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Taxi market equilibrium with third-party hailing service

Xinwu Qian, Satish V. Ukkusuri*

Lyles school of Civil Engineering, Purdue University, West Lafayette, IN 47907, USA

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ABSTRACT

With the development and deployment of new technologies, the oligopolistic taxi industry is transforming into a shared market with coexistence of both traditional taxi service (TTS) and app-based third-party taxi service (ATTS). The ATTS is different from TTS in both entry policy and fare setting, and brings competition into the market. To account for the revolution of the taxi industry, in this study, we analyze the characteristics of the TTS and ATTS, model the taxi market as a multiple-leader-follower game at the network level, and investigate the equilibrium of taxi market with competition (TMC Equilibrium). In particular, passengers are modeled as the leaders who seek to minimize their travel cost associated with taxi rides. Followers involve TTS and ATTS drivers, who compete for passengers to maximize their revenue. The network model captures selfish behavior of passengers and drivers in the taxi market, and we prove the existence of TMC Equilibrium for the proposed model using variational inequality formulations. An iterative algorithm is further developed to find the TMC Equilibrium, which corresponds to the strongly stationary point of the multi-leader-follower game. Based on numerical results, it is observed that fleet size and pricing policy are closely associated with the level of competition in the market and may have significant impact on total passengers cost, average waiting time, and fleet utilization.

1. Introduction

Urban taxi offers great flexibility and mobility via its door-to-door service and 7/24 availability, and plays an important role in urban transportation system. By the end of 2014, there were over 50,000 taxi drivers serving 600,000 passengers daily in New York City (NYCTLC, 2014). Moreover, in other large cities such as Tokyo and Paris, there is one taxicab per hundred population (Consultant, 2014). In light of the size and significance of urban taxi industry, framing regulation policies is of key concern for stakeholders in order to correct market imperfection (Schaller, 2007) and maintain level of service. To help frame regulations, economists have developed aggregate demand and supply models to examine the effectiveness and consequences of policies mainly associated with entry and fare controls (Douglas, 1972; De Vany, 1975; Beesley and Glaister, 1983; Häckner and Nyberg, 1995; Cairns and Liston-Heyes, 1996; Shreiber, 1981; Dempsey, 1996; Çetin and Yasin Eryigit, 2011). Nevertheless, as suggested by Yang and Wong (1998), since taxi service takes place over space, aggregate models are oversimplified, and are incapable of capturing the influence of road network structure and the equilibrium nature of taxi service. This motivates to model taxi service at the network level.

* Corresponding author.
E-mail addresses: qian39@purdue.edu (X. Qian), sukkusur@purdue.edu (S.V. Ukkusuri).

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The first work which models urban taxi service at the network level was exemplified by Yang and Wong in 1998 (Yang and Wong, 1998), where a network model was developed to characterize the movement of vacant and occupied taxis on the network. It was assumed that passengers will always receive the service and any driver will eventually find a passenger, and taxi waiting time and utilization rate were analyzed with the change of taxi fleet size. Based on the network model, Wong and Yang developed an efficient algorithm to solve the taxi network movement problem as an optimization problem Wong and Yang (1998), Wong et al. (2001) improved the model by incorporating network congestion and demand elasticity, Wong et al. further proposed a sensitivity-based solution algorithm to solve their congestion model more efficiently (Wong et al., 2002), and the basic taxi network model was implemented to study how different regulations may affect the demand-supply equilibrium (Yang et al., 2002). While drivers' behavior is the main focus of the aforementioned studies, Yang et al. (2010) proposed to model the behavior of drivers and passengers jointly, where the Cobb-Douglas function (Varian, 1992) was implemented to characterize trip waiting time as a function of the choices made by passengers and drivers. While the taxi service model (Yang and Wong, 1998) serves as the base for all these studies, they are observed to share two fundamental assumptions: (1) the taxi service is well-regulated and the market is monopolistic, and (2) all passengers will be serviced and all drivers will eventually find a trip during the modeling period.

However, as Uber launched their transportation service in 2009, it breaks the convention of traditional taxi service (TTS), where street hailing is the main way of getting a taxi ride. Instead, it offers app-based third-party taxi service (ATTS), which allows passengers to request taxi service using smartphones. In a few years, we have witnessed the revolution of taxi industry globally with the entrance of other ATTS providers such as Lyft and Didi, and the market property has been complicated as it transforms from oligopoly to shared economy. Consequently, to understand the nature of the new market, there is an emerging need of a network model which accounts for the coexistence of TTS and ATTS. And modeling the market with TTS and ATTS is the objective of the study.

Specifically, the importance of the study can be understood from two aspects. First, the entrance of ATTS provides passengers with an alternative taxi service, and therefore introduces competition into the market. Therefore, it is no longer appropriate to model taxi service as monopolistic, and we need to develop new methodologies to account for the consequence of competition in the market and investigate the equilibrium of taxi market with competition (TMC Equilibrium). Second, within 5 years, Uber has attracted over 8,000,000 users globally, and is serving 400 cities, with 50,000 new drivers added monthly (DMR, 2015). Meanwhile, similar ATTS providers emerge all over the world and become legitimate in more and more cities. Admittedly, ATTS is becoming indispensable in taxi market, and caution should be exercised as how to regulate the market with both TTS and ATTS appropriately. This in return requires a model to analyze the impact of different regulation strategies on the market.

To model the taxi market with TTS and ATTS, it is essential to understand the underlying differences between TTS and ATTS. Unlike TTS, which mainly serves passenger via street hailing, ATTS offers rides through smartphone apps, which helps to narrow the information gap between passengers and drivers. Operationally, there are two major differences between TTS and ATTS. First, as a revenue-driven business, ATTS will charge passengers premiums on top of the time-distance based fare setting when there is high level of demand. This indicates that ATTS is not regulated in terms of pricing policy. Second, a smartphone may turn any licensed car owner into an ATTS driver. Hence, there is no restriction on the fleet size of ATTS and drivers have the freedom to join or leave the market at anytime. Moreover, due to joint-effect of the two properties, ATTS drivers are observed to join the market strategically based on the amount of premiums, known as 'chasing the surge' (Cook, 2015). On the contrary, TTS drivers purchase the medallion to enter the market, and usually keep running for their entire shift in order to compensate the entry cost and make revenue.

In this paper, we build the taxi network model for the coexistence of TTS and ATTS. In particular, the interactions among passengers, TTS drivers and ATTS drivers are mathematically formulated as a taxi market multi-leader follower game (TM-L/F game), and passenger equilibrium and driver equilibrium are modeled as generalized Nash equilibrium problems (GNEPs). For the GNEPs, passengers choose from TTS and ATTS to minimize their trip cost. And TTS and ATTS drivers need to decide where to pick up the next passengers to maximize their revenue. The study contributes to the literature in the following ways: (1) a network service model is formulated to characterize passenger and driver behavior in the taxi market with both TTS and ATTS, (2) competition in the market is modeled as a multi-leader-follower game and the definition of TMC Equilibrium is introduced, (3) the structure of the game is analyzed and variational inequality is used to prove the existence of market equilibrium for the taxi market with competition, (4) an iterative algorithm is developed to find the strongly stationary point of the multi-leader-follower game, and (5) we explore the consequences of change of market attributes including fleet size and pricing policy, and provide insights for regulating the taxi market with competition.

The rest of the paper is organized as follows. Next section introduces mathematical notation and preliminaries of taxi market, followed by formulations of passenger and driver behavior in taxi market with TTS and ATTS. The third section casts the network model into the TM-L/F game, discusses structural properties of the game, and proves the existence of TMC Equilibrium. The fourth part relaxes the GNEPs in both levels of the game as augmented Nash equilibrium problems (NEPs), and presents an iterative algorithm for solving the TM-L/F game. Finally, the numerical experiments are presented and results are discussed, and summary and future studies are provided.

Table 1
Notation list of fixed parameters and variables.

Notation	Description
Fixed Parameters	
α^s	Coefficient which captures perceived cost per unit trip time for passengers with mode s
β^s	Coefficient which captures perceived revenue per unit trip time for drivers with mode s
$\mathcal{N}$	The collection of zonal areas in the network
N^s	Base number of drivers for trip service s
Ω	The set of all trip pairs (i, j), where $i, j \in \mathcal{N}$
s	Trip service mode $s \in \{I, II\}$, where I refers to trips by TTS and trip mode II is the ATTS.
$t_{(i,j)}$	Trip time from zone i to zone j. $(i, j) \in \Omega$
Variables	
$c^s_{(i,j)}(y)$	Trip costs net of revenues for drivers traveling from i to j with service s, $c : R^{\lvert\Omega\rvert}_+ \rightarrow R$
F^s_i	Average trip nest revenue of service s at zone i
N^{II}_i	Induced number of ATTS drivers at zone i
S^s_i	Surge price of mode s at zone i, $i \in \mathcal{I}$
$u^s_{(i,j)}(x)$	Travel cost function for passengers traveling from i to j with service s, $u : R^{\lvert\Omega\rvert}_+ \rightarrow R$
w^I_i	Average searching time for TTS drivers at zone i to find a passenger
w^{II}_i	Average searching time for ATTS drivers at zone i to find a passenger
W^s_i	Average passenger waiting time of service s at zone i.
$x^s_{(i,j)}$	Passenger trips from zone i to zone j with service s, $x \in R^{\lvert\Omega\rvert}_+$
$y^s_{(i,j)}$	Empty trips from zone i to zone j $(i \neq j)$with service s, $y \in R^{\lvert\Omega\rvert - \lvert\mathcal{N}\rvert}_+$
$y^s_{(i,i)}$	Excessive drivers at zone i, $y \in R^{\lvert\mathcal{N}\rvert}_+$.

2. Formulation

2.1. Notation

Before discussing the model in detail, we first present the set of notations used in the study (Table 1):

2.2. Market preliminaries

We develop a network model for taxi market with TTS and ATTS. In the network model, the set of nodes refers to trip zones, which may serve as trip origins and destinations. The set of links represents trip pairs and the flow carried by links is the amount of trips between each pair of nodes. The overall market network can be further decomposed into three inter-dependent sub-networks: the network of passengers, the network of TTS drivers, and the network of ATTS drivers. The inter-dependency among the three sub-networks lies in that passengers' cost is determined by how drivers are distributed in the network, and drivers' distribution is largely affected based on where passengers are located. Self-loops are allowed and represent intra-zone trips. An example of the taxi market network is given in Fig. 1.

We consider a taxi market with independent dispatching platforms for TTS and ATTS. Passengers have equal access to both TTS and ATTS platforms to obtain the estimated arrival time of drivers. TTS and ATTS drivers can only access their specific platform to receive dispatching orders, which contain the information of trip origin and destination. Moreover, the platform will only send passenger orders to nearby drivers. In the taxi market, the operation strategy for TTS drivers is considered as the combination of street hailing trips and orders from a dispatching platform. They can either pick up a passenger by the curb side, or accept a dispatched order during cruising. On the other hand, ATTS drivers are only allowed to serve orders sent from their platform.

Before discussing the mathematical formulations, we first present the main assumptions in this study:

1. Drivers and passengers are assumed to be selfish. Passengers will always choose the service with the lowest cost, and drivers will maximize their revenue by determining the place to pick up next passengers after each drop-off.
2. Both passengers and drivers are assumed to have perfect information. Passengers know exactly how long they need to wait for their service and the total trip cost from both platforms. Also, the platforms provide drivers with the location of passengers as well as the trip revenue.
3. We assume that the average searching time for ATTS drivers is no longer than that of TTS drivers in the same zone under same supply and demand level, since orders are more optimally assigned by the ATTS platform.
4. Similarly, we assume that the average waiting time for ATTS passengers is no longer than that of TTS passengers in the same zone under same supply and demand level.
5. TTS is well regulated, with fixed fleet size and pricing policy. On the contrary, ATTS market is unregulated, with no caps on the fleet size and surge price is applied. Moreover, ATTS drivers will behave strategically based on the number of passengers in the market and the amount of surge premiums.
6. For simplicity, it is assumed that the trip cost and revenue are proportional to travel time, since trip time and distance are closely correlated in a given time interval. Such relationship is also revealed from the real-world taxi trip data (Zhan et al., 2013).

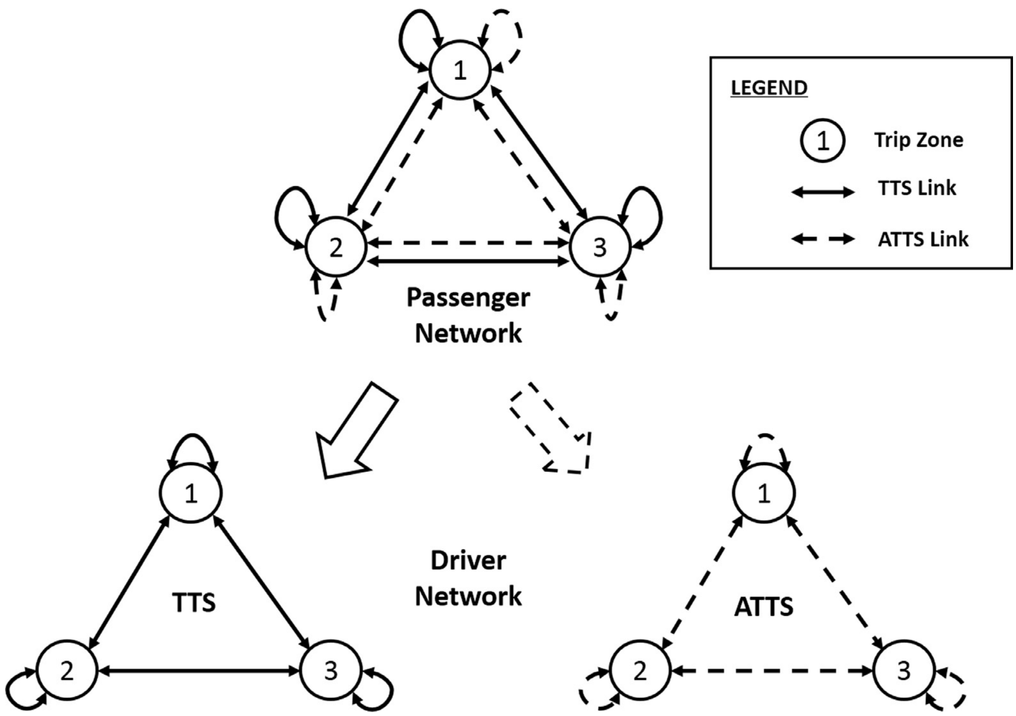


Fig. 1. A sample network for the taxi market with TTS and ATTS.

For taxi passengers, their goal is to minimize their travel costs, which is affected by the decisions of other passengers. Similarly, drivers' objective is to maximize revenue by deciding where to pick up passengers, and the decision is closely associated with the choices of other drivers. Consequently, the behavior of taxi drivers and passengers can be modeled as two Nash games on the taxi network. For passenger game, the players are links in passenger network, which reflect the decision of individual passengers by the amount of flow on the link. The game reaches equilibrium state when there is no single passenger who can improve his or her trip cost by unilaterally altering trip mode. For driver game, the players are links in TTS (ATTS) network, where the strategies of drivers are represented by the amount of driver flow on the link. The driver game reaches equilibrium state when no single driver can improve his or her trip revenue by unilaterally altering his or her next pick-up location. The overall framework of the taxi market game between TTS and ATTS, and the inter-dependency between passenger and driver games are described in Fig. 2. As shown in Fig. 2, TTS and ATTS drivers' distributions are the results of the Nash games with the passenger flow as input. Similarly, the flow pattern of TTS and ATTS drivers determines the passenger trip cost, and eventually leads to new passenger flow pattern. Additionally, the surge price introduces another layer of impact on both driver and passenger games through changing the supply level of ATTS driver, and subsequently affecting ATTS passengers' cost. The mathematical formulations of this modeling framework are presented in the following two sections, where cost functions and constraints are developed based on the behavior of drivers and passengers in this taxi market game.

2.2.1. Driver movement

We consider drivers having two states: occupied and available. When a driver is occupied, he will drive all the way to passengers' destination. Upon completion of a trip, the driver becomes available and needs to decide the next pick-up location which maximizes his or her perceived revenue. We define the costs net of revenues for TTS and ATTS drivers moving from zone i to zone j as:

$$c^{I}_{(i,j)}(y^{I}_{(i,j)}, y^{I}_{-(i,j)}) = \beta^{I}(t_{(i,j)} + w^{I}_{j}) - S^{I}_{j}F^{I}_{j} \tag{2.1}$$

$$c^{II}_{(i,j)}(y^{II}_{(i,j)}, y^{II}_{-(i,j)}) = \beta^{II}(t_{(i,j)} + w^{II}_{j}) - S^{II}_{j}F^{II}_{j} \tag{2.2}$$

The flow vector $(y^{s}_{(i,j)}, y^{s}_{-(i,j)})$ represents the strategy of player (i, j) when decisions of all other players $y^{s}_{-(i,j)}$ are known. The function for costs net of revenues contains three components: $t_{(i,\,j)}$ is the travel time of running empty from zone i to zone j, w^{s}_{j} is the average searching time for finding the next passenger at zone j, and $S^{s}_{j}F^{s}_{j}$ is the revenue a driver may expect by picking up a passenger at zone j. In particular, the trip revenue F^{s}_{j} is calculated as the average trip fare of all passenger

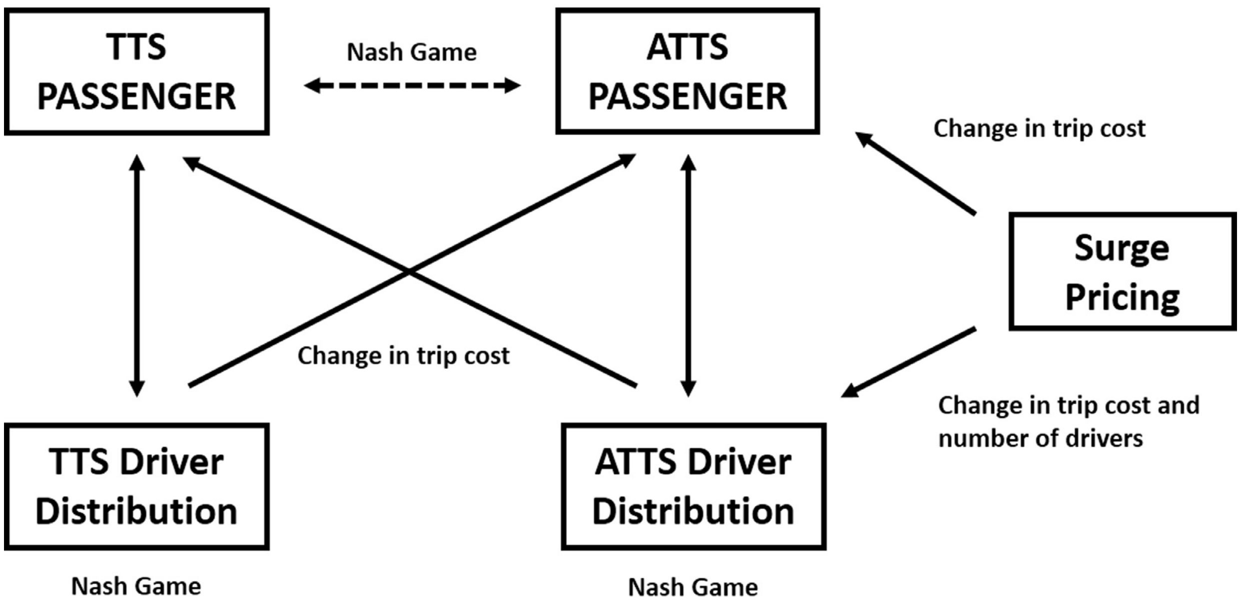


Fig. 2. Illustration of the TTS-ATTS competition.

trips originated from zone j, e.g. $F^I_j = \frac{\sum_j a^I x^I_{(i,j)} t_{(i,j)}}{\sum_j x^I_{(i,j)}}$ for TTS. Note that while surge price may be applicable to both TTS and ATTS, we only model such factor for ATTS and S^I_j is set to 1 for the rest of the study.

The average TTS searching time is derived as

$$w^I_j = \frac{A^I_j}{1 + \sum_{k\in\mathcal{N}} x^I_{(j,k)}} \frac{y^I_{(j,j)} + \sum_{k\in\mathcal{N}} x^I_{(j,k)}}{\sum_{k\in\mathcal{N}} x^I_{(j,k)}} \tag{2.3}$$

where A^I_j is zonal cruising-dispatching parameter for TTS, and its value depends on the size and road length of zone j. It can be interpreted as the time that is required for TTS drivers to travel across the entire zone following their cruising-dispatching strategy and no passenger is found. We assume that passengers are distributed uniformly in the trip zone. Therefore, the first component of w_j states that average searching time decreases hyperbolically as the number of passengers increases, if demand and supply levels are perfectly matched. However, oftentimes the supply level may exceed the number of available passengers. Consequently, the second component captures this situation by scaling up the average searching time due to competitions from excessive drivers $y^I_{(j,j)}$. If $y^I_{(j,j)} = 0$, then the amount of passengers and drivers are balanced, and the searching time is determined by only the first component.

For ATTS, the average searching time is derived following the similar form as of TTS:

$$w^{II}_j = \frac{A^{II}_j}{1 + \sum_{k\in\mathcal{N}} x^{II}_{(j,k)}} \frac{y^{II}_{(j,j)} + \sum_{k\in\mathcal{N}} x^{II}_{(j,k)}}{\sum_{k\in\mathcal{N}} x^{II}_{(j,k)}} \tag{2.4}$$

where A^{II}_j is named as the zonal dispatching parameter. The difference of ATTS searching time compared to TTS searching time is that the value of coefficient A^{II}_j should be no greater than that of A^I_j. This corresponds to our third assumption and reflects the technology advances of ATTS in matching drivers with passengers.

For each zone i, the following flow conservation constraints should be satisfied:

$$\sum_m x^s_{(m,i)} = \sum_{n\in\mathcal{N}/i} y^s_{(i,n)} - y^s_{(i,i)}, \quad \forall i, s \tag{2.5}$$

$$\sum_{m\in\mathcal{N}/i} y^s_{(m,i)} - y^s_{(i,i)} = \sum_n x^s_{(i,n)}, \quad \forall i, s \tag{2.6}$$

where $y^s_{(i,n)}$, $i \neq n$ represents empty trips traveling from zone i to zone n, $x^s_{(i,n)}$ refers to occupied trips, and $y^s_{(i,i)}$ is the excessive drivers at zone i. When a driver completes a trip, he or she needs to decide the next zone to search for passengers. Two choices are immediately available: (1) moving to zone j without passenger on board, which is characterized by $y^s_{(i,j)}$, and (2) staying at zone i (assuming he/she finishes the trip at zone i). The second option may result in two outcomes: (a) the driver successfully finds a passenger and is captured by $x^s_{(i,i)}$, and (b) the driver fails to find a trip after searching across zone i, which corresponds to $y^s_{(i,i)}$. These give rise to the two conservation constraints above. In particular, Eq. (2.5) is the departing driver flow constraint, suggesting that the summation of all arriving passenger flow into zone i and the excessive drivers at zone i should be equal to the total number of vacant drivers departed from zone i. Eq. (2.6) is the arriving driver flow constraint, implying that vacant drivers arrived at zone i will either leave zone i occupied or fail to find a passenger and become excessive drivers. If there is no excessive drivers and $y^s_{(i,i)} = 0$, the flow conservation constraints are consistent with those in Yang and Wong (1998), where all drivers are assumed to be able to find a passenger.

Finally, since we model the taxi market for one-hour time period, it is therefore restricted that the total service hour (occupied trip + empty trip time) constraints should not be violated:

$$\sum_{(i,j)\in\Omega} [x^I_{(i,j)}(t_{(i,j)} + w^I_i) + y^I_{(i,j)}t_{(i,j)}] = N^I \tag{2.7}$$

$$\sum_{(i,j)\in\Omega} [x^{II}_{(i,j)}(t_{(i,j)} + w^{II}_i) + y^{II}_{(i,j)}t_{(i,j)}] = N^{II} + \sum_{i\in\mathcal{N}} \bar{N}^{II}_i \tag{2.8}$$

The first component on the left-hand-side of the Eq. (2.7) is the total taxi hour spent on occupied trips, which comprises trip time and searching time. The second component on the left-hand-side of the equation denotes the total taxi hours spent on vacant movement between (within) zones. Compared with Eq. (2.7), Eq. (2.8) has an extra term $\bar{N}^{II}_i$, which captures the entrance of additional drivers due to incentives of high surge price.

2.2.2. Choice of passenger

For any given OD pair (i, j), passengers have equal access to TTS and ATTS. Consequently, passengers will choose the service which gives the minimum travel cost. The choices are closely associated with the competition between TTS and ATTS. We define passenger cost functions as:

$$u^I_{(i,j)}(x^I_{(i,j)}, x^I_{-(i,j)}) = \alpha^I(t_{(i,j)} + W^I_i) \tag{2.9}$$

$$u^{II}_{(i,j)}(x^{II}_{(i,j)}, x^{II}_{-(i,j)}) = S^{II}_i\alpha^{II}t_{(i,j)} + \alpha^{II}(z^{II}_i + W^{II}_i) - \phi \tag{2.10}$$

The flow vector $(x^s_{(i,j)}, x^s_{-(i,j)})$ represents the strategy of player (i, j) when decisions of all other players $x^s_{-(i,j)}$ are known. Eqs. (2.9) and (2.10) state that waiting time and in-vehicle travel time are the two main components of passenger cost. The term z^{II}_i is a constant representing order confirmation time after a passenger orders the ATTS service using app. As it is assumed that ATTS service is always available to passengers, the z^{II}_i is set to a small value. ϕ is a constant that accounts for the monetary value of extra benefits for ATTS, such as technology advance, and newer and cleaner vehicles. While we use a single term to capture the benefits, a more comprehensive expression including heterogeneity of different features can be developed by collecting passengers' behavior data using a survey instrument.

For each OD pair, passenger demand should satisfy the flow conservation constraint where the summation of demand served by TTS and ATTS should equal to total passenger demand, as follows

$$x^{II}_{(i,j)} + x^I_{(i,j)} = x_{(i,j)} \tag{2.11}$$

Different from the regulated TTS, the ATTS is free entry and the surge price scheme is also applied. In reality, the scheme uses a surge price multiplier (SPM) where the value is set to 1 (no change in price) if demand and supply are balanced, or if there is more supply than demand. When there is a shortage of supply, the SPM will be set to a higher value greater than 1 to generate extra profits. In our study, we also allow the SPM to be less than one, which offers trip discount for passengers. Additional ATTS drivers will enter the market based on the value of SPM and the demand level (Hall et al., 2015), and this behavior can be captured using the following function:

$$\bar{N}^{II}_i = \pi \sum_j x^{II}_{(i,j)}(S^{II}_i - 1) \tag{2.12}$$

where π is the coefficient which controls the entrance rate of drivers with respect to the amount of passengers, and a linear relationship is specified with respect to the increase in SPM.

It is well-accepted that passenger waiting time is inversely associated with total vacant taxi hours in each zone (Arnott, 1996). Following the same idea, we consider that passenger waiting time is inversely proportional to the number of available drivers and the amount of excessive drivers in each zone. It is intuitive that more excessive drivers at zonal level will lead to more vacant taxi hours. This assumption helps to maintain the mathematical simplicity while at the same time captures the underlying mechanism of taxi market. For TTS, the waiting cost is calculated as:

$$W^I_i = \frac{B^I_i}{1 + \sum_{k\in\mathcal{N}/i} y^I_{(k,i)} - y^I_{(i,i)}} \frac{\sum_{j\in\mathcal{N}} x^I_{(i,j)}}{\sum_{k\in\mathcal{N}/i} y^I_{(k,i)}} \tag{2.13}$$

where B^I_i is the zonal coefficient which captures waiting time for passengers using TTS. Similar to our discussion related to Eq. (2.3), the first component of Eq. (2.13) captures the average passenger waiting time if the demand and supply level are balanced. Following flow conservation constraint (2.6), $\sum_{k\in\mathcal{N}/i} y^I_{(k,i)}$ captures exactly the portion of occupied drivers among all arrived available drivers, and more occupied drivers imply lower waiting time. As for the second component, it deals with the existence of excessive drivers. The more the amount of excessive drivers, the smaller the second component will be, which leads to reduced passenger waiting time.

Similar passenger waiting time function for ATTS service can be constructed as

$$W^{II}_i = \frac{B^{II}_i}{1+\sum_{k\in\mathcal{N}/i} y^{II}_{(k,i)} - y^{II}_{(i,i)}} \frac{\sum_{j\in\mathcal{N}} x^{II}_{(i,j)}}{\sum_{k\in\mathcal{N}/i} y^{II}_{(k,i)}} \tag{2.14}$$

and B^{II}_i is no greater than B^I_i to reflect that ATTS passengers will experience lower waiting time compared with TTS passengers at the same zone, under same demand and supply level.

3. Model properties

Based on previous discussions, it can be verified that the taxi market with TTS and ATTS includes three inter-dependent games: the passenger game for choosing the service provider, the TTS driver game on where to cruise for next passenger, and the ATTS driver game on which zone to move to for receiving the next order. While for each game drivers and passengers perform non-cooperatively to maximize their revenue and minimize trip cost, the inter-dependency among the games results in the Stackelberg competition in the taxi market. In particular, passengers who are the demand side of the market can be viewed as the leaders of the game, and drivers who are the supply side of the market will respond to passengers' actions. Nevertheless, different from the Stackelberg game, the taxi market has more than one leader with the same set of followers competing non-cooperatively, which results in the form of the multi-leader-follower game. The corresponding mathematical model for the taxi market leader-follower (TM-L/F) game is known as the equilibrium problem with equilibrium constraints (EPEC). Attacking the multiple-leader-follower game is intrinsically difficult, as suggested by Pang and Fukushima (2005), since the solution of the multi-leader-follower game may not exist and solving the problem may be computationally intractable, due to the non-convexity of the followers' feasible set. In this section, we analyze the properties of the TM-L/F game and prove the existence of the equilibrium for this particular game.

We first introduce the definition for the equilibrium in taxi market with competition (TMC Equilibrium).

Definition 3.1 (TMC Equilibrium)**.** The TM-L/F game reaches TMC Equilibrium when no driver can improve his or her trip revenue by unilaterally changing his or her strategy, given the equilibrium responses from all passengers. Moreover, no individual passenger can reduce his or her travel cost by unilaterally changing his or her strategy, given the equilibrium responses from TTS and ATTS drivers.

Definition 3.1 stipulates that the market equilibrium is achieved when all three games arrive at their equilibria simultaneously.

Note that for the TM-L/F game, the cost functions for each leader (Eqs. (2.9) and (2.10)) and follower (Eqs. (2.1) and (2.2)) are not only the function of themselves, but also rely on the strategy of other leaders and followers. And the same is observed for the constraint set for each leader and follower. This results in a non-symmetric Hessian and therefore the problem can no longer be formulated and solved as a standard optimization problem. It is well-known that when each player of a Nash game solves a convex program, the Nash game can be formulated and solved as a finite-dimensional variational inequality (VI) problem (Harker and Pang, 1990). Moreover, when each player's set depends on the strategy set of other players, the game is known as the GNEP and can be written as a quasi-variation inequality (QVI) problem (Harker, 1991). Consequently, in this study, the leader game and the follower games are formulated as QVIs.

For cleaner mathematical representation, in the following analysis, we denote Eqs. (2.1) and (2.2) as $c^s_{(i,j)}(y^s_{(i,j)}, \bar{y}^s_{-(i,j)}, \bar{x})$, and constraints (2.5) to (2.8) as $h^s_{(i,j)}(y^s_{(i,j)}, \bar{y}^s_{-(i,j)}, \bar{x})$, where $\bar{y}^s_{-(i,j)}$ is the fixed vector representing the empty trips on all other links except (i, j), and $\bar{x}$ is the response of passengers which is also fixed. Denote $y^{s*}_{(i,j)}$ as the optimal strategy of player (i, j), we can rewrite the QVIs for TTS and ATTS drivers as

$$[Follower\,Game] \qquad (y^s_{(i,j)} - y^{s*}_{(i,j)}) c^s_{(i,j)}(y^{s*}_{(i,j)}, \bar{y}^{s*}_{-(i,j)}, \bar{x}) \geq 0, \quad \forall y^s_{(i,j)} \in H^s_{(i,j)}(\bar{y}^s_{-(i,j)}, \bar{x}),\ \forall s \in \{I, II\} \tag{3.1}$$

where $H^s_{(i,j)}(\bar{y}^s_{-(i,j)}, \bar{x}) = \{y^s_{(i,j)} \in R_+ | h^s_{(i,j)}(y^s_{(i,j)}, \bar{y}^s_{-(i,j)}, \bar{x}) = 0\}$ is the point-to-set mapping which can be understood as the moving strategy set of each follower, and R_+ denotes the non-negative orthant.

Similarly, denoting cost function of leader (i, j) as $u^s_{(i,j)}(x^{s*}_{(i,j)}, \bar{x}^{s*}_{-(i,j)}, \bar{y})$ and the constraint set as $K^s_{(i,j)}(\bar{x}^s_{-(i,j)})$, we can rewrite the leader game as

$$[Leader\,Game] \qquad (x^s_{(i,j)} - x^{s*}_{(i,j)}) u^s_{(i,j)}(x^{s*}_{(i,j)}, \bar{x}^{s*}_{-(i,j)}, \bar{y}) \geq 0, \quad \forall x^s_{(i,j)} \in K^s_{(i,j)}(\bar{x}^s_{-(i,j)}), \forall s \in \{I, II\} \tag{3.2}$$

where $K^s_{(i,j)}(\bar{x}^s_{-(i,j)}) = \{x^s_{(i,j)} \in R_+ | x^I_{(i,j)} + x^{II}_{(i,j)} = x_{(i,j)}\}$ is the point-to-set mapping which refers to the moving strategy set of each leader.

For each service mode s, it is observed that the mapping $c^s_{(i,j)}(y)$ and the feasible region $H^s_{(i,j)}(y)$ are parameterized by the fixed passenger flow vector $\bar{x}$ and the fixed strategy set of other followers $\bar{y}^s_{-(i,j)}$. As for each leader (i, j), the mapping $u^s_{(i,j)}(x)$ relies on the response of followers and the strategy of other leaders, and the constraint set depends only on other leaders' strategies. Since TTS and ATTS drivers can be viewed as two independent firms, we denote the follower games as two independent QVIs parameterized by $\bar{x}$: $QVI^I(\bar{x})$ and $QVI^{II}(\bar{x})$. Moreover, we denote the solution set of the two QVIs as $SOL(QVI^I(\bar{x}))$ and $SOL(QVI^{II}(\bar{x}))$, which represent the response of TTS and ATTS drivers to the strategy of passengers. The TM-L/F game can be expressed as one single QVI as:

$$[TM-L/F\ Game] \begin{array}{l} (x^s_{(i,j)} - x^{s*}_{(i,j)})u^s_{(i,j)}(x^{s*}_{(i,j)}, \bar{x}^{s*}_{-(i,j)}, y^I, y^{II}) \geq 0, \forall\,(i,j) \in \Omega, \forall s \in \{I, II\} \\ (x^s_{(i,j)}, y^I, y^{II}) \in K^s_{(i,j)}(\bar{x}^s_{-(i,j)}) \times SOL(QVI^I(x)) \times SOL(QVI^{II}(x)) \end{array} \quad (3.3)$$

where $\times$ is the Cartesian product.

We next prove the existence of solution for the TM-L/F game (3.3).

Proposition 3.1. *Given the strategy vector x of all leaders , the solution set of followers $SOL(QVI^I(x))$ and $SOL(QVI^{II}(x))$ are nonempty, compact and upper semi-continuous.*

Proof. For each player in the follower game, the feasible region $H^s_{(i,j)}(\bar{y}^s_{-(i,j)}, \bar{x})$ consists of affine equality constraints, which are parameterized by $\bar{x}$ and $\bar{y}^s_{-(i,j)}$. Therefore, $H^s_{(i,j)}$ is convex and compact with respect to $y^s_{(i,j)}$. The feasible region of the follower game is simply the Cartesian product of the feasible regions of all players:

$$\mathcal{H}^s(\bar{x}) = \prod_{(i,j)\in\Omega} H^s_{(i,j)}(\bar{y}^s_{-(i,j)}, \bar{x})$$

It is easy to verify that $\mathcal{H}^s$ is nonempty, as we can always find the driver flow to match passenger flow under the mild assumption that all passengers will eventually get a ride. We can further conclude $\mathcal{H}^s$ is compact following the Bolzano-Weiertrass theorem on the Cartesian product of compact sets. Moreover, the mapping $c^s_{(i,j)}(x, y)$, which is parameterized by x, is continuous on y. Consequently, the sets $SOL(QVI^I(\bar{x}))$ and $SOL(QVI^II(\bar{x}))$ are nonempty and compact. The upper semi-continuity of the solution sets comes naturally from Theorem 7 in Berge (1963). This completes the proof. □

Theorem 3.2 (Corollary 1, Harker and Pang, 1988)**.** *Let $f : R^{n+m} \to R$ be continuous, an optimal solution to the problem*

$$\text{Minimize} \quad f(x, y), \text{ subject to } x \in X \text{ and } y \in Y(x)$$

exists if X is a nonempty subset of R^n and Y: $X \to R^m$ is a nonempty-valued, compact-valued, upper semi-continuous point-to-set map.

For each leader (i, j), $K^s_{(i,j)}(x)$ simply involves OD-demand conservation constraint and $SOL(QVI^I(x))$ and $SOL(QVI^{II}(x))$ are point-to-set maps having properties as in Proposition 3.1. It follows readily from Theorem 3.2 that an optimal strategy exists for each player in game (3.3). Nevertheless, the existence of optimal strategy of individual player does not guarantee the existence of optimal strategy (possibly local) for the TM-L/F game. Denote (x^*, y^*) as the equilibrium solution to the TM-L/F game, the solution set of the TM-L/F game is given by the intersection of the solution sets of each player, which may be empty. Fortunately, we can utilize the property of joint feasible set to show that the TM-L/F game indeed possesses an equilibrium.

Definition 3.2. A GNEP is a GNEP with Jointly Convex Shared Constraints (JCSC) if the feasible sets are defined as:

$$\mathcal{K}_i(x_{-i}) \triangleq \{x_i \in \bar{\mathcal{K}}_i : g(x_i, x_{-i}) \leq 0\}$$

where $\bar{\mathcal{K}}_i$ is the closed and convex set of individual constraints of player i and $g(x_i, x_{-i})$ is the set of shared coupling constraints which has the same form for all players and is jointly convex. Moreover, a GNEP is a GNEP with Jointly Shared Constraints (JSC) if the shared coupling constraints may not be convex. Note that JSC-GNEP is a general case of JCSC-GNEP with weaker condition on the shared coupling constraint.

The TM-L/F game can be formulated as a GNEP with JSC with the mapping function:

$$U(x, y) = [u^I_{(1,2)}, u^{II}_{(1,2)}, .., u^{II}_{(i,j)}, \ldots]^T$$

and we need to find the flow vector $x^* \in \mathcal{K}(x, y)$ such that

$$[JSC\,TM - L/F\ Game] \qquad (x - x^*)^T U(x^*, y^*) \geq 0, \forall x \in \mathcal{K}(x, y) \quad (3.4)$$

where $\mathcal{K}(x, y) \triangleq \{(x, y) | x \in \mathcal{X},\ y \in SOL(QVI(x))\}$ and $\mathcal{X} \triangleq \{x \in R^{|\Omega|}_+ | x^I_{(i,j)} + x^{II}_{(i,j)} = x_{(i,j)},\ \forall (i,j) \in \Omega\}$. The set $\mathcal{X}$ refers to the OD-demand conservation constraints which are satisfied by all players simultaneously.

Proposition 3.3. *The solution of the JSC TM-L/F game (3.4) is a solution of the TM-L/F game (3.3).*

Proof. Let (x^*, y^*) be the optimal solution of the JSC TM-L/F game. Since $(x^*, y^*) \in \mathcal{K}(x^*, y^*)$, it follows readily that every $(x^{s*}_{(i,j)}, x^{s*}_{-(i,j)}, y^*) \in K^s_{(i,j)}(x^{s*}_{-(i,j)}, y^*)$. Moreover, $(x - x^*)^T U(x^*, y^*) \geq 0$ implies that

$$(x^s_{(i,j)} - x^{s*}_{(i,j)})u^s_{(i,j)}(x^{s*}_{(i,j)}, \bar{x}^*_{-(i,j)}, y^{I*}, y^{II*}) \geq 0, \forall x^s_{(i,j)} \in K_{(i,j)}(x^{s*}_{-(i,j)}, y^*)$$

Consequently, (x^*, y^*) is a solution to the TM-L/F game. □

Proposition 3.4. *The TM-L/F game has an TMC equilibria.*

Proof. It is easy to verify that $\mathcal{X}$ is non-empty, convex, and compact, and the mapping $U(x, y)$ is continuous. By applying Theorem 3.2, we arrive at the conclusion that the JSC GNEP admits a solution. Consequently, **we conclude that the TM-L/F game admits an TMC equilibria.** □

4. Solution algorithm

Recall that the TM-L/F game is generally non-convex due to the non-convex feasible set of follower games. As a non-convex problem, finding an optimal solution was proved to be NP-hard (Murty and Kabadi, 1987) and is therefore computationally intractable. Hence, the focus of the study is therefore to find a stationary point of the TM-L/F game, where the TMC Equilibrium Definition 3.1 may also be applicable. Before presenting the algorithm, we first write the EPEC form of the TM-L/F game and introduce preliminaries for identifying the strongly stationary point.

First, consider the following formulations for a particular player (i, j) of mode s in the TM-L/F game 3.3, which is a Stackelberg game expressed as a mathematical program with equilibrium constraint (MPEC)

$$\begin{aligned}
&(x^s_{(i,j)} - x^{s*}_{(i,j)})u^s_{(i,j)}(x^{s*}_{(i,j)}, \bar{x}^{s*}_{-(i,j)}, y^I, y^{II}) \geq 0 \\
&(x^s_{(i,j)}, y^I, y^{II}) \in K^s_{(i,j)}(\bar{x}^s_{-(i,j)}) \times SOL(QVI^I(x)) \times SOL(QVI^{II}(x))
\end{aligned} \tag{4.1}$$

For the MPEC (4.1), the Abadie constraint qualification always holds for the follower game given the response x from the leader game, and we can rewrite the $SOL(QVI^I(x))$ and $SOL(QVI^{II}(x))$ using the Karush-Kuhn-Tucker (KKT) condition, which takes the form of a non-linear complementarity problem parameterized by x. Therefore, we can create the following mathematical program with complementarity constraints:

$$\begin{aligned}
(x^s_{(i,j)} - x^{s*}_{(i,j)})u^s_{(i,j)}(x^{s*}_{(i,j)}, \bar{x}^{s*}_{-(i,j)}, y^I, y^{II}) &\geq 0 \\
x^s_{(i,j)} \in K^s_{(i,j)}(\bar{x}^s_{-(i,j)})& \\
0 \leq y^I \perp d^I &\geq 0 \\
C^I(y^I, x) + \nabla_{y^I} H^I(y^I, x)^T \lambda^I_{(i,j)} - d^I &= 0 \\
H^I(y^I, x) &= 0 \\
0 \leq y^{II} \perp d^{II} &\geq 0 \\
C^{II}(y^{II}, x) + \nabla_{y^{II}} H^{II}(y^{II}, x)^T \lambda^{II}_{(i,j)} - d^{II} &= 0 \\
H^{II}(y^{II}, x) &= 0
\end{aligned} \tag{4.2}$$

where d^I and d^{II} are slack variables.

As suggested by Su (2005), the EPEC form of the TM-L/F game can be created by concatenating the MPEC (4.2) for all leaders as:

$$\begin{aligned}
(x^s_{(i,j)} - x^{s*}_{(i,j)})u^s_{(i,j)}(x^{s*}_{(i,j)}, \bar{x}^{s*}_{-(i,j)}, y^I, y^{II}) &\geq 0, \forall s \in \{I, II\}, \forall (i,j) \in \Omega \\
x^s_{(i,j)} \in K^s_{(i,j)}(\bar{x}^s_{-(i,j)})&, \forall s \in \{I, II\}, \forall (i,j) \in \Omega \\
0 \leq y^I \perp d^I &\geq 0 \\
C^I(y^I, x) + \nabla_{y^I} H^I(y^I, x)^T \lambda^I_{(i,j)} - d^I &= 0, \forall (i,j) \in \Omega \\
H^I(y^I, x) &= 0 \\
0 \leq y^{II} \perp d^{II} &\geq 0 \\
C^{II}(y^{II}, x) + \nabla_{y^{II}} H^{II}(y^{II}, x)^T \lambda^{II}_{(i,j)} - d^{II} &= 0, \forall (i,j) \in \Omega \\
H^{II}(y^{II}, x) &= 0
\end{aligned} \tag{4.3}$$

Finally, the KKT condition for the EPEC (4.3) can be formulated following (Leyffer and Munson, 2010):

$$\begin{aligned}
\nabla_{x^s_{(i,j)}} \theta^s_{(i,j)} + \nabla_{x^s_{(i,j)}} P^s_{(i,j)} \mu^s_{(i,j)} - \chi^s_{(i,j)} &\geq 0, \forall s \in \{I, II\}, \forall (i,j) \in \Omega \\
\nabla_{y^s} \theta^s_{(i,j)} + \nabla_{y^s} P^s_{(i,j)} \mu^s_{(i,j)} - \psi^s_{(i,j)} + d^s \circ \xi^s_{(i,j)} &\geq 0, \forall s \in \{I, II\}, \forall (i,j) \in \Omega \\
\mu^s_{(i,j)} - \sigma^s_{(i,j)} + y^s \circ \xi^s_{(i,j)} &= 0, \forall s \in \{I, II\}, \forall (i,j) \in \Omega \\
P^s_{(i,j)} - d^s &= 0, \forall s \in \{I, II\}, \forall (i,j) \in \Omega \\
0 \leq x^s_{(i,j)} \perp \chi^s_{(i,j)} &\geq 0, \forall s \in \{I, II\}, \forall (i,j) \in \Omega \\
0 \leq y \perp \psi^s_{(i,j)} &\geq 0, \forall s \in \{I, II\}, \forall (i,j) \in \Omega \\
0 \leq d^s \perp \sigma^s_{(i,j)} &\geq 0, \forall s \in \{I, II\}, \forall (i,j) \in \Omega
\end{aligned}$$

$$0 \leq -y^s \circ d^s \perp \xi^s_{(i,j)} \geq 0, \forall s \in \{I, II\}, \forall (i,j) \in \Omega \tag{4.4}$$

where $\theta^s_{(i,j)} = \int u^s_{(i,j)} dx^s_{(i,j)}$, $P^s_{(i,j)}$ is the Cartesian product of all equality constraints of EPEC (4.3), and $\circ$ represents the Hadamard product.

Definition 4.1. A solution $(x^*, y^*, d^*, \chi^*, \lambda^*, \mu^*, \psi^*, \sigma^*, \xi^*)$ of (4.4) is called a strongly stationary point of the EPEC (4.3).

Definition 4.2. Individual MPEC (4.1) is said to satisfy an MPEC linear independent constraint qualification (MPEC-LICQ) if the MPCC (4.2) without the complementarity conditions satisfies an LICQ.

Proposition 4.1. *If (x^*, y^*) is an equilibrium point of the TM-L/F game (3.3), and if every MPEC (4.1) satisfies MPEC-LICQ, then there exists multipliers $(\chi^*, \lambda^*, \mu^*, \psi^*, \sigma^*, \xi^*)$ so that the KKT system (4.4) holds.*

Proof. The statement follows directly from Proposition 3.1 in Leyffer and Munson (2010). □

Definition 4.1 stipulates that any TMC Equilibrium of the TM-L/F game is a strongly stationary point. Though it is intractable to obtain an optimal solution, finding a strongly stationary point may lead to the TMC Equilibrium and will reveal the stationary properties of the taxi market. Moreover, if MPEC-LICQ holds, Proposition 4.1 indicates that a solution of the KKT system (4.4) corresponds to the strongly stationary point of TM-L/F game, and the KKT multipliers always exist. Consequently, in order to obtain a strongly stationary solution, it is equivalent to find the point and corresponding multipliers which satisfy the KKT system (4.4).

We next present the following iterative algorithm to find the stationary point of the TM-L/F game:

The main philosophy of the Algorithm 1 is to iterate and solve the leader and follower game successively, and the algorithm stops when an optimal solution is found. The following proposition proves that the output of the algorithm is **a strongly stationary point of the TM-L/F game.**

Algorithm 1 Iterative algorithm for TM-L/F game.

Initialize a feasible leader strategy x_0, e.g., using all or nothing assignment. Set iteration number $i = 0$. Initialize step size d.
while Stopping criteria is not met **do**
  Initialize a starting strategy for the follower game.
  $y^I_i, y^{II}_i \leftarrow$ Solve the follower game 3.1 which is parameterized by x^i.
  $x'_i \leftarrow$ Solve the leader game 3.2 which is parameterized by y^I_i, y^{II}_i.
  $x_{i+1} \leftarrow x_i + d_i(x'_i - x_i)$
  $i \leftarrow i + 1$
end while

Proposition 4.2. *Let (x_k, y_k) be the sequence of solutions generated by Algorithm 1 and let (x_k, y_k) converge to (x^*, y^*) as $k \to \infty$. Then (x^*, y^*) is a strongly stationary point of the TM-L/F game.*

Proof. At convergence, the tuple (x^*, y^*) is an optimal solution of the leader game. Therefore, for the KKT system 4.4,simply take $\chi^s_{(i,j)} = \nabla_{x^s_{(i,j)}} \theta^s_{(i,j)} + \nabla_{x^s_{(i,j)}} P^s_{(i,j)} \mu^s_{(i,j)}$, and $\psi^s_{(i,j)} - d^{sT} \xi^s_{(i,j)} = \nabla_{y^s} \theta^s_{(i,j)} + \nabla_{y^s} P^s_{(i,j)} \mu^s_{(i,j)} \mu^s_{(i,j)}$, evaluated at (x^*, y^*), and we have the following complementarity conditions hold:

$$0 \leq x^s_{(i,j)} \perp \chi^s_{(i,j)} \geq 0$$
$$0 \leq y \perp \psi^s_{(i,j)} \geq 0$$

Moreover, since the solution is also optimal for the follower game and all constraints are satisfied, we have $P^s_{(i,j)} - d^s = 0$ and $y^s \circ d^s = 0$ satisfied. Consequently, the remaining active equations of the KKT system (4.4) are

$$d^s \circ \xi^s_{(i,j)} \geq 0$$
$$\mu^s_{(i,j)} - \sigma^s_{(i,j)} + y^s \circ \xi^s_{(i,j)} = 0$$
$$0 \leq d^s \perp \sigma^s_{(i,j)} \geq 0$$

And this system of equations has infinite number of solutions by properly choosing σ and ξ. This implies that we are always able to find the KKT multiplier for the tuple (x^*, y^*) to satisfy the KKT system (4.4). Consequently, (x^*, y^*) is a strongly stationary point of the TM-L/F game. □

The difficulty of the algorithm lies in that it is nontrivial to solve the leader and follower game directly, since leader and follower game take the form of GNEPs. As suggested by Pang and Fukushima (2005), while NEPs are very well studied, there were only handful results regarding the solution existence of GNEPs and the development of convergent algorithm is still in its infancy. More importantly, even for a GNEP with convex mapping and convex constraint set, it may not admit a unique solution. As solving GNEPs is the intermediate step of our problem, the existence of multiple solutions may result in inconsistency among multiple executions of the algorithm and it is therefore meaningless to compare the outputs for different scenarios.

Nevertheless, recall the Definition 3.2, instead of attacking the GNEPs directly, we can convert the leader game and the follower game as two JCSC GNEPs and solve the JCSC GNEPs to get the solution for the leader game and follower game. An appealing feature of this modification is that the JCSC GNEPS can be further relaxed as augmented NEPs by moving the coupling constraints into the objective function using penalty terms and solving NEPs can be much easier. Moreover, the relaxed NEP admits a unique solution as it solves a convex problem. In particular, the JCSC for leader game is to have all trip pairs satisfy the OD flow constraints simultaneously, and the JCSC for the follower game is to have flow conservation constraints and taxi hour constraints hold for all players at the same time. Therefore, we can rewrite the leader game (3.2) and the follower game (3.1) as:

$$[JCSC\,Follower\,Game] \qquad (y-y^*)^T C(\bar{x}, y^*) \geq 0 \quad \forall y \in \bar{\mathcal{H}}(\bar{x}) \tag{4.5}$$

$$[JCSC\,Leader\,Game] \qquad (x-x^*)^T U(x^*, \bar{y}) \geq 0 \quad \forall x \in \bar{\mathcal{K}} \tag{4.6}$$

and we obtain the augmented VIs by relaxing the JCSC GNEPsc as:

$$[VI-JCSC\,Follower\,Game] \qquad (y-y^*)^T C(\bar{x}, y^*, \lambda) \geq 0 \quad \forall y \in \bar{\mathcal{H}}(\bar{x}) \tag{4.7}$$

$$[VI-JCSC\,Leader\,Game] \qquad (x-x^*)^T U(x^*, \bar{y}, \gamma) \geq 0 \quad \forall x \in \bar{\mathcal{K}} \tag{4.8}$$

where $C(\bar{x}, y^*, \lambda) = C(\bar{x}, y^*) + \lambda \nabla_y \mathcal{H}(\bar{x})$, and $U(x^*, \bar{y}) = U(x^*, \bar{y}, \gamma) + \gamma \nabla_x \mathcal{K}(\bar{y})$.

Lemma 4.3. *Every solution of the VI(K,F) is a solution of the JCSC GNEP if the following assumption holds:*

1. *each mapping $f_i(\bullet)$ is C^2 on $\bar{\mathcal{K}}$ and convex in $x_i \in \bar{\mathcal{K}}_i$ for every fixed x_{-i}.*
2. *constraint $g(\bullet)$ is C^1 on $\bar{\mathcal{K}}$.*

Following Lemma 4.3, it is obvious that solutions of augmented leader VI (4.8) and follower VI (4.7) are solutions of the JCSC game, since the mappings and constraints are affine functions. Therefore, the solutions of (4.8) and (4.7) are also the solutions for the original leader and follower games (3.2) and (3.1). Moreover, affine mappings and constraints indicate that both augmented VIs can be converted into the mixed linear complementarity problems (MLCPs) as:

$$\begin{aligned} &0 \leq y \perp C(\bar{x}, y, \lambda) \geq 0 \\ &\qquad h(\bar{x}, y) = 0, \quad \lambda \in R^{|h|} \end{aligned} \tag{4.9}$$

$$\begin{aligned} &0 \leq x \perp U(x, \bar{y}, \gamma) \geq 0 \\ &\qquad k(x) = 0, \quad \gamma \in R^{|k|} \end{aligned} \tag{4.10}$$

Consequently, the two subproblems in Algorithm 1 reduce to solving the two MLCPs. Solving MLCP is well-studied and MLCP can be solved using commercial solvers such as KNITRO. Readers may refer to Harker and Pang (1990) for an overview of algorithms to solve MLCP problems.

5. Numerical experiments

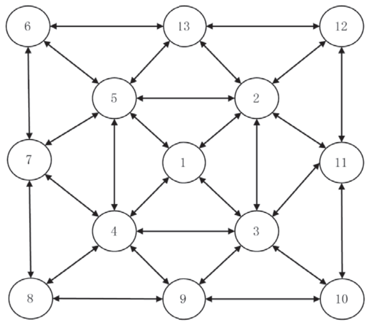
5.1. Experiment setup

We first discuss the algorithm performance and solution quality of the TMC equilibrium model. In particular, we conduct experiments on three different networks. The first test network is similar to that in Yang and Wong (1998). In addition, four self-loops are added to represent trips within the same zone. Consequently, we have 4 zones and 16 links. TTS and ATTS are available for each link, which gives the total of 32 OD pairs. The second network resembles the typical shape of urban areas with single city center, as shown in Fig. 3(a). The network has a center node as the city center, which attracts and generates highest demand. The network parameters are designed in the way so that the center node has low searching time and high travel time, and peripheral nodes have both high searching time and travel time. The third network enlarges the single center network by introducing a second center area, and the network structure is presented in Fig. 3(b). The single center network has 138 OD pairs, and the dual center network has 254 OD pairs respectively. The network configurations and parameter settings can be found in Appendix section.

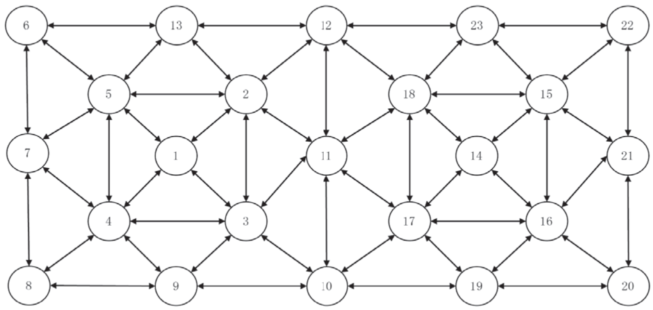
The convergence criteria is set as the relative gap of passenger flow, which can be represented as:

$$\delta_{k+1} = \frac{\sqrt{\sum_s \sum_{(i,j)} (x^s_{(i,j),k+1} - x^s_{(i,j),k})^2}}{\sum_s \sum_{(i,j)} x^s_{(i,j),k+1}}$$

The numerator is the root mean squared error which captures the passenger flow difference between successive iterations on each link for each service, and the denominator represents the total passenger flow at current iteration. While passenger



(a) Single center nerwork



(b) Dual center network

Fig. 3. Example of two taxi market networks.

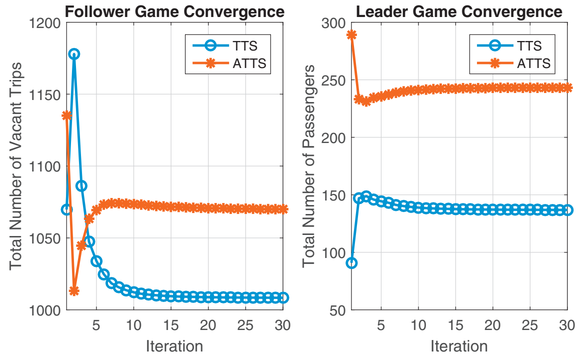
flow pattern is the output of the leader game, it is highly sensitive to changes in the lower level so that the convergence of both leader and follower games are captured simultaneously. The algorithm will stop if the relative gap is smaller than 1e-3.

5.2. Discussion

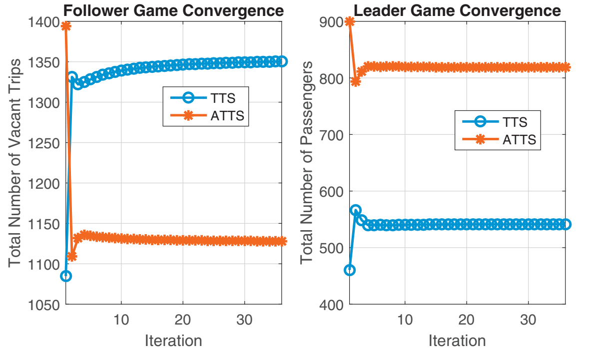
5.2.1. Algorithm convergence

All experiments are conducted on a desktop with 3.5 GHz six-core processor and 32GB RAM memory. KNITRO is called in TOMLAB optimization toolbox (Holmström, 1999) using interior point method to solve the two subproblems in each iteration. We choose the step size sequence d in Algorithm 1 as $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots, \frac{1}{k}, \frac{1}{k}, \ldots\}$, which can be found in Nagurney and Zhang (2012) as an efficient step size sequence for the method of successive average. In terms of computational efficiency, it takes 0.05 s, 0.47 s, and 1.63 s for completing each iteration in 4-node, single center, and dual center network respectively. The convergence patterns for all three networks are shown in Fig. 4. For each scenarios, the figure on the left shows the convergence of the follower game, which is characterized by the change in number of vacant trips for TTS and ATTS. The right figure is the convergence of the leader game, which is described as the change in total number of passengers served by each mode. While it converges in a few iterations on small network, the large network with two centers will require around 90 iterations to reach the stopping criteria. In general, the converge pattern of the iterative algorithm is seen to be smooth and iteration algorithm is capable of solving the TMC equilibrium problem efficiently for lager urban areas.

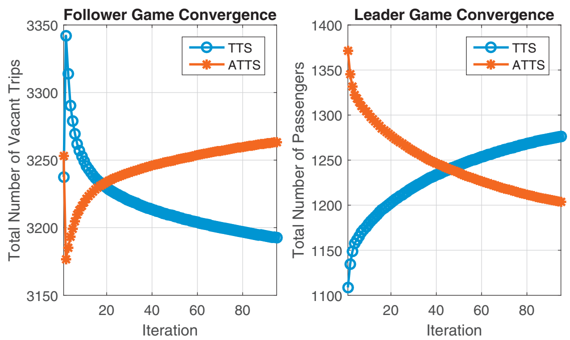
Further, it is noticed that the convergent speed is closely associated with the level of competition between TTS and ATTS in the taxi market. We set the same supply level as in Fig. 4 and test with two different SPMs. The SPM = 1.2 refers to a highly competitive taxi market, where the inflated trip cost of ATTS is close to the fixed cost of TTS. The SPM = 2.3 implies a taxi market dominated by TTS, since the trip cost for ATTS is far beyond that of TTS. For these two scenarios, the relationship between number of iterations and the error rate is presented in Fig. 5. It can be verified that the algorithm achieves the same level of error rate much faster when the market is less competitive. For SPM = 2.3, it takes less than 10 iterations for 4-node and single center networks to reach error rate of 1e−03, but it requires 30 iterations to get the same level when the SPM = 1.2. After 100 iterations, the error rate can be as low as 10e−8 for SPM = 2.3, and the more competitive market gets an error rate that is 100 times greater.



(a) 4-node network, S=1,NTTS=200,NATTS=200

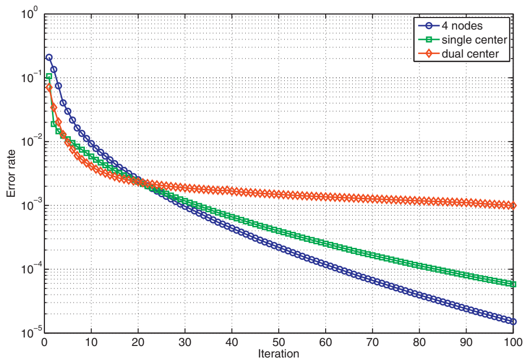


(b) single center network, S=1,NTTS=1000,NATTS=1000

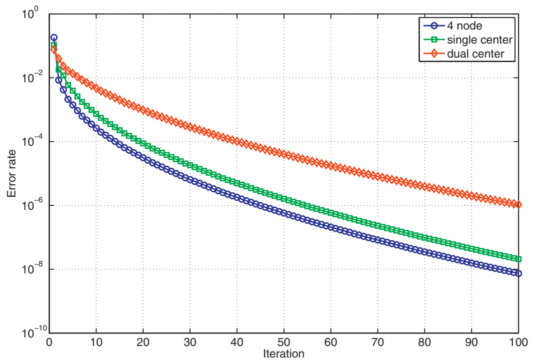


(c) dual center network, S=1,NTTS=2000,NATTS=2000

Fig. 4. Convergence pattern of iterative algorithm on three networks.



(a) High competition, S=1.2

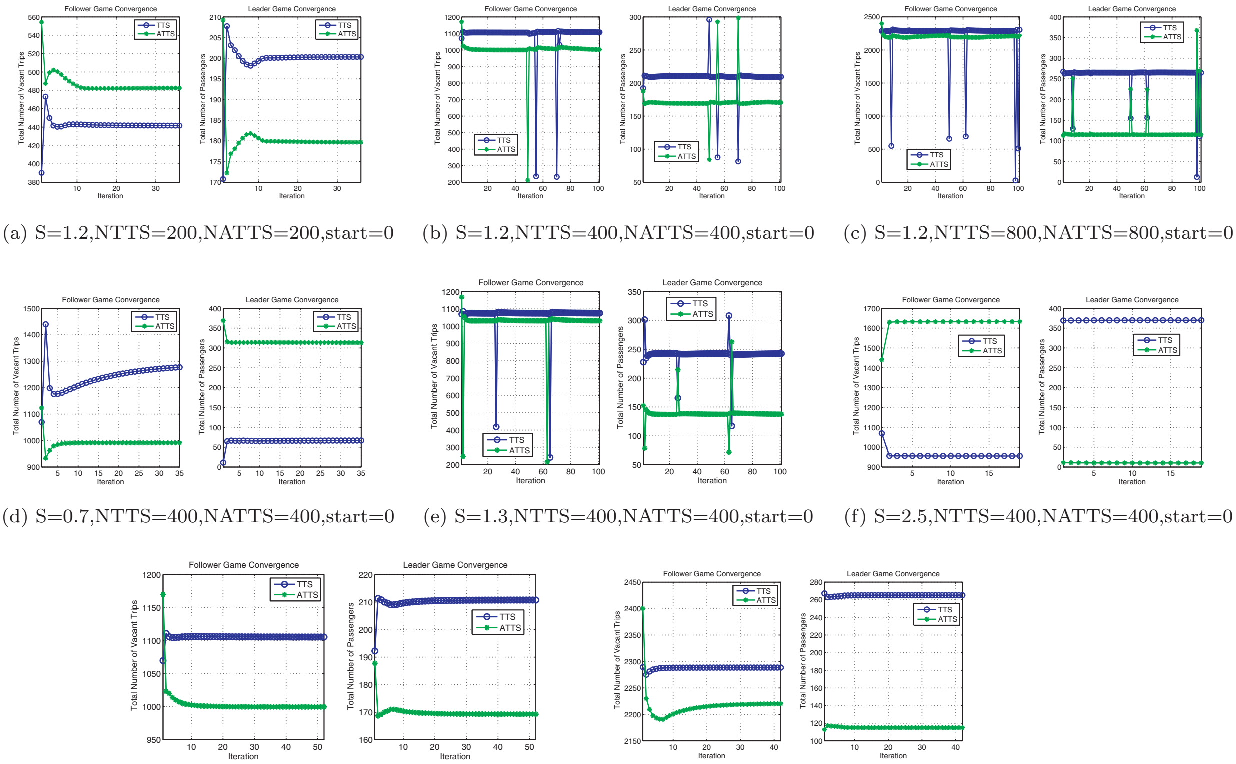


(b) Low competition, S=2.3

Fig. 5. Algorithm convergence on three different networks.

5.2.2. Choice of starting point

Moreover, though in general the algorithm convergence is not guaranteed for the TM-L/F Game and remains an open problem in the literature, all numerical tests are found to converge in our study if the initial solution is selected properly. For the leader game, it can be analogous to the well studied traffic assignment problem and the initial solution is generated by all-or-nothing (AON) assignment, where all passenger flow is assigned to the service with the lowest cost for each OD pair. However, for the lower level game, generating a feasible start point requires solving the system of linear equations in each iteration, which may be computationally expensive when the network is large. Also, since the linear system has more variables than equations, there are infinite number of feasible points to choose from, and we observe that even a feasible start point may make the algorithm fail to converge. We illustrate this convergence issue by comparing two different start



(a) S=1.2,NTTS=200,NATTS=200,start=0 (b) S=1.2,NTTS=400,NATTS=400,start=0 (c) S=1.2,NTTS=800,NATTS=800,start=0

(d) S=0.7,NTTS=400,NATTS=400,start=0 (e) S=1.3,NTTS=400,NATTS=400,start=0 (f) S=2.5,NTTS=400,NATTS=400,start=0

(g) S=1.2,NTTS=400,NATTS=400,start=balance (h) S=1.2,NTTS=800,NATTS=800,start=balance

Fig. 6. Algorithm convergence patterns under two different choices of starting point.

point selections for the follower game on the 4-node network: all zero flow vector and a balance-loaded network flow. In particular, the balance-loaded network flow is computed as follows:

1. Calculate the average travel time $\bar{t}$ of all links.
2. Let l denote the number of links in the network and N is the fleet size, load each link with flow

$$f = \frac{N}{\bar{t}l}$$

where it assumes that there is no passengers on the network and each link is equally loaded with the average amount of empty trips.

Fig. 6 presents the convergence results for the two start points under various scenarios. Fig. 6(a)–(f) shows the convergence starting with all zero flow vector. It is observed that it converges smoothly in three cases, but fails to reach convergence and has several spikes in the other three cases. And the key difference among these six scenarios is the level of competition between TTS and ATTS. Fig. 6(d) and (f) correspond to scenarios where most passengers choose TTS or ATTS when the SPM is high or low, and (a) refers to the scenario where there is insufficient supply to serve all passengers by one service mode and TTS and ATTS are therefore sharing the market. The commonality of these scenarios is that little competition is involved and therefore the searching space may be smooth. On the contrary, as the level of competition increases, a slight change in passenger distribution will result in significant different driver flow pattern, which is reflected by the spikes in the figures. Consequently, choosing a start point far from the stationary point will make the iterative algorithm unstable and fail to converge. Nevertheless, by choosing the balance-loaded network flow as the initial solution, we observe empirically that it always converges smoothly and is easy to compute, as shown in Fig. 6(g)–(h). Therefore, the balance-loaded network flow vector is recommended as a start point for the follower game.

5.2.3. *Passenger flow pattern under market equilibrium*

Fig. 7 presents the equilibrium passenger flow pattern on the 4-node network, for the case where the fleet size is 300 for both TTS and ATTS. The dashed lines give the travel cost between OD pairs and the bar plot shows the number of passengers that take TTS and ATTS between each OD pair. The results comply with the equilibrium pattern in all cases, where passengers consistently choose the service with lower cost. For instance, in Fig. 7(b), TTS offers lower cost for link

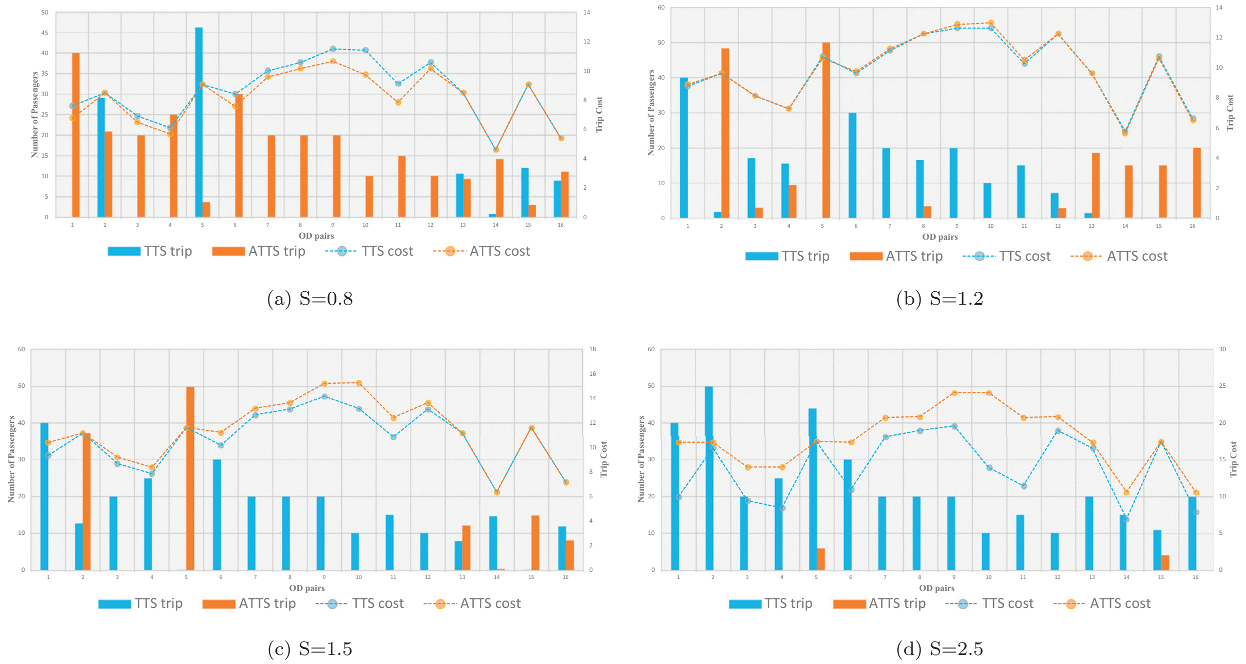


Fig. 7. Change in market equilibrium with NTTS = 300 and NATTS = 300.

Table 2
Results of modeling stochasticity.

Scenario I: Number of TTS = 200, Number of ATTS = 200, SPM = 1.2						
	15% error		30% error		45% error	
Sample size	Expected total cost	std	Expected total cost	std	Expected total cost	std
100	4.56E+03	150.94	4.53E+07	311.32	4.46E+03	537.98
500	4.57E+03	149.98	4.52E+03	312.01	4.47E+03	509.25
deterministic	4.58E+03	–	–	–	–	–
Scenario II: Number of TTS = 200, Number of ATTS = 200, SPM = 2						
	15% error		30% error		45% error	
Sample size	Expected total cost	std	Expected total cost	std	Expected total cost	std
100	5.34E+03	100.94	5.30E+03	286.11	5.23E+03	474.97
500	5.34E+03	103.98	5.30E+03	282.24	5.25E+03	455.39
deterministic	5.33E+03	–	–	–	–	–

1,6,7,9,10,11 and all passengers are assigned to TTS respectively, and ATTS is cheaper for OD pair 5,14,15,16. For the other OD pairs, both services have the same price and share the market accordingly. The figure also reveals the level of competition in the market. The highest level of competition is observed for S = 1.2, where there is very minor differences between the travel cost curves of the two services for all OD pairs. Moderate competitions are observed for S = 0.8 and S = 1.5, where some of the OD pairs have close travel cost. When S is set to 2.5, there is little competition in the market due to excessive cost of riding ATTS, and almost all passengers are assigned to TTS.

5.2.4. Modeling stochasticity

Finally, while perfect information is assumed and the TMC is modeled deterministically, it is possible that uncertainty may arise when the platform estimates passenger trip cost, travel time, and distance to drivers. To model the uncertainty, the constant parameters A_j^s and B_j^s can be changed to random variates. The mean value can be used to denote the expected searching time and waiting time, and different standard deviations can be incorporated to account for various levels of uncertainty. While formulating and solving the stochastic model is not the focus of the study, we conduct additional sets of experiments to understand the robustness of our deterministic model to stochasticity using simulation approach. For simplicity, we consider the searching parameter A_i^s and waiting parameter B_j^s are uniformly distributed random variates, and test 15%, 30%, and 45% of uncertainty for the parameters. The results are given in Table 2. The experiments are conducted on the 4-node network and cover two different scenarios with different levels of market competition. We randomly sample 100 and 500 sets of parameters, and the expected total costs are calculated by taking the average of the total cost from

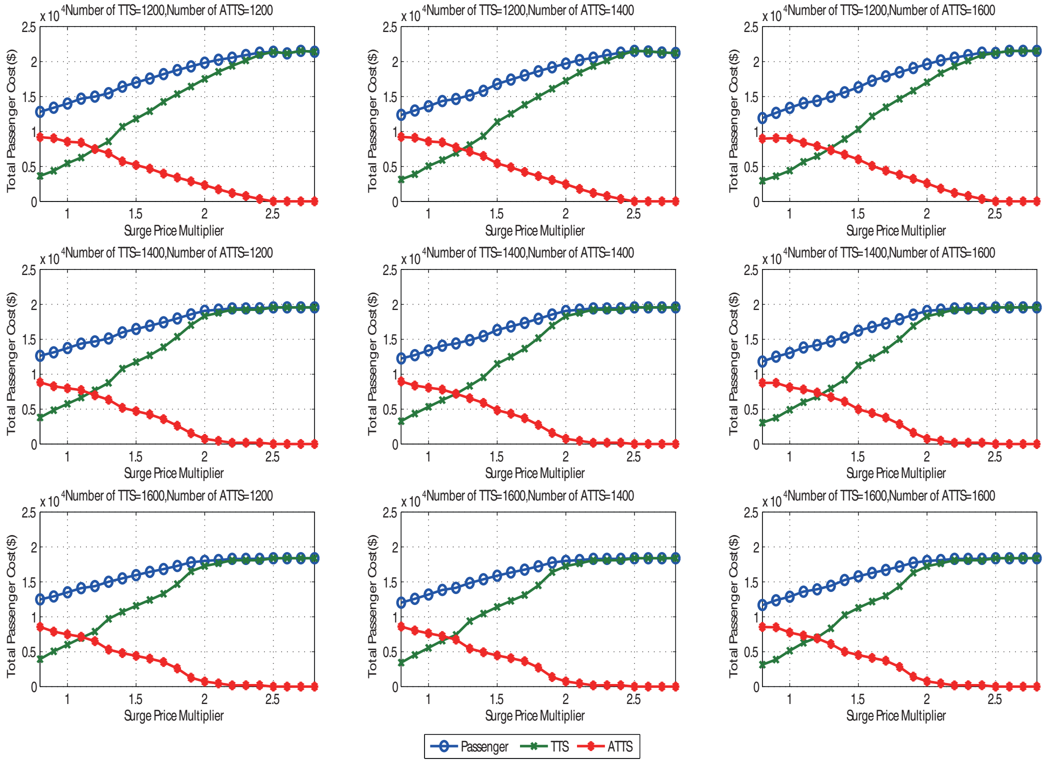


Fig. 8. Sensitivity analysis of passengers' costs with respect to fleet size and SPM.

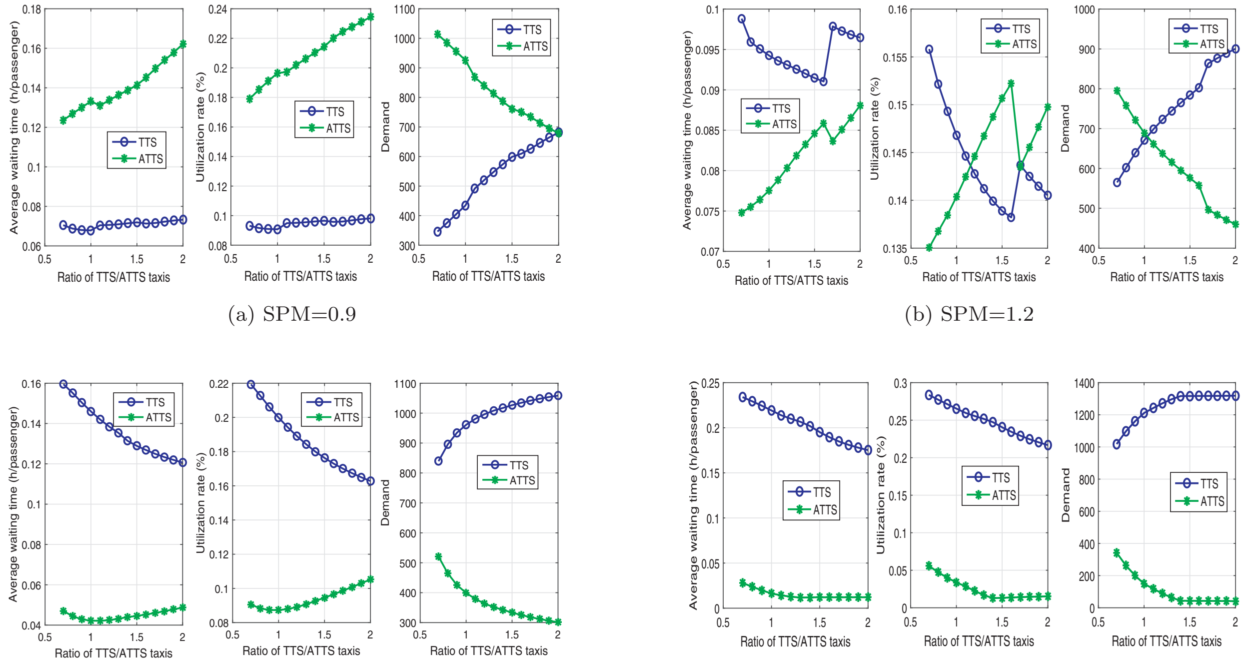
these samples. While large sample size gives closer solution to the true mean value, only minor differences between 100 and 500 samples are observed in terms of the expected total cost. When the error rate is bounded by 15%, the obtained expected total cost is almost identical to the value of the deterministic case. As the error rate increases, the expected total cost gets decreased, and the value is still close to that of deterministic case. Moreover, the gaps are even closer for market with less competition. Consequently, the results suggest that our deterministic model is highly robust to stochasticity due to possible system estimation uncertainty.

5.2.5. Market performance with TTS and ATTS

More comprehensive understanding of the market condition with coexistence of TTS and ATTS at equilibrium can be obtained by analyzing passengers' costs, trip waiting time and taxi fleet utilization. In particular, the fleet size of TTS and ATTS, and the surge price policy made by ATTS are observed to have significant impact on the equilibrium states of the market. All results discussed in this section are obtained from experiments on the single center network.

Fig. 8 presents a side-by-side comparison for the impact of both taxi fleet size and surge price policy on the cost of passenger. The experiments are conducted on the single center network. Total passenger cost consists of two parts: the waiting cost before the arrival of drivers and the travel cost from trip origin to trip destination. The results help to understand the change in total passenger cost with various regulation settings, and how TTS and ATTS contribute to the total cost separately. In general, the increase in SPM will result in passengers shifting to TTS, and passengers are observed to prefer TTS to ATTS when SPM is greater than 2.5 under the experiment setting. In particular, compared with scenarios with 1200 TTS taxis, passengers shift to TTS faster when there are 1600 TTS taxis considering reduced waiting time. Moreover, given the same SPM, passengers cost decreases as the fleet size of TTS and ATTS increase. For majority of the scenarios, TTS and ATTS are found to contribute equally to the total passenger cost when SPM $= 1.2$, and ATTS is more popular when SPM is lower while TTS is more popular when SPM is higher. An exception is observed for the case with 1200 TTS taxis and 1600 ATTS taxis, where passengers are willing to pay trip premiums for the saved waiting time. Based on the change in total passenger cost, passengers are observed to benefit most from ATTS during off-peak hours, where the surge price is usually at low level, and the total passenger cost may be increased by over 50% if we keep increasing the SPM until all passengers shifted to TTS. One important implication from the model is that, while in reality ATTS only charges premium on passengers, ATTS drivers may expect more revenue and passengers may also spend less if trip discount is offered. Meanwhile, when SPM is higher than 1.5, additional increase in SPM may lead to higher marginal loss for total ATTS cost.

We next assess the impacts of fleet size on SPMs and passenger waiting time, fleet utilization rate and passenger demand. Fig. 9 shows how passenger waiting time and fleet utilization may vary with the change of fleet size and different SPMs. The total passenger demand is 1360, the total fleet size is set to 2500, and we compare various split ratios between TTS and



(a) SPM=0.9 (b) SPM=1.2

(c) SPM=1.5 (d) SPM=2

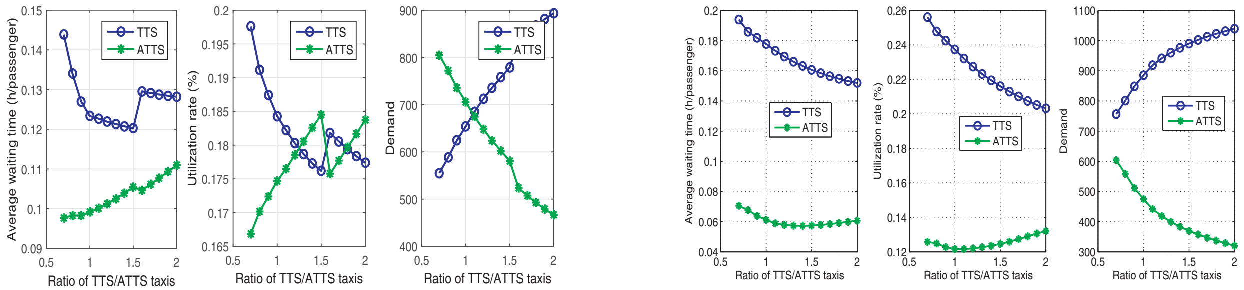
Fig. 9. Sensitivity of passenger waiting time, fleet utilization rate and passenger demand, with total 2500 taxis and 1360 passengers.

ATTS, and 4 different SPM values. When SPM is 0.9, more passengers are observed to take ATTS, and ATTS passenger waiting time and utilization of ATTS increase with higher TTS/ATTS ratio, which corresponds to smaller ATTS fleet size. This is because that ATTS takes the lead in the market and most passengers prefer ATTS due to low travel cost. They gradually shift to TTS as smaller fleet size results in higher waiting time. Meanwhile, no significant change is observed for the utilization rate and passenger waiting time for TTS. The reason is that the number of TTS passengers increases almost linearly with more TTS vehicles. On the other hand, when SPM = 2 and TTS starts to dominate the market, we observe that the utilization rate and passenger waiting time for both TTS and ATTS actually decrease as the TTS/ATTS ratio increases. The decrease in TTS passenger time is due to higher TTS trip density, and the decrease in ATTS passenger time is caused by a higher reduction speed of ATTS passengers compared with that of ATTS fleet size, which can be verified by the convex shape for change in ATTS demand.

An interesting phenomenon is observed for the scenario of SPM = 1.2, where abrupt changes are observed for passenger waiting time and fleet utilization rate. Moreover, such change breaks the monotonically decreasing (increasing) trend of the passenger waiting time and taxi utilization rate. The reason for the abrupt changes is due to the competition between TTS and ATTS in the market. It is observed in Fig. 8 that TTS and ATTS have similar contributions to the total system cost when SPM is around 1.2, where the market undergoes the highest level of competition between the two modes as they have similar trip cost and waiting time. Due to greedy behavior of drivers and passengers, a slight change in fleet size may result in sudden surge or drop of passenger demand due to mode shift, which can be verified by the corresponding patterns for passenger demand when the abrupt changes take place. As for this particular case, the change in number of passengers when TTS/ATTS ratio increases from 1.6 to 1.7 is almost 7 times greater than that when TTS/ATTS ratio increases from 1.5 to 1.6. This implies that the TMC equilibrium is highly sensitive when there is a high level of competition in the market, and a small market fluctuation may lead the market to a significantly different state. Same market pattern is also revealed when we change the total number of taxis, as shown in Fig. 10(a). These abrupt changes disappear when the competition level gets reduced, as shown by increasing SPM to 1.5. Moreover, the shape of ATTS passenger waiting time and utilization rate patterns are observed to be convex when SPM = 1.5. This is because the number of ATTS passengers drops faster than the decrease in ATTS fleet initially, and later on fewer ATTS passengers shift to TTS since the waiting time is much lower.

6. Conclusion

In this paper, we model the taxi market with TTS and ATTS as the multi-leader-follower game at the network level and investigate the TMC Equilibrium. Specifically, taxi drivers and passengers are considered as players in the game and are assumed to have greedy behavior. We formulated the problem using quasi-variational inequality, analyzed the game structure and model properties, and proved that the TM-L/F game has a TMC Equilibrium. Seeing the difficulties of solving multi-leader-follower game, we decomposed the problem as solving two parameterized augmented VIs, and developed an iterative algorithm to find the strongly stationary point of the problem.



(a) SPM=1.2 (b) SPM=1.5

Fig. 10. Sensitivity of passenger waiting time, fleet utilization rate and passenger demand, with total 2000 taxis and 1360 passengers.

Based on the results, we find that the iterative algorithm may converge smoothly to the final stationary point if starting points are chosen properly. While AON can be used to identify the start point for the leader game, empirical experiments suggest that loading the follower network with balanced driver flow as an initial solution helps to reach better convergence. We further conduct a set of sensitivity analysis to comprehensively understand how market nature is affected by the fleet size and pricing policy of TTS and ATTS. The results indicate that the proposed model is capable of capturing the influence of the level of competition in the market on total passengers cost. It is also able to characterize the change in passengers waiting time, taxi utilization, and how passengers are distributed between the two modes with respect to different fleet sizes and SPMs.

Considering that ATTS is becoming much more popular in recent years, there is no doubt that the market nature becomes much more complicated and there is an emerging need to understand the corresponding complex nature. The proposed model makes the first attempt to bridge this gap, and is deemed to help stakeholders to understand the market states of various scenarios and therefore helps to frame proper regulation policies based on different objectives. However, there are several drawbacks of the model in the study, which may be addressed in future studies for more accurate market modeling. First, while we consider that passengers have equal access to TTS and ATTS, the model may be extended to account for the situation where there are passengers who will only ride TTS (ATTS). And to model this only requires minor changes to the current model, which is to revise the passenger flow conservation constraint by removing these groups of riders. For followers, the number of passengers who stick to TTS or ATTS will become constant parameters for the cost function and driver flow conservation constraint. In general, this will lead to different formulations but certainly not more difficult to solve compared with our current formulations. Second, there are several variations of service types for TTS and ATTS, such as uberBlack of premium vehicles and ridesharing for multiple passengers, which may be modeled in the future works as well. For the case of ridesharing, passengers will enjoy lower trip cost, but at the cost of longer travel time, waiting time, and less comfort, which can be reflected by setting different model parameters. And there will be dedicated followers (drivers) who serve shared taxi trips. However, this modification is different compared with the first extension, since it will change the network structure. For instance, our current model has two links between each pair of OD for ATTS and TTS flow. Adding a new service mode such as ride sharing is equivalent as introducing a third link. Similarly, we will have an additional follower game that is independent of the ATTS and TTS game. Therefore, this modification will increase the problem size and eventually increase computation load. Third, nonlinear functions may be incorporated to better capture the relationship between waiting time and the level of supply and demand. While this may significantly change the model properties, new solution approaches may also need to be developed for the nonlinear TM-L/F game. Finally, the experiments and parameters in this study are hypothetical due to limited data availability. As more data become available in the future, efforts can be made to recalibrate model parameters, derive new results with updated parameters, and compare the experiment results with real-world cases for validation purposes.

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Appendix

The parameter settings and network configurations can be found in the following tables.

Table 3
Parameter setting of 4-node network.

A_i^I	A_i^{II}	B_i^s	α^s	β^s	π	z^{II}	ϕ
[0.25,0.2,0.3,0.3]	[0.2,0.15,0.25,0.25]	[0.35,0.3,0.3,0.3]	[30,27]	[30,30]	0.1	0.01	0.05

Table 4
Link travel time and network OD demand of 4-node network.

Link	O	D	Demand	Time(h)
1	2	1	40	0.25
2	1	2	50	0.25
3	4	2	20	0.2
4	2	4	25	0.2
5	3	4	50	0.25
6	4	3	30	0.25
7	1	3	20	0.3
8	3	1	20	0.3
9	1	4	20	0.35
10	4	1	10	0.35
11	2	3	15	0.3
12	3	2	10	0.3
13	1	1	20	0.25
14	2	2	15	0.15
15	3	3	15	0.25
16	4	4	20	0.15

Table 5
Parameter setting of single center network.

Variable	Value
A_i^I	[0.15 0.2 0.2 0.2 0.2 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25]
A_i^{II}	[0.2 0.25 0.25 0.25 0.25 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3]
B_i^I	[0.15 0.2 0.2 0.2 0.2 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25]
B_i^{II}	[0.2 0.25 0.25 0.25 0.25 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3]

Table 6
Link travel time and network OD demand of single center network.

Link	O	D	Demand	Time(h)	Link	O	D	Demand	Time(h)
1	1	2	40	0.25	36	11	2	20	0.3
2	2	1	40	0.25	37	2	12	20	0.3
3	1	3	40	0.25	38	12	2	20	0.3
4	3	1	40	0.25	39	2	13	20	0.3
5	1	4	40	0.25	40	13	2	20	0.3
6	4	1	40	0.25	41	6	7	10	0.3
7	1	5	40	0.25	42	7	6	10	0.3
8	5	1	40	0.25	43	6	13	10	0.3
9	2	3	30	0.2	44	13	6	10	0.3
10	3	2	30	0.2	45	8	7	10	0.3
11	2	5	30	0.2	46	7	8	10	0.3
12	5	2	30	0.2	47	8	9	10	0.3
13	4	5	30	0.2	48	9	8	10	0.3
14	5	4	30	0.2	49	10	9	10	0.3
15	4	3	30	0.2	50	9	10	10	0.3
16	3	4	30	0.2	51	10	11	10	0.3
17	5	6	20	0.3	52	11	10	10	0.3
18	6	5	20	0.3	53	12	11	10	0.3
19	5	7	20	0.3	54	11	12	10	0.3
20	7	5	20	0.3	55	12	13	10	0.3
21	5	13	20	0.3	56	13	12	10	0.3
22	13	5	20	0.3	57	1	1	20	0.15
23	4	7	20	0.3	58	2	2	15	0.2
24	7	4	20	0.3	59	3	3	15	0.2
25	4	8	20	0.3	60	4	4	15	0.2
26	8	4	20	0.3	61	5	5	15	0.2
27	4	9	20	0.3	62	6	6	10	0.3
28	9	4	20	0.3	63	7	7	10	0.3
29	3	9	20	0.3	64	8	8	10	0.3
30	9	3	20	0.3	65	9	9	10	0.3
31	3	10	20	0.3	66	10	10	10	0.3
32	10	3	20	0.3	67	11	11	10	0.3
33	3	11	20	0.3	68	12	12	10	0.3
34	11	3	20	0.3	69	13	13	10	0.3
35	2	11	20	0.3					

Table 7
Parameter setting of dual center network.

Variable	Value
A_i^I	[0.15 0.2 0.2 0.2 0.2 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.15 0.2 0.2 0.2 0.2 0.25 0.25 0.25 0.25 0.2]
A_i^{II}	[0.2 0.25 0.25 0.25 0.25 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.2 0.25 0.25 0.25 0.25 0.3 0.3 0.3 0.3 0.3]
B_i^I	[0.15 0.2 0.2 0.2 0.2 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.15 0.2 0.2 0.2 0.2 0.25 0.25 0.25 0.25 0.25]
B_i^{II}	[0.2 0.25 0.25 0.25 0.25 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.2 0.25 0.25 0.25 0.25 0.3 0.3 0.3 0.3 0.3]

Table 8
Link travel time and network OD demand of dual center network.

Link	O	D	Demand	Time(h)	Link	O	D	Demand	Time(h)	Link	O	D	Demand	Time(h)
1	1	2	40	0.25	46	7	8	10	0.3	91	17	16	20	0.2
2	2	1	40	0.25	47	8	9	10	0.3	92	16	15	20	0.2
3	1	3	40	0.25	48	9	8	10	0.3	93	15	16	20	0.2
4	3	1	40	0.25	49	10	9	10	0.3	94	16	19	20	0.2
5	1	4	40	0.25	50	9	10	10	0.3	95	19	16	20	0.2
6	4	1	40	0.25	51	10	11	10	0.3	96	16	21	20	0.2
7	1	5	40	0.25	52	11	10	10	0.3	97	21	16	20	0.2
8	5	1	40	0.25	53	12	11	10	0.3	98	16	20	20	0.2
9	2	3	30	0.2	54	11	12	10	0.3	99	20	16	20	0.2
10	3	2	30	0.2	55	12	13	10	0.3	100	15	21	20	0.2
11	2	5	30	0.2	56	13	12	10	0.3	101	21	15	20	0.2
12	5	2	30	0.2	57	1	1	20	0.15	102	15	22	20	0.2
13	4	5	30	0.2	58	2	2	15	0.2	103	22	15	20	0.2
14	5	4	30	0.2	59	3	3	15	0.2	104	15	23	20	0.2
15	4	3	30	0.2	60	4	4	15	0.2	105	23	15	20	0.2
16	3	4	30	0.2	61	5	5	15	0.2	106	10	19	20	0.2
17	5	6	20	0.3	62	6	6	10	0.3	107	19	10	20	0.2
18	6	5	20	0.3	63	7	7	10	0.3	108	19	20	20	0.2
19	5	7	20	0.3	64	8	8	10	0.3	109	20	19	20	0.2
20	7	5	20	0.3	65	9	9	10	0.3	110	20	21	20	0.2
21	5	13	20	0.3	66	10	10	10	0.3	111	21	20	20	0.2
22	13	5	20	0.3	67	11	11	10	0.3	112	21	22	20	0.2
23	4	7	20	0.3	68	12	12	10	0.3	113	22	21	20	0.2
24	7	4	20	0.3	69	13	13	10	0.3	114	22	23	20	0.2
25	4	8	20	0.3	70	14	15	40	0.25	115	23	22	20	0.2
26	8	4	20	0.3	71	15	14	40	0.25	116	23	12	20	0.2
27	4	9	20	0.3	72	14	16	40	0.25	117	12	23	20	0.2
28	9	4	20	0.3	73	16	14	40	0.25	118	14	14	20	0.15
29	3	9	20	0.3	74	14	17	40	0.25	119	15	15	15	0.2
30	9	3	20	0.3	75	17	14	40	0.25	120	16	16	15	0.2
31	3	10	20	0.3	76	14	18	40	0.25	121	17	17	15	0.2
32	10	3	20	0.3	77	18	14	40	0.25	122	18	18	15	0.2
33	3	11	20	0.3	78	18	11	20	0.2	123	19	19	10	0.3
34	11	3	20	0.3	79	11	18	20	0.2	124	20	20	10	0.3
35	2	11	20	0.3	80	18	12	20	0.2	125	21	21	10	0.3
36	11	2	20	0.3	81	12	18	20	0.2	126	22	22	10	0.3
37	2	12	20	0.3	82	18	23	20	0.2	127	23	23	10	0.3
38	12	2	20	0.3	83	23	18	20	0.2					
39	2	13	20	0.3	84	17	10	20	0.2					
40	13	2	20	0.3	85	10	17	20	0.2					
41	6	7	10	0.3	86	17	11	20	0.2					
42	7	6	10	0.3	87	11	17	20	0.2					
43	6	13	10	0.3	88	17	19	20	0.2					
44	13	6	10	0.3	89	19	17	20	0.2					
45	8	7	10	0.3	90	16	17	20	0.2					

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