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## Time-varying correlations in global real estate markets: A multivariate GARCH with spatial effects approach

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### Abstract

The present study applies the multivariate generalized autoregressive conditional heteroscedasticity (MGARCH) with spatial effects approach for the analysis of the time-varying conditional correlations and contagion effects among global real estate markets. A distinguishing feature of the proposed model is that it can simultaneously capture the spatial interactions and the dynamic conditional correlations compared with the traditional MGARCH models. Results reveal that the estimated dynamic conditional correlations have exhibited significant increases during the global financial crisis from 2007 to 2009, thereby suggesting contagion effects among global real estate markets. The analysis further indicates that the returns of the regional real estate markets that are in close geographic and economic proximities exhibit strong co-movement. In addition, evidence of significantly positive leverage effects in global real estate markets is also determined. The findings have significant implications on global portfolio diversification opportunities and risk management practices.

**Keywords:** Time-varying correlation; MGARCH; Spatial effects; Contagion effects; Real estate markets

### 1. Introduction

The analysis of the linkages between the volatilities and co-volatilities of the global financial markets, especially the global real estate markets, is a critical issue on global portfolio diversification opportunities and risk management practices. Estimations of correlations between the asset returns are relevant for predicting time-varying beta coefficients in capital asset pricing model, obtaining an optimal estimation of hedge ratio, and forecasting the Value-at-Risk of a portfolio strategy. Thus, studies on the estimation of time-varying correlations across financial assets, mostly by modeling the time-series structure of financial asset returns and volatilities, have increased [1–5]. Notably, a popular approach to the modeling of multivariate asset volatility dynamics is the conditional variance–covariance matrix estimation method. Over the past three decades, various parameterizations of the

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conditional variance–covariance matrix have been developed in the multivariate generalized autoregressive conditional heteroscedasticity (MGARCH) model applied in the literature.

One of the most popular models for the estimation of multivariate asset correlation dynamics is that of Engle [6], who proposed a dynamic conditional correlation GARCH (DCC–GARCH) model in which the conditional correlation matrix is time varying. Similarly, Tse and Tsui [7] formulated the conditional correlation matrix as a weighted average of the past correlations. The basic specification of the DCC–GARCH modeling approach includes two aspects. One aspect is the univariate variance process, and another aspect is the time-varying correlation process. Several recent extensions of the DCC–GARCH model developed by Engle [6] have been developed to provide more flexibility to the modeling of the second aspect, i.e., the time-varying correlation process [6]. Several recent typical examples are the corrected DCC model by Aielli [8], the non-scalar DCC model by Bauwens, Grigoryeva, and Ortega [9], the volatility threshold DCC model by Kasch and Caporin [10], and the asymmetric DCC model by Tamakoshi and Hamori [11]. For recent reviews of MGARCH models, see Ref. [12] and for other applications of such models, see Refs. [13–15].

These MGARCH models provide a useful tool for understanding how financial volatilities move together over time and across markets. However, a regional financial market can be characterized by locality and segregation on the basis of the fixed location of countries. Financial applications typically consider the correlations between the pairs of returns of financial indices observed in different countries, without including the spatial effects in the model adopted. Thus, a natural extension is to incorporate the spatial effects in the appropriate MGARCH models. However, two kinds of problems must be solved before we can achieve the idea of such extension. One problem is on dimensionality and the other is the choice of an appropriate MGARCH model that is able to include spatial effects. If we directly incorporate the spatial effects into the MGARCH models, providing feasible estimates of the models would be difficult because they contain numerous unknown parameters and require the conditional covariance matrix to be positive definite [12]. To solve the dimensionality problem, Otranto [16] proposed a clustering algorithm to detect groups of homogeneous time series in terms of one extensively used DCC model. Following this line of thought, a natural choice of appropriate MGARCH models is the family of the DCC models. Moreover, as stressed by Ref. [17], the most appropriate DCC model specification that can include spatial effects is the DCC model developed by Tse and Tsui [7]. Therefore, the starting point of this article is the contribution of Otranto, Mucciardi, and Bertuccelli [17], which introduced spatial effects to the analysis of dynamic conditional correlation models, and thus to the

estimation and measurement of contagion effects. However, we take a step further in several aspects.

First, we considered a compound spatial weight matrix, which is a combination of the geographic distance and economic distance spatial weight matrices used by Case, Rosen, and Hines [18] and Zhu, Füss, and Rottke [19] to effectively demonstrate the effects of geographic and economic indicators on the global real estate markets. Our main motivation for studying spatial effects is that a country's real estate market is prone to be affected by its nearby countries not only because of geographical proximity but also because of economic and financial similarities [19–22]. In terms of the specification of the geographic distance spatial weight matrix, we regarded all spatial units as the neighbor of each other and made use of a Gaussian kernel function form because we believe that the interactions between nearer neighbors are larger than that of those between farther neighbors. Moreover, we employed the Mahalanobis distance to construct the economic distance spatial weight matrix using relevant economic indicators, such as per-capita gross domestic products (GDP), population, national unemployment rates, and imports and exports of the country.<sup>1</sup> We also believe that regions with similar economic development conditions will exhibit strong co-movement because the Mahalanobis distance considers the correlations of economic indicators. Notably, the kernel bandwidth is the key controlling parameter and can be specified either by a fixed bandwidth or by an adaptive bandwidth [19]. In contrast to Ref. [19], we chose an adaptive bandwidth rather than a fixed bandwidth. The optimal adaptive bandwidth is determined by the maximum log-likelihood function value and the significance of the regression coefficients (see Section 2.3 for details).

Second, another important contribution of the study is on the methodological side. We started by generalizing the model of Ref. [17] with the introduction of an augmented ARMA (1, 1)–GJR–GARCH (1, 1) model specification. With this specification, we can ascertain whether leverage effects, i.e., asymmetric responses to positive and negative shocks, exist among global real estate markets and assess how the recent global financial crisis (GFC) from 2007 to 2009 influences the estimated dynamic correlations across global real estate markets. The motivation for the analysis of the influence of GFC is that the outbreak of GFC from 2007 to 2009 once again showed that global financial markets, particularly the global real

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<sup>1</sup> Mahalanobis distance is defined as a dissimilarity measure between two random column vectors  $x$  and  $y$  of the same distribution with the covariance matrix  $V$ :  $d(x, y) = \sqrt{(x - y)^T V^{-1} (x - y)}$ , where the superscript  $T$  denotes the matrix transpose and the matrix  $V^{-1}$  represents the inverse of the matrix  $V$ . This dissimilarity measure is extensively used in cluster analysis and other classification techniques (see e.g., Ref. [19]).

estate markets, are more interconnected among cities, states, regions, and countries [22–24]. In fact, an increasing amount of recent studies on the spread of financial crises have identified significant increases in the dynamic conditional correlations among stock markets (Refs. [23, 25–26]), real estate markets (Refs. [27–28]), currency markets (Ref. [11]), and even real economy sectors (Ref. [29]). One representative example is Tamakoshi and Hamori [11], who showed that significant increases in cross-market correlations tend to occur particularly during crisis. We attempted to examine the existence of such increased correlations among global real estate markets by modeling the estimated DCC using an ARMA (1, 1)–GJR–GARCH (1, 1) model with dummy variables in the variance process. Specifically, we employed a crisis dummy to identify how the GFC from 2007 to 2009 affected the estimated dynamic conditional correlations.

We also presented a portfolio in-sample analysis that shows the potential benefits deriving from our proposed approach compared with the traditional MGARCH models. The two adopted portfolios are minimum-variance and hedged portfolios, which include two assets. Our empirical results highlighted the role of time-varying correlations and the importance of the GFC from 2007 to 2009 on portfolio performances. The motivation for comparing the alternative models in terms of portfolio-management performance criteria rather than statistical criteria lies in the fact that global portfolio diversifications are becoming increasingly important due to the rapid development in increasing integration among global financial markets (Refs. [30–31]). These results hold important implications for global portfolio managers. For instance, if the correlations of the global real estate markets are driven not only by geographic proximities but also by economic similarities, then formulating asset allocation strategies by considering only the geographic adjacency factors regardless of the economic similarity factors will lead to biases in investment decision making.

The remainder of this paper is organized as follows. In Section 2, we briefly introduce the proposed MGARCH model with spatial effects, describe the maximum likelihood estimation procedures, and discuss the specification rule of the spatial weight matrix. Section 3 describes the data and discusses the empirical results. Section 4 compares the performance of the alternative models in the asset allocation framework. Conclusions are presented in Section 5.

## 2. Empirical methods

### 2.1. Model specifications

We use a MGARCH model with spatial effects to assess the time-varying correlations among global real estate markets. Let  $y_{i,t}$  denote the return on asset  $i$  at time  $t$ . We consider

the volatilities and correlations for  $N$  assets with their returns collected in the  $N$ -dimensional vector  $y_t = (y_{1,t}, y_{2,t}, \dots, y_{N,t})'$  for  $t = 1, 2, \dots, T$ . We assume that the return vector  $y_t$  is described by the stochastic vector process as follows:

$$y_t = \mu_t + \varepsilon_t, \quad (1)$$

where  $\mu_t = \mu_t(\theta)$  is an  $N \times 1$  vector, which is parameterized by a vector  $\theta$  and represents the conditional mean of  $y_t$ , i.e.,  $E_{t-1}(y_t | I_{t-1}) = \mu_t$ ,  $E(\cdot | \cdot)$  denotes a conditional expectation operator,  $I_{t-1}$  is the information set at time  $t-1$ , and  $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t}, \dots, \varepsilon_{N,t})'$  is an  $N \times 1$  vector of unpredictable residuals.

Assuming the predictable conditional mean and volatility of  $y_t$ , Eq. (1) is rewritten as follows:

$$\begin{aligned} y_t &= \mu_t + H_t^{1/2} z_t, \\ \mu_t &= E_{t-1}(y_t | I_{t-1}), \quad E(z_t) = 0, \quad \text{Var}(z_t) = I_N. \end{aligned} \quad (2)$$

where  $H_t$  is an  $N \times N$  positive definite matrix representing the conditional variance–covariance matrix of  $y_t$ , the matrix  $H_t^{1/2}$  is the Cholesky factorization of the matrix  $H_t$ ,  $\text{Var}(\cdot)$  is a variance operator,  $z_t$  is a vector of independent, identically distributed, and standardized residuals, and  $I_N$  is an identity matrix of order  $N$ .

To refine the specification of the conditional mean vector  $\mu_t$ , we assume that the conditional mean process is modeled separately for each real estate index return to allow us to estimate each autoregressive moving average (ARMA) model independently as follows:

$$y_t = \kappa + \sum_{i=1}^p a_i y_{t-i} + \sum_{j=1}^q b_j \varepsilon_{t-j} + \varepsilon_t, \quad (3)$$

where  $\kappa$  is an  $N \times 1$  vector of the unknown parameters,  $a_i$  and  $b_j$ , are diagonal matrices containing unknown parameters. We fitted the best of the univariate ARMA ( $p$ ,  $q$ )–GJR–GARCH (1, 1) models to each real estate index return series based on the Bayesian information criterion (BIC).<sup>2</sup>

As for the variance–covariance matrix process  $H_t$ , two popular methods can be applied, namely, modeling the conditional variance–covariance matrix  $H_t$  directly (e.g., the vector error correction [VEC]–GARCH model of Ref. [32]) and modeling the conditional correlation

<sup>2</sup> For the sake of brevity and based on the BIC, we adopted the augmented ARMA (1, 1)–GJR–GARCH (1, 1) model to fit each national property index return series.

matrix indirectly using a correlation matrix (e.g., DCC–GARCH models). In this study, we adopted the latter method and assume that the multivariate conditional variance–covariance matrix  $H_t$  is specified as follows:

$$H_t = E(\varepsilon_t \varepsilon_t') = D_t R_t D_t, \quad (4)$$

where  $D_t = D_t(\phi)$  is an  $N \times N$  diagonal matrix of the time-varying volatilities  $\sigma_{i,t}$ , parameterized by a vector  $\phi$ ,  $R_t = R_t(\varphi)$  is the conditional correlation matrix, parameterized by a vector  $\varphi$ . To examine the effect of the recent GFC from 2007 to 2009 on conditional volatilities, the element in  $D_t$  is assumed to follow the augmented conditional variance equation given as follows:

$$\sigma_{i,t}^2 = \omega_{i0} + \omega_{i1} \varepsilon_{i,t-1}^2 + \eta_i I_{i,t} \varepsilon_{i,t-1}^2 + \omega_{i2} \sigma_{i,t-1}^2 + \xi_i Crisis_t, \quad i = 1, 2, \dots, N, \quad (5)$$

where  $I_{i,t} = 1$  if  $\varepsilon_{i,t-1} < 0$ , and  $I_{i,t} = 0$  otherwise;  $\phi_i = (\omega_{i0}, \omega_{i1}, \omega_{i2}, \eta_i, \xi_i)$  is the unknown coefficient vector; the dummy variable  $Crisis_t$  is included into the variance equation from August 9, 2007 to November 4, 2009.  $\varepsilon_{i,t-1} < 0$  represents bad news or negative shocks, whereas  $\varepsilon_{i,t-1} > 0$  represents good news or positive shocks. Thus, the effects of these two factors on conditional variance  $\sigma_{i,t}^2$  are different. Particularly, the effect of good news on conditional variance  $\sigma_{i,t}^2$  is  $\omega_{i1}$ , while the influence of bad news on conditional variance  $\sigma_{i,t}^2$  is  $\omega_{i1} + \eta_i$ . Therefore, the significantly positive sign of  $\eta_i$  suggests that leverage effects exist in real estate markets. The coefficients must satisfy the following constraints to ensure positive and stable conditional variances:

$$\omega_{i0} > 0, \omega_{i1} > 0, \omega_{i2} > 0 \quad \text{and} \quad \omega_{i1} + \frac{1}{2}\eta_i + \omega_{i2} > 0, \quad (6)$$

By obtaining the conditional standard deviations from Eq. (5), we can model the correlation of returns  $y_t$ , which is the same as the correlation of the residuals  $z_t$ . The evolution of the multivariate dynamic conditional correlation model is provided as follows:

$$R_t = (1 - \alpha - \beta - \gamma)R + \alpha R_{t-1} + \beta \Psi_{t-1} + \gamma W \circ \Psi_{t-1}, \quad (7)$$

where  $W$  is a spatial weight matrix, which is fixed subjectively using several criteria and will be discussed in detail in the following section; the  $\circ$  symbol represents element-by-element product multiplication;  $\Psi_{t-1}$  is a square positive definite matrix; and  $\alpha, \beta, \gamma$  are unknown scalar parameters. The positive definiteness of  $R_t$  is ensured by the following conditions:

$$\alpha \geq 0, \beta \geq 0, \gamma \geq 0 \quad \text{and} \quad \alpha + \beta + \gamma < 1, \quad (8)$$

Equation (7) is an extension of DCC model developed by Tse and Tsui [7], which incorporates spatial effects into the DCC structure. We can obtain the initial model introduced by Ref. [7] if  $\gamma \equiv 0$ . However, a minor distinction between the two models lies in the fact that the correlation matrix  $R$  can be estimated by the sample correlation of the standardized residuals in the original specification of Ref. [7] under the hypothesis that the expected value of  $R_t$  and  $\Psi_t$  is  $R$ , while in our model such assumption on the correlation matrix  $R$  is impossible because the expected value of  $(W \circ \Psi_{t-1})$  is not  $R$ . Specifically, the correlation matrix  $R$  in Eq. (7) is a positive definite matrix with  $N(N-1)/2$  unknown parameters. Moreover, we suggested adopting the following parameterized specification of the correlation matrix  $R$  to avoid the dimensionality problem due to a larger  $N$ .

$$R = cc' + \Delta, \quad (9)$$

where  $c = (c_1, c_2, \dots, c_N)'$  is a vector of unknown parameters with  $0 < |c_i| \leq 1$  for each  $i$ , and  $\Delta$  is a diagonal matrix with the elements on the diagonal equal to  $(1 - c_i^2)$ . Such a specification provides an invertible matrix with its diagonal elements equal to 1 and the off-diagonal elements between  $-1$  and  $1$ . Therefore, the correlation matrix  $R$  is an  $N \times N$  correlation matrix containing  $N$  unknown parameters.

Denoting  $\Psi_t = \{\psi_{ij,t}\}$  and following Ref. [7], the specification for the elements in the matrix  $\Psi_{t-1}$  is as follows:

$$\psi_{ij,t-1} = \frac{\sum_{m=1}^M z_{i,t-m} z_{j,t-m}}{\sqrt{\left(\sum_{m=1}^M z_{i,t-m}^2\right) \left(\sum_{m=1}^M z_{j,t-m}^2\right)}}, \quad 1 \leq i < j \leq N, \quad (10)$$

where  $z_{i,t}$  is the standardized residuals for each  $i$  and  $M$  is a positive integer. To guarantee that the matrix  $\Psi_t$  is the positive definite, a necessary condition is  $M \geq N$ .

## 2.2. Model estimation method

The estimation of parameters is usually done in two steps by quasi-maximum likelihood, assuming that the innovations  $z_t$  are Gaussian. We split the joint log-likelihood function of the proposed model into two parts and maximized it sequentially following the lines of Refs. [6–7]. Assuming the conditional normality of the innovations  $z_t$ , the joint log-likelihood function of the model has the following form:



$$\begin{aligned}
L(\theta, \phi, \varphi) &= -\frac{1}{2} \sum_{t=1}^T \left[ N \log(2\pi) + \log |H_t| + \varepsilon_t' H_t^{-1} \varepsilon_t \right] \\
&= -\frac{1}{2} \sum_{t=1}^T \left[ N \log(2\pi) + \log |D_t R_t D_t| + (y_t - \mu_t)' D_t^{-1} R_t^{-1} D_t^{-1} (y_t - \mu_t) \right],
\end{aligned} \tag{11}$$

where  $\theta = (\kappa, a_1, \dots, a_p, b_1, \dots, b_q)$  stands for the coefficient vector in Eq. (3),  $\phi = (\phi_1, \phi_2, \dots, \phi_N)$  represents the set of all unknown parameters to be estimated in Eq. (5),  $\varphi = (\alpha, \beta, \gamma)$ ,  $N$  is the number of dependent variables, and  $T$  is the number of observations.

Let  $z_t = D_t^{-1}(y_t - \mu_t)$  be the standardized residual vector. Equation (13) can be rewritten as follows:

$$\begin{aligned}
L(\theta, \phi, \varphi) &= -\frac{1}{2} \sum_{t=1}^T \left[ N \log(2\pi) + \log |D_t|^2 + \log |R_t| + z_t' R_t^{-1} z_t \right] \\
&= L_v(\theta, \phi) + L_c(\theta, \phi, \varphi),
\end{aligned} \tag{12}$$

where the volatility part is as follows:

$$L_v(\theta, \phi) = -\frac{1}{2} \sum_{t=1}^T \left[ N \log(2\pi) + \log |D_t|^2 + z_t' z_t \right], \tag{13}$$

and the conditional correlation part is as follows:

$$L_c(\theta, \phi, \varphi) = -\frac{1}{2} \sum_{t=1}^T \left[ \log |R_t| + z_t' R_t^{-1} z_t - z_t' z_t \right]. \tag{14}$$

First, we estimate the parameters of the conditional mean and volatility processes, i.e., parameters in Eq. (13), where the estimates of  $(\theta, \phi)$  are given as follows:

$$(\hat{\theta}, \hat{\phi}) = \arg \max_{\theta, \phi} L_v(\theta, \phi). \tag{15}$$

Once  $\hat{\theta}$  and  $\hat{\phi}$  are obtained, the unknown parameters of the conditional correlation process, i.e., parameters in Eq. (14), are estimated as follows:

$$\hat{\varphi} = \arg \max_{\varphi} L_c(\hat{\theta}, \hat{\phi}, \varphi). \tag{16}$$

### 2.3. Specifications of spatial weight matrices

#### 2.3.1. Compound spatial weight matrix definition

In spatial econometrics, the spatial weight matrix is a core element that will reflect the possible relations between spatial locations. Thus, modeling the spatial volatility spillovers between the markets need to correctly specify first the spatial weight matrix. Generally, a spatial weight matrix can be defined as a spatial correlation function of geographic, economic, or social distances between cross-sectional units [18, 22]. Given that one aims of the current

study is to choose the most appropriate spatial weight matrix that can correctly capture the spatial interaction patterns among real estate markets, a spatial weight matrix, which can reflect the geographic relations and economic linkages between the markets, will be a reasonable choice. Therefore, we follow Refs. [18–19] and use the following compound spatial weight matrix, which is a combination of the geographic and economic distance spatial weight matrices.

$$W = aW_G + (1-a)W_E, \quad 0 \leq a \leq 1, \quad (17)$$

where  $W_G$  represents the geographic distance spatial weight matrix,  $W_E$  denotes the economic distance spatial weight matrix, and  $a$  is the proportion of the geographic weight in the compound spatial weight matrix.

With respect to the geographic distance spatial weight matrix, the longer distance the two markets hold, the more interaction cost, such as travel time, will be. Thus, when constructing  $W_G$ , we regard all spatial locations as the neighbor of each other but consider the interactions between nearer neighbors to be larger than that between farther neighbors. As such, we used a Gaussian kernel function form, as follows:

$$w_{ij}^G = \begin{cases} \exp\left\{-0.5\left[d_{ij}^G/d_G\right]^2\right\} & i \neq j, \\ 1 & i = j, \end{cases} \quad (18)$$

where  $w_{ij}^G$  represents the geographic weight between country  $i$  and country  $j$ ,  $d_{ij}^G$  is the straight-line distance between country  $i$  and country  $j$ , and parameter  $d_G$  refers to the kernel bandwidth and reflects the strength of diffusion between two countries. The kernel bandwidth is the key controlling parameter and can be specified either by a fixed bandwidth or by an adaptive bandwidth [33]. In our empirical illustration, we chose an optimal adaptive bandwidth, which is determined by the maximum log-likelihood function value and the significance of the regression coefficients (see Subsection 2.3.2 for details).

In addition, we supported the view that the strong spatial dependencies exist among those countries with more similar economic development conditions. Thus, following Ref. [19], we adopted the Mahalanobis distance to construct our economic distance spatial weight matrix using the macro-economic relevant indicators, such as per-capita GDP, population, national unemployment rates, and imports and exports of the given country. In particular, the element in the economic distance spatial weight matrix  $W_E$  is defined as follows:

$$w_{ij}^E = \begin{cases} \exp\left\{-\left[d_{ij}^E/d_E\right]^2\right\} & i \neq j, \\ 1 & i = j, \end{cases} \quad (19)$$

where parameter  $d_E$  is the kernel bandwidth, whose definition is similar to that of parameter  $d_G$ ;  $d_{ij}^E = \sqrt{(E_i - E_j)'V^{-1}(E_i - E_j)}$  measures the Mahalanobis distance between two countries;  $E_i$  and  $E_j$  represent the economic indicator vector for countries  $i(i=1,2,\dots,N)$ , and  $j(j=1,2,\dots,N)$ , respectively;  $V$  denotes the sample variance–covariance matrix; and the matrix  $V^{-1}$  is the inverse of the matrix  $V$ .

The economic relevant indicators adopted in this study consist of per-capita GDP, population, national unemployment rates, and imports and exports of the countries. However, in contrast with the work in Ref. [19], these variables are the median values covering the period from 2002 to 2014. Table 1 provides the descriptive statistics of these variables. The data were taken from the Chinese stock market and accounting research (CSMAR) financial database. This database is compiled by Shenzhen GTA Information Technology Company Limited; it is a famous economic database containing most of the historical and recent trading data for listed Chinese firms.

**Table 1**  
Descriptive statistics of the variables from 2002 to 2014.

Variable	Mean	Median	Std. deviation	Minimum	Maximum
Economic distance variables					
Per-capita GDP(in \$)	377.809	402.769	176.690	10.090	880.293
Populations (in millions)	59.302	33.250	74.707	4.110	318.860
Unemployment rates (in %)	7.440	6.300	5.012	2.600	27.200
Imports (in \$ billions)	431.743	357.417	362.083	29.723	1620.532
Exports (in \$ billions)	474.866	359.000	475.559	29.267	2412.547
Geographic distance variables					
Distances (in miles)	4813.1	5405	2946.8	108	10552

Notes: The data of the economic distance variables were taken from CSMAR, the data of the geographic distance variables were extracted from timeanddate.com (<http://www.timeanddate.com>).

### 2.3.2. Estimations of the optimal kernel bandwidth parameters

To estimate the optimal kernel bandwidth parameters  $d_G$  and  $d_E$  and the proportion parameter  $a$ , we adopt the following iterative process.

Denoting  $\lambda = (a, d_G, d_E)$ , the compound spatial weight matrix  $W$  is parameterized by vector  $\lambda$ . Then, we substitute  $W$  in Eq. (17) into Eq. (7), and we obtain the following equation:

$$\tilde{R}_t = (1 - \alpha - \beta - \gamma)R + \alpha\tilde{R}_{t-1} + \beta\Psi_{t-1} + \gamma[aW_G + (1-a)W_E] \circ \Psi_{t-1}, \quad (20)$$

Notably, matrix  $\tilde{R}_t$  is parameterized by vectors  $\phi$  and  $\lambda$ . Next, we substitute the matrix  $\tilde{R}_t$  in Eq. (20) into Eq. (12), and the Eq. (12) can be rewritten as follows:

$$\begin{aligned} \tilde{L}(\theta, \phi, \varphi, \lambda) &= -\frac{1}{2} \sum_{t=1}^T \left[ N \log(2\pi) + \log |D_t|^2 + \log |\tilde{R}_t| + z_t' \tilde{R}_t^{-1} z_t \right] \\ &= L_v(\theta, \phi) + \tilde{L}_c(\theta, \phi, \varphi, \lambda), \end{aligned} \quad (21)$$

where the volatility part  $L_v(\theta, \phi)$  has been described in Eq. (13) and the conditional correlation part  $\tilde{L}_c(\theta, \phi, \varphi, \lambda)$  has the following form:

$$\tilde{L}_c(\theta, \phi, \varphi, \lambda) = -\frac{1}{2} \sum_{t=1}^T \left[ \log |\tilde{R}_t| + z_t' \tilde{R}_t^{-1} z_t - z_t' z_t \right]. \quad (22)$$

Therefore, following the lines of Section 2.2, we first obtain the estimates  $(\hat{\theta}, \hat{\phi})$  by maximizing the log-likelihood function value  $L_v(\theta, \phi)$  in Eq. (13). Once  $(\hat{\theta}, \hat{\phi})$  are obtained, the unknown parameters  $(\varphi, \lambda)$  in Eq. (22) are estimated as follows:

$$(\hat{\varphi}, \hat{\lambda}) = \arg \max_{\varphi, \lambda} \tilde{L}_c(\hat{\theta}, \hat{\phi}, \varphi, \lambda). \quad (23)$$

To obtain the estimations of all the unknown parameters, we first translate the constrained maximization problem in Eqs. (15) and (23) into the unconstrained minimization problem. Subsequently, we use the “fminunc” function from the Matlab optimization toolbox. This function finds an unconstrained minimum of a scalar function of several variables.

### 3. Empirical illustration

#### 3.1. Data description

Our data comprise daily prices of real estate index securities in 15 countries for four geographical regions (i.e., Asia Pacific, Europe, Africa, and Latin America) denominated in dollars. The data extracted from the Global Property Research (GPR) database (<http://www.globalpropertyresearch.com>) covers the period from January 3, 2002 to May 31, 2016, thereby leading to a sample size of 3,709 observations. In particular, the data used for empirical analysis are the historical data of 15 country indices from the GPR 250 index database. The GPR 250 index is a market-weighted total return index that is composed of the 250 most liquid listed property companies in the country concerned and effectively represents of the global real estate market. Thus, the GPR database is a sound choice for investigating the time-varying correlations across real estate markets from an international perspective.

Another motivation for collecting data from the database is that it has not been excessively used and may avoid a substantial risk of data-snooping due to the same datasets [28].

The study covers the following 15 national property markets: Australia (AUS), Belgium (BEL), Canada (CAN), France (FRA), Germany (GER), Hong Kong (HKG), Japan (JPN), Netherlands (NED), Philippines (PHI), Singapore (SIN), South Africa (RSA), Sweden (SWE), Switzerland (SWZ), the United Kingdom (GBR), and the United States (US). In this study, we chose these country indices for research because only such indices have a complete sample set in the GPR 250 index during the sample period from 2002 to 2016.

For each property index, the continuously compounded return is estimated as  $r_t = 100 \times [\ln p_t - \ln p_{t-1}]$ , where  $p_t$  is the closing price on day  $t$ . Following, for instance, Ref. [34], we use two-day rolling-average returns in our analysis. Two-day average returns are mindfully utilized that the markets around the world are not open at the same periods.

Table 2 presents the country indices investigated in the current study along with descriptive statistics on the two-day rolling average property index returns. We divided the countries into four different regions, namely, Asia Pacific, Europe, Africa, and Latin America, which helps in reflecting the vast differences in property markets that are located in different regions/areas. Table 2 clearly demonstrates that all the real estate markets exhibit an average positive return. As shown by the Jarque–Bera tests, all the property index return series exhibit non-normal characteristics. The characteristics of left skewed and fat tails are also exhibited for all (but four) the property index return series. In addition, the Ljung–Box test shows that all the property index return series exhibit volatility clustering.

**Table 2**  
Descriptive statistic on two-day rolling average property index returns.

Region/country	Mean	Std. dev.	Minimum	Maximum	Skewness	Kurtosis	Jarque–Bera	LB (20)
<i>Asia Pacific</i>								
AUS	0.039	1.183	-10.160	8.690	-0.621	10.700	9400.78***	1089.16***
HKG	0.036	1.121	-5.983	5.390	0.021	5.616	1058.07***	1041.84***
JPN	0.045	1.316	-9.189	7.904	0.184	6.456	1866.79***	903.07***
PHI	0.057	1.465	-7.983	7.529	0.026	5.085	672.46***	977.01***
SIN	0.045	1.084	-6.225	6.650	-0.001	6.963	2426.70***	1034.22***
<i>Europe</i>								
BEL	0.038	0.869	-5.495	5.316	-0.160	6.659	2084.55***	961.67***
FRA	0.064	1.151	-5.767	5.317	-0.320	5.786	1263.10***	1020.33***
GER	0.014	1.285	-9.435	6.522	-0.577	7.745	3685.50***	1216.48***
NED	0.042	1.087	-6.263	5.348	-0.579	6.447	2042.74***	1192.47***
SWE	0.071	1.293	-7.114	6.549	-0.392	6.645	2147.64***	1072.21***
SWZ	0.049	0.753	-4.658	6.766	0.207	8.043	3956.73***	973.88***

GBR	0.031	1.217	-8.266	7.438	-0.382	8.535	4825.08***	1095.81***
<i>Africa</i>								
RSA	0.066	1.198	-14.383	5.946	-0.886	11.784	12408.60***	1139.65***
<i>Latin America</i>								
CAN	0.050	0.929	-9.777	5.452	-1.075	12.971	16079.26***	1366.43***
US	0.036	1.241	-11.561	10.069	-0.585	16.998	30492.77***	794.96***

Notes: The table presents descriptive statistics for the fifteen property indices' returns during the sample period (January 3, 2002–May 31, 2016). *LB* (20) refers to Ljung–Box statistic with up to 20-day lags. The signs \*\*\*, \*\*, and \* denote significant coefficients at 1%, 5%, and 10% levels, respectively.

### 3.2. Estimation results

We first fitted the best of the univariate ARMA ( $p, q$ )–GJR–GARCH (1, 1) models to each series of the property index returns. Based on the BIC value, we found that the augmented ARMA (1, 1)–GJR–GARCH (1, 1) is most appropriate to fit the data set.

Table 3 presents the estimation results of the mean and the augmented variance equations, i.e., the estimation results of the Eqs. (3) and (5) of the DCC's two-step estimation procedure. In Table 3,  $\omega_1$  and  $\omega_2$  represent the ARCH and GARCH effects, while  $\eta$  represents the leverage effects. For all of the 15 property markets, the coefficients of the GARCH components ( $\omega_1$  and  $\omega_2$ ) are statistically significant and standard ( $\omega_1 + \omega_2 < 1$ ) in terms of magnitude. Coefficient  $\eta$  is statistically significant for all property markets, indicating the evidence of leverage effects in the real estate markets. The existence of leverage effects suggests that in real estate markets, investors react more strongly to bad news than to good news. Moreover, the p-values of the Ljung–Box statistics, *LB* (20) is much larger than 1% for all 15 property markets, thereby suggesting no autocorrelation up to order 20 for standardized residuals of each series.

In addition, to examine the effects of GFC from 2007 to 2009 on the conditional variances, we used the dating method suggested by Refs. [11, 20] and specified the phase of GFC from August 9, 2007 to November 4, 2009. The reason for tracing the beginning of GFC to August 9, 2007 is that the BNP Paribas, a major global investment bank, suspended its funds affected by the US's subprime mortgage liabilities on August 9, 2007. We identified November 4, 2009 as the major breakpoint for the end of GFC considering that the outbreak of the European sovereign debt crisis (ESDC) began on November 5, 2009, triggered by the Greek debt problem, and thus our post-crisis sample can partially reflect the influences of ESDC on the parameter estimation results. The results indicate that all of the coefficients  $\xi$  weights on the recent GFC are positive and statistically significant at 1% level, thereby suggesting that the conditional variances increase significantly during the GFC period.

**Table 3**

Estimation results from the augmented ARMA (1, 1)–GJR–GARCH (1, 1) model.

Region	Mean Equations			Variance Equations					$R^2$	LL	LB(20)
	$\kappa$	$a_1$	$b_1$	$\omega_0$	$\omega_1$	$\omega_2$	$\eta$	$\xi$			
<i>Asia Pacific</i>											
AUS	0.070*** (0.019)	0.020 (0.018)	0.988*** (0.003)	0.008*** (0.002)	0.016* (0.009)	0.920*** (0.009)	0.033*** (0.010)	0.082*** (0.014)	0.514	-3650	0.142
HKG	0.043* (0.024)	0.075*** (0.017)	0.998*** (0.001)	0.006*** (0.001)	0.030*** (0.006)	0.931*** (0.007)	0.015** (0.006)	0.049*** (0.009)	0.527	-3705	0.336
JPN	0.025 (0.021)	0.042** (0.020)	0.994*** (0.002)	0.011*** (0.003)	0.059*** (0.009)	0.909*** (0.010)	0.017** (0.009)	0.037*** (0.012)	0.505	-4517	0.971
PHI	0.074*** (0.028)	0.009 (0.018)	0.990*** (0.002)	0.048*** (0.014)	0.047*** (0.013)	0.881*** (0.025)	0.024* (0.013)	0.050*** (0.016)	0.500	-5231	0.108
SIN	0.054*** (0.016)	0.049** (0.020)	0.996*** (0.001)	0.005*** (0.001)	0.041*** (0.010)	0.919*** (0.012)	0.010* (0.006)	0.058*** (0.012)	0.515	-3499	0.330
<i>Europe</i>											
BEL	0.058*** (0.017)	-0.032* (0.018)	0.956*** (0.006)	0.008*** (0.002)	0.029*** (0.010)	0.910*** (0.013)	0.011** (0.004)	0.068*** (0.013)	0.468	-3088	0.801
FRA	0.088*** (0.018)	0.025 (0.017)	0.965*** (0.004)	0.012*** (0.002)	0.023*** (0.008)	0.915*** (0.011)	0.021*** (0.008)	0.072*** (0.012)	0.495	-4005	0.665
GER	0.051*** (0.020)	0.062*** (0.019)	0.989*** (0.003)	0.007*** (0.001)	0.041*** (0.008)	0.929*** (0.008)	0.025*** (0.008)	0.031*** (0.010)	0.532	-4101	0.331
NED	0.070*** (0.019)	0.066*** (0.016)	0.972*** (0.004)	0.007*** (0.001)	0.019** (0.009)	0.917*** (0.010)	0.017*** (0.006)	0.082*** (0.013)	0.518	-3554	0.392
SWE	0.106*** (0.018)	0.032* (0.017)	0.990*** (0.002)	0.010*** (0.002)	0.024*** (0.008)	0.917*** (0.009)	0.031*** (0.010)	0.074*** (0.012)	0.514	-4174	0.171
SWZ	0.070*** (0.013)	-0.030* (0.017)	0.991*** (0.002)	0.009*** (0.002)	0.027*** (0.007)	0.907*** (0.016)	0.016*** (0.005)	0.049*** (0.014)	0.491	-2685	0.154
GBR	0.060*** (0.019)	0.049*** (0.018)	0.982*** (0.003)	0.010*** (0.002)	0.031*** (0.009)	0.908*** (0.011)	0.038*** (0.013)	0.070*** (0.014)	0.513	-3799	0.551
<i>Africa</i>											
RSA	0.083*** (0.023)	0.082*** (0.018)	0.975*** (0.004)	0.020*** (0.004)	0.029*** (0.009)	0.896*** (0.015)	0.010** (0.006)	0.080*** (0.015)	0.531	-4179	0.380
<i>America</i>											
CAN	0.055*** (0.012)	0.096*** (0.030)	0.999*** (0.001)	0.005*** (0.001)	0.044** (0.011)	0.892*** (0.012)	0.010** (0.004)	0.091*** (0.016)	0.565	-2689	0.316
US	0.045*** (0.015)	-0.003 (0.019)	1.000*** (0.000)	0.009*** (0.001)	0.071*** (0.014)	0.855*** (0.013)	0.059*** (0.017)	0.097*** (0.019)	0.433	-3348	0.921

Notes: The numbers in the parentheses are robust standard error and the signs \*\*\*, \*\*, and \* denote significant coefficients at 1%, 5%, and 10% levels, respectively.  $R^2$  is the goodness-of-fit. LL is the log value of maximized likelihood, and LB (20) refers to Ljung–Box statistic with up to 20-day lags for standardized residuals.

Once the estimates of the unknown parameters in the augmented ARMA ( $p$ ,  $q$ )–GJR–GARCH (1, 1) model are obtained, we can estimate the time-varying conditional volatilities of each country's property index return. The time-varying conditional volatilities of the property index return for Japan, the United Kingdom, South Africa, and the US are illustrated in Fig. 1, where the three periods of crisis are added to visualize the regional

conditional volatility differences during the GFC.<sup>3</sup> As shown in Fig. 1, the conditional variances of each country's property index return increase markedly during the GFC period, thereby suggesting the existence of contagion effects among global real estate markets. These findings are consistent with the results represented in Table 3.

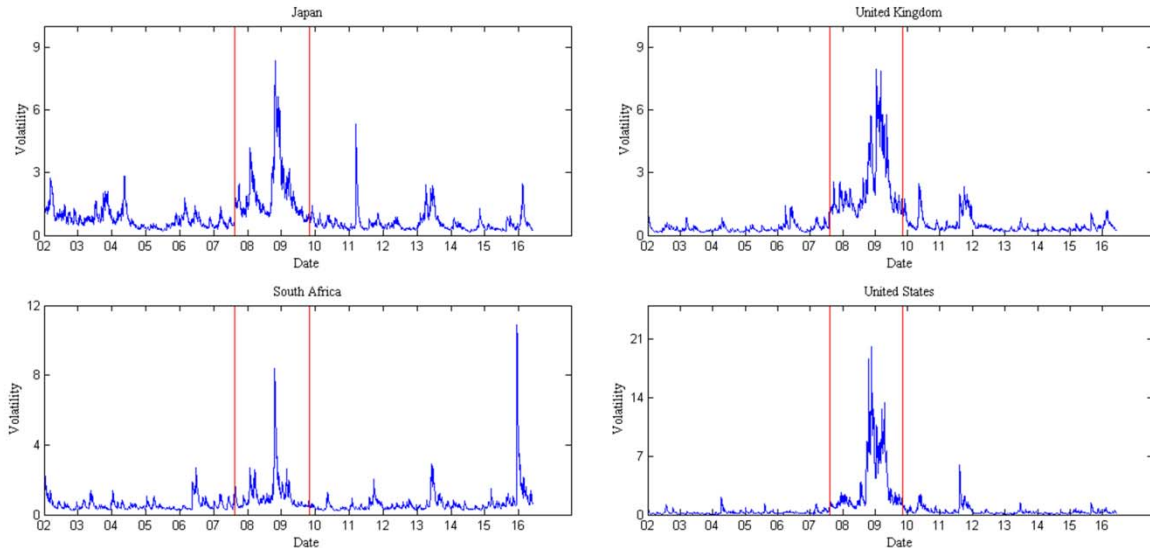


Fig. 1. Time-varying conditional volatilities of global real estate markets.

Table 4 presents the estimation results of the time-varying conditional correlations based on different model specifications. Specification 1 introduces a non-spatial model, which does not contain spatial weight matrix, i.e., the coefficient  $\gamma \equiv 0$  in Eq. (7). In Specifications 2 and 3, we allow for the incorporation of the geographic and economic distance spatial weight matrices into the proposed DCC–GARCH model, respectively. The three specifications are our benchmark models. Specification 4 is the estimation results of our proposed model using the compound spatial weight matrix specification form in Eq. (17).

As shown in Table 4, all the coefficients are statistically significant at 1% level. In particular, a significant coefficient  $\alpha$  indicates the existence of the autoregressive component of the conditional correlation. The coefficients  $\gamma$  are significant in the Specifications 2, 3, and 4 that show that the spatial effects exist among global real estate markets. In terms of fitting, the log-likelihood value of Specification 4 is the largest, which suggests that our proposed model seems to be the best one for capturing the time-varying conditional correlations among global real estate markets.

<sup>3</sup> Notably, we chose Japan, the United Kingdom, South Africa, and the US to represent the regions of Asia Pacific, Europe, Africa, and Latin America, respectively. The trends of the conditional volatilities for the other 11 countries' property index returns are almost the same as the results in Fig. 1; we then do not report these results in this study. These results are available from the authors upon request.



**Table 4**

Estimation results of the time-varying conditional correlations based on different model specifications.

Parameter	Specification 1		Specification 2		Specification 3		Specification 4	
	P.E.	S.E.	P.E.	S.E.	P.E.	S.E.	P.E.	S.E.
$\alpha$	0.961***	0.001	0.939***	0.003	0.960***	0.001	0.938***	0.003
$\beta$	0.018***	0.001	0.006***	0.001	0.016***	0.001	0.006***	0.001
$\gamma$	—	—	0.026***	0.002	0.004***	0.001	0.028***	0.002
$\psi_G$	—	—	3048.49***	131.30	—	—	3057.78***	133.55
$\psi_E$	—	—	—	—	2.589***	0.527	1.872***	0.485
$a$	—	—	—	—	—	—	0.934***	0.034
$c_1$	0.525***	0.020	0.876***	0.022	0.545***	0.021	0.886***	0.024
$c_2$	0.410***	0.025	0.628***	0.021	0.444***	0.027	0.640***	0.022
$c_3$	0.280***	0.026	0.467***	0.025	0.315***	0.028	0.480***	0.026
$c_4$	0.177***	0.026	0.307***	0.026	0.203***	0.028	0.316***	0.027
$c_5$	0.471***	0.022	0.736***	0.018	0.498***	0.023	0.744***	0.019
$c_6$	0.759***	0.011	0.758***	0.011	0.763***	0.012	0.763***	0.012
$c_7$	0.886***	0.007	0.889***	0.007	0.900***	0.009	0.898***	0.010
$c_8$	0.723***	0.012	0.729***	0.013	0.805***	0.018	0.759***	0.021
$c_9$	0.884***	0.007	0.882***	0.007	0.901***	0.009	0.893***	0.011
$c_{10}$	0.756***	0.011	0.795***	0.012	0.784***	0.014	0.810***	0.016
$c_{11}$	0.635***	0.015	0.655***	0.015	0.690***	0.018	0.678***	0.020
$c_{12}$	0.744***	0.012	0.766***	0.012	0.765***	0.013	0.777***	0.014
$c_{13}$	0.480***	0.021	0.772***	0.022	0.542***	0.023	0.793***	0.026
$c_{14}$	0.534***	0.020	0.758***	0.019	0.547***	0.020	0.766***	0.021
$c_{15}$	0.374***	0.024	0.479***	0.024	0.417***	0.027	0.493***	0.026
$LL$	11438		11906		11460		11910	

Notes: P.E. stands for parameter estimates. S.E. stands for standard errors.  $LL$  is the log value of maximized likelihood. The signs \*\*\*, \*\*, and \* denote significant coefficients at 1%, 5%, and 10% levels, respectively.

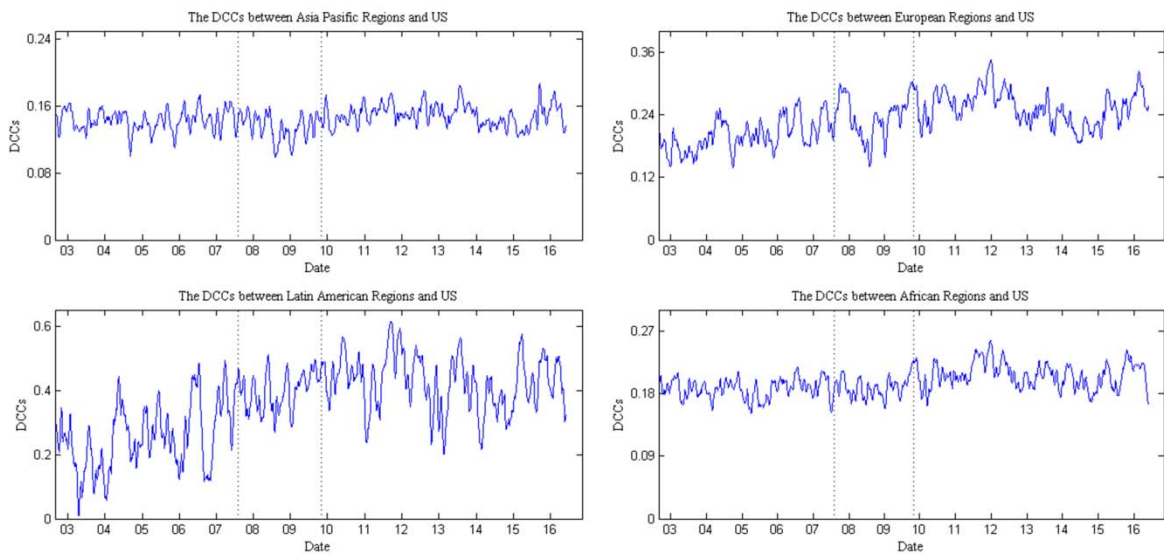
Table 5 provides the average conditional correlation results derived from the space–time model in Specification 4 in Table 4. As shown in Table 5, the countries with the highest conditional correlation coefficient in the time span considered are between the France and the Netherlands (on an average of 0.784), whereas the one with the lowest conditional correlation coefficient in the time span considered are between the Philippines and the US (on an average of 0.070). The conditional correlation coefficient is on an average larger than 0.6 for eight pairs of countries. One of the interesting findings is that all of these eight pairs of countries are located in Europe. This finding indicates that co-movements or contagion effects among European countries' property markets are stronger than those countries' property markets located in other regions.

**Table 5**

Average conditional correlations derived from the spatial GARCH model in Specification 4.

	HKG	JPN	PHI	SIN	BEL	FRA	GER	NED	SWE	SWZ	GBR	RSA	CAN	USA
AUS	0.335	0.259	0.178	0.420	0.342	0.397	0.332	0.397	0.363	0.302	0.348	0.361	0.344	0.211
HKG		0.284	0.208	0.465	0.246	0.292	0.249	0.292	0.279	0.217	0.264	0.254	0.242	0.154
JPN			0.157	0.261	0.192	0.220	0.189	0.221	0.206	0.174	0.197	0.189	0.184	0.110
PHI				0.211	0.120	0.141	0.118	0.138	0.132	0.106	0.125	0.130	0.117	0.070
SIN					0.297	0.346	0.295	0.348	0.324	0.261	0.309	0.322	0.286	0.181
BEL						0.661	0.553	0.687	0.579	0.528	0.547	0.336	0.382	0.209
FRA							0.635	0.784	0.655	0.564	0.671	0.397	0.436	0.255
GER								0.643	0.568	0.498	0.548	0.339	0.369	0.248
NED									0.666	0.577	0.654	0.390	0.439	0.256
SWE										0.518	0.578	0.352	0.401	0.238
SWZ											0.472	0.309	0.328	0.179
GBR												0.343	0.395	0.237
RSA													0.310	0.197
CAN														0.360

Once the estimates of the unknown parameters are obtained, we can estimate the time-varying conditional correlations for each pair of countries. The time-varying regional conditional correlations with the US are illustrated in Fig. 2, where the three periods of crisis are added to visualize regional differences in the property market correlations during the GFC. As shown in Fig. 2, the time path of the DCC series fluctuates over the entire sample period for all pairs, thereby suggesting that the assumption of constant correlations may be inappropriate. As shown in Fig. 2, the overall conditional correlations between Latin America and the US seem to be the largest one, whereas the smallest one is the one between the Asia Pacific and the US. These findings are consistent with the results presented in Table 5.



**Fig. 2.** Time-varying regional conditional correlations with the US market. The solid lines illustrate the dynamic conditional correlations (average value for the regions) between the US and the 14 other countries.

#### 4. Portfolio in-sample estimation

To compare among the alternative models proposed in our study, these models should be compared in terms of portfolio-management performance criteria than that of the statistical criteria. Therefore, we showed the implications of the estimation results of our proposed model in Table 4 on portfolio optimization. Specifically, we considered minimum-variance and hedged portfolios made up of two assets. First, the properties of the estimated conditional correlations are used to compute the time-varying conditional covariance structure between the global property index returns. Then, we evaluated the in-sample performance of the minimum-variance and the hedged portfolios based on the estimated value of conditional covariance.

In practice, an investor's objective is often aimed at minimizing the risk of his two asset portfolios while keeping the same expected returns. Therefore, the optimal portfolio return  $r$  of the two assets  $(i, j)$  is often a linear combination of each asset as follows:

$$r_{p,t} = w_t r_{i,t} + (1 - w_t) r_{j,t}, \quad (24)$$

$$w_t = \frac{\sigma_{j,t}^2 - Cov(r_{i,t}, r_{j,t})}{\sigma_{i,t}^2 + \sigma_{j,t}^2 - 2Cov(r_{i,t}, r_{j,t})}, \text{ where } Var(r_t | I_{t-1}) = H_t,$$

where the time-varying weights  $w_t$  stand for the optimal proportion of each asset in a portfolio based on the forecast of the time-varying variance-covariance matrix  $H_t$ .

In addition, the optimal hedges considered in this study are constructed by minimizing the variance of the portfolio return. Following the lines of Ref. [34], we used the same approach for each pair of assets by holding one and shorting another to obtain a hedged portfolio with an optimal minimum variance as follows:

$$r_{p,t} = r_{i,t} - \beta_{i,j,t} r_{j,t}, \text{ where } \beta_{i,j,t} = \frac{Cov(r_{i,t}, r_{j,t})}{\sigma_{j,t}^2}. \quad (25)$$

We adopted an in-sample evaluation framework to evaluate the portfolios' efficiency and a hedged portfolio is efficient only if the variance of portfolio return is smallest. The proposed hedged portfolios are constituted to all possible combinations of pairs of the fifteen property market indexes, i.e., a total of 105 portfolios. The performance of the minimum-variance and the optimal hedge ratio procedures are examined in the full estimation, the pre-crisis, the in-crisis, and the post-crisis periods by computing the covariance matrices with four different model specifications in Table 4. The full sample covers the period from January 3, 2002 to May 31, 2016 (3,709 observations). The pre-crisis period captures the period from January 3,

2002 until August 8, 2007 (1,444 observations). The in-crisis period lasts from August 9, 2007 to November 4, 2009 (571 observations). The post-crisis period varies from November 5, 2009 through May 31, 2016 (1,694 observations).

The estimation results of the optimized portfolios (minimum-variance and hedged portfolios) are reported in Table 6. The accounted annualized volatility is 11.56% for the property index returns over the entire estimation period, and the respective volatilities are 9.51%, 19.56%, and 10.41% over the pre-crisis, in-crisis and post-crisis periods, respectively. Apparently, all the portfolios have decreased the overall variance. The estimation results further demonstrate that the differences between the portfolio variances using different specifications are small. This observation indicates that our proposed model performs well in terms of constructing portfolio strategy when compared with the alternative models. The findings also suggest that the geographic adjacencies and the similarities in the economic development conditions can affect the effectiveness of the portfolio strategy. However, the geographic adjacencies are more important than that of the similarities in economic development conditions.

In addition, to dynamically show how the optimal portfolio variances and the corresponding portfolio weights vary with time, we plot Figs. 3 and 4, respectively.<sup>4</sup> Notably, the results presented in Fig. 4 are similar to those in Fig. 3, except that the former focuses on a minimum-variance portfolio. Figures 3 and 4 demonstrate that the optimal portfolio variances (minimum-variance and hedged portfolios) and optimal portfolio weights are time-varying. Optimal portfolio variances increase markedly during the GFC period, thereby indicating that the GFC indeed increases the portfolio investment risk.

**Table 6**  
Averaged annualized standard deviations of the optimized portfolios.

Portfolio annualized std.	Specification 1	Specification 2	Specification 3	Specification 4
<i>Full estimation period</i>				
Min. var.	0.0877 (0.7015)	0.0880 (0.7032)	0.0878 (0.7016)	0.0880 (0.7032)
Hedge	0.0899 (0.8103)	0.0899 (0.8047)	0.0899 (0.8096)	0.0899 (0.8047)
<i>Pre-crisis period</i>				
Min. var.	0.0703 (0.6972)	0.0708 (0.7021)	0.0703 (0.6975)	0.0708 (0.7022)
Hedge	0.0742 (0.8296)	0.0739 (0.8173)	0.0742 (0.8284)	0.0739 (0.8172)
<i>In-crisis period</i>				
Min. var.	0.1499 (0.7131)	0.1499 (0.7129)	0.1499 (0.7132)	0.1499 (0.7128)
Hedge	0.1522 (0.8006)	0.1523 (0.7990)	0.1522 (0.8002)	0.1524 (0.7990)
<i>Post-crisis period</i>				
Min. var.	0.0799 (0.7008)	0.0799 (0.7008)	0.0799 (0.7008)	0.0799 (0.7007)
Hedge	0.0807 (0.7990)	0.0808 (0.7971)	0.0807 (0.7987)	0.0808 (0.7972)

<sup>4</sup> The dynamics of the optimal portfolio variances and the corresponding portfolio weights under the other three specifications (Specifications 1, 2 and 3) are almost the same as the results in Figs. 3 and 4; we then do not report them in this paper. These results are available from the authors upon request.

Notes: The portfolios are constituted of all possible combinations of pairs of the property indices' returns accounting to total amount of 105 portfolios for each period. The averaged optimal weights of the portfolios, which have lower variance in the combined pair-wise asset allocations, are reported in the parenthesis.

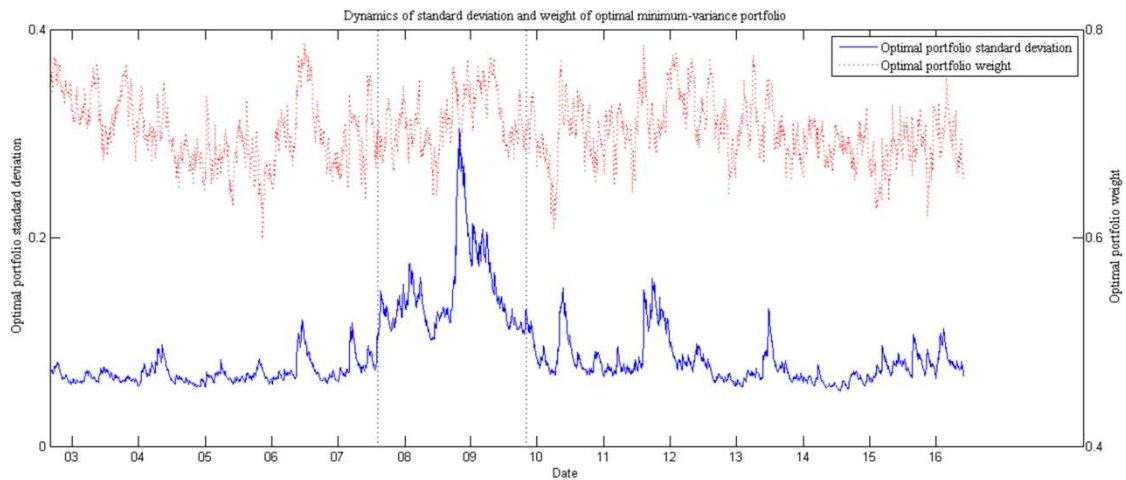


Fig. 3. Dynamics of standard deviation and weight of optimal minimum-variance portfolio under Specification 4.

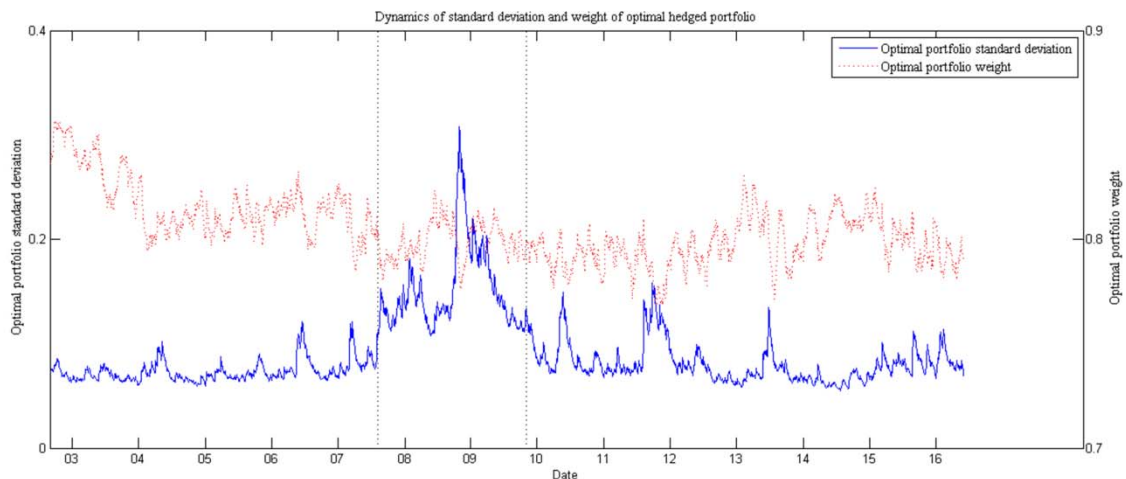


Fig. 4. Dynamics of standard deviation and weight of optimal hedged portfolio under Specification 4.

## 5. Conclusions

In this study, we have employed the MGARCH with spatial effects approach to examine the daily returns of 15 national property market indices from January 3, 2002 to May 31, 2016, taking into account the structural breaks of each time series linked to the recent GFC from 2007 to 2009. A distinguishing feature of the proposed model is that it can simultaneously capture the spatial interactions and the dynamic conditional correlations compared with the traditional MGARCH models, like DCC–GARCH models. The main findings are presented as follows.

First, this study investigates whether leverage effects exist among global real estate markets and examines the effects of the recent GFC from 2007 to 2009 on the estimated dynamic conditional correlations. The analysis of the issues has been conducted in the context of an extended DCC model, which contains spatial effects, using an augmented ARMA (1, 1)–GJR–GARCH (1, 1) specification in the first stage. In terms of the leverage effects, the significant positive coefficient of leverage effects indicates that the leverage effects exist in global real estate markets. The existence of leverage effects suggests that in real estate markets, investors react more strongly to bad news than to good news. The estimated dynamic conditional correlations have exhibited significant increases during the GFC from 2007 to 2009, thereby suggesting contagion effects exist among global real estate markets.

Second, the results reveal that the estimated conditional correlations across global real estate markets are time varying, and the US property market has a lead-lag effect, which is in accordance with the findings of Refs. [23, 25]. The estimated average dynamic conditional correlation results indicate that in global real estate markets, co-movements across markets appear among regions with geographic adjacencies, like those between the France and the Netherlands, or with similar economic development conditions, like those between the United Kingdom and the Australia. However, strong forms of the co-movement occur among countries/regions, which are located close to one another. For example, the co-movements among European countries' property markets are stronger than that of countries' property markets located in other regions.

Third, we further evaluated the performance of our proposed model by adopting a two-asset portfolio allocation framework. The portfolios' efficiency is estimated in-sample and achieving the smallest variance of portfolio return is the only criterion for success or for achieving an efficient portfolio. The estimation results demonstrate that the differences between the portfolio variances using different specifications are small. This result suggests that the geographic adjacencies and the similarities in economic development conditions can affect the effectiveness of the portfolio strategy because our compound spatial weight matrix is a combination of the geographic and economic distance spatial weight matrices.

Therefore, the consideration of spatial effects in the DCC model can increase the fitting of the data and the performance of the model in portfolio allocation. We believe that these results provide important implications on global portfolio diversification opportunities and risk management practices.

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## References

- [1] G.-D. Chang, C.-S. Chen, Evidence of contagion in global REITs investment, *Int. Rev. Econ. Financ.* 31 (2014) 148–158.
- [2] D.Y. Kenett, X. Huang, I. Vodenska, S. Havlin, H.E. Stanley, Partial correlation analysis: applications for financial markets, *Quant. Financ.* 15 (2015) 569–578.
- [3] P.N. Rotta, P.L. Valls Pereira, Analysis of contagion from the dynamic conditional correlation model with markov regime switching, *Appl. Econ.* 48 (2016) 2367–2382.
- [4] D. Stošić, D. Stošić, T. Stošić, H.E. Stanley, Multifractal analysis of managed and independent float exchange rates, *Physica A* 428 (2015) 13–18.
- [5] M. Yang, Z.-Q. Jiang, The dynamic correlation between policy uncertainty and stock market returns in China, *Physica A* 461 (2016) 92–100.
- [6] R. Engle, Dynamic conditional correlation, *J. Bus. Econ. Stat.* 20 (2002) 339–350.
- [7] Y.K. Tse, A.K.C. Tsui, A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations, *J. Bus. Econ. Stat.* 20 (2002) 351–362.
- [8] G.P. Aielli, Dynamic conditional correlation: on properties and estimation, *J. Bus. Econ. Stat.* 31 (2013) 282–299.
- [9] L. Bauwens, L. Grigoryeva, J.-P. Ortega, Estimation and empirical performance of non-scalar dynamic conditional correlation models, *Comput. Stat. Data Anal.* 100 (2016) 17–36.
- [10] M. Kasch, M. Caporin, Volatility threshold dynamic conditional correlations: an international analysis, *J. Financ. Econom.* 11 (2013) 706–742.
- [11] G. Tamakoshi, S. Hamori, Co-movements among major European exchange rates: a multivariate time-varying asymmetric approach, *Int. Rev. Econ. Financ.* 31 (2014) 105–113.
- [12] L. Bauwens, S. Laurent, J.V.K. Rombouts, Multivariate GARCH models: a survey, *J. Appl. Econom.* 21 (2006) 79–109.
- [13] L. Bauwens, E. Otranto, Modeling the dependence of conditional correlations on market volatility, *J. Bus. Econ. Stat.* 34 (2016) 254–268.
- [14] T. Bodnar, N. Hautsch, Dynamic conditional correlation multiplicative error processes, *J. Empir. Financ.* 36 (2016) 41–67.
- [15] M. Caporin, P. Paruolo, Proximity-structured multivariate volatility models, *Econom. Rev.* 34 (2013) 559–593.
- [16] E. Otranto, Identifying financial time series with similar dynamic conditional correlation, *Comput. Stat. Data Anal.* 54 (2010) 1–15.
- [17] E. Otranto, M. Mucciardi, P. Bertuccelli, Spatial effects in dynamic conditional correlations, *J. Appl. Stat.* 43 (2016) 604–626.
- [18] A.C. Case, H.S. Rosen, J.R. Hines, Budget spillovers and fiscal policy interdependence, *J. Public Econ.* 52 (1993) 285–307.

- [19] B. Zhu, R. Füss, N.B. Rottke, Spatial linkages in returns and volatilities among U.S. regional housing markets, *Real Estate Econ.* 41 (2013) 29–64.
- [20] P. Gong, Y. Weng, Value-at-Risk forecasts by a spatiotemporal model in Chinese stock market, *Physica A* 441 (2016) 173–191.
- [21] M.C. Münnix, R. Schäfer, O. Grothe, Estimating correlation and covariance matrices by weighting of market similarity, *Quant. Financ.* 14 (2011) 931–939.
- [22] Y. Weng, P. Gong, Modeling spatial and temporal dependencies among global stock markets, *Expert Syst. Appl.* 43 (2016) 175–185.
- [23] O. Hemche, F. Jawadi, S.B. Maliki, A.I. Cheffou, On the study of contagion in the context of the subprime crisis: a dynamic conditional correlation–multivariate GARCH approach, *Econ. Model.* 52 (2016) 292–299.
- [24] G.-J. Wang, C. Xie, Z.-Q. Jiang, H.E. Stanley, Extreme risk spillover effects in world gold markets and the global financial crisis, *Int. Rev. Econ. Financ.* 46 (2016) 55–77.
- [25] S. Mollah, A.M.M.S. Quoreshi, G. Zafirov, Equity market contagion during global financial and Eurozone crises: evidence from a dynamic correlation analysis, *J. Int. Financ. Mark. Institutions Money.* 41 (2016) 151–167.
- [26] J.C. Reboredo, A.K. Tiwari, C.T. Albuлесcu, An analysis of dependence between Central and Eastern European stock markets, *Econ. Syst.* 39 (2015) 474–490.
- [27] C.-B. Tse, T. Rodgers, J. Niklewski, The 2007 financial crisis and the UK residential housing market: did the relationship between interest rates and house prices change?, *Econ. Model.* 37 (2014) 518–530.
- [28] G.-J. Wang, C. Xie, Correlation structure and dynamics of international real estate securities markets: a network perspective, *Physica A* 424 (2015) 176–193.
- [29] D. Kenourgios, D. Dimitriou, Contagion effects of the global financial crisis in US and European real economy sectors, *Panoeconomicus.* 61 (2014) 275–288.
- [30] P. Christoffersen, V. Errunza, K. Jacobs, X. Jin, Correlation dynamics and international diversification benefits, *Int. J. Forecast.* 30 (2014) 807–824.
- [31] Y. Wang, L. Liu, Crude oil and world stock markets: volatility spillovers, dynamic correlations, and hedging, *Empir. Econ.* 50 (2016) 1481–1509.
- [32] T. Bollerslev, R.F. Engle, J.M. Wooldridge, A capital asset pricing model with time-varying covariances, *J. Polit. Econ.* 96 (1988) 116–131.
- [33] D.C. Wheeler, A. Páez. Geographically weighted regression, in: M.M. Fischer, P. Nijkamp (Eds.), *Handb. Reg. Sci.*, Springer Berlin Heidelberg, Berlin, Heidelberg, 2014, pp. 1435–1459.
- [34] J. Kotkatvuori-Örnberg, J. Nikkinen, J. Äijö, Stock market correlations during the financial crisis of 2008–2009: evidence from 50 equity markets, *Int. Rev. Financ. Anal.* 28 (2013) 70–78.



## Highlights

- We present a novel multivariate GARCH with spatial effects model.
- We examine spatial effects on time-varying correlations in real estate markets.
- We analyze both the geographic relations and economic linkages on correlations.
- We demonstrate the efficiency of our proposed model in portfolio allocation.