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IMF Mode Demixing in EMD for Jitter Analysis

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Highlights

- The problem of mode mixing in the EMD is addressed.
- Noise cancellation based on demixing of Intrinsic Mode Functions constructed by EMD is proposed.
- Application of the proposed method for jitter analysis is presented.

Abstract—We propose a novel noise cancellation method based on the scale-adaptive remixing and demixing of Intrinsic Mode Functions (IMFs) constructed using Empirical Mode Decomposition (EMD). The method addresses the problem of mode mixing in the EMD by performing IMF mode demixing using a heuristic algorithm that minimizes correlation between subsets of second order IMFs generated from partial sums of first order IMFs. An illustrative example using the proposed method for jitter analysis of a noisy random binary sequence is presented. The proposed approach allows achieving better denoising results (evaluated using correlation, Peak-to-Peak value and predictability with AR(4) model) than the classic first IMF discarding approach.

Keywords—digital signal processing; signal denoising; empirical mode decomposition; mode mixing; jitter analysis.

I. INTRODUCTION

Signal decomposition (or source separation) is important in many domains of application (such as telecommunications, speech recognition, biomedical signal analysis, computer vision, seismic signal processing, time series forecasting), where several signals (including noise) have been mixed together into a combined signal and the aim is to recover the original constituents from the mixture signal. Several approaches can be used to perform signal decomposition such as Wavelet Transform [1], Independent Component Analysis (ICA) [2], Prony decomposition [3], Vold-Kalman filter (VKF) [4], blind decorrelation [5], multichannel decorrelation [6], Time Delayed Correlations [7], Denoising Source Separation [8], second generation wavelets [9], generalized eigenvalue decomposition [10], Quadratic Component Analysis [11], Fractal decomposition [12], Singular Spectrum Decomposition (SSD) [13], Biorthogonal Wavelet Decomposition (BWD) [14], Fourier Decomposition (FD) [15], Structural Sparse Decomposition [16], Regular Decomposition [17], Principle Phase Decomposition [18], Singular Value Decomposition (SVD) [19], Intrinsic Mode Decomposition [20], Pulse-Amplitude Modulated (PAM) decomposition [21], Variational Mode Decomposition [22], Transient Decomposition [23], Scattering Decomposition [24], Autoregressive Decomposition [25] and to separate informative and noise signal components. However, the existing methods for signal decomposition have a number of limitations and shortcomings. For example, while Fourier Transform is effective when applied to stationary signals, it makes little physical sense when applied to non-stationary and noisy data.

One of the areas, where signal decomposition is applied is jitter analysis. Jitter is generally defined as the deviation of a timing event of a signal from its ideal position. Jitter affects overall system performance and can be introduced by every circuit element used to convey and to receive a signal. Jitter analysis and decomposition is important for identifying root sources of jitter such as electromagnetic interference, crosstalk and bandwidth limitation, and is crucial in proper design, diagnosing and evaluating high-speed serial communications systems (e.g. PCI-express, SATA) [26]. As communication speeds reach multi-gigabits per second and voltage swings shrink to conserve power, the timing jitter in a communication system becomes a fundamental performance limit nowadays [27]. Jitter is also one of the most common elements in determining the overall bit error rate of a chip design. As a result, jitter generally affects long-term device stability, and understanding the amount of jitter introduced by each element of a system is imperative for predicting overall system performance.

The Total Jitter (TJ) can be decomposed to two characteristic components: Random Jitter (RJ) and Deterministic Jitter (DJ). DJ consists of several subcomponents, the main of which is Sinusoidal Jitter (SJ), which is caused by systematic and data dependent sources such as electromagnetic interference, crosstalk, signal reflection and etc. RJ is caused by unpredictable noise sources such as thermal noise or flicker noise (random motion of particles within a device or transmission media) typically assumed to be characterized by Gaussian distribution with unbounded Gaussian probability density function (PDF). DJ is characterized by its bounded peak-to-peak (PP) value, while RJ is characterized by its root mean square (RMS) value.

Understanding the characteristics of these components helps to identify the root cause of a jitter and establish whether the jitter poses a problem. Historically, jitter analysis relied on visual inspection using eye diagram, statistics histogram, time trend and jitter spectrum. In recent years, there have been several other methods for jitter analysis and decomposition proposed.

Known approaches for jitter decomposition include jitter time-frequency trend analysis using the tail of the probability density function (PDF) [28], wavelet transform [27], time lag correlation (TLC) [29], Fast Fourier Transform with TLC [30], derivatived Gaussian wavelet transform and Fisher's information [31], incoherent undersampling and linear regression [32].

Recently, Hilbert–Huang Transform (HHT) [33] has been widely used to analyze non-linear and non-stationary signals in various applications such as seismic and biomedical signal processing [34]. In HHT, the Empirical Mode Decomposition (EMD) [35] is the key component for decomposing natural signals into intrinsic mode functions (IMFs). The major advantage of EMD is that mother wavelet functions are derived

from the signal itself. Hence, the analysis is adaptive, which is different from the wavelet-based approaches whose mother wavelet functions are fixed. Signal denoising based on EMD has a wide range of applications, such as in biomedical signals, acoustic and ionospheric signals [36]. An EMD-based approach for jitter analysis and decomposition has been proposed by Zhu [26]. According to the simulation experiments, the EMD technique can not only analyze and measure jitter with adaptive and high resolution in the time frequency plain but also decompose and estimate jitter components of short sample length accurately in the time domain. Jitter identification is performed heuristically by identifying the IMF that visually is most similar to the sinusoidal signal.

In this paper, we apply EMD for jitter analysis and decomposition. Since the EMD suffers from mode-mixing problem, the mode demixing method is proposed to solve this problem. This paper is an extended version on [37], where the same problem was addressed, too.

The structure of the remaining parts of the paper is as follows. Section II describes the EMD method and discusses the problem of mode mixing in EMD. Section III describes the proposed mode demixing method. Section IV describes the experimental results and evaluates the results. Finally, section V presents conclusions.

II. EMD AND PROBLEM OF MODE MIXING

A. Preliminaries: classical EMD method

Empirical Mode Decomposition (EMD) is a self-adaptive signal-processing method, which has been applied for analysis and processing of non-stationary signals. The EMD method is based upon the local characteristics of data in the time domain and it can decompose the complicated signal function into a number of its constituent mono-component signals, called Intrinsic Mode Functions (IMFs), which reflect the intrinsic information of the analysed signal. The EMD method is based on the concept of instantaneous frequency defined as the derivative of the phase of an analytic signal. A mono-component signal will have positive and well-defined instantaneous frequency. A signal with multiple modes of oscillation could be decomposed into several IMFs. The aim of EMD is to recognize these oscillatory modes of the signal.

The steps comprising the EMD method are as follows:

1. Identify local maxima and minima of signal $s(t)$, where t is time.
2. Perform cubic spline interpolation between maxima and minima to obtain envelopes $E_{max}(t)$ and $E_{min}(t)$.
3. Calculate the mean of the envelopes as $M(t) = (E_{max}(t) + E_{min}(t)) / 2$.
4. Calculate the difference between a signal and the mean of its envelopes as $c_1(t) = s(t) - M(t)$.
5. IF the number of local extrema of $c_1(t)$, is equal to or differs from the number of zero crossings by 1, and the average of $c_1(t)$ is close to 0, THEN $IMF_1 = c_1(t)$; ELSE repeat steps 1-4 on $c_1(t)$, until new $c_1(t)$ satisfies the conditions of an IMF in Step 5.
6. Calculate residue $R_1(t) = s(t) - c_1(t)$.
7. If residue $R_1(t)$ is above a threshold, then repeat steps 1-6 on $R_1(t)$ to get the next IMF and a new residue.

As a result, n orthogonal IMFs are obtained from which the original signal may be reconstructed by summation:

$$S(t) = \sum_i IMF_i(t) + R(t). \quad (1)$$

When applied to signal denoising, a common approach before reconstructing the signal is to discard the first IMF that contains primarily noise [38]. The method has been criticized, since the first IMF may contain a mixture of noise and meaningful signal components.

B. Problems of EMD

EMD as a method for signal separation has several problems, which have been addressed by several extensions of the EMD method as follows:

- Low frequency resolution – EMD can resolve only distant spectral components differing by more than octave [39], while EMD does not perform well for smaller amplitudes of the second harmonics and cannot distinguish frequencies that are close to each other [40]. For those close frequencies the Hilbert Vibration Decomposing (HVD) [41] method and its improvements [42] is more suitable as they provide better frequency resolution for adaptive decomposition of non-stationary and AM modulated signals.

- Mode mixing – which is defined as a single IMF either consisting of signals of widely disparate scales, or a signal of a similar scale residing in different IMF components as a consequence of signal intermittency [43], or a single mode is fragmented into different IMFs. When mode mixing occurs, the resulting IMFs can be devoid of any physical meaning. Furthermore, mode mixing renders the EMD algorithm unstable as any small perturbation of the signal may result in a completely different set of IMFs.

- Decomposition into false artificial components not present in the initial signal could occur as a result of mode mixing, however, the problem sometimes could be solved using Ensemble Empirical Mode Decomposition (EEMD) [43].

- Unseparability of noise and informative components of a signal – the problem has been addressed by second order decomposition of IMFs [44].

Other problems of EMD such as the end effect [45] and the IMF stopping criterion [46] have been mentioned, too.

Several approaches have been proposed to deal with EMD problems such as additional mathematical operators based on differentiation and integration [40]. The signal is differentiated, EMD applied, and the result is integrated. However, the integration process is adding errors, which can corrupt the patterns in the decomposed signal. Mode mixing also has been addressed by Variational Mode Decomposition [47], which performs decomposition into band-limited IMFs, and Analytical Mode Decomposition [48], which allows to extend the EMD approach to the decomposition of a non-stationary and nonlinear signal with two or more amplitude-decaying and frequency-changing components. In [49], frequency shifting is proposed as a preprocessing technique to be applied before EMD, which involves non-linear mixing of the distorted signal with a pure tone of frequency greater than the highest frequency present in the distorted signal. In [50], the mode mixing problem is addressed by providing a new definition of the envelope and introducing an envelope approximation algorithm. In [51], the sinusoidal-like functions are added into data to help the signal extraction, while in [43] a noise is added to the data to provide a uniformly distributed reference scale for IMFs, and in [52] energy conservation criterion is used to cancel pseudo mode components (false IMFs).

III. PROPOSED MODE DEMIXING METHOD

The classic EMD is an example of Single-Input Multiple-Output (SIMO) system: for the single input of data signal X it produces multiple IMFs which constitute the decomposition of X (see Fig. 1). The obtained IMFs, however, suffer from the mode mixing problem.

The proposed approach applies the ideas from wireless communication domain to deal with interference (crosstalk) problem in multi-channel digital transmission systems. In a variety of contexts, observations are made of the outputs of an unknown multiple-input multiple-output (MIMO) linear system, from which it is of interest to identify the unknown system and to recover the input signals [5].

Following this idea, we propose a post-processing stage for the classic EMD approach to perform the IMF demixing. The method consists of the consecutive application of the following stages for each pair of adjacent

IMFs as follows: remixing (summation), application of the classic EMD, splitting of new IMFs into non-overlapping subsets with a minimal correlation, and summation to obtain the demixed IMFs. The splitting is performed by finding a cut in the set of IMFs that provides the smallest correlation between the sum of IMFs with higher frequency and the sum of IMFs with lower frequency as follows:

$$\arg \min_{1 < k < n} \text{corr} \left(\sum_{1 \leq i \leq k} \overline{IMF}_i, \sum_{k < i \leq n} \overline{IMF}_i \right), \quad (2)$$

here n is the number of IMFs derived from the signal.

The algorithm is summarized in Fig. 2.

ALGORITHM: DemixIMFs

BEGIN

Sort IMFs by decreasing frequency

FOR ALL pairs of adjacent IMFs: IMF_i and IMF_{i+1}

LET S be the sum of IMF_i and IMF_{i+1}

Apply EMD to S

LET \overline{IMF} be the IMFs of S

Split \overline{IMF} into non-overlapping subsets \overline{IMF}' and

\overline{IMF}'' with smallest correlation between them

LET IMF_i^* be the sum of \overline{IMF}'

LET IMF_{i+1}^* be the sum of \overline{IMF}''

END FOR ALL

END ALGORITHM

Fig. 1. Algorithm of IMF mode demixing

The method is depicted graphically in Fig. 3.

IMF mode demixing using 2x2 MIMO approach.

IV. EXPERIMENTS

We used MATLAB to generate a signal and perform the jitter decomposition. The signal generated in the experiment was modelled as a random binary sequence consisting of 512 symbols (sampling rate – 128 samples per symbol). The signal was filtered using a Gaussian filter and a Gaussian noise with 20 db signal-to-noise ratio (SNR) was added.

The aim of the experiment is to compare the classic approach of the EMD-based denoising, which discards the first IMF (further – IMF_1), with the approach proposed in this paper (discarding of the demixed IMF_1). Fig. 4 shows a fragment of the original (noise-free) signal and Fig. 5 shows a signal with jitter added as a time series of length 1024 (representing a binary sequence of 10111011).

Fig. 6 shows the simulated representation of the noisy signal on the oscilloscope screen. Constant binary 1 and 0 levels are shown, as well as transitions from 0 to 1 and from 1 to 0. Note the noisiness of the signal representation.

The IMFs obtained using the classic EMD approach are given in Fig. 7, while the IMFs after mode demixing using the proposed approach are presented in Fig. 8.

Fig. 9 shows the sample of the signal denoised using classic approach, while Fig. 10 shows the sample of the signal after performing denoising using the proposed approach. Note that decreased noisiness of the signal using the proposed approach.

Fig. 11 shows the eye diagram of the signal after performing denoising using the classic approach, while Fig. 12 shows the eye diagram of the signal after performing denoising using the proposed approach. Note that reduced noisiness of the diagram as compared to Fig. 6.

Sample of signal denoised using classic approach.

Fig. 2. IMFs of noisy signal after IMF mode demixing.

Eye diagram of the signal after denoising using classic approach.

Fig. 3. Eye diagram of the signal after denoising using proposed approach.

As a random variable, random jitter should have little correlation with deterministic jitter [16]. The effectiveness of denoising was evaluated numerically using the following metrics: average correlation of IMF₁ with noise (0.7472 for classic IMF₁ and 0.8788 for proposed IMF₁; higher value is better), and average PP of the denoised signal (1.1437 for classic EMD-based denoising and 1.0908 for the proposed approach; lower value is better).

The value distributions of noise, IMF₁ from classic EMD and IMF₁ from the proposed approach using an estimate of probability density function (PDF) based on a Gaussian kernel function (see Fig. 13). Estimates were obtained by segmenting the signal into segments of length 2048 and calculating the IMFs for each segment separately. Note that IMF₁ from the classic EMD approach clearly has the bimodal distribution suggesting that it has been produced by a mixture of signals from two independent sources, while the IMF₁ from the proposed approach has a unimodal distribution and follows the PDF of noise closely (correlation of classic IMF₁ with noise PDF is 0.8841, while for demixed IMF₁ it is 0.9853).

PDFs of noise, IMF₁ from classic EMD, and IMF₁ from the proposed approach.

Fig. 4. Correlation (with confidence intervals) of noise with IMF₁ vs noise SNR using classic EMD and the proposed approach.

For a more extensive validation, we have performed experiments with different values of noise SNR. The results are presented in Figs. 14 and 15 for the correlation and PP values with confidence intervals corresponding to one standard deviation given, respectively. In case of correlation, the Fisher's z-transformation [53] was applied to normalize the data.

The proposed approach is consistently better at noise vs IMF₁ correlation (grand mean is 0.875 ± 0.003 vs 0.745 ± 0.002 of the classic EMD) and does not depend on the SNR of noise ($R^2 = -0.0018$ and $R^2 = -0.0009$, respectively). While for the PP value, the approach is consistently better for smaller noise SNR values and decays exponentially ($R^2 = -0.8594$ and $R^2 = -0.8081$, respectively).

The significant statistical difference between two correlation populations has been confirmed by the two sample t-test, which has rejected the hypothesis that the samples came from normal distributions with equal means ($p = 5.9 \cdot 10^{-44}$).

PP values (with confidence intervals) of denoised signal vs noise SNR using classic EMD and the proposed approach.

Finally, predictability of IMF_1 was evaluated and compared with that of the noise signal using the autoregressive AR(4) model (the order of the model was selected based on the autoregression function plot). Since the noise component is generated randomly, it has low predictability (evaluated using the goodness-of-fit of the AR model).

The results (in terms of goodness-of-fit) are presented in Fig. 16. The results show that the predictability of the IMF_1 after the application of the proposed demixing approach has been reduced (grand mean fit 3.3%) meaning that it is capturing the original noise signal more effectively than IMF_1 derived using the classical EMD (grand mean fit 3.6%), but it is still worse than predictability of the original noise component (grand mean fit 1.8%).

Predictability of demixed IMF_1 vs classical IMF_1 and noise.

In order to ensure the completeness and orthogonality of EMD, all IMFs should reconstruct the original data set and be orthogonal to each other [54]. However, in practice the derived IMFs may not fulfil the requirement of orthogonality. To evaluate the orthogonality of decomposition Huang *et al.* [54] proposed the Total Orthogonality Index (OI_T) and the Partial Orthogonality Index (OI_P). Here we use OI_P to evaluate the contribution of IMF_1 for total orthogonality of IMFs obtained using the proposed demixing approach.

The partial orthogonality results are presented in Fig. 17. The Partial Orthogonality of IMF_1 after demixing using the proposed approach has increased as compared to IMF_1 obtained using the classical EMD.

Summarizing, the simulation results show that the proposed method works better both in terms of decorrelation (the obtained RJ correlates better with the original noise signal in the jitter) and Peak-to-Peak (PP) value (PP of the denoised signal is closer to the PP of the original noise-free signal) than the classical EMD-based denoising approach. The demixed IMF_1 is closer to noise in terms of its predictability (fit) evaluated using the autoregressive AR(4) model. Also the demixing of IMFs has allowed to improve the orthogonality of the entire set of IMFs.

Orthogonality of classic IMFs vs demixed IMFs.

V. CONCLUSIONS

A novel method has been proposed to eliminate the phenomenon of mode mixing that exists in Empirical Mode Decomposition (EMD). The method remixes the adjacent Intrinsic Mode Functions (IMFs) and repeats the EMD procedure to derive IMFs that are more separated in terms of correlation (i.e., are less correlated). The proposed method has been verified to be effective for analyzing and denoising the simulated jitter signal. Using the proposed method, we can analyze jitter signal and decompose jitter into random jitter (RJ) (the first IMF – IMF_1) and deterministic jitter (DJ) (the sum of the remaining IMFs) in the time domain.

Furthermore, the proposed method can be used for deriving more physically meaningful IMFs. However more research still is needed in analyzing the results of the method when applied to different types of jitter. In future work, we intend to apply the proposed method for decomposition, analysis and denoising of complex noisy biological signals such as EEG, EMG, ECG and gaze tracking data.

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Fig 1 Classical EMD: SIMO (Single Input Multiple Output).

Fig 2 Algorithm of IMF mode demixing

Fig 3 IMF mode demixing using 2x2 MIMO approach.

Fig 4 Signal as time series plot.

Fig 5 Signal with jitter.

Fig 6 Eye diagram of the noisy signal.

Fig 7 Classic IMFs of noisy signal.

Fig 8 IMFs of noisy signal after IMF mode demixing.

Fig 9 Sample of signal denoised using classic approach.

Fig 10 IMFs of noisy signal after IMF mode demixing.

Fig 11 Eye diagram of the signal after denoising using classic approach.

Fig 12 Eye diagram of the signal after denoising using proposed approach.

Fig 13 PDFs of noise, IMF_1 from classic EMD, and IMF_1 from the proposed approach.

Fig 14 Correlation (with confidence intervals) of noise with IMF_1 vs noise SNR using classic EMD and the proposed approach.

Fig 15 PP values (with confidence intervals) of denoised signal vs noise SNR using classic EMD and the proposed approach.

Fig 16 Predictability of demixed IMF_1 vs classical IMF_1 and noise.

Fig 17 Orthogonality of classic IMFs vs demixed IMFs.