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# Ubiquitous inequality: The home market effect in a multicountry space[☆]

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## HIGHLIGHTS

- We reveal an economic mechanism for spatial inequalities of both nominal income and real income in a multicountry space.
- We derive a non-monotonic relation between income inequalities and trade integration.
- The HME is examined in a multicountry space.
- Two HMEs in terms of firm share and labor wage, are observed and equivalent.
- The HME in terms of trade pattern is not equivalent to other two.

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ABSTRACT

We show that spatial inequalities in an economic space of multiple countries in terms of both nominal income and real income are ubiquitous in the sense that they appear when countries are differentiated by population only. A new trade theory model is constructed without any freely traded homogeneous good, so that we can examine the home market effect (HME) and the non-monotonic relation between income inequalities and globalization. Meanwhile, there are three HME definitions for a two-country space in terms of firm share, labor wage, and trade pattern. The first two remain applicable in a multicountry space, and they are shown to be equivalent. However, a natural extension of the third is not equivalent.

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## 1. Introduction

Two kinds of inequality are known in our society, natural or physical and ethical or political. The former is attributed to different skills and abilities and is considered acceptable, while the latter results from specific economic systems and is considered detrimental because it makes economies inefficient and unstable. This naturally leads to a question, is the second inequality avoidable?

This paper focuses on spatial inequality, which appears not only between countries but also within them, exhibiting uneven economic development. Globalization is one of the reasons for spatial income inequalities (Anand and Segal, 2008). The heterogeneity of space (uneven distribution of technologies, natural resources, and amenities) results in such inequalities, whose linkage has been extensively explored by traditional trade theory. Meanwhile, by a model of New Trade Theory (NTT), a recent paper of Takahashi et al. (2013) shows that the income inequality between two countries always occur in the spatial equilibrium even when the countries are homogeneous and differ only in size. Moreover, such a spatial inequality initially rises and then falls when globalization deepens. Consequently, the inequality appears even when there are no relative advantages in technology, resource endowment, and geographic feature.

However, the analysis of Takahashi et al. (2013) is limited to the case of two countries. In a space of two countries, there is only one way the countries can interact. Moving away from one

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country automatically implies that firms go to the other country. Whereas in the case of more than two countries, there are many ways in which these countries can interact.[1] So it is not clear how the two-country analysis extends. We need a multicountry model to incorporate complex feedback, which is also important for empirical studies.

The primary purpose of this paper is to generalize Takahashi et al. (2013) into an economic space of multiple countries, showing the non-monotonic curve of income inequality between any two countries when trade is more integrated. Some empirical studies supporting this fact are known. For example, Williamson (1965) observes a non-linear relation between regional inequalities and national development in the United States; Barrios and Strobl (2009) observe bell-shaped spatial inequalities in the European Union; and Sala-i-Martin (2006) concludes that the poverty rate in Latin America decreased in 1970s but increased slightly in 1980s and 1990s. By contrast, theoretical studies are not sufficient. To the best of our knowledge, Behrens et al. (2009) is the only NTT paper considering firm locations in an economic space of multiple countries.[2] However, as in most NTT papers, a freely traded good is assumed there which equalizes the wages all over the world if there is no Ricardian productivity difference among the countries. For this reason, their model cannot be applied to examine the spatial inequality of nominal income, especially its non-monotonicity.

The income inequality between any pair of countries is an economic concern in general, which is observable in our two-factor model (labor and capital) without a freely traded good. Due to the assumption of homogeneous labor and capital endowment, the income inequality is measured by the wage differential between the countries in our model. We derive the wage rates by investigating the home market effect (HME), which plays a central role in NTT. The HME is formally defined as a phenomenon in which a country with a larger local demand attracts a more-than-proportionate share of manufacturing firms (Krugman, 1980, Section III; Helpman and Krugman, 1985, Section 10.4). Note that in a perfect competitive market with a technology of constant returns to scale, the firm share in a country is exactly the same as the population share there. Thus, the HME discloses an agglomeration force resulting from the monopolistic competition and the technology of increasing returns to scale, which is known as the second-nature force.[3] The HME is closely related to trade costs, or globalization level. Moreover, two other definitions for the HME are known in relevant literature. One is based on wage rates. Other things being equal, the wage is higher in a larger country (Krugman, 1991, p. 491; Behrens et al., 2009, footnote 1). The existence of this HME implies the wage inequality, and we are also interested in how this inequality depends on trade integration. The third definition is based on trade pattern: the large country is a net exporter of manufactured goods (Krugman, 1995, p. 1261; Davis, 1998, p. 1271). Note that capital is mobile across countries which generates returns repatriated to the owners, a trade surplus in manufactured goods is possible in our model.

While different definitions are applied in different studies, they are shown to be equivalent in the two-country framework of Takahashi et al. (2013). Another purpose of this paper is to examine their relation when there are more than two countries. We find that the HME definitions in terms of firm share and wage are equivalent in this more general setting as well, but the definition in terms of trade pattern is not. So our multicountry model offers some clarification for the alternative HME definitions. Essentially, the definition in firm share measures the ratio of firm number to population, but the definition in trade pattern measures the differential between firm number and population. They become different when more than two countries are involved. Since the real world consists of many countries, this theoretical result is important for empirical studies on the HME.

We emphasize that the extension to many countries requires a far more delicate mathematical idea beyond the two-country model technique. Due to the simpler structure of a two-country space, the existence of a spatial equilibrium can be explored by the intermediate value theorem. However, to ensure the equilibrium existence in a space of multiple countries, we need to construct a suitable mapping to utilize the fixed point theorem. It is noteworthy that the wages in our model are determined by wage equations implicitly. In order to establish the non-monotonic shape of wage differential, we have to investigate the wage equations to clarify the shape of wage curves by the implicit function theorem. These techniques are new in the literature of both new trade theory and inequality, which constitute a technical contribution of this paper.

The remainder of the paper is organized as follows. In Section 2, we establish the model of $n$ countries. Section 3 provides the equilibrium analysis. The main results are given in Sections 4 and 5. While Section 4 studies the HME definitions and its existence, Section 5 examines the ubiquitous inequalities in terms of both nominal income and real income. Finally, Section 6 summarizes the conclusions.

## 2. The model

Countries in the real world are different in many respects, making it difficult to build a multicountry model. Since the HME aims to clarify the role of second nature and size effect in trade pattern, we assume all countries are symmetric except for their sizes.[4] In particular, we exclude first-nature features such as technological difference, resource abundance, and geographical advantage. This does not mean that these factors are unimportant. To the contrary, as shown in Behrens et al. (2009), the excluded factors do impact on economic activities. However, including them in a model would obscure the size effect.[5]

Specifically, the global economy consists of $n$ countries $i = 1, 2, \dots, n$, which have the same conditions except for their population sizes. Let the population share in country $i$ be $\theta_i \in (0, 1)$. We label these countries such that $1 > \theta_1 \geq \theta_2 \geq \cdots \geq \theta_n > 0$. We assume two factors for production: labor and capital.[6] There are $L$ units of labor and $K$ units of capital in total. We rule

[1] Analogous to game theory, an $m \times n$ matrix game is much more complicated than a $2 \times 2$ matrix game.

[2] An earlier paper by Tabuchi et al. (2005) considers the economic activity of multiple regions by a core–periphery model, where skilled workers are mobile and regions have the same number of unskilled workers.

[3] Borrowed from Cronon (1991), "first nature" is a force by which firms locate according to local natural advantages while "second nature" is a force by which firms locate according to an advantage stemming from the presence of other firms. These terms are adopted in a series of new economic geography papers including Krugman (1993) and Redding (2010), etc.

[4] Although the country size is a kind of first-nature feature, in a perfect competition market with a production technology of constant returns to scale, the number of firms in a country is proportional to its population size when trade costs are positive. Therefore, in the new trade theory, it is common to assume different country size and examine how the number of firms in a country is disproportional to the country size.

[5] For the same reason, the assumption of two symmetric regions is imposed in the core–periphery model of Krugman (1991). Such an assumption makes our model far from the real world, but it is the only way to capture the essence of the second nature. To analogize, we need to peel an orange to taste the flesh; otherwise, we will not know whether the taste is from the peel or the flesh.

[6] According to Takatsuka and Zeng (2012a,b), capital is an important production factor to be included in HME analysis.

out the comparative advantage of resource abundance by assuming that each resident holds the same amount of capital: the capital owner share in country $i$ is also $\theta_i$. Capital is mobile and its rents are repatriated to the capital owners even if the capital is employed in another country. Workers are immobile and their wages are not necessarily equal across countries. However, there is no income inequality within a country.

Our model assumes only one (manufacturing) sector, whose production is under a technology of increasing returns to scale. Consumers gain utility only from manufacturing goods, having a continuum of varieties. The utility in country $i$ is

$$U_i = \left[\int_0^N d_i(v)^{\frac{\sigma-1}{\sigma}} dv\right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ represents the elasticity of substitution between two manufactured varieties, $N$ is the number of varieties, and $d_i(v)$ is the demand for a typical manufactured good $v$ in country $i$. The varieties are supposed to be symmetric, so we can omit the variety name and simply use $d_i$ to indicate the demand for each variety.

Maximization of consumers' utility function derives the following demand functions

$$d_{ij} = \frac{p_{ij}^{-\sigma}}{P_j^{1-\sigma}} Y_j \tag{1}$$

where $d_{ij}$ is the demand for a variety made in country $i$ and sold in country $j$, $p_{ij}$ is its price, and $Y_i$ is the national income in country $i$. Finally, $P_i$ is the manufacturing price index in country $i$ given by $P_i = \left[\int_0^N p_i(v)^{1-\sigma} dv\right]^{\frac{1}{1-\sigma}}$, where $p_i(v)$ is the price of variety $v$ sold in country $i$.

As in the literature of NTT (see Baldwin et al., 2003, p. 74), we simply assume that, in each country, $\theta_i$ of its employed capital belongs to country $i$ for any distribution of firms. Therefore, the average capital returns of individuals in all countries are always the same, and the spatial inequality of nominal income takes the form of wage differential between countries.

Next, we turn to the production side of the economy. We assume the same technology in all countries. Fixed input of one unit of capital and a marginal input of $(\sigma-1)/\sigma$ units of labor are required in production everywhere. Let the capital share employed in country $i$ be $k_i$, which is different from the endowed capital share $\theta_i$ because of capital mobility. The number of firms in $i$ is then $k_iK$.

International shipment of any variety incurs costs, which evidently depend on the geographical distance between the origin and destination countries. As shown in Redding and Venables (2004) and Behrens et al. (2009), the geographical features of countries might produce a hub effect having a significant impact on trade patterns. We rule out the geographic advantage among countries by assuming that the transport costs are the same for all pairs of countries. Specifically, $\tau \geq 1$ units of a manufactured good must be shipped for one unit to arrive between any two countries. Then we have $p_{ij} = \tau_{ij} p_{ii}$ for any $i, j = 1, \ldots, n$ where

$$\tau_{ij} = \begin{cases} \tau & \text{if } i \neq j \\ 1 & \text{if } i = j. \end{cases} \tag{2}$$

A firm located in country $i$ sets its price to maximize its profit

$$\pi_i = \sum_{j=1}^n \left(p_{ij} d_{ij} - \frac{\sigma-1}{\sigma} w_i d_{ij} \tau_{ij}\right) - r_i \tag{3}$$

where $r_i$ is the capital rent in country $i$. The first-order condition to maximize (3) gives prices

$$p_{ij} = w_i \tau_{ij}. \tag{4}$$

Therefore, the price indices are simplified as

$$P_i = \left(\sum_j K k_j w_j^{1-\sigma} \phi_{ij}\right)^{\frac{1}{1-\sigma}}, \tag{5}$$

where $\phi_{ij} \equiv \tau_{ij}^{1-\sigma} \in [0, 1]$ is the trade freeness between countries $i$ and $j$. By the assumption of (2), it holds that

$$\phi_{ij} = \phi_{ji} \quad \text{for } i, j = 1, \ldots, n. \tag{6}$$

Because of the free-entry condition of firms, the profit of firms is zero in equilibrium. Consequently, the capital rent is

$$r_i = \frac{1}{\sigma K} \sum_{j=1}^n \frac{w_i^{1-\sigma} \phi_{ij} Y_j}{\sum_{k=1}^n k_k w_k^{1-\sigma} \phi_{jk}}. \tag{7}$$

Next, the national income of country $i$ is the sum of the capital rent and wages,

$$Y_i = \theta_i K \sum_j k_j r_j + w_i \theta_i L. \tag{8}$$

Finally, we choose the labor in country 1 as numéraire so that $w_1 = 1$.

## 3. Equilibrium

This section examines the spatial equilibrium by deriving some equations, which are used to determine the equilibrium wage rates and firm shares in all countries.

Firms choose the country providing higher capital returns. In an interior equilibrium, firms are located in all countries, so the capital rents are equal in all countries: $r_1 = \cdots = r_n$. By Eqs. (6) and (7), the capital rents satisfy

$$\begin{aligned} r_l &= \sum_{i=1}^n k_i r_i = \frac{1}{\sigma K} \sum_{i=1}^n \sum_{j=1}^n k_i w_i^{1-\sigma} \frac{\phi_{ij} Y_j}{\sum_{k=1}^n k_k w_k^{1-\sigma} \phi_{jk}} \\ &= \frac{1}{\sigma K} \sum_{j=1}^n Y_j \frac{\sum_{i=1}^n k_i w_i^{1-\sigma} \phi_{ij}}{\sum_{k=1}^n k_k w_k^{1-\sigma} \phi_{jk}} = \frac{1}{\sigma K} \sum_{j=1}^n Y_j \equiv r \end{aligned} \tag{9}$$

for all $l = 1, \ldots, n$. Let $\overline{w} = \sum_{i=1}^n \theta_i w_i$, which is a weighted sum of all nominal wage rates. Adding up Eq. (8) for all countries, we obtain the world revenue $\sum_{i=1}^n Y_i = Kr + L\overline{w}$. Substituting it into the last equality of (9) obtains

$$(\sigma - 1)Kr = L\overline{w}. \tag{10}$$

The above equation reveals an important relation between the fixed and marginal production costs. Since each firm uses one unit of capital as the fixed input of production, $Kr$ in the LHS of (10) is the global fixed cost in the world. Meanwhile, the RHS is the worldwide labor wage, which is also the global marginal cost of production. Therefore, (10) concludes that the ratio of the fixed input to the marginal costs is $1 : (\sigma - 1)$. This is also true for each country. In fact, the markup of a firm in each country is $\sigma/(\sigma - 1)$ since the marginal input is normalized to $(\sigma - 1)/\sigma$ units of labor and the price is given by (4). For this reason,

$$(\sigma - 1)(k_i K)r = (\theta_i L)w_i \tag{11}$$

holds from the free-entry condition. Eqs. (10) and (11) produce an important result showing how wages and firm shares are linked in each country:

$$k_i = \frac{\theta_i w_i}{\overline{w}}. \tag{12}$$

Eq. (12) indicates that the size of the manufacturing sector in a country and the wage rate there are positively related. Intuitively, since the supply of labor is perfectly inelastic, the competition on the local labor market intensifies as firms agglomerate in a country, which increases the labor wage. This in turn generates a higher demand for the manufactured goods, making this country more attractive to the firms of other countries.

By use of (9) and (10), the national income of (8) can be rewritten as

$$Y_i = L\theta_i\left(w_i + \frac{\overline{w}}{\sigma - 1}\right). \tag{13}$$

Because of the simple relation in (12), to solve the model, we only need to determine the wage rates, which can be done by examining the full employment condition in each country:

$$k_i K \frac{\sigma - 1}{\sigma} \sum_j \tau_{ij} d_{ij} = L\theta_i.$$

Let $\phi = \tau^{1-\sigma} \in [0, 1]$. According to (1), (4), (5), (12) and (13), the above equality can be written as

$$\sum_{j=1}^{n} H_j(\boldsymbol{w}, \phi)\phi_{ij} = w_i^{\sigma-1}, \quad i = 1, \ldots, n, \tag{14}$$

where

$$H_j(\boldsymbol{w}, \phi) \equiv \frac{(\sigma-1)K}{\sigma L\overline{w}} \frac{Y_j}{P_j^{1-\sigma}} \tag{15}$$

$$= \frac{\frac{\sigma-1}{\sigma} w_j + \frac{1}{\sigma}\overline{w}}{(1-\phi)w_j^{2-\sigma} + \phi \sum_{l=1}^{n} \frac{\theta_l}{\theta_j} w_l^{2-\sigma}},$$

$$\text{for } \boldsymbol{w} \in \mathbb{R}^n_{++},\ j = 1, \ldots, n. \tag{16}$$

Although (14) looks complicated, these $n$ equations contain wage variables $\boldsymbol{w}$ only. One of the equations is redundant and $w_1 = 1$ because the labor in country 1 is the chosen numéraire.

In the case of two countries, there is only one wage variable $w_2$, which is determined implicitly by the above equation. As done in Takahashi et al. (2013), the intermediate value theorem is powerful enough to examine the properties of $w_2$. In contrast, in the case of multiple countries, we have $n - 1$ variables, requiring a more powerful tool, the Brouwer fixed point theorem. For this purpose, we need to construct a suitable mapping as follows.

As shown in Appendix A, Eq. (14) can be reformulated as the mapping $\mathcal{M}(\boldsymbol{w}, \phi) = \boldsymbol{w}$ of $\boldsymbol{w} \in \{1\} \times \mathbb{R}^{n-1}_{++}$, where the $i$th component of $\mathcal{M}(\boldsymbol{w}, \phi)$ is

$$\mathcal{M}_i(\boldsymbol{w}, \phi) = \begin{cases} \left\{(1-\phi)[H_i(\boldsymbol{w}, \phi) - H_1(\boldsymbol{w}, \phi)] + 1\right\}^{\frac{1}{\sigma-1}}, \\ \quad \text{if } \sigma \geq 2, \\ w_i^{2-\sigma}\left\{(1-\phi)[H_i(\boldsymbol{w}, \phi) - H_1(\boldsymbol{w}, \phi)] + 1\right\}, \\ \quad \text{if } \sigma \in (1, 2), \end{cases} \tag{17}$$

for $i = 2, \ldots, n$. Because of (12), $k_j w_j^{1-\sigma}$ in (5) is proportional to $w_j^{2-\sigma}$, obtaining terms of wages raised to the power of $2 - \sigma$ in the denominator in (16). Since $w_k^{2-\sigma}$ increases in $w_k$ when $\sigma \in (1, 2)$ but decreases when $\sigma \in (2, \infty)$, we have to construct different mappings for these two cases in (17). The reader can find details in Appendix A.

As $w_1 = 1$, we write $\boldsymbol{w} = (1, \boldsymbol{w}_{-1})$, where $\boldsymbol{w}_{-1} = (w_2, \ldots, w_n) \in \mathbb{R}^{n-1}_{++}$. Then, equilibrium wage $\boldsymbol{w}_{-1}(\phi)$ is a fixed point of mapping

$$\mathcal{M}_{-1}(\boldsymbol{w}, \phi) = (\mathcal{M}_2, \ldots, \mathcal{M}_n) \quad \text{from } \mathbb{R}^{n-1}_{++} \text{ to } \mathbb{R}^{n-1}_{++}.$$

The following result ensures the existence of such equilibrium wages.

**Lemma 1.** *For any $\sigma \in (1, \infty)$, there exist equilibrium wage rates $\boldsymbol{w}(\phi)$ satisfying (14) with range $w_i(\phi) \in [\phi^{\frac{1}{\sigma-1}}, 1]$.*

**Proof.** See Appendix A.

In Lemma 1, the wages are bounded below by $1/\tau$, generalizing the case of two countries ((9) of Takahashi et al., 2013). A similar result of $w \geq \tau^{(1-\sigma)/\sigma}$ is known for one-factor models of Hanson and Xiang (2004, p. 1111) and Takatsuka and Zeng (2012b, p. 312).

## 4. Existence of the HME

### 4.1. Three definitions of the HME

As mentioned previously, three definitions of the HME in a two-country space are known in the extant literature on this topic. They are in terms of firm share, wage and trade pattern.

Behrens et al. (2009) first explore the HME of multiple countries. Labor is the only factor of production there. They successfully generalize the HME definition in terms of firm share. For countries with size order $\theta_1 \geq \cdots \geq \theta_n$, the HME is the phenomenon of

$$\frac{k_1}{\theta_1} \geq \cdots \geq \frac{k_n}{\theta_n}. \tag{18}$$

Stated differently, the order of firm share reflects the order of countries' market size. This means that the country of a larger market size always accommodates a relatively larger share of firms.

The framework of Behrens et al. (2009) is unable to explore the HME in terms of wage due to the assumption of a costlessly traded agricultural good. However, it is easy to extend the definition into a multicountry space: the wage is higher in a larger country. In our notation, it is written as

$$1 = w_1 \geq w_2 \geq \cdots \geq w_n. \tag{19}$$

By removing the assumption of free transportation of the agricultural good, we are fortunately able to see the direct link between firm share and wage. This link discloses the fact that the two HME definitions are generally equivalent in a multicountry space.

**Proposition 1.** *Two definitions (18) and (19) of the HME are equivalent.*

**Proof.** By reformulating equality in (12) as

$$\frac{k_i}{\theta_i} = \frac{w_i}{\overline{w}},$$

we can see the equivalence of (18) and (19) clearly. □

Since wage is an important part of income, the market in a country with a higher wage rate is larger, attracting more firms to locate there. On the other hand, because labor is a production factor, a higher wage rate in a country increases the production costs there, pushing firms out of the country. The above relation between wage rate and firm share is a balance of these two opposing forces. The simplicity comes from some properties of the CES preference and the production structure.

The last definition of the HME is based on trade pattern: the larger country is a net exporter of manufactured goods (Krugman, 1995, p. 1261; Davis, 1998, p. 1271). In the case of two countries, it is evident that the trade surpluses of manufactured goods are actually ranked by their sizes according to Takahashi et al. (2013). Therefore, a natural extension of this definition is that the export volumes are ranked by the country sizes. Unfortunately, this extended definition is not equivalent to the previous two in the case of multiple countries. We will examine this issue again in Section 4.3.

Because of the equivalence result of Proposition 1, we employ the definitions in terms of firm share and wage for the HME in this paper.

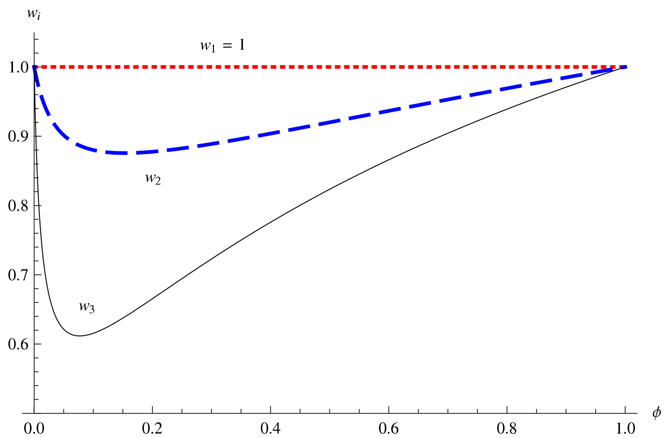


**Fig. 1.** Wages $w_i$.

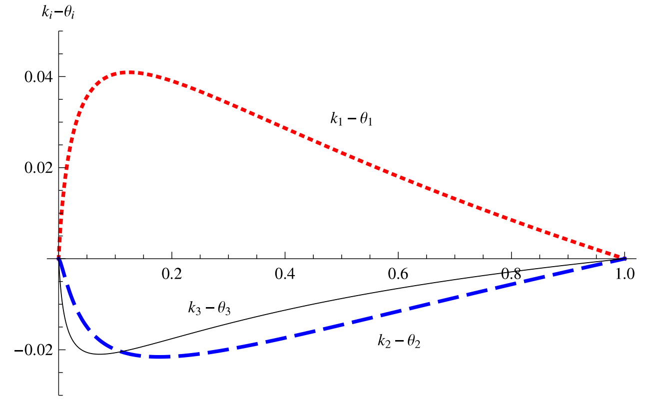


**Fig. 2.** Net flows of capital rent.

*4.2. The existence*

Proposition 1 only clarifies the equivalence of inequalities (18) and (19). We are able to conclude that both of them are true. In other words, generalizing the result with two countries, the HME holds when there are multiple countries.

**Proposition 2.** *For any $\phi \in (0, 1)$, if $\theta_1 > \theta_2 > \cdots > \theta_n$, then the wages are ordered as $1 = w_1 > w_2 > \cdots > w_n$ and the HME occurs.*

**Proof.** See Appendix B. □

Mapping (17) is complicated and we have no analytical form for its fixed point. To form an image of the wage curves, we perform a numerical simulation for the case of three countries.[7] Fig. 1 shows how wages change with $\phi$ when $n = 3, \theta_1 = 0.62, \theta_2 = 0.32, \theta_3 = 0.06, \sigma = 3$, where $w_1 = 1$ is the dotted line, $w_2$ is the dashed line and $w_3$ is the solid line. We can see that a larger country indeed provides a higher wage rate.

To understand the result, it is noteworthy that in countries with high wages each firm reduces its labor share in its total cost, resulting in a higher firm share in a larger country due to the full-employment condition. The HME then emerges naturally.

*4.3. Trade surplus*

According to the balance of payment in countries at equilibrium, the trade surplus of manufactured goods in each country is equal to the net flow of capital rent, which is calculated as $(k_i - \theta_i)Kr$ for country $i$. Therefore, the definition of the HME in terms of trade pattern is written as

$$k_i - \theta_i \geq k_{i+1} - \theta_{i+1}, \quad \text{for } i = 1, \ldots, n-1. \tag{20}$$

If $k_i > \theta_i$, we then have

$$k_i - \theta_i = \theta_i\left(\frac{k_i}{\theta_i} - 1\right) > \theta_{i+1}\left(\frac{k_i}{\theta_i} - 1\right) > \theta_{i+1}\left(\frac{k_{i+1}}{\theta_{i+1}} - 1\right) = k_{i+1} - \theta_{i+1},$$

where the second inequality is from (18), Propositions 1 and 2. Therefore, (20) is true if at least one of countries $i$ and $i + 1$ has a positive net capital inflow.

However, such a relation does not hold for countries having negative net capital inflows. According to (B.1) and (C.1) of Appendices B and C, the slope of $k(\phi)$ at $\phi = 0, 1$ is given by

$$k_i'(0) = \sigma(\theta_i n - 1), \qquad k_i'(1) = \frac{\theta_i}{\sigma - 1}\left(-\theta_i + \sum_{j=1}^{n}\theta_j^2\right)$$

from (12). While $k_1'(0) \geq k_2'(0) \geq \cdots \geq k_n'(0)$ holds, $k_i'(1)$ is not necessarily monotone in $i$. Fig. 2 illustrates an example of net capital inflows in a space of three countries. We use the same parameters of Fig. 1. In Fig. 2, the dotted curve is $k_1 - \theta_1$, the dashed curve is $k_2 - \theta_2$ and the solid curve is $k_3 - \theta_3$. We can see that $k_1'(0) > 0 > k_2'(0) > k_3'(0)$ and $k_2 - \theta_2 > k_3 - \theta_3$ hold when $\phi$ is small. However, $k_1'(1) < 0 < k_3'(1) < k_2'(1)$ and $k_2 - \theta_2 < k_3 - \theta_3$ hold when $\phi$ is large. Therefore, $k_2 - \theta_2$ and $k_3 - \theta_3$ intersect somewhere inside (0, 1). Accordingly, the order of (20) is not generally true in a multicountry space.

Generally speaking, when $\phi$ is small, serving the local market is most important, and the volatility of capital flow is relatively low. Both $k_i/\theta_i \geq k_{i+1}/\theta_i$ and $k_i - \theta_i \geq k_{i+1} - \theta_{i+1}$ hold even they are negative. However, when $\phi$ is large, it is easy to serve foreign markets and the volatility of capital flow is relatively high. While $k_i/\theta_i \geq k_{i+1}/\theta_{i+1}$ remains true, inequality $k_i - \theta_i \geq k_{i+1} - \theta_{i+1}$ might be reversed when both of them are negative and $\theta_{i+1}$ is much smaller than $\theta_i$.

In Fig. 2, the largest country is a net exporter of manufactured goods and the smallest country is a net importer. They are generally true and consistent with the results of two-country models like those in Zeng and Kikuchi (2009) and Oyama et al. (2011). However, we have no determinate result regarding other countries. For example, Fig. 3 plots a simulation result of $k_2 - \theta_2$ in a three-country case with parameters $\theta_1 = 0.4, \theta_2 = 0.34, \theta_3 = 0.26$ and $\sigma = 3$. In this example, country 2 is a net exporter of manufactured goods when $\phi$ is small but a net importer when $\phi$ is large. This fact makes the trade-pattern definition difficult to apply in HME empirical studies.

As noticed in Section 1, Proposition 1 does not hold in a single-factor model nor in a setting with an outside good. Therefore, capital is an important production factor needs to be included in NTT models if we do not rely on the homogeneous good. On the other hand, the inequivalence result of the trade-pattern definition is a reminder that we need to be careful about the exact definition in empirical studies on the HME since the real world consists of many countries.

## 5. Ubiquitous inequalities

Inequality is a sensitive issue because some politicians and economists of small regions or countries worry about whether

[7] The simulations of this paper are performed by Mathematica 8.0. The programs are available from the corresponding author.

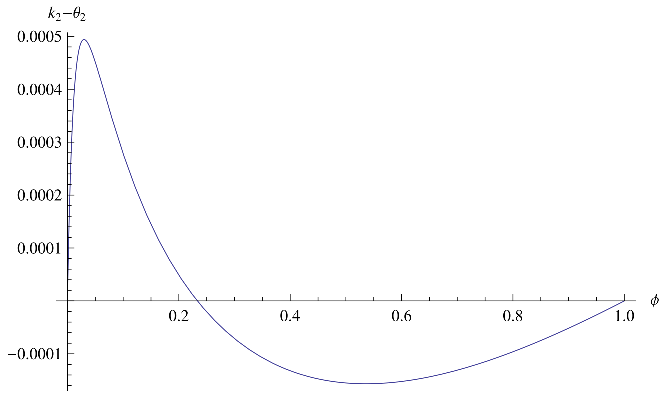


**Fig. 3.** Indeterminate capital flow of country 2.

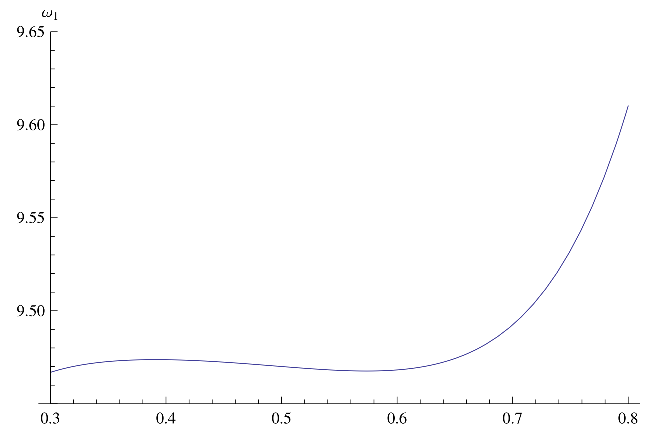


**Fig. 4.** Real income $V_1$.

their economies could collapse as a result of increased economic integration.

Regional inequality can be examined by both nominal and real incomes. Most papers in the literature, like Krugman and Venables (1990, 1995), Venables (1996), Puga and Venables (1997) and Zeng and Kikuchi (2009), discuss the inequality of real income only since their assumption on the agricultural sector mutes the nominal income inequality. Fortunately, the wage rates in our framework are endogenously determined, which depend on trade freeness $\phi$. This section aims to clarify the evolution of wage rates and firm shares, and explore the nominal and real income inequalities together.

### 5.1. Nominal income inequality

Because each resident has the same capital endowment, the nominal income inequality is attributed to the nominal wage inequality. Proposition 2 shows that the nominal income inequality occurs whenever $\phi \neq 0, 1$. Put differently, the inequality between any two countries occurs even when there is no relative advantage in technology, resource endowment and geography among countries.

Fig. 1 indicates how nominal wages behave when trade is more integrated in the case of three countries. The curves of (relative) wages $w_2(\phi) = w_2(\phi)/w_1(\phi)$ and $w_3(\phi) = w_3(\phi)/w_1(\phi)$, initially decrease and then increase in $\phi$. This non-monotonic fact turns out to be quite general.

**Proposition 3.** *If $\theta_1 > \theta_2 > \cdots > \theta_n$, the relative wage $w_i/w_j$ between any two countries $i > j$ increases in $\phi$ when $\phi$ is small and decreases in $\phi$ when $\phi$ is large.*

**Proof.** See Appendix C.

The intuition provided by Fujita and Thisse (2013, p. 371) is helpful. Because workers are immobile, a higher concentration of firms in a country increases wages there, resulting in two opposing forces. One is a backward linkage: final demand increases because of consumers' higher income, which is a centripetal force encouraging agglomeration; The other is a forward linkage: a higher wage rate increases the labor costs of firms, which is a centrifugal force discouraging agglomeration. When $\phi$ is small, the centripetal force is weaker because serving the local market is important, which results in a dispersion stage of firms. When $\phi$ is large, the centrifugal force is stronger, forming another dispersion stage of firms.

In the case of two countries, Takahashi et al. (2013) show that $w_2/w_1$ exhibits a U shape. Our simulation results suggest that the U shape is true even in a multicountry space.

Despite the close relation between $k_i$ and $w_i$ shown in (12), $k_i(\phi)$ and $w_i(\phi)$ may have different responses to $\phi$. In the numerical example of Fig. 3, $w_2$ has a U shape while $k_2$ exhibits a more complicated pattern.

### 5.2. Real income inequality

The real income (indirect utility) $V_i$ of a country $i$ depends on the nominal income and the price index there, and the latter is determined by the firm distribution.

In a space of free trade, the nominal wages and price indices are all equalized across the countries. Therefore, the real incomes in all countries are also equal and there is no income inequality in this economic space. However, the real income inequality occurs whenever $\phi < 1$. Note that the trade surplus of manufactured goods are not necessarily ranked by the country labels as in Fig. 2, one may wonder whether the price indices are of non-monotonic ranking. Surprisingly, the following proposition concludes that both the real income and the price index are ranked exactly by the country size. The benefit of a relatively more varieties in a larger country is greater than the loss of higher prices of domestically produced manufactured goods.

**Proposition 4.** *At the equilibrium, we have $P_i < P_{i+1}$ and $V_i > V_{i+1}$ for all $\phi \in [0, 1)$ and $i = 1, \ldots, n-1$ if $\theta_1 > \theta_2 > \cdots > \theta_n$.*

**Proof.** See Appendix D.

While the real incomes converge when trade is fully integrated, they are not necessarily monotonic when $\phi$ increases. An example is given below as a simulation result with parameters $\theta_1 = 0.85, \theta_2 = 0.12, \theta_3 = 0.03, \sigma = 1.1$. The real income of country 1 plotted in Fig. 4 is not monotone when $\phi \in [0, 3, 0.8]$.

Although the real income inequality is also ubiquitous, all countries gain from trade liberalization if $\phi$ is either small or large with different slopes.

**Proposition 5.** *Real income $V_i$ in country $i$ increases in $\phi$ when $\phi$ is either small or large. Furthermore, a smaller country gains from globalization less than a larger country when $\phi$ is small but gains more when $\phi$ is large.*

**Proof.** See Appendix D.

Note that in the process of trade integration, the number of firms changes in each country, which impacts on the price index directly. However, the real income in each country is improved since the gain from cheaper imported goods dominates when $\phi$ is either large or small.

## 6. Conclusion

This paper reveals the close relation between the country size and economic activity in a multicountry space when there are no relative advantages in technology, resource endowment and geographic feature. Unlike most NTT papers, our framework does not assume a freely traded homogeneous good, so wage rates are endogenously determined, deriving the spatial income inequality. The results show that the spatial income inequalities in terms of both nominal income and real income are ubiquitous.

The income inequality is examined by the HME concept. There are three equivalent definitions of the HME in a two-country space according to Takahashi et al. (2013). We prove that two of them, in terms of firm share and wage, are equivalent in a multicountry space. Both of them are observed in our model. However, the definition in terms of trade pattern is not equivalent to the other two.

The real world consists of many countries and the wage rates are different across countries. By clarifying a simple mechanism producing income inequality in the absence of relative advantage of technology, resource endowment and geography, this paper contributes to the theoretical inequality analysis which may shed some light on empirical studies in the future.

To explore the role of increasing returns to scale, NTT models typically make a number of other restrictive assumptions: the same CES taste for all consumers, the iceberg transportation etc. While those hypotheses make elegant algebra possible, the models are far away from the economic reality. More interesting results are expectable by relaxing them step by step.

## Appendix A. Existence of equilibrium wages

We form mapping $\mathcal{M}(\boldsymbol{w}, \phi)$ of (17) and show that it has a fixed point in $[\phi^{\frac{1}{\sigma-1}}, 1]^n$ for any $\phi \in [0, 1]$.

Since $w_1 = 1$, Eq. (14) for $i = 1$ gives

$$\phi \sum_{j=1}^{n} H_j(\boldsymbol{w}, \phi) = 1 - (1-\phi)H_1(\boldsymbol{w}, \phi), \tag{A.1}$$

and other $n-1$ equations in (14) are

$$\phi \sum_{j=1}^{n} H_j(\boldsymbol{w}, \phi) + (1-\phi)H_i(\boldsymbol{w}, \phi) = w_i^{\sigma-1}, \quad i = 2, \ldots, n. \tag{A.2}$$

The equations of (A.1) and (A.2) imply

$$(0 <)H_i(\boldsymbol{w}, \phi) = \frac{w_i^{\sigma-1}}{1-\phi} - \left(\frac{1}{1-\phi} - H_1(\boldsymbol{w}, \phi)\right), \quad i = 1, \ldots, n, \tag{A.3}$$

which can also be rewritten as

$$\left\{(1-\phi)[H_i(\boldsymbol{w}, \phi) - H_1(\boldsymbol{w}, \phi)] + 1\right\}^{\frac{1}{\sigma-1}} = w_i, \quad \text{for } i = 2, \ldots, n. \tag{A.4}$$

Equations of (A.4) are evidently equivalent to

$$w_i^{2-\sigma}\{(1-\phi)[H_i(\boldsymbol{w}, \phi) - H_1(\boldsymbol{w}, \phi)] + 1\} = w_i, \quad \text{for } i = 2, \ldots, n,$$

obtaining mapping (17).

We now provide some properties of functions $H_i(\boldsymbol{w}, \phi)$.

**Lemma A.1.** *It holds that $H_1(\boldsymbol{w}, \phi) \le 1$ for $\boldsymbol{w}_{-1} \in [\phi^{\frac{1}{\sigma-1}}, 1]^{n-1}$.*

**Proof.** The inequalities

$$\frac{1}{\sigma}(1-\overline{w}) \ge 0 \ge \phi\left(-\sum_{k=2}^{n} \frac{\theta_k}{\theta_1} w_k^{2-\sigma}\right)$$

imply

$$\frac{\sigma-1}{\sigma} + \frac{1}{\sigma}\overline{w} \le 1 - \phi + \phi \sum_{k=1}^{n} \frac{\theta_k}{\theta_1} w_k^{2-\sigma},$$

which can be rewritten as $H_1(\boldsymbol{w}, \phi) \le 1$. □

**Lemma A.2.** *For $\boldsymbol{w}_{-1} \in [\phi^{\frac{1}{\sigma-1}}, 1]^{n-1}$, $i = 2, \ldots, n$, we have (i) $H_i(\boldsymbol{w}, \phi) \le H_1(\boldsymbol{w}, \phi)$ when $\sigma \ge 2$ and (ii) $(1-\phi)[H_i(\boldsymbol{w}, \phi) - H_1(\boldsymbol{w}, \phi)] + 1 \le w_i^{\sigma-2}$ when $\sigma \in (1, 2)$.*

**Proof.** (i) For $\sigma \ge 2$ and $w_i \le 1$, we have $w_i^{2-\sigma} \ge 1$. Therefore, (16) implies $H_i(\boldsymbol{w}, \phi) \le H_1(\boldsymbol{w}, \phi)$ directly.

(ii) For $\sigma \in (1, 2)$ and $w_i \le 1$, we have

$$\begin{aligned} w_i^{2-\sigma} H_i(\boldsymbol{w}) &= \frac{\frac{\sigma-1}{\sigma} w_i + \frac{1}{\sigma}\overline{w}}{1 - \phi + \frac{\phi}{w_i^{2-\sigma}\theta_i} \sum_{k=1}^{n} \theta_k w_k^{2-\sigma}} \\ &\le \frac{\frac{\sigma-1}{\sigma} + \frac{1}{\sigma}\overline{w}}{1 - \phi + \frac{\phi}{\theta_1} \sum_{k=1}^{n} \theta_k w_k^{2-\sigma}} \\ &= H_1(\boldsymbol{w}) \le 1. \end{aligned}$$

Meanwhile, $(1 - w_i^{2-\sigma})H_1(\boldsymbol{w}) \le 1 - w_i^{2-\sigma}$ holds from $w_i^{2-\sigma} \le 1$, which implies that

$$\begin{aligned} &(1-\phi)[H_i(\boldsymbol{w}, \phi) - H_1(\boldsymbol{w}, \phi)] + 1 \\ &\quad \le (1-\phi)(w_i^{\sigma-2} - 1)H_1(\boldsymbol{w}, \phi) + 1 \\ &\quad \le w_i^{\sigma-2}(1 - w_i^{2-\sigma}) + 1 = w_i^{\sigma-2}. \quad \square \end{aligned}$$

**Proof of Lemma 1.** We prove that $\mathcal{M}(\boldsymbol{w}, \phi)$ has a fixed point in $[\phi^{\frac{1}{\sigma-1}}, 1]$. Given $\boldsymbol{w}_{-1} \in [\phi^{\frac{1}{\sigma-1}}, 1]^{n-1}$, Lemma A.1, on one hand, shows that

$$\begin{aligned} &\left\{(1-\phi)[H_i(\boldsymbol{w}, \phi) - H_1(\boldsymbol{w}, \phi)] + 1\right\}^{\frac{1}{\sigma-1}} \\ &\quad \ge \left[1 - (1-\phi)H_1(\boldsymbol{w}, \phi)\right]^{\frac{1}{\sigma-1}} \\ &\quad \ge \phi^{\frac{1}{\sigma-1}}, \quad \text{for } \sigma \ge 2, \\ &w_i^{2-\sigma}\left\{(1-\phi)[H_i(\boldsymbol{w}, \phi) - H_1(\boldsymbol{w}, \phi)] + 1\right\} \\ &\quad \ge w_i^{2-\sigma}[1 - (1-\phi)H_1(\boldsymbol{w}, \phi)] \\ &\quad \ge \phi^{\frac{2-\sigma}{\sigma-1}}\phi = \phi^{\frac{1}{\sigma-1}}, \quad \text{for } \sigma \in (1, 2). \end{aligned}$$

Lemma A.2, on the other hand, concludes that $\mathcal{M}_{-1}(\boldsymbol{w}, \phi) \le 1$ for all $\sigma \in (1, \infty)$. Therefore, $\mathcal{M}_{-1}$ is a map of $\boldsymbol{w}_{-1}$ from $[\phi^{\frac{1}{\sigma-1}}, 1]^{n-1}$ to itself. The Brouwer fixed point theorem then tells us that Eq. (A.4) has a solution of $\boldsymbol{w}_{-1}^* \in [\phi^{\frac{1}{\sigma-1}}, 1]^{n-1}$.

## Appendix B. Existence of the HME

It is easy to show that $w_1 = \cdots = w_n = 1$ is the only solution of (14) when $\phi = 1$. The following result clarifies the case of $\phi = 0$.

**Lemma B.1.** *For $\phi \in [0, 1)$ and $\theta_i \ne \theta_j$, equality $w_i = w_j$ holds iff $\phi = 0$.*

**Proof.** *Necessity*: when $w_i = w_j$, (A.4) implies $H_i(\boldsymbol{w}, \phi) = H_j(\boldsymbol{w}, \phi)$, deriving

$$\phi\left(\frac{1}{\theta_i} - \frac{1}{\theta_j}\right) \sum_{k=1}^{n} \theta_k w_k^{2-\sigma} = 0.$$

Then $\phi = 0$ because $\theta_i \ne \theta_j$.

*Sufficiency*: for $\phi = 0$, definition (16) gives

$$H_i(\boldsymbol{w}, 0) = \frac{(\sigma-1)w_i + \overline{w}}{\sigma w_i^{2-\sigma}},$$

and (A.1) is simplified as $\overline{w} = 1$, implying $w_i = w_j$. □

**Proof of Proposition 2.** We rewrite (16) as

$$\left[(1-\phi)w_j^{2-\sigma}+\phi\sum_{k=1}^{n}\frac{\theta_k}{\theta_j}w_k^{2-\sigma}\right]H_j(\boldsymbol{w},\phi)$$
$$=\frac{\sigma-1}{\sigma}w_j+\frac{1}{\sigma}\sum_{k=1}^{n}\theta_k w_k.$$

Taking the total differential of two sides with respect to $\phi$ at $\phi=0$ (where equilibrium wage $w_j=1$ for all $j$), we obtain:

$$\left.\frac{dH_j(\boldsymbol{w}(\phi),\phi)}{d\phi}\right|_{\phi=0}=\left(\sigma-1-\frac{1}{\sigma}\right)w_j'(0)+\frac{1}{\sigma}\sum_{k=1}^{n}\theta_k w_k'(0)+1-\frac{1}{\theta_j},\quad \text{for } j=1,\ldots,n.$$

Recalling that $w_1(\boldsymbol{w})=1$ holds for all $\phi$, the above equation for $j=1$ implies

$$\left.\frac{dH_1(\boldsymbol{w}(\phi),\phi)}{d\phi}\right|_{\phi=0}=\frac{1}{\sigma}\sum_{k=1}^{n}\theta_k w_k'(0)+1-\frac{1}{\theta_1}.$$

Now we take the total differential of (A.4) with respect to $\phi$ at $\phi=0$, obtaining

$$w_j'(0)=\frac{1}{\sigma-1}\left[\left.\frac{dH_j(\boldsymbol{w}(\phi),\phi)}{d\phi}\right|_{\phi=0}-\left.\frac{dH_1(\boldsymbol{w}(\phi),\phi)}{d\phi}\right|_{\phi=0}\right]$$
$$=\frac{1}{\sigma-1}\left[\left(\sigma-1-\frac{1}{\sigma}\right)w_j'(0)+\frac{1}{\theta_1}-\frac{1}{\theta_j}\right].$$

Therefore, we have

$$w_j'(0)=\sigma\left(\frac{1}{\theta_1}-\frac{1}{\theta_j}\right)\quad\text{for } j=2,\ldots,n, \tag{B.1}$$

so that $w_1'(0)>w_2'(0)>\cdots>w_n'(0)$. On the other hand, curves $w_i(\phi)$ do not cross in $\phi\in(0,1)$ according to Lemma B.1. Therefore, $1>w_2(\phi)>\cdots>w_n(\phi)$ holds for all $\phi\in(0,1)$. □

## Appendix C. Non-monotonic wage curves

**Proof of Proposition 3.** (a) First, we prove the case of $j=1$, by showing that $w_i(\phi)/w_j(\phi)=w_i(\phi)$ increases when $\phi$ is close to 0 and decreases when $\phi$ is close to 1. It is enough to show that $w_i'(1)>0$ and $w_i'(0)<0$.

Taking the total differential of (A.4) with respect to $\phi$ and let $\phi\to 1$, we obtain

$$w_i'(1)=\frac{1}{\sigma-1}\left[H_1(\boldsymbol{w}(1),1)-H_i(\boldsymbol{w}(1),1)\right]$$
$$=\frac{\theta_1-\theta_i}{\sigma-1}>0,\quad\text{for } i=2,\ldots,n. \tag{C.1}$$

Meanwhile, according to (B.1), $w_i'(0)<0$ holds for $i=2,\ldots,n$.

(b) We now show that the conclusion generally holds for any two countries $j=1,\ldots,n-1$ and $i=j+1,\ldots,n$. We replace the mapping $\mathcal{M}(\boldsymbol{w},\phi)$ with $\mathcal{M}\left(\frac{\boldsymbol{w}}{w_j},\phi\right)$. Then the above proof in Step (a) can be rewritten to show that $w_i/w_j$ evolves in the same way.

## Appendix D. Real income inequality

**Proof of Proposition 4.** The results are evidently true for $\phi=0$, since country $i$ has more varieties than country $i+1$ while nominal wages are equal across the countries. For $\phi\in(0,1)$, to the contrary, we assume that $P_i\geq P_{i+1}$ holds for some $\phi$ and $i=1,\ldots,n-1$. Then it holds that

$$\left(\frac{w_i}{w_{i+1}}\right)^{\sigma-1}<\frac{\frac{w_i^{\sigma-1}}{1-\phi}-\left(\frac{1}{1-\phi}-H_1(\boldsymbol{w},\phi)\right)}{\frac{w_{i+1}^{\sigma-1}}{1-\phi}-\left(\frac{1}{1-\phi}-H_1(\boldsymbol{w},\phi)\right)}=\frac{H_i(\boldsymbol{w},\phi)}{H_{i+1}(\boldsymbol{w},\phi)}$$
$$=\frac{Y_i}{Y_{i+1}}\left(\frac{P_{i+1}}{P_i}\right)^{1-\sigma}$$
$$=\frac{Y_i}{Y_{i+1}}\frac{(1-\phi)\theta_{i+1}w_{i+1}^{2-\sigma}+\phi\sum_{l=1}^{n}\theta_l w_l^{2-\sigma}}{(1-\phi)\theta_i w_i^{2-\sigma}+\phi\sum_{l=1}^{n}\theta_l w_l^{2-\sigma}}$$
$$\leq\frac{Y_i}{Y_{i+1}}\frac{\theta_{i+1}w_{i+1}^{2-\sigma}}{\theta_i w_i^{2-\sigma}},$$

where the first inequality and the first equality are due to Proposition 2, Lemma A.1 and (A.3), the second equality is from (15), the third equality is from (5) and (12), and the last inequality is from the assumption of $P_i\geq P_{i+1}$ so that $P_{i+1}^{1-\sigma}\geq P_i^{1-\sigma}$. The relation between the first and the last terms gives

$$\frac{\theta_i w_i}{\theta_{i+1}w_{i+1}}<\frac{Y_i}{Y_{i+1}}=\frac{\theta_i\left(w_i+\frac{K}{L}r\right)}{\theta_{i+1}\left(w_{i+1}+\frac{K}{L}r\right)},$$

indicating $w_i<w_{i+1}$, which contradicts Proposition 2.

Since the nominal income in country $i$ is higher than that in country $i+1$, the real incomes are ranked as $V_1>V_2>\cdots>V_n$.

**Proof of Proposition 5.** The real income in country $i$ is

$$V_i=\left(w_i+\frac{\overline{w}}{\sigma-1}\right)\left(\theta_i w_i^{2-\sigma}\frac{1-\phi}{\overline{w}}+\sum_{j=1}^{n}\theta_j w_j^{2-\sigma}\frac{\phi}{\overline{w}}\right)^{\frac{1}{\sigma-1}},$$

so that

$$\frac{dV_i}{d\phi}=\frac{V_i}{w_i+\frac{\overline{w}}{\sigma-1}}\left(\frac{dw_i}{d\phi}+\frac{1}{\sigma-1}\frac{d\overline{w}}{d\phi}\right)+\frac{V_i}{\sigma-1}$$
$$\times\frac{\overline{w}}{\theta_i w_i^{2-\sigma}(1-\phi)+\sum_{j=1}^{n}\theta_j w_j^{2-\sigma}\phi}$$
$$\times\left\{\theta_i\left[-\frac{w_i^{2-\sigma}}{\overline{w}}-\frac{w_i^{2-\sigma}(1-\phi)}{\overline{w}^2}\frac{d\overline{w}}{d\phi}+\frac{w_i^{1-\sigma}(2-\sigma)(1-\phi)}{\overline{w}}\frac{dw_i}{d\phi}\right]\right.$$
$$\left.+\sum_{j=1}^{n}\left[\theta_j\left(\frac{w_j^{2-\sigma}}{\overline{w}}-\frac{w_j^{2-\sigma}\phi}{\overline{w}^2}\frac{d\overline{w}}{d\phi}+\frac{w_j^{1-\sigma}(2-\sigma)\phi}{\overline{w}}\frac{dw_j}{d\phi}\right)\right]\right\}.$$

By use of (B.1) and (C.1), we have

$$\left.\frac{dV_i}{d\phi}\right|_{\phi=0}=\frac{n-1}{\sigma-1}V_i(0)>0, \tag{D.1}$$

$$\left.\frac{dV_i}{d\phi}\right|_{\phi=1} = V_i(1)\left[\frac{\theta_i + \sigma(1-2\theta_i)}{\sigma(\sigma-1)} + \frac{1}{\sigma}\sum_{j=1}^{n}\theta_j^2\right]$$
$$\geq V_i(1)\frac{1-\theta_i}{\sigma(\sigma-1)}[\sigma - \theta_i(\sigma-1)] > 0. \quad \text{(D.2)}$$

Note that $V_1(0) > V_2(0) > \cdots > V_n(0)$, $V_1(1) = V_2(1) = \cdots = V_n(1)$ hold. Accordingly, the equalities of (D.1) and (D.2) imply

$$\left.\frac{dV_i}{d\phi}\right|_{\phi=0} > \left.\frac{dV_{i+1}}{d\phi}\right|_{\phi=0} \quad \text{and} \quad \left.\frac{dV_i}{d\phi}\right|_{\phi=1} < \left.\frac{dV_{i+1}}{d\phi}\right|_{\phi=1}$$

for $i = 1, \ldots, n-1$.

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