Journal of Manufacturing Systems 38 (2016) 1–12

Contents lists available at ScienceDirect

Journal of Manufacturing Systems

journal homepage: www.elsevier.com/locate/jmansys

Analyzing a lost-sale stochastic inventory model with Markov-modulated demands: A simulation-based optimization study

Rafael Diaz[a,*], Mike P. Bailey[b], Sameer Kumar[c,1]

[a] MIT Global Scale, MIT-Zaragoza International Logistics Program, Zaragoza Logistics Center, Zaragoza 50197, Spain
[b] Combat Development and Integration, U.S. Marine Corps Combat Development Command, Quantico, VA 22134, USA
[c] Operations and Supply Chain Management, Opus College of Business, University of St. Thomas, 1000 LaSalle Avenue, Minneapolis, MN 55403-2005, USA

ARTICLE INFO

Article history:
Received 5 January 2015
Received in revised form 16 July 2015
Accepted 16 September 2015

Keywords:
Stochastic inventory model
Autocorrelation
Simulation-based optimization
Simulated annealing
Ranking and selection methods
Pattern search

ABSTRACT

A consumer demand that presents auto-correlated components is a class of demand commonly found in competitive markets in which consumers may develop preferences for certain products which influence their willingness to purchase them again. This behavior may be observed in inventory systems whose products are subject to promotion plans in which mechanisms that incentivize the demand are implemented. Inventory systems that ignore these dependency components may severely impair their performance. This paper analyzes a stochastic inventory model where the control review system is periodic, is categorized as a lost-sale case, and is exposed to this class of auto-correlated demand pattern. The demand for products is characterized as a discrete Markov-modulated demand in which product quantities of the same item may relate to one another according to an empirical probability distribution. A simulation-based optimization that combines simulated annealing, pattern search, and ranking and selection (SAPS&RS) methods to approximate near-optimal solutions to this problem is employed. Lower and upper bounds for a range of near-optimal solutions are determined by the pattern search step enhanced by ranking and selection—indifferent zone. Results indicate that inventory performance significantly declines as the autocorrelation increases and is disregarded.

1. Introduction

A supply chain refers to all parts that are systematically involved in fulfilling a customer demand [1]. As such, a supply chain entails the coordination of resources to move goods or provide services from firms to consumers. In this sense, consumer demand is a critical component of the supply chain that has significant implications for the operational and strategic goals of the firm. Thus, understanding the effects of uncertainty on demand is a long-standing interest from practical and theoretical perspectives.

Demand uncertainties may have an impact on supply chain performance, and therefore, it may compromise firm's ability to control costs and make profits. These demand uncertainties may be substantial in some settings. Such demand fluctuations oblige operational and strategic managers to frequently review decisions related to sales and operations planning. Firms that possess the ability to adapt and change to these demand fluctuations are more likely to succeed in highly competitive environments.

Recognizing and addressing issues related to planning and controlling operations subject to uncertain demand may be a differentiator that distinguishes successful supply chain and others. Understanding both relevant sources of uncertainty and their effects is imperative for addressing these issues. This necessarily requires the use of a capable framework that allows managers to process this information and create solutions that minimize the adverse effects on the firm. This information may enable the firm to address these negative effects, and hence, maximize opportunities to better match the supply with the demand.

While uncertain demands continuously change over time, the arrangement of supply chain assets commonly shows high degrees of rigidity. Generating solutions to address fluctuating demand issues in this environment may be challenging. Managers employ supply chain management techniques (e.g., reduce inventory) to effectively manage resources and products to maximize supply chain surpluses and minimize risks [1]. From the supply perspective, inventory and capacity management have been recognized as

* Corresponding author. Tel.: +34 976 077 600.
E-mail addresses: diazr@mit.edu (R. Diaz), michael.bailey@usmc.mil (M.P. Bailey), sameerkumar724@gmail.com (S. Kumar).
[1] Tel.: +1 651 962 4350.

http://dx.doi.org/10.1016/j.jmsy.2015.09.007
0278-6125/

effective tools for balancing the supply and demand. Conversely, from the demand angle, advertising and marketing tools may be used to promote increases in the demand. Furthermore, information, sourcing, and pricing are considered cross-functional levers that enhance supply chain management.

Pricing is a cornerstone for spurring revenue increases that stem from supply chain assets utilization. Clearly, the amount of product required by the uncertain consumer demand is shaped by pricing decisions that invariably leads to revenue generation. Pricing decisions are central in promoting sales, and therefore, in the ability to generate revenues by managers. However, promotion policies are one of the most important contributors of the "bullwhip effect" [2]. Bullwhip effect refers to the increased variance in demand that is observed as we move upstream in a supply chain. Thus, the demand variance shows to be greater in supplier orders than in retailer orders or sales. Supply chain members may be misled in making inventory decisions, and hence, experience significant losses as the information from these orders are inaccurate. However, the use of promotions is prevalent in supply chains as many organizations are inclined to use pricing or quantity discounts to spur revenues while engaging in riskier decisions that may erode prospective profits.

An accurate analysis of the effects of promotion or another type of pricing vehicle to increase the demand over other logistical levers such as inventories may be needed. Anticipating the inventory and capacity policies that support the implementation of such mechanisms is critical in maintaining a firm's competitive edge. Pricing promotion and quantity discount decisions that are frequently practiced in retail environments may be improved from properly considering the effects that such incentives may have on inventory management. In this environment, it is well known that advertising campaigns promote the buying of a product (or combination of products) while receiving another number of items free of charge or at discount. These promotions may create a dependency effect (e.g., induced autocorrelation) on the probabilistic demand that may be determined by analyzing and modeling its stochastic pattern.

Ref. [3] shows that a positive autocorrelation is commonly found among a large number of retail products in varying proportions. However, most inventory models developed in the literature, assume that demand can be described as a continuous function whose observations are identically independently distributed (IID). This assumption may be misleading as the performance of inventory systems that fail to consider dependency components declines as increases in demand variability produces stockouts that quickly compromise acceptable service levels [2,4]. Some authors acknowledge this dependency and formulate demands as serially correlated characterizations (e.g., Miller [5,6], and [3,7]. The use of continuous formulations is largely predominant among the few inventory models that consider auto-correlated demands (e.g., Kurata and Liu [8], and Diaz and Ezell [9] employ Autoregressive AR(1)).

Dependency patterns may be complex and largely dependent on the promotion scheme that affects customer behavior. For example, a consumer requesting a product may additionally request two more identical items if the promotion policy offers a benefit according to a probability distribution. Likewise, for another set of customers, the promotion policy might vary, and hence, induce a different demand patterns for the consumption of the same product. Customer segmentation that demands the same product at different rates has been largely studied in the marketing literature. These different auto-correlated demand patterns for the same product may have important consequences for the inventory. One way that consumer demand with the described autocorrelation patterns can be formulated consists of using a Discrete Markov-modulated Chain (DMC) formulation. Finite quantity demanded can be modeled as discrete states that are connected through transition probabilities that generate auto-correlated dependencies constrained by limits in the quantities demanded by the promotion. The mathematical formulation of inventory systems like this are deemed as intractable due to complicated multivariate integration. Supply chain literature that explores the performance of inventory systems that are subject to auto-correlated demand viewed as DMC that consider multiple probabilities such those induced by promotion schemes is scant.

The purpose of this paper is to characterize and study an inventory system that is conditioned to DMC demand frequently found in competitive markets that use promotions to induce consumption in segmented markets. This work extends the work of Diaz and Ezell [9] as it employs a simulation based-optimization approach to approximate solutions to a lost-sale inventory system in a different stochastic inventory environment. When probabilistic distributions, as the one considered in this paper, are intractable, simulation-based optimization such as Markov Chain Monte Carlo overcomes this limitation by generating a sample sequence where each decision point has the desired distribution [10]. This study generates policy solutions to inventory systems whose demand contains dependent components that can be described as DMC. Furthermore, the main effects and interactions over holding, ordering, and stockout costs that define the performance of these systems are investigated through statistical analysis as the auto-correlated demand increases.

The stochastic inventory problem involves a single-item whose replenishment takes place over the next business day and not as a perpetual inventory review policy. The analyzed inventory control system assumes a (s, S, R) periodic review policy where s is reorder point, S is the targeted inventory level and R is the reviewing period. The simulation optimization method employed to analyze this inventory management problem is based on a technique that combines simulated annealing (SA) approach with pattern search (PS) and ranking and selection (R&S). This method estimates solutions to the objective function that randomly generate a location in the feasible space and apply randomized (SA) and deterministic (PS and R&S) rules to select whether to move to a new location on the path to a solution. As the auto-correlated component increases, the R&S component play a more relevant role since more replications are needed to assess prescribed levels of user-defined Indifference Zone (IZ). We now offer a brief review of the three procedures used in this study.

Simulated annealing is a probabilistic method proposed in Kirkpatrick et al. [11] and Cerny [12] for finding the global minimum of a cost function, in a large search space, that may possess several local minima. It is often used when the search space is discrete. It works by emulating the physical process whereby a solid is slowly cooled so that when eventually its structure is frozen, this happens at a minimum energy configuration. The notion of slow cooling is implemented in the simulated annealing algorithm as a slow decrease in the probability of accepting worse solutions as it explores the solution space.

Pattern search is a family of numerical optimization methods that do not require the gradient of the problem to be optimized. Hence PS can be used on functions that are not continuous or differentiable. The name pattern search, was coined by [13]. An early PS variant is attributed to Fermi and Metropolis as described by Davidon [14] who summarized the algorithm as: Varying one theoretical parameter at a time by steps of the same magnitude and when no such increase or decrease in any one parameter further improved the fit to the experimental data, step sizes are halved and the process is repeated until the steps are deemed sufficiently small.

Ranking and Selection is a group of statistical techniques developed to address the optimization problem associated with the goal of selecting the "best" simulated system configuration (s) from a given solution space, where "best" refers to the maximization or minimization of some user-specified performance measure [15]. These procedures act as a guideline for the sampling process, specifying how many samples are required to ensure the desired probability of selecting the best alternative as well as determining when alternatives can be designated as inferior and eliminated from further sampling. One of the statistical techniques used in R&S to evaluate the simulation optimization problem is the Indifference Zone (IZ) selection. IZ selection is a group of statistical procedures designed to select the true best alternative (s) from a population of competing alternatives, based on the estimation of their expected performance, and with a user-specified probability. The selection is made through the use of a sample size calculation based on an indifference parameter, which indicates the user-specified practically significant difference that the experimenter is indifferent to. These procedures are synonymous to a power calculation, in that they are used to determine the number of samples required to detect this practically significant difference and select the true best alternative with a given probability.

This paper is structured as follows. In Section 2, a brief literature review of Markov-modulated demand and autocorrelation studies in the inventory and operational management context is presented. Section 3 briefly describes the Discrete Markov Chain model for managing inventory. Section 4 deals with the computational study that numerically analyzes the performance of the proposed integrated model composed of various sub-models and heuristics. Finally, conclusions and managerial implications are discussed in Sections 5 and 6, respectively. In Appendix A the inventory model is described and briefly presented the diverse components that shape the heuristic employed to solve the inventory problem.

2. Literature review

A brief overview of relevant literature that considers auto-correlated demands and those that have been modeled as different Markov-modulated classes is presented below.

Serially and cross-correlated demands are two types of dependency components that have been identified in some consumer demands [3,16]. A few significant recent studies on correlated demands that influence inventory management decisions are reviewed next.

Inventory models that consider positive auto-correlated demands have been studied before. Providing solutions to inventory systems whose demands contain auto-correlated components may be difficult since complicated multivariate integration may be required [17]. Some authors have studied similar systems and derive heuristics which assist in solving this class of problems. Particularly, [18] and Diaz and Bailey [4] consider auto-correlated demands in stochastic inventory models involving lost sale. Also, [19] consider deterministic EOQ with partial backordering and correlated demand caused by cross-selling. Zhang et al. [20] extend this model to make it more applicable to dealing with the inventory replenishment problem for multiple associated items. These authors consider the joint replenishment problem with complete backordering and correlated demand. Diaz and Ezell [9] employ Autoregressive AR(1) to characterize auto-correlated demand in a lost sale stochastic inventory model. A simulation-based optimization approach is used to solve this complex problem.

Several authors have different classes of Markov-modulated structures to characterize demand requirements. A few studies are highlighted here. For instance, Cheng and Sethi [21] study optimality of state-dependent (s, S) policies in lost sales inventory models whose demand is represented by Markov-modulated demand. Another one is Chen and Song [22] who analyze a multistage serial inventory system subject to a Markov-modulated demand. The authors show that the optimal policy is an echelon base-stock policy with state dependent order-up-to levels and suggest an algorithm for determining the optimal base-stock levels. Muharremoglu and Tsitsiklis [23] show the optimality of state dependent echelon base stock policies in uncapacitated serial inventory systems with Markov modulated demand and Markov modulated stochastic lead times in the absence of order crossing.

The methods described above are exact or bound approximations developed to provide solutions to the inventory problem. As a result, when faced by more complex situations, these methods contain restrictive assumptions. Thus, as recommended by Silver and Peterson [24], near-optimal methods can be used to solve these representations with a high probability of converging to reliable solutions.

Unlike studies described above whose goal involves comparing the performance of different inventory systems, the scope of this paper includes generating solutions to a particular class of inventory policies while providing an in-depth analysis of the performance of the studied system. The class of inventory problem considered in this paper involves a Discrete Markov-modulated Chain demand in the lost-sale case which is subject to various levels of dependency in its demand component. As this demand component becomes stronger, this study investigates its effect on total costs, order quantities, and different cost components. Statistical analysis of the significance of the variation found is conducted. Given the well-known complexities in finding solutions to this type of problem, this study uses a metaheuristic approach that approximates near-optimal solutions that satisfy problems constraints. Other methods used to generate dependent demands and approximate solutions to a similar family of problems might include a discrete Ornstein–Uhlenbeck process which traditionally can be seen as an AR(1) process as the ones previously mentioned. In addition, [25] refer to the work of Zheng and Federgruen [26] who developed a fast algorithm for finding the optimal s and S for given R assuming demand distributions dissimilar than the one considered in this paper.

The proposed study extends the work of Diaz and Ezell [9] to consider a Discrete Markov-modulated demand in which different levels of a single-item product required by consumer demand is constrained by a finite number of states that relate one another to capture demand autocorrelation. The application of a simulation based-optimization approach to approximate solutions to a lost-sale inventory system that faces a Discrete Markov-modulated demand pattern as the one previously described is the focus of this paper. The proposed approach is based on a heuristics that combines simulated annealing (SA) with pattern search (PS) and ranking and selection (R&S). Furthermore, the main effects and interactions of measurements of the inventory performance are investigated. Similar to Diaz and Ezell [9], the stochastic inventory problem involves a single-item whose replenishment takes place over the next business day and not as a perpetual inventory review policy.

3. Discrete Markov-modulated chain model

This study considers a Discrete Markov-modulated Chain with four states that generates different auto-correlated demand characterized by the sequence presented in Fig. 1 and values presented in Table 1. Stationary time series demand is generated given the values of the transition probability distribution p_{ij} and discrete stochastic demand which is assumed to be the probability of mass function (PMF).

As already described earlier, simulated annealing may be described as an iterative procedure in which candidate solutions

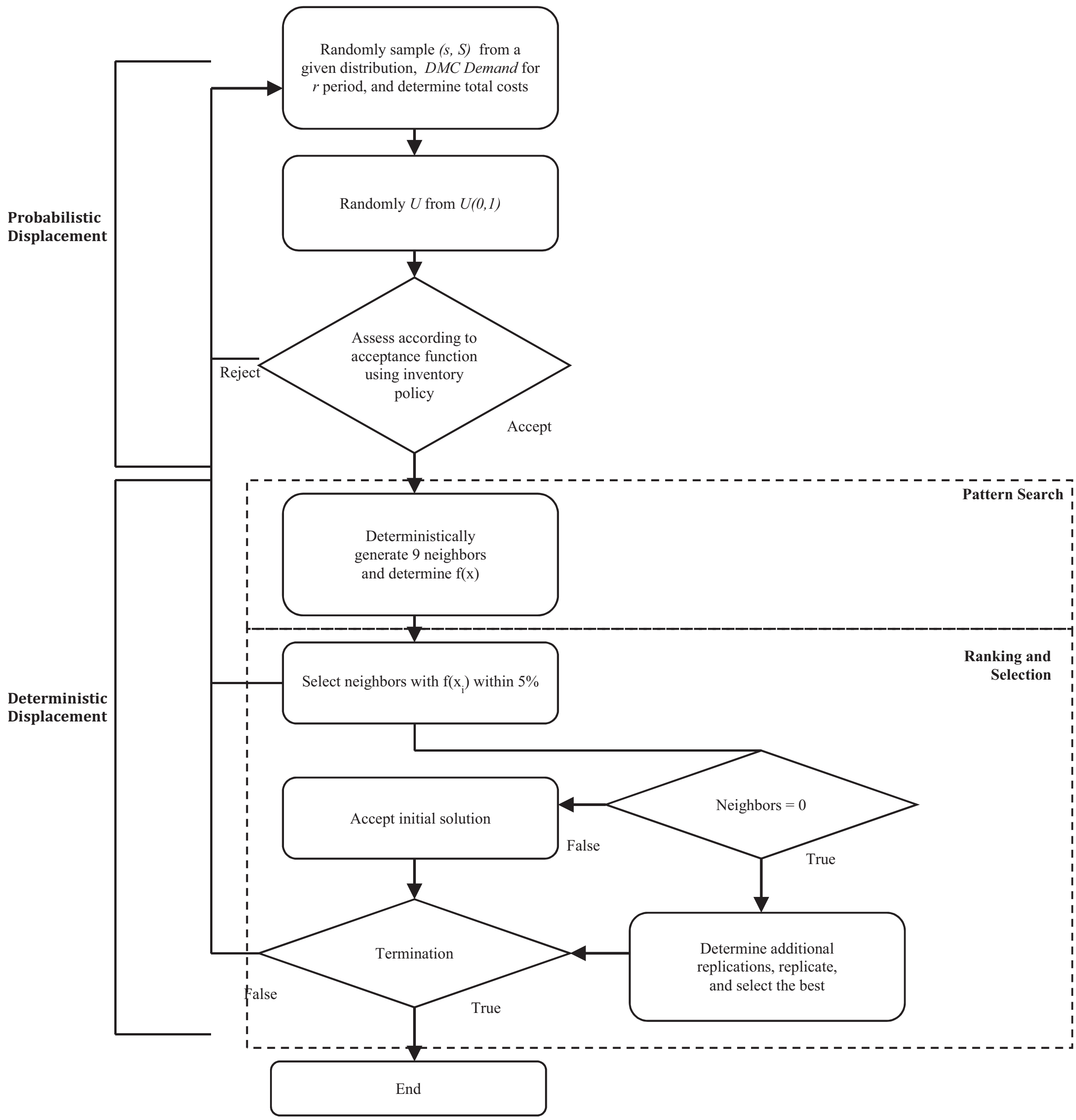

Fig. 1. Procedure integration—flow diagram.

are nominated and accepted or rejected in agreement with a certain evaluation function and a temperature schedule. Some authors have enhanced the simulated annealing procedure to include additional heuristics that improve its performance [9,27,28]. The entire procedure was implemented in C++ while the analysis of the results was performed using SAS 9.2. The procedure improves the process of selecting and evaluating candidate solutions by exploring the neighborhood of a nominated solution accepted by SA. This procedure uses a PS to deterministically produce supplementary neighbors around an accepted candidate solution. It also uses R&S

Table 1
Invariant distribution values derived from given Transition probability distribution values (Ind: individual; Cum: cumulative).

	Transition probabilities																	
$p_{12} = p_{21}$	0.1		0.2		0.3		0.4		0.5		0.6		0.7		0.8		0.9	
$p_{23} = p_{10}$	0.9		0.8		0.7		0.6		0.5		0.4		0.3		0.2		0.1	
	Invariant probabilities																	
	Ind	*Cum*	*Ind*	*Cum*	*Ind*	*Cum*	*Ind*	*Cum*	*Ind*	*Cum*	*Ind*	*Cum*	*Ind*	*Cum*	*Ind*	*Cum*	*Ind*	*Cum*
π_0	0.237		0.222		0.206		0.188		0.167		0.143		0.115		0.083		0.045	
π_1	0.263	0.5	0.278	0.5	0.294	0.5	0.313	0.5	0.333	0.5	0.357	0.5	0.385	0.5	0.417	0.5	0.455	0.5
π_2	0.263	0.763	0.278	0.778	0.294	0.794	0.313	0.813	0.333	0.833	0.357	0.857	0.385	0.885	0.417	0.917	0.455	0.955
π_3	0.237	1	0.222	1	0.206	1	0.188	1	0.167	1	0.143	1	0.115	1	0.083	1	0.045	1

to assess such neighbors. The PS and R&S improves the stochastic local search of solutions as they require less replications than traditional Markov Chain Monte Carlo methods [4].

The procedure's first step involves randomly producing a candidate solution. The second step includes randomly selecting to accept the nominated solution. This procedure is complemented with two additional steps as follows. Once a candidate solution has been accepted at the end of the second step, the third step entails a systematic and deterministic production of accepted candidate's neighbors. This is accomplished by using a pattern search procedure. Finally, the accepted candidate and its complementary neighbors are assessed by using a common Ranking and Selection process. Thus, this approach integrates PS and R&S in a typical SA procedure to improve the quality and selection of candidate solutions. One of the most prominent features of this procedure is that it employs a Markov Chain Monte Carlo procedure that describes an ergodic Markov Chain which entails producing near-optimal solutions. Additionally PS and R&S steps limit the number of replications required to accomplish this goal. Details of the foundations of the Simulated Annealing algorithm may be found in Diaz and Ezell [9].

In the proposed DMC model, random policies are generated considering inventory constraints that include maximum and minimum reorder and inventory level allowed. Then, considering the given probabilistic distribution parameters, the correlation-free and serially-correlated demands are generated. Lower and upper bounds for range of near-optimal solutions are provided by the PS step. Final near-optimal candidate solution selection is executed according to criteria embedded in the indifferent zone at the R&S step.

The details of Discrete Markov-modulated Chain inventory model with auto-correlated demand which uses the proposed heuristics for generating near-optimal solution are described in Appendix A.

3.1. Generating autocorrelated demands

A DMC is a model that allows the representation of discrete values according to a transition and invariant probability distributions [29]. Markov-modulated demand modeling has been considered in many other domains such as acquisition sequences analysis [30], describing priority demands [31], and analyzing the effects of promotions in a periodic inventory model [21]. In this paper, a DMC demand is considered to describe the discrete case of the stochastic demand where autocorrelations can be modeled from the transition probability matrix [32]. Specifically, for each given transitional distribution matrix that characterizes a particular auto-correlated case, a stationary (invariant) distribution is derived using common properties of ergodic Markov chains [33].

The form of this transitional probability expresses the degree of autocorrelation among their states. Estimating autocorrelation in a simple Markov chain has been studied in the past (e.g., Basawa [34]). In this paper, autocorrelation values are determined from the DMC configuration and according to different combinations of the values presented in the transitional probabilities. As previously indicated, in order to analyze whether ignoring dependency has an effect on estimating the minimal costs and the (s, S) policy, the correlation-free representation of the DMC must be derived.

4. Numerical analysis

The main purpose of this section is to present and conduct experimental procedures designed in terms of the experimental design, analysis and evaluation of results. To examine and assess the impact of ignoring auto-correlated components on the demand, the analysis process was subdivided into four stages as follows:

(1) *Experimental design*. It provides direction to determine the importance and behavior of factors and interactions on the studied inventory system. The varying factors and potential interactions are defined in terms of the cost structure and autocorrelation factors while the responses are quantified in terms of average total costs and control policies.
(2) *Analyzing and evaluating responses*. Responses are obtained by applying the SAPSR&S algorithm to the inventory problem.
(3) *Main effects and two-way interactions.* The main effects and two-way interactions of the costs structure (ordering, shortage, and holding) are determined. ANOVA tests are conducted to gauge the significance of interactions and effects.
(4) *Evaluating significance of main effects and two-way interactions.* Assess the significance of the main effects and two-way interactions in terms of costs structure.

The integration of the DMC demand, the inventory, and the simulated annealing extension that approximate solutions to this class of problems is shown in Fig. 1.

4.1. Experimental design

4.1.1. Determining dependent and independent variables

Three independent variables and three dependent variables are adopted in a series of simulation experiments. The independent variables include ordering costs, shortage cost, and holding cost. The dependent variables involve the average total cost of the inventory system and the near-optimal policy that minimizes the average total cost. The near-optimal policy is composed of two variables: the reorder point "s" and the maximum inventory level "S." In order to conveniently analyze the inventory system, instead of considering ordering "up-to-S" it is convenient to re-parameterize the decision in terms of order quantity D [35]. The order quantity is defined as the difference between the "up-to-S" level and the reorder point "s".

4.1.2. Design of experiments

The class of stochastic auto-correlated demands considered in this paper to design these experiments is DMC. DMC is considered for modeling a stochastic discrete demand where each discrete value is represented by a state. Based on the demand and probabilistically generated control policies (s, S, R) using the algorithm SAPSR&S, near-optimal policies and average total cost are determined.

The inventory level at the reorder points triggers for adjusting the ordered quantities in order to minimize total inventory costs. The experiments are designed to evaluate the effects of each cost component per autocorrelation factor. A full-factorial design for these factors and levels required a 2^3 design which implies a 32 trial experiment per correlation factor (simple factors and interactions). Ordering costs (2 levels), shortage costs (2 levels), holding costs (2 levels), auto-correlation levels (18 levels

Table 2
Experiment design.

	Factors					
Experiment	c	$p+C$	h	$c\times(p+C)$	$c\times h$	$(p+C)\times h$
A	–	–	–	+	+	+
B	+	–	–	–	–	+
C	–	+	–	–	+	–
D	+	+	–	+	–	–
E	–	–	+	+	–	–
F	+	–	+	–	+	–
G	–	+	+	–	–	+
H	+	+	+	+	+	+

Table 3
Design factors.

Factor	Name	–	+
1	Ordering cost (c)	1	2
2.1	Shortage cost (p)	5	19
2.2	Cost @ inventory of 0	100	200
3	Holding cost (h)	0.5	2.5

for MC subdivided into correlated (9 levels) and correlation-free cases (9 levels)).

Each simulation execution was of 20,000 periods that depends upon the autocorrelation level and the reaching of the termination criteria. The simulation run length corresponded to the stage length of the SAPSR&S algorithm. The runs included five replications of all combinations. The value for h was selected based on [3] where a retail inventory is analyzed assuming an auto-correlated demand. The magnitude of the shortage cost p was derived using the approach of service level based on critical ratio [24,36]. From assuming a service level with a critical ratio near one led the system to provide a service level of about 97% while a shortage cost of \$19 per unit. Then, a relaxation of this condition, which portrays a situation where an inventory system selects a critical ratio that is not close to one, derived in a lower shortage cost of \$5 per unit. Taha [37] asserts that it is not rational to purchase an item whose shortage cost is higher than the ordering cost. As a result, based on literature statements, ordering costs c was assumed to be lower than p, with its holding cost h lowest level at \$0.5 per unit and its highest level at \$ 2.5 per unit. Table 2 summarizes the experimental design factors. Table 3 shows values for each design factor.

The specific input variables integrated in the simulation model are specified in Table 4 as follows. Notice that two levels of input data, particularly, for the inventory model and the SAPSR&S algorithm such that given information is processed and output data is generated.

The arbitrary policy selected in this model is produced considering inventory constraints that include maximum and minimum reorder and inventory level allowed. Considering the given probabilistic distribution parameters, the correlated and correlation-free demands are generated. Based upon the demand, the inventory control is determined. Inventory levels fluctuate with the values of demand in each period, therefore, making it a stochastic function. Specific input data for the stochastic distribution include:

- Consider the inventory formulation from Appendix A.4 where four types of arbitrary discrete demands can occur and may be represented according to a given transition probability matrix as illustrated in Fig. 2. The discrete values include $\xi_D = \{1000, 2000, 3000, 4000\}$. It behaves according to a given p_{ij}, where $p_{10} = p_{23}$, $p_{12} = p_{21}$, and $[0, p_{10}) = p_{32} = 1$.
- Invariant distributions were derived from their transition probability distribution. Autocorrelation values were obtained from each given transition probability distribution.
- Values used in the transition probability distribution and the derived invariant distribution, are presented in Table 1.

Table 4
Input data.

Input type	Description
(1) Inventory model	(1.1) Demand distribution: discrete demand modeled as Markov Chain (1.2) Costs: (1.2.1) Ordering (1.2.2) Holding (1.2.3) Shortage (1.3) Maximum/minimum inventory level allowed in the system ($s = 1000$; $S = 8000$)
(2) SAPSR&S algorithm	(2.1) SA (2.1.1) Maximum temperature (based in acceptance ≥98%) (2.1.2) Temperature Gradient $\tau_i = 0.85 \times \tau_{i-1}$ (2.1.3) Length of the stage (20,000 periods) (2.1.4) Stopping criteria (combination of (s, S)±10%, average costs ±5%, and $\tau_i < 100$ units) (2.2) Pattern search (2.2.1) Step Size for reorder $\delta_s \pm 15\%$ and resupply level $\delta_S \pm 15\%$ (2.2.2) Number of neighbors to explore per iteration = 9 (2.3) Ranking and selection (2.3.1) Indifference zone value 5% (2.3.2) h based on the indifference value and the number of neighbor to explore 3.619 (2.3.3) Initial number of replications $n_0 = 20$

Table 5
Effects of auto-correlated DMC demands for various transition and invariant probabilities.

P01	ϕ	Cost_Dep	s_Dep	Sdep	Cost_CF	s_CF	S CF	D_Dep	D_CF
0.10	−0.15	6394.22	1626	3002	6505.39	2501	3001	1376	500
0.20	0.13	6509.96	1562	3002	6716.83	2468	3001	1440	533
0.30	0.29	6608.50	1507	3002	6895.95	2501	3001	1494	500
0.40	0.38	6690.88	1528	3002	7050.32	2349	3001	1474	652
0.50	0.45	6766.83	1480	3003	7184.79	2502	3002	1523	500
0.60	0.49	6831.02	1425	3003	7299.11	2418	3001	1578	584
0.70	0.53	6887.74	1485	3004	7403.02	2400	3003	1519	603
0.80	0.56	6713.82	805	3004	7492.90	2502	3002	2199	500
0.90	0.64	6541.69	595	3011	7574.97	2444	3001	2415	557

Table 6
p-Values for Markovian Modulated Demand for various transition and invariant probabilities.

		p-Value	Hypothesis		p-Value	Hypothesis		p-Value	Hypothesis	
P01	ϕ	Cost	Ho	Ha	s	Ho	Ha	D	Ho	Ha
0.10	−0.15	6.26E − 07	Reject	Accept	0.00066	Reject	Accept	0.000594	Reject	Accept
0.20	0.13	4.65E − 11	Reject	Accept	0.000263	Reject	Accept	0.000243	Reject	Accept
0.30	0.29	3.06E − 16	Reject	Accept	1.89E − 08	Reject	Accept	2E − 08	Reject	Accept
0.40	0.38	2.8E − 13	Reject	Accept	0.003486	Reject	Accept	0.003612	Reject	Accept
0.50	0.45	1.75E − 16	Reject	Accept	0.001825	Reject	Accept	0.001844	Reject	Accept
0.60	0.49	2.5E − 16	Reject	Accept	6.17E − 06	Reject	Accept	6.17E − 06	Reject	Accept
0.70	0.53	5.25E − 15	Reject	Accept	0.003887	Reject	Accept	0.003768	Reject	Accept
0.80	0.56	4.4E − 13	Reject	Accept	3.84E − 05	Reject	Accept	4.26E − 05	Reject	Accept
0.90	0.64	6.65E − 13	Reject	Accept	1.75E − 10	Reject	Accept	1.77E − 10	Reject	Accept

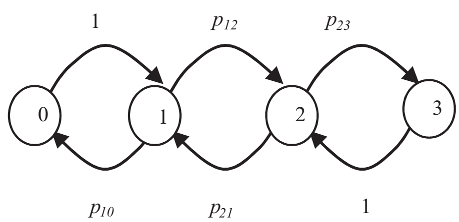

Fig. 2. Transition probability graphical representation.

4.2. Analyzing and evaluating responses

One of the central objectives of this research is to report and assess the effects of auto-correlated demands on average cost and near-optimal (s, S) policy. This is achieved by calculating the near-optimal policy, determining the response of the system, and testing whether there is a significant difference in the minimum average total cost and the near-optimal inventory policy between the correlated and correlation-free cases.

To demonstrate the ability of the model, an experiment is selected and presented. Column P01 in Table 5 corresponds to the values of the transition probability. As previously discussed,

Table 7
Obtained values of DMC demands main effect and two-way interaction for various transition and invariant probabilities.

P_01	ϕ	Effect	CostDep	sDep	Ddep	CostCF	sCF	DCF
0.1	−0.149	1	2453.90	109.80	−0.55	2453.98	−3.55	−0.10
		2	449.51	516.00	499.15	449.43	441.05	500.10
		3	4646.70	−2105.60	−2008.20	5004.14	−1493.80	−2003.60
		1 × 2	21.32	−3.90	0.25	22.79	41.05	0.40
		1 × 3	1226.33	88.50	0.55	1226.66	78.25	0.20
		2 × 3	178.31	−631.30	−496.65	178.48	−421.65	−498.00
0.2	0.13	1	2416.16	109.20	−2.55	2414.79	12.55	−5.25
		2	613.90	592.20	493.65	613.77	463.95	492.65
		3	6001.35	−1757.20	−2029.00	6659.59	−1503.80	−2035.00
		1 × 2	41.82	−14.80	2.45	40.25	−41.55	5.65
		1 × 3	1207.97	69.70	2.65	1206.41	−34.45	5.65
		2 × 3	534.25	−443.70	−489.85	535.37	−449.65	−489.65
0.3	0.29	1	2411.51	105.60	1.00	2414.21	−70.30	2.40
		2	794.32	81.90	7.30	793.09	37.30	6.20
		3	7305.62	−3588.80	−3982.40	8222.55	−2959.60	−3980.00
		1 × 2	28.16	10.10	0.40	29.91	15.70	2.40
		1 × 3	1162.44	−5.60	0.30	1164.38	15.20	1.90
		2 × 3	793.33	−16.90	4.60	794.34	−28.70	6.20
0.4	0.38	1	2426.17	−96.10	0.95	2427.70	−13.05	−0.45
		2	780.62	459.80	499.65	779.79	476.95	500.05
		3	7758.03	−1603.60	−2007.80	8896.77	−1910.60	−2001.40
		1 × 2	72.81	−55.10	−0.15	71.57	15.65	0.45
		1 × 3	1177.95	8.80	−0.15	1178.90	−4.45	−0.65
		2 × 3	780.54	533.10	500.15	780.54	479.65	499.75
0.5	0.45	1	2414.04	−75.75	1.55	2427.92	66.85	6.35
		2	701.10	527.15	498.75	689.60	350.85	494.45
		3	7883.03	−1682.60	−2009.80	9163.88	−1845.40	−2025.80
		1 × 2	85.02	−90.75	−0.15	94.63	61.85	4.55
		1 × 3	1167.31	−114.05	−0.15	1178.79	61.95	4.65
		2 × 3	699.51	489.15	499.45	689.49	319.05	494.95
0.6	0.49	1	2405.29	−66.20	2.10	2405.57	−30.00	0.00
		2	632.79	538.50	498.10	633.45	438.00	500.10
		3	7987.97	−2275.20	−2011.60	9481.70	−1716.40	−2003.20
		1 × 2	93.70	−260.50	−0.60	93.20	−27.80	0.20
		1 × 3	1156.84	−214.00	0.10	1156.17	−60.20	−0.70
		2 × 3	632.65	541.10	500.80	633.71	407.90	499.80
0.7	0.53	1	2396.18	27.70	0.60	2396.88	−27.30	0.20
		2	572.68	551.10	499.10	572.69	461.20	500.30
		3	8084.41	−1954.80	−2002.80	9725.73	−1714.40	−2002.40
		1 × 2	101.95	63.70	0.20	102.49	18.50	0.60
		1 × 3	1147.72	95.70	0.00	1147.51	−13.30	0.50
		2 × 3	573.09	571.30	498.50	572.98	404.60	498.80
0.8	0.56	1	2330.04	−196.35	−6.50	2387.84	−24.85	0.05
		2	576.42	901.35	504.80	519.30	391.65	500.05
		3	7955.18	−2564.60	−1980.40	9938.72	−1771.00	−2005.40
		1 × 2	167.25	81.55	−9.40	110.88	−24.65	0.25
		1 × 3	1085.67	−134.35	−9.30	1139.10	−24.65	0.25
		2 × 3	574.31	683.35	508.60	519.13	391.45	499.85
0.9	0.64	1	2336.33	−197.45	3.60	2380.78	−12.35	0.10
		2	579.05	871.55	492.60	471.36	402.95	500.00
		3	7140.06	−3898.20	−2008.00	10133.48	−1837.80	−2003.60
		1 × 2	144.81	106.25	−3.60	118.47	8.75	−0.30
		1 × 3	1106.43	45.45	−3.20	1132.44	−20.05	−0.60
		2 × 3	562.01	67.95	498.80	471.43	371.45	500.10

the autocorrelation value of ϕ was obtained from each transition probabilities distribution presented in Table 1. Table 5 shows the minimum average cost and near-optimal policy obtained for the correlated (Dep) and correlation-free (CF) cases.

It is clear that differences between the average total costs "Cost_Dep" and "Cost_CF" are substantial. The difference between the ordered quantities "D_Dep" and "D_CF" for both cases is considerable as well. As the autocorrelation amplifies, the differences between costs and reorder points for the correlated and CF case also increase. Similar results are obtained for the other of the treatments. Notice that the order quantity "D_Dep" increases by reducing the reorder point s while maintaining the up-to level S quantity to comparable levels for both the CF and the Dep cases.

The null hypothesis for the test in one-way analysis claims that the k populations (represented by the k samples) all have the same mean value while the alternative hypothesis claims that they are not all the same. Thus, the alternative hypothesis indicates the autocorrelated demand does change the average total cost, the reorder point, and the difference of the inventory system is tested.

ANOVA analyses are conducted for each response while results are collected. To illustrate the ANOVA test results, significance values from the F test is presented in Table 6.

Considering a $p > 0.05$ significance level, the following conclusions may be drawn. The difference between the correlation-free and correlated case for the total cost, near-optimal inventory policy, and order quantity are all highly significant. Thus, an in-depth exploration and analysis is subsequently presented.

4.3. Main effects and two-way interactions

The approach employed to analyze main effects and interactions follows the traditional analysis approach, in which the main effects are determined first followed by determining two-way interactions [38]. A total of eight experiments are conducted per correlation factor for two levels of variable changes in ordering, shortage, and holding costs. Each treatment is performed five times per correlation factor.

4.3.1. Determining main effects and two-way interaction of the experiment per correlation factor

The main effects assess the average change in the response as a result of a change in an individual factor, with this average calculated over all possible combinations. However, the effect of a given factor may depend in some way on the level of some other factor. Changes in these factors and their interactions may be significant and have an effect on the average cost and the selected (s, S, R) policy.

Table 7 report the main effects and two-way interaction values obtained. Further, these effects and interactions are stated in terms of the average total cost, the reorder points, and the order quantities per autocorrelation factor.

4.4. Evaluating significance of main effects and two-way interactions

Hypothesis tests are performed to determine the significance of the levels of the individual factors and their interactions. The null hypothesis for the one-way analysis test claims that the effects of individual factors and their interactions have the same mean value while the alternative hypothesis claims that not all are the same. Results of the ANOVA tests conducted for each factor and their interactions are presented in Table 8.

For each of the observed main effects and two-way interactions, it is important to understand the direction and the magnitude of the effects.

Table 8
DMC demands main effect and two–way interaction for various transition probabilities.

P_01	Effect/Interaction	COST	REORDER	D
0.1	1	0.99402	0.65782	0.65569
	2	0.99317	0.76959	0.76615
	3	1.40E − 90	0.01752	0.01807
	1 × 2	0.72385	0.67464	0.67433
	1 × 3	0.93641	0.92372	0.92600
	2 × 3	0.96751	0.05121	0.04876
0.2	1	0.89366	0.70566	0.71278
	2	0.98940	0.61629	0.61811
	3	1.34E − 142	0.32252	0.30994
	1 × 2	0.70542	0.80270	0.77874
	1 × 3	0.70757	0.33115	0.31534
	2 × 3	0.78729	0.95568	0.95399
0.3	1	0.79285	0.49200	0.48741
	2	0.90437	0.86162	0.86464
	3	1.50E − 172	0.01460	0.01471
	1 × 2	0.67365	0.95828	0.97306
	1 × 3	0.64091	0.84595	0.85706
	2 × 3	0.80888	0.91223	0.89996
0.4	1	0.88163	0.74551	0.74071
	2	0.93600	0.94656	0.94766
	3	1.94E − 192	0.23095	0.22019
	1 × 2	0.76413	0.50890	0.51066
	1 × 3	0.81832	0.90150	0.90479
	2 × 3	0.99942	0.61768	0.61879
0.5	1	0.17756	0.57743	0.58931
	2	0.26349	0.49102	0.50051
	3	2.66E − 203	0.52477	0.56524
	1 × 2	0.02138	0.15500	0.16621
	1 × 3	0.00607	0.10123	0.10929
	2 × 3	0.01646	0.11313	0.12131
0.6	1	0.97818	0.88749	0.88069
	2	0.94917	0.69452	0.68797
	3	1.48E − 217	2.98E − 02	3.19E − 02
	1 × 2	0.90503	0.03063	0.03048
	1 × 3	0.87151	0.15179	0.14792
	2 × 3	0.79835	0.21424	0.21569
0.7	1	0.94553	0.82980	0.83058
	2	0.99946	0.72535	0.72112
	3	2.59E − 226	0.34792	0.34744
	1 × 2	0.89701	0.67293	0.66885
	1 × 3	0.96001	0.30918	0.30487
	2 × 3			
0.8	1	5.51E − 08	0.50289	0.51821
	2	7.70E − 08	0.04736	0.04885
	3	5.90E − 244	2.16E − 03	1.52E − 03
	1 × 2	7.52E − 31	0.32174	0.27775
	1 × 3	1.39E − 28	0.30609	0.34792
	2 × 3	6.29E − 30	0.00686	0.00841
0.9	1	2.28E − 05	0.46967	0.46013
	2	4.40E − 21	0.06808	0.06318
	3	2.07E − 282	5.08E − 14	4.95E − 14
	1 × 2	1.21E − 09	0.36289	0.34481
	1 × 3	1.87E − 09	0.54082	0.52308
	2 × 3	1.20E − 56	4.97E − 03	4.97E − 03

The magnitude of the effects of the cost structure on the average total cost in descending order was the holding costs (3), the ordering cost (1), and the shortage cost (2). The magnitude of the interactions, in descending order, was ordering and holding costs (1 × 3), shortage and holding costs (2 × 3), and ordering and shortage costs (1 × 2). However, in general, as the autocorrelation increased, from the perspective of main effects, the effect of holding costs (3) became not only stronger but also very significant. From the point of view of two-way interactions, the shortage and holding costs (2 × 3) interaction become stronger and significant as the autocorrelation increases. Ordering and holding costs (1 × 3) and ordering and shortage costs (1 × 2) demonstrated high

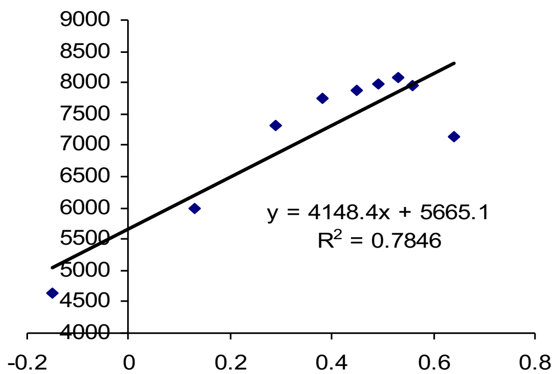

Fig. 3. Behavior of holding costs per autocorrelation factor.

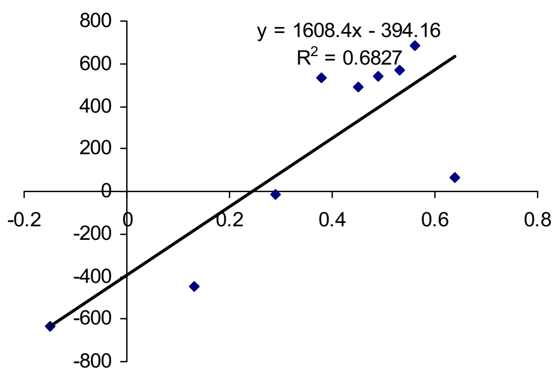

Fig. 4. Behavior of two-way interaction 2 × 3 on average reorder point per autocorrelation factor.

level of significance at higher levels of autocorrelation factors. Figs. 3 and 4 illustrate the behavior of the most relevant main effects (holding costs) and two-way interaction (shortage and holding costs) in this experiment. Mathematical expressions show the linear approximations that describe the behavior of these data points as the autocorrelation increases. Additionally, the strength of this relationship shows that these linear approximations capture the behavior of the system when the autocorrelation changes values.

5. Conclusions

Auto-correlated demands are frequently found in competitive markets. In depth examination of the effects of serially correlated demands on probabilistic lost-sale inventory model is the central goal of this paper. This paper investigates the results of the formal study in this class of inventory systems whose control policy involves a (*s*, *S*, *R*) scheme, where *R* is fixed. Auto-correlated demands are modeled in this paper as a Discrete Markov-modulated demand. This formulation may allow modeling a given demand as subject to a promotion plan that seeks to spur demand for products. A simulation-based optimization built on an enhanced simulated annealing algorithm is employed to determine near-optimal solutions to this lost-sale inventory problem. The enhanced simulated annealing involves combining features from pattern search and ranking and selection. The simulated annealing step of the procedure randomly generates an inventory-policy candidate solution that minimizes a cost function. The pattern search step generates additional candidate solutions considering the neighborhood of the candidate solution. This may be viewed as generating lower and upper bounds stemming from the candidate solution. Finally, the ranking and selection step evaluates these bounds by generating additional replications that allows comparing and determining the final candidate solution. These three steps are sequentially and recurrently repeated until improvements to solutions of the studied lost-sale inventory problem are marginal. The study offers elaborate designed experiments and provides the novel use and integration of various disparate tools in the proposed methodology in examining a widely encountered lost-sale inventory problem in industry and by academics.

Among the significant outcomes of modeling offered in this paper include some of the following. It is found that differences between the average total inventory costs for the auto correlated and auto correlation free demand cases are extensive. The differences between the ordered quantities for both cases are also huge. As the autocorrelation amplifies, the differences between costs and reorder points for the auto correlated and autocorrelation free demand cases also increase. The order quantity in the auto correlated demand case increases by reducing the reorder point, s while maintaining up-to level S quantity to comparable levels for both the autocorrelation free and the auto correlated cases. The difference between the two cases for the total cost, near optimal inventory policy and order quantity are all highly significant.

In the auto correlated demand case, the following results obtained from the model were noteworthy. The magnitude of the effects of the cost structure on the average total inventory cost in descending order was the holding cost, the ordering cost, and the shortage cost. As the demand autocorrelation increased, from the perspective of main effects, the effect of holding costs became not only stronger but also very significant. Finally, as the demand autocorrelation increased, from the point of view of two way interactions, the shortage and holding cost interaction became stronger and significant.

6. Managerial implications

Consistent with other research that considers inventory problems facing auto-correlated demands; this study finds that ignoring serially-correlated components leads to severe and significant errors which may negatively impact inventory performance. Implications for managers, knowledgeable in business statistics and simulation, who believe their inventory demands contain auto-correlated components include the need to:

- Observe the behavior of the demand and determine if it contains auto-correlated components. Visual inspection, Durbin–Watson statistics, and calculating the sample auto-correlation function are generally recommended techniques to detect auto-correlated components [4,17,39].
- If autocorrelation components are identified, the manager may use the simulation-based optimization method presented in this study to mitigate the effects of autocorrelation.
- Engage in the following adjustments by the manager to:
  - Reduce reorder points.
  - Increase order quantities.
  - Monitor stockouts and increase replenishment rates if needed.

As indicated earlier, in inventory systems with similar characteristics, holding costs become stronger and highly significant as autocorrelation levels increase. Thus, a reduction in minimum stock or reorder points implies a decline in holding cost. Order quantity increases lead to ordering costs increases as well. Conversely, the impact of ordering costs on total cost declines as the autocorrelation rises. Hence, as the results from the application of the simulation-optimization suggests, reduction in the reorder point combined with an increase in the ordered quantity reported better performance of the inventory system. In addition, stockouts should be monitored since they may be potentially significant, so replenishment rate should be kept high such that desired service level may be maintained.

Empirical results obtained in this study indicate that managers may obtain a better performance of their inventory system by acknowledging that auto-correlated components may be present in the inflow demand and by following the described actions.

Appendix A.

A.1. Modeling the inventory and auto-correlated demand

This section provides details of the inventory model which uses auto-correlated demand implemented through a heuristics.

A.2. Notation and assumptions

In order to develop the mathematical model of the problem, let us first introduce the notations and assumptions as follows:

i	period
n	period planned horizon
s	minimum reorder point for inventory
S	maximum inventory level
R	time to review inventory and place an order
x_i	initial inventory level at the beginning of period i
z_i	quantity of items ordered to resupply inventory at period i
w_i	the amount of inventory on-hand at period i
y_i	inventory level after ordering (x_i+z_i) at period i
d_i	demand at period i
c	cost of ordering per inventory unit
h	holding cost per inventory unit
h_0	minimal holding cost
p	penalty cost per inventory unit
$f(\xi_i)$	probability mass function of the stochastic demand ξ_i
$\Phi(a)$	cumulative distribution function of the demand
λ	delivery lag
$C(y_i)$	the expected total costs for the period i if the amount of inventory on-hand at the beginning of period i is x_i.

A.3. Setting

The inventory problem that is considered in this paper consists of a single item that is demanded over an infinite horizon. Positive auto-correlated demands considered which are commonly observed in consumer products, electronic retail industry and many other business situations. At the end of each period, the inventory level is reviewed and an order may be placed. The inventory policy considered in this paper is the (s, S, R). As in many practical situations including the examples mentioned, R is fixed and chosen by trading partners based on convenience and logistical feasibility [24]. Every R unit of time, the inventory position is reviewed and if it is at or below the reorder point s, then an order large enough to raise it to S is generated. If the inventory position is above s, no order is placed until at least the next review instance. The only decision to be made is to choose s and S assuming R is fixed. In such industry setting, dynamic (s, S) policy is widely used for a fixed R. For this reason, dynamic (s, S, R) policy is not relevant for the industry setting being studied in this paper and thus not explored.

A.4. Description of the inventory model

The generic formulation of this problem in a period is presented in Eq. (1). The expected cost for the period, $E\left\{C\left(y_i\right)\right\}$, is expressed as:

$$E\left\{C\left(y_i\right)\right\} = c\left(y_i-x_i\right)+\sum_{\xi_i=0}^{y_i} L_i^0\left(y_i,\xi_i\right) f\left(\xi_i\right) + \sum_{\xi_i=y_i}^{\infty} L_i^0\left(y_i,\xi_i\right) f\left(\xi_i\right). \tag{1}$$

Ordering calculation is given by:

$$z_i = \begin{cases} S-x_i & \text{if } x_i \le s \\ 0 & \text{elsewhere} \end{cases}. \tag{2}$$

Holding and shortage costs are determined as:

$$L_i^0\left(y_i,\xi_i\right) = \begin{cases} h\left(y_i-\xi_i\right) & \text{if } \left(y_i-\xi_i\right)>0 \\ p\left(\xi_i-y_i\right)+h(0) & \text{if } \left(\xi_i-y_i\right)\ge 0 \end{cases} \tag{3}$$

Inventory on hand after demand has been satisfied (total or partial) but before placing an order:

$$x_{i+1} = \max\left(y_i-\xi_i, 0\right) \tag{4}$$

Additional constraints that reflect space requirements as well as minimum ordering include:

$$s \ge \ell, \quad S \le u, \quad S \ge ms, \quad \xi_i \ge 0 \tag{5}$$

where l and u are lower and upper limits for reorder point for inventory and maximum inventory level, respectively and m is a multiple greater than 1.

A brief description of these equations follows. For the discrete random variable ξ_i, $f(\xi_i)$ is the probability mass function of the stochastic demand while $\Phi(a)$ is the cumulative mass function of the demand (CMF) that implies $\Phi(a)=\sum_0^a f\left(\xi_i\right)$. x_i is the initial inventory point at the beginning of period i, z_i is the number of items ordered to replenish, and (x_i+z_i) provides the inventory level after ordering. The holding and shortage costs when y_i items are on hand and delivery lag is zero is given by $L_i^0\left(y_i,\xi_i\right)$. The demand for withdrawing units from inventory is assumed to be stochastic ξ_i. The probability distribution is known to be Discrete Markov-modulated. An excess of demand is lost, if a stockout occurs before the order is received. Thus, the demand may be partially covered with existing inventory while the unfilled portion is lost. In this formulation, an ordering cost is incurred each time that an order is placed. Discount costs are not considered in this problem. The cost of the ordering c is proportional to the order quantity z_i. h is a holding cost incurred for each unit in inventory. A shortage cost, p, is incurred for each unit lost per unit time when a stockout occurs. Every time that the inventory reaches the zero level, a minimal holding cost $h(0)$ is generated.

Since a balance is required between the risk of being short and having an excess, minimizing the expected value of the total costs is indicated [24]. This is achieved by minimizing the expected total cost function. This function depends upon the probability distribution of the demand. Normally, a representation of this probability distribution is difficult to find. Thus, the optimal service level is obtained by minimizing the function. This value can be found either by solving its mathematical expression or by finding the area under the curve by simulation optimization. In this research, the simulation optimization approach is used to approximate the function.

Constraints that mirror both storage-space limitations and minimum ordering quantity policies in place have been embedded in this formulation (Eq. (5)). Specifically, restrictions that involve arbitrary maximum and minimum acceptable inventory levels and constraints in their relationship are considered. The consideration of such constraints provide more pragmatism to this policy-making exercise.

A.5. SAPSR&S heuristics for the inventory problem

In order to develop the heuristics for the problem that uses Discrete Markov Chain Monte Carlo procedure, let us first introduce the notations as follows.

i	iteration
r	period planned horizon
x_i	accepted state (accepted candidate solution)
y	nominated state (proposed candidate solution)
$H(x_i)$	objective cost function evaluating state x_i
T	temperature
α	acceptance function
χ	decision space
$\chi_{\min}$	a small region in decision space χ for minimum objective cost function value
τ_1	maximum temperature
l_k	stage length kth stage
Z	accepted candidate solution for specific x_i value
δ	step length
h	a constant that depends on the number of alternatives
A	number of alternatives
p_{ij}	transition probability
$1-\theta$	desired confidence level
n_0	initial number of replications
N_i	additional replications
S_i^2	sample variance of the n_0 observations
d^*	Significant difference specified by the user
IZ	indifference zone.

The objective is to estimate the minimization of the objective function, $f_{\min} = H(x_i)$, and an element, $Z = x_i$, in decision space, $\chi_{\min}$. We are given: acceptance function α, stage length $\left\{l_1 < l_2 < \ldots\right\}$, the number of stages k, maximum temperature τ_1, step length δ, and the Ranking and Selection parameters h and n_0.

Various steps involved in the proposed SAPSR&S heuristics are outlined as follows:

(1) $i = 1$ and $k = 1$

(2) Assign an initial state x_0, and $\hat{f}_{\min} = f(x_0)$

(3) Repeat:
SA:
a. while $k \le r$:
b. while $i \le l_k$
c. Randomly sample y from the given distribution
d. Randomly sample U from $U(0, 1)$
e. If $U \le \min\left\{1, e^{-[f(y)-f(x_{i-1})]/\tau_k}\right\}$, $x_i = y$
PS:
f. Deterministically generate n additional neighbors (test points) to x using step length δ
g. Simulate and obtain $\hat{f}(y)$ per potential neighbor
R&S:
h. Select y such that the performance of y is no more than 5% greater than the performance of x_i.
i. Determine the sample variance S_i^2 of the n_0 observations.
j. Check the number of observations n_0 to be independent and normally distributed.
k. Determine additional replications N_i per test point.
l. Execute additional replications per each competing alternative
m. Select the best y
n. If $f(x_i) < \hat{f}_{\min}$, $\hat{f}_{\min} = H(x_i)$, and $Z = x_i$
o. $i = i + 1$,
p. $k = k + 1$,
q. $\tau_k = \alpha\tau_{k-1}$ until termination criteria is satisfied or $k > r$

(4) $(H_{\min}, Z)$ is the estimated solution, where $H_{\min}$ is the minimum cost value and Z is the candidate solution

Number of additional replications N_i was computed based on the following formula developed by Rinot [40]:

$$N_i = \max\left\{n_0, \left\lceil \left(\frac{hS_i}{d^*}\right)^2 \right\rceil\right\}.$$

Given the constraints stemming from the inventory model presented in Appendix A.2, an arbitrary initial policy candidate solution is generated. The DMC procedure is employed to generate demands. The total cost using such policy is evaluated accordingly. A random number drawn from $U(0,1)$ is generated to assess the probabilistic displacement given by $U \le \min\left\{1, e^{-\left[f(s_i,S_i)-f(s_{i-1},S_{i-1})\right]/T}\right\}$. If a candidate solution is accepted, the local neighborhood of the candidate solution is further explored by systematically generating additional pairs of policy solutions. Inventory costs are then determined using each new policy. Each pair whose costs are no more than 5% greater than the original candidate solution is selected (Indifference Zone). If all new costs are above the original, the original policy is accepted as a solution for such iteration. Otherwise, additional replications are determined per each new policy that reported cost improvements. Thus, new replications are performed for each pair that reported lower costs. If obtained costs are below the original costs, then the new policy that reported lower cost is selected, otherwise, the original inventory policy is accepted as final. The process is repeated until stop criteria is satisfied.

References

[1] Chopra S, Meindl P. Supply chain management—strategy, planning, and operation. New Jersey: Prentice Hall; 2010.
[2] Lee HL, Padmanabhan V, Whang S. Information distortion in a supply chain: the bullwhip effect. Manage Sci 1997;43(4):546–58.
[3] Lee HL, So KC, Tang CS. The value of information sharing in a two-level supply chain. Manage Sci 2000;46(5):626–43.
[4] Diaz R, Bailey M. Building knowledge to improve enterprise performance from inventory simulation models. Int J Prod Econ 2011;134(1):108–13.
[5] Miller BL. Scarf's state reduction method, flexibility, and a dependent demand inventory model. Oper Res 1986;34(1):83–90.
[6] Kahn JA. Inventories and the volatility of production. Am Econ Rev 1987:667–79.
[7] Urban TL. A periodic-review model with serially-correlated, inventory-level-dependent demand. Int J Prod Econ 2005;95(3):287–95.
[8] Kurata H, Liu JJ. Optimal promotion planning – depth and frequency – for a two-stage supply chain under Markov switching demand. Eur J Oper Res 2007;177(2):1026–43.
[9] Diaz R, Ezell BC. A Simulation-based optimization approach to a lost sale stochastic inventory model. Int J Oper Res Inf Syst (IJORIS) 2012;3(2):46–63.
[10] Fishman G. A first course of Monte Carlo simulation. Thompson Learning; 2005.
[11] Kirkpatrick S, Gelett CD, Vecchi MP. Optimization by simulated annealing. Science 1983;220:621–30.
[12] Cerny V. A thermodynamic approach to the traveling salesman problem. J Optim Theory Appl 1985;45:41–51.
[13] Hookes R, Jeeves TA. Direct search solution of numerical and statistical problems. J Assoc Comput Mach 1961;8(2):212–29.
[14] Davidon WC. Variable metric method for minimization. SIAM J Optim 1991;1(1):1–17.
[15] Goldsman D, Nelson BL. Ranking, selection and multiple comparisons in computer simulation. In: Simulation conference proceedings. Winter. 1994. p. 1994.
[16] Hausman WH, Erkip NK. Multi-echelon vs. single-echelon inventory control policies for low-demand items. Manage Sci 1994;40(5):597–602.
[17] Neter J, Wasserman W, Kutner MH. Applied linear statistical models: regression, analysis of variance, and experimental designs. Homewood, IL: Richard D. Irwin, Inc; 1985.
[18] Bao J, Lee C, Lee L. The value of electronic marketplace in a perishable product inventory system with auto-correlated demand. OR Spectr 2007;29(4):627–41.
[19] Zhang R-q, Kaku I, Xiao Y-y. Deterministic EOQ with partial backordering and correlated demand caused by cross-selling. Eur J Oper Res 2011;210(3):537–51.
[20] Zhang R, Kaku I, Xiao Y. Model and heuristic algorithm of the joint replenishment problem with complete backordering and correlated demand. Int J Prod Econ 2012;139(1):33–41.
[21] Cheng F, Sethi SP. Optimality of state-dependent (s, S) policies in inventory models with Markov-modulated demand and lost sales. Prod Oper Manage 1999;8(2):183–92.
[22] Chen F, Song J-S. Optimal policies for multiechelon inventory problems with Markov-Modulated demand. Oper Res 2001;49(2):226–34.
[23] Muharremoglu A, Tsitsiklis JN. A single-unit decomposition approach to multiechelon inventory systems. Oper Res 2008;56(5):1089–103.

[24] Silver EA, Peterson R. Decision systems for inventory management and production planning 1985. John Wiley and Sons; 1985.
[25] Yuan K-J, Chang S-H, Li R-K. Enhancement of theory of constraints replenishment using a novel generic buffer management procedure. Int J Prod Res 2003;41(4):725–40.
[26] Zheng Y-S, Federgruen A. Finding optimal (*s, S*) policies is about as simple as evaluating a single policy. Oper Res 1991;39(4):654–65.
[27] Marinari E, Parisi G. Simulated tempering: a new Monte Carlo scheme. EPL (Europhys Lett) 1993;19(6):451.
[28] Ahmed MA, Alkhamis TM. Simulation-based optimization using simulated annealing with ranking and selection. Comput Oper Res 2002;29(4):387–402.
[29] Durrett R, Durrett R. Essentials of stochastic processes. New York, NY: Springer; 1999.
[30] Prinzie A, Van den Poel D. Investigating purchasing-sequence patterns for financial services using Markov, MTD and MTDg models. Eur J Oper Res 2006;170(3):710–34.
[31] Cohen MA, Kleindorfer PR, Lee HL. Service constrained (*s, S*) inventory systems with priority demand classes and lost sales. Manage Sci 1988;34(4):482–99.
[32] Diaz R. Methodology for analyzing and characterizing errors generation in presence of autocorrelated demands in stochastic inventory models. Old Dominion University; 2007 (Doctor of philosophy).
[33] Behrends E. Introduction to Markov chains. Vieweg; 2000.
[34] Basawa I. Estimation of the autocorrelation coefficient in simple Markov chains. Biometrika 1972;59(1):85–9.
[35] Law AM, Kelton D. Simulation modeling and analysis. McGraw Hill; 2000.
[36] Treharne JT, Sox CR. Adaptive inventory control for nonstationary demand and partial information. Manage Sci 2002;48(5):607–24.
[37] Taha HA. Operations research, an introduction. Prentice Hall; 2002.
[38] Kleijnen JP. Design and analysis of simulation experiments. Springer Publishing Company; 2007 (Incorporated).
[39] Zinn W, Marmorstein H, Charnes J. The effect of auto correlated demand on customer service. J Bus Logist 1992;13(1):173–93.
[40] Rinott Y. On two-stage selection procedures and related probability-inequalities. Commun Stat-Theory Methods 1978;7(8):799–811.