Journal of International Economics 93 (2014) 92–107

Contents lists available at ScienceDirect

Journal of International Economics

journal homepage: www.elsevier.com/locate/jie

Trade policy: Home market effect versus terms-of-trade externality

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ARTICLE INFO

Article history:
Received 19 December 2011
Received in revised form 11 December 2013
Accepted 15 December 2013
Available online 25 January 2014

JEL classification:
F12
F13
F42

Keywords:
Home market effect
Terms of trade
Tariffs and subsidies

ABSTRACT

We study trade policy in a two-sector Krugman (1980) trade model, allowing for wage, import and export subsidies/taxes. We study non-cooperative trade policies, first for each individual instrument and then for the situation where all instruments can be set simultaneously, and contrast those with the efficient allocation. We show that in this general context there are four motives for non-cooperative trade policies: the correction of monopolistic distortions; the terms-of-trade manipulation; the delocation motive for protection (home market effect); the fiscal-burden-shifting motive. The Nash equilibrium when all instruments are available is characterized by first-best-level wage subsidies, and inefficient import subsidies and export taxes, which aim at relocating firms to the other economy and improving terms of trade. Thus, the dominating incentives for non-cooperative trade policies are the fiscal-burden-shifting motives and terms-of-trade effects.

1. Introduction

The aim of this paper is to study optimal trade policy in the canonical two-sector Krugman (1980) model, where one sector is characterized by monopolistic competition, increasing returns and iceberg trade costs, while the other features perfect competition and constant returns. Within this framework we allow for wage, import and export subsidies/taxes. We study non-cooperative trade policies, first for each individual instrument and then for the case where all instruments are set simultaneously, and contrast those with the efficient allocation.

The common wisdom of the literature[1] (Venables, 1987; Helpman and Krugman, 1989; Ossa, 2011) is that in this model unilateral trade policy is set so as to agglomerate firms in the domestic economy in order to reduce transport costs. This reduces the domestic price index thereby increasing domestic welfare.[2] According to the literature, this delocation motive (also called home market effect) provides a reason for protectionist and ultimately welfare detrimental unilateral trade policy in the Krugman (1980) model and, as argued by Ossa (2011), gives an alternative theoretical justification to the neoclassical terms-of-trade externality explanation (Johnson, 1953–1954; Grossman and Helpman, 1995; Bagwell and Staiger, 1999) as to why countries need to sign trade agreements. Similarly, the same mechanism also provides a theoretical justification for the World Trade Organization (WTO)'s limitation of production and export subsidies,[3] which cannot be explained within the neoclassical framework.[4]

By considering a situation where countries can simultaneously choose all three policy instruments (wage, import and export taxes), we contribute to the literature in three ways. First, we show that in this more general setting there are four motives behind non-cooperative trade policies: the correction of monopolistic distortions,[5] the terms-of-trade manipulation, the delocation motive for protection,

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[1] A detailed review of the literature is provided in the next section.

[2] An import tariff makes foreign differentiated goods more expensive relative to domestic ones so that domestic consumers shift expenditure towards domestic differentiated goods. This triggers entry into the domestic differentiated sector and exit out of the foreign differentiated sector, thereby reducing the domestic price index – since now less of the domestically consumed goods are subject to transport costs – and increasing the foreign one. Similarly, a production or an export subsidy also renders the domestic market a more attractive location and reduces the domestic price index at the expense of increasing the foreign one.

[3] See, e.g., WTO (2006). GATT Article XVI and the Uruguay Round Subsidies Code prohibit the use of export subsidies, while the latter also establishes that countervailing duties can be imposed on countries using production subsidies subject to an injury test.

[4] Production and export subsidies are puzzling within the neoclassical framework because they increase foreign welfare at the expense of domestic welfare.

[5] Observe that monopolistic distortions arise because there are two sectors in the model, so that monopolistic markups lead to too low a provision of variety in the monopolistically competitive sector. In their seminal paper, Dixit and Stiglitz (1977) show that the market solution is not first-best Pareto optimal in such a model, and that subsidies on fixed costs and on marginal costs are required to implement it. Thus, policymakers try to improve the use of domestic resources by increasing entry into the differentiated sector.

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http://dx.doi.org/10.1016/j.jinteco.2013.12.010

and the fiscal-burden-shifting motive. The last motive arises when countries use wage subsidies in order to correct for the monopolistic distortions. When this is the case, there is an incentive to relocate firms to the foreign economy, so as to shift the fiscal burden of the subsidy to the other country. Second, and most importantly, we show that the Nash equilibrium is characterized by the first-best level of wage subsidies, and inefficient import subsidies and export taxes. This result has several implications. It shows that, in contrast to the previous literature, the delocation motive for protection is not the dominating motive for strategic trade policy in the Krugman (1980) model once a sufficient number of policy instruments are available. This is so because countries choose to *subsidize* imports and *tax* exports with the intention to relocate firms to the other economy. It also shows that when all three policy instruments are available, the Krugman (1980) model cannot rationalize why countries would set import tariffs and export subsidies in the absence of trade agreements. Finally, following Bagwell and Staiger (1999, 2009), we consider which international externalities countries try to remedy by signing trade agreements. We do so by looking at the *politically optimal policy*, which is defined as the one that noncooperative policymakers would choose if they did not try to manipulate their terms of trade. We find that the politically optimal policy is still distortive. This implies that, differently from Bagwell and Staiger (2009) – who consider simultaneous choice of import and export taxes in the Krugman (1980) model – terms-of-trade externalities are not the only source of inefficiencies which trade agreements try to solve. Instead, the fiscal-burden-shifting motive – which leads to import subsidies and export taxes – is an additional externality that can be eliminated with international trade agreements. Similarly to Bagwell and Staiger (2009), we also find that the delocation motive is not an externality which needs to be corrected by international trade agreements, when all three policy instruments are available.

To clarify policymakers' incentives, we start by considering wage subsidies/taxes as the only available policy instrument. A wage subsidy increases profits of firms in the domestic differentiated sector, and triggers a relocation of firms from the foreign to the domestic economy, thereby reducing monopolistic distortions and exploiting the delocation motive. However, this comes at the cost of a negative terms-of-trade effect because the wage subsidy reduces the international price of domestically produced varieties. We show that the balance always tips in favor of the terms-of-trade effect before monopolistic distortions are eliminated: the non-cooperative outcome is a wage subsidy that is always lower than the first-best one. Thus, the delocation effect does not induce inefficiently large wage subsidies. Instead, the terms-of-trade effect leads to an inefficiently low subsidy level.

The result on wage subsidies makes it clear that the desire to eliminate monopolistic distortions is an important motive for non-cooperative policy choice. Keeping this in mind, we next study import subsidies/tariffs. First, when starting from the (inefficient) free trade allocation, both monopolistic distortions and the delocation motive for protection call for a tariff, which reduces the domestic price level. This is the case studied by Ossa (2011). Next, we consider a situation where monopolistic distortions have been eliminated by appropriate wage subsidies, so that the market allocation is first-best efficient. In this case the motives for import policy are the delocation motive and the fiscal-burden-shifting effect. It turns out that the optimal non-cooperative import policy entails import subsidies, which aim at relocating firms to the foreign economy and thereby shifting part of the subsidy burden to the other country. Thus, the fiscal-burden-shifting effect dominates the delocation motive.

A similar result holds for non-cooperative export policy. When starting from the (inefficient) free trade allocation, non-cooperative policymakers set export subsidies, which are intended to induce entry into the domestic differentiated sector by relocating firms from the foreign economy and thus reduce monopolistic distortions and exploit the delocation effect. These motives dominate the negative terms-of-trade effect of export subsidies. In contrast, when monopolistic distortions have been eliminated by appropriate wage subsidies, the prevailing incentives are terms-of-trade effects and the fiscal-burden-shifting motive. Indeed, in this case the Nash equilibrium is characterized by an export tax, which aims at improving domestic terms of trade and shifting the fiscal burden of the subsidy to the other country.

Finally, we analyze a situation where countries can set wage, import and export policy instruments simultaneously. This is the relevant situation if one wants to address the question of why countries need to sign trade agreements, given that in the absence of such agreements the set of tax instruments that can be used strategically is not limited to a single wage tax or trade tax instrument. In line with the above results for single instruments, we find that non-cooperative policymakers choose the level of wage subsidies that exactly offsets the monopolistic distortions, and that they set import subsidies and export taxes, which aim at improving domestic terms of trade and shifting the subsidy burden to the other country. This result is important since it clarifies that in the Krugman (1980) model, the role of international trade agreements is to solve international externalities due to both terms-of-trade effects and fiscal-burden-shifting motives. Delocation effects only become a relevant motive for trade policy, once the set of policy instruments is restricted.

1.1. Related literature

Our results differ markedly from those of the previous literature on trade policy in the two-sector Krugman (1980) model (Venables, 1987; Helpman and Krugman, 1989 chapter 7; Ossa, 2011). All these contributions find that in this model non-cooperative trade policy is driven by delocation effects, leading to inefficiencies compared to free trade. In particular, Venables (1987) studies unilateral incentives to set, alternatively, tariffs, production or export subsidies and shows that any of those can improve domestic welfare compared to free trade due to the delocation effect. However, he does not study the welfare consequences of a strategic game. Helpman and Krugman (1989) limit their discussion to unilaterally set tariffs, while Ossa (2011) considers a tariff game, where positive tariffs are set in equilibrium due to the delocation effect. While we also find that non-cooperative import policy leads to tariffs, this is true only when wage subsidies and export taxes are not available. Moreover, we find that strategically set production (= wage) subsidies are welfare enhancing compared to free trade.

Closely related to our paper is Bagwell and Staiger (2009), who consider a two-sector Krugman (1980) model with quasi-linear utility allowing policymakers to simultaneously choose import and export taxes. They show that in this case Nash-equilibrium policy choices are explained exclusively by the terms-of-trade effects and not by the delocation motive, because import-tariff-induced delocation effects are counterbalanced by export-subsidy-induced delocation effects. Compared to their work, we use the same utility specification as in Ossa (2011), thus allowing for income effects, and add wage subsidies to the set of policy instruments available to policymakers. We show that when all three policy instruments can be set strategically and income effects are allowed for, there is a new international externality – the fiscal-burden-shifting effect – that can be solved by trade agreements, in addition to the terms-of-trade externality.

Other related work is Gros (1987), who studies an import tariff game in the one-sector variant of the Krugman (1980) model. In that version of the model relocation effects are absent and the free trade allocation is Pareto optimal. He finds that in the Nash equilibrium policymakers set import tariffs which aim at increasing domestic wages due to terms-of-trade effects. Finally, Flam and Helpman (1987) and Helpman and Krugman (1989) chapter 7 discuss a production efficiency effect of trade policy. Since with imperfect competition prices are set above marginal costs, domestic consumption of any given variety is too low. Thus, an import tariff (or a production or export subsidy), which shifts demand towards domestic varieties, can reduce monopolistic distortions. However, their effect refers to a change in average cost induced by a change in firm size and not to a change in the number of domestic firms as in the present paper. Since firm size provided by the market is optimal in the Krugman (1980) model, there is no room for a production efficiency effect in their sense.

The paper proceeds as follows. In the next section we set up the model. In Section 3 we compare the market allocation with the planner solution and discuss the non-cooperative policymakers' problems and incentives. Sections 4 and 5 are dedicated to the study of individual policy instruments: wage taxes/subsidies, import tariffs/subsidies and export taxes/subsidies. In Section 6 we consider simultaneous choice of all policy instruments and the last section presents our conclusions.

2. The model

The setup is exactly as in Venables (1987) and Ossa (2011). The only difference is that we allow for transfers. The world economy consists of two countries: Home (H) and Foreign (F). Each country produces a homogeneous good and a continuum of differentiated goods. All goods are tradable but only the differentiated goods are subject to transport costs. The differentiated goods sector is characterized by monopolistic competition, while there is perfect competition in the homogeneous good sector. Both countries are identical in terms of preferences, production technology, market structure and size. All variables are indexed such that the first sub-index refers to the location of consumption and the second subindex to the location of production. Finally, varieties in the differentiated sector are indexed with i, while countries are indexed with j.

2.1. Households

Households' utility function in the Home country is given by:

$$U(C_H, Z_H) \equiv C_H^{\alpha} Z_H^{1-\alpha}, \tag{1}$$

where C_H aggregates over the varieties of differentiated goods, Z_H represents consumption of the homogeneous good and α is the expenditure share of the differentiated bundle in the aggregate consumption basket. While the homogeneous good is identical across countries, each country produces a different subset of differentiated goods. In particular, N_H varieties are produced in the Home country while N_F are produced by Foreign. The differentiated varieties produced in the two countries are aggregated with a CES function:

$$C_H = \left[\int_0^{N_H} c_{HH}(i)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_0^{N_F} c_{HF}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2}$$

where $c_{HH}(i)$ denotes domestic consumption of a domestically produced variety, $c_{HF}(i)$ is domestic consumption of a Foreign produced variety and $\varepsilon > 1$ is the elasticity of substitution between different varieties. Analogous definitions hold for Foreign consumption bundles.

Given the Dixit–Stiglitz structure of preferences, the households' maximization problem can be solved in two stages. At the first stage, households choose how much to consume of each Home and Foreign variety. The optimality conditions imply the following domestic demand functions and domestic price index:

$$c_{HH}(i) = \left[\frac{p_{HH}(i)}{P_H}\right]^{-\varepsilon} C_H \qquad c_{HF}(i) = \left[\frac{p_{HF}(i)}{P_H}\right]^{-\varepsilon} C_H \tag{3}$$

$$P_H = \left[\int_0^{N_H} p_{HH}(i)^{1-\varepsilon} di + \int_0^{N_F} p_{HF}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}, \tag{4}$$

where P_H is the domestic price index of the differentiated bundle, and $p_{HH}(i)$ ($p_{HF}(i)$) is the domestic price of variety i produced by Home (Foreign).

In the second stage, households choose how to allocate income between the homogeneous good and the differentiated bundle. Thus, they maximize Eq. (1) subject to the following budget constraint:

$$P_H C_H + p_{ZH} Z_H = I_H, \tag{5}$$

where $I_H = W_H L + T_H$, L is the total labor available in each country, W_H is the domestic wage rate, p_{ZH} is the domestic price of the homogeneous good, and T_H is a lump-sum transfer which depends on the tax scheme adopted by the domestic government. The solution to the domestic consumer problem implies that the marginal rate of substitution between the homogeneous good and the differentiated bundle equals their relative price:

$$\frac{\alpha}{1-\alpha}\frac{Z_H}{C_H} = \frac{P_H}{p_{ZH}}. \tag{6}$$

Foreign households solve a symmetric problem.

2.2. Firms

Firms in the differentiated sector operate under monopolistic competition. They pay a fixed cost in terms of labor, f, and then produce with linear technology:

$$y_H(i) = L_{CH}(i) - f, \tag{7}$$

where $L_{CH}(i)$ is the amount of labor allocated to the production of variety i in the differentiated sector. Goods sold in the Foreign market are subject to an iceberg transport cost $\tau > 1$. The government of each country $j \in \{H, F\}$ disposes of three fiscal instruments. A wage tax/subsidy (τ_{Wj}) on firms' fixed and marginal costs,[6] a tariff/subsidy on imports (τ_{Ij}) and a tax/subsidy on exports (τ_{Xj}). Note that τ_{mj} indicates a gross tax for $m \in \{W, I, X\}$, i.e., $\tau_{mj} < 1$ indicates a subsidy and $\tau_{mj} > 1$ indicates a tax. In what follows, we will use the word tax whenever we refer to a policy instrument without specifying whether τ_{mj} is smaller or larger than one. We assume that taxes are paid directly by the firms. Given the constant price elasticity of demand, optimal prices charged by Home firms in the domestic market ($p_{HH}(i)$) are a fixed markup over their perceived marginal cost ($\tau_{WH} W_H$), and optimal prices paid by Foreign consumers for Home produced varieties ($p_{FH}(i)$) equal domestic prices augmented by transport costs and trade taxes:[7]

$$p_{HH}(i) = \tau_{WH}\frac{\varepsilon}{\varepsilon-1} W_H \qquad p_{FH}(i) = \tau_{IF}\tau_{XH}\tau p_{HH}(i). \tag{8}$$

Foreign firms adopt symmetric optimal pricing rules:

$$p_{FF}(i) = \tau_{WF}\frac{\varepsilon}{\varepsilon-1} W_F \qquad p_{HF}(i) = \tau_{IH}\tau_{XF}\tau p_{FF}(i). \tag{9}$$

The homogenous good is produced in both countries j with identical production technology:

$$Q_{Zj} = L_{Zj}, \tag{10}$$

where L_{Zj} is the amount of labor allocated to producing the homogeneous good. Since the good is sold in a perfectly competitive market without trade costs, price equals marginal cost and is the same in both countries. We assume that the homogeneous good is produced in both

[6] Wage taxes are levied on both fixed and marginal costs. This assumption is necessary to keep firm size unaffected by wage taxes, which turn out to be optimal, as we will show in Section 3.1.

[7] Following the previous literature (Venables, 1987; Ossa, 2011), we assume that tariffs and export taxes are charged ad valorem on the factory gate price augmented by transport costs. This implies that transport services are taxed.

countries in equilibrium. Given the production technology, this implies factor price equalization:

$$p_{ZH} = p_{ZF} = W_H = W_F. \tag{11}$$

For convenience, we normalize $P_{ZH} = 1$.

Using the optimal pricing rules just derived, it is possible to rewrite the domestic price index of the differentiated bundle as:

$$P_H = \left[N_H\left(\frac{\varepsilon}{\varepsilon-1}\tau_{WH}\right)^{1-\varepsilon} + N_F\left(\frac{\varepsilon}{\varepsilon-1}\tau_{WF}\tau_{IH}\tau_{XF}\tau\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{12}$$

Note that trade policy can reduce the price index through three different channels. First, because of Dixit–Stiglitz preferences, increasing the total number of varieties reduces the price level. This is the so-called love for variety effect. Second, by increasing N_H at the expense of N_F, the policymaker lowers the price level since Home households can now consume a larger fraction of goods for which they do not pay transport costs. This is the so-called *delocation effect*. Finally, trade policy can reduce the price level through the direct effect of subsidies on the prices of individual varieties.

2.3. Government

All government revenues are redistributed to consumers through a lump-sum transfer T_j. The government is assumed to run a balanced budget. Hence, the domestic government's budget constraint is given by:

$$\begin{aligned}&(\tau_{IH}-1)\int_0^{N_F}\tau_{XF}\tau p_{FF}(i)c_{HF}(i)di + (\tau_{XH}-1)\int_0^{N_H}\tau p_{HH}(i)c_{FH}(i)di\\&+(\tau_{WH}-1)\int_0^{N_H}W_H(y_H(i)+f)di = T_H.\end{aligned} \tag{13}$$

Government income consists of import tax revenues charged on imports of differentiated goods gross of transport costs and Foreign export taxes (thus, tariffs are charged on CIF values of Foreign exports); export tax revenues charged on exports gross of transport costs; and wage tax revenues from taxes on marginal and fixed costs. The Foreign government has a symmetric budget constraint.

2.4. Market clearing conditions

The market clearing condition for a differentiated variety produced at Home is given by:

$$y_H(i) = c_{HH}(i) + \tau c_{FH}(i). \tag{14}$$

A similar condition holds for Foreign varieties. Free entry in the differentiated sector implies that monopolistic producers make zero profit in equilibrium[8] and that production of each differentiated variety is fixed: $y_H(i) \equiv y = (\varepsilon - 1)f$.[9] Moreover, given that firms share the same production technology, the equilibrium is symmetric: all firms in the differentiated sector of a given country charge the same price and produce the same quantity. Using symmetry, the demand function (3) and the fact that the production of each variety is equal to $(\varepsilon - 1)f$, we can rewrite the market clearing condition of domestically produced differentiated varieties (Eq. (14)) as:

$$(\varepsilon-1)f = p_{HH}^{-\varepsilon}\left[P_H^{\varepsilon}C_H + \tau^{1-\varepsilon}(\tau_{IF}\tau_{XH})^{-\varepsilon}P_F^{\varepsilon}C_F\right]. \tag{15}$$

Using the demand functions, the market clearing condition for the homogeneous good – $Q_{ZH} + Q_{ZF} = Z_H + Z_F$ – can be written as:

$$Q_{ZH} + Q_{ZF} = \frac{(1-\alpha)}{\alpha}[P_HC_H + P_FC_F]. \tag{16}$$

Equilibrium in the labor market implies that $L = L_{CH} + L_{ZH}$ with $L_{CH} = N_H\ L_{CH}\ (i)$. Making use of Eqs. (7) and (10), labor market clearing can be written as:

$$Q_{ZH} = L - N_H\varepsilon f. \tag{17}$$

Finally, we assume that there is no trade in financial assets, so trade is balanced. The balanced trade condition is given by:[10]

$$(Q_{ZH}-Z_H) + \tau\tau_{XH}N_Hp_{HH}c_{FH} = \tau\tau_{XF}N_Fp_{FF}c_{HF}. \tag{18}$$

The left hand side of Eq. (18) is the sum of the net export value of the homogeneous goods and the value of exports of differentiated varieties (at CIF inclusive international prices), while the right hand side is the value of imports of differentiated varieties (at CIF inclusive international prices).

As is standard in the trade literature (see e.g., Helpman and Krugman, 1989), we define the *terms-of-trade effect* as a change of the international price of exports $\left(\tau_{XH}p_{HH} = \tau_{XH}\tau_{WH}\frac{\varepsilon}{\varepsilon-1}\right)$ relative to that of imports $\left(\tau_{XF}p_{FF} = \tau_{XF}\tau_{WF}\frac{\varepsilon}{\varepsilon-1}\right)$ of individual varieties. This implies that only wage and export taxes have terms-of-trade effects, while import taxes cannot affect international prices in this model. In particular, a domestic wage or export tax increases the international price of exports one to one and improves domestic terms of trade, while a Foreign export tax or wage tax increases the international price of imports and worsens domestic terms of trade.

2.5. Equilibrium

The optimal pricing rules (Eq. (8)), the good market clearing condition for Home's differentiated varieties (Eq. (15)), the labor market clearing condition (Eq. (17)), the corresponding conditions for Foreign, and the balanced trade condition (Eq. (18)), together with the expressions for the price indices, fully characterize the equilibrium of the economy.

It is possible to solve this system explicitly for N_H and N_F as functions of the trade policy instruments:

$$N_H = \frac{L(A_{2H}-A_{1F})}{A_{2F}A_{2H}-A_{1H}A_{1F}} \qquad N_F = \frac{L(A_{2F}-A_{1H})}{A_{2F}A_{2H}-A_{1H}A_{1F}}, \tag{19}$$

where A_{1H}, A_{2H}, A_{1F} and A_{2F} are non-linear functions of Home policy instruments $\Lambda_H \equiv \{\tau_{WH}, \tau_{IH}, \tau_{XH}\}$ and Foreign policy instruments $\Lambda_F \equiv \{\tau_{WF}, \tau_{IF}, \tau_{XF}\}$. The expressions for these coefficients, as well as the derivation of the equilibrium allocation, can be found in Appendix A.

Let the superscript FT denote the market allocation in the absence of trade policies (free trade allocation). We already showed that production of each differentiated variety is fixed, thus for both countries $y^{FT} = (\varepsilon - 1)f$. Given the assumption of symmetric countries, the equilibrium allocation is also symmetric and Eq. (19) simplifies to $N^{FT} = \frac{\alpha L}{\varepsilon f}$. In the next section, we compare the free trade allocation with the first-best allocation and show how the first-best allocation can be implemented. We then lay out the general structure of the policymakers' problems and discuss the incentives that determine their trade policy choices.

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[8] $\Pi_H(i) = c_{HH}(i)[p_{HH}(i) - \tau_{WH}] + c_{FH}(i)[\tau p_{HH}(i) - \tau\tau_{WH}] - f\tau_{WH} = 0$.

[9] Note that wage taxes on fixed costs are necessary for this result, as can be easily verified from the free entry condition.

[10] Import taxes are collected directly by the governments at the border so they do not enter into this condition.

3. Trade policy

3.1. The first-best allocation and its implementation

The first-best allocation constitutes the natural benchmark against which one can compare the equilibrium outcomes under different policy regimes. The social planner chooses an allocation that maximizes total world welfare subject to the technology constraints and full employment in each country:[11]

$$\max_{C_H, C_F, Z_H, Z_F} C_H^{\alpha} Z_H^{1-\alpha} + C_F^{\alpha} Z_F^{1-\alpha} \tag{20}$$

subject to Eqs. (7), (10) and (14), $Q_{ZH} + Q_{ZF} = Z_H + Z_F$, $L = L_{CH} + L_{ZH}$, the definitions of consumption indices and the corresponding constraints for Foreign.

Proposition 1 presents the solution to this problem, compares it with the free trade allocation and states how the first-best allocation can be implemented with the available tax instruments:[12]

Proposition 1. The first-best allocation and its implementation

(1) *The first-best allocation entails the same firm size but more varieties than the free trade allocation. Formally,* $y^{FB} = f(\varepsilon - 1) = y^{FT}$ and $N^{FB} = \frac{\alpha L}{(\varepsilon - 1 + \alpha) f} > N^{FT} = \frac{\alpha L}{\varepsilon f}$.

(2) *The first-best allocation can be implemented by setting wage subsidies equal to the inverse of the markup and choosing trade taxes such that* $\tau_I^{FB} \cdot \tau_X^{FB} = 1$. *Formally,* $\tau_W^{FB} = \frac{\varepsilon - 1}{\varepsilon}$, $\tau_I^{FB} \cdot \tau_X^{FB} = 1$ and $N = N^{FB}$.

The first part of the proposition replicates Dixit and Stiglitz's (1977) finding that the market provides optimal firm size but too little variety. Because of monopolistic competition in the differentiated sector, individual free trade prices are too high. As a consequence, there is too little demand for the differentiated goods and thus too little entry in the differentiated sector. Therefore, the free trade equilibrium is characterized by a *monopolistic distortion*: both countries would be better off by simultaneously shifting some of their labor force from the homogenous sector to the differentiated sector. The first-best allocation can then be achieved by setting a wage subsidy on marginal and fixed costs equal to the inverse of the markup in both countries. Simultaneously, trade instruments ($\tau_I^{FB} = \tau_X^{FB} = 1$) are either not used, or set in a way that does not distort the prices of imports and exports relative to domestically produced varieties ($\tau_I^{FB} \cdot \tau_X^{FB} = 1$).[13]

3.2. Optimal policy problems

We now turn to the description of the optimal policy problems. First, we assume that policymakers choose only one policy instrument at a time. In this way, we can clarify for each policy instrument which is the driving incentives for policymakers' decisions. Subsequently, we allow all three policy instruments to be available simultaneously. For each case, we study non-cooperative policies and compare them with the first-best allocation.

Note that given Cobb–Douglas utility, Home welfare, represented by the indirect utility function, can be written as:

$$V_H(P_H(\Lambda_H, \Lambda_F), I_H(\Lambda_H, \Lambda_F)) = -\alpha \log(P_H(\Lambda_H, \Lambda_F)) + \log(I_H(\Lambda_H, \Lambda_F)) \tag{21}$$

where P_H and I_H are functions of the policy instruments Λ_H and Λ_F.

The non-cooperative policymaker chooses the domestic trade policy instruments Λ_H in order to maximize Home welfare, given the level of the Foreign trade policy instruments:

$$\max_{\lambda_H} V_H(P_H(\Lambda_H, \Lambda_F), I_H(\Lambda_H, \Lambda_F)) \tag{22}$$

where $\lambda_H \in \{\tau_{WH}, \tau_{IH}, \tau_{XH}, \Lambda_H\}$.

3.3. Policymakers' incentives

We now provide some intuition for the incentives that drive non-cooperative trade policy choices. We have already pointed out that *monopolistic distortions* lead to prices of individual varieties which are too high and hence to too little entry into the differentiated sector. As a consequence, the domestic price level is too high from the single country's perspective. Thus, domestic policymakers will try to set policy instruments in order to reduce prices of individual varieties and to increase entry into the domestic differentiated sector, both of which lead to a fall in the domestic price level. More entry into the domestic differentiated sector can be achieved in different ways: by setting a wage subsidy; by setting an import tariff, which shifts demand towards domestically produced varieties; by setting an export subsidy, which makes the domestic market a more attractive location for firms.

Second, there is the *delocation motive* for protection, which has first been highlighted by Venables (1987) and has more recently been emphasized by Ossa (2011). This channel operates through changes in N_H and N_F that reduce the domestic price level by increasing the fraction of varieties produced domestically, since domestic consumers do not have to incur transport costs on these varieties. Again, entry into the domestic differentiated sector can be achieved by setting, alternatively, wage subsidies, import tariffs, or export taxes. Such policies impose a delocation externality on the other country by leading to exit of firms from its differentiated sector and thereby increasing the Foreign price level.

Third, there is the classical *terms-of-trade externality*, whereby a country tries to increase its income by manipulating international prices in its favor. In the present model both wage taxes and export taxes have positive terms-of-trade effects, since they increase international prices of individual varieties one to one. Import taxes, on the other hand, have no effect on international prices. Observe that exploiting the terms-of-trade externality always comes at the cost of reducing the number of domestically produced varieties, thereby making monopolistic distortions more severe and imposing a negative delocation effect on the domestic economy.

Finally, there is what we call a *fiscal-burden-shifting externality*. This externality exists only conditional on wage subsidies eliminating the monopolistic distortion being in place in both countries. In this case, the domestic policymaker has an incentive to reduce their own subsidy bill at the other country's expense by implementing policies that induce relocation of firms to the Foreign economy. Import subsidies and export taxes can both achieve this aim. We now turn to a detailed discussion of non-cooperative tax policies.

4. Wage taxes

In this section we study non-cooperative wage subsidies/taxes, assuming that they are the only available policy instruments, i.e., $\tau_{IH} = \tau_{IF} = \tau_{XH} = \tau_{XF} = 1$.

Unilateral setting of wage taxes does not lead to the first-best outcome but rather to a wage subsidy which is too low compared to the first-best level. This can be verified by computing the derivative of

[11] More generally, there exists a whole set of Pareto-efficient allocations such that no country can be made better off, without making the other one worse off, which can be traced out by varying the welfare weights in the planner problem. We choose the point on the frontier that corresponds to equal weights of both countries because we always study symmetric allocations, which seems natural given that both countries are identical.

[12] All proofs can be found in the Appendix.

[13] One can show that when wage subsidies are not available, cooperatively set import subsidies or export subsidies can improve upon the free trade allocation in a second-best fashion. Such subsidies can partially eliminate the monopolistic distortions, but they cannot achieve the first-best allocation. This is so, since the markups on varieties produced and consumed in the same country cannot be eliminated with those instruments.

indirect utility with respect to the wage tax at two points.[14] First, when both countries set the first-best subsidy level, i.e. when $\tau_{WH} = \tau_{WF} = \frac{\varepsilon - 1}{\varepsilon}$, a unilateral deviation to a lower subsidy increases domestic welfare, since in this case $\frac{\partial V_H}{\partial \tau_{WH}} > 0$. Still, the domestic policymaker chooses to set a positive level of subsidy: when we evaluate the unilateral deviation at the free trade allocation, i.e. $\tau_{WH} = \tau_{WF} = 1$, we find that $\frac{\partial V_H}{\partial \tau_{WH}} < 0$.

This result can be understood as follows. There are two forces that push for setting a wage subsidy: first, the monopolistic distortion, which requires to increase the total number of differentiated varieties above the level provided by the market and, second, the delocation motive, which pushes for a larger fraction of the differentiated sector being located at Home in order to reduce transport costs and thus the domestic price level. Conversely, the terms-of-trade effect, which calls for making domestic varieties more expensive internationally, calls for a wage tax. At the free trade allocation the monopolistic distortion and the delocation motive prevail on the terms-of-trade effect, while at the first best allocation the terms-of-trade effect induces a reduction in the wage subsidy.

The incentives from the unilateral deviations translate into strategic outcomes as follows.

Proposition 2. Nash-equilibrium wage subsidies

In the Nash equilibrium both countries set a wage subsidy. However, this subsidy is smaller than that needed to implement the first-best allocation. The equilibrium number of varieties is larger than in the free trade allocation, but lower than the first-best level. Formally,

(1) $\tau_W^{FB} < \tau_W^{Nash} < 1$ *and* $N^{FT} < N^{Nash} < N^{FB}$.

Thus, single-country policymakers never over-subsidize domestic wages, as would be required if the delocation effect were the dominant incentive for non-cooperative policy choice. Instead, the trade-off between the delocation motive and the monopolistic distortions, on the one hand, and terms-of-trade effects on the other hand, leads policymakers to choose an inefficiently low level of wage subsidies. This is an important result, because it contradicts the standard wisdom that in the two-sector Krugman model countries have an incentive to over-subsidize the domestic differentiated sector in order to attract more firms at the expense of the other country (Venables, 1987). We now turn to a discussion of trade policy instruments.

5. Import and export taxes

Here, we assume that the only strategic trade policy instrument available is either an import tariff/subsidy or an export tax/subsidy.[15] Given the results of the previous section, where we pointed out the importance of the monopolistic distortions, we study non-cooperative import and export taxes under two scenarios. In the first scenario, monopolistic distortions are present (i.e., $\tau_{WH} = \tau_{WF} = 1$), while in the second scenario wage subsidies have already been set in a non-strategic fashion such as to eliminate monopolistic distortions and to implement the first-best allocation (i.e., $\tau_{WH} = \tau_{WF} = \frac{\varepsilon - 1}{\varepsilon}$).

Again, we can obtain some intuition for the policymakers' incentives by computing the change in domestic welfare of unilaterally setting a trade tax. First, we evaluate the change in domestic indirect utility when we start from the free trade allocation, i.e., when $\tau_{WH} = \tau_{WF} = 1$. In this case a small import tariff is welfare enhancing, i.e., $\frac{\partial V_H}{\partial \tau_{IH}} > 0$, and so is a small export subsidy, i.e., $\frac{\partial V_H}{\partial \tau_{XH}} < 0$. However, the opposite is true when wage subsidies are set at the first-best level, i.e., when $\tau_{WH} = \tau_{WF} = \frac{\varepsilon - 1}{\varepsilon}$. In this case we find that domestic welfare is reduced by setting an import tariff or an export subsidy, i.e., $\frac{\partial V_H}{\partial \tau_{IH}} < 0$ and $\frac{\partial V_H}{\partial \tau_{XH}} > 0$.

How can the difference in outcomes depending on whether first-best wage subsidies are present or not be understood? In the absence of wage subsidies, the delocation effect and the monopolistic distortions push for an import tariff or an export subsidy, both of which reduce the domestic price index. While tariffs do not have terms-of-trade effects, since they cannot impact on international prices, an export subsidy worsens domestic terms of trade. In this case there are no fiscal-burden-shifting effects since $\tau_{WH} = \tau_{WF} = 1$. Overall, monopolistic distortions and the delocation motive are the dominant effects, leading to import tariffs or export subsidies. However, when $\tau_{WH} = \tau_{WF} = \frac{\varepsilon - 1}{\varepsilon}$, an import subsidy or an export tax, can shift the subsidy burden to the other country by reducing the number of domestic firms and increasing the Foreign one. Thus, in the presence of first-best wage subsidies, monopolistic distortions are absent, fiscal-burden-shifting effects call for an import subsidy or an export tax, and terms-of-trade effects also push for an export tax. Taken together, these incentives dominate the delocation motive.

In order to check the intuition that the fiscal-burden-shifting effect is crucial for the difference in outcomes, we perform the following experiment.[16] Assume that wage subsidies are set at their first-best level in both countries, but subsidy costs are split evenly, independently of firms' location. Indeed, we find that in this case – which shuts down the fiscal-burden-shifting effect – $\frac{\partial V_H}{\partial \tau_{IH}} > 0$, so that unilateral policymakers have an incentive to set a tariff.[17]

Having gained intuition for the incentives from unilateral deviations, we now provide the corresponding results for Nash trade policies, when policymakers can either set import taxes or export taxes.

Proposition 3. Nash-equilibrium import tariffs/subsidies and export taxes/subsidies

(1) *Let* $\tau_{XH} = \tau_{XF} = 1$. *When starting from the free trade allocation, the Nash equilibrium entails a tariff, implying fewer varieties than the free trade allocation. In contrast, when starting from the first-best allocation implemented with wage subsidies, the Nash-equilibrium policy consists of an import subsidy, implying more varieties than the first-best allocation Formally, if* $\tau_{WH} = \tau_{WF} = 1$, *then there exists a* $\tau_I^{Nash} > 1$ *such that* $N^{Nash} < N^{FT} < N^{FB}$. *If* $\tau_{WH} = \tau_{WF} = \frac{\varepsilon - 1}{\varepsilon}$, *then* $\tau_I^{Nash} < 1$ *and* $N^{FT} < N^{FB} < N^{Nash}$.

(2) *Let* $\tau_{IH} = \tau_{IF} = 1$. *When starting from the free trade allocation the Nash equilibrium entails an export subsidy, implying more varieties than the free trade allocation. In contrast, when starting from the first-best allocation, the Nash-equilibrium policy consists of an export tax, implying fewer varieties than the first-best allocation. Formally, if* $\tau_{WH} = \tau_{WF} = 1$, *then* $\tau_X^{Nash} < 1$ *and* $N^{FT} < N^{Nash} < N^{FB}$. *If* $\tau_{WH} = \tau_{WF} = \frac{\varepsilon - 1}{\varepsilon}$, *then* $\tau_X^{Nash} > 1$ *and* $N^{Nash} < N^{FB}$.

In the absence of wage subsidies, strategic trade policy leads to import tariffs or export subsidies, which aim at exploiting the delocation motive and reducing monopolistic distortions. In the symmetric Nash equilibrium with import tariffs, no country reaches its objective since symmetric tariffs actually reduce entry in the differentiated sector in both countries. Strategic export subsidies instead do increase entry in the differentiated sector, thus getting closer to the first-best number of varieties. The result on tariffs confirms the finding of Venables

[14] All proofs for the unilateral policies with no retaliation can be found in the Appendix.
[15] In other words, in this section when we study import (export) taxes we set $\tau_{IH} = \tau_{IF} = 1$ ($\tau_{XH} = \tau_{XF} = 1$).

[16] See Appendix A.4 for the layout of the model in this case.
[17] Alternatively, one can also implement the first-best allocation with consumption subsidies equal to the inverse of the markup $\tau_C = \frac{\varepsilon - 1}{\varepsilon}$. Since consumption subsidies are independent of the location of production, they do not induce any fiscal-burden-shifting externality. In this case too we find that when we start from the first-best allocation implemented with consumption subsidies, $\frac{\partial V_H}{\partial \tau_{IH}} > 0$.

(1987) and Ossa (2011) that in the absence of other policy instruments, countries set welfare reducing import tariffs. In contrast, when monopolistic distortions have been eliminated with wage subsidies, strategic import subsidies or export taxes aiming at exploiting the fiscal-burden-shifting incentive (and in the case of export taxes also the terms-of-trade effect) are set. Again in the Nash equilibrium no country achieves its objectives. Moreover, import subsidies increase entry beyond the first-best level, while export taxes reduce it below the efficient level. We now turn to the scenario where policymakers can choose wage taxes, import taxes and export taxes simultaneously.

6. Simultaneous policy choice

In this section we allow for simultaneous strategic choice of all three policy instruments. Proposition 4 presents our main result:

Proposition 4. Nash-equilibrium policy instruments

The Nash-equilibrium policy consists of the first-best level of wage subsidies, and inefficient import subsidies and export taxes. Formally,

(1) $\tau_W^{Nash} = \tau_W^{FB} = \frac{\varepsilon - 1}{\varepsilon}, \tau_I^{Nash} < 1$ *and* $\tau_X^{Nash} > 1$.

The result that wage subsidies are set so as to completely offset monopolistic distortions is an application of the principle of targeting in public economics (Dixit, 1985). It states that an externality or distortion is best countered with a tax instrument that acts directly on the appropriate margin. The trade policy instruments are instead used to deal with the other incentives for policy intervention: the terms-of-trade effect, the delocation motive, and the fiscal-burden-shifting motive, which is a consequence of wage subsidies being in place. Import taxes address both the delocation motive and the fiscal-burden-shifting motive. The second motive dominates the first one, thus leading to an import subsidy. In the case of exports, terms of trade considerations and the fiscal-burden-shifting motive dominate over the delocation motive, thus leading to an export tax.

Finally, we consider which externalities are addressed by international trade agreements in this model. In particular, Ossa (2011) has highlighted the delocation effect as the relevant international externality solved by trade agreements when tariffs are the only available policy instrument. In contrast, Bagwell and Staiger (2009) emphasize that when both import and export taxes (but not wage subsidies) can be set strategically, the only remaining international externality is the terms-of-trade effect. To make their point they define the concept of politically optimal trade policies (see also Bagwell and Staiger, 1999). These are the levels of tax rates which non-cooperative policymakers would set in a Nash equilibrium if they did not try to manipulate international prices of individual varieties (i.e., if they disregarded the terms-of-trade effect). They show that the politically optimal trade taxes coincide with those that a cooperative policymaker, who maximizes total world welfare, would choose.

To define the concept of politically optimal taxes in our context, we follow Bagwell and Staiger (2009) and write welfare exclusively in terms of destination-level (local) prices ($p_{HH}, p_{HF}, p_{FH}, p_{FF}$) and international prices (which include transport costs, wage taxes and export taxes but not import taxes — $p_{WH} = \tau\tau_{XH}p_{HH}$ and $p_{WF} = \tau\tau_{XF}p_{FF}$). First, observe that terms-of-trade effects operate exclusively through income. Second, note that domestic income can be written as: $I_H = L + N_H f\varepsilon(\tau_{WH} - 1) + NF(\tau_{IH} - 1)\tau p_{WF}c_{HF} + N_H(p_{WH} - p_{HH})\tau c_{FH}$. Finally, define the politically optimal taxes as those maximizing indirect utility (Eq. (22)) with respect to the three policy instruments when the terms $\frac{\partial p_{WH}}{\partial \tau_{WH}}$ and $\frac{\partial p_{WH}}{\partial \tau_{XH}}$ are set equal to zero in the first-order conditions. Proposition 5 studies the welfare effects of unilateral deviations from the first-best allocation once terms-of-trade effects are not taken into consideration.

Proposition 5. Politically optimal policy instruments

The politically optimal policy is not efficient. Formally,

(1) $\Delta_{\tau_{WH}}\big|_{\tau_{WH}=\tau_{WF}=\frac{\varepsilon-1}{\varepsilon},\tau_{IH}=\tau_{IF}=1,\tau_{XH}=\tau_{XF}=1} > 0$;
(2) $\Delta_{\tau_{IH}}\big|_{\tau_{WH}=\tau_{WF}=\frac{\varepsilon-1}{\varepsilon},\tau_{IH}=\tau_{IF}=1,\tau_{XH}=\tau_{XF}=1} < 0$;
(3) $\Delta_{\tau_{XH}}\big|_{\tau_{WH}=\tau_{WF}=\frac{\varepsilon-1}{\varepsilon},\tau_{IH}=\tau_{IF}=1,\tau_{XH}=\tau_{XF}=1} > 0$;

where $\Delta_{\tau_{jH}}$ is defined as the derivative of V_H with respect to τ_{jH} when $\frac{\partial p_{WH}}{\partial \tau_{WH}} = 0$.

The derivatives of the indirect utility evaluated at the first-best levels of taxes are all different from zero, implying that the politically optimal taxes do not coincide with those which would implement the first-best allocation. This result implies that when the set of available policy instruments consists of wage taxes, import taxes and export taxes, terms-of-trade effects are not the only international externality which can be addressed by trade agreements. The fiscal-burden-shifting motive – which leads to import subsidies and export taxes – is an additional externality that needs to be addressed.

7. Conclusions

In this paper we have studied first-best and Nash trade policies in a two-sector Krugman (1980) model of intra-industry trade, considering wage, import and export taxes as policy instruments. It is common wisdom that in this model non-cooperative trade policies are set in order to try to agglomerate firms in the domestic economy, which reduces transport costs for domestic consumers thus lowering the domestic price level (*delocation motive*).

Contrary to the results of the previous literature, we show that in this model the delocation effect is not a dominating motive for non-cooperative trade policy choices once policymakers are allowed to use wage, import and export taxes strategically. Instead, they are driven by the desire to eliminate *monopolistic distortions*, by the *terms-of-trade externality* and by the *fiscal-burden-shifting externality*. Indeed, due to monopolistic competition, in the free trade equilibrium there are too few firms in the differentiated sector and this affects policymakers' incentives in a crucial way. Thus, when wage taxes are available, non-cooperative policymakers increase efficiency by setting wage subsidies. However, these subsidies are lower than the first-best ones due to negative terms-of-trade effects. When only import (export) tax instruments are available, non-cooperative policymakers use tariffs (export subsidies), which reduce the domestic price level through entry of firms in the domestic economy; thereby policymakers mitigate monopolistic distortions and exploit the delocation effect. However, once monopolistic distortions have been offset by appropriate wage subsidies, results reverse: policymakers set import subsidies (export taxes), which shift the fiscal burden of the wage subsidy to the other country and – in the case of export taxes – also improve domestic terms of trade. Finally, when policymakers can set all three policy instruments simultaneously, they choose to set wage subsidies, which exactly offset monopolistic distortions. Moreover, they set import subsidies and export taxes, which aim at shifting the subsidy burden and improving domestic terms of trade. The implications of our findings are important: in the Krugman (1980) model, both terms-of-trade externalities and fiscal-burden-shifting effects are reasons why countries need to sign trade agreements. Our result also shows that when all three policy instruments are available, the Krugman (1980) model cannot rationalize why countries would set import tariffs and export subsidies in the absence of trade agreements. In reality, of course, there may be other reasons why non-cooperative policy makers may want to set tariffs – such as political economy motives for protection (see, e.g., Grossman and Helpman,

1994) – which are not present in the model. We leave such an analysis for future research.[18]

Acknowledgments

We would especially like to thank Miklós Koren for the helpful suggestions. We also thank Marius Brülhart, Gino Gancia, Jordi Galí, Luisa Lambertini, Giovanni Maggi and Ralph Ossa as well as the seminar participants at the University of Lausanne, University of Nottingham, University of Vienna, Central European University, Magyar Nemzeti Bank and participants at the 2009 SSES meeting, the 2009 SED meeting, the 2009 ETSG conference, the 2009 FIW conference and the 2010 ITSG conference for the helpful discussions. Chiara Forlati gratefully acknowledges financial support from the Swiss National Science Foundation [CRSI11-133058]. The views expressed here are solely our own and do not necessarily reflect those of the Hungarian National Bank.

Appendix A. Equilibrium

A.1. Equilibrium allocation and prices

Substituting the optimal pricing rules (8) and (9) into the definition of Home (Eq. (4)) (and Foreign) aggregate price indices we obtain:

$$P_H = \frac{\varepsilon}{\varepsilon-1}\left[N_H\tau_{WH}^{1-\varepsilon} + N_F(\tau_{IH}\tau_{XF}\tau\tau_{WF})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{23}$$

$$P_F = \frac{\varepsilon}{\varepsilon-1}\left[N_F\tau_{WF}^{1-\varepsilon} + N_H(\tau_{IF}\tau_{XH}\tau\tau_{WH})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Combining the market clearing condition (15) with the analogous one for Foreign and substituting out the expressions for the prices (Eq. (23)), gives:

$$C_H = \frac{fP_H^{-\varepsilon}(\varepsilon-1)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}\tau^{\varepsilon}\left[-\tau\tau_{WF}^{\varepsilon} + (\tau\tau_{WH}\tau_{IF}\tau_{XH})^{\varepsilon}\right](\tau_{IH}\tau_{XF})^{\varepsilon}}{\tau^{2\varepsilon}(\tau_{IF}\tau_{XH}\tau_{IH}\tau_{XF})^{\varepsilon}-\tau^2} \tag{24}$$

$$C_F = \frac{fP_F^{-\varepsilon}(\varepsilon-1)\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon}\tau^{\varepsilon}\left[-\tau\tau_{WH}^{\varepsilon} + (\tau\tau_{WF}\tau_{IH}\tau_{XF})^{\varepsilon}\right](\tau_{IF}\tau_{XH})^{\varepsilon}}{\tau^{2\varepsilon}(\tau_{IF}\tau_{XH}\tau_{IF}\tau_{XF})^{\varepsilon}-\tau^2}. \tag{25}$$

Using the trade balance condition (18), the labor market clearing condition (17), the equivalent equations for Foreign, and the expressions for C_H, C_F, P_H and P_F just derived, we obtain the following system of equations in N_H and N_F:

$$A_{1H}N_H + A_{2H}N_F - L = 0 \tag{26}$$

$$A_{2F}N_H + A_{1F}N_F - L = 0. \tag{27}$$

The solution to this system is:

$$N_H = \frac{L(A_{2H}-A_{1F})}{A_{2F}A_{2H}-A_{1H}A_{1F}} \qquad N_F = \frac{L(A_{2F}-A_{1H})}{A_{2F}A_{2H}-A_{1H}A_{1F}} \tag{28}$$

where:

$$A_{1H} = \frac{f\varepsilon\tau_{WH}^{-\varepsilon}\tau^{2\varepsilon}(\tau_{WH}\tau_{IH}\tau_{IF}\tau_{XH}\tau_{XF})^{\varepsilon}(\alpha+(1-\alpha)\tau_{WH})}{\alpha(\tau^{2\varepsilon}(\tau_{IH}\tau_{IF}\tau_{XH}\tau_{XF})^{\varepsilon}-\tau^2)} + \frac{f\varepsilon\tau_{WH}^{-\varepsilon}\tau\left[\alpha\tau\tau_{WH}^{\varepsilon}(\tau_{WH}\tau_{XH}-1)-\tau_{WH}(\tau\tau_{WF}\tau_{IH}\tau_{XF})^{\varepsilon}(1-\alpha+\alpha\tau_{XH})\right]}{\alpha(\tau^{2\varepsilon}(\tau_{IH}\tau_{IF}\tau_{XH}\tau_{XF})^{\varepsilon}-\tau^2)} \tag{29}$$

$$A_{2H} = \frac{f\varepsilon\tau\tau_{XF}\tau_{WF}^{1-\varepsilon}(-\alpha-(1-\alpha)\tau_{IH})\left[\tau\tau_{WF}^{\varepsilon}-(\tau\tau_{WH}\tau_{IF}\tau_{XH})^{\varepsilon}\right]}{\alpha(\tau^{2\varepsilon}(\tau_{IH}\tau_{IF}\tau_{XH}\tau_{XF})^{\varepsilon}-\tau^2)} \tag{30}$$

$$A_{1F} = \frac{f\varepsilon\tau_{WF}^{-\varepsilon}\tau^{2\varepsilon}(\tau_{WF}\tau_{IH}\tau_{IF}\tau_{X}\tau_{XF})^{\varepsilon}(\alpha+(1-\alpha)\tau_{WF})}{\alpha(\tau^{2\varepsilon}(\tau_{IH}\tau_{IF}\tau_{XH}\tau_{XF})^{\varepsilon}-\tau^2)} + \frac{f\varepsilon\tau_{WF}^{-\varepsilon}\tau\left[\alpha\tau\tau_{WF}^{\varepsilon}(\tau_{WF}\tau_{XF}-1)-\tau_{WF}(\tau\tau_{WH}\tau_{IF}\tau_{XH})^{\varepsilon}(1-\alpha+\alpha\tau_{XF})\right]}{\alpha(\tau^{2\varepsilon}(\tau_{IH}\tau_{IF}\tau_{XH}\tau_{XF})^{\varepsilon}-\tau^2)} \tag{31}$$

$$A_{2F} = \frac{f\varepsilon\tau\tau_{XH}\tau_{WH}^{1-\varepsilon}(-\alpha-(1-\alpha)\tau_{IF})\left[\tau\tau_{WH}^{\varepsilon}-(\tau\tau_{WF}\tau_{IH}\tau_{XF})^{\varepsilon}\right]}{\alpha(\tau^{2\varepsilon}(\tau_{IH}\tau_{IF}\tau_{XH}\tau_{XF})^{\varepsilon}-\tau^2)}. \tag{32}$$

A.2. Free trade allocation

Let $\tau_{WH} = \tau_{WF} = \tau_{IH} = \tau_{IF} = \tau_{XH} = \tau_{XF} = 1$. Then Eq. (28) simplifies to:

$$N_H = N_F = \frac{\alpha L}{\varepsilon f} \equiv N^{FT}. \tag{33}$$

A.3. Some definitions

For some of the proofs we find it useful to define the following consumption indices:

$$C_{HH} = \left[\int_0^{N_H} c_{HH}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad C_{HF} = \left[\int_0^{N_F} c_{HF}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{34}$$

Note that $C_H = \left[C_{HH}^{\frac{\varepsilon-1}{\varepsilon}} + C_{HF}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$ gives us back consumption as defined in Eq. (2). Solving the standard expenditure minimization problem we obtain:

$$c_{HH}(i) = \left[\frac{p_{HH}(i)}{P_{HH}}\right]^{-\varepsilon}C_{HH} \qquad C_{HH} = \left[\frac{P_{HH}}{P_H}\right]^{-\varepsilon}C_H \tag{35}$$

$$c_{HF}(i) = \left[\frac{p_{HF}(i)}{P_{HF}}\right]^{-\varepsilon}C_{HF} \qquad C_{HF} = \left[\frac{P_{HF}}{P_H}\right]^{-\varepsilon}C_H \tag{36}$$

where again P_H coincide with the one defined in Eq. (4):

$$P_H = \left[P_{HH}^{1-\varepsilon} + P_{HF}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{37}$$

$$P_{HH} = \left[\int_0^{N_H} p_{HH}(i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}} \qquad P_{HF} = \left[\int_0^{N_F} p_{HF}(i)^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}}. \tag{38}$$

A similar thing can be done for Foreign consumption (C_{FF} and C_{FH}) and price (P_{FF} and P_{FH}) indices.

A.4. Fiscal-burden-shifting model

In this section we eliminate the fiscal-burden-shifting motive by assuming that the wage subsidy is implemented by a central planner who shares the global cost of the subsidy equally among the two countries, independently of where firms are located.

[18] In particular, in their seminal paper Grossman and Helpman (1994) explain that lobby groups have an interest in setting institutions which limit the choice of instruments available for redistribution to the most distortive ones (e.g., trade taxes) even when less distortive instruments (such as production subsidies) are in principle available, because more distortive instruments maximize the political power of lobby groups.

The only two equations affected by this change are the income and the trade balance. All the other first-order and equilibrium conditions are unchanged. The global cost of implementing the wage subsidy is given by:

$$T_{world} = (\tau_{WH}-1)N_H(y_H+f)+(\tau_{WF}-1)N_F(y_F+f). \quad (39)$$

Recalling that $y_H = y_F = (\varepsilon - 1)f$ therefore, we can rewrite the global cost of the subsidy as:

$$T_{world} = \varepsilon f[(\tau_{WH}-1)N_H+(\tau_{WF}-1)N_F]. \quad (40)$$

If T_{world} is equally split between the two countries, the Home income can be written as:

$$I_H = L+(\tau_{IH}-1)\tau_{XF}\tau N_F p_{FF} c_{HF}+(\tau_{XH}-1)\tau N_H p_{HH} c_{FH} + \frac{\varepsilon f}{2}[(\tau_{WH}-1)N_H+(\tau_{WF}-1)N_F]. \quad (41)$$

Note that it is convenient to rewrite income in the following way:

$$I_H = L+(\tau_{IH}-1)\tau_{XF}\tau N_F p_{FF} c_{HF}+(\tau_{XH}-1)\tau N_H p_{HH} c_{FH}+(\tau_{WH}-1)N_H\varepsilon f + \frac{\varepsilon f}{2}(\tau_{WF}-1)N_F - \frac{\varepsilon f}{2}(\tau_{WH}-1)N_H. \quad (42)$$

so that the first line corresponds to the definition of income for the baseline model, and the second line contains the transfers between Home and Foreign taking place in this new version.

Given the new income, we can derive the new balanced trade condition:

$$(Q_{ZH}-Z_H)+\tau\tau_{XH}N_H p_{HH} c_{FH} = \tau\tau_{XF}N_F p_{FF} c_{HF} + \frac{\varepsilon f}{2}(\tau_{WH}-1)N_H - \frac{\varepsilon f}{2}(\tau_{WF}-1)N_F. \quad (43)$$

Appendix B. The planner's problem

Proposition 1. The first-best allocation and its implementation

(1) *The first-best allocation entails the same firm size but more varieties than >the free trade allocation. Formally,* $y^{FB} = f(\varepsilon-1) = y^{FT}$ *and* $N^{FB} = \frac{\alpha L}{(\varepsilon-1+\alpha)f} > N^{FT} = \frac{\alpha L}{\varepsilon f}$.

(2) *The first-best allocation can be implemented by setting wage subsidies equal to the inverse of the markup and choosing trade taxes such that* $\tau_I^{FB}\cdot\tau_X^{FB} = 1$. *Formally,* $\tau_W^{FB} = \frac{\varepsilon-1}{\varepsilon}, \tau_I^{FB}\cdot\tau_X^{FB} = 1$ *and* $N = N^{FB}$.

Proof of Proposition 1.

(1) The Lagrangian for the planner's problem is:

$$\mathcal{L} = \left[\int_0^{N_H} c_{HH}(i)^{\frac{\varepsilon-1}{\varepsilon}}di + \int_0^{N_H} c_{HF}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon\alpha}{\varepsilon-1}} Z_H^{1-\alpha} + \left[\int_0^{N_F} c_{FH}(i)^{\frac{\varepsilon-1}{\varepsilon}}di + \int_0^{N_F} c_{FF}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon\alpha}{\varepsilon-1}} Z_F^{1-\alpha} + \int_0^{N_H}\lambda_1(i)[L_{CH}(i)-f-c_{HH}(i)-\tau c_{FH}(i)]di + \int_0^{N_F}\lambda_2(i)[L_{CF}(i)-f-c_{FF}(i)-\tau c_{HF}(i)]di + \lambda_3\left[L_H+L_F-\int_0^{N_H}L_{CH}(i)di-\int_0^{N_F}L_{CF}(i)di-Z_H-Z_F\right].$$

The first-order conditions are:

$$\frac{\partial\mathcal{L}}{\partial c_{HH}(i)} = 0: \alpha C_H^{\alpha}\left[\int_0^{N_H} c_{HH}(i)^{\frac{\varepsilon-1}{\varepsilon}}di + \int_0^{N_F} c_{HF}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{-1} Z_H^{1-\alpha} c_{HH}(i)^{\frac{-1}{\varepsilon}} = \lambda_1(i) \quad (44)$$

$$\frac{\partial\mathcal{L}}{\partial c_{HF}(i)} = 0: \alpha C_H^{\alpha}\left[\int_0^{N_H} c_{HH}(i)^{\frac{\varepsilon-1}{\varepsilon}}di + \int_0^{N_F} c_{HF}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{-1} Z_H^{1-\alpha} c_{HF}(i)^{\frac{-1}{\varepsilon}} = \tau\lambda_2(i) \quad (45)$$

$$\frac{\partial\mathcal{L}}{\partial Z_H} = 0: (1-\alpha)C_H^{\alpha}Z_H^{-\alpha} = \lambda_3 \quad (46)$$

$$\frac{\partial\mathcal{L}}{\partial L_{CH}(i)} = 0: \lambda_1(i) = \lambda_3 \quad (47)$$

$$\frac{\partial\mathcal{L}}{\partial N_H} = 0: \alpha\frac{\varepsilon}{\varepsilon-1}\left\{C_H^{\alpha}Z_H^{1-\alpha}\left[\int_0^{N_H} c_{HH}(i)^{\frac{\varepsilon-1}{\varepsilon}}di + \int_0^{N_F} c_{HF}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{-1} c_{HH}(N_H)^{\frac{\varepsilon-1}{\varepsilon}} + C_F^{\alpha}Z_F^{1-\alpha}\left[\int_0^{N_H} c_{FH}(i)^{\frac{\varepsilon-1}{\varepsilon}}di + \int_0^{N_F} c_{FF}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{-1} c_{FH}(N_H)^{\frac{\varepsilon-1}{\varepsilon}}\right\} = \lambda_3 L_{CH}(N_H), \quad (48)$$

where in the last condition we have already used the fact that $\lambda_1(N_H)[L_{CH}(N_H) - f - c_{HH}(N_H) - \tau c_{FH}(N_H)] = 0$.

The first-order conditions with respect to Foreign variables are completely symmetric and are thus omitted for the sake of space. By imposing symmetry we find $\lambda_1(i) = \lambda_2(i)$. Combining Eqs. (44) and (45) we obtain:

$$c_{HF}(i) = c_{HH}(i)\tau^{-\varepsilon}. \quad (49)$$

Combining Eqs. (44), (47) and (48) we obtain:

$$\frac{\varepsilon}{\varepsilon-1}\left[c_{HH}(i)^{\frac{\varepsilon-1}{\varepsilon}} + c_{HF}(i)^{\frac{\varepsilon-1}{\varepsilon}}\right] = L_{CH}(i)c_{HH}(i)^{\frac{1}{\varepsilon}}. \quad (50)$$

Combining Eqs. (49) and (50), we obtain:

$$c_{HH}(i) = \frac{\varepsilon}{\varepsilon-1}\left[1+\tau^{1-\varepsilon}\right]^{-1}. \quad (51)$$

Substituting the expression for $c_{HH}(i)$ and $c_{HF}(i)$ into the resource condition for domestic varieties $L_{CH}(i) = f + c_{HH}(i) + \tau c_{FH}(i)$, we get $L_{CH}(i) = \varepsilon f$ and using the production function $y_H(i) = L_{CH}(i) - f$ we obtain $y^{FB} = (\varepsilon - 1)f$. Moreover, $c_{HH}^{FB}(i) = (\varepsilon - 1)f[1 + \tau^{1-\varepsilon}]^{-1}$ and $c_{HF}^{FB}(i) = (\varepsilon - 1)f\tau^{-\varepsilon}[1 + \tau^{1-\varepsilon}]^{-1}$.

Using the resource condition for Z_H, we obtain $Z_H = L - N_H\varepsilon f$.

Finally, combining Eqs. (44), (46) and (47):

$$(1-\alpha)C_H^{\frac{\varepsilon-1}{\varepsilon}} = \alpha Z_H c_{HH}(i)^{-\frac{1}{\varepsilon}}. \quad (52)$$

Substituting the expressions for Z_H, C_H, $c_{HH}^{FB}(i)$ and $c_{HF}^{FB}(i)$ into Eq. (52), we can solve for $N_H = N_F \equiv N^{FB} = \frac{\alpha L}{f(\varepsilon+\alpha-1)}$.

(2) In the symmetric equilibrium where $\tau_W^{FB} = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_I^{FB}\tau_X^{FB} = 1$, policymakers exactly eliminate the price markup charged by the monopolistic firms in the differentiated sector. Indeed, from Eq. (8) we see that individual domestic varieties are now priced at their marginal costs i.e., $p_{HH}(i) = 1$ and $p_{FH}(i) = \tau$, and the same holds for the Foreign country. Substituting $\tau_W^{FB} = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_I^{FB}\tau_X^{FB} = 1$ into Eq. (28), we obtain $N_H = N_F = \frac{\alpha L}{f(\varepsilon-1+\alpha)} \equiv N^{FB}$. Intuitively, if $\tau_W^{FB} = \frac{\varepsilon-1}{\varepsilon}$, any policy such that $\tau_I^{FB}\tau_X^{FB} = 1$ allows the social optimum to be reached since the effects of import tariffs/subsidies are exactly offset by those of export subsidies/taxes. ■

Appendix C. Wage taxes

In this section we set $\tau_{IH} = \tau_{IF} = \tau_{XH} = \tau_{XF} = 1$.

Lemma A1. Unilaterally set wage subsidies

The optimal unilateral deviation entails a reduction in the wage subsidy when starting from the efficient allocation. When starting from the free trade allocation, the optimal unilateral deviation entails a wage subsidy. Formally,

(1) *If* $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}, \frac{\partial V_H}{\partial \tau_{WH}} > 0.$

(2) *If* $\tau_{WH} = \tau_{WF} = 1, \frac{\partial V_H}{\partial \tau_{WH}} < 0.$

Proof of Lemma A1.

(1) If $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$, then:

$$\frac{\partial V_H}{\partial \tau_{WH}} = \frac{\alpha\varepsilon^2\tau(\tau^\varepsilon+\tau)}{(\varepsilon-1)(\tau^\varepsilon-\tau)(\alpha(\tau+\tau^\varepsilon)+(\varepsilon-1)(\tau^\varepsilon-\tau))} > 0.$$

(2) If $\tau_{WH} = \tau_{WF} = 1$, then:

$$\frac{\partial V_H}{\partial \tau_{WH}} = -\frac{\alpha((1-\alpha)\tau^\varepsilon+\tau(\alpha+\varepsilon-1))}{(\varepsilon-1)(\tau^\varepsilon-\tau)} < 0. \quad (53)$$

■

Proposition 2. Nash-equilibrium wage subsidies

In the Nash equilibrium both countries set a wage subsidy. However, this subsidy is smaller than the one needed to implement the first-best allocation. The equilibrium number of varieties is larger than in the free trade allocation, but lower than the first-best level. Formally,

(1) $\tau_W^{FB} < \tau_W^{Nash} < 1$ *and* $N^{FT} < N^{Nash} < N^{FB}$.

Proof of Proposition 2. First, we prove that $\tau_W^{FB} < \tau_W^{Nash} < 1$. The Nash solution of this game will be symmetric due to the symmetry assumption for the two countries. Therefore, to derive τ_W^{Nash} it is enough to compute the best reply of Home, $\frac{\partial V_H(P_H(\tau_{WH},\tau_{WF}), I_H(\tau_{WH},\tau_{WF}))}{\partial \tau_{WH}} = 0$, and then impose symmetry, i.e., $\tau_{WH} = \tau_{WF} = \tau_W$. Here, $P_H(\tau_{WH}, \tau_{WF})$ is given by Eq. (23), which is implied by the equilibrium expressions for $N_H(\tau_{WH}, \tau_{WF})$ and $N_F(\tau_{WH}, \tau_{WF})$, Eq. (28). Moreover, $I_H(\tau_{WH}, \tau_{WF})$ is given by $L + (\tau_{WH} - 1)\varepsilon f N_H(\tau_{WH}, \tau_{WF})$. In so doing, we obtain a quadratic expression in τ_W^{Nash}:

$$a\left(\tau_W^{Nash}\right)^2 + b\tau_W^{Nash} + c = 0 \quad (54)$$

where $a \equiv \alpha(1-\alpha)\varepsilon\tau^\varepsilon[(3-2\varepsilon-\alpha)\tau-(1-\alpha)\tau^\varepsilon]$, $b \equiv \alpha[(\varepsilon-1+\alpha)\tau^2+(1-\alpha)(\varepsilon-1-\alpha(2\varepsilon-1))\tau^{2\varepsilon}+(2\varepsilon-2+\alpha)(\varepsilon-1-\alpha(2\varepsilon-1))\tau^{1+\varepsilon}]$ and $c \equiv \alpha^2(\varepsilon-1)\tau^\varepsilon((2\varepsilon-1+\alpha)\tau+(1-\alpha)\tau^\varepsilon)$.

Note that $a < 0$ and $c > 0$. To prove that $a < 0$ it suffices to see that:

(i) $\tau^\varepsilon > \tau \; \forall \, \varepsilon > 1$ and $\forall \, \tau > 1$;
(ii) $1-\alpha > 3-2\varepsilon-\alpha \; \forall \, \varepsilon > 1$.

Hence, Eq. (54) has two real solutions, one positive and one negative $\forall \, \varepsilon > 1, \alpha \in (0, 1)$ and $\tau > 1$. Then, since $\tau_W^{Nash} \in [0, \infty)$, Eq. (54) implies that the Nash solution always exists and is unique. As a consequence:

(i) At $\tau_W = 1$ we have: $a\tau_W^2 + b\tau_W + c = -\alpha(\tau^\varepsilon-\tau)[(\varepsilon+\alpha-1)\tau + (1-\alpha)\tau^\varepsilon] < 0$, implying that $\tau_W^{Nash} < 1$ since $a < 0$.

(ii) At $\tau_W^{FB} = \frac{\varepsilon-1}{\varepsilon}$ we have: $a\tau_W^2 + b\tau_W + c = \frac{\alpha(\varepsilon-1)(\varepsilon+\alpha-1)\tau(\tau+\tau^\varepsilon)}{\varepsilon} > 0$, implying $\tau_W^{FB} < \tau_W^{Nash}$.

Second, we show that $N^{FT} < N^{Nash} < N^{FB}$. This follows from $\tau_W^{FB} < \tau_W^{Nash} < 1$ and $dN_H > 0$ when $d\tau_W < 0$. Indeed, in the symmetric equilibrium:

$$dN_H = \frac{\partial N_H}{\partial \tau_{WH}} d\tau_{WH} + \frac{\partial N_H}{\partial \tau_{WF}} d\tau_{WF} = \left(\frac{\partial N_H}{\partial \tau_{WH}}\Big|_{\tau_{WH}=\tau_W} + \frac{\partial N_F}{\partial \tau_{WF}}\Big|_{\tau_{WF}=\tau_W}\right) d\tau_W = -\frac{L(1-\alpha)\alpha}{f\varepsilon[\alpha-(\alpha-1)\tau_W]^2} d\tau_W \quad (55)$$

■

Appendix D. Import and export taxes

Lemma A2. Unilaterally set import tariffs/subsidies

Let $\tau_{IH} = \tau_{IF} = \tau_{XH} = \tau_{XF} = 1$. *The optimal unilateral deviation entails an import subsidy when starting from the first-best allocation implemented by a wage subsidy, and an import tariff when starting from the free trade allocation. Formally,*

(1) *If* $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$, *then* $\frac{\partial V_H}{\partial \tau_{IH}} < 0$.
(2) *If* $\tau_{WH} = \tau_{WF} = 1$, *then* $\frac{\partial V_H}{\partial \tau_{IH}} > 0$.

Proof of Lemma A2.

(1) If $\tau_{IH} = \tau_{IF} = 1$ and $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$, it is easy to show that:

$$\frac{\partial V_H}{\partial \tau_{IH}} = -\frac{\alpha\tau^2((\alpha+2\varepsilon-1)\tau^\varepsilon+(1-\alpha)\tau)}{((\alpha(\tau^\varepsilon+\tau)+(\varepsilon-1)(\tau^\varepsilon-\tau))(\tau^{2\varepsilon}-\tau^2)} < 0.$$

(2) If $\tau_{IH} = \tau_{IF} = 1$ and $\tau_{WH} = \tau_{WF} = 1$, it is easy to show that:

$$\frac{\partial V_H}{\partial \tau_{IH}} = \frac{\alpha\tau((\alpha+\varepsilon-1)\tau^\varepsilon+(1-\alpha)\tau)}{(\varepsilon-1)(\tau^{2\varepsilon}-\tau^2)} > 0.$$

■

Lemma A3. Unilaterally set export taxes/subsidies

Let $\tau_{IH} = \tau_{IF} = \tau_{XH} = \tau_{XF} = 1$. *The optimal unilateral deviation entails an export tax when starting from the first-best allocation implemented by a wage subsidy, and an export subsidy when starting from the free trade allocation. Formally,*

(1) *If* $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$, *then* $\frac{\partial V_H}{\partial \tau_{XH}} > 0$.
(2) *If* $\tau_{WH} = \tau_{WF} = 1$, *then* $\frac{\partial V_H}{\partial \tau_{XH}} < 0$.

Proof of Lemma A3.

(1) If $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$, then:

$$\frac{\partial V_H}{\partial \tau_{XH}} = \frac{\alpha\tau\left((1-\alpha)\tau^{\varepsilon+1}+(\alpha+\varepsilon-1)\tau^{2\varepsilon}+\varepsilon\tau^2\right)}{(\tau^{2\varepsilon}-\tau^2)(\alpha(\tau^\varepsilon+\tau)+(\varepsilon-1)(\tau^\varepsilon-\tau))} > 0$$

(2) If $\tau_{WH} = \tau_{WF} = 1$, then:

$$\frac{\partial V_H}{\partial \tau_{XH}} = -\frac{\alpha\tau\left(\tau(\alpha+\varepsilon-1)+(1-\alpha)\tau^{\varepsilon}\right)}{(\varepsilon-1)\left(\tau^{2\varepsilon}-\tau^2\right)} < 0$$

■

Lemma A4. Unilaterally set import tariffs when FBS motive is absent

Let $\tau_{IH} = \tau_{IF} = \tau_{XH} = \tau_{XF} = 1$ and $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$. When the cost of the wage subsidy is equally shared between Home and Foreign, the optimal unilateral deviation entails an import tariff. Formally,

(1) *Let income be defined as in (42), and trade balance be defined as in (43). Then, $\frac{\partial V_H}{\partial \tau_{IH}} > 0$.*

Proof of Lemma A4.

(1) If $\tau_{IH} = \tau_{IF} = 1$ and $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$, it is easy to show that:

$$\frac{\partial V_H}{\partial \tau_{IH}} = \frac{\alpha\tau\left((1-\alpha)\tau+(\alpha+2\varepsilon-1)\tau^{\varepsilon}\right)}{2(\varepsilon-1)\left(\tau^{2\varepsilon}-\tau^2\right)} > 0.$$

■

Proposition 3. Nash-equilibrium import tariffs/subsidies and export taxes/subsidies

(1) *Let $\tau_{XH} = \tau_{XF} = 1$. When starting from the free trade allocation, the Nash equilibrium entails a tariff, implying fewer varieties than the free trade allocation. In contrast, when starting from the first-best allocation implemented with wage subsidies, the Nash-equilibrium policy consists of an import subsidy, implying more varieties than the first-best allocation Formally, if $\tau_{WH} = \tau_{WF} = 1$, then there exists a $\tau_I^{Nash} > 1$ such that $N^{Nash} < N^{FT} < N^{FB}$. If $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$, then $\tau_I^{Nash} < 1$ and $N^{FT} < N^{FB} < N^{Nash}$.*

(2) *Let $\tau_{IH} = \tau_{IF} = 1$. When starting from the free trade allocation the Nash equilibrium entails an export subsidy, implying more varieties than the free trade allocation. In contrast, when starting from the first-best allocation, the Nash-equilibrium policy consists of an export tax, implying less varieties than the first-best allocation. Formally, if $\tau_{WH} = \tau_{WF} = 1$, then $\tau_X^{Nash} < 1$ and $N^{FT} < N^{Nash} < N^{FB}$. If $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$, then $\tau_X^{Nash} > 1$ and $N^{Nash} < N^{FB}$.*

Proof of Proposition 3.

(1) Let $\tau_{XH} = \tau_{XF} = 1$ In the case of tariffs, the non-cooperative policymaker maximizes:

$$\max_{\tau_{IH}} V_H(P_H(\tau_{IH},\tau_{IF}), I_H(\tau_{IH},\tau_{IF})) \tag{56}$$

where $P_H(\tau_{IH}, \tau_{IF})$ is given by Eq. (23) once we substitute in $N_H(\tau_{IH}, \tau_{IF})$ and $N_F(\tau_{IH}, \tau_{IF})$ as implicitly determined by Eq. (19). $I_H(\tau_{IH}, \tau_{IF})$ is equal to $L + (\tau_{WH} - 1)N_H(\tau_{IH}, \tau_{IF})\varepsilon f + (\tau_{IH} - 1)\tau P_{FF}(\tau_{IH}, \tau_{IF}) C_{HF}(\tau_{IH}, \tau_{IF})$ where $P_{FF}(\tau_{IH},\tau_{IF}) = \frac{\varepsilon}{\varepsilon-1}\tau_{WF}(N_F(\tau_{IH},\tau_{IF}))^{\frac{1}{1-\varepsilon}}$, $C_{HF}(\tau_{IH}, \tau_{IF}) = P_{HF}(\tau_{IH}, \tau_{IF})^{-\varepsilon} P_H(\tau_{IH}, \tau_{IF})^{\varepsilon} C_H(\tau_{IH}, \tau_{IF})$, $P_{HF}(\tau_{IH},\tau_{IF}) = \frac{\varepsilon}{\varepsilon-1}\tau\tau_{IH} \tau_{WF}(N_F(\tau_{IH},\tau_{IF}))^{\frac{1}{1-\varepsilon}}$ and finally $C_H(\tau_{IH}, \tau_{IF})$ is given by its equilibrium value in Eq. (24).

(I) By taking the derivative of Eq. (56) with respect to τ_{IH} and imposing symmetry i.e., $\tau_{IH} = \tau_{IF} = \tau_I$, the first-order condition evaluated at $\tau_{WH} = \tau_{WF} = 1$ can be written as:

$$\frac{A_I^{Nash}(\tau_I)}{B_I^{Nash}(\tau_I)} = 0$$

where:

$$\begin{aligned}
A_I^{Nash}(\tau_I) \equiv\ & \alpha\Big(\tau^{2\varepsilon+3}\tau_I^{2\varepsilon}\Big(\tau_I\Big((\alpha-1)(\varepsilon+1)\tau_I(\alpha+\varepsilon-1)-\alpha^2(2\varepsilon+1)\\
& -2\alpha(\varepsilon-1)\varepsilon+(\varepsilon-1)\varepsilon+1)\\
& +\alpha\varepsilon(\alpha+\varepsilon-1))+\varepsilon\tau^{\varepsilon+4}((\alpha-1)\tau_I-\alpha)(\varepsilon\tau_I-\varepsilon+1)\tau_I^{\varepsilon}\\
& -\varepsilon\tau^{3\varepsilon+2}\tau_I^{3\varepsilon}(\tau_I(\alpha+\varepsilon-1)-\alpha-\varepsilon)\\
& +(-\alpha-\varepsilon+1)\tau^{4\varepsilon+1}((\varepsilon-1)\tau_I-\varepsilon)\tau_I^{4\varepsilon}-(\alpha-1)\tau^5\tau_I((\alpha-1)\tau_I-\alpha)(\varepsilon\tau_I-\varepsilon+1))\\
B_I^{Nash}(\tau_I) \equiv\ & (\varepsilon-1)\tau_I(\tau^{\varepsilon}\tau_I^{\varepsilon}+\tau\tau_I)\\
& \left(\tau^{2\varepsilon}\tau_I^{2\varepsilon}-\tau^2\right)((\alpha-1)\tau\tau_I+\tau^{\varepsilon}\tau_I^{\varepsilon}-\alpha\tau)(\tau_I(\tau-\alpha\tau)+\tau^{\varepsilon}\tau_I^{\varepsilon}+\alpha\tau).
\end{aligned}$$

We need to show that: (i) there exist at least one Nash equilibrium of the policy game for which $\tau_I^{Nash} > 1$; (ii) for such a $\tau_I^{Nash} > 1$, we have $N^{Nash} < N^{FT} < N^{FB}$:

(i) To show this point considers that:
  (a) $A_I^{Nash}(\tau_I)$ is a continuous function of τ_I;
  (b) If $\tau_I = 1, A_I^{Nash} = \tau(\tau^{\varepsilon} - \tau)(\tau^{\varepsilon} + \tau)^2\left[(\alpha + \varepsilon - 1)\tau^{\varepsilon} - \alpha\tau + \tau\right] > 0$;
  (c) If $\tau_I = \frac{\varepsilon}{\varepsilon-1}$:

$$\begin{aligned}
A_I^{Nash}\left(\frac{\varepsilon}{\varepsilon-1}\right) =\ & -\frac{\varepsilon\tau^2}{(\varepsilon-1)^3}\Big[(\varepsilon-1)\tau(\alpha(\varepsilon-\alpha)+\alpha(1-\alpha)\\
& +(\varepsilon-1)(2\varepsilon-1))\left(\frac{\varepsilon\tau}{\varepsilon-1}\right)^{2\varepsilon}+(1-\alpha)\\
& \times(2\varepsilon-1)\tau^3(\varepsilon-\alpha)+(2\varepsilon-1)\\
& \times(\varepsilon-1)\tau^2(\varepsilon-\alpha)\left(\frac{\varepsilon\tau}{\varepsilon-1}\right)^{\varepsilon}+\alpha(\varepsilon-1)^{2-3\varepsilon}(\varepsilon\tau)^{3\varepsilon}\Big] < 0.
\end{aligned}$$

Therefore, by the intermediate value theorem there exists a $\tau_I^{Nash} \in \{1, \frac{\varepsilon}{\varepsilon-1}\}$ such that $A_I^{Nash}(\tau_I^{Nash}) = 0$.

(ii) To prove this statement note that if $\tau_I < \frac{\varepsilon}{\varepsilon-1}$ then:

$$\left.\frac{\partial N_H}{\partial \tau_{IH}}\right|_{\tau_{IH}=\tau_I} + \left.\frac{\partial N_H}{\partial \tau_{IF}}\right|_{\tau_{IF}=\tau_I} = -\frac{L(1-\alpha)\alpha\tau\left((\varepsilon(1-\tau_I)+\tau_I)\tau^{\varepsilon}\tau_I^{\varepsilon}+\tau\tau_I\right)}{f\varepsilon\tau_I\left(\tau_I\tau(1-\alpha)+\tau^{\varepsilon}\tau_I^{\varepsilon}+\alpha\tau\right)^2} < 0.$$

Then at the symmetric equilibrium, $dN_H = \frac{\partial N_H}{\partial \tau_{IH}} d\tau_{IH} + \frac{\partial N_H}{\partial \tau_{IF}} d\tau_{IF} = \left(\frac{\partial N_H}{\partial \tau_{IH}} + \frac{\partial N_H}{\partial \tau_{IF}}\right) d\tau_{IH} > 0$ for all $\tau_I < \frac{\varepsilon}{\varepsilon-1}$ and $d\tau_{IH} = d\tau_{IF} < 0$.

Hence, from (i) we can be sure that there exists a solution $\tau_I^{Nash} \in \{1, \frac{\varepsilon}{\varepsilon-1}\}$ such that $N^{Nash} < N^{FT} < N^{FB}$.

(II) If $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_{IH} = \tau_{IF}$, the first-order condition of (56) with respect to τ_{IH} can be written as:

$$\frac{A_I^{Nash}(\tau_I)}{B_I^{Nash}(\tau_I)} = 0$$

where:

$$\begin{aligned}
A_I^{Nash}(\tau_I) \equiv\ & \alpha(\varepsilon-2)\varepsilon^2\tau^{\varepsilon+3}\tau_I^{\varepsilon}+(\alpha-1)\varepsilon\left(\alpha+\varepsilon^2-1\right)\tau^{\varepsilon+3}\tau_I^{\varepsilon+2}\\
& +(\alpha-1)\left(\varepsilon^2+\varepsilon-1\right)(\alpha+\varepsilon-1)\tau^{2\varepsilon+2}\tau_I^{2\varepsilon+2}\\
& -(\alpha+\varepsilon-1)\left(\alpha\varepsilon+\alpha+\varepsilon^2+\varepsilon-1\right)\tau^{3\varepsilon+1}\tau_I^{3\varepsilon+1}\\
& +\left((1-2\alpha)\varepsilon^3+2(\alpha-1)\varepsilon^2-(\alpha-1)\alpha\varepsilon+(\alpha-1)^2\right)\tau^{\varepsilon+3}\tau_I^{\varepsilon+1}\\
& +\varepsilon(\alpha(\varepsilon-1)-1)(\alpha+\varepsilon-1)\tau^{2\varepsilon+2}\tau_I^{2\varepsilon}\\
& +\varepsilon(\alpha+\varepsilon-1)^2\tau^{3\varepsilon+1}\tau_I^{3\varepsilon}-\varepsilon(\alpha+\varepsilon-1)((2\alpha-1)\varepsilon+2)\tau^{2\varepsilon+2}\tau_I^{2\varepsilon+1}\\
& +\varepsilon(\alpha+\varepsilon-1)^2\tau^{4\varepsilon}\tau_I^{4\varepsilon}-\varepsilon(\alpha+\varepsilon-1)^2\tau^{4\varepsilon}\tau_I^{4\varepsilon+1}-(\alpha-1)^2(\varepsilon-1)\varepsilon\tau^4\tau_I^3\\
& +(\alpha-1)(\varepsilon-1)\tau^4\tau_I^2(\alpha(2\varepsilon-1)-\varepsilon+1)+(1-\alpha)\alpha(\varepsilon-2)\varepsilon\tau^4\tau_I\\
B_I^{Nash}(\tau_I) \equiv\ & \tau_I(\tau^{\varepsilon}\tau_I^{\varepsilon}+\tau\tau_I)\left(\tau^{2\varepsilon}\tau_I^{2\varepsilon}-\tau^2\right)((\alpha+\varepsilon-1)\tau^{\varepsilon}\tau_I^{\varepsilon}+(\alpha-1)\\
& (\varepsilon-1)\tau\tau_I-\alpha(\varepsilon-2)\tau)\\
& ((\alpha+\varepsilon-1)\tau^{\varepsilon}\tau_I^{\varepsilon}+\tau\tau_I(-\alpha\varepsilon+\alpha+\varepsilon-1)+\alpha\varepsilon\tau).
\end{aligned}$$

We have to show that if $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$: (i) there is no solution of the Nash equilibrium of the non-cooperative policy game for $\tau_I > 1$; (ii) if $\varepsilon > 2$, there exists a solution of the non-cooperative policy game for $\tau_I < 1$; (iii) $N^{Nash} > N^{FB} > N^{FT}$.

(i) In order to show that no Nash equilibrium exists, we need to prove that there are no zeros of $A_I^{Nash}(\tau_I)$ for $\tau_I > 1$. This is so because: (a) A_I^{Nash} is a second-order polynomial in α; (b) if $\alpha = 0$ or $\alpha = 1$, $A_I^{Nash}(\tau_I) < 0$; (c) $\frac{\partial A_I^{Nash}(\tau_I)}{\partial \tau_I}\Big|_{\alpha=0} < 0$.

(a) It is straightforward to see that A_I^{Nash} is quadratic in α.

(b) If $\alpha = 0$ and $\tau_I > 1$:

$$\begin{aligned} A_I^{Nash}(\tau_I) = & -(\varepsilon-1)\Big(\tau^{\varepsilon+3}\tau_I^{\varepsilon+1}((\varepsilon+1)\varepsilon\tau_I-\varepsilon(\varepsilon-1)+1\Big) \\ & +\tau^{2\varepsilon+2}\tau_I^{2\varepsilon}\Big(\Big(\varepsilon^2+\varepsilon-1\Big)\tau_I^2-(\varepsilon-2)\varepsilon\tau_I+\varepsilon\Big) \\ & +\tau^{3\varepsilon+1}\tau_I^{3\varepsilon}\Big(\Big(\varepsilon^2+\varepsilon-1\Big)\tau_I-(\varepsilon-1)\varepsilon\Big) \\ & +\tau^{4\varepsilon}\tau_I^{4\varepsilon}(\varepsilon-1)\varepsilon(\tau_I-1) \\ & +\tau^4\tau_I^2(\varepsilon(\tau_I-1)+1))<0. \end{aligned}$$

If $\alpha = 1$ and $\tau_I > 1$:

$$\begin{aligned} A_I^{Nash}(\tau_I) = & -\varepsilon^2\tau^\varepsilon\tau_I^\varepsilon(\tau^\varepsilon\tau_I^\varepsilon+\tau)(2\tau^{\varepsilon+1}\tau_I^{\varepsilon+1}+\varepsilon(\tau_I-1)\tau^{2\varepsilon}\tau_I^{2\varepsilon} \\ & +\tau^2(\varepsilon\tau_I-\varepsilon+2))<0. \end{aligned}$$

(c) To see why $\partial A_I^{Nash}(\tau_I)/\partial\tau_I < 0$, first consider that if $\alpha = 0$:

$$\begin{aligned} \frac{\partial A_I^{Nash}(\tau_I)}{\partial \tau_I} = & \tau_I^{4\varepsilon}\tau^{4\varepsilon}\kappa_1+\tau^{3\varepsilon+1}\tau_I^{3\varepsilon}\kappa_2+\tau^{2\varepsilon+2}\tau_I^{2\varepsilon}\kappa_3 \\ & +\tau^{\varepsilon+3}\tau_I^{\varepsilon}\kappa_4+\tau^4\tau_I\kappa_5 \end{aligned}$$

where:

$$\begin{aligned} \kappa_1 &\equiv -2(\varepsilon-1)\varepsilon(\tau_I-1) \\ \kappa_2 &\equiv -\Big(\Big(2\varepsilon^2+\varepsilon-2\Big)\tau_I-2(\varepsilon-1)\varepsilon\Big) \\ \kappa_3 &\equiv (\varepsilon-2)\Big(\varepsilon^2+\varepsilon-1\Big)\tau_I^2+((3-2\varepsilon)\varepsilon-2)\varepsilon\tau_I+(\varepsilon-2)\varepsilon^2 \\ \kappa_4 &\equiv \tau_I\Big[\Big(\varepsilon^2-2\Big)\varepsilon\tau_I-2(\varepsilon-1)\varepsilon^2+\varepsilon-2\Big]+(\varepsilon-2)\varepsilon^2 \\ \kappa_5 &\equiv (\varepsilon-1)\tau_I(2\varepsilon\tau_I-3\varepsilon+2)+(\varepsilon-2)\varepsilon. \end{aligned}$$

First, we show that $\partial A_I^{Nash}(\tau_I)/\partial\tau_I < 0$ for $\varepsilon < 2$. Under this assumption $\kappa_1 < 0$, $\kappa_2 < 0$, $\kappa_3 < 0$ and κ_3–$\kappa_4 < 0$. In this case it is sufficient to show that $\tau_I^{4\varepsilon}\tau^{4\varepsilon}\kappa_1 + \tau^{3\varepsilon+1}\tau_I^{3\varepsilon}\kappa_2 + \tau^4\tau_I\kappa_5 < 0$. Note that $\tau_I^{4\varepsilon}\tau^{4\varepsilon}\kappa_1 + \tau^{3\varepsilon+1}\tau_I^{3\varepsilon}\kappa_2 + \tau^4\tau_I\kappa_5 < \delta(\tau_I)$ where $\delta(\tau_I) \equiv (\kappa_1 + \kappa_2)\tau_I^{2\varepsilon} + \kappa_5$. It can be shown that $\delta'(\tau_I) < 0$. It follows then from $\delta(\tau_I) = -2\varepsilon$ at $\tau_I = 1$ that $\delta(\tau_I) < 0$.

Second, we show that $\partial A_I^{Nash}(\tau_I)/\partial\tau_I < 0$ for $\varepsilon > 2$. Under this assumption $\kappa_1 < 0$, $\kappa_2 < 0$ and $\kappa_5 < 0$. Therefore, in this case it suffices to show that $\tau_I^{4\varepsilon}\tau^{4\varepsilon}\kappa_1 + \tau^{2\varepsilon+2}\tau_I^{2\varepsilon}\kappa_3 < 0$ and $\tau^{3\varepsilon+1}\tau_I^{3\varepsilon}\kappa_2 + \tau^{\varepsilon+3}\tau_I^{\varepsilon}\kappa_4 + \tau^4\tau_I\kappa_5 < 0$ or alternatively that $\delta_1(\tau_I) \equiv \kappa_1\tau_I^{2\varepsilon} + \kappa_3 < 0$ and $\delta_2(\tau_I) \equiv \kappa_2\tau_I^{2\varepsilon} + \kappa_4 + \kappa_5 < 0$. These last conditions are always satisfied because at $\tau_I = 1$, $\delta_1(\tau_I) = 2 - 5\varepsilon$ and $\delta_2(\tau_I) = -2 - 3\varepsilon$ and it can be proved that $\delta_2'(\tau_I) < 0$ and $\delta_1'(\tau_I) < 0$.

(ii) This is equivalent to show that there is at least one zero of $A_I^{Nash}(\tau_I)$ for $\tau_I < 1$. A sufficient condition for the existence of a Nash solution is $\varepsilon > 2$. To see why this is the case, consider that: a) $A_I^{Nash}(\tau_I)$ is a continuous function in τ_I; b) $A_I^{Nash}(1) = -\tau(\alpha + \varepsilon - 1)(\tau^\varepsilon + \tau)^2((1 - \alpha)\tau + (\alpha + 2\varepsilon - 1)\tau^\varepsilon) < 0$; c) $A_I^{Nash}(0) = 0$ and $\partial A_I^{Nash}(0)/\partial\tau_I = (1 - \alpha)\alpha(\varepsilon - 2)\varepsilon\tau^4 > 0$ for $\varepsilon > 2$. Then, by the intermediate value theorem there exists a value $\tau_I \in (0, 1)$ such that $A_I^{Nash}(\tau_I) = 0$.

(iii) To prove this statement recall that if $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$, then:

$$\frac{\partial N_H}{\partial \tau_{IH}}\Big|_{\tau_{IH}=\tau_I}+\frac{\partial N_H}{\partial \tau_{IF}}\Big|_{\tau_{IF}=\tau_I} = -\frac{L(1-\alpha)\alpha(\varepsilon-1)\tau\big(\tau^\varepsilon(\varepsilon(1-\tau_I)+\tau_I)\tau_I^\varepsilon+\tau\tau_I\big)}{f\tau_I\big((\alpha+\varepsilon-1)\tau^\varepsilon\tau_I^\varepsilon+\tau\tau_I(1-\alpha)(\varepsilon-1)+\alpha\varepsilon\tau\big)^2}<0$$

for all $\tau_I \le 1$. We have already proven at points (i) and (ii) that when $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$, $\tau_I^{Nash} < 1$. As a consequence, $N^{Nash} > N^{FB} > N^{FT}$ since at the symmetric equilibrium $dN_H = \frac{\partial N_H}{\partial \tau_{IH}} d\tau_{IH} + \frac{\partial N_H}{\partial \tau_{IF}} d\tau_{IF} = \Big(\frac{\partial N_H}{\partial \tau_{IH}} + \frac{\partial N_H}{\partial \tau_{IF}}\Big) d\tau_{IH} > 0$ for all $\tau_I \le 1$ and $d\tau_{IH} = d\tau_{IF} < 0$.

(2) Let $\tau_{IH} = \tau_{IF} = 1$. In the case of export taxes, the non-cooperative policymaker maximizes:

$$\max_{\tau_{XH}} V_H(P_H(\tau_{XH},\tau_{XF}), I_H(\tau_{XH},\tau_{XF})) \tag{57}$$

where $P_H(\tau_{XH}, \tau_{XF})$ is given by Eq. (23), which is implied by the equilibrium expressions for $N_H(\tau_{XH}, \tau_{XF})$ and $N_F(\tau_{XH}, \tau_{XH})$, Eq. (28). Moreover, $I(\tau_{XH}, \tau_{XF}) = L + (\tau_{XH} - 1)\tau P_{HH}(\tau_{XH}, \tau_{XF}) C_{FH}(\tau_{XH}, \tau_{XF}) + (\tau_{WH} - 1) N_H(\tau_{XH}, \tau_{XF})\varepsilon f$, where $P_{HH}(\tau_{XH}, \tau_{XF}) = \frac{\varepsilon}{\varepsilon-1}\tau_{WH} N_H(\tau_{XH}, \tau_{XF})^{\frac{1}{1-\varepsilon}}$, $C_{FH} = P_{FH}(\tau_{XH}, \tau_{XF})^{-\varepsilon} P_F(\tau_{XH}, \tau_{XF})^\varepsilon C_F(\tau_{XH}, \tau_{XF})$, $P_{FH}(\tau_{XH}, \tau_{XF}) = \frac{\varepsilon}{\varepsilon-1}\tau\tau_{XH}\tau_{WH} N_H(\tau_{XH}, \tau_{XF})^{\frac{1}{1-\varepsilon}}$ and finally $C_F(\tau_{XH}, \tau_{XF})$ is given by its equilibrium value in Eq. (24).

(I) By taking the derivative of Eq. (57) with respect to τ_{XH} and then imposing symmetry i.e., $\tau_{XH} = \tau_{XF} = \tau_X$, the first-order conditions at the symmetric Nash equilibrium evaluated at $\tau_{WH} = \tau_{WF} = 1$ can be written as:

$$\frac{A_X^{Nash}(\tau_X)}{B_X^{Nash}(\tau_X)} = 0 \tag{58}$$

where:

$$\begin{aligned} A_X^{Nash}(\tau_X) \equiv \alpha\{ & \tau^{\varepsilon+4}\tau_X^{\varepsilon+1}[\tau_X\tau_X\Big(\varepsilon-\alpha^2\varepsilon\Big)+2\alpha^2\varepsilon+(\alpha-1)\alpha-\varepsilon^2 \\ & +\varepsilon)-\alpha^2(\varepsilon+1)+\alpha+(\varepsilon-1)\varepsilon]+\tau^{2\varepsilon+3}\tau_X^{2\varepsilon} \\ & [\tau_X(\tau_X\Big((\alpha-1)\varepsilon^2-\alpha+\varepsilon+1\Big)+\alpha\Big(-2\varepsilon^2+\varepsilon-1\Big) \\ & +(\varepsilon-1)^2)+\alpha(\varepsilon-1)\varepsilon]+\tau^{3\varepsilon+2}\tau_X^{3\varepsilon}[\tau_X(\alpha(\varepsilon-1)\tau_X(\alpha+\varepsilon-1) \\ & -(2\alpha+1)\varepsilon^2-2(\alpha-2)\alpha\varepsilon+(\alpha-1)\alpha)+\varepsilon(\alpha(\alpha+\varepsilon-2) \\ & +\varepsilon-1)]+\tau^{4\varepsilon+1}\tau_X^{4\varepsilon}(\varepsilon(\alpha+\varepsilon-2)-(\varepsilon-1)\tau_X(\alpha+\varepsilon-1)) \\ & +\tau^5\tau_X^2(\alpha+\varepsilon-1)\Big\} \end{aligned}$$

$$\begin{aligned} B_X^{Nash}(\tau_X) \equiv & (\varepsilon-1)\tau_X(\tau^\varepsilon\tau_X^\varepsilon+\tau\tau_X)\Big(\tau^{2\varepsilon}\tau_X^{2\varepsilon}-\tau^2\Big)(-(\alpha+1)\tau\tau_X+\tau^\varepsilon\tau_X^\varepsilon+\alpha\tau) \\ & (\tau_X(\tau-\alpha\tau)+\tau^\varepsilon\tau_X^\varepsilon+\alpha\tau) \end{aligned}$$

(i) In order to show that there exists a solution with $\tau_X < 1$, we first show that when $\tau_X = 1$, $A_X^{Nash}(1)$ is negative. This is so given that $A_X^{Nash}(1) = \tau(\tau^\varepsilon - \tau)(\tau^\varepsilon + \tau)^2[(\alpha - 1)\tau^\varepsilon + \tau(-\alpha - \varepsilon + 1)] < 0$.

(ii) Next, we show that for $\varepsilon > 2$ there exists a $\tau_X \in \{0, 1\}$ with $A_X^{Nash}(\tau_X) > 0$. By continuity of $A_X^{Nash}(\tau_X)$ this is enough to guarantee the existence of a solution. Consider $\tau_X = \frac{\varepsilon-2}{\varepsilon}$. Then, $A_X^{Nash}\left(\frac{\varepsilon-2}{\varepsilon}\right) = \frac{\tau}{\varepsilon^2}[(\varepsilon-2)^2\tau^4(\alpha+\varepsilon-1) + (\varepsilon-2)^{4\varepsilon}\varepsilon^{1-4\varepsilon}(2+2\varepsilon^2-5\varepsilon+3\alpha\varepsilon-2\alpha)\tau^{4\varepsilon} + (\varepsilon-2)^{1+\varepsilon}\varepsilon^{-\varepsilon}(4+2\alpha-6\alpha^2+3(\varepsilon-2)\varepsilon)\tau^{3+\varepsilon} + \left(\frac{\varepsilon-2}{\varepsilon}\right)^{2\varepsilon}(\alpha(6\varepsilon-4)+(\varepsilon-2)(\varepsilon^2-2))u^{2+2\varepsilon} + \tau(\alpha^2(6\varepsilon-4)+2\alpha(\varepsilon-2)(2\varepsilon-1)+\varepsilon^3)\left(\frac{(\varepsilon-2)\tau}{\varepsilon}\right)^{3\varepsilon}] > 0$ since each of the coefficients is positive for $\varepsilon > 2$. This proves that a solution with $\tau_X < 1$ exists.

(iii) Finally, we show that $N^{Nash} < N^{FB}$.

(a) Let $\tau_X^{Nash} = f(\alpha, \varepsilon, \tau)$ and $\tau_X^{FB} = g(\alpha, \varepsilon, \tau)$ be, respectively, the Nash equilibrium export subsidy and the export subsidy that implements the first-best number of varieties. First we show that there is no intersection between the set of τ_X^{Nash} and the set of τ_X^{FB} in the interval [0,1]. If $\tau_X = \tau_X^{Nash}$, $A_X^{Nash}(\tau_X^{Nash}) = 0$. At the same time τ_X^{FB} is such $N = \frac{L\alpha\left(\tau+\left(\tau\tau_X^{FB}\right)^\varepsilon\right)}{f\varepsilon\left(\alpha\tau+\tau(1-\alpha)\tau_X^{FB}+\left(\tau\tau_X^{FB}\right)^\varepsilon\right)} = \frac{L\alpha}{f(\varepsilon+\alpha-1)} = N^{FB}$. This last condition can be rewritten as $(\tau\tau_X)^{\varepsilon} = -\varepsilon\tau\tau_X + \tau(\varepsilon - 1)$. Note that when combined, these two conditions are a system of two equations in τ_X. We now investigate if there exists a τ_X such that both conditions are satisfied simultaneously. Once we substitute the above condition into A_X^{Nash} we obtain a fifth-order polynomial in τ_X which can be factorized into two polynomials. The first polynomial is $-\varepsilon\tau^5(\tau_X - 1)^2(\alpha + \varepsilon - 1)$, with solutions $\tau_X = \{1,1\}$. None of these solutions solves $(\tau\tau_X)^{\varepsilon} = -\varepsilon\tau\tau_X + \tau(\varepsilon - 1)$. The second polynomial is cubic and we call it A_{Xmod}^{Nash}. It can be shown that there exists at most one real solution of A_{Xmod}^{Nash}. However, evaluating A_{Xmod}^{Nash} at $\tau_X = 1$ and $\tau_X = 0$ we find that both $A_{Xmod}^{Nash}(1) < 0$ and $A_{Xmod}^{Nash}(0) < 0$. Thus, by continuity of A_{Xmod}^{Nash}, either there exists no real solution or there are at least two zeros of $A_{Xmod}^{Nash} = 0$ that are real. Since there exists at most one real solution of $A_{Xmod}^{Nash} = 0$ in [0,1], we can conclude that there is no intersection between the set of τ_X^{Nash} and the set of τ_X^{FB} in the interval [0,1].

(b) The second step is to show that $\tau_X^{FB} < \tau_X^{Nash}$ in the interval [0,1]. To this end, recall that f and g are two continuous functions in the space $\{0 < \alpha < 1, \tau > 1, \varepsilon > 1\}$, given that the derivatives of τ_X^{FB} and τ_X^{Nash} with respect to the three parameters always exists in the permitted parameter space. In point (a) we proved that there is no intersection between g and f. As a consequence, we either have $\tau_X^{FB} < \tau_X^{Nash}$ or the other way around. We evaluate both functions at $\{\alpha = 0.5, \varepsilon = 2, \tau = 1.5\}$ and find $\tau_X^{FB} = 0.39 < 0.82 = \tau_X^{Nash}$. Thus, the noncooperative export subsidy is always smaller than that needed to implement the first-best number of varieties.

(c) Finally, note that in the symmetric equilibrium $\tau_{XH} = \tau_{XF} = \tau_X$ and $d\tau_{XH} = d\tau_{XF}$:

$$dN_H = \frac{\partial N_H}{\partial \tau_{XH}} d\tau_{XH} + \frac{\partial N_H}{\partial \tau_{XF}} d\tau_{XF} = \left(\frac{\partial N_H}{\partial \tau_{XH}} + \frac{\partial N_F}{\partial \tau_{XH}}\right) d\tau_{XH}.$$

When $0 < \tau_X \le 1$ the following derivative is negative:

$$\frac{\partial N_H}{\partial \tau_{XH}} + \frac{\partial N_F}{\partial \tau_{XH}} = -\frac{L(1-\alpha)\alpha\tau\left(\tau_X\tau + (\tau_X\tau)^{\varepsilon}(\tau_X + \varepsilon(1-\tau_X))\right)}{f\tau_X\varepsilon(\alpha\tau + (\tau_X\tau)^{\varepsilon} + \tau_X\tau(1-\alpha))^2} < 0$$

which implies that $dN_H = dN_F > 0 \Leftrightarrow d\tau_{HX} = d\tau_{XF} < 0$ i.e., by symmetrically increasing the export subsidy in both countries policymakers increase the number of varieties. It then follows that $\tau_X^{FB} < \tau_X^{Nash} \Rightarrow N^{Nash} < N^{FB}$.

(II) By taking the derivative of Eq. (57) with respect to τ_{XH} and then imposing symmetry i.e., $\tau_{XH} = \tau_{XF} = \tau_X$, the first-order conditions at the symmetric Nash equilibrium evaluated at $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$ can be written as:

$$\frac{A_X^{Nash}(\tau_X)}{B_X^{Nash}(\tau_X)} = 0 \qquad (59)$$

where:

$$\begin{aligned}
A_X^{Nash}(\tau_X) \equiv \alpha\tau\Big\{ & -\tau^{\varepsilon+3}\tau_X^{\varepsilon+1}\Big[\tau_X\Big(\big(\alpha^2-1\big)(\varepsilon-1)\varepsilon\tau_X + \big(-2\alpha^2+\alpha-2\big)\varepsilon^2 + (\alpha-1)^2 + \varepsilon^3\Big) \\
& + \varepsilon\Big((\alpha-1)\alpha\varepsilon + (\alpha-1)\alpha - \varepsilon^2 + \varepsilon\Big)\Big] \\
& + \tau^{2\varepsilon+2}\tau_X^{2\varepsilon}\Big[\tau_X\Big(\varepsilon\tau_X\Big(\alpha^2 + \alpha(\varepsilon-1)^2 - (\varepsilon-2)(\varepsilon-1)\Big) \\
& -\alpha^2(\varepsilon+1) + 2\alpha\Big(-\varepsilon^3+\varepsilon^2+1\Big) + (\varepsilon-1)\Big(\varepsilon^2+1\Big)\Big) + \alpha\varepsilon^3\Big] \\
& + \tau^{3\varepsilon+1}\tau_X^{3\varepsilon}(\alpha+\varepsilon-1)\Big[(\varepsilon-1)\tau_X(\tau_X(\alpha(\varepsilon-1)+1) - (2\alpha+1)\varepsilon) + (\alpha+1)\varepsilon^2\Big] \\
& -\tau_X^{4\varepsilon}\tau^{4\varepsilon}(\alpha+\varepsilon-1)^2[(\varepsilon-1)\tau_X - \varepsilon] + \tau^4\tau_X^2\varepsilon(\alpha+\varepsilon-1)\Big\}
\end{aligned}$$

$$\begin{aligned}
B_X^{Nash}(\tau_X) \equiv {} & \tau_X(\tau^{\varepsilon}\tau_X^{\varepsilon} + \tau\tau_X)\Big(\tau^{2\varepsilon}\tau_X^{2\varepsilon} - \tau^2\Big)\Big[(\alpha+\varepsilon-1)\tau^{\varepsilon}\tau_X^{\varepsilon} - (\alpha+1)(\varepsilon-1)\tau\tau_X + \alpha\varepsilon\tau\Big] \\
& \Big[(\alpha+\varepsilon-1)\tau^{\varepsilon}\tau_X^{\varepsilon} + \tau\tau_X(-\alpha\varepsilon + \alpha + \varepsilon - 1) + \alpha\varepsilon\tau\Big].
\end{aligned}$$

(i) We first show that no solution with $\tau_X < 1$ exists. Focusing on the numerator of the first-order condition, this is so since all terms of $A_X^{Nash}(\tau_X)$ are positive for $\tau_X < 1$.

(ii) Next, we show that there exists at least one solution with $\tau_X > 1$.

(a) For $\tau_X = 1$, $A_X^{Nash}(1) = (\varepsilon + \alpha - 1)(\tau^{\varepsilon} + \tau)^2[(1 - \alpha)\tau^{\varepsilon}\tau + \varepsilon\tau^2 + \tau^{2\varepsilon}(\varepsilon - 1 + \alpha)] > 0$;

(b) $lim_{\tau_X \to \infty} A_X^{Nash}(\tau_X) = -\infty$;

(c) Thus, by continuity of $A_X^{Nash}(\tau_X)$, there exists a $\tau_X^{Nash} > 1$ such that $A_X^{Nash}(\tau_X^{Nash}) = 0$.

(iii) It remains to show that if $\tau_X^{Nash} > 1$, then $N^{Nash} < N^{FB}$. When $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$ and after imposing symmetry i.e., $\tau_{XH} = \tau_{XF} = \tau_X$ and $d\tau_{XH} = d\tau_{XF}$:

$$\begin{aligned}
dN_H &= \frac{\partial N_H}{\partial \tau_{XH}} d\tau_{XH} + \frac{\partial N_H}{\partial \tau_{XF}} d\tau_{XF} = \left(\frac{\partial N_H}{\partial \tau_{XH}} + \frac{\partial N_H}{\partial \tau_{XF}}\right) d\tau_{XH} \\
&= \frac{L(1-\alpha)\alpha(\varepsilon-1)\tau\left[\tau^{\varepsilon}\tau_X^{\varepsilon}((\varepsilon-1)\tau_X - \varepsilon) - \tau\tau_X\right]}{f\tau_X\left[(\alpha+\varepsilon-1)\tau^{\varepsilon}\tau_X^{\varepsilon} + \tau\tau_X(1-\alpha)(\varepsilon-1) + \alpha\varepsilon\tau\right]^2} d\tau_{XH}.
\end{aligned}$$

Note that $d\tau_{XH} > 0 \Rightarrow dN_H \ge 0 \Leftrightarrow \tau^{\varepsilon}\tau_X^{\varepsilon}[(\varepsilon - 1)\tau_X - \varepsilon] - \tau\tau_X \ge 0$. Let us define the following two continuous and monotonic functions $f(\tau_X) \equiv (\varepsilon - 1)\tau^{\varepsilon}\tau_X^{\varepsilon+1}$ and $g(\tau_X) \equiv \varepsilon\tau^{\varepsilon}\tau_X^{\varepsilon} + \tau\tau_X$ with $f'(\tau_X) > 0$, $f''(\tau_X) > 0$, $g'(\tau_X) > 0$ and $g''(\tau_X) > 0$. Note that $f(1) - g(1) < 0$ implying that $dN_H < 0$ when $\tau_X = 1$. By continuity and monotonicity of the two functions, only two cases are possible. They either never cross, in which case $dN_H < 0\ \forall\ \tau_X \in [1, \infty)$ and consequently $N^{Nash} < N^{FB}$. Or, they cross only once. That implies that $\exists \overline{\tau}_X > 1$ such that $f(\tau_X) \ge g(\tau_X)$, $\forall\ \tau_X \ge \overline{\tau}_X$ implying $dN_H > 0 \Leftrightarrow \tau_X \in (\overline{\tau}_X, \infty)$. However note that:

$$\lim_{\tau_X -> \infty} N_H = \lim_{\tau_X -> \infty} \frac{L\alpha(\tau^{\varepsilon}\tau_X^{\varepsilon} + \tau)}{f\left((\alpha+\varepsilon-1)\tau^{\varepsilon}\tau_X^{\varepsilon} + \tau\tau_X(\alpha(-\varepsilon) + \alpha + \varepsilon - 1) + \alpha\varepsilon\tau\right)} = N^{FB}$$

implying that also in this case $N^{Nash} < N_X^{FB}$. ■

Proposition 4. Nash-equilibrium policy instruments

The Nash-equilibrium policy consists of the first-best level of wage subsidies, and inefficient import subsidies and export taxes. Formally,

(1) $\tau_W^{Nash} = \tau_W^{FB} = \frac{\varepsilon-1}{\varepsilon}$, $\tau_I^{Nash} < 1$ *and* $\tau_X^{Nash} > 1$.

Proof of Proposition 4. Maximizing indirect utility w.r.t. all three instruments, as we did for the previous propositions, results in an intractable policy problem. To prove Proposition 4 we thus follow an alternative approach. First, we find it useful to rewrite utility as follows:

$$\begin{aligned} U(C_H, Z_H) &= C_H^{\alpha} Z_H^{1-\alpha} \\ &= C_H^{\alpha}\left(\frac{1-\alpha}{\alpha} P_H C_H\right)^{1-\alpha} \\ &= \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} P_H^{-\alpha} P_H C_H \\ &= \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} P_H^{-\alpha}(\tau\tau_{IH}\tau_{XF} P_{FF} C_{HF} + P_{HH} C_{HH}). \end{aligned}$$

Next, we maximize utility subject to the equilibrium conditions.[19] The non-cooperative policymaker maximizes domestic utility subject to the goods market clearing conditions, the trade balance and the demand functions of the domestic and Foreign economy. The Lagrangian associated with the optimal policy problem of the non-cooperative policymaker can be formulated as:

$$\begin{aligned} \mathcal{L} = {} & P_H^{-\alpha}(\tau\tau_{IH}\tau_{XF} P_{FF} C_{HF} + P_{HH} C_{HH}) \\ & + \lambda_1\left[f(\varepsilon-1) N_H^{\frac{\varepsilon}{\varepsilon-1}} - C_{HH} - \tau C_{FH}\right] + \lambda_2\left[f(\varepsilon-1) N_F^{\frac{\varepsilon}{\varepsilon-1}} - C_{FF} - \tau C_{HF}\right] \\ & - \lambda_3\left[\frac{(1-\alpha)}{\alpha}(P_{HH} C_{HH} + \tau\tau_{IH}\tau_{XF} P_{FF} C_{HF}) + \tau\tau_{XF} P_{FF} C_{HF} - \tau\tau_{XH} P_{HH} C_{FH} - Q_H\right] \\ & - \lambda_4\left[\frac{(1-\alpha)}{\alpha}(P_{FF} C_{FF} + \tau\tau_{IF}\tau_{XH} P_{HH} C_{FH}) + \tau\tau_{XH} P_{HH} C_{FH} - \tau\tau_{XF} P_{FF} C_{HF} - Q_F\right] \\ & - \lambda_5\left[P_{HH}^{\varepsilon} C_{HH} - (\tau\tau_{IH}\tau_{XF})^{\varepsilon} P_{FF}^{\varepsilon} C_{HF}\right] - \lambda_6\left[P_{FF}^{\varepsilon} C_{FF} - (\tau\tau_{IF}\tau_{XH})^{\varepsilon} P_{HH}^{\varepsilon} C_{FH}\right] \end{aligned}$$

where P_H and P_{HH} are defined consistently with Eqs. (4), (8) and (38) and their Foreign counterparts. Making use of the constraints and rearranging the first-order conditions of L with respect to C_{HH}, C_{HF}, C_{FF}, C_{FH}, N_H, N_F, τ_{WH}, τ_{IH} and τ_{XH}, which we evaluate at the symmetric equilibrium, we obtain, respectively:

$$\begin{aligned} P_{HH}^{1-\alpha}\left[1+(\tau\tau_I\tau_X)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}} &= \lambda_1 + \frac{1-\alpha}{\alpha}\lambda_3 + \gamma_5 \\ \tau\tau_I\tau_X P_{HH}^{1-\alpha}\left[1+(\tau\tau_I\tau_X)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}} &= \lambda_2\tau + \lambda_3\left(\tau\tau_X + \frac{1-\alpha}{\alpha}\tau\tau_I\tau_X\right) - \gamma_4\tau\tau_X - \gamma_5(\tau\tau_I\tau_X)^{\varepsilon} \\ 0 &= \lambda_2 + \frac{1-\alpha}{\alpha}\gamma_4 + \gamma_6 \\ 0 &= \lambda_1\tau - \lambda_3\tau\tau_X + \gamma_4\left(\tau\tau_X + \frac{1-\alpha}{\alpha}\tau\tau_I\tau_X\right) - \gamma_6(\tau\tau_I\tau_X)^{\varepsilon} \\ 0 &= \lambda_1 - \lambda_3\frac{\varepsilon-1}{\varepsilon\tau_W} \\ (1-\alpha)(\tau\tau_I\tau_X)^{1-\varepsilon} P_{HH}^{1-\alpha}\left[1+(\tau\tau_I\tau_X)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}} &= \lambda_2\varepsilon\left[1+\tau^{1-\varepsilon}(\tau_I\tau_X)^{-\varepsilon}\right] + \lambda_3\left(\frac{1-\alpha}{\alpha} + \frac{1}{\tau_I}\right)(\tau\tau_I\tau_X)^{1-\varepsilon} \\ & \quad + \gamma_4\left\{\frac{1-\alpha}{\alpha} - \frac{\varepsilon-1}{\tau_W}\left[1+\tau^{1-\varepsilon}(\tau_I\tau_X)^{1-\varepsilon}\right] - \tau_I^{-\varepsilon}(\tau\tau_X)^{1-\varepsilon}\right\} \\ & \quad - \gamma_5\varepsilon + \gamma_6\varepsilon \\ (1-\alpha) P_{HH}^{1-\alpha}\left[1+(\tau\tau_I\tau_X)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}} &= \lambda_3\left[\frac{1-\alpha}{\alpha} + \tau_I^{-\varepsilon}(\tau\tau_I\tau_X)^{\varepsilon-1}\right] + \gamma_4\left(\frac{1-\alpha}{\alpha} + \frac{1}{\tau_I}\right)(\tau\tau_I\tau_X)^{1-\varepsilon} \\ & \quad - \gamma_5\varepsilon + \gamma_6\varepsilon \\ (1-\alpha) P_{HH}^{1-\alpha}\left[1+(\tau\tau_I\tau_X)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}} &= \lambda_3\frac{1-\alpha}{\alpha} + \gamma_5\varepsilon(\tau\tau_I\tau_X)^{\varepsilon-1} \\ 0 &= \lambda_3\tau_I^{-\varepsilon}(\tau\tau_X)^{1-\varepsilon} - \gamma_4\left[\frac{1-\alpha}{\alpha}(\tau\tau_I\tau_X)^{1-\varepsilon} + \tau_I^{-\varepsilon}(\tau\tau_X)^{1-\varepsilon}\right] \\ & \quad + \gamma_6\varepsilon \end{aligned} \tag{60}$$

where $\gamma_3 \equiv \lambda_3 P_{HH}$, $\gamma_4 \equiv \lambda_4 P_{HH}$, $\gamma_5 \equiv \lambda_5 P_{HH}^{\varepsilon}$ and $\gamma_6 \equiv \lambda_6 P_{HH}^{\varepsilon}$.

Combining the previous equations, we can solve for τ_W and the multipliers:

$$\begin{aligned} \tau_W &= \frac{\varepsilon-1}{\varepsilon} \\ \lambda_1 &= \gamma_3 = P_{HH}^{1-\alpha}\alpha\left[1+(\tau\tau_I\tau_X)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}} \\ \gamma_4 &= \frac{P_{HH}^{1-\alpha}\alpha^2(1-\alpha)(\varepsilon(\tau_X-1)-\tau_X)\left[1+(\tau\tau_I\tau_X)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}}}{(\varepsilon-1)(\alpha+(1-\alpha)\tau_I)\tau_X} \\ \gamma_5 &= 0 \\ \gamma_6 &= -\frac{P_{HH}^{1-\alpha}\alpha\tau^{1-\varepsilon}(\tau_I\tau_X)^{-\varepsilon}\left[1+(\tau\tau_I\tau_X)^{1-\varepsilon}\right]^{-\frac{\alpha}{1-\varepsilon}}}{\varepsilon-1} \\ \lambda_2 &= -\gamma_6 - \frac{\gamma_4}{\alpha}. \end{aligned} \tag{61}$$

The first condition in Eq. (61) already states that the Nash equilibrium wage subsidy completely offsets the monopolistic distortion. What remains to show is that $\tau_I^{Nash} < 1$ and $\tau_X^{Nash} > 1$. Substituting the expressions for the multipliers and the solution for τ_W in the first-order conditions and simplifying, we are left with two equations, the derivative with respect to C_{HF} and the one with respect to N_F. The derivative with respect to C_{HF} is given by:

$$A_1(\tau_I, \tau_X) + A_2(\tau_I, \tau_X) + A_3(\tau_I, \tau_X) = 0 \tag{62}$$

[19] This approach is similar to that used in the public finance literature. See Lucas and Stokey (1983) and Chari and Kehoe (1999).

where:

$$A_1(\tau_I,\tau_X) \equiv -(\varepsilon-1)(1-\tau_I)\tau_X^2(\alpha+(1-\alpha)\tau_I)$$
$$A_2(\tau_I,\tau_X) \equiv -(\varepsilon-(\varepsilon-1)\tau_X)(\alpha\tau_X+1-\alpha)$$
$$A_3(\tau_I,\tau_X) \equiv -(\alpha+(1-\alpha)\tau_I)(\tau\tau_X)^{1-\varepsilon}\tau_I^{-\varepsilon}.$$

Note that:

(i) $A_3(\tau_I, \tau_X) < 0$ always;
(ii) $A_1(\tau_I, \tau_X) < 0 \Leftrightarrow \tau_I < 1$;
(iii) $A_2(\tau_I, \tau_X) < 0 \Leftrightarrow \tau_X < \frac{\varepsilon}{\varepsilon-1}$.

Thus, a necessary condition for τ_I and τ_X to solve Eq. (62) is that if $\tau_X < \frac{\varepsilon}{\varepsilon-1}$ then $\tau_I > 1$. By combining (62) with the first-order condition with respect to N_F we obtain a second condition:

$$B_1(\tau_I,\tau_X)+B_2(\tau_I,\tau_X)+B_3(\tau_I,\tau_X)=0 \tag{63}$$

where:

$$B_1(\tau_I,\tau_X) \equiv -\tau_X^2(\varepsilon-1)(\alpha+(1-\alpha)\tau_I)(1-\varepsilon(1-\tau_I))$$
$$B_2(\tau_I,\tau_X) \equiv (-\varepsilon+\tau_X(\varepsilon-1))(\varepsilon-(1-\alpha))\tau^{\varepsilon-1}(\tau_I\tau_X)^{\varepsilon}$$
$$B_2(\tau_I,\tau_X) \equiv -\alpha(-\varepsilon+\tau_X(\varepsilon-1))^2.$$

Note that:

(i) $B_3(\tau_I, \tau_X) < 0$ always;
(ii) $B_1(\tau_I, \tau_X) < 0 \Leftrightarrow \tau_I > \frac{\varepsilon-1}{\varepsilon}$;
(iii) $B_2(\tau_I, \tau_X) < 0 \Leftrightarrow \tau_X < \frac{\varepsilon}{\varepsilon-1}$.

Thus, a necessary condition for τ_I and τ_X to solve Eq. (63) is that if $\tau_X < \frac{\varepsilon}{\varepsilon-1}$ then $\tau_I < \frac{\varepsilon-1}{\varepsilon}$. Note that this condition contradicts the one needed for Eq. (62). Therefore, the only possible solution is $\tau_X^{Nash} > \frac{\varepsilon}{\varepsilon-1}$ i.e., this proves that $\tau_X^{Nash} > 1$.

We now have to show that $\tau_I^{Nash} < 1$. We will prove this by contradiction. First, we show that a necessary condition for $\tau_I^{Nash} > 1$ is that $\tau_I^{Nash} \cdot \tau_X^{Nash} < 1$. Second, we show that if $\tau_I^{Nash} > 1$ it must be that $\tau_X^{Nash} < 1$, which contradicts the fact that $\tau_X^{Nash} > 1$. In order to show the first point, it is useful to rewrite Eq. (62) as follows:

$$\begin{aligned}&-(\varepsilon-1)(1-\tau_I)\tau_X(\alpha\tau_X+(1-\alpha)\tau_{IX})+((\varepsilon-1)\tau_X-\varepsilon)\\&\quad\times(\alpha\tau_X+1-\alpha)-(\alpha\tau_X+(1-\alpha)\tau_{IX})\left(\tau^{1-\varepsilon}\tau_{IX}^{-\varepsilon}\right)\\&\quad=0\end{aligned}$$

where $\tau_{IX} \equiv \tau_I\tau_X$. If we solve the previous equation for τ_I we obtain:

$$\tau_I^{Nash} = \frac{C_1(\tau_X,\tau_{IX})}{C_2(\tau_X,\tau_{IX})} \tag{64}$$

where:

$$\begin{aligned}C_1(\tau_X,\tau_{IX}) \equiv\;& (\tau_{IX}\tau)^{-\varepsilon}(\tau_{IX}\tau(1-\alpha)+(\tau_{IX}\tau)^{\varepsilon}\varepsilon(1-\alpha-\tau_X+2\alpha\tau_X)\\&+\alpha\tau_X\tau+(\tau_{IX}\tau)^{\varepsilon}\tau_X(1-\alpha)+\tau_{IX}^{1+\varepsilon}\tau^{\varepsilon}\tau_X(\varepsilon-1)(1-\alpha))\end{aligned}$$
$$C_2(\tau_X,\tau_{IX}) \equiv (\varepsilon-1)\tau_X(\tau_{IX}(1-\alpha)+\alpha\tau_X)$$

Now suppose that $\tau_I > 0$ and $C_2(\tau_X, \tau_{IX}) > 0$, it must also be the case that $C_1(\tau_X, \tau_{IX}) > 0$. Moreover, for τ_I^{Nash} to be greater than 1, $C_1(\tau_X, \tau_{IX}) > 0$–$C_2(\tau_X, \tau_{IX}) > 0$ should be greater than 0:

$$\begin{aligned}&C_3(\tau_I,\tau_X) \equiv C_1(\tau_X,\tau_{IX})-C_2(\tau_X,\tau_{IX}) =\\&(\tau_{IX}\tau)^{-\varepsilon}(\tau_{IX}\tau(1-\alpha)+\alpha\tau\tau_X+(\tau_{IX}\tau)^{\varepsilon}((1-\alpha)\varepsilon+\\&(1-\alpha+2\tau_{IX}(1-\alpha)(\varepsilon-1))-\varepsilon+2\alpha\varepsilon)\tau_X+\alpha(\varepsilon-1)\tau_X^2).\end{aligned}$$

Note that:

(i) $C_3(\tau_I, \tau_X)$ is linear in α;
(ii) $\alpha = 0$ and $\tau_{IX} > 1$ implies $C_3 = -(\tau_{IX}\tau)^{-\varepsilon}(\tau_{IX}\tau + (\tau_{IX}\tau)^{\varepsilon}(\varepsilon + (\varepsilon - 1)(\tau_{IX} - 1) + \tau_{IX}\tau_X(\varepsilon - 1))) < 0$;
(iii) $\alpha = 1$ and $\tau_{IX} > 1$ implies $C_3 = -(\tau_{IX}\tau)^{-\varepsilon}(\tau\tau_X + (\tau\tau_{IX})^{\varepsilon}(\varepsilon\tau_X + (\varepsilon - 1)\tau_X^2)) < 0$;
(iv) By continuity, $\forall\, \alpha \in (0, 1)$ $\tau_{IX} > 1 \Rightarrow C_3 < 0 \Rightarrow \tau_I^{Nash} < 1$;
(v) Thus, a necessary condition for $\tau_I^{Nash} > 1$ is $\tau_{IX} < 1$.

However, we have already proven that $\tau_X^{Nash} > 1$ thus, it cannot be that $\tau_I^{Nash} > 1$ and $\tau_{IX}^{Nash} \equiv \tau_I^{Nash}\tau_X^{Nash} < 1$. Therefore, it has to be that $\tau_I^{Nash} < 1$.■

Proposition 5. Politically optimal policy instruments

The politically optimal policy is not efficient. Formally,

(1) $\Delta_{\tau_{WH}}|_{\tau_{WH}=\tau_{WF}=\frac{\varepsilon-1}{\varepsilon},\tau_{IH}=\tau_{IF}=1,\tau_{XH}=\tau_{XF}=1} > 0$;
(2) $\Delta_{\tau_{IH}}|_{\tau_{WH}=\tau_{WF}=\frac{\varepsilon-1}{\varepsilon},\tau_{IH}=\tau_{IF}=1,\tau_{XH}=\tau_{XF}=1} < 0$;
(3) $\Delta_{\tau_{XH}}|_{\tau_{WH}=\tau_{WF}=\frac{\varepsilon-1}{\varepsilon},\tau_{IH}=\tau_{IF}=1,\tau_{XH}=\tau_{XF}=1} > 0$;

Proof. The politically optimal policy is defined as in Bagwell and Staiger (2009). First, we rewrite income in terms of local and international prices:

$$\begin{aligned}I_H &= L+N_Hf\varepsilon(\tau_{WH}-1)+N_F(\tau_{IH}-1)\tau\tau_{XF}\tau_{WF}\frac{\varepsilon}{\varepsilon-1}c_{HF}+N_H\tau\tau_{XH}\tau_{WH}\frac{\varepsilon}{\varepsilon-1}c_{FH}\\I_H &= L+N_Hf\varepsilon(\tau_{WH}-1)+N_F(\tau_{IH}-1)\tau p_{WF}c_{HF}+N_H(p_{WH}-p_{HH})\tau c_{FH}\end{aligned} \tag{65}$$

where $p_{WH} = \tau\tau_{XH}p_{HH}$ and $p_{WF} = \tau\tau_{XF}p_{FF}$ are the two international prices. Second, we define $\Delta_{\tau_{jH}}$ as the derivative of V_H with respect to τ_{jH} when $\frac{\partial p_{WH}}{\partial \tau_{WH}} = 0$. Then we find that

(1) To show Eq. (1) consider that:

$$\begin{aligned}&\Delta_{\tau_{WH}}|_{\tau_{WH}=\tau_{WF}=\frac{\varepsilon-1}{\varepsilon},\tau_{IH}=\tau_{IF}=1,\tau_{XH}=\tau_{XF}=1}\\&=\varepsilon\tau\left[\frac{\varepsilon}{(\varepsilon-1)(\tau^{\varepsilon}-\tau)}-\frac{\alpha}{(\varepsilon-1)(\tau+\tau^{\varepsilon})}-\frac{\varepsilon}{(1+\alpha-\varepsilon)\tau+(\varepsilon-1+\alpha)\tau^{\varepsilon})}\right]\end{aligned} \tag{66}$$

Note that $\Delta_{\tau_{WH}}|_{\tau_{WH}=\tau_{WF}=\frac{\varepsilon-1}{\varepsilon},\tau_{IH}=\tau_{IF}=1,\tau_{XH}=\tau_{XF}=1} = \varepsilon\tau A/B$ where:

$$\begin{aligned}A &= \varepsilon(\alpha(\tau^{\varepsilon}+\tau)+(\varepsilon-1)(\tau^{\varepsilon}-\tau))(\tau+\tau^{\varepsilon})-\alpha(\tau^{\varepsilon}-\tau)(\alpha\tau^{\varepsilon}+\tau)+(\varepsilon-1)(\tau^{\varepsilon}-\tau))\\&\quad-\varepsilon(\varepsilon-1)(\tau^{\varepsilon}-\tau)(\tau^{\varepsilon}+\tau)\\B &= (\varepsilon-1)(\tau^{\varepsilon}-\tau)(\tau^{\varepsilon}+\tau)(\alpha(\tau^{\varepsilon}+\tau)+(\varepsilon-1)(\tau^{\varepsilon}-\tau)).\end{aligned}$$

Moreover, $B > 0$ always while $A = \alpha((1 + \alpha)\tau^2 + (1 - \alpha)\tau^{2\varepsilon} + (4\varepsilon - 2)\tau^{1+\varepsilon})$, implying that $A > 0$ too. Therefore, $\Delta_{\tau_{WH}} > 0$, when $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$, $\tau_{IH} = \tau_{IF} = 1$ and $\tau_{XH} = \tau_{XF} = 1$.

(2) To prove Eq. (2) recall that:

$$\begin{aligned}&\Delta_{\tau_{IH}}|_{\tau_{WH}=\tau_{WF}=\frac{\varepsilon-1}{\varepsilon},\tau_{IH}=\tau_{IF}=1,\tau_{XH}=\tau_{XF}=1}\\&=\frac{\alpha\tau^2(\tau-\alpha\tau+(\varepsilon-1+\alpha+\varepsilon)\tau^{\varepsilon})}{((1+\alpha-\varepsilon)\tau+(\alpha+\varepsilon-1)\tau^{\varepsilon})(\tau^2-\tau^{2\varepsilon})}\end{aligned}$$

with $\tau - \alpha\tau + (\varepsilon - 1 + \alpha + \varepsilon)\tau^{\varepsilon} = \tau(1 - \alpha) + (\alpha + 2\varepsilon - 1)\tau^{\varepsilon} > 0$ and $(1 + \alpha - \varepsilon)\tau + (\alpha - 1 + \varepsilon)\tau^{\varepsilon} = (\varepsilon - 1)(\tau^{\varepsilon} - \tau) + \alpha(\tau^{\varepsilon} + \tau) > 0$. As a consequence, $\Delta_{\tau_{IH}} < 0$ when $\tau_{WH} = \tau_{WF} = \frac{\varepsilon-1}{\varepsilon}$, $\tau_{IH} = \tau_{IF} = 1$ and $\tau_{XH} = \tau_{XF} = 1$.

(3) Finally, to see why Eq. (3) holds note that:

$$\Delta_{\tau_{XH}}\big|_{\tau_{WH}=\tau_{WF}=\frac{\varepsilon-1}{\varepsilon},\tau_{IH}=\tau_{IF}=1,\tau_{XH}=\tau_{XF}=1}=\frac{\alpha\tau^2\left((1+\alpha)\tau-(1+\alpha-2\varepsilon)\tau^{\varepsilon}\right)}{\left((1+\alpha-\varepsilon)\tau+(\alpha+\varepsilon-1)\tau^{\varepsilon}\right)\left(\tau^{2\varepsilon}-\tau^2\right))}$$

where $(1+\alpha)\tau-(1+\alpha-2\varepsilon)\tau^{\varepsilon}=(1+\alpha)\tau+(2\varepsilon-(1+\alpha))\tau^{\varepsilon}>0$ and $(1+\alpha-\varepsilon)\tau+(-1+\alpha+\varepsilon)\tau^{\varepsilon}=(\varepsilon-1)(\tau^{\varepsilon}-\tau)+\alpha(\tau^{\varepsilon}+\tau)>0$. Hence, $\Delta_{\tau_{XH}}>0$ if $\tau_{WH}=\tau_{WF}=\frac{\varepsilon-1}{\varepsilon}$, $\tau_{IH}=\tau_{IF}=1$ and $\tau_{XH}=\tau_{XF}=1$. ■

References

Bagwell, Kyle, Staiger, Robert W., 1999. An economic theory of GATT. Am. Econ. Rev. 89 (1), 215–248.

Bagwell, Kyle, Staiger, Robert W., 2009. Delocation and trade agreements in imperfectly competitive markets. NBER working paper, p. 15444.

Chari, V.V., Kehoe, Patrick J., 1999. Optimal fiscal and monetary policy. Handb. Macroecon. 1, 1671–1745.

Dixit, Avinash K., 1985. Tax policy in open economies. In: Auerbach, Allan J., Feldstein, Martin (Eds.), Handbook of Public Economics, vol. 1. Elsevier.

Dixit, Avinash K., Stiglitz, Joseph E., 1977. Monopolistic competition and optimum product diversity. Am. Econ. Rev. 67 (3), 297–308.

Flam, Harry, Helpman, Elhanan, 1987. Industrial policy under monopolistic competition. J. Int. Econ. 22 (1–2), 79–102.

Gros, Daniel, 1987. A note on the optimal tariff retaliation and the welfare loss from tariff wars in a framework with intra-industry trade. J. Int. Econ. 23 (3–4), 357–367.

Grossman, Gene M., Helpman, Elhanan, 1994. Protection for sale. Am. Econ. Rev. 84 (4), 833–850.

Grossman, Gene M., Helpman, Elhanan, 1995. Trade wars and trade talks. J. Polit. Econ. 103, 675–708.

Helpman, Elhanan, Krugman, Paul, 1989. Trade Policy and Market Structure. MIT Press.

Johnson, Harry G., 1953–1954. Optimum tariffs and retaliation. Rev. Econ. Stud. 21 (2), 142–153.

Krugman, Paul, 1980. Scale economics, product differentiation, and pattern of trade. Am. Econ. Rev. 70 (5), 950–959.

Lucas, Robert E., Stokey, Nancy L., 1983. Optimal fiscal and monetary policy in an economy without capital. J. Monet. Econ. 12, 55–93.

Ossa, Ralph, 2011. A new trade theory of GATT/WTO negotiations. J. Polit. Econ. 119 (1), 122–152.

Venables, Anthony, 1987. Trade and trade policy with differentiated products: a Chamberlinian–Ricardian model. Econ. J. 97 (387), 700–717.

WTO, 2006. World Trade Report 2006. World Trade Organization.