supply points, the intermediate level includes reprocessing facilities, and the third level includes demand points. In the reverse product flow, we assume that consumers leave their used products at collection centers that group the used products to be reprocessed, such as computers, cell phones and copy machines.
The second part of the process involves a triage of the collected materials, which are then sent to reprocessing sites where they will be cleaned, disassembled and separated according to their condition. Then, the parts are sent to demand points and/or are adequately discarded according to current regulations. Consider a structure in which clients $j \in J$ are the sites of demand for reprocessed products; the set $i \in I$ represents the collection centers for used products, which act as supply points; and $k \in K$ represents the candidate locations for reprocessing facilities.

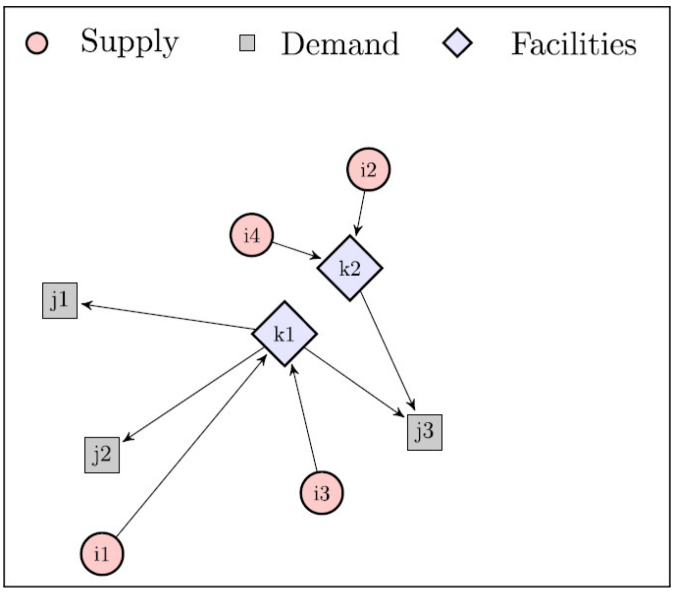

Figure 1: Structure: Supply - Facilities – Demand

As shown in Figure 1, the products received from supply nodes $I = \{i1, i2, i3, i4\}$ arrive at reprocessing facilities $K = \{k1, k2\}$, where they are reprocessed and redistributed to demand locations $J = \{j1, j2, j3\}$.

In this model, the flow that leaves the supply locations I is considered indivisible and is intended for only one reprocessing location. Once reprocessed, the products are sent to demand locations or are discarded, according to their condition. It is worth noting that the demand for used products depends on the success of collection activities. However, supply can be estimated based on the consumption of new products. Each company that is interested in remanufacturing can establish the amount to be supplied in the reverse flow of their products according to their sales and to the end of life of their products. The proposed mathematical formulation aims to minimize both the variable transportation costs of these products and the fixed costs of installing and managing reprocessing facilities. Equations (1) to (8) define a model for CPL-RL. The formulation is based on classic PLC models, in two levels which attends the flow indivisibility from the collection centers.

$$Min \sum_{k \in K} f_k w_k + \sum_{i \in I} \sum_{k \in K} (c_{ik}^0 + f_k^m) a_i x_{ik} + \sum_{k \in K} \sum_{j \in J} c_{kj}^r y_{kj} \tag{1}$$

$$\sum_{i \in I} a_i x_{ik} \leq m_k w_k, \qquad \forall k \in K \tag{2}$$

$$\sum_{i \in I} a_i x_{ik} = \sum_{j \in J} y_{kj}, \qquad \forall k \in K \tag{3}$$

$$\sum_{k \in K} y_{kj} = b_j, \qquad \forall j \in J \tag{4}$$

$$\sum_{k \in K} x_{ik} = 1, \qquad \forall i \in I \tag{5}$$

$$y_{kj} \geq 0, \qquad \forall j \in J, \forall k \in K \tag{6}$$

$$x_{ik} \in \{0,1\}, \qquad \forall i \in I, \forall k \in K \tag{7}$$

$$w_k \in \{0,1\}, \qquad \forall k \in K \tag{8}$$

The sets, constants and variables of the model are described below:

$\mathrm{I} = \{1 \ldots \mathrm{I}\}$, set of supply points;

$\mathrm{J} = \{1 \ldots \mathrm{J}\}$, set of demand points for remanufactured products;

$\mathrm{K} = \{1 \ldots \mathrm{K}\}$, set of candidate locations for the installation of reprocessing facilities;

a_i, amount of supplied products in $i \in I$;

b_j, demand in $j \in J$;

f_k, fixed cost of deploying the reprocessing facility in the intermediate level $k \in K$;

f_k^m, variable cost per unit of reprocessed product in the candidate facility $k \in K$;

c_{ik}^0, transportation cost per unit of product from supply point $i \in I$ to facility $k \in K$;

c_{kj}^r, transportation cost per unit of reprocessed product from facility $k \in K$ to demand point $j \in J$;

m_k , reprocessing facility capacity located at $k \in K$;

w_k, binary variable $w_k = 1$ if facility k is open, $w_k = 0$ otherwise;

x_{ik}, binary variable $x_{ik} = 1$ if the supply node $i \in I$ serves reprocessing facility $k \in K$, $x_{ik} = 0$ otherwise;

y_{kj}, flow of reprocessed products from facility $k \in K$ to demand point $j \in J$;

The objective function (1) minimizes the sum of the installation costs of facilities K, the total transportation costs of products from I to J passing by K and the cost of reprocessing products at K. The constraints (2) ensure that the amount of products supplied in I pass only by open reprocessing facilities and that their capacity limits are respected. The constraints (3) ensure the flow conservation and all products collected in I will pass through K and will be sent to demand points J. The constraints (4) imply that all the demand in J will be met by reprocessing facilities. The constraints (5) guarantee that the products offered in $i \in I$ will be sent to a single reprocessing facility. The constraints (6, 7 and 8) define the non-negativity and integrality of variables.

The proposed model can be solved by a commercial solver which uses exact algorithms. However it is limited to small and not realistic instances and not always converges to optimal solutions. To overcome these limitations a possibility is using some specific exact algorithm for the MIP, such as Benders' Algorithm. Or instead use any of these approaches, is possible solve the problem using greedy heuristics, based on reduction tests, for example.

The first proposal is to apply Benders Method adapted for the CPLP-RL. A second option is initially apply reduction tests, to fix open or closed a subset of facilities, and posteriorly uses Benders method to solve the remaining problem.

However, the presented model is a two-level CPLP, it is a MILP problem that will be solved using a two-steps algorithm. In the first step, reduction tests will be performed; with cuts on the set of feasible solutions. These tests usually determine which facilities will be open or closed, see section 3. In the second step of the algorithm, we use Benders' method to solve the remaining problem, see section 4.

3. REDUCTION TESTS

The complexity of the CLP is related to the number of integer variables, specially the location variables w_k. The main objective of the reduction tests is exactly to fix "a priori" if a facility will be open ($w_k = 1$) or not ($w_k = 0$) in the optimal solution. In this case, the dimension and complexity of the integer problem will be reduced, and the convergence improved.

Using the reduction tests, it is possible to determine if any of the facilities must be open or closed in an optimal solution. Initially, Efroymson and Ray (1966) and Khumawala (1972) worked under the assumption that it was possible to solve relaxed location problems by inspection. Aiming to accelerate the algorithms for the CPLP, Akinc and Khumawala (1977), Mateus and Bornstein (1991), Van Roy (1986) and Jacobsen (1983) introduced exact and approximate reduction tests. These tests can be used to decrease the size of the problem when The complexity of the CLP is related to the number of integer variables, specially the location variables w_k. The main objective of the reduction tests is exactly to fix "a priori" if a facility will be open ($w_k = 1$) or not ($w_k = 0$) in the optimal solution. In this case, the dimension and complexity of the integer problem will be reduced, and the convergence improved. Van Roy (1986) and Akinc and Khumawala (1977), for example, combined reduction tests with the branch-and-bound algorithm. Although tests can decrease the size of the original problem by decreasing the number of integer variables, they usually do not fix all integer variables.

The CPL-RL differs from the traditional facility location problem because, in addition to having a level of intermediate nodes between the supply and demand nodes, this problem considers other costs associated with managing the facilities and the indivisibility of the supply flows. The algorithm developed has the reduction tests in the first step and applies Benders' decomposition to address the remaining problem. The results obtained show that the combined use of the two techniques is successful. In some instances, the optimal solution is obtained only with the application of reduction tests. In others, the decreased size of the problem provides agility to Benders' method and the problems are solved in reduced computation time.

3.1. Exact Reduction Tests

Let V be the set of all feasible solutions x and y for problem (1) to (8) and a vector $w \in \{0,1\}$ with $|K|$ dimension. Consider the set $\mathrm{K} = \{1, 2, .., \mathrm{k}\}$ of candidate sites for a location facility and the subset $S \subset K$ such that the Z function defined in S, $Z(S)$ is minimal. The function $Z(S)$ is defined by equation (9):

$$Z(S) = \begin{cases} Min_{S \subseteq K} \sum_{k \in S} f_k + \sum_{k \in S} \sum_{i \in I} \left(c_{ik}^0 + f_m^k\right) a_i x_{ik} + \sum_{k \in S} \sum_{j \in J} c_{kj}^r y_{kj}, \\ \quad if\ S \neq \emptyset \quad \exists x,y \in V\ and\ \ w_k = 1, \forall\ k \in S \\ \infty, \ if\ S \neq \emptyset \quad \nexists x,y \in V\ and\ \ w_k = 1, \forall k \in S \\ 0, \ \ if\ S = \emptyset \end{cases} \tag{9}$$

To simplify the notation, we will adopt $C_{ik} = c_{ik}^0 + f_k^m$ for every pair(i,k); therefore, $\mathrm{Z}(S)$ can be rewritten by the following formulation:

$$Z(S) = Min_{w \in V} \sum_{k \in S} f_k + \sum_{i \in I} C_{ik} a_i x_{ik} + \sum_{k \in S} \sum_{j \in J} c_{kj}^r y_{kj}$$

Dividing the $Z(S)$ function into two other functions, $W(S)$ and $F(S)$, we have:

$$W(S) = \begin{cases} Min \sum_{k \in S} \sum_{i \in I} C_{ik} a_i x_{ik} + \sum_{k \in S} \sum_{j \in J} c_{kj}^r y_{kj}, \\ \quad if\ S \neq \emptyset \mid \exists x,y \in V\ and\ \ w_k = 1, \forall\ k \in S \\ \infty, \ \ if\ S \neq \emptyset \mid \nexists x,y \in V\ and\ \ w_k = 1, \forall k \in S \\ 0, \ \ if\ S = \emptyset \end{cases} \tag{10}$$

and

$$F(S) = \begin{cases} Min \sum_{k \in S} f_k, \quad if\ S \neq \emptyset \\ \ 0, \ \ if\ S = \emptyset \end{cases}$$

The $F(S)$ and $W(S)$ functions are linear and convex, and the value of the $W(S)$ function coincides with the optimal solution of the minimum cost flow (MCF) problem, with the additional restriction that the flow from each node $i \in I$ from the first layer must be sent to node $k \in K$ only. Therefore, $F(S)$ and $W(S)$ are submodular because $W(S)$ is the value of the objective function for an MCF problem for $S \subseteq K$, and $F(S)$ is the sum of constants f_k. Thus, the function $Z(S)$ is also submodular due to being a linear combination of submodular functions, Nemhauser et al. (1978).
Thus, $Z(S)$, $W(S)$ and $F(S)$ satisfy the following properties:

$$Z(R \cup \{k\}) - Z(R) \leq Z(S \cup \{k\}) - Z(S), \forall R \subseteq S, k \in S \tag{11}$$

$$Z(S - \{k\}) - Z(S) \leq Z(R - \{k\}) - Z(R), \forall R \subseteq S, k \in R \quad (12)$$

The set of K candidate facilities are partitioned into the following subsets:

- $K0$ - Set of closed fixed facilities $w_k = 0$, initially empty .
- $K1$ - Set of open fixed facilities $w_k = 1$, initially empty.
- $K2$ - Set of not fixed facilities, initially $K2 = K$.

Using the definitions of Z in (9) and W in (10) for the sets $K0$, $K1$ and $K2$, we can define the following problems:

$$W(K1 \cup K2) = \begin{cases} Min \sum_{k \in K1 \cup K2} \sum_{i \in I} C_{ik} a_i x_{ik} + \sum_{k \in K1 \cup K2} \sum_{j \in J} c_{kj}^{r} \end{cases} \quad (13)$$

$$Z(K1 \cup K2) = \begin{cases} Min \sum_{k \in K1 \cup K2} f_k + \sum_{k \in K1 \cup K2} \sum_{i \in I} C_{ik} a_i x_{ik} + \sum_{k \in K1 \cup K2} \sum_{j \in J} c_{kj}^{r} \end{cases} \quad (14)$$

The minimum value of function (14) occurs for a subset $S \subseteq K1 \cup K2$ and must be such that $S \supseteq K1$.

The idea behind reduction tests is to determine, by comparing variable and fixed costs, whether there are any advantages to opening (or closing) each of the candidate facilities $k \in K2$. To determine whether a facility $k \in K2$ must be open, we calculate the difference between the objective function for the $MCF(K1 \cup K2)$ problem and the same problem without the facility k, $MCF(K1 \cup K2 - \{k\})$. If this difference Δ_k is greater than or equal to the fixed cost f_k, then the facility k must be fixed open.

Conversely, to determine which facilities must be closed, we calculate the difference between the objective function for the problem $MCF(K1)$ for facilities that are already open and the same problem by adding a facility $k \in K2$, $MCF(K1 \cup \{k\})$. If this difference Ω_k is less than or equal to the fixed cost, facility k must be fixed close.

3.1.1. TEST TO OPEN

For every $k \in K2$,

$$\Delta_k = W(K1 \cup K2 - \{k\}) - W(K1 \cup K2)$$

Δ_k is the variation in the transportation and management costs of the $MCF(K1 \cup K2)$ associated with deactivating the candidate facility k.

Theorem 1. If $\Delta_k \geq f_k$ then the facility k is fixed open, i.e., $w_k = 1$.

Proof: By definition, $(K1 \cup K2)$ is a non-increasing set in terms of the number of elements. Moreover the function $W(.)$ is submodular, according to property (12), and non-decreasing. Thus, Δ_k represents the smallest possible economy, in terms of variable costs, associated with opening the candidate facility k. Therefore, if this minimum economy is higher than the fixed cost the facility k can be fixed open.

3.1.2. TEST TO CLOSE

For every $k \in K2$,

$$\Omega_k = W(K1) - W(K1 \cup \{k\})$$

Ω_k is the variation in the transportation and management costs of the MCF(K1) when activating the empty facility $k \in K2$. It is worth noting that the feasibility of the problem $MCF(K1)$ is a prerequisite of the test to close. Thus, it is required that $\sum_{i \in I} a_i \leq \sum_{k \in K1} m_k$ and $\sum_{k \in K1} m_k \geq \sum_{j \in J} b_j$.

Theorem 2. If $\Omega_k \leq f_k$ then the facility k is fixed close, i.e., $w_k = 0$.
Proof: According to the definition, the set $K1$ is non-decreasing in terms of the number of elements. The function $W(.)$ is submodular, according to property (11), and non-increasing. Thus, Ω_k is the maximum economy obtained, relative to variable costs, when opening the facility k. Thus, if this maximum economy is less than the fixed cost (f_k) of this facility, then k can be fixed close, i.e., $w_k = 0$.

3.2 Algorithm

According to the criteria from theorems (1) and (2), the reduction tests can be applied to set which facilities should be open or close in an optimal solution for the CPL-RL . These tests can be run iteratively: we first run the test to open, followed by the test to close. If any facility is closed, the test to open can be used again. The test to close can also be used again if any new facility is opened and included in $K1$. In general, after the reduction tests are performed, it is not possible to conclude that the facilities that remain in $K2$ will be fixed open (or close). In this case, Benders' method will be used to solve the resulting problem. The higher the number of facilities fixed in the first step of the algorithm, the smaller the problem to be solved in the second step. Although it is not the objective of reduction tests, sometimes the problem can be solved in the first step, when the set $K2 \neq \emptyset$; and the solution obtained with $K1$ is the optimal solution for CPL-RL . Algorithm (1) presents the pseudo-code of this algorithm.

```
Algorithm 1
Initialization   K0 = ∅
                 K1 = ∅
                 K2 = K
Step 1           For all k ∈ K2 do
                     Δ_k = W(K1 ∪ K2 − {k}) − W(K1 ∪ K2)
                     If Δ_k ≥ f_k Then
                         K1 ← K1 ∪ {k}
                         K2 ← K2 − {k}
Step 2           If Σ_{k∈S} m_k ≥ Σ_{i∈I} a_i and Σ_{k∈S} m_k ≥ Σ_{j∈J} b_j Then
                 For all k ∈ K2 do
                         Ω_k = W(K1) − W(K1 ∪ {k})
                     If Ω_k ≤ f_k Then
                         K0 ← K0 ∪ {k}
                         K2 ← K2 − {k}
Step 3           If K2 = ∅        Then Optimal Solution was found.
                 Else       Use Benders' Algorithm for solve the
                            remaining problem.
```

The result, in section (6), demonstrates that the chosen methodology was adequate for CPL-RL. In some cases was not necessary using the Benders' Algorithm, because the problem was completely solve by the Reduction Tests, see Table 2.

4. BENDERS' METHOD

Benders' decomposition (Benders, 1962) is a classic method for solving combinatorial optimization problems based on projections of complicating variables and the generation of constraints. In this method, the model to be solved is separated into two simpler formulations: a master problem (MP) and a dual subproblem (SP). The master problem is a relaxed version of the original problem that contains only a subset of variables and constraints. The subproblem is the original problem in which the variables considered in the master problem are fixed.

Benders' method is based on the iterative solution of the master problem and the subproblem to obtain the optimal solution to the original model. The solution of the master problem provides a dual limit. These variables are then sent to the subproblem, whose solution creates a valid cut for the master problem. The cut is called the optimal cut if the partial solution found for the master problem is a primal limit, whereas it is called feasibility cut when the determination of the partial solution for the master problem implies an unfeasible problem. This procedure is repeated until the GAP defined by the difference between the dual and primal limits is satisfactorily small. When considering the structure of the CPL-RL problem, we observe that the integer variables w and the decision variables x are coupled in constraint (2). Determining the integer variables x and w results in a linear problem in y, i.e., a transportation problem between the facilities and the demand points that can be easily solved. Using this structure in Benders' decomposition, the master problem includes the binary variables and the subproblem determines the flows of products from the facilities already selected to be open to the demand points.

4.1. Decomposition of the CPL-RL Model

In this subsection, Benders' decomposition for the CPL-RL is presented. This model has two set of integer variables x and w; by fixing both of them as (x,w), we can rewrite CPL-RL as:

$$\sum_{k \in K} f_k \overline{w}_k + \sum_{i \in I}\sum_{k \in K} (c^0_{ik} + f^m_k) a_i \overline{x}_{ik} + Min \sum_{k \in K}\sum_{j \in J} c^r_{kj} y_{kj} \quad (15)$$

$$\sum_{j \in J} y_{kj} = \sum_{i \in I} a_i \overline{x}_{ik}, \quad \forall k \in K \quad (16)$$

$$\sum_{k \in K} y_{kj} = b_j, \quad \forall j \in J \quad (17)$$

$$y_{kj} \geq 0, \quad \forall k \in K, \forall j \in J \quad (18)$$

$$x_{ik} \in V', \quad \forall i \in I, \forall k \in K \quad (19)$$

$$\overline{w}_k \in V', \quad \forall k \in K \quad (20)$$

in which $V' \subset V$ is the following set:

$$V' = \begin{cases} \sum_{i \in I} a_i \overline{x}_{ik} \leq m_k \overline{w}_k, & \forall k \in K \\ \sum_{k \in K} \overline{x}_{ik} = 1, & \forall i \in I \end{cases} \quad (21)$$

If the dual variables q_k and r_j are associated with constraints (16) and (20), respectively, the dual problem associated with the problem (15 to 21) is determined by the following formulation:

Dual Sub-problem

$$\sum_{k\in K} f_k\overline{w}_k + \sum_{k\in K}\sum_{i\in I}\left(c_{ik}^0 + f_k^m\right)a_i\overline{x}_{ik} + Max\sum_{k\in K}\sum_{i\in I}a_i\overline{x}_{ik}q_k + \sum_{j\in J}b_jr_j \tag{22}$$

$$r_j + q_k \le c_{kj}^r, \quad \forall j \in J, \forall k \in K \tag{23}$$

$$q_k, r_j \quad free, \quad \forall k \in K, \forall j \in J \tag{24}$$

According to the duality theory, if the primal problem has a feasible solution for fixed values of x and w then the dual problem is limited and its solution is an extreme point of the polyhedron defined by equations (16) to (20). Thus, the optimal cuts are defined according to equation (25):

$$\eta \ge \sum_{k\in K}\sum_{i\in I}a_ix_{ik}q_k + \sum_{j\in J}b_jr_j \tag{25}$$

If the primal solution is unfeasible, the dual problem has an unlimited solution, and an extreme ray of the polyhedron (16 to 20) is generated. Thus, the feasibility cuts can be defined (26) for the CPL-RL as follows:

$$0 \ge \sum_{k\in K}\sum_{i\in I}a_ix_{ik}q_k + \sum_{j\in J}b_jr_j \tag{26}$$

Using the cuts defined in equations (25 and 26), the master problem for the CPL-RL problem is defined by the following formulation:

Master Problem

$$Min\sum_{k\in K} f_kw_k + \sum_{k\in K}\sum_{i\in I}\left(c_{ik}^0 + f_k^m\right)a_ix_{ik} + \eta \tag{27}$$

$$\eta \ge \sum_{k\in K}\sum_{i\in I}a_ix_{ik}q_k + \sum_{j\in J}b_jr_j \tag{28}$$

$$0 \ge \sum_{k\in K}\sum_{i\in I}a_ix_{ik}q_k + \sum_{j\in J}b_jr_j \tag{29}$$

$$w_k, x_{ik} \in V \tag{30}$$

4.2 Algorithm

The algorithm for Benders' method is an iterative procedure that is repeated until the GAP between the upper and lower limits is sufficiently small or lower then ε. Algorithm (2) presents the pseudo-code of this algorithm.

Algorithm 2

Initialization $LB = -\infty$

$UB = +\infty$

$\overline{x}$ and $\overline{w}$ valids

While $UB - LB \leq \varepsilon$ **do**

Solve SP

If SP has Optimal solution **Then**

$Optimality\ Cut$

$UB \leftarrow DOF^{*}$

Else If SP is Unbounded **Then**

$Feasibility\ Cut$

Solve MP $LB \leftarrow POF^{**}$

End

**DOF is the Dual Objective function value* ***POF is the primal Objective function value*

The application of Benders' method to the CPL-RL problem, according to the decomposition presented in this section, obtained good results. Only three instances could not be solved using Benders' algorithm, for the set of 60 instances. The results of section (6) demonstrate that, in addition to saving computation time, all the proposed problems could be solved using the combination of the reduction tests and Benders' method.

5. DATA GENERATION

The set of randomly generated data contains 60 instances with different dimensions between their parameters. The results shows that the techniques used are adequate, reaching the optimal solution for each of the proposed instances.

Following a procedure similar to that found in a study by Jayaraman et al. (2003), data for CPL-RL were generated as follows:

• Two datasets were generated, the first with instances with 100 nodes ($|I| + |J| + |K| = 100$) and the second with 200 nodes ($|I| + |J| + |K| = 200$).

• Each demand location j, reprocessing site k and supply location i were randomly located in a square that measured $100x100$.

• The transportation variable costs of products (c_{ik}^{0} and c_{kj}^{r}) were equivalent to the Euclidian distances between the nodes i and k and between the nodes k and j respectively.

• The amounts of supply a_i and demand b_j were determined according to a uniform distribution between an upper bound (UB) and a lower bound (LB), according to Table 1. To ensure equality, or $\Sigma a_i = \Sigma b_j$, manual adjustment was performed by randomly changing the demands b_j of some facilities.

• The instances dimensions in relation to the numbers of nodes i, j, k were chosen to examine the different possible configurations of the problem associated with large, medium and small numbers of candidate facilities in relation to the supply/demand nodes.

• For instances with the same number of nodes, the parameters are identical, except for costs f_k, f_k^m and capacities m_k that vary between low (L), balanced (B), and high (H) values.

• For instances with high (H) fixed costs, each facility has a capacity that varies uniformly between $[0{,}1\ \Sigma_{i \in I} a_i - 0{,}2 \Sigma_{i \in I} a_i]$.

• For instances with balanced (B) fixed costs, the capacity of each facility ranges between $[0{,}2\ \Sigma_{i \in I} a_i - 0{,}3 \Sigma_{i \in I} a_i]$.

• For instances with low (L) fixed costs, the capacity of each facility ranges uniformly between $[0{,}3\ \Sigma_{i\in I}a_i - 0{,}4\Sigma_{i\in I}a_i]$.

Instance	$\lvert I\rvert x\lvert J\rvert x\lvert K\rvert$	f_k	a_i	b_j	Instance	$\lvert I\rvert x\lvert J\rvert x\lvert K\rvert$	f_k	a_i	b_j
110	50x10x40	H	[700 - 1000]	[800 - 1200]	210	100x20x80	H	[600 - 1200]	[800 - 1400]
111	50x15x35	H	[100 - 600]	[400 -600]	211	100x30x70	H	[400 - 800]	[700 - 1000]
112	40x20x40	H	[100 - 400]	[100 - 400]	212	80x40x80	H	[300 - 700]	[300 - 700]
113	40x25x35	H	[200 - 600]	[100 - 700]	213	80x50x70	H	[300 - 1000]	[500 - 1000]
114	60x10x30	H	[800 - 1500]	[1500 - 2500]	214	120x20x60	H	[1000 - 3000]	[3000 - 5000]
115	60x15x25	H	[100 - 500]	[600 - 1200]	215	120x30x50	H	[1000 - 2000]	[2000 - 5000]
116	70x10x20	H	[1500 - 3000]	[7000 - 9000]	216	140x20x40	H	[1000 - 2000]	[4000 - 7000]
117	70x5x25	H	[800 - 2000]	[2500 - 5000]	217	140x10x50	H	[1000 - 2000]	[3000 - 6000]
118	80x10x10	H	[100 - 500]	[2000 - 2800]	218	160x20x20	H	[300 - 500]	[2000 - 6000]
119	80x5x15	H	[2000 - 8000]	[20000 - 45000]	219	160x10x30	H	[500 - 900]	[3000 - 5000]
120	50x10x40	B	[700 - 1000]	[800 - 1200]	220	100x20x80	B	[600 - 1200]	[800 - 1400]
121	50x15x35	B	[100 - 600]	[400 -600]	221	100x30x70	B	[400 - 800]	[700 - 1000]
122	40x20x40	B	[100 - 400]	[100 - 400]	222	80x40x80	B	[300 - 700]	[300 - 700]
123	40x25x35	B	[200 - 600]	[100 - 700]	223	80x50x70	B	[300 - 1000]	[500 - 1000]
124	60x10x30	B	[800 - 1500]	[1500 - 2500]	224	120x20x60	B	[1000 - 3000]	[3000 - 5000]
125	60x15x25	B	[100 - 500]	[600 - 1200]	225	120x30x50	B	[1000 - 2000]	[2000 - 5000]
126	70x10x20	B	[1500 - 3000]	[7000 - 9000]	226	140x20x40	B	[1000 - 2000]	[4000 - 7000]
127	70x5x25	B	[800 - 2000]	[2500 - 5000]	227	140x10x50	B	[1000 - 2000]	[3000 - 6000]
128	80x10x10	B	[100 - 500]	[2000 - 2800]	228	160x20x20	B	[300 - 500]	[2000 - 6000]
129	80x5x15	B	[2000 - 8000]	[20000 - 45000]	229	160x10x30	B	[500 - 900]	[3000 - 5000]
130	50x10x40	L	[700 - 1000]	[800 - 1200]	230	100x20x80	L	[600 - 1200]	[800 - 1400]
131	50x15x35	L	[100 - 600]	[400 -600]	231	100x30x70	L	[400 - 800]	[700 - 1000]
132	40x20x40	L	[100 - 400]	[100 - 400]	232	80x40x80	L	[300 - 700]	[300 - 700]
133	40x25x35	L	[200 - 600]	[100 - 700]	233	80x50x70	L	[300 - 1000]	[500 - 1000]
134	60x10x30	L	[800 - 1500]	[1500 - 2500]	234	120x20x60	L	[1000 - 3000]	[3000 - 5000]
135	60x15x25	L	[100 - 500]	[600 - 1200]	235	120x30x50	L	[1000 - 2000]	[2000 - 5000]
136	70x10x20	L	[1500 - 3000]	[7000 - 9000]	236	140x20x40	L	[1000 - 2000]	[4000 - 7000]
137	70x5x25	L	[800 - 2000]	[2500 - 5000]	237	140x10x50	L	[1000 - 2000]	[3000 - 6000]
138	80x10x10	L	[100 - 500]	[2000 - 2800]	238	160x20x20	L	[300 - 500]	[2000 - 6000]
139	80x5x15	L	[2000 - 8000]	[20000 - 45000]	239	160x10x30	L	[500 - 900]	[3000 - 5000]

Table 1. Instances Parameters

The set contains 60 instances, 30 with 100 nodes and 30 with 200 nodes. Of the total number of instances, one-third have high fixed costs compared with their variable costs, one-third have balanced fixed and variable costs and the remainder have low fixed costs. The reprocessing capacity of each facility is inversely proportional to its fixed costs; for example, instances that have higher fixed costs have lower reprocessing capacity. This choice may seem contradictory, but according to Alumur et al. (2012), the available sites for installing facilities that are close to large urban centers are smaller and have higher fixed costs. Another configuration with a directly proportional relationship between capacities and costs underwent preliminary testing. The results are similar in most aspects, but the following difference was observed: problems involving facilities with more relaxed capacities are more easily solved. Therefore, the higher the installation cost, the lower the reprocessing capacity and the more computationally complex the problem becomes.

6. COMPUTACIONAL RESULTS

This section presents the computational results of the application of Benders' algorithm (Benders) and the two-steps algorithm (Tests+Benders) whose first step involves reduction tests and whose second step (when required) involves Benders' method. We will discuss the advantages of combining the two techniques and the scope for future studies involving location and reverse logistics problems.

The algorithms were implemented using AMPL programming language, and the linear problems (SP, MP) were solved using CPLEX software, version 12.6. The computational tests were performed using a 64-bit machine with Ubuntu version 13.10 operating system, 7.8 GB RAM memory and Intel core i7-2600 CPU@3.40 GHz x 8.

Table 2 presents the cardinality of sets $|K0|$, $|K1|$ and $|K2|$ after the reduction tests application in the first step. Note that two of the proposed problems (110 and 113) are solved at this step; thus, there is no need to run Benders' algorithm. The remaining problems are partially solved, with the majority of the facilities fixed open in the set $K1$. Thus, applying the reduction tests decreases the number of binary variables and problem constraints, making the problem less costly and more easily solved.

Instances	$\|K\|$	$\|K0\|$	$\|K1\|$	$\|K2\|$	Instances	$\|K\|$	$\|K0\|$	$\|K1\|$	$\|K2\|$
110	10	2	8	0	210	20	0	3	17
111	15	0	7	8	211	30	0	4	26
112	20	0	12	8	212	40	0	21	19
113	25	22	3	0	213	50	0	4	46
114	10	0	6	4	214	20	0	12	8
115	15	0	3	12	215	30	0	13	17
116	10	0	5	5	216	20	0	11	3
117	5	0	1	4	217	10	0	4	6
118	10	0	0	10	218	20	0	6	14
119	5	0	1	4	219	10	0	7	3

Table 2. Cardinality of sets $|K0|$, $|K1|$ and $|K2|$ after reduction tests step.

Table 3 show the problems results which facilities with high fixed costs and low reprocessing capacity (Instances 110-119 and 210-219). Tables 4 and 5 show the problems results of the remaining set of instances (Instances 120-139 and 220-239).

In Table 3 left column are presented the results of Benders' algorithm; the columns are organized as follows: Instance - number of the instance according to data from Table 1; Iterations - the number of iterations performed using Benders' method, i.e., the number of times the MP and SP were solved; Initial GAP - the GAP between UB and LB in the first iteration of Benders' method $GAP = \left(\left[\frac{UB - LB}{UB}\right].100\right)\%$; Total Time - the total running time of the algorithm until the optimal solution for the proposed problem was obtained. On the last line of the table, Average is the arithmetic mean of each parameter measured for the set of instances.

The Benders' algorithm obtains the optimal solution for instances with 100 nodes. The same can be observing when the commercial solver CPLEX is used. Although, for some realistic instances with 200 nodes (211, 213 and 214) the Benders' algorithm and CPLEX did not reach the optimality. In both cases the maximal computational time was fixed in 48 hours. The increase in the number of nodes and the decrease in the capacity of facilities are complicating factors for CPL-RL. When instances with such characteristics are encountered,

alternatives should be sought to overcome these difficulties.
The combination of reduction tests and Benders' method proved very suitable for CPL-RL. This algorithm solved all instances with 100 and 200 nodes while decreasing the running time. Therefore, more complex problems that could not be solved by other methods are solved using this combination.

Benders' algorithm				Tests+Benders					
Instance	Iterations	Initial GAP	Total Time (s)	Instance	Tests time(s)	Benders time (s)	Initial GAP	Benders Iterations	Total time(s)
110	2	9.93%	184.81	110	158.82	-	-	-	158.82
111	2	13.35%	27.82	111	21.28	25.51	0.44%	2	46.79
112	2	22.54%	600.14	112	189.17	222.06	2.06%	2	411.23
113	8	33.29%	0.48	113	1.01	-	-	-	1.01
114	5	13.15%	25.14	114	10.27	11.14	3.53%	3	21.41
115	3	41.36%	3.13	115	11.2	4.67	24.10%	2	15.87
116	3	24.18%	3.19	116	1.11	1.62	2.37%	2	2.73
117	2	25.41%	0.07	117	0.56	0.09	11.38%	2	0.65
118	2	15.58%	0.71	118	0.7	0.76	12.91%	3	1.46
119	7	9.27%	3.66	119	0.33	3.01	2.86%	6	3.34
Average	**3.60**	**20.81%**	**84.92**	**Average**	**39.4**	**33.61**	**7.46%**	**2.75**	**66.33**
210	6	47.45 %	12.81	210	11.67	5.83	26.74%	3	17.5
211	-	-	-	211	339.64	11476.74	14.95%	3	11816.38
212	4	20.55%	7431.94	212	1106.71	1904.2	4.07%	2	3010.91
213	-	-	-	213	62247.77	14725.2	27.89%	4	76972.97
214	-	-	-	214	77060.3	19283.04	5.34%	5	96343.34
215	5	47.77%	46089.57	215	32605.46	843.43	18.96%	3	33448.89
216	4	40.53%	119861.6	216	81351.49	11709.48	8.32%	3	93060.97
217	6	28.19%	14.32	217	6.14	4.9	12.93%	3	11.04
218	3	19.25%	11382.53	218	203.12	11246.51	8.91%	3	11449.63
219	2	13.82%	11178.54	219	9804.69	848.03	1.64%	2	10652.72
Average	**4.29**	**28.35%**	**32659.76**	**Average***	**17869.9**	**3794.626**	**0.116529**	**2.714286**	**21664.52**

Table 3. Benders' algorithm and Tests+Benders results

Table 3 right columns present the results of the two-steps algorithm Tests+Benders. In this case, the total running time of the algorithm is the direct sum of the running time of the reduction tests (Tests Time) and Benders' algorithm (Benders Time). The information provided in these tables reveals a decrease in the running times of the algorithms. The mean time spent by Benders' algorithm on problems with 200 nodes is 32,659.76 seconds, whereas the mean time for Tests+Benders is only 21,664.52 seconds. A comparison of the time spent by Benders' algorithm after the facilities are fixed and the time spent by Benders' algorithm with no previous reduction tests application reveals a reduction of almost 90%, from 32,659.76 seconds to 3,794.63 seconds. The number of iterations and the initial GAP, shown in Table 3, are also decreased comparing the values between Benders' algorithm and Tests+Benders.
The remaining tables present the results of problems with balanced costs (instances 120-129 and 220-229) and with low fixed costs (instances 130-139 and 230-239). According to theorems 1 and 2, the comparison of the variable costs and the fixed costs of a given facility (k), determines whether the facility will be considered open/close in reduction tests. Therefore, when the fixed costs are not as high as the variable costs in the problem, we expect that it will be possible to fix a larger number of variables. In this case was exactly what occurred: the lower the fixed costs, the more problems could be solved running only the first step of the Tests+Benders algorithm.

Instance	Benders Iterations	Initial GAP	Total Time (s)	Instance	Benders Iterations	Initial GAP	Total Time (s)
120	3	30.30%	0.66	220	11	32.89%	1.73
121	3	22.48%	1.57	221	48	35.71%	434.21
122	10	31.82%	23.73	222	6	50.16%	137.09
123	3	45.92 %	1.58	223	5	43.36%	111.8
124	3	47.92 %	0.45	224	30	46.75%	128.53
125	12	36.45 %	29.29	225	21	48.25%	48.52
126	5	26.16 %	1.01	226	17	40.81%	12534.14
127	5	17.62 %	0.12	227	17	29.08%	102.14
128	4	31.07 %	5.4	228	4	49.04%	304.59
129	4	26.21 %	0.52	229	8	35.15%	23.33
Average	**5.2**	**28.20%**	**6.43**	**Average**	**16.7**	**41.12%**	**1382.61**
130	5	33.77%	0.33	230	25	46.04%	3.65
131	19	44.48%	2.77	231	25	51.75%	24.79
132	12	43.13%	1.49	232	13	57.24%	11.35
133	15	45.70%	2.26	233	15	46.68%	51.93
134	7	49.92%	0.53	234	14	50.60%	175.63
135	12	54.84%	3.14	235	43	64.00%	24.21
136	6	39.99%	0.29	236	33	43.99%	52.61
137	4	32.27%	0.1	237	7	43.31%	0.85
138	8	52.43%	0.43	238	5	44.26%	18631.78
139	5	44.11%	0.17	239	7	42.75%	18.34
Average	**9.3**	44.06%	**1.15**	**Average**	**18.7**	**49.06%**	**1899.51**

Table 4. Benders' Algorithm – Balanced Fixed Costs – Low Fixed Costs

Table 4 shows the results of Benders' algorithm for the remaining dataset. All the problems (120-139 and 220-239) could be solved with optimal solutions. Table 5 contains data from the execution of the Tests+Benders algorithm for the same dataset. A comparison of Tables 4 and 5 reveals that most of the problems are solved in the first step of the Tests+Benders algorithm. This result indicates a certain sensitivity of reduction tests to the fixed installation costs. Although the purpose of these tests is not to completely solve the problem, parameters with more balanced dimensions cause the fixing of variables to become more significant.
For problems with 100 nodes (120-139), the running time of the Tests+Benders algorithm is greater compared to Benders' algorithm. Although these smaller problems are simpler, the reduction tests requires that a MCF problem be solved for each tested facility; for this reason, there is a small increase in the mean running time. Conversely, problems with 200 nodes (220-239) highlighted the advantages of using the two-steps algorithm.
These last datasets are generated to diversify the sample and did not have close similarities to real problems. Most of the time, reprocessing facilities has high installation costs and this process is limited by severe budget restrictions. Planning the implementation costs of a reverse network involves several external variables and interferences. The present study provides a general analysis of the problem and may provide a basis for future case studies.

Instance	Tests time(s)	Benders time (s)	Initial GAP	Benders Iterations	Total time(s)	Instance	Tests time(s)	Benders time (s)	Initial GAP	Benders Iterations	Total time(s)
120	17.07	-	-	-	17.07	220	2.87	-	-	-	2.87
121	18.36	-	-	-	18.36	221	12.2	414.47	4.73%	47	426.67
122	2.52	24.32	18.96%	10	26.84	222	80.35	118.63	19.73%	6	198.98
123	1.32	-	-	-	1.32	223	76.98	67.55	16.46%	3	144.53
124	5.17	-	-	-	5.17	224	16.21	15.81	2.14%	13	32.02
125	8.45	6.1	6.04%	12	14.55	225	26.72	-	-	-	26.72
126	1.62	-	-	-	1.62	226	3.89	1899.94	16.54%	20	1903.83
127	0.29	-	-	-	0.29	227	1.63	9.62	2.57%	6	11.25
128	6.71	-	-	-	6.71	228	31.7	123.06	31.82%	3	154.76
129	1.45	-	-	-	1.45	229	1.58	2.74	10.55%	4	4.32
Average	**6.30**	**15.21**	**12.50%**	**11**	**9.34**	**Average**	**25.41**	**331.48**	**13.07%**	**12.75**	**290.60**
130	1.31	-	-	-	1.31	230	6.33	2.38	0.60%	19	8.71
131	1.87	-	-	-	1.87	231	27.22	-	-	-	27.22
132	1.32	1.38	1.94%	11	2.7	232	82.05	4.83	0.66%	6	86.88
133	2.7	1.09	2.20%	9	3.79	233	20.11	19.92	9.50%	14	40.03
134	1.82	-	-	-	1.82	234	121.03	-	-	-	121.03
135	6.88	-	-	-	6.88	235	11.69	-	-	-	11.69
136	0.66	-	-	-	0.66	236	7.06	-	-	-	7.06
137	0.48	-	-	-	0.48	237	2.35	-	-	-	2.35
138	0.64	-	-	-	0.64	238	3648.8	-	-	-	3648.8
139	0.37	-	-	-	0.37	239	22.55	-	-	-	22.55
Average	**1.81**	**1.24**	**2.07%**	**10**	**2.05**	**Average**	**394.92**	**9.04**	**3.59%**	**13**	**397.63**

Table 5. Two-steps Algorithm results - Balanced Fixed Costs - Low Fixed Costs

To improve the analysis of the results found using the two-steps algorithm Tests+Benders, figures (2) to (7) present graphical comparisons of the mean values shown in Tables 2 to 5. For instances with high fixed costs, the mean time spent by the Tests+Benders algorithm is much lower than that used by Benders' algorithm. Figures (2 and 3) show that the difference in time is very significant. Although Benders spend less time than Tests+Benders algorithm as shown in Figure 2 for balanced and low fixed costs. That occurs because the instances are easily solved and the Tests execution time add some seconds at solution process. The same is note true for instances with 200 nodes.

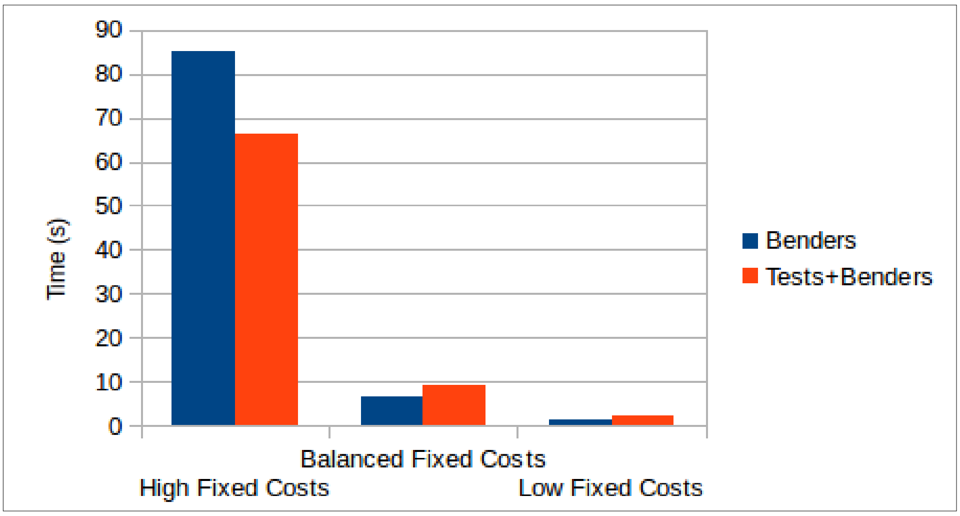

Figure 2: Mean time in seconds - instances with 100 nodes

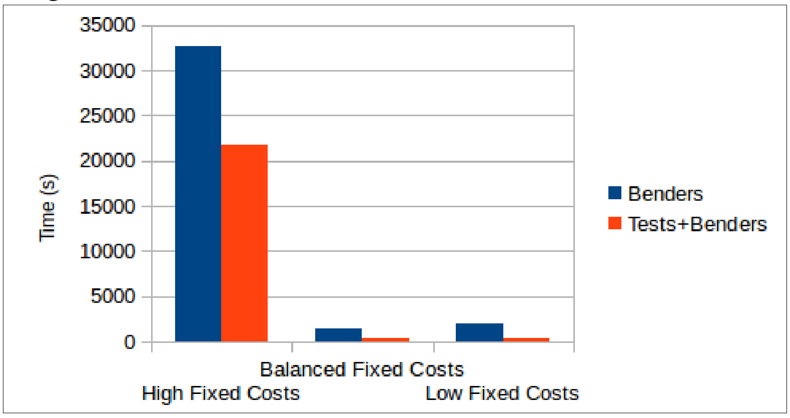

Figure 3: Mean time in seconds - instances with 200 nodes

In addition to time, other important parameters can be analyzed. Figures (4 and 5) show the initial GAPs and the number of Benders' algorithm iterations when reduction tests is performed first. For all instances the GAP are lower when the reduction tests are applied. Moreover, the computational time required to solve the instances is reduced in most cases once the number of iterations and linear problems decrease. Although, in some cases, the average value of iterations has increased to Tests+Benders algorithm. This average referred to few instances, because in this set of problems, many instances are resolved in the first step of the algorithm. In Table 3, case by case, after the Reduction Tests application, the number of Benders algorithm iterations decreased or remained unchanged for all instances.

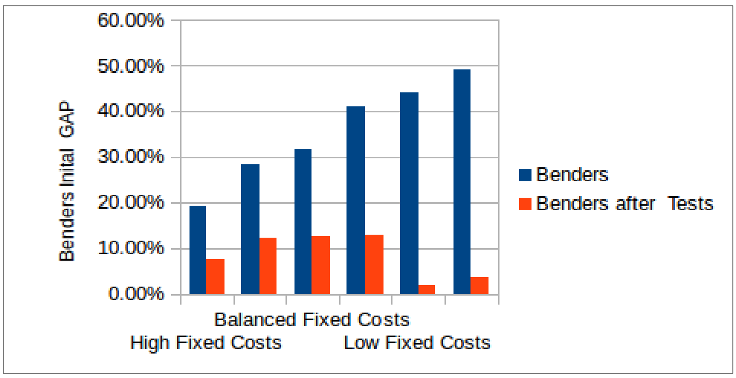

Figure 4: GAP between UB and LB in the first iteration of Benders' algorithm with and without the prior application of reduction tests

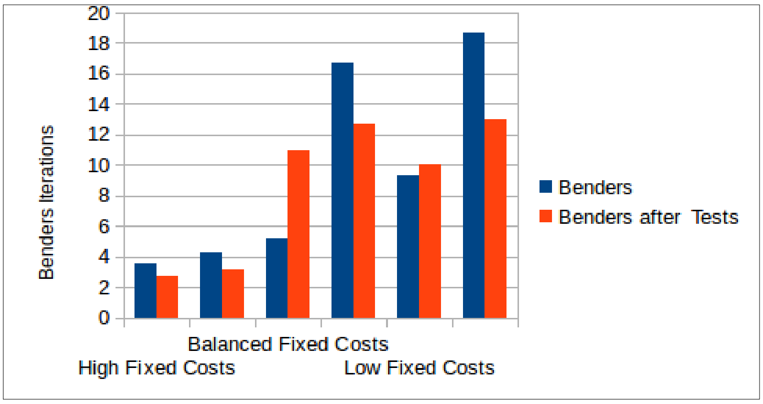

Figure 5: Iterations of Benders' algorithm with and without the prior application of reduction tests

Finally, figures (6 and 7) compare the mean running times of Benders' algorithm with and without the prior application of reduction tests. Decreases in the initial GAP and in the number of iterations of Benders' method in the two-steps algorithm are associated with reductions in the running time of the algorithm.

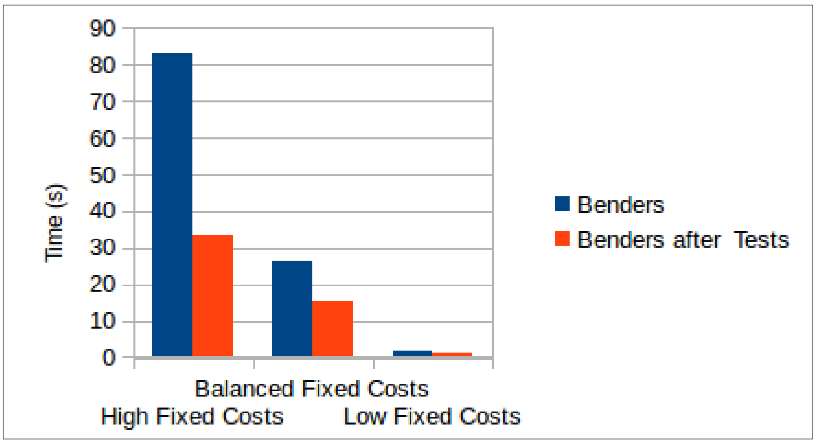

Figure 6: Mean times of Benders' algorithm with and without the prior application of reduction tests - 100 nodes

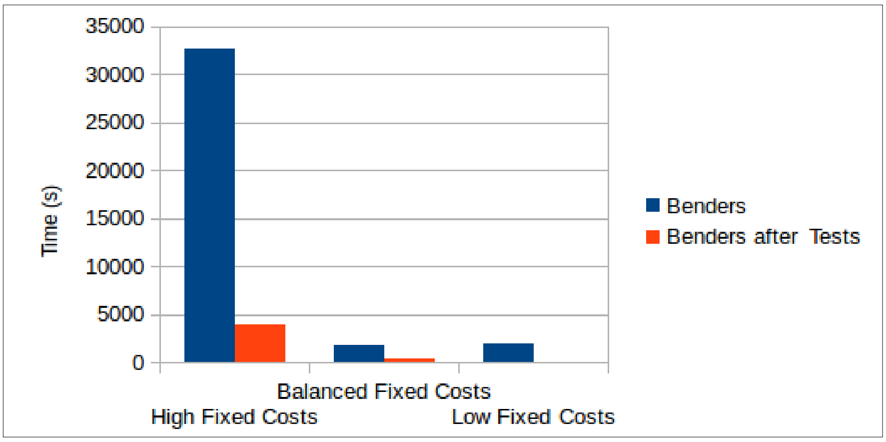

Figure 7: Mean time of Benders' algorithm with and without the prior application of reduction tests - 200 nodes

7. CONCLUSIONS

In the present paper, the capacitated location problem in reverse logistics is analyzed. This problem is a MILP with two levels: supply points ($i \in I$), reprocessing facilities ($k \in K$) and demand points ($j \in J$). The objective is to identify the optimal sites to install reprocessing facilities to minimize the variable costs of transportation and management as well as the fixed costs of installing these facilities. To solve the problem, we propose a Benders' decomposition algorithm and a combination of Benders' method and tests for fixing variables in a single two-steps algorithm. A set of 60 instances was randomly generated, and the performance of both algorithms is evaluated using this set of problems. It became clear that the performance of the Tests+Benders algorithm is superior to Benders' method in terms of computation time and effort. The difficulties imposed by the generated dataset are diversified and the efficiency of the proposed algorithm is demonstrated, considering that this method allows all of the testing problems to be solved, including those that could not be solved directly by the CPLEX solver and Benders' algorithm without reduction tests.

In addition to computational aspects, the present study has social relevance: it provides models and methods to help public, private and mixed organizations plan their reverse logistics networks. Any efforts in these areas, whether in the remanufacturing, recycling or reconditioning of products, prevent the generation of waste. Although some companies try to implement the reverse flow of products with the goal of complying with current legislation (Fleischmann et al., 2000), if the business plan is well executed, this segment can become a very lucrative activity for companies (Blackburn, 2004).

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REFERENCES

Akinc, U., Khumawala, B. M., 1977. An efficient branch and bound algorithm for the capacitated warehouse location problem. Management Science 23 (6), 585–594.
URL http://dx.doi.org/10.1287/mnsc.23.6.585

Alumur, S. A., Nickel, S., da Gama, F. S., Verter, V., 2012. Multi-period reverse logistics network design. European Journal of Operational Research 220 (1), 67 – 78.
URL http://www.sciencedirect.com/science/article/pii/S037722171200032X

Amin, S., Zhang, G., 2012. A proposed mathematical model for closed-loop network configuration based on product life cycle. The International Journal of Advanced Manufacturing Technology 58 (5-8), 791–801.
URL http://dx.doi.org/10.1007/s00170-011-3407-2

Barbosa-P´ovoa, A. P., Gomes Salema, M. I., Novais, A. Q., 2010. Design and Planning of Closed-Loop Supply Chains. Wiley-VCH Verlag GmbH & Co. KGaA, pp. 187–218.
URL http://www.sciencedirect.com/science/article/pii/S0959652616307855

Benders, J., 1962. Partitioning procedures for solving mixed-variables programming problems. Numerische Mathematik 4 (1), 238–252.
URL http://dx.doi.org/10.1007/BF01386316

Blackburn, J.D., G. V.-S. G. V. W. L., 2004. Reverse supply chains for commercial returns. Numerische Mathematik 46 (2), 6–22.
URL http://www.jstor.org/stable/41166207

Bornstein, C. T., Azlan, H. B., 1998. The use of reduction tests and simulated annealing for the capacitated plant location problem. Location Science 6 (1–4), 67 – 81.
URL http://www.sciencedirect.com/science/article/pii/S096683499800062X

Campêlo, M., Bornstein, C. T., 2001. Add/drop procedures for the capacitated plant location problem. Electronic Notes in Discrete Mathematics 7 (0), 102 – 105, brazilian Symposium on Graphs, Algorithms and Combinatorics.
URL http://www.sciencedirect.com/science/article/pii/S1571065304002355

Daskin, M., Snyder, L., Berger, R., 2005. Facility location in supply chain design. In: Langevin, A., Riopel, D. (Eds.), Logistics Systems: Design and Optimization. Springer US, pp. 39–65.
URL http://dx.doi.org/10.1007/0-387-24977-X_2

Dekker, R., Fleishmann, M., Inderfurth K., Van Wassenhove, L.N., 2004. Reverse Logistics, 1st Edition. Springer-Verlag Berlin Heidelberg, Ch. Quantitative Models for Closed-Loop Supply Chains, p. 436.
URL http://www.springer.com/br/book/9783540406969#aboutBook

Domschke, W., Drexl, A., 1985. Add-heuristics' starting procedures for capacitated plant location models. European Journal of Operational Research 21 (1), 47 – 53.
URL http://www.sciencedirect.com/science/article/pii/0377221785900864

Efroymson, M. A., Ray, T. L., 1966. A branch-bound algorithm for plant location. Operations Research 14 (3), 361–368.
URL http://dx.doi.org/10.1287/opre.14.3.361

Eiselt, H., Marianov, V., 2011. Foundations of Location Analysis. International Series in Operations Research & Management Science. Springer.
URL https://books.google.com.br/books?id=6bQ8JJ_Rx6sC

Fleischmann, M., Krikke, H. R., Dekker, R., Flapper, S. D. P., 2000. A characterisation of logistics networks for product recovery. Omega 28 (6), 653 – 666.
URL http://www.sciencedirect.com/science/article/pii/S0305048300000220

Giutini, R., Gaudette, K., 2003. Remanufacturing: The next great opportunity for boosting US productivity. Business Horizons 46 (6), 41 – 48.
URL http://www.sciencedirect.com/science/article/pii/S0007681303000879

Glover, F., Klingman, D., Phillips, N.V., 2011. Network Models in Optimization and Their Applications in Practice. John Wiley & Sons, 304 pages.

Huabo Duan, T. Reed Miller, J. G. R. K., 2013. Quantitative characterization of domestic and transboundary flows of used electronics. Tech. rep., StEP (Solving the E-waste Problem).

Jacobsen, S. K., 1983. Heuristics for the capacitated plant location model. European Journal of Operational Research 12 (3), 253 –261.
URL http://www.sciencedirect.com/science/article/pii/0377221783901959

Jayaraman, V., Patterson, R. A., Rolland, E., 2003. The design of reverse distribution networks: Models and solution procedures. European Journal of Operational Research 150 (1), 128 – 149, O.R. Applied to Health Services.
URL http://www.sciencedirect.com/science/article/pii/S0377221702004976

Kannan, D., Diabat, A., Alrefaei, M., Govindan, K., Yong, G., 2012. A carbon footprint based reverse logistics network design model. Resources, Conservation and Recycling 67 (0), 75 – 79.
URL http://www.sciencedirect.com/science/article/pii/S0921344912000511

Kerr, W., Ryan, C., 2001. Eco-efficiency gains from remanufacturing: A case study of photocopier remanufacturing at fuji xerox Australia. Journal of Cleaner Production 9 (1), 75 – 81.
URL http://www.sciencedirect.com/science/article/pii/S0959652600000329

Khumawala, B. M., 1972. An efficient branch and bound algorithm for the warehouse location problem. Management Science 18 (12), B–718–B–731.
URL http://dx.doi.org/10.1287/mnsc.18.12.B718

Lee, H. B., Cho, N. W., Hong, Y. S., 2010. A hierarchical end-of-life decision model for determining the economic levels of remanufacturing and disassembly under environmental regulations. Journal of Cleaner Production 18 (13), 1276 – 1283.
URL http://www.sciencedirect.com/science/article/pii/S0959652610001617

Li, N., Jan 2011. Research on location of remanufacturing factory based on particle swarm optimization. In: Management Science and Industrial Engineering (MSIE), 2011 International Conference on. pp. 1016–1019.

Lu, Z., Bostel, N., 2007. A facility location model for logistics systems including reverse flows: The case of remanufacturing activities.
Computers & Operations Research 34 (2), 299 – 323, reverse Logistics Reverse Logistics.
URL http://www.sciencedirect.com/science/article/pii/S0305054805000973

Lund, R., 1996. The Remanufacturing Industry: Hidden Giant. Boston University.
URL http://books.google.com.br/books?id=tFBSGwAACAAJ

Lund, R., January 2010. Remanufacturing - an American perspective. IET Conference Proceedings, 1–6(5).
URL http://digital-library.theiet.org/content/conferences/10.1049/cp.2010.0404

Mateus, G., Luna, H., 1992. Decentralized decision-making and capacitated facility location. The Annals of Regional Science 26 (4), 361–377.
URL http://dx.doi.org/10.1007/BF01581867

Mateus, G., Thizy, J., 1999. Exact sequential choice of locations in a network. Annals of Operations Research 86 (0), 199–219.
URL http://dx.doi.org/10.1023/A%3A1018950617642

Mateus, G. R., Bornstein, C. T., 1991. Dominance criteria for the capacitated warehouse location problem. The Journal of the Operational Research Society 42 (2), 145–149.
URL http://www.jstor.org/stable/2583179

Min, H., Ko, H. J., Ko, C. S., 2006. A genetic algorithm approach to developing the multi-echelon reverse logistics network for product returns. Omega 34 (1), 56 – 69.
URL http://www.sciencedirect.com/science/article/pii/S030504830400115X

Mutha, A., Pokharel, S., 2009. Strategic network design for reverse logistics and remanufacturing using new and old product modules. Computers & Industrial Engineering 56 (1), 334 – 346.
URL http://www.sciencedirect.com/science/article/pii/S0360835208001241

Nemhauser, G., Wolsey, L., Fisher, M., 1978. An analysis of approximations for maximizing submodular set functions—i. Mathematical Programming 14 (1), 265–294.
URL http://dx.doi.org/10.1007/BF01588971

Pishvaee, M. S., Farahani, R. Z., Dullaert, W., 2010. A memetic algorithm for bi-objective integrated forward/reverse logistics network design. Computers & Operations Research 37 (6), 1100 – 1112.
URL http://www.sciencedirect.com/science/article/pii/S0305054809002366

Rogers, D., Tibben-Lembke, R., University of Nevada, R. C. f. L. M., 1999. Going Backwards: Reverse Logistics Trends and Practices. Reverse Logistics Executive Council.
URL http://books.google.com.br/books?id=rFJwMgAACAAJ

Rommert, D., A., V. D. L. E., 1999. Gestion des stocks pour la fabrication et la refabrication simultanées : synthèse de résultats récents. Logistique Management 7 (13), 61 – 66.
URL http://www.logistique-management.com/index.php?page=articleart=139

Saavedra, Y. M., Barquet, A. P., Rozenfeld, H., Forcellini, F. A., Ometto, A. R., 2013. Remanufacturing in brazil: case studies on the automotive sector. Journal of Cleaner Production 53 (0), 267 – 276.
URL http://www.sciencedirect.com/science/article/pii/S0959652613001765

Salema, M. I. G., Barbosa-Povoa, A. P., Novais, A. Q., 2010. Simultaneous design and planning of supply chains with reverse flows: A generic modelling framework. European Journal of Operational Research 203 (2), 336 – 349.
URL http://www.sciencedirect.com/science/article/pii/S0377221709005426

Santibanez-Gonzalez, E. D., Diabat, A., 2013. Solving a reverse supply chain design problem by improved Benders decomposition schemes. Computers & Industrial Engineering 66 (4), 889 – 898.
URL http://www.sciencedirect.com/science/article/pii/S0360835213002982

Santibanez-Gonzalez, E. D. R., Luna, H. P., 2012. An evolutionary scheme for solving a reverse supply chain design problem. The International Conference on Artificial Intelligence, 844–850.
URL http://worldcomp-proceedings.com/proc/p2012/ICA2668.pdf

Steeneck, D. W., Sarin, S. C., 2013. Pricing and production planning for reverse supply chain: a review. International Journal of Production Research 51 (23-24), 6972–6989.
URL http://dx.doi.org/10.1080/00207543.2013.836780

Sun, M., 2012. A tabu search heuristic procedure for the capacitated facility location problem. Journal of Heuristics 18 (1), 91–118.
URL http://dx.doi.org/10.1007/s10732-011-9157-3

Tragantalerngsak, S., Holt, J., Ronnqvist, M., 2000. An exact method for the two-echelon, single-source, capacitated facility location problem. European Journal of Operational Research 123 (3), 473 – 489.
URL http://www.sciencedirect.com/science/article/pii/S0377221799001058

Van Roy, T. J., Jan. 1986. A cross decomposition algorithm for capacitated facility location. Oper. Res. 34 (1), 145–163.
URL http://dx.doi.org/10.1287/opre.34.1.145

Zeng, X., Song, Q., Li, J., Yuan, W., Duan, H., Liu, L., 2015. Solving e-waste problem using an integrated mobile recycling plant. Journal of Cleaner Production 90 (0), 55 – 59.
URL http://www.sciencedirect.com/science/article/pii/S0959652614010683

Zhang, Q., Shah, N., Wassick, J., Helling, R., van Egerschot, P., 2014. Sustainable supply chain optimisation: An industrial case study. Computers & Industrial Engineering 74 (0), 68 – 83.
URL http://www.sciencedirect.com/science/article/pii/S0360835214001478

ZHOU, Y., WANG, S., 2008. Generic model of reverse logistics network design. Journal of Transportation Systems Engineering and Information Technology 8 (3), 71 – 78.
URL http://www.sciencedirect.com/science/article/pii/S1570667208600252