



## Interdependence of foreign exchange markets: A wavelet coherence analysis<sup>☆</sup>



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### ABSTRACT

Based on the wavelet decomposition approach, we study co-movement among foreign exchange markets using the returns of exchange rates (GBP/USD, EUR/USD, and JPY/USD). We focus on the interdependence among returns of exchange rates during the recent global financial crisis and European debt crisis. We use a wavelet analysis because of its ability to decompose signals into high and low frequencies. This approach allows us to study shorter time periods independently of longer time periods. The results reveal strong interdependence between the euro and pound sterling at all frequency bands of scale over the sample period. With regard to the yen-pound pairwise, covariation is localized at high scales. Further, we find that interdependence is more pronounced during crises.

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### 1. Introduction

A major issue facing international investors is to identify whether observed financial market fluctuations are mainly due to contagion or fundamentals.<sup>1</sup> This is because when increasing volatility and cross-market linkages are due to a contagion, they disappear after a few days. However, if increasing fluctuation and co-movements are due to fundamental variables, they are likely to continue for a long time. Thus, an investigation into financial contagion is critical because of its damaging impact on the global economy in relation to portfolio risk management, the formulation of monetary and fiscal policy, and strategic asset allocation and pricing.

Recently, authors have subdivided financial market “linkage” into “interdependence” and “contagion.” Interdependence represents a state of stable period relationships that are driven by fundamentals (Jung and Maderitsch, 2014). This interdependence theory emphasizes real linkage and fundamental integration as channels for transmission shocks between two markets in crisis and non-crisis periods. In this regard, Flavin and Sheenan (2015) define interdependence as market correlations that exist in all global conditions and arise because of standard asset market linkages and exposure to common risk sources. Ahmad et al. (2013) examine financial contagion and interdependence

in the stock markets of Brazil, Russia, India, Indonesia, China, South Korea, and South Africa (BRICKS) during the eurozone crisis. The study reports that Brazil, Russia, India, China, and South Africa were strongly affected by the contagion shock during the crisis; however, Indonesia and South Korea reported only interdependence and not contagion. Further, Shen et al. (2015) estimate a time-varying parameter correlation coefficient for the stock returns of the eurozone and China. This tests whether the European debt crisis was contagious and identifies interdependence and pure contagion across countries.

In contrast, the state of contagion is characterized by strong and sudden changes in measured market linkages. Despite the large amount of literature on financial market contagion, disagreement exists about the exact definition of what constitutes contagion and how we should measure it. In this study, we follow the definition of Forbes and Rigobon (2002) who define contagion as a significant increase in cross-market linkages after the occurrence of a shock in one country. In this context, Jung and Maderitsch (2014) provide evidence of volatility transmission among international financial markets and find the volatility spillover effect from foreign markets. Kenourgios and Dimitriou (2015) investigate the contagion effects of the global financial crisis (2007–2009) by examining 10 sectors in six developed and emerging regions and indicate that the most severe contagion effects existed after the failure of Lehman Brothers, thereby limiting the effectiveness of portfolio diversification. Loaiza-Maya et al. (2015) test contagion among the exchange rates of the six largest Latin American countries by implementing a regular vine copula approach. Their study reports that they find evidence of contagion among the Brazilian, Chilean, Colombian, and Mexican exchange rates and that there are differences in contagion during periods of large exchange rate depreciation and appreciation. Thus, understanding the direction of financial contagion not

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<sup>1</sup> Baele and Inghelbrecht (2010) and Kallberg and Pasquariello (2008) assume that market linkages are driven by fundamentals.

only has important implications for cross-market risk management and international asset pricing; governments must also design economic policies to diminish the negative effects caused by external crises. Particularly in recent years, in the context of U.S. subprime issues and subsequent eurozone disturbance, distinguishing between contagion and interdependence, and examining the direction and degree of financial contagion, is of significant concern. Thus, these topics form the basis of our research.

Interdependence or contagion, in contrast to linear correlation, leads to causality relationships across financial markets. Moreover, the conventional Pearson correlation is not appropriate for measuring dependence across financial markets with different time horizons. In our analysis, we consider wavelet analysis because it is a useful analytical tool for studying the properties of multi-resolution.<sup>2</sup> The multi-resolution decomposition and property of the wavelet transform provide a different perspective on the empirical problem of identifying contagion and interdependence by using frequency domain analysis. Examples of prior studies that test for contagion by associating contagion and interdependence with distinct frequency ranges (high and low frequencies respectively) are relatively recent (see [Aloui et al., 2015](#); [Bodart and Candelon, 2009](#); [Breitung and Candelon, 2006](#); [Orlov, 2009](#)).

Several studies use wavelet variance and wavelet correlation to investigate interdependence among markets that work to different timescales. For example, [Dajcman et al. \(2012\)](#) investigate the dynamics of stock market return co-movements among individual central and eastern European countries and developed European stock markets from 1997 to 2010. In this regard, they apply a maximal overlap discrete wavelet correlation and a running correlation technique. The study shows that the developed European stock markets of France, the UK, Germany, and Austria were more interdependent in the observed period than the central and eastern European (CEE) stock markets. [Gallegati \(2012\)](#) applies a wavelet-based approach in order to test whether contagion occurred during the U.S. subprime crisis of 2007 and to identify the contagion and interdependence of the original return series. [Gallegati \(2012\)](#) uses the discrete type of wavelet transformation, while in this paper we provide a richer analysis by employing wavelet coherence analysis in order to capture the dependence structure across different timescales and the causality relationships.<sup>3</sup> Numerous recent works have also researched cross-market linkages using wavelet coherence analysis. For example, [Rua and Nunes \(2009\)](#) examine co-movement among international stock markets and characterize simultaneously how international stock returns relate in the time and frequency domains. [Rua and Nunes' \(2009\)](#) study finds that co-movement among markets is stronger at the lower frequencies and that the strength of co-movement in the time-frequency space varies across countries and sectors. A similar methodology is applied by [Ranta \(2013\)](#) to the data of major world equity markets. [Ranta's \(2013\)](#) study reports that short timescale co-movements increase during a major crisis while long timescale co-movements remain approximately at the same level and gradually increase interdependence among markets. However, these studies only confirm that interdependence among markets is scale-dependent, thereby exhibiting the strength of interdependence. No general conclusions have been reached about distinguishing interdependence and contagion. We believe that our study is the first research that not only distinguishes between interdependence and contagion but also captures the degree and direction, or causality relationship, of contagion by applying wavelet coherence analysis. Further, our contagion analysis can be conducted more effectively than prior analyses and in a straightforward manner.

Over the past decades, a number of studies have investigated the interdependence and contagion of exchange rate series. Generally, three basic types of study focus on the issue of interdependence in foreign exchange markets. An example of the first type is a study by [Engle et al. \(1990\)](#), who contend that exchange rates react not only to shocks in individual markets but also to shocks transmitted across markets. This study is based on the generalized autoregressive conditional heteroskedasticity (GARCH) model, and since then many papers have discussed the interdependence of exchange rate returns based on the GARCH framework. For example, [Pérez-Rodríguez \(2006\)](#) finds, from research based on the dynamic conditional correlation (DCC) GARCH model, that the correlation between the EUR/USD and GBP/USD is particularly high. Moreover, [Tamakoshi and Hamori \(2014\)](#) find an asymmetric response in the correlations among the GBP, EUR, and CHF currencies by employing the asymmetric dynamic conditional correlation (ADCC) GARCH model.

The second type of study is one that considers the cause-and-effect relationship among different currencies. For example, [Spagnolo et al. \(2005\)](#) provide the causality relationship among forward and spot exchange rates by employing a Markov switching model and instrumental variables. Further, [Nikkinen et al. \(2006\)](#) find, by applying the vector autoregressive model and Granger causality tests, that the implied volatility of the EUR affects the GBP and the CHF. In addition, [Inagaki \(2007\)](#) discovers a unidirectional causality-in-variance from the EUR to the GBP based on the cross-correlation function. [Beirne and Gieck \(2014\)](#) also find, using a global vector autoregression (VAR) model, that the interdependence of foreign exchange markets is notable in developed markets.

The third type of study considers non-linear dependence based on the copula functions. For example, [Patton \(2006\)](#), by employing a time-varying copula model, provides evidence that the dependence between the DEM/USD and JPY/USD exchange rates is asymmetric. He also finds that the degree of dependence when the currencies depreciate is higher than when they appreciate. Further, [Dias and Embrechts \(2010\)](#) model the dependence of the EUR/USD and the JPY/USD returns based on the copula-GARCH model. They find that a time-varying copula with the proposed interdependence specification gives better results than alternative dynamic benchmark models.

Modeling the interdependence of exchange rates is often of interest in the areas of risk management, asset pricing, and portfolio management. Appropriate action to rebalance a portfolio and adjust exchange rate exposure can be initiated accordingly. However, few studies cover the most important issue in risk management: the interdependence of exchange rates over different timescales. For example, [Nekhili et al. \(2002\)](#) explore and compare the empirical distribution of the USD/DEM exchange rate returns with well-known continuous-time processes at different frequencies. In addition, [Nikkinen et al. \(2011\)](#) employ the cross-wavelet approach to analyze interdependence and provide the lead-lag relationship for exchange rates over different timescales. They find that the three major currencies (the euro, British pound, and Japanese yen) vis-à-vis the U.S. dollar are closely linked over different timescales and that there are significant lead-lag relationships between the expected exchange rate probability densities. However, the authors fail to provide dynamic interdependence maps and causality relationships in the foreign exchange markets. Thus, our paper, in contrast to [Beirne and Gieck \(2014\)](#) and [Nikkinen et al. \(2011\)](#), employs wavelet coherence analysis to explore the dynamic interdependence maps of exchange rates over different timescales with the causality test. Moreover, by inputting the GBP/USD returns into our analysis, we can consider more time horizons and draw more information about the foreign exchange markets than [Dias and Embrechts \(2010\)](#), who investigate the dependence of the EUR/USD and JPY/USD exchange rates in six different time horizons.

Our contribution can be summarized as multifold. First, we compensate for the lack of academic studies on the dynamic interdependence maps of exchange rates in the foreign exchange markets over different

<sup>2</sup> For a thorough review of wavelet analysis, see [Gençay et al. \(2001a, b\)](#) and [Percival and Walden \(2000\)](#).

<sup>3</sup> For the applications of wavelet coherence analysis, please refer to [Aloui and Hkiri \(2014\)](#) and [Uddina et al. \(2013\)](#).

timescales. Second, we show the differences between interdependence and contagion among currencies and measure their degree and direction by applying a wavelet coherence analysis. Third, our sample period covers the recent global financial crisis (the global factor) and the European debt crisis (the local factor). Thus, by comparing and contrasting the different impacts from these two crises, we can provide useful implications for investors related to risk management across different regimes. Moreover, understanding the interdependence of a foreign currency provides meaningful information on carry trade.

The rest of this paper is organized as follows: Section 2 discusses the methodology used; Section 3 describes the data and statistical issues; Section 4 provides the empirical results; and Section 5 concludes the study.

## 2. Methodology – wavelet analysis

### 2.1. The wavelet

We employ wavelet methodology to generate a data structure containing segments of various lengths.<sup>4</sup> The various applications of wavelet analysis in economics and finance have already been documented by Gençay et al. (2001a, 2001b) and Percival and Walden (2000) and are not repeated here. However, one advantage of wavelet analysis is that it can decompose a time series into more elementary functions that contain information on a series. Based on the different scales of time series, useful information can be retrieved from the signals (the raw data). For example, In and Kim (2013) provide an overview of how wavelets can be applied in research on economics and finance.<sup>5</sup> In this paper, we investigate the interdependence of the foreign exchange markets based on different timescales in order to analyze the degree of interaction and the contagion effect in recent financial crises.

Generally, two types of wavelet can be identified based on different normalization rules; namely, father wavelets  $\phi$  and mother wavelets  $\psi$ . The father wavelet integrates to 1 ( $\int \phi(t)dt = 1$ ) and the mother wavelet integrates to 0 ( $\int \psi(t)dt = 0$ ). The father wavelet denotes the smooth and low frequency parts of a signal (the raw data), and the mother wavelet denotes the detail and the high-frequency components.

By transforming any function  $y(t)$  in  $L^2(\mathbb{R})$  (the space for square summable functions) into different frequency components through a resolution matched to its scale, the wavelet function can be built as a sequence of projections onto the father and mother wavelets generated from  $\phi$  and  $\psi$  through scaling and translation as follows:

$$\phi_{j,k}(t) = 2^{-j/2} \phi(2^{-j}t - k), \tag{1}$$

$$\psi_{j,k}(t) = 2^{-j/2} \psi(2^{-j}t - k), \tag{2}$$

where  $j = 1, 2, \dots, J$  is the scaling parameter in a  $J$ -level decomposition and  $k$  is a translation parameter. Thus, the wavelet representation of the signal  $y(t)$  in  $L^2(\mathbb{R})$  can be expressed as:

$$y(t) = \sum_k s_{j,k} \phi_{j,k}(t) + \sum_k d_{j,k} \psi_{j,k}(t) + \dots + \sum_k d_{1,k} \psi_{1,k}(t) \tag{3}$$

where  $s_{j,k} = \int y(t) \phi_{j,k}(t) dt$  and  $d_{j,k} = \int y(t) \psi_{j,k}(t) dt$ . In Eq. (3),  $J$  denotes the number of multi-resolution components,  $s_{j,k}$  denotes the smooth coefficients, and  $d_{j,k}$  denotes the detail coefficients. The value of

coefficients  $(s_{j,k}, d_{j,k})$  measures the contribution of the corresponding wavelet function relative to the total signal.

The scale factor  $2^j$  in Eqs. (1) and (2) denotes the dilation factor, while the translation parameter  $2^j k$  refers to the location parameter. The larger the index  $j$ , the larger the value of the scale factor  $2^j$ . Thus, the function becomes wider and more spread out. As the functions  $\phi_{j,k}(t)$  and  $\psi_{j,k}(t)$  become wider, their translation parameters  $2^j k$  also rise correspondingly.

The decomposed signals for a multi-resolution decomposition are represented as follows:

$$S_j(t) = \sum_k s_{j,k} \phi_{j,k}(t), \tag{4}$$

$$D_j(t) = \sum_k d_{j,k} \psi_{j,k}(t). \tag{5}$$

The functions  $S_j(t)$  and  $D_j(t)$  in Eqs. (4) and (5) are the smooth signals and the detail signals, respectively. They constitute a decomposition of a signal into orthogonal components at different scales. Thus, a signal  $y(t)$  can be rewritten as:

$$y(t) = S_j(t) + D_j(t) + D_{j-1}(t) + \dots + D_1(t). \tag{6}$$

The highest-level approximation  $S_j(t)$  is the smooth signal, and the detail signals  $D_1(t), D_2(t), \dots, D_j(t)$  are associated with oscillations of lengths  $2-4, 4-8, \dots, 2^j - 2^{j+1}$ , respectively. A real-valued function  $y(t)$  for the discrete wavelet transform (DWT) is defined as follows:

$$\omega = Wy, \tag{7}$$

where the coefficients are ordered from coarse scales to fine scales in the vector  $\omega$ .  $W$  is introduced as a set of low-pass a filter and  $y$  is called by band-pass filter.  $W$  and  $y$  are orthogonal vectors with  $N \times 1$  elements. The coefficients in the filter are determined by the type of the mother wavelet. As  $n$  is divisible by  $2^j$ ,  $\omega$  can be specified as:

$$\omega = \begin{pmatrix} s_j \\ d_j \\ d_{j-1} \\ \vdots \\ d_1 \end{pmatrix} \tag{8}$$

where,

$$s_j = (s_{j,1}, s_{j,2}, \dots, s_{j,n/2^j})',$$

$$d_j = (d_{j,1}, d_{j,2}, \dots, d_{j,n/2^j})',$$

$$d_{j-1} = (d_{j-1,1}, d_{j-1,2}, \dots, d_{j-1,n/2^j})',$$

.....

$$d_1 = (d_{1,1}, d_{1,2}, \dots, d_{1,n/2^j})'.$$

Each set of coefficients  $s_j, d_j, d_{j-1}, \dots, d_1$  is called a crystal in which the wavelet coefficients correspond to a set of translated wavelet functions arranged on a regular lattice.

### 2.2. The continuous wavelet

To investigate the joint behavior of time series for both frequency and time, we employ wavelet coherence analysis using Morlet's specification in order to examine interdependence among foreign exchange

<sup>4</sup> See In and Kim (2013).

<sup>5</sup> There is a large amount of literature on wavelets that deals with the decomposition of undiversifiable risk across time horizons (Gençay et al., 2003; Gençay et al., 2005), multi-scale decomposition (Gençay et al., 2001a, b; Nekhili et al., 2002; Xu and Gençay, 2003), unit root tests (Fan and Gençay, 2010), serial correlation and jump test wavelets as variance ratios (Gençay and Signori, 2015; Xue et al., 2014), and hedging and diversification across time horizons (Conlon et al., 2015).

markets. The wavelet is defined as follows:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-u}{s}\right), \varphi(\cdot) \in L^2(\mathbb{R}) \quad (9)$$

where  $1/\sqrt{s}$  is the normalization factor to ensure the unit variance of the wavelet,  $|\psi_{u,s}|^2 = 1$ ,  $u$  is the location parameter that provides the exact position of the wavelet, and  $s$  is the scale-dilation parameter of the wavelet. Morlet's wavelet can be expressed by:

$$\psi^M(t) = \frac{1}{\pi^{1/4}} e^{i\omega_0 t} e^{-t^2/2} \quad (10)$$

where  $\omega_0$  is the central frequency of the wavelet. Following Grinsted et al. (2004), Rua and Nunes (2009), and Vacha and Barunik (2012),  $\omega_0$  is usually set to 6. In addition, following the studies of Rua and Nunes (2009) and Vacha and Barunik (2012), the continuous wavelet transform (CWT) is given by:

$$W_x(u, s) = \int x(t) \frac{1}{\sqrt{s}} \psi\left(\frac{t-u}{s}\right) dt. \quad (11)$$

$W_x(u, s)$  is calculated by projecting the specific wavelet  $\psi(\cdot)$  onto the selected time series. Thus, the CWT can decompose and then consequently reconstruct the function  $x(t) \in L^2(\mathbb{R})$ :  $x(t) \in L^2(\mathbb{R})$

$$x(t) = \frac{1}{C_\psi} \int_0^\infty \left[ \int_0^\infty W_x(u, s) \psi_{u,s}(t) du \right] \frac{ds}{s^2}, s > 0. \quad (12)$$

Correspondingly, the variance for the power spectrum analysis is represented as follows:

$$\|x\|^2 = \frac{1}{C_\psi} \int_0^\infty \left[ \int |W_x(u, s)|^2 du \right] \frac{ds}{s^2}, s > 0. \quad (13)$$

### 2.3. Wavelet squared coherence

Based on the foregoing discussion, we employ wavelet-squared coherence to investigate the joint behavior of both time and frequency in foreign exchange markets. However, before doing so, we must introduce the cross-wavelet transform (XWT). Following Torrence and Compo (1998), we express the XWT for the two signals  $x(t)$  and  $y(t)$  as follows:

$$W_{x,y}(u, s) = W_x(u, s) W_y^*(u, s) \quad (14)$$

where  $u$  denotes the position,  $s$  is the scale, and  $*$  denotes the complex conjugate. The XWT shows the area in the timescale space where the time series has a high common power. In other words, the timescale space represents the local covariance between the two signals at each scale. Following the study of Torrence and Webster (1999), the squared absolute value of the smoothed cross-wavelet power spectrum of each selected time series is the expression of the wavelet coherence. Hence, the squared wavelet coefficient is defined as:

$$R^2(u, s) = \frac{|S(s^{-1} W_{xy}(u, s))|^2}{S(s^{-1} |W_x(u, s)|^2) S(s^{-1} |W_y(u, s)|^2)} \quad (15)$$

where  $s$  denotes the smoothing parameter. The value of squared wavelet coherence is in the range  $0 \leq R^2(u, s) \leq 1$ . If its value is close to zero, weak interdependence will be identified and vice versa. By calculating the value of Eq. (15), we can compare and contrast the degree of interdependence among these currencies by comparing correlations across timescales. Moreover, we can also investigate the contagion effect across the different currencies by comparing high-frequency interdependence with low-frequency interdependence. If the former

**Table 1**  
Descriptive statistics for raw data.

	Euro	Yen	Pound
Mean	-0.000139	-7.05E-05	-4.13E-05
Median	-0.000110	5.53E-05	-8.24E-05
Std. Dev.	0.006246	0.006588	0.006027
Skewness	-0.082894	-0.261466	0.099648
Kurtosis	5.855190	7.043117	7.394020
Jarque-Bera <sup>a</sup>	1067.096	2168.250	2523.995
Observations	3131	3131	3131

Note: Our sample period is from January 1, 2002 to December 31, 2013 at daily frequency.  
<sup>a</sup> The results of the Jarque-Bera (JB) test show that the null hypothesis of normal distribution (unconditionally) is rejected in all cases at the 1% significance level.

increase sharply in a certain period and the latter do not, we can state that contagion occurs. Based on the above logic, we use exchange rate returns data to answer the three research questions posed in the Introduction.

### 2.4. The phase pattern

Using the property of wavelet coherence, we cannot distinguish whether dependence is positive or negative because the wavelet coherence is squared. Thus, we employ the phase difference tool to investigate the dependence and causality relationships among foreign exchange markets. Since the CWT is complex and can be divided into a real part and an imaginary part according to the mother wavelet (the Morlet wavelet), the phase difference between  $x(t)$  and  $y(t)$  is defined as follows (Bloomfield et al., 2004):

$$\phi_{xy} = \tan^{-1} \left( \frac{\Im \{S(s^{-1} W_{xy}(u, s))\}}{\Re \{S(s^{-1} W_{xy}(u, s))\}} \right), \text{ with } \psi_{xy} \in [-\pi, \pi] \quad (16)$$

where  $\Im$  and  $\Re$  are the imaginary and real parts of the smoothed XWT respectively. Further, based on the researches of Aguiar-Conraria et al. (2008); Aguiar-Conraria and Soares (2011a, b); Bloomfield et al. (2004), and Voiculescu and Usoskin (2012), the phase pattern based on the instantaneous time lag between  $x(t)$  and  $y(t)$  can be estimated as:

$$(\Delta t)_{xy} = \frac{\psi_{xy}}{2\pi f} \quad (17)$$

where  $2\pi f$  is the angular frequency with respect to the timescale  $s$  and the usual Fourier frequency  $f$  is given by  $f = \omega_{\psi}/2\pi s$ . By substituting  $\omega_0 = 6$  into  $f = \omega_{\psi}/2\pi s$ , we have  $f = 6/2\pi s \approx 1/s$ . Thus, the time lag is  $(\Delta t)_{xy}$ , given by:

$$(\Delta t)_{xy} = \frac{\psi_{xy} s}{2\pi}. \quad (18)$$

In this paper, the phase patterns are represented by the direction of the arrows in the wavelet coherence plots. By introducing the phase patterns, we can investigate the causality relationship between two

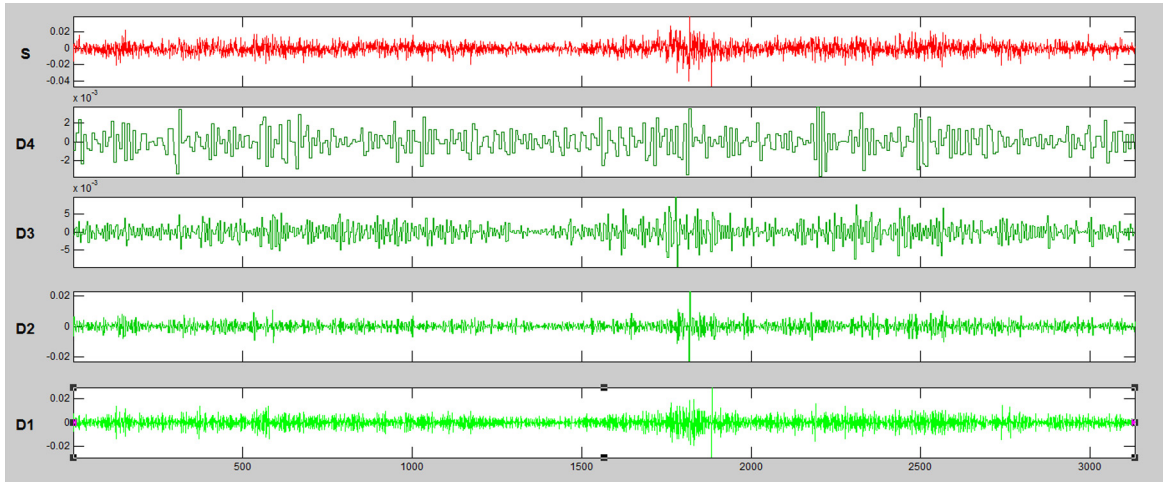
**Table 2**  
Pearson correlations of raw data for foreign exchange markets.

	Euro	Yen	Pound
Euro	1		
Yen	0.28065 (0.000)	1	
Pound	0.69361 (0.000)	0.12126 (0.000)	1

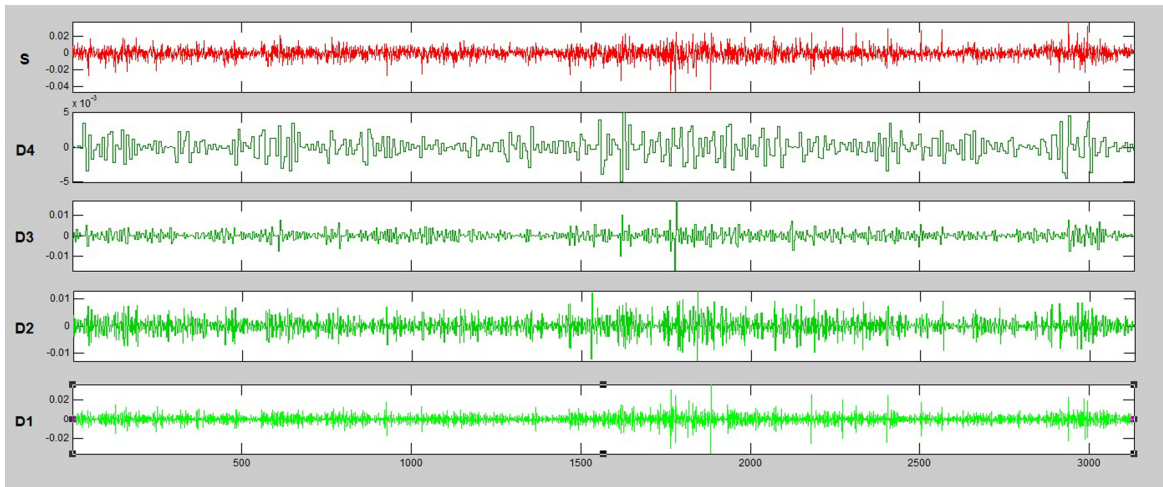
Note: The numbers in parentheses are  $p$ -values for the correlations based on the permutation test.

variables (Grinsted et al., 2004; Tiwari et al., 2013). The significance of the direction of the arrows is as follows. Arrows pointing to the right mean that  $x(t)$  and  $y(t)$  are in phase (or positively related). Arrows

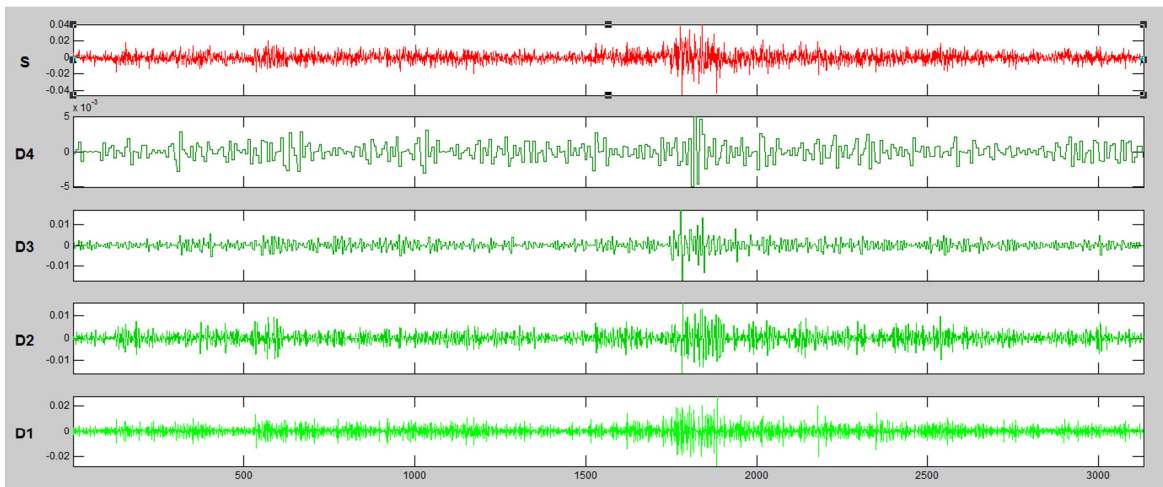
pointing to the left mean that  $x(t)$  and  $y(t)$  are anti-phase (or negatively related). Arrows pointing in other directions indicate lags or leads between  $\{x(t)$  and  $y(t)\}$ ; for example, an arrow pointing straight down



(a) Euro



(b) Yen



(c) Pound

**Fig. 1.** The raw data, and the DWT of the raw data, for the euro, yen, and pound. Note: This figure plots the raw data and the DWT for the euro, yen, and pound respectively, based on the Haar wavelet, from January 1, 2002 to December 31, 2013 at daily frequency.  $D1$ ,  $D2$ ,  $D3$ , and  $D4$  denote frequencies of two days, four days (one week), eight days (half-month), and 16 days (one month) respectively.

**Table 3**

Correlations of the DWT between the euro and the yen.

	Euro D1	Euro D2	Euro D3	Euro D4
Yen D1	0.288441 (0.000)	1.09E−19 (1.000)	−1.58E−20 (1.000)	7.18E−20 (1.000)
Yen D2	2.73E−20 (1.000)	0.253550 (0.000)	2.69E−18 (1.000)	−2.77E−05 (1.000)
Yen D3	−3.14E−19 (1.000)	−2.31E−18 (1.000)	0.241481 (0.000)	−7.06E−19 (1.000)
Yen D4	−9.00E−21 (1.000)	0.000104 (1.000)	−6.01E−20 (1.000)	0.258153 (0.000)

Note: D1, D2, D3, and D4 denote frequencies of two days, four days (one week), eight days (half-month), and 16 days (one month) respectively. Our sample period is from January 1, 2002 to December 31, 2013 at daily frequency. The numbers in parentheses are  $p$ -values for the correlations.

means that  $y(t)$  leads  $x(t)$ , and an arrow pointing straight up means that  $x(t)$  leads  $y(t)$ .

### 3. Data and statistical issues

Our data comprise daily spot exchange rates based on the US dollar for the euro, the Japanese yen, and the British pound, covering the sample period January 1, 2002 to December 31, 2013. The sample period starts from January 1, 2002 when the euro entered the foreign exchange markets. The data set was obtained from DataStream. We computed the returns from the prices of one U.S. dollar expressed in euros, pounds, and yen (i.e., EUR/USD, GBP/USD, and JPY/USD) based on the first difference of the logarithm of daily average prices. The descriptive statistics are reported in Table 1. The results of the Jarque–Bera (JB) test show that the null hypothesis of normal distribution is rejected in all cases. The linear correlations matrix among these currencies is reported in Table 2. As shown in Table 2, the degree of Pearson correlation between the pound and the euro is the highest while the degree of correlation between the yen and the pound is the lowest.

### 4. Empirical findings

#### 4.1. The discrete wavelet transform (DWT)

In this subsection, we report the results of the DWT of the returns on the currencies. In order to evaluate the degree of foreign exchange market integration, we investigated these correlations at different timescales. Thus, we decomposed the raw data into four timescale components; namely, D1, D2, D3, and D4. The finest scale component D1 represents short-term or high frequency variations due to shocks that occur at frequencies of two days, while D2 accounts for variations at frequencies of four days (corresponding to the working days of a week). Similarly, the D3 and D4 components represent the mid-term (half-month) variations at frequencies of eight and 16 days respectively.

Variations in exchange rate returns most often occur in the short term (as reflected by D1 and D2). The panels in Fig. 1 illustrate the raw data, and the DWT of the raw data, for the euro, yen, and pound respectively. We note that these three currencies show the highest variation, at different timescales, around 2008, when the international financial crisis occurred. Tables 3, 4, and 5 provide the correlation matrixes for the DWT of the raw data for the euro, yen, and pound. These three tables show that the degree of correlation among the foreign exchange markets increases as the timescale rises. This trend is consistent with our expectation that the spot exchange rate is hard to predict compared with the mid-term or long-term exchange rates, thereby causing

**Table 4**

Correlations of the DWT between the yen and the pound.

	Yen D1	Yen D2	Yen D3	Yen D4
Pound D1	0.173198 (0.000)	2.37E−19 (1.000)	2.44E−19 (1.000)	3.65E−20 (1.000)
Pound D2	−3.84E−20 (1.000)	0.147041 (0.000)	−1.25E−18 (1.000)	1.27E−05 (1.000)
Pound D3	2.51E−19 (1.000)	2.42E−18 (1.000)	0.020388 (0.254)	3.52E−19 (1.000)
Pound D4	−2.91E−20 (1.000)	−3.05E−05 (1.000)	3.75E−19 (1.000)	0.092097 (0.000)

Note: D1, D2, D3, and D4 denote frequencies of two days, four days (one week), eight days (half-month), and 16 days (one month) respectively. Our sample period is from January 1, 2002 to December 31, 2013 at daily frequency. The numbers in parentheses are  $p$ -values for the correlations.

a high degree of correlation at the longest timescale, since market participants have similar expectations about these currencies.

#### 4.2. The continuous wavelet transform (CWT)

Fig. 2 illustrates the raw data variations based on the CWT. The red area at the bottom (top) of the continuous power spectra represents strong variation at low (high) frequencies while the red area on the left-hand (right-hand) side indicates significant variation at the beginning (end) of the sample period. As before, the frequency is based on daily data.

According to Fig. 2, the euro shows significant high variation at a half-year scale (128 days) around 2008 and at a two-year scale (512 days) from 2007 to 2011. However, the pound shows significant high variation only around 2008 at a two-year scale (512 days). With regard to the yen, we note high variation at a three-year scale (1024 days) from 2007 to 2009 during the global financial crisis. A high variation also occurs from 2011 to 2013 during the European debt crisis. All these results indicate that the global financial crisis had a significant impact on foreign exchange markets.

Further, the XWTs for the pairs are summarized on the left-hand side of Fig. 3 and the wavelet transform coherences (WTCs) for the pairs are summarized on the right-hand side. The interpretation of Fig. 3 is similar to that of Fig. 2. However, Fig. 3 also provides a relative phasing of two time series by using phase arrows, which indicate the cause–effect relationships among foreign exchange markets. Arrows pointing right indicate in-phase pairs, such as the euro and the pound. Arrows pointing left represent anti-phase pairs, such as the pound and the yen. An arrow pointing straight down means that the right side leads the left side. If an arrow points straight up, the left-hand side leads the right-hand side.

This figure shows that the euro–pound pair is in phase in the significant area, indicating that the exchange rate movement of the euro largely mirrors that of the pound at a two-year scale (512 days) from 2007 to 2011. However, an anti-phase pattern is observed in the yen–pound pair at the one-year scale (256 days) from 2007 to 2009, which indicates that the exchange rate movement of the pound largely mirrors that of the yen. Moreover, the yen–euro pair shows little significant area in the XWTs.

#### 4.3. Wavelet coherence

To investigate the interdependence among foreign exchange markets, we plot the wavelet coherence results for each pair on the right-hand side of Fig. 3. In a similar way to Fig. 2, the red area at the bottom

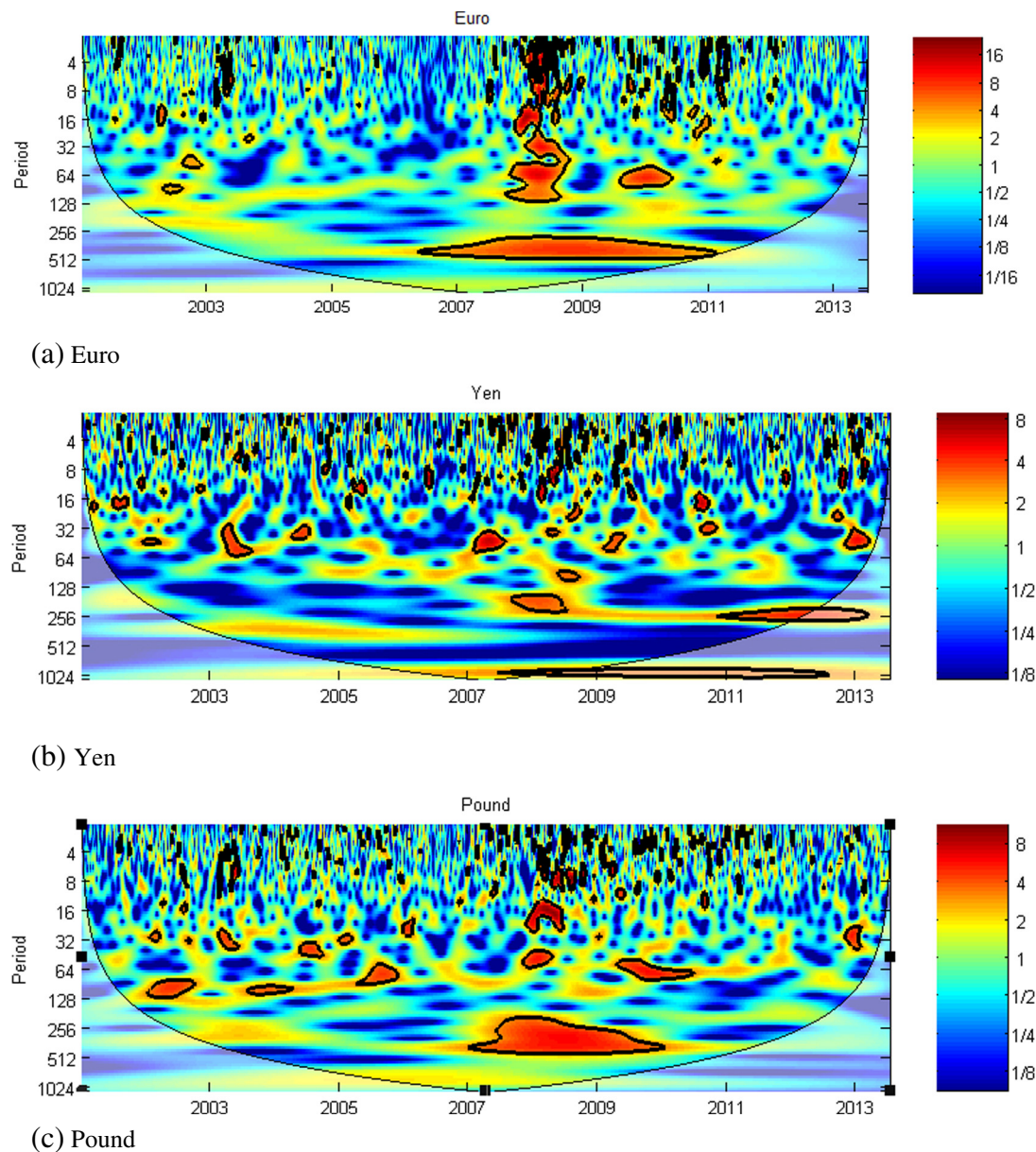
**Table 5**  
Correlations of the DWT between the euro and the pound.

	Euro D1	Euro D2	Euro D3	Euro D4
Pound D1	0.694648 (0.000)	$-3.94E-19$ (1.000)	$-3.23E-19$ (1.000)	$-8.23E-20$ (1.000)
Pound D2	$-1.47E-19$ (1.000)	0.665705 (0.000)	$-2.24E-18$ (1.000)	$2.54E-05$ (1.000)
Pound D3	$-1.68E-19$ (1.000)	$3.49E-18$ (1.000)	0.700968 (0.000)	$-1.78E-19$ (1.000)
Pound D4	$1.20E-19$ (1.000)	0.000231 (1.000)	$3.45E-19$ (0.000)	0.611549 (0.000)

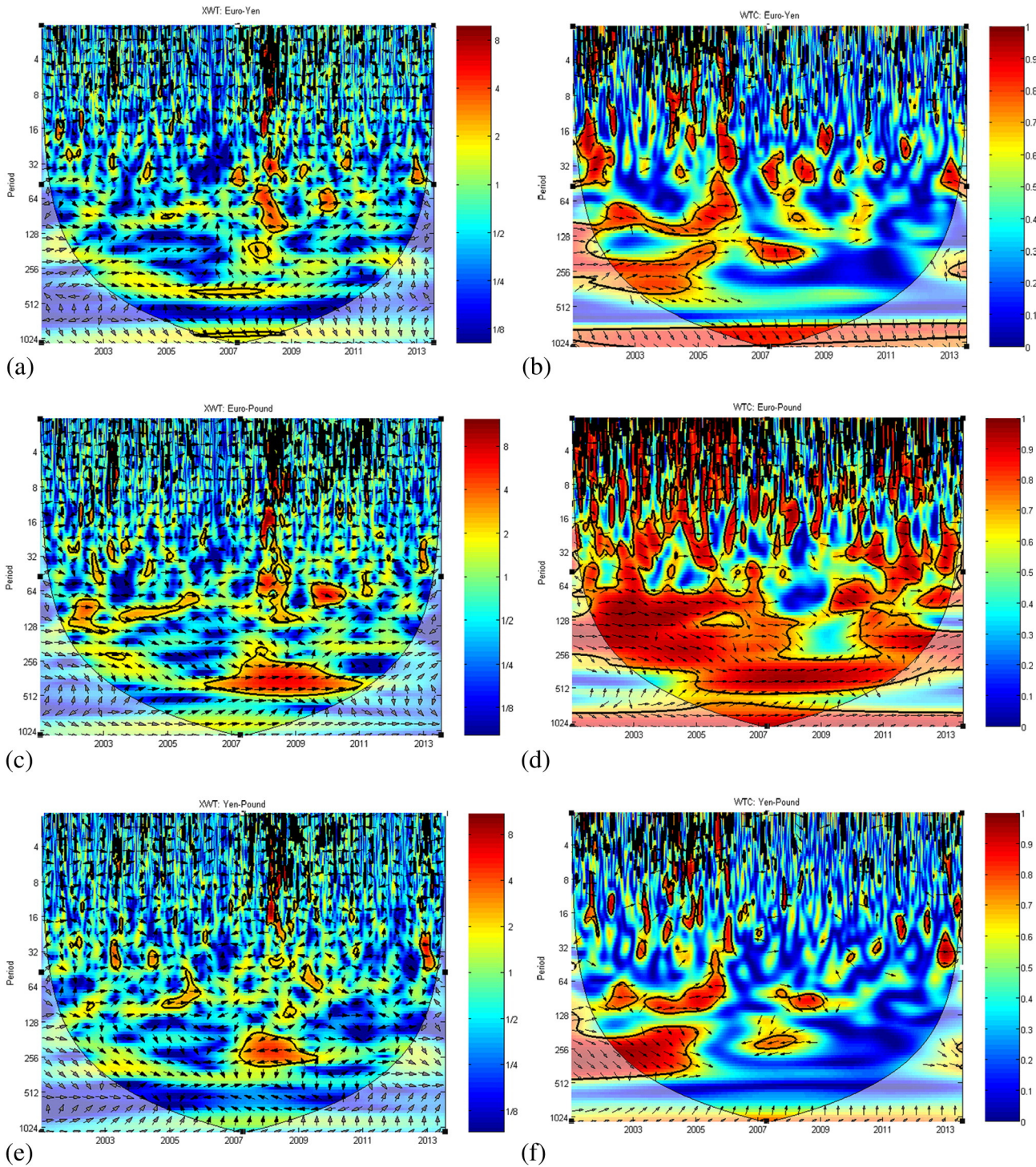
Note: D1, D2, D3, and D4 denote frequencies of two days, four days (one week), eight days (half-month), and 16 days (one month) respectively. Our sample period is from January 1, 2002 to December 31, 2013 at daily frequency. The numbers in parentheses are  $p$ -values for the correlations.

(top) of the wavelet coherence represents strong interdependence at low (high) frequencies, while the red area on the left-hand (right-hand) side signifies significant interdependence at the beginning (end) of the sample period. Thus, it is possible to identify contagion or interdependence. Strong wavelet coherence between two foreign exchange markets at the higher frequency tends to be indicated as “contagion,” whereas strong wavelet coherence at the lower frequency tends to be classified as “interdependence” (see Bodart and Candelon, 2009; Saiti et al., 2014).

The right-hand side in Fig. 3 illustrates areas of significant interdependence among the currencies. Specifically, for the euro-yen pair, we note that the significant area covers the entire sample period at the three-year scale (1024 days), indicating that long-term exchange rates can be predicted accurately based on the interest differential. Moreover, the arrow points straight down, indicating that the yen leads the euro. This may be attributable to the large holdings of European assets in the Japanese banking system. Meanwhile, a significant area is observed at the mid-term (one- or two-year) scale before 2006 with an in-phase



**Fig. 2.** The continuous power spectra of the euro, yen, and pound. Note: This figure plots the CWT of the euro, yen, and pound respectively, based on Morlet's wavelet, from January 1, 2002 to December 31, 2013 at daily frequency. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)



**Fig. 3.** XWT and WTC. This figure plots the XWT on the left-hand side and the WTC on the right-hand side from January 1, 2002 to December 31, 2013 at daily frequency. Fig. 3a represents the XWT for the euro-yen; Fig. 3b represents the WTC for the euro-yen; Fig. 3c represents the XWT for the euro-pound; Fig. 3d represents the WTC for the euro-pound; Fig. 3e represents the XWT for the yen-pound; and Fig. 3f represents the WTC for the yen-pound. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

pattern. Noticeably, we observe that the arrow points straight up around 2008 at the two-year scale (512 days), indicating that the global financial crisis spread from the EU to Japan. High wavelet coherences can also be seen at the low time scale of 32–64 days around 2008 and 2013. This implies that the yen and the euro were contagious during the global financial crisis and the European debt crisis. With regard to the euro and the pound, we find that a large significant area covers different scales with an in-phase pattern, thereby explaining the high

integration of the European foreign exchange market. As in *Beirne and Gieck (2014)*, the degree of correlation between the euro and the pound increased during the two financial crises, indicating the effect of financial contagion. For the yen and the pound, the significant area is observed at the mid-term (one- or two-year) scale before 2006, with the arrow pointing straight down. This result indicates that the pound leads the yen at the mid-term scale, which could probably explain the important role of the London interbank offered rate (LIBOR).



Moreover, the contagion effect between the pound and the yen seems modest. Comparing our findings with those of Dias and Embrechts (2010), we find that the correlation between the euro and the yen decreases significantly for all timescales after 2007.

## 5. Conclusion

Distinguishing between interdependence and contagion is a key issue from the perspective of portfolio diversification, especially during periods of high volatility. By making a distinction, investors can derive important information that enables them to develop rational asset allocation strategies and select optimal portfolios. Similarly, policymakers can create reliable crisis management policies in order to avoid risk.

This paper investigates the co-movements of foreign exchange markets at different timescales. Using wavelet coherence analysis, we identify contagion and interdependence on the basis of frequency domain and establish that lower timescales are associated with contagion and higher timescales with interdependence. We also find evidence of financial contagion between the euro and the yen and between the euro and the pound in foreign exchange markets during the global financial crisis and the European debt crisis, while there is no obvious evidence of the contagion effect between the yen and the pound. Moreover, we discover a high degree of interdependence between the pound and the euro for all timescales and between the euro and the yen for the long timescale. We also find a low degree of interdependence between the yen and the pound for the long timescale. In addition, the phase pattern indicates that the pound led the yen at the mid-term scale before 2006, while the other pairs appeared to mimic each other. Further, our findings show that interest rate parity is useful in order to predict the long-term exchange rate of the yen–euro and the pound–euro pairs, but is unsuitable for predicting the exchange rate of the yen–pound pair.

The results of our investigation provide at least three implications for investors. First, interest rate parity is still useful for predicting exchange rates in the long term and provides a useful tool to hedge the risk of variation from the exchange rate. Second, by illustrating the phase patterns, we can clearly observe the landscape of the asset price transmission channel, thus providing useful information to execute carry trade. Third, the local factor may have an important role in determining the interdependence of foreign exchange markets. Moreover, by illustrating the interdependence map across the different scales, it can provide meaningful information to construct a portfolio and diversify risk across different currencies (Beirne and Gieck, 2014; Dias and Embrechts, 2010; Nikkinen et al., 2011).

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