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On the optimal diversification of social networks in frictional labour markets with occupational mismatch☆

Anna Zaharieva

Center for Mathematical Economics, Department of Business Administration and Economics, Bielefeld University, 33501 Bielefeld, Germany

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ABSTRACT

This paper incorporates social networks into a frictional labour market framework. There are two worker types and two occupations, which are subject to correlated fluctuations in output. The equilibrium is characterized by occupational mismatch which is associated with a wage penalty. Every worker has a fixed number of social contacts in the network. The fraction of contacts of the same occupational type defines homophily of the social network, so this paper investigates the optimal level of network homophily. Workers are risk-neutral and take aggregate variables as given, so their optimal individual choice is full homophily. This is different from the social planner's perspective. The planner internalizes external effects of workers' network choices on aggregate variables, so there exists a unique interior value of network homophily maximizing the present value of income. On the one hand, higher homophily is associated with lower occupational mismatch. But on the other hand, higher homophily separates the two groups of workers, prevents exchange of information about open vacancies, and leads to more unemployment, especially in recessions. So it is the trade-off between these two effects and not the desire to reduce income volatility, as in standard portfolio theory, which gives rise to network diversification. Comparative statics shows that optimal network homophily is lower and diversification is stronger with a lower wage penalty from mismatch, lower unemployment benefit and negative correlation in output fluctuations.

1. Introduction

This paper investigates the link between social networks and welfare in the context of a frictional labour market with occupational mismatch. Empirical studies show that 30-60% of new hires find jobs via referrals.[1] At the same time, there is evidence that up to 47% of workers in some occupations are mismatched (Robst, 2007). Thus it is natural to ask whether the network channel of job search contributes to higher occupational mismatch. From a theoretical perspective, Bentolila et al. (2010) and Horvath (2014b) show that social networks with weak homophily may generate more mismatch compared to the formal channel of search. Weak homophily here means that workers have many social contacts in occupations other than their own. However, more mismatch is not equivalent to lower welfare, especially in the presence of volatile output. On the contrary, it may be optimal for workers to diversify their networks across occupations in order to reduce the risk of unemployment even if this strategy is associated with more mismatch. This study fills the gap in the analysis of network implications for social welfare and investigates the optimal level of network diversification in a setting with stochastic output.

The ingredients of the model are as follows. There are two worker types and two occupations. The type of worker is given by the initial training in one of the two occupations. Every worker can be unemployed, employed in the primary occupation or mismatched, which is associated with a wage penalty. Output fluctuations are described by a time-homogeneous transition matrix and are correlated between the two occupations. In the benchmark case there are binary fluctuations in each occupation, e.g. a period of expansion with high output and many vacancies and a period of recession with low output and fewer vacancies. Every worker has a fixed total number of social contacts, which is the network size. The level of network homophily is characterized by the proportion of contacts with other workers of the same type. Thus a higher level of homophily implies a less diversified network and vice versa.

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E-mail address: azaharieva@wiwi.uni-bielefeld.de.

[1] The incidence of referrals is 34–36% in France (Margolis and Simmonet (2003) and Delattre and Sabatier (2007)), 47% in Italy and Portugal (Pistaferri (1999) and Addison and Portugal (2002)), 50–56% in the United States (Granovetter (1995) and Bentolila et al. (2010)).

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In this setting the primary contribution of the paper is a detailed characterization of the optimal network diversification level from the individual and social perspective. Given that workers are risk-neutral and take aggregate indicators as given, they choose full homophily of the social network under some realistic conditions. This is different when the problem is considered from the perspective of social welfare. The social planner internalizes externalities that individual network choices impose on other labour market participants (other workers and firms) and takes into account changes in aggregate variables, such as the equilibrium unemployment rate and vacancies. So for a large range of parameter values, there exists an interior homophily level which is maximizing workers' expected present value of income. Hence this paper supports policies targetting stronger occupational diversification (i.e. interdisciplinary projects and educational programms) as a means of reducing network homophily.

First, the model is considered in a setting with exogenous vacancies and no on-the-job search. In this setting there are two counteracting effects of higher homophily on the expected income. On the one hand, higher homophily reduces the average fraction of time workers spend in mismatch. This is a positive effect on the expected income since workers are less likely to suffer from a wage penalty. On the other hand, stronger homophily implies that the two groups of workers are increasingly separated from each other, which prevents information exchange about open positions. Thus unemployment is higher especially in the times of low labour demand. This is a negative effect on the expected income of workers since unemployment is also associated with a temporary drop in income. This trade-off leads to the optimality of a diversified social network. Note that this diversification result is not driven by the risk aversion of workers and their desire to reduce the volatility of income as in standard portfolio theory (Markowitz, 1970).

Optimal diversification level is robust to parameter changes in the comparative statics analysis. For example, I find that the optimal homophily level is higher and diversification is weaker if labour demand fluctuations are positively correlated. This is because with positive correlation it is less likely that the mismatch occupation has high labour demand while it is low in the primary occupation of the worker. Thus the gain from diversification is reduced. On the contrary, the optimal homophily level is lower and diversification is stronger with a lower unemployment benefit. Consider the situation when the primary occupation has low labour demand and the person becomes unemployed. If unemployment insurance is relatively low, it is optimal for the worker to have more contacts in the mismatch occupation in order to leave the state of unemployment as soon as possible. Lower wage penalty from mismatch is also associated with stronger diversification since the cost of diversification is reduced.

Second, the paper is extended to allow for on-the-job search in the state of mismatch. On the one hand, searching on-the-job is a valuable option for workers which should raise the present value of income. But on the other hand, searching mismatched workers reduce job-finding chances of unemployed workers, which raises unemployment and has a negative effect on income. I find that this negative effect is dominating especially if social networks are relatively well diversified. Thus the optimal level of homophily is higher and diversification is weaker with on-the-job search. Nevertheless, this negative effect of on-the-job search on income is partially mitigated when job creation is endogenized in the final version of the model. The optimal homophily is estimated at 0.8, which means 80% of contacts in the primary occupation and 20% in the mismatch occupation. The welfare gain from diversification is equal to 1.3% compared to a fully diversified network and 0.6% compared to a fully homophilous network.

This paper is closely related to the literature on social networks in the labour market. The first idea to introduce a separate homophily parameter into an economic model is due to Montgomery (1991). This author and later Simon and Warner (1992) emphasize the point that friends and acquaintances are likely to have similar skills and ability (homophily by skills). Thus referrals from high ability employees reveal positive information to the firm about the quality of the match. This idea is empirically confirmed by Hensvik and Skans (2013) who show that in Sweden entrants are more likely to be linked to high ability incumbent employees than to low ability incumbents (defined from test scores or wages). Stupnytska and Zaharieva (2015) extend this idea by separating family and professional contacts in their model. In the equilibrium there is a self-selection of low ability workers into family referrals and high ability workers into professional referrals, which generates a U-shape hiring pattern. Overall, transmitting information about applicants' characteristics to the employer is a first influence channel of social networks, which is particularly important in a setting with heterogeneous workers.

Ioannides and Soetevent (2006) and Fontaine (2008) describe a second influence channel which is based on the transmission of information about vacancies between connected workers. In the former study better connected workers experience lower unemployment rates and receive higher wages. Fontaine (2008) considers a frictional labour market and shows that differences in networks can generate wage dispersion among equally productive workers. Other studies incorporating networks into the search and matching framework include Kugler (2003), Cahuc and Fontaine (2009), Zaharieva (2013) and Galenianos (2014). Kugler (2003) suggests that referees may exert peer pressure on newly hired workers, whereas Zaharieva (2013) shows that bargained wages are inefficiently high in the equilibrium because workers do not internalize the positive externality on their network connections. Galenianos (2014) predicts and confirms empirically a positive correlation between referral hiring and matching efficiency across industries. Cahuc and Fontaine (2009) is a first study incorporating an explicit structure of the network into the matching function. Their network approach is also used in the present study but there is no mismatch and diversification in their model. To the best of my knowledge, there are only two studies combining social networks and on-the-job search. Both Horvath (2014a) and Zaharieva (2015) consider a setting with heterogeneous firms, hence employed workers accept job offers from more productive employers and forward other offers from less productive employers to their network connections. This setup implies that referral offers are associated with wage penalties. This feature is also present in the current study as mismatch jobs pay low wages and there is an incentive for workers to continue searching on-the-job in the hope of better payment in the primary occupation. Despite this similarity neither Horvath (2014a) nor Zaharieva (2015) consider network diversification in the presence of output fluctuations.

Most other studies on occupational mismatch are empirical and distinguish between vertical and horizontal mismatch. The former approach investigates whether workers are over- or underqualified for the job. In contrast, horizontal mismatch appears in situations when the worker doesn't have the “right type” of education to perform the job successfully, thus this approach is about the degree of correspondence between the field of study and the occupational choice. This latter idea is also used in the present work. A number of early empirical papers on horizontal mismatch include Allen and van der Velden (2001), Wolbers (2003) and Robst (2007). For example, Wolbers (2003) considers data on school graduates in a number of Western European Economies and finds that school-leavers from humanities, arts and agriculture are more likely to be mismatched than those from engineering, manufacturing, business and law. He also considers the business cycle perspective and reports that in times of high unemployment school-leavers more often have to accept a job that does not fit their field of education. Another interesting finding of that paper is that for school-leavers with a job mismatch, the odds of looking for another job is 1.4 times larger than for the properly matched school-leavers. Robst (2007) finds similar results for college graduates in the United States, where 27–47% of workers in arts, social sciences, psychology, languages and biology are mismatched. He also reports that horizontal mismatch is associated with a wage loss of about 10%.

More recent empirical studies on occupational mismatch include

Allen and de Weert (2007), Nordin et al. (2008), and Beduwe and Giret (2011). The former study finds that the proportion of workers employed with an appropriate level of schooling but in a different field is 6% in Spain, 10% in Germany, 11% in the Netherlands and 18% in the UK. Moreover, this study confirms that mismatched German and British respondents are significantly more likely to look for other work. Nordin et al. (2008) finds that in Sweden 23% of men are strongly mismatched and 16% are weakly mismatched in their job. In addition, they find a very large penalty associated with occupational mismatch in Sweden equal to 32%. People with dentist, police and law education are least often mismatched, whereas those with a biology, psychology or artistic education are again more often mismatched. Finally, Beduwe and Giret (2011) investigate job characteristics of workers who accomplished vocational training and find that 30% of them are vertically matched but horizontally mismatched in France. They also find wage penalties of 2–3% for mismatched workers. Overall, empirical evidence suggests that horizontal mismatch is a frequent phenomenon. Moreover, mismatched workers are more likely to be involved in on-the-job search. There is also a significant wage penalty associated with occupational mismatch.

The plan of the paper is as follows. Section 2 explains notation and the economic environment. Section 3 presents the model with two occupations and output fluctuations. Section 4 contains numerical results and comparative statics analysis. Section 5 concludes the paper.

2. The framework

Consider the model with two groups of infinitely lived risk neutral workers and two occupations. Workers of type A obtained training in occupation A, which is their primary occupation, but they can also work in occupation B, which is a mismatch occupation for them. In a similar way, occupation B is a primary occupation for type B workers, whereas there is mismatch if type B workers are employed in occupation A. Each group of workers is a continuum of measure 1.

There are m macroeconomic states of the world in this economy, $i = 1\ldots m$. Every state i is characterised by the output vector $\{y_A^i, y_B^i\}$ produced by workers employed in their primary occupation. Mismatched workers produce lower output $y_{0j}^i = y_j^i - \Delta y,\ j = A, B$. I assume that Δy is positive but sufficiently small so that firms accept mismatch applications. Firms can open vacancies in each of the two occupations and pay the flow cost c for every open position. Endogenous variables v_A^i and $v^i{}_B$ denote stocks of vacancies open in occupations A and B respectively. In order to keep the model tractable I assume that there is only one channel of job search by means of referrals. Thus hiring takes place if employed workers who get information about open vacancies recommend their contacts for the job. More specifically, firms with open vacancies in occupation j contact type j workers employed in their occupation at rate s and ask them to recommend a friend for the job. This is an exogenous search intensity of employers.

Every worker can be either employed or unemployed and employed workers can be properly matched or mismatched. Let u_j^i denote the measure of unemployed workers of type j and $e^i{}_j$ – the measure of employed type j workers, thus $u_j^i + e_j^i = 1$ in every state i. Variables $e^i{}_{AA}$ and $e^i{}_{BB}$ denote the measures of properly matched employees, whereas $e^i{}_{AB}$ and $e^i{}_{BA}$ are mismatched employees. This notation implies that $e_{AA}^i + e_{AB}^i = e_A^i$ and $e_{BB}^i + e_{BA}^i = e_B^i$. Workers employed in their primary occupation receive a high wage w, whereas mismatched workers receive a low wage $w_0 < w$. These wages are the same in the two occupations and independent of the state. This reflects the idea that workers and firms sign long-term contracts and there is no possibility of renegotiation. $\Delta w = w - w_0$ denotes a wage penalty associated with mismatch. I consider both settings with and without on-the-job search by mismatched workers. Jobs are destroyed at an exogenous rate δ. After the job destruction shock workers become unemployed and firms exit the market. Unemployed workers receive the unemployment benefit $z < w_0$. Workers and firms discount future cash flows at rate r.

Next consider the social structure of the population. Every worker has n social contacts; γn of the same type and $(1-\gamma)n$ contacts of the different type. Variable $\gamma \in [0.5\,..\,1]$ can be interpreted as a level of homophily in the society. Montgomery (1991) refers to it as an "inbreeding bias" by type. If $\gamma = 1$ the society is homophilous as only workers of the same type are connected in networks. In contrast, if $\gamma = 0.5$ the two groups are strongly mixed and there is no "inbreeding bias". In general, homophily refers to the fact that people are more prone to maintain relationships with people who are similar to themselves. There can be homophily by age, race, gender, religion or profession and it is generally a robust observation in social networks (see McPherson et al. (2001) for an overview of research on homophily). The focus of this paper is on the latter type of homophily by profession or occupation. Jackson (2008) distinguishes between homophily due to opportunity and due to choice. In this respect, homophily by occupation is likely to arise due to the fact that workers with the same profession studied or worked together in the beginning of their career. Thus it is rather a limited opportunity of meeting workers from different professions which generates homophily rather than an explicit choice.

At Poisson rate ϕ the macroeconomic state of the economy may change according to the time-homogeneous discrete Markov chain with a transition matrix Π:

$$\Pi = \begin{pmatrix} \pi_1 & \cdots & \pi_m \\ \vdots & \cdots & \vdots \\ \pi_1 & \cdots & \pi_m \end{pmatrix} \quad \text{where} \quad \sum_{i=1}^{m} \pi_i = 1$$

Intuitively, this means that $\phi\pi_i$ is a constant arrival rate of state i with a vector of outputs $\{y_A^i, y_B^i\}$, $i = 1,\ldots,m$. Variable ϕ measures the frequency of output fluctuations. For example, a higher value of ϕ implies more frequent fluctuations, while a lower value of ϕ is associated with higher persistence of economic states.

For the purpose of illustrating the model it is convenient to analyze the case of binary output leading to m=4 states. In this economy every occupation $j = A, B$ can have high output and labour demand (expansion) or low output and labour demand (recession). Let the probability of expansion be identical in the two occupations and equal to 0.5. In addition, let ρ be the correlation coefficient between the two occupations. With this notation, the four transition probabilities $\pi_1, \pi_2, \pi_3, \pi_4$ can be defined and expressed as follows:

The derivation of this matrix is presented in Appendix A. Note that $\rho = 0$ corresponds to the case of independent fluctuations in output. $\rho > 0$ leads to more probability mass on the diagonal, thus symmetric states with two expansions or two recessions become more likely. In contrast, when $\rho < 0$ asymmetric states become more likely. This case is convenient for analyzing comparative statics results with respect to ρ.

2.1. Network matching

This subsection is dealing with the job-finding and the job-filling rates for workers and firms. Even though these transition rates are state-dependent because they depend on the numbers of vacancies and (un)employed workers, I will suppress the upper index i to simplify the notation. In order to model referral hiring I follow the approach of Cahuc and Fontaine (2009). Recall that firms with open vacancies in occupation j contact type j workers employed in their occupation at rate s per unit time and ask them to recommend a friend for the job. I assume that firms never ask mismatched employees for a recommendation, so the average probability of being contacted for properly matched type A workers is sv_A/e_{AA}. Consider some unemployed type A worker x with a fraction of type A contacts in the network equal to γ_x, whereas γ continues to denote the network composition of all other workers in the market. Since unemployed and mismatched contacts are not useful in finding type A jobs, the person has $\gamma_x n e_{AA}$ properly matched type A friends. Multiplying this term with sv_A/e_{AA} yields $\gamma_x nsv_A$, which is the number of friends of person x contacted by type A firms.

Note that person x is not the only contact of the friend, thus there is competition for job information. Suppose that the friend has k other unemployed type A contacts apart from person x. This happens with probability $u_A^k(1-u_A)^{\gamma n-1-k}$ multiplied by the corresponding binomial coefficient. Since all friends are treated equally, the probability of getting a job offer for person x is given by $1/(k+1)$. Since k is a random variable, the final job-finding rate λ_{AA}^x for person x in occupation A is given by:

$$\begin{aligned}\lambda_{AA}^x &= \gamma_x nsv_A \sum_{k=0}^{\gamma n-1}\binom{\gamma n-1}{k}u_A^k(1-u_A)^{\gamma n-1-k}\frac{1}{k+1}\\ &= \gamma_x nsv_A\frac{(1-(1-u_A)^{\gamma n})}{\gamma n u_A} = \frac{\gamma_x}{\gamma}\frac{sv_A}{u_A}(1-e_A^{\gamma n}) \end{aligned} \tag{1}$$

The derivation of this equation is presented in Appendix A. This expression shows the following. If person x wants to increase the fraction of type A contacts γ_x, this has a positive impact on the job-finding rate in occupation A, but only if no other worker does the same. On the contrary, if all other workers also increase their fraction γ, the positive effect of higher γ_x will be completely dissolved. This already reveals differences in the privately optimal decisions of workers and a socially optimal composition of the network. Further I assume that all workers are identical, that is $\gamma_x = \gamma$, and analyze the optimal composition of the social network from a social perspective. So the main research question here is which level of γ will maximize the present value of expected income for all workers of a given group? Is it a network with $\gamma = 0.5$, $\gamma = 1$ or an intermediate value of γ? Differences between privately and socially optimal γ will be discussed in later sections of the paper.

If all workers are identical, the job-finding rates λ_{AA} and λ_{BB} as well as the two job-filling rates ψ_{AA} and ψ_{BB} for firms become:

$$\lambda_{AA} = \frac{sv_A}{u_A}(1-e_A^{\gamma n}) \quad \psi_{AA} = s(1-e_A^{\gamma n}) \quad \lambda_{BB} = \frac{sv_B}{u_B}(1-e_B^{\gamma n}) \quad \psi_{BB} = s(1-e_B^{\gamma n})$$

These equations have an easy intuitive interpretation. Every type A employee has γn type A contacts, where every contact is employed with probability e_A. So the probability that there is at least one unemployed contact of type A who is willing to take the job is equal to $1-e_A^{\gamma n}$. Thus, the number of matches between workers and vacancies of type A is equal to $sv_A(1-e_A^{\gamma n})$.

Next I proceed with identifying the mismatch transition rates λ_{AB}, λ_{BA} and ψ_{AB}, ψ_{BA}. These rates are different depending on whether on-the-job search by mismatched workers is included in the model or not. Consider first, the situation without on-the-job search.

Suppose the firm is asking a randomly chosen employee of type A to recommend a friend for the job. If all type A contacts of the chosen employee are also employed, which happens with probability $e_A^{\gamma n}$, this employee may recommend a contact of type B. The probability that there is at least one unemployed type B contact out of $(1-\gamma)n$ is equal to $1-e_B^{(1-\gamma)n}$. This is because type B workers are employed with probability e_B. Hence, the total number of matches between type B workers and type A vacancies is equal to $sv_A e_A^{\gamma n}(1-e_B^{(1-\gamma)n})$. With this information one can find the job arrival rate to workers B in occupation A, which is denoted by λ_{BA}. This rate is equal to the ratio between the number of corresponding matches per unit time and the number of searching unemployed workers u_B. In a similar way, one can find the job arrival rate to workers A getting jobs in occupation B, that is λ_{AB}.

$$\lambda_{BA} = \frac{sv_A}{u_B}\cdot e_A^{\gamma n}\cdot(1-e_B^{(1-\gamma)n}) \quad \lambda_{AB} = \frac{sv_B}{u_A}\cdot e_B^{\gamma n}\cdot(1-e_A^{(1-\gamma)n})$$

Multiplying these rates by u_j/v_j gives the corresponding job-filling rates $\psi_{BA} = u_B\lambda_{BA}/v_A$ and $\psi_{AB} = u_A\lambda_{AB}/v_B$ for firms. Note that the first subindex defines the worker type.

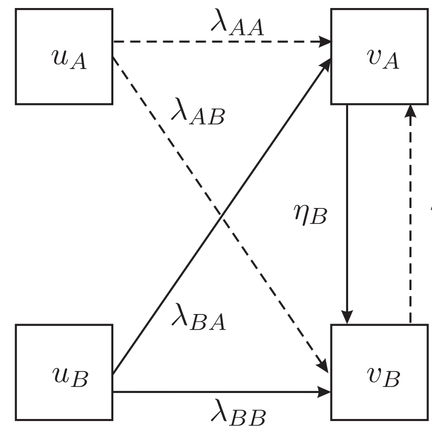

Fig. 1. Labour market transitions with on-the-job search.

2.2. On-the-job search

It is intuitive to think that mismatched workers may continue searching for jobs in their primary occupation. Let η_A denote the job arrival rate to workers A employed in the mismatch occupation B and associated with on-the-job search. In a similar way, η_B denotes the job arrival rate to workers B associated with on-the-job search (see Fig. 1).

If on-the-job search is included in the model, the incumbent employee contacted by the firm can choose not only among unemployed friends of both types, but may also forward the job offer to mismatched friends. I assume that this employee acts according to the following scheme:

- If there is at least one unemployed type A contact, forward the offer to this person;
- If there are no unemployed type A contacts, check if there is at least one mismatched type A contact and forward the offer to this person;
- If none of type A contacts is either unemployed or mismatched, check if there is at least one unemployed type B contact and forward the offer to this person;
- If none of type A contacts is either unemployed or mismatched and none of type B contacts is unemployed the offer is lost.

In addition, the employee should randomize if there are several contacts in the chosen group. This sequence of actions implies that workers always first try to recommend their contacts of the same type, regardless of whether they are unemployed or mismatched, and only afterwards forward the offer to the unemployed contacts of the opposite type. One rationale for this assumption is that workers with the same training are often working in teams and their knowledge is complementary to each over. Thus there may be private productivity gains from recommending workers of the same type. Nevertheless, no specific technology assumptions are necessary for the purpose of this paper. When choosing between unemployed and mismatched workers of the same type, it is intuitive to think that the employee is maximizing utility of the network's member and puts unemployed friends first in the order of priority.

Given the described priority order there is no change in the expressions for λ_{AA} and λ_{BB} compared to the model without on-the-job search. Next suppose the worker doesn't have any unemployed type A contacts which happens with probability $e_A^{\gamma n}$. Conditional on being employed, a given contact is working in occupation A with probability e_{AA}/e_A and is mismatched with a counterprobability e_{AB}/e_A. Thus with probability $(e_{AA}/e_A)^{\gamma n}$ all employed contacts are working in occupation A. This means that $1-(e_{AA}/e_A)^{\gamma n}$ is the probability that there is at least one mismatched employee out of γn employed type A contacts. This reasoning allows me to calculate the job arrival rate η_A in the following way:

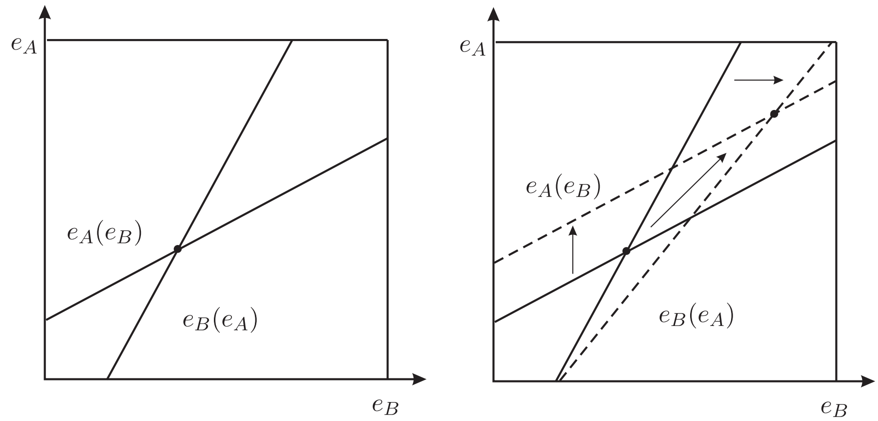

Fig. 2. Left panel: equilibrium. Right panel: positive shock in v_A.

$$\eta_A = \frac{sv_A}{e_{AB}} \cdot e_A^{\gamma n} \cdot \left(1 - \left(\frac{e_{AA}}{e_A}\right)^{\gamma n}\right) = \frac{sv_A}{e_{AB}} \cdot (e_A^{\gamma n} - e_{AA}^{\gamma n})$$

Further, note that with probability $e_{AA}^{\gamma n} = e_A^{\gamma n} \cdot (e_{AA}/e_A)^{\gamma n}$ neither of type A contacts of the incumbent employee is unemployed nor mismatched. However, with probability $1 - e_B^{(1-\gamma)n}$ there is at least one unemployed type B contact, so the job-finding rate λ_{BA} becomes:

$$\lambda_{BA} = \frac{sv_A}{u_B} \cdot e_{AA}^{\gamma n} \cdot (1 - e_B^{(1-\gamma)n})$$

In a similar way, one can derive the job-finding rate λ_{AB} and the job arrival rate to mismatched type B workers associated with on-the-job search η_B:

$$\lambda_{AB} = \frac{sv_B}{u_A} \cdot e_{BB}^{\gamma n} \cdot (1 - e_A^{(1-\gamma)n}) \quad \text{and} \quad \eta_B = \frac{sv_B}{e_{BA}} \cdot (e_B^{\gamma n} - e_{BB}^{\gamma n})$$

Considering the problem from the perspective of firms, let variables τ_j, $j = A, B$ denote the probability of filling the position with a type j worker who was previously in mismatch employment and was searching on-the-job. Then $\tau_A = \eta_A e_{AB}/v_A$ and $\tau_B = \eta_B e_{BA}/v_B$. Finally, note that the two job-filling rates ψ_{BA} and ψ_{AB} are formally again given by $\psi_{BA} = u_B \lambda_{BA}/v_A$ and $\psi_{AB} = u_A \lambda_{AB}/v_B$ but they are different from the expressions without on-the-job search.

3. The model

Sections 3.1–3.3 are dedicated to the analysis of unemployment rates u_A and u_B, where the upper index i is again suppressed, whereas Section 3.4 explains the present value equations.

3.1. Unemployment rates without on-the-job search

In order to find the equilibrium unemployment rates u_A and u_B I follow the approach by Hall and Milgrom (2008). Although in principle both unemployment rates are separate state variables, they move so much faster than i that one can use the equilibrium values as close approximations of the actual values of unemployment rates. Thus in every state i, the inflow of workers into unemployment should be equal to the outflow of workers. On the one hand, at rate δ every worker loses the job, thus, the inflow of type A workers is equal to $\delta(1 - u_A)$. On the other hand, every unemployed type A worker finds some job at rate $\lambda_{AA} + \lambda_{AB}$, either in the primary occupation or in the mismatched occupation. So the differential equation for the unemployment rate of type A workers becomes: $\dot{u}_A = \delta(1 - u_A) - u_A(\lambda_{AA} + \lambda_{AB})$. And the differential equation for the unemployment rate of type B workers is: $\dot{u}_B = \delta(1 - u_B) - u_B(\lambda_{BB} + \lambda_{BA})$. In the equilibrium it should hold $\dot{u}_A = 0$ and $\dot{u}_B = 0$, so that:

$$u_A = \frac{\delta}{\delta + \lambda_{AA} + \lambda_{AB}} \qquad u_B = \frac{\delta}{\delta + \lambda_{BA} + \lambda_{BB}} \tag{2}$$

Inserting values for λ_{AA}, λ_{AB}, λ_{BB} and λ_{BA} into (2) produces a system of two equations in two employment variables e_A and e_B. This is summarized in the following lemma:

Lemma 1. *For every state i, $i = 1, \ldots, m$, the equilibrium employment rates e_A and e_B are uniquely determined from the following system of equations*:

$$sv_A(1 - e_A^{\gamma n}) + sv_B \cdot e_B^{\gamma n} \cdot (1 - e_A^{(1-\gamma)n}) = \delta e_A \quad \Rightarrow \quad e_A(e_B) \tag{3}$$

$$sv_A \cdot e_A^{\gamma n} \cdot (1 - e_B^{(1-\gamma)n}) + sv_B \cdot (1 - e_B^{\gamma n}) = \delta e_B \quad \Rightarrow \quad e_B(e_A) \tag{4}$$

A higher number of vacancies v_A leads to higher employment in both occupations e_A and e_B and higher job-finding rates λ_{AA}, λ_{AB}, λ_{BB} and λ_{BA}. The same is true for a higher v_B.

Proof. Appendix A.□.

Consider Eq. (3). The right-hand side of this equation is a linear function increasing from 0 to δ when e_A is increasing from 0 to 1. The left-hand side of this equation is decreasing down to zero when e_A=1. So there exists a unique intersection between these curves. A larger value e_B raises the left-hand side of equation (3), thereby increasing e_A. Thus, one can write e_A as an increasing function of e_B, where $e_A(0) > 0$ and $e_A(1) < 1$. This relation highlights spillovers between the two occupations. If a larger fraction of type B workers is employed (i.e. e_B is rising), then more type B workers will be recommending their type A social contacts for jobs in occupation B. So the equilibrium employment rate of type A workers is higher. The same holds true for a higher employment rate e_A, which has a positive effect on the employment rate of type B workers. This follows from equation (4), where $e_B(0) > 0$ and $e_B(1) < 1$. Hence the equilibrium values of e_A and e_B can be obtained at the intersection between the two positively sloping curves $e_A(e_B)$ and $e_B(e_A)$ which is illustrated on the left panel of Fig. 2.

Next consider a positive change in vacancies v_A which is illustrated on the right panel of Fig. 2. First, there is a direct positive effect on the employment of type A workers as finding jobs in occupation A becomes easier. Graphically this corresponds to the upward shift in $e_A(e_B)$. Second, a higher employment of type A workers brings more type B workers into jobs, so there is a downward rotation in the curve $e_B(e_A)$. Combining these two shifts together one can see that due to networks a higher number of vacancies in one occupation is propagating employment in the other occupation. However, this network multiplier is not necessarily a desirable feature for the labour market as it will unambiguously amplify the rise of unemployment in the economy-wide recession. As a final remark in this subsection, notice that the corner case $\gamma = 1$, corresponding to full homophily, implies that $e_A(e_B)$ is a horizontal line independent of e_B and $e_B(e_A)$ is a vertical line independent of e_A. In this case, unemployment shifts in response to vacancy fluctuations are not amplified as the two types of workers are not connected.

3.2. Unemployment rates with on-the-job search

Differential equations for unemployment don't change with on-the-job search, thus $\dot{u}_j = \delta e_j - (\lambda_{jA} + \lambda_{jB})(1 - e_j),\ j = A, B$. Next consider changes in variable e_{AA}. The inflow of workers into this category consists of unemployed type A workers $\lambda_{AA} u_A$ and mismatched workers coming from occupation B, that is $\eta_A e_{AB}$. Given that $e_{AB} = e_A - e_{AA}$, $u_A = 1 - e_A$ and repeating the analysis for type B workers one gets the following steady state conditions for e_{AA} and e_{BB}:

$$e_{AA}(\eta_A + \delta) = \lambda_{AA}(1 - e_A) + \eta_A e_A \quad e_{BB}(\eta_B + \delta) = \lambda_{BB}(1 - e_B) + \eta_B e_B$$

Solving these two equations jointly with the steady state equations $\dot{u}_A = 0$ and $\dot{u}_B = 0$ with respect to variables $\{e_A, e_{AA}, e_B, e_{BB}\}$ gives rise to Lemma 2:

Lemma 2. *For every state i, $i = 1, ..., m$, the equilibrium rates e_A, e_{AA}, e_B and e_{BB} are uniquely determined from the following system of equations*:

$$sv_A(1 - e_A^{\gamma n}) + sv_B \cdot e_{BB}^{\gamma n} \cdot (1 - e_A^{(1-\gamma)n}) = \delta e_A \quad \text{where} \quad \delta e_{BB} = sv_B(1 - e_{BB}^{\gamma n}) \tag{5}$$

$$sv_A \cdot e_{AA}^{\gamma n} \cdot (1 - e_B^{(1-\gamma)n}) + sv_B \cdot (1 - e_B^{\gamma n}) = \delta e_B \quad \text{where} \quad \delta e_{AA} = sv_A(1 - e_{AA}^{\gamma n}) \tag{6}$$

A higher number of vacancies v_j leads to higher employment in both occupations e_A and e_B, $j = A, B$. However, a higher v_A raises the fraction of mismatched type B workers e_{BA}, while a higher v_B raises the fraction of mismatched type A workers e_{AB}. For the same state, employment in both occupations is lower in the model with on-the-job search.

These findings are illustrated on Fig. 3. Note that e_{BB} is the point where function $e_B(e_A)$ is crossing the horizontal axis, so that $e_{BB} = e_B(0)$. This means that $e_A = e_A(e_B(0))$. Intuitively, on-the-job search reduces employment of type A workers as their chances of finding jobs in occupation B are getting worse. In a similar way, e_{AA} is the point where function $e_A(e_B)$ is crossing the vertical axis, so that $e_{AA} = e_A(0)$. This means that $e_B = e_B(e_A(0))$, thus the equilibrium employment of type B workers is also lower with on-the-job search.

Further, consider a higher number of vacancies v_A which is illustrated on the right panel of Fig. 3. This vacancy shock is associated with an upward shift of the curve $e_A(e_B)$ and a downward rotation of the curve $e_B(e_A)$, thus triggering a rise of employment rates e_A and e_B. However, there is no change in the number of type B workers employed in their primary occupation, that is e_{BB} remains unchanged with on-the-job search. If occupation A has low labour demand, then relatively many type B workers find jobs in their primary occupation directly from unemployment and relatively few workers go through the state of intermediate mismatch employment in occupation A. In contrast, when occupation A has high labour demand, then relatively many type B workers go through the state of intermediate employment in occupation A and relatively few of them find jobs directly from unemployment. However, the sum of the two inflows (from unemployment and mismatched jobs) remains unchanged whatever the situation in occupation A, leading to the unchanged rate e_{BB}. Hence, the number of mismatched type B workers $e_{BA} = e_B - e_{BB}$ is unambiguously higher when occupation A has high labour demand.

Table 1
Definition of transition probabilities π_i, $i = 1, 2, 3, 4$.

	Expansion in B	Recession in B
Expansion in A	$\pi_1 = 0.25(1 + \rho)$	$\pi_2 = 0.25(1 - \rho)$
Recession in A	$\pi_3 = 0.25(1 - \rho)$	$\pi_4 = 0.25(1 + \rho)$

3.3. Numerical example

This subsection illustrates the theoretical result from Lemma 1 by means of a numerical example. As a starting point the analysis is focused on the model with exogenous vacancies and binary fluctuations in labour demand. This is the model with m=4 states (see Table 1), where every occupation can have high labour demand (with a vacancy rate equal to 0.052) or low labour demand (with a vacancy rate equal to 0.045). Also note that the two occupations are completely symmetric with this parameter choice. Other parameters are set in order to reproduce realistic features of a stylized labour market and include r=0.02, $\delta = 0.1$, s=2, n=40. These parameters will be used everywhere throughout the paper (Table 2).

Consider the economy in state 4 which is an economy-wide recession (v_A=0.045, v_B=0.045). At rate $\phi\pi_2$ occupation A recovers which is captured by the rise in vacancies v_A from 0.045 to 0.052. In accordance with Lemma 1 this is associated with a rise of employment of type A workers e_A from 0.896 to 0.930. In addition, there is a positive spillover effect on type B workers whose job-finding rate in the mismatch occupation λ_{BA} is increasing from 0.085 to 0.251 and their employment rate e_B is increasing from 0.896 to 0.922. At rate $\phi\pi_1$ occupation B recovers and the economy is moving from state 2 to state 1. This is associated with a moderate rise of employment e_B from 0.922 to 0.942 followed by a small increase of employment e_A from 0.930 to 0.942 due to the network spillover. Finally, note that 21.9% of workers are mismatched in state 1. This is due to the fact that vacancy stocks

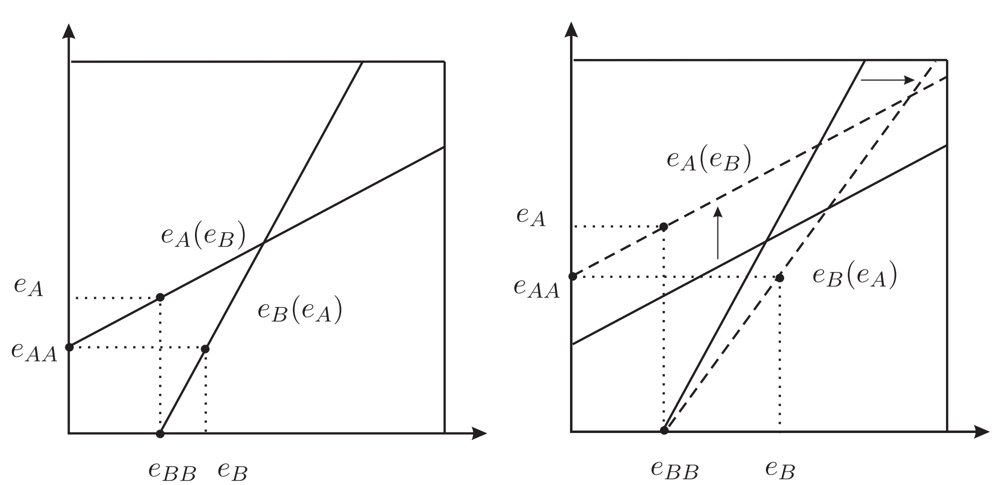

Fig. 3. Left panel: equilibrium with OJS. Right panel: positive shock in vacancies v_A.

Table 2
The model with m=4 states, $\gamma = 0.5$.

	Expansion in B, υ_B=0.052	Recession in B, υ_B=0.045
Expansion in A υ_A=0.052	State 1 with prob. π_1 $e_A = e_B = 0.942$ $e_{AB} = e_{BA} = 0.219$ $\lambda_{AA} = \lambda_{BB} = 1.245$ $\lambda_{AB} = \lambda_{BA} = 0.376$	State 2 with prob. π_2 e_A=0.930, e_B=0.922 $e_{AB} = 0.138$, $e_{BA} = 0.196$ $\lambda_{AA} = 1.135$, $\lambda_{BB} = 0.935$ $\lambda_{AB} = 0.197$, $\lambda_{BA} = 0.251$
Recession in A υ_A=0.045	State 3 with prob. π_3 e_A=0.922, e_B=0.930 $e_{AB} = 0.196$, $e_{BA} = 0.138$ $\lambda_{AA} = 0.935$, $\lambda_{BB} = 1.135$ $\lambda_{AB} = 0.251$, $\lambda_{BA} = 0.197$	State 4 with prob. π_4 $e_A = e_B = 0.896$ $e_{AB} = e_{BA} = 0.089$ $\lambda_{AA} = \lambda_{BB} = 0.773$ $\lambda_{AB} = \lambda_{BA} = 0.085$

are relatively high in both occupations and workers prefer to be mismatched rather than unemployed. In contrast, the mismatch level is only 8.9% in state 4. High fractions of mismatched workers in all states show that the state of mismatch is a stumbling block for workers on their way to proper employment. One reason for this is the impossibility to continue searching on-the-job in the considered setting, so next I turn to the characterisation of the model with on-the-job search in the state of mismatch.

When on-the-job search is permitted, the equilibrium employment rates are calculated according to Lemma 2 and presented in Table 3. First, with on-the-job search unemployment is higher and employment is lower in every state. This is because unemployed workers of the opposite type are now less likely to hear about a job as priority is given to unemployed and mismatched workers of the same type. Even more striking is that the fraction of mismatched workers is also lower. For example, in state 1 when both occupations have expansion periods, mismatched workers constitute only 2.2% of their group. This is much lower than 21.9% in the model without on-the-job search. These stock variables can be low for two reasons. Either because the inflow of mismatched workers is relatively low or because the outflow is too high. Focusing on type A workers, Table 3 reveals that the outflow rate of mismatched type A workers ($\eta_A = 0.377$) is almost three times larger than the inflow rate $\lambda_{AB} = 0.141$. Thus the stock of mismatched workers is low mostly due to the high outflow rate. Hence many workers accept jobs in the mismatch occupation with a fast transition to their primary occupation thereafter. So in this setting mismatch employment is rather a stepping stone on the way to proper employment.

Further, as follows from Table 3 mismatch can still be significant despite on-the-job search in the asymmetric states 2 and 3. For example, in state 2 we can see that 4.2% of type B workers are mismatched. This is intuitive as these workers experience difficulties finding jobs in their primary occupation B and accept jobs in the mismatch occupation at rate $\lambda_{BA} = 0.120$. However, the corresponding transition rate from A to B is relatively low, $\eta_B = 0.174$, and 4.2% of type B workers remain mismatched on average in state 2.

Table 3
The model with m=4 states, $\gamma = 0.5$.

	Expansion in B, υ_B=0.052	Recession in B, υ_B=0.045
Expansion in A υ_A=0.052	State 1 with prob. π_1 $e_A = e_B = 0.925$ $e_{AB} = e_{BA} = 0.022$ $\eta_A = \eta_B = 0.377$ $\lambda_{AA} = \lambda_{BB} = 1.090$ $\lambda_{AB} = \lambda_{BA} = 0.141$	State 2 with prob. π_2 e_A=0.912, e_B=0.903 $e_{AB} = 0.008$, $e_{BA} = 0.042$ $\eta_A = 0.326$, $\eta_B = 0.174$ $\lambda_{AA} = 0.989$, $\lambda_{BB} = 0.816$ $\lambda_{AB} = 0.043$, $\lambda_{BA} = 0.120$
Recession in A υ_A=0.045	State 3 with prob. π_3 e_A=0.903, e_B=0.912 $e_{AB} = 0.042$, $e_{BA} = 0.008$ $\eta_A = 0.174$, $\eta_B = 0.326$ $\lambda_{AA} = 0.816$, $\lambda_{BB} = 0.989$ $\lambda_{AB} = 0.120$, $\lambda_{BA} = 0.043$	State 4 with prob. π_4 $e_A = e_B = 0.879$ $e_{AB} = e_{BA} = 0.018$ $\eta_A = \eta_B = 0.130$ $\lambda_{AA} = \lambda_{BB} = 0.694$ $\lambda_{AB} = \lambda_{BA} = 0.035$

One remaining question in this section is whether mismatch is pro- or countercyclical. As mentioned in the introduction, empirical studies (e.g. Wolbers (2003) and Moscarini and Vella (2008)) report that in recessions workers are more likely to exit unemployment into mismatch, thus mismatch seems to be countercyclical. Concerning the model, recall that transition probabilities π_i are given by: $\pi_1 = \pi_4 = 0.25(1 + \rho)$ and $\pi_2 = \pi_3 = 0.25(1 - \rho)$. The fraction of mismatched type A workers is equal to 0.022 in state 1 and 0.008 in state 2 when occupation A has high labour demand. In contrast, when labour demand is low in occupation A the fraction of mismatched type A workers is equal to 0.042 in state 3 and 0.018 in state 4, which yields the following averages:

$$\frac{0.022\pi_1}{\pi_1 + \pi_2} + \frac{0.008\pi_2}{\pi_1 + \pi_2} = 0.03 + 0.014\rho \qquad \frac{0.042\pi_3}{\pi_3 + \pi_4} + \frac{0.018\pi_4}{\pi_3 + \pi_4} = 0.06 - 0.024\rho$$

The first (second) term above is the fraction of mismatched type A workers in the times of high (low) labour demand in their occupation. Comparing these two values, one can see that type A workers are more likely to be mismatched when labour demand is low in their primary occupation if $\rho < 0.789$. For example, when $\rho = -1$, we know that 4.2% of type A workers are mismatched in the recession (state 3) and only 0.8% of them are mismatched in the expansion (state 2). Given that the two populations of workers are symmetric the same conclusions hold for type B workers. Thus the model is inline with empirical evidence if the correlation coefficient is not too large. Only in the extreme case when the two occupations are almost perfectly positively correlated ($\rho > 0.789$), mismatch becomes pro-cyclical in the model.

3.4. Present value equations

Next consider present value equations for workers of type A. Let U_A^i denote the present value of unemployed type A workers in state i and U_A be the corresponding column-vector containing values $U^i{}_A$. When unemployed, all workers receive the unemployment benefit z, so let ζ denote a column-vector of z-values, which has dimension $m \times 1$. In addition, let variables $\mathrm{W}^i{}_{AA}$ and $\mathrm{W}^i{}_{AB}$ denote the present values of type A employees in occupations A and B respectively with the corresponding column-vectors of present values W_{AA} and W_{AB}. Concerning wages let ω, ω_0 be the $(m \times 1)$ vectors of wages w and w_0 respectively. Unemployed type A workers can find a job in occupation A, which corresponds to the matrix of job-finding rates Λ_{AA} of dimension $m \times m$, and a mismatch job in occupation B, which corresponds to the matrix of job-finding rates Λ_{AB}. In order to analyse the implications of on-the-job search I use an indicator vector $\mathbb{1}$ with all values equal to 1 if on-the-job search is included in the model and all values equal to 0 otherwise. With this notation the present value equations in matrix form can be written as:

$$\begin{aligned}
rU_A &= \zeta + \Lambda_{AA}(W_{AA} - U_A) + \Lambda_{AB}(W_{AB} - U_A) + \phi(\Pi - I)U_A \\
rW_{AA} &= \omega - \delta(W_{AA} - U_A) + \phi(\Pi - I)W_{AA} \\
rW_{AB} &= \omega_0 - \delta(W_{AB} - U_A) + \mathbb{1}N_A(W_{AA} - W_{AB}) + \phi(\Pi - I)W_{AB} \\
U_j &= \begin{pmatrix} U_j^1 \\ \vdots \\ U_j^m \end{pmatrix} \quad W_{Aj} = \begin{pmatrix} W_{Aj}^1 \\ \vdots \\ W_{Aj}^m \end{pmatrix} \quad \Lambda_{Aj} = \begin{pmatrix} \lambda_{Aj}^1 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & \lambda_{Aj}^m \end{pmatrix} \quad N_j = \begin{pmatrix} \eta_j^1 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & \eta_j^m \end{pmatrix}
\end{aligned}$$

where $j = A, B$, parameters r, δ, ϕ are scalars, I is the identity matrix and N_j is the $m \times m$ matrix containing the job arrival rates $\eta^i{}_j$ on the main diagonal. So the term $N_A(W_{AA} - W_{AB})$ reflects the fact that type A workers employed in occupation B may change the job at rate $\eta^i{}_A$ and return to their primary occupation.[2] Note that at rate ϕ the macro-

[2] One implicit assumption here is that the mismatch wage w_0 is sufficiently high, so that accepting mismatch jobs is optimal for workers. For the detailed characterisation of the mismatch reservation wage see the working paper version of this article (IMW working paper Nr. 547, Bielefeld 2015)

economic state of the economy may change according to the transition matrix Π. Further, one can see that:

$$W_{AA} = M(\omega + \delta U_A) \quad W_{AB} = M_A(\omega_0 + \delta U_A + \mathbb{1}N_A M(\omega + \delta U_A))$$

where $M = [(r+\delta+\phi)I - \phi\Pi]^{-1}$ and $M_j = [(r+\delta+\phi)I - \phi\Pi + \mathbb{1}N_j]^{-1}$ are the auxiliary matrices, $j = A, B$. Inserting these expressions into the equation for U_A and repeating the same procedure for workers of type B, I obtain the following results (see Appendix B):

$$\begin{aligned} U_A &= [(r+\phi)I - \phi\Pi + \Lambda_{AA}(I - \delta M) + \Lambda_{AB}(I - \delta M_A(I + \mathbb{1}N_A M))]^{-1} \\ &\quad \times (z + \Lambda_{AB}M_A w_0 + (\Lambda_{AA} + \mathbb{1}\Lambda_{AB}M_A N_A)Mw) \\ U_B &= [(r+\phi)I - \phi\Pi + \Lambda_{BB}(I - \delta M) + \Lambda_{BA}(I - \delta M_B(I + \mathbb{1}N_B M))]^{-1} \\ &\quad \times (z + \Lambda_{BA}M_B w_0 + (\Lambda_{BB} + \mathbb{1}\Lambda_{BA}M_B N_B)Mw) \end{aligned}$$

Note that the job-finding matrices Λ_{Aj} and Λ_{Bj}, $j = A, B$ depend on the homophily parameter γ, thus the present value of searching workers also depends on γ which allows me to investigate the question whether full homophily $\gamma = 1$ will maximize the present value of income, or it will be maximized for some interior value of γ, implying a diversified (heterophilous) network.

Next consider the present value equations for firms. Let variables V^i_A and V^i_B denote the present values of open positions for firms operating in occupations A, B and V_A, V_B be the corresponding column-vectors. When opening the position firms pay an identical flow cost c stacked in a column-vector σ. At rate ψ^i_{AA} firms of type A are matched with a worker of type A and at rate ψ^i_{BA} they hire an applicant of type B. Let the corresponding matrices of job-filling rates be denoted by Ψ_{AA} and Ψ_{BA}. Further variables J^i_{AA} and J^i_{BA} denote present values of profits for firms in occupation A hiring type A and type B workers respectively. The corresponding column-vectors are denoted by J_{AA} and J_{BA}. In the former case the worker is properly matched and produces a high output y^i_A with the flow profit $\nu^i_A = y^i_A - w$. In the latter case the worker is mismatched and produces a low output y^i_{0A} with the flow profit $\nu^i_{0A} = y^i_{0A} - w_0$. In this respect let column-vectors ν_j, ν_{0j} denote flow profits of firms from hiring properly matched and mismatched workers, $j = A, B$. This yields:

$$\begin{aligned} rV_A &= -\sigma + \Psi_{AA}(J_{AA} - V_A) + \Psi_{BA}(J_{BA} - V_A) + \mathbb{1}T_A(J_{AA} - V_A) \\ &\quad + \phi(\Pi - I)V_A \\ rJ_{AA} &= \nu_A - \delta(J_{AA} - V_A) + \phi(\Pi - I)J_{AA} \\ rJ_{BA} &= \nu_{0A} - \delta(J_{BA} - V_A) - \mathbb{1}N_B(J_{BA} - V_A) + \phi(\Pi - I)J_{BA} \end{aligned}$$

$$V_j = \begin{pmatrix} V^1_j \\ \vdots \\ V^m_j \end{pmatrix} \quad J_{jA} = \begin{pmatrix} J^1_{jA} \\ \vdots \\ J^m_{jA} \end{pmatrix} \quad \Psi_{jA} = \begin{pmatrix} \psi^1_{jA} & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & \psi^m_{jA} \end{pmatrix} \quad T_j = \begin{pmatrix} \tau^1_j & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & \tau^m_j \end{pmatrix}$$

where $j = A, B$. These equations reveal the role of on-the-job search. On the one hand, it is easier for firms in occupation A to hire type A workers, even if they are temporarily mismatched in occupation B. This effect is captured by matrix T_A containing state-specific hiring rates τ^i_A. On the other hand, type B workers employed in occupation A may quit their jobs to return to their primary occupation. This effect is captured by matrix N_B containing state-specific quitting rates η^i_B. Equations for V_B, J_{BB} and J_{AB} can be obtained in a similar way.

In every state i there is free-entry of firms, which means that in the equilibrium V_A=0 and V_B=0. These $2 \times m$ free-entry conditions can be rewritten as:

$$\sigma = (\Psi_{AA} + \mathbb{1}T_A)\nu_A M + \Psi_{BA}\nu_{0A}M_B \quad \sigma = (\Psi_{BB} + \mathbb{1}T_B)\nu_B M + \Psi_{AB}\nu_{0B}M_A \tag{7}$$

Finally, solving these $2 \times m$ free-entry conditions together with $2 \times m$ equilibrium equations for unemployment (3)–(4) (or Eqs. (5)–(6) if on-the-job search is permitted) yields $4 \times m$ equilibrium values of vacancies and employment rates v^i_A, v^i_B, e^i_A and e^i_B $\forall\ i = 1..m$.

3.5. Economy without output fluctuations

This section deals with the simplest version of the model with symmetric occupations but without output fluctuations and on-the-job search. This model serves for the purpose of clarifying the main mechanism giving rise to network diversification. Since the two occupations are symmetric, let v denote the level of vacancies (that is $v = v_j$) and e – the equilibrium employment level for each worker group (that is $e = e_j$, $j = A, B$). Focusing on type A workers, one can show that the value of unemployment rU is a weighted average of income levels z, w and w_0:

$$\begin{aligned} rU_A &= \frac{z(r+\delta) + w\lambda_{AA} + w_0\lambda_{AB}}{r + \delta + \lambda_{AA} + \lambda_{AB}} \\ &= \frac{z(r+\delta)(1-e) + wsv(1 - e^{\gamma n}) + w_0 sve^{\gamma n}(1 - e^{(1-\gamma)n})}{(r+\delta)(1-e) + sv(1 - e^n)} \end{aligned}$$

where the last equality obtains by inserting the job-finding rates λ_{AA} and λ_{AB}. It is straightforward to show that rU_A is increasing in λ_{AA} and λ_{AB} as long as the mismatch wage w_0 is acceptable to workers.[3] Then it follows from Lemma 1 that it is also increasing in the number of vacancies v and in the equilibrium employment e. Further, we know that $sv(1 - e^{\gamma n})$ unemployed workers are matched to firms in their primary occupation and δe_{AA} workers lose jobs and become unemployed. So the equilibrium number of properly employed workers (of a given type) is given by $e_{AA} = sv(1 - e^{\gamma n})/\delta$. Similar logic implies that the equilibrium number of mismatched workers is given by $e_{AB} = sve^{\gamma n}(1 - e^{(1-\gamma)n})/\delta$. So the equilibrium employment level $e = e_{AA} + e_{AB}$ can be obtained from equation $sv(1 - e^n) = \delta e$. Inserting this in the denominator of rU_A in the above equation and considering the limit $r \to 0$ yields the following:

$$\begin{aligned} \lim_{r\to 0} rU_A &= z(1-e) + w\frac{sv}{\delta}(1 - e^{\gamma n}) + w_0\frac{sv}{\delta}e^{\gamma n}(1 - e^{(1-\gamma n)}) \\ &= z(1-e) + we_{AA} + w_0 e_{AB} \end{aligned}$$

which is the social welfare of type A workers. This means that if r is sufficiently small, then maximizing rU_A is equivalent to maximizing expected income of type A workers. Rearranging this equation to separate the effect of γ on rU_A gives the following:

$$\lim_{r\to 0} rU_A = z(1-e) + \frac{sv}{\delta}(w - w_0 e^n) - \frac{sv}{\delta}(w - w_0)e^{\gamma n}$$

Since $e^{\gamma n}$ is decreasing in γ, one can immediately see that expected income rU_A is increasing in the homophily parameter γ. This is because with higher γ workers are less likely to be mismatched and mismatch is associated with a wage penalty $w - w_0$. Note also that the equilibrium employment level e obtains from equation $sv(1 - e^n) = \delta e$. This means that employment is independent of the network composition γ in the setting with fixed vacancies. The reason is that with higher γ more workers are properly matched e_{AA} and less workers are mismatched e_{AB}, but the sum of the two remains unchanged. Given that we consider the setting with symmetric occupations, expected income of both worker types is identical (U_A=U_B), thus one can conclude that social welfare of workers is maximized at full homophily of the social network ($\gamma^* = 1$) in a setting with fixed and exogenous vacancies.

What are the implications of endogenous job creation? The free-entry condition for firms in occupation A in the decentralized economy can be written as:

$$\begin{aligned} c &= \psi_{AA}\frac{y - w}{r + \delta} + \psi_{BA}\frac{y_0 - w_0}{r + \delta} = s(1 - e^{\gamma n})\frac{y - w}{r + \delta} \\ &\quad + se^{\gamma n}(1 - e^{(1-\gamma)n})\frac{y_0 - w_0}{r + \delta} \end{aligned} \tag{8}$$

[3] Accepting w_0 requires that $rU_A > w_0$ which is equivalent to $(w_0 - z)(r + \delta) > (w - w_0)\lambda_{AA}$. This condition guarantees that rU_A is increasing in the job-finding rate λ_{AB}.

where the last equality obtains by inserting the job-filling rates $\psi_{AA} = s(1 - e^{\gamma n})$ and $\psi_{AB} = se^{\gamma n}(1 - e^{(1-\gamma)n})$. Rearranging terms on the right-hand side one gets:

$$(r + \delta)\frac{c}{s} = (y - w) - (y_0 - w_0)e^n - (\Delta y - \Delta w)e^{\gamma n}$$

This equation shows that expected profits of firms are increasing in the homophily parameter γ but decreasing in the employment level e if $\Delta y > \Delta w$. Intuitively this condition means that hiring workers with occupation-specific training yields higher profits $y - w$ than hiring mismatched workers with a flow profit $y_0 - w_0$. This is typically the case in labour intensive occupations with specific human capital. If this condition is satisfied, than stronger homophily γ reduces the probability of hiring mismatched workers and boosts firm profits. From the job creation condition one can see that it is associated with higher profits, higher employment level e and more vacancies v. This implies that expected income of workers is again maximized at full homophily $\gamma^* = 1$ since rU_A is increasing in vacancies and employment.

With endogenous vacancies the total social welfare is given by the welfare of workers and firms.[4] To calculate the welfare of firms I follow the approach of Acemoglu and Shimer (1999) (Appendix A), which implies that the welfare of firms in occupation A can be written as:

$$\lambda_{AA}u\frac{y - w}{r + \delta} + \lambda_{BA}u\frac{y_0 - w_0}{r + \delta} - cv = v\left[\psi_{AA}\frac{y - w}{r + \delta} + \psi_{BA}\frac{y_0 - w_0}{r + \delta} - c\right]$$

Here the first term represents firms' payoff from newly created jobs with type A workers. The number of new jobs is the number of unemployed workers times the probability that each is hired $\lambda_{AA}u$. The second term represents firms' payoff from new matches with type B workers and cv are the costs of opening vacancies. Since $\lambda_{jA}u/v = \psi_{jA}$, $j = A, B$ it follows that the welfare of firms is equal to zero in the steady state given that the free-entry condition is satisfied in each occupation. This implies that total social welfare of workers and firms is maximized at full homophily ($\gamma^* = 1$) if $\Delta y > \Delta w$.

Next consider the case $\Delta y - \Delta w < 0$. This is the situation in capital-intensive occupations, where worker's productivity is largely driven by the quality of equipment rather than occupation-specific human capital. In this case firms gain higher profits by hiring mismatched workers. Even though the overall effect of employment e on profits is now ambiguous, it is still negative if $\Delta y - \Delta w$ is sufficiently small in absolute terms. Thus in this parameter setting it is possible that higher network homophily γ leads to less vacancies v and lower employment e. For workers this implies that higher γ, on the one hand, reduces the probability of being mismatched and suffering a wage penalty $w - w_0$, but on the other hand, it generates fewer vacancies v and higher unemployment $1 - e$. This trade-off may lead to the interior solution γ^* maximizing the social welfare. Thus one can conclude that $\Delta y - \Delta w < 0$ is a necessary condition for network diversification in a setting without output fluctuations.

So far the analysis was focused on γ^* which is optimal from the social perspective neglecting the individual decisions of workers. To capture these, consider some worker x with a network homophily γ_x. The present value of unemployment for this worker can be written as:

$$rU_A(\gamma_x, \gamma) = z + \frac{(w - z)\frac{\gamma_x}{\gamma}sv(1 - e^{\gamma n}) + (w_0 - z)\frac{(1 - \gamma_x)}{(1 - \gamma)}sve^{\gamma n}(1 - e^{(1-\gamma)n})}{(r + \delta)(1 - e) + \frac{\gamma_x}{\gamma}sv(1 - e^{\gamma n}) + \frac{(1 - \gamma_x)}{(1 - \gamma)}sve^{\gamma n}(1 - e^{(1-\gamma)n})}$$

The first order condition for maximizing $rU_A(\gamma, \gamma_x)$ with respect to γ_x is presented in Appendix C. This maximization is performed for fixed networks of other workers γ and given equilibrium employment level e and vacancies v, which a single worker can not influence. The numerator of this first order condition doesn't depend on the homophily level γ_x. Moreover, it is positive if the marginal gain from proper employment $(w - z)sv(1 - e^{\gamma n})/\gamma$ is larger than the marginal gain from mismatch employment $(w_0 - z)sve^{\gamma n}(1 - e^{(1-\gamma)n})/(1 - \gamma)$, so the optimal homophily level is a corner solution $\gamma_x^* = 1$. Assuming that this condition, holds one can see that individual optimum coincides with the social optimum and implies full homophily of the social network ($\gamma^* = \gamma_x^*$) if vacancies are fixed or if vacancies are endogenous and $\Delta y > \Delta w$. However, when $\Delta y < \Delta w$ and its absolute value is sufficiently small, there exists an interior social optimum $\gamma^* < 1$. In this situation workers choosing full homophily of their network γ_x^* act optimally from their individual perspective, however, they don't internalize the negative impact of the network on the endogenous profits of firms. In contrast, the social planner would take into account that higher network homophily γ reduces the expected profits of firms, which leads to less vacancies and lower employment and would recommend a diversified social network.[5]

This section sheds some light on the optimal network diversification from the individual and the social perspective, however, conclusions are limited to the case of a one-state economy without fluctuations in output. Next section continues this analysis and clarifies the role of output fluctuations for network diversification.

3.6. Economy with binary output fluctuations

In order to investigate the role of output fluctuations for network diversification I consider the model with binary fluctuations, so the matrix of transition probabilities is given by Table 1. In the extreme case of perfect negative correlation $\rho = -1$ the model is reduced from 4 to only 2 states (m=2): state 2 with probability 0.5 – expansion in occupation A, recession in occupation B, and state 3 with probability 0.5 – recession in occupation A, expansion in occupation B.

As shown in the previous section, maximizing welfare of a given worker type is equivalent to maximizing the present value of income if r is sufficiently small. However, when output is fluctuating the interests of the two worker groups are not aligned. Ultimately, one is interested in the homophily level γ, which will maximize total social welfare of all workers, that is $0.5U_A^i + 0.5U_B^i$. But it is clear that this homophily level will be a compromise solution respecting interests of both groups. So the first step on the way to this goal is to gain better understanding of the monopoly outcomes of every worker group. From an economic perspective this step is of larger importance because it reveals the group-specific interests of the two worker types and uncovers the underlying mechanism of network diversification. Thus the focus of this paper is on the group-specific optimal diversification level, whereas the second step of aggregating preferences is considered in the end of the paper. Without loss of generality, the problem will be addressed from the perspective of type A workers. The following proposition shows the present value of unemployment U_A^3 in the model without on-the-job search:

[4] Considering $r \to 0$ one can see that the free-entry condition of firms (8) in occupation A can be rewritten as: $cv = e_{AA}(y - w) + e_{AB}(y_0 - w_0)$, which implies that in the equilibrium flow profits of firms on the right-hand side should be equal to the total costs of open vacancies on the left-hand side. This also means that the flow income of employed workers is equal to the flow output minus the costs of job creation: $e_{AA}w + e_{AB}w_0 = e_{AA}y + e_{AB}y_0 - cv$. Inserting this expression into the social welfare function one gets: $\lim_{r\to 0} rU_A = z(1 - e) + e_{AA}y + e_{AB}y_0 - cv$, which shows another way of interpreting social welfare as flow output and leisure net of the job creation costs.

[5] There is also another case when the marginal case from proper employment $(w - z)sv(1 - e^{\gamma n})/\gamma$ is much smaller than the marginal gain from mismatch employment $(w_0 - z)sve^{\gamma n}(1 - e^{(1-\gamma)n})/(1 - \gamma)$, however, this case is ignored in the above analysis for being unrealistic

Proposition. *In the case of binary fluctuations in output, perfect negative correlation between the occupations ($\rho = -1$) and no on-the-job search, the present value of unemployment is a weighted average between wages w, w_0 and the unemployment benefit z:*

$$rU_A^i(\gamma) = z(1 - f_{AA}^i(\gamma) - f_{AB}^i(\gamma)) + wf_{AA}^i(\gamma) + w_0 f_{AB}^i(\gamma)$$

where the weights $f_{AA}^i, f_{AB}^i \in [0..1]$, i=2,3 are given by:

$$f_{Aj}^2(\gamma) = \frac{a\lambda_{Aj}^2 + b\lambda_{Aj}^3}{(r+\delta)(a+b) + a\lambda_{AA}^2 + b\lambda_{AA}^3 + a\lambda_{AB}^2 + b\lambda_{AB}^3} \quad j = A, B$$

$$f_{Aj}^3(\gamma) = \frac{c\lambda_{Aj}^2 + d\lambda_{Aj}^3}{(r+\delta)(c+d) + c\lambda_{AA}^2 + d\lambda_{AA}^3 + c\lambda_{AB}^2 + d\lambda_{AB}^3} \quad j = A, B$$

and variables a, b, c, d are provided in the appendix.

Proof. Appendix D.□.

Consider the problem from the perspective of type A workers. Variable f^i_{AA} is a fraction of time that type A workers are employed in their primary occupation and receive a high wage w. Variable f^i_{AB} is a fraction of time these workers are mismatched and receive a low wage w_0. So the remaining fraction of time $1 - f_{AA}^i - f_{AB}^i$ workers spend unemployed and receive the unemployment benefit z. As shown in the previous section, maximizing social welfare of type A workers can be achieved by maximizing their present value of unemployment rU^i_A. Then the optimal level of diversification γ^* can be found from the following first order condition:

$$\underbrace{-\frac{\partial f_{AA}^i(\gamma^*)}{\partial\gamma}(w-z)}_{\text{Marginal cost of diversification}} = \underbrace{\frac{\partial f_{AB}^i(\gamma^*)}{\partial\gamma}(w_0-z)}_{\text{Marginal gain of diversification}}$$

where $\partial f_{AA}^i/\partial\gamma > 0$ and $\partial f_{AB}^i/\partial\gamma < 0$. Consider diversifying the network by reducing γ. The left-hand side of this condition is the marginal cost of diversification, which is reflected in worse employment chances in the primary occupation for type A workers if gamma is lower. The right-hand side of this equation is the marginal gain, as lower γ implies a higher number of type B contacts and better employment chances in occupation B. Note that the marginal gain of diversification strongly depends on the difference $w_0 - z$. When accepting mismatched jobs in occupation B type A workers increase their flow income from z to w_0 but give up the option of finding a job in their primary occupation (recall that this section investigates the model without on-the-job search) This means that diversification of social contacts is never optimal if $w_0 = z$. In this situation there are no gains from diversification. Thus diversification is more likely with a higher value of w_0 and a lower value of z.

The optimal diversification level γ^* above was derived from the aggregate perspective of type A workers in the initial state i. Given that the explicit analytical solution for γ^* is not feasible, next section is dedicated to the numerical analysis of the model with binary output. Without loss of generality further analysis will be performed for state i=3 as the initial state. One can expect that network diversification is quantitatively more important when output fluctuations are negatively correlated, which increases the frequency of states 2 and 3, and when the primary occupation of workers has low output. This leaves state 3 as a preferred candidate for the initial state. The underlying mechanisms are qualitatively similar for other initial states, however, corner solutions of γ^* are more likely in initial states with higher output.

4. Comparative statics analysis

4.1. Model 1: exogenous vacancies and no on-the-job search

This section presents numerical results for the model with exogenous vacancies and no on-the-job search. Keeping the same parameters as before implies that the vacancy rate is equal to 0.052 in the times of high labour demand and 0.045 in the times of low labour demand.[6] Fig. 4 illustrates the shape of the unemployment present value U_A^3 for different values of the correlation coefficient ρ. The corresponding transition probabilities are given by $\pi_1 = \pi_4 = 0.25(1+\rho)$ and $\pi_2 = \pi_3 = 0.25(1-\rho)$. The left panel of this figure shows that the present value of income is maximized for some interior homophily level. For example, if the two occupations are perfectly negatively correlated. i.e. $\rho = -1$, type A workers achieve a maximum present value when $\gamma^* = 0.825$, which is the proportion of contacts in their primary occupation (see the green solid curve). This corresponds to 33 contacts out of n=40. At the same time, it is optimal for them to have $(1-\gamma)n = 7$ contacts in the mismatch occupation in order to exit unemployment in the times of low labour demand in the primary occupation.

Note that the optimal γ^* is increasing with the correlation coefficient. For example, when the two occupations are perfectly positively correlated, i.e. $\rho \to 1$, the optimal level of homophily is equal to 0.875 (see the purple solid curve). The right panel of Fig. 4 shows expected income U_A^3 as a function of the mismatch wage w_0. One can see that γ^* is sensitive to this parameter and falls down to 0.65 when the mismatch wage is raised to 0.89. This implies a wage penalty of about 1% (0.9/0.89 − 1), whereas it is about 6% in the benchmark specification of the model (0.9/0.85 − 1). This result is intuitive since a higher mismatch wage increases the marginal gain from network diversification.

To understand the reason for diversification consider the left panel of Fig. 5, which illustrates unemployment rates u_A^3 and u^2_A for different levels of homophily. If both occupations have high labour demand (state 1), then the unemployment rate u^1_A is equal to 0.058 for any value of γ. The same is true when both occupations have low labour demand (state 4), so the unemployment rate u^4_A is equal to 0.104 for any value of γ. In contrast, the level of homophily has a strong impact on the unemployment rates if the two occupations are in the opposite states. As follows from Fig. 5, increasing homophily from $\gamma = 0.5$ to 1 is associated with a drop in unemployment from 0.070 to 0.058 in state 2 (see the purple solid curve). Moreover, the same increase in γ is associated with a dramatic rise of unemployment from 0.078 to 0.104 in state 3 (see the green solid curve). Now recall that $rU_A^3(\gamma)$ can be rewritten as (when $\rho = -1$):

$$rU_A^3(\gamma) = w - (w-z)(1 - f_{AA}^3(\gamma) - f_{AB}^3(\gamma)) - (w - w_0)f_{AB}^3(\gamma)$$

The sharp rise in unemployment in times of low labour demand implies that workers spend more and more time on average unemployed as γ is increasing. This is captured by the rise in fraction $(1 - f_{AA}^3 - f_{AB}^3)$, which is illustrated on the right panel of Fig. 5, right axis. So the average income drop from unemployment $(w-z)(1 - f_{AA}^3(\gamma) - f_{AB}^3(\gamma))$ is increasing and achieves its maximum at $\gamma = 1$. This is intuitive since higher network homophily reduces the average chances of workers to exit unemployment; it is a negative effect of higher γ on $rU_A^3(\gamma)$. Nevertheless, the same figure shows that the fraction of time workers spend mismatched, that is f_{AB}^3, is decreasing in γ (left axis). This means that the income drop from mismatch $(w - w_0)f_{AB}^3(\gamma)$ is maximized at $\gamma = 0.5$. This is a positive effect of higher γ on $rU_A^3(\gamma)$. So the optimal γ^* is an interior solution, such that the income drop from unemployment is not yet too large, while the income drop from mismatch is already relatively low.

4.2. Model 2: exogenous vacancies and on-the-job search

This subsection is dedicated to the numerical analysis of the model with on-the-job search, but vacancies are still fixed and exogenous in this subsection. Fig. 6 (right panel) illustrates changes in the unemployment rate for type A workers when on-the-job search is included

[6] More specifically, $v_A^1 = v_A^2 = v_B^1 = v_B^3 = 0.052$ and $v_A^3 = v_A^4 = v_B^2 = v_B^4 = 0.045$

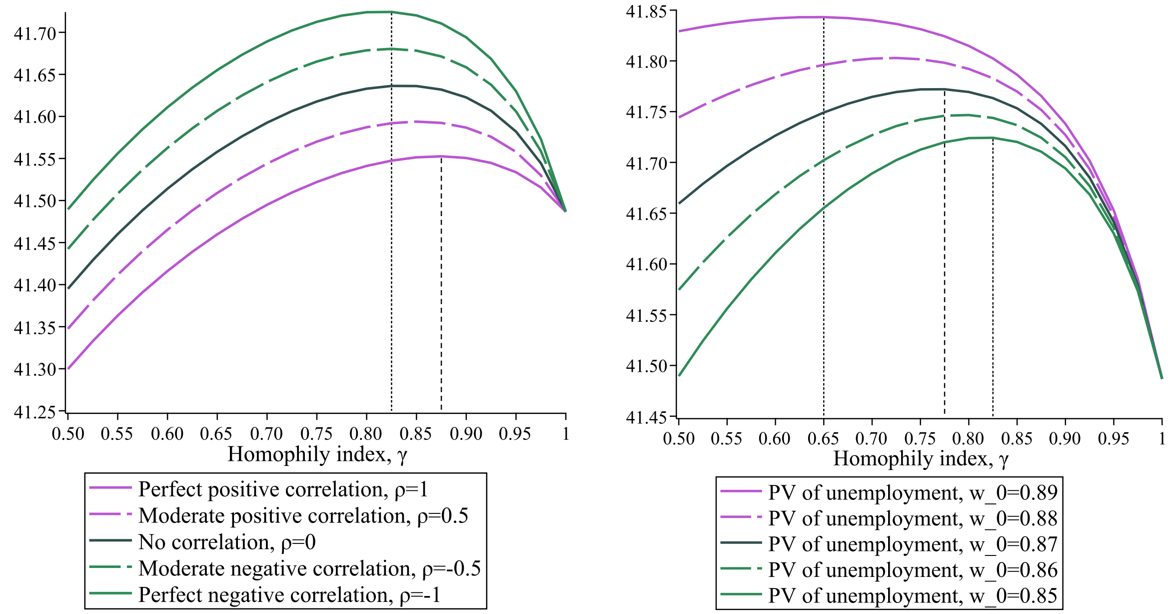

Fig. 4. Left panel: PV of unemployment U_A^3 as a function of the correlation coefficient ρ, w_0=0.85. Right panel: PV of unemployment $U^3{}_A$ as a function of the mismatch wage w_0, $\rho = -1$. Parameters: z=0.2, w=0.9. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

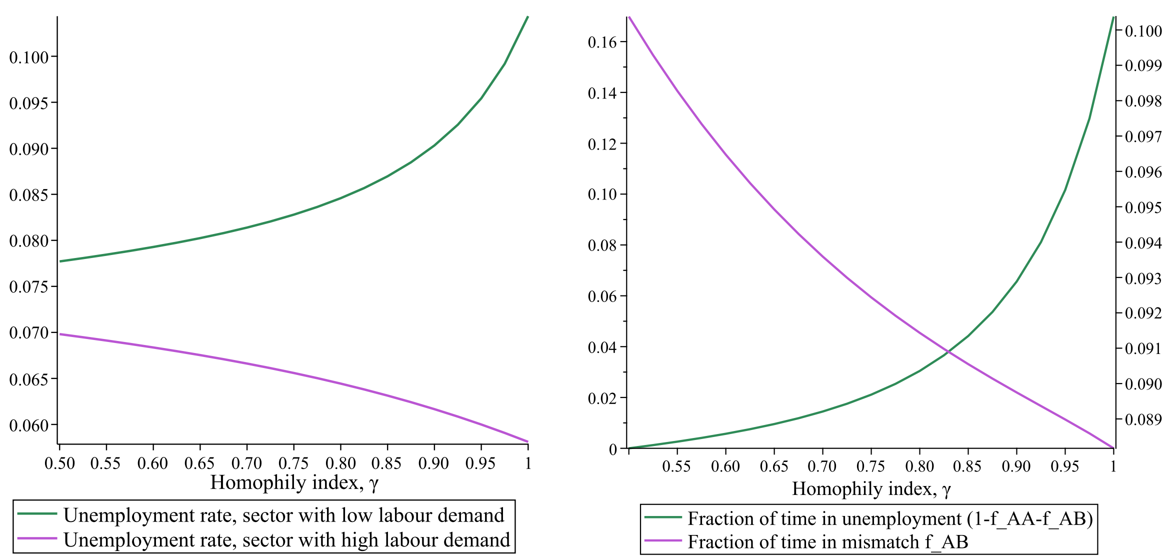

Fig. 5. Left panel: Unemployment rates with low labour demand (u_A^3) and high labour demand ($u^2{}_A$). Right panel: Average fraction of time in unemployment ($1 - f_{AA}^3 - f_{AB}^3$) and in mismatch f_{AB}^3. Parameters: $\rho = -1$, z=0.2, w_0=0.85, w=0.9. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

into the model. In this setting unemployed workers of the different type are ranked behind mismatched workers of the same type, thus the probability of leaving unemployment is lower with on-the-job search and unemployment is higher. For example, for $\gamma = 0.5$ the unemployment rate of type A workers increases from 0.070 to 0.088 in the times of high labour demand (state 2) and from 0.078 to 0.096 in the times of low labour demand (state 3).

Changes in the unemployment rate reveal that there are two counteracting effects of on-the-job search on the present value of income. On the one hand, on-the-job search is beneficial as the option to continue searching in the mismatched state always has a positive value. However, this effect is based on the assumption of unchanged job-finding rates which is not the case in the present model. In contrast, unemployment is increasing and the probability of finding a job for unemployed workers is falling with on-the-job search because some job offers are forwarded to mismatched employees. This is a negative effect on the present value of income. The left panel of Fig. 6 shows that the negative effect is dominating, especially for low values of γ, since the rise in unemployment is particularly pronounced for strong diversification levels. Note that the present value U_A^3 on Fig. 6 is compared to its counterpart without on-the-job search for the case of perfect negative correlation $\rho = -1$, however it is qualitatively similar for all values of ρ.

The optimal level of diversification is decreasing (i.e. higher γ^*) with on-the-job search but the change is very moderate. For example, γ^* goes up from 0.825 to 0.85 in the case of perfect negative correlation ($\rho = -1$), which means that workers should have 34 social contacts of the same type and only 6 contacts of the opposite type. Underlying this result is the fact that primary contacts become more important if the worker wants to leave a mismatch job and come back to the primary occupation. Thus the cost of diversification is higher and the gain is lower.

Fig. 7 shows the average fractions of time workers spend in unemployment ($1 - f_{AA}^3 - f_{AB}^3$) and mismatch f_{AB}^3 in model 2. One can

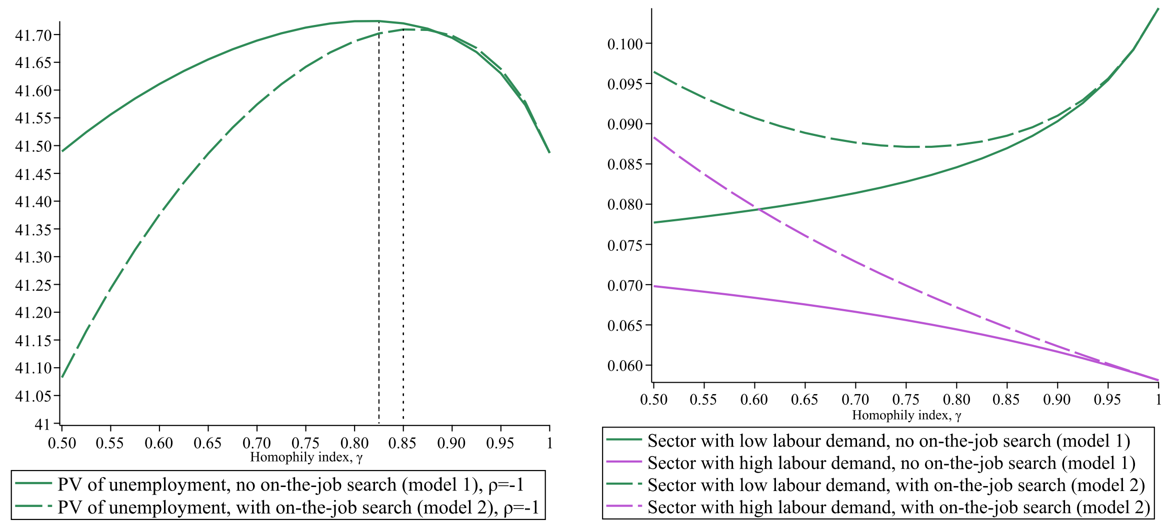

Fig. 6. Left panel: PV of unemployment U_A^3 with and without on-the-job search. Right panel: Unemployment in the sector with low labour demand ($u^3{}_A$) and high labour demand ($u^2{}_A$) with and without on-the-job search. Parameters: $\rho = -1$, z=0.2, w_0=0.85, w=0.9.

see that on-the-job search sharply reduces the time workers are mismatched (from 0.170 down to 0.030 when $\gamma = 0.5$), because the probability of making a job-to-job transition back to the primary occupation is relatively high. However, the fact that mismatched workers receive many job offers from the primary occupation dramatically reduces the chances of finding a job out of unemployment and raises the fraction of time workers spend unemployed. It is even the case that the fraction of time in unemployment with $\gamma = 0.5$ is now larger than with $\gamma = 1$ (compare 0.088 and 0.110). Also the time in unemployment is no longer a monotonous function of γ. Thus full diversification $\gamma = 0.5$ is now suboptimal not because of the sharp income drop due to mismatch as it was in model 1 but rather because the risk of unemployment is too high. As γ is increasing from 0.5 to 1 the average risk of unemployment is falling because less workers are mismatched and more job offers reach unemployed workers. But when γ is already sufficiently high, the probability of mismatch becomes low and unimportant, while the risk of unemployment starts increasing again because the two groups of workers are more and more separated. This effect is the same in models 1 and 2.

Finally, there remains a question whether the drop in the present values of income in model 2 can be attributed to the fact that vacancies are fixed and exogenous throughout this section. Indeed, one can expect that on-the-job search can be beneficial for firms, making it easy to hire workers, hence firms may create more vacancies when on-the-job search is permitted. This question is treated in the next subsection.

4.3. Model 3: endogenous vacancies and on-the-job search

This section is dealing with endogenous job creation. In order to guarantee comparability of the three models, output variables y_j^i and the cost parameter c are chosen such that endogenous vacancies coincide with exogenous vacancies when the two occupations are separated, that is when $\gamma = 1$. Consider occupation A. When $\gamma = 1$ the state space is reduced to only two states from the perspective of firms, namely, the high output state (i=1 or i=2) and the low output state (i=3 or i=4). The high output is set equal to 0.95, which implies a profit mark-up for firms equal to 5.5% over the wage w=0.9. The low output is equal to 0.94. This implies that output drops by slightly more than 1% in the recession. The fall in output of mismatched workers is assumed to be the same and equal to 0.01. This means: $y_A^2 = 0.95$, $y_A^3 = 0.94$ and $y_{0A}^3 = y_{0A}^2 - 0.01$. Thus there remain two unknown parameters: the output of mismatched workers in the good times y_{0A}^2 and the flow cost of an open vacancy c. Restricting endogenous vacancies to be equal to exogenous vacancies from the previous sections and solving two free-entry conditions (7) jointly with two equations for the equilibrium unemployment (5)–(6) for firms A in the good and the

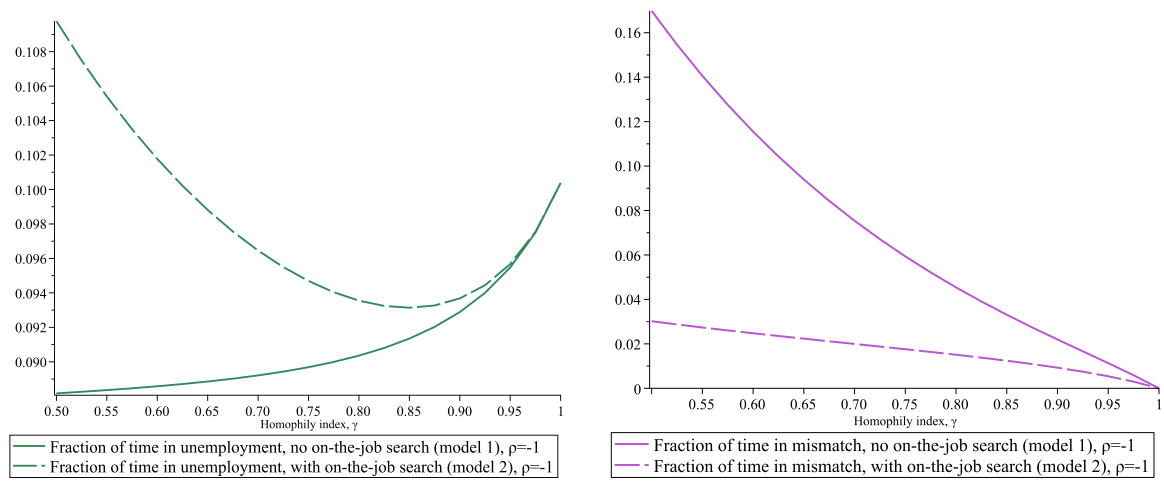

Fig. 7. Left panel: Average fraction of time in unemployment $(1 - f_{AA}^3 - f_{AB}^3)$ with and without on-the-job search. Right panel: Average fraction of time in mismatch f_{AB}^3 with and without on-the-job search. Parameters: $\rho = -1$, z=0.2, w_0=0.85, w=0.9.

bad states yields $y_{0A}^2 = 0.92$ and c=0.71. All output variables for occupation B are set symmetrically.[7] These parameters imply that all three models coincide in the case of full homophily and guarantee that model 3 is comparable to models 1 and 2.

The left panel of Fig. 8 shows vacancies as a function of the homophily parameter γ with endogenous job creation. One can see that in the good times (high output, $y_A^2 = 0.95$) firms increase vacancies above the exogenous level 0.052 whenever $\gamma < 1$. On the contrary, in the bad times (low output $y_A^3 = 0.94$) firms decrease their job creation below the exogenous level 0.045. Note that the correlation coefficient is set at $\rho = -1$. This means when output is high in occupation A (state 2), it is necessarily low in occupation B. Since firms in occupation B reduce their job creation, then (1) type A workers are less likely to exit their primary occupation and (2) type B workers are more likely to enter employment in occupation A. This is gainful from the perspective of type A firms, which leads to more job creation/vacancies in this occupation. Moreover, the advantage for firms A is stronger the more interrelated the two populations are, that is lower γ. However, when output is low in occupation A, it is necessarily high in occupation B. Thus type A workers are more likely to leave their primary occupation accepting type B jobs and there will be less type B applicants. Both of these effects reduce expected profits of firms in occupation A, which leads to less job creation/vacancies in this occupation. Again, this disadvantage is strongest when the two populations are more interrelated. So the divergence in vacancies compared to their exogenous levels is maximal for $\gamma = 0.5$.

The right panel of Fig. 8 shows changes in the unemployment rates, here models 2 and 3 are compared. Both models include on-the-job search. From the perspective of unemployed workers, on the one hand, in the good times there are more vacancies in the primary occupation but less vacancies in the mismatch occupation if vacancies are endogenous. On the other hand, in the bad times there are more positions in the mismatch occupation but less vacancies in the primary occupation. The figure shows that in both cases unemployment rates (u_A^2 and ${u^3}_A$) are lower in model 3. This means that in the good times (high output) unemployment ${u^2}_A$ is lower because of more intensive job creation in the primary occupation, whereas in the bad times unemployment ${u^3}_A$ is lower because it easier to get jobs in the mismatch occupation.

This result has implications for the fractions of time workers spend unemployed and mismatched. They are illustrated on Fig. 9. Compared to model 2 workers spend less time unemployed but more time mismatched if vacancies are endogenous. The same figure also shows comparison with model 1. One can see that for low values of γ (below 0.725), the rise in the time workers spend unemployed due to on-the-job search is relatively strong, so even with endogenous vacancies workers spend more time unemployed if on-the-job search is permitted. However, for larger values of homophily (above 0.725), the impact of on-the-job search on unemployment is moderated, thus workers spend less time unemployed in model 3 compared to model 1.

These effects shape the present value of income, which is illustrated on Fig. 10. One can see that U_A^3 is higher in model 3 compared to model 2. This is because workers spend less time unemployed. However, comparing model 3 and model 1 shows the following: when homophily is relatively low ($\gamma < 0.6$) then the present value of income is highest in model 1, but when homophily is relatively high ($\gamma > 0.6$), then the present value of income is highest in model 3. Overall, the optimal γ^* is falling down to 0.8 when vacancies are endogenized. Thus even though the optimal homophily level γ^* is not very sensitive to the model specification, the gain in income associated with raising γ from 0.5 to the optimum strongly depends on the inclusion of on-the-job search and endogenous job creation into the model. For example, this gain is equal to 1.3% ((41.84/41.32) − 1) in model 3, while it is two times smaller in model 1 ((41.72/41.49) − 1).

Finally, the right panel of Fig. 10 shows income of type B workers in state 3, that is U_B^3. High output in occupation B leads to high labour demand, so type B workers enjoy higher present value of income relative to type A workers and their gain from diversification is lower. This leads to the fact that their expected income is maximized at higher γ equal to 0.85 (see the purple curve). The average welfare of both groups ($0.5U_A^3 + 0.5U_B^3$) is a black curve inbetween. One can see that the total welfare is maximized at the compromise solution γ equal to 0.825.

5. Conclusions

This paper develops a search model with two worker types, two occupations and correlated output fluctuations. The model is used to analyze the effect of social networks on occupational mismatch and workers' expected income. Given that workers are risk-neutral and take aggregate macroeconomic variables as given they choose a fully homophilous network. This is different when the problem is considered from a social planner perspective. On the one hand, stronger homophily leads to a lower probability of mismatch. This is a positive effect on the expected income. On the other hand, stronger homophily leads to a higher risk of unemployment, especially when output and labour demand are low in the primary occupation of the worker. This trade-off generates an interior optimal network homophily from the perspective of the planner who is accounting for the response of aggregate variables. Hence this paper supports policies targetting stronger occupational diversification of social networks (i.e. interdisciplinary projects). Comparative statics shows that optimal network diversification is stronger with lower mismatch penalty and lower unemployment benefit but it is weaker if outputs in the two occupations are positively correlated. On-the-job search reduces expected income and raises the equilibrium unemployment rate. This is due to the fact that job information is less likely to reach unemployed workers. However, this negative effect is partially neutralized by endogenous job creation.

Finally, some limitations of the presented model should be discussed. One simplifying assumption is that formal job search is not included in the model. This is because the purpose of this paper is to investigate the qualitative properties of a stylized labour market with occupational mismatch and the underlying mechanism leading to network diversification. In this respect formal job search doesn't affect the mechanism, but only the quantitative predictions. Nevertheless, it would be essential to include formal job search for a proper empirical estimation of the model. Another limitation is that the total size of the network is fixed and exogenously given. This assumption can be addressed by introducing a cost of maintaining social contacts. Intuitively, this cost could be justified by the fixed time endowment workers can spend communicating with their friends. However, as long as the cost of maintaining contacts is the same for both worker groups, the two decisions – network composition and network size – can be analyzed sequentially. The more interesting but also the more complex case of different costs would imply that the two decisions are interrelated and it is left for future research.

[7] More precisely, $y_A^1 = y_A^2 = y_B^1 = y_B^3 = 0.95$, $y_A^3 = y_A^4 = y_B^2 = y_B^4 = 0.94$, $y_{0A}^1 = y_{0A}^2 = y_{0B}^1 = y_{0B}^3 = 0.92$, $y_{0A}^3 = y_{0A}^4 = y_{0B}^2 = y_{0B}^4 = 0.91$

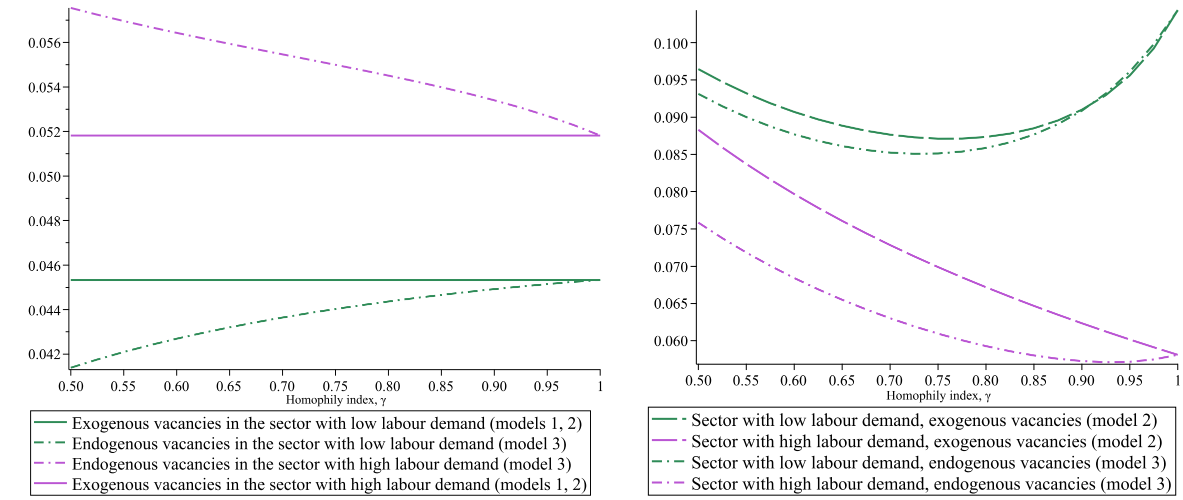

Fig. 8. Left panel: Vacancies in good times v_A^2 and in bad times $v^3{}_A$. Right panel: Unemployment rates in good times $u^2{}_A$ and in bad times $u^3{}_A$ with exogenous and endogenous job creation. Parameters: $\rho = -1$, z=0.2, w_0=0.85, w=0.9.

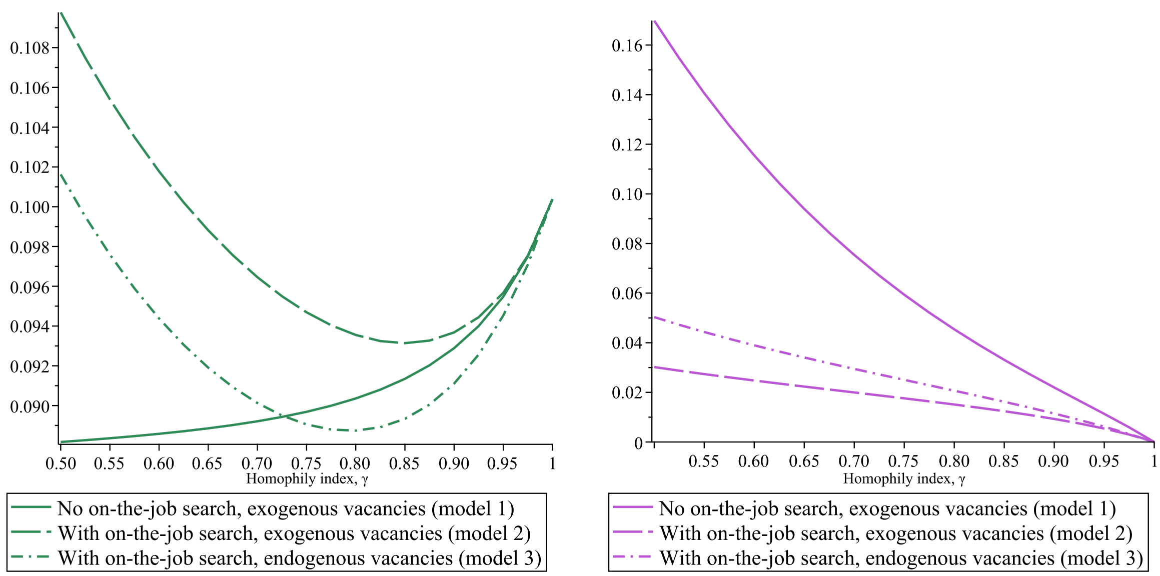

Fig. 9. Left panel: Average time in unemployment $(1 - f_{AA}^3 - f_{AB}^3)$. Right panel: Average time in mismatch f_{AB}^3. Parameters: $\rho = -1$, z=0.2, w_0=0.85, w=0.9.

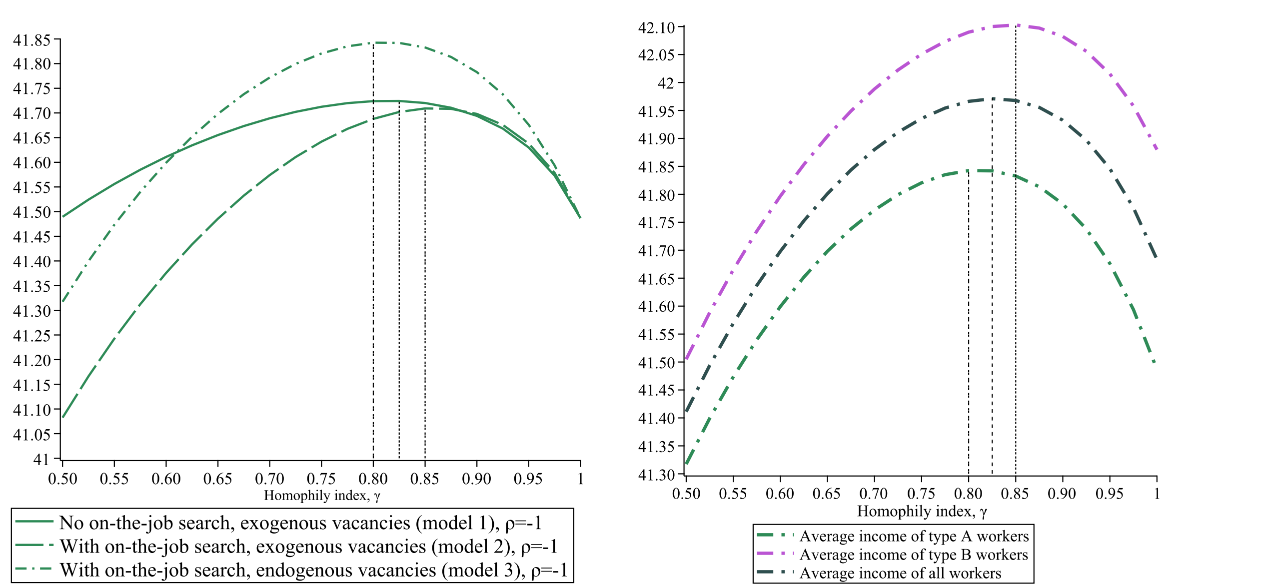

Fig. 10. Left panel: PV of unemployment U_A^3 for three model specifications, $\rho = -1$. Right panel: PV of unemployment $U^3{}_A$ in model 3 for different values of ρ. Parameters: $p = q = 0.5$, z=0.2, w_0=0.85, w=0.9. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

Appendix A. Appendix

Derivation of the transition matrix Π. Consider two indicator functions x_j each taking value 1 if occupation j, ($j = A, B$) has high output (with probability 0.5) and 0 otherwise. Then the means of these indicator functions are given by 0.5 and variances by 0.25. Further, we know that $\pi_1 + \pi_2 = 0.5$ and $\pi_1 + \pi_3 = 0.5$, which implies that $\pi_2 + \pi_3 = 1 - 2\pi_1$. Given that all probabilities add up to 1, we get $\pi_4 + \pi_1 = 1 - \pi_2 - \pi_3 = 2\pi_1$, so that $\pi_4 = \pi_1$. The covariance between the two indicator functions is:

$$\begin{aligned} cov(x_A, x_B) &= (1-0.5)(1-0.5)\pi_1 + (1-.05)(0-0.5)(0.5-\pi_1) + (0-0.5)(1-0.5)(0.5-\pi_1) \\ &\quad + (0-0.5)(0-0.5)\pi_1 = \pi_1 - 0.25 \end{aligned}$$

So the correlation coefficient ρ, which gives rise to Table 1, can be expressed as:

$$\rho = \frac{cov(x_A, x_B)}{0.25} = \frac{\pi_1 - 0.25}{0.25} \quad \Rightarrow \quad \pi_1 = 0.25(1+\rho)$$

Derivation of expression (1):

$$\begin{aligned} \sum_{k=0}^{\gamma n-1} \binom{\gamma n-1}{k} u_A^k (1-u_A)^{\gamma n-1-k} \frac{1}{k+1} &= \sum_{k=0}^{\gamma n-1} \frac{(\gamma n-1)!}{k!(\gamma n-1-k)!} u_A^k (1-u_A)^{\gamma n-1-k} \frac{1}{k+1} \\ &= \sum_{k=0}^{\gamma n-1} \frac{(\gamma n)!}{(k+1)!(\gamma n-(k+1))!\gamma n} u_A^k (1-u_A)^{\gamma n-1-k} = \frac{1}{\gamma n} \sum_{k=0}^{\gamma n-1} \binom{\gamma n}{k+1} u_A^k (1-u_A)^{\gamma n-1-k} \\ &= \frac{1}{\gamma n} \sum_{l=1}^{\gamma n} \binom{\gamma n}{l} u_A^{l-1} (1-u_A)^{\gamma n-l} = \frac{1}{\gamma n u_A} \left[\sum_{l=1}^{\gamma n} \binom{\gamma n}{l} u_A^l (1-u_A)^{\gamma n-l} \pm \binom{\gamma n}{0} (1-u_A)^{\gamma n} \right] \\ &= \frac{1}{\gamma n u_A} \left[\sum_{l=0}^{\gamma n} \binom{\gamma n}{l} u_A^l (1-u_A)^{\gamma n-l} - (1-u_A)^{\gamma n} \right] = \frac{1-(1-u_A)^{\gamma n}}{\gamma n u_A} \end{aligned}$$

Proof of lemma 1.

$$\begin{aligned} \frac{\partial \lambda_{AA}}{\partial e_A} &= \frac{s v_A}{(1-e_A)^2} [-\gamma n e_A^{\gamma n-1}(1-e_A) + 1 - e_A^{\gamma n}] = \frac{s v_A}{(1-e_A)^2}(1-\sigma(e_A)) \geq 0, \\ &\quad \text{where} \quad \sigma(e_A) = e_A^{\gamma n-1}(\gamma n(1-e_A) + e_A) \leq 1 \end{aligned}$$

Note that auxiliary function $\sigma(e_A)$ is always smaller than 1 for $e_A < 1$. This is because $\sigma(0) = 0$, $\sigma(1) = 1$ and $\sigma'(e_A) = (\gamma n - 1)e_A^{\gamma n-2}\gamma n(1-e_A) > 0$. Therefore, all job-finding rates are increasing in v_A and v_B due to the direct effect and the indirect effect through higher employment rates e_A and e_B.□

Appendix B. Appendix

Let $M = [(r+\delta+\phi)I - \phi\Pi]^{-1}$ and $M_A = [(r+\delta+\phi)I - \phi\Pi + N_A]^{-1}$, so that:

$$\begin{aligned} &((r+\delta)I - \phi(\Pi - I))W_{AA} = w + \delta U_A \quad \Rightarrow \quad W_{AA} = M(w + \delta U_A) \\ &((r+\delta)I - \phi(\Pi - I) + N_A)W_{AB} = \omega_0 + \delta U_A + N_A W_{AA} \end{aligned}$$

Thus the vectors of present values W_{AB} and U_A can be expressed as:

$$\begin{aligned} W_{AB} &= M_A(\omega_0 + \delta U_A + N_A W_{AA}) = M_A(\omega_0 + N_A M w + \delta(I + N_A M)U_A) \quad \Rightarrow \\ rU_A &= \zeta + \Lambda_{AA}[M(w + \delta U_A) - U_A] \\ &\quad + \Lambda_{AB}[M_A(\omega_0 + N_A M w + \delta(I + N_A M)U_A) - U_A] + \phi(\Pi - I)U_A \\ &\quad \Rightarrow [(r+\phi)I - \phi\Pi + \Lambda_{AA}(I - \delta M) + \Lambda_{AB}(I - \delta M_A(I + N_A M))]U_A \\ &= \zeta + \Lambda_{AB} M_A \omega_0 + (\Lambda_{AA} + \Lambda_{AB} M_A N_A) M w \end{aligned}$$

Inverting the matrix in the square bracket on the left-hand side produces the final equation for U_A.

Appendix C. Appendix

Let $\bar{\lambda}_{AA} = sv(1 - e^{\gamma n})/\gamma$ and $\bar{\lambda}_{AB} = sve^{\gamma n}(1 - e^{(1-\gamma)n})/(1-\gamma)$. The numerator of the derivative $\partial rU_A(\gamma_x, \gamma)/\partial\gamma_x$ is given by:

$$\begin{aligned} &[(w-z)\bar{\lambda}_{AA} - (w_0 - z)\bar{\lambda}_{AB}][(r+\delta)(1-e) + \gamma_x\bar{\lambda}_{AA} + (1-\gamma_x)\bar{\lambda}_{AB}] \\ &- [(w-z)\gamma_x\bar{\lambda}_{AA} + (w_0 - z)(1-\gamma_x)\bar{\lambda}_{AB}][\bar{\lambda}_{AA} - \bar{\lambda}_{AB}] \\ &= (w-z)\bar{\lambda}_{AA}(r+\delta)(1-e) + (w-z)\gamma_x\bar{\lambda}_{AA}^2 + (w-z)\bar{\lambda}_{AA}(1-\gamma_x)\bar{\lambda}_{AB} \\ &- (w_0 - z)\bar{\lambda}_{AB}(r+\delta)(1-e) - (w_0 - z)\bar{\lambda}_{AB}\gamma_x\bar{\lambda}_{AA} - (w_0 - z)(1-\gamma_x)\bar{\lambda}_{AB}^2 \\ &- (w-z)\gamma_x\bar{\lambda}_{AA}^2 + (w-z)\bar{\lambda}_{AA}\gamma_x\bar{\lambda}_{AB} - (w_0 - z)\bar{\lambda}_{AB}(1-\gamma_x)\bar{\lambda}_{AA} + (w_0 - z)(1-\gamma_x)\bar{\lambda}_{AB}^2 \\ &= (r+\delta)(1-e)[(w-z)\bar{\lambda}_{AA} - (w_0 - z)\bar{\lambda}_{AB}] + (w - w_0)\bar{\lambda}_{AB}\bar{\lambda}_{AA} \end{aligned}$$

Appendix D. Appendix

Proof of the proposition. When $\rho = -1$, transition matrix Π can be expressed as:

$$\Pi = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} \quad \Rightarrow \quad (r+\delta+\phi)I - \phi\Pi = \begin{pmatrix} r+\delta+\phi 0.5 & -\phi 0.5 \\ -\phi 0.5 & r+\delta+\phi 0.5 \end{pmatrix}$$

$$M = \frac{1}{(r+\delta)(r+\delta+\phi)} \begin{pmatrix} r+\delta+\phi 0.5 & \phi 0.5 \\ \phi 0.5 & r+\delta+\phi 0.5 \end{pmatrix}$$

$$U_A = \frac{1}{ad-bc} \begin{pmatrix} a & b \\ c & d \end{pmatrix} (\zeta + \Lambda_{AA} M\omega + \Lambda_{AB} M\omega_0)$$

$$a = r + \tfrac{r(\lambda_{AA}^3+\lambda_{AB}^3)}{r+\delta} + c \qquad b = \varphi 0.5 + \tfrac{(\lambda_{AA}^2+\lambda_{AB}^2)}{r+\delta}\tfrac{\delta\varphi 0.5}{r+\delta+\varphi}$$

$$c = \varphi 0.5 + \tfrac{(\lambda_{AA}^3+\lambda_{AB}^3)}{r+\delta}\tfrac{\delta\varphi 0.5}{r+\delta+\varphi} \quad d = r + \tfrac{r(\lambda_{AA}^2+\lambda_{AB}^2)}{r+\delta} + b$$

$$\begin{aligned} ad - bc = \; & ad - \left(d - r - \frac{r(\lambda_{AA}^2+\lambda_{AB}^2)}{r+\delta}\right)\left(a - r - \frac{r(\lambda_{AA}^3+\lambda_{AB}^3)}{r+\delta}\right) \\ = \; & ad - ad + d\left(r + \frac{r(\lambda_{AA}^3+\lambda_{AB}^3)}{r+\delta}\right) + \left(r + \frac{r(\lambda_{AA}^2+\lambda_{AB}^2)}{r+\delta}\right)\left(a - r - \frac{r(\lambda_{AA}^3+\lambda_{AB}^3)}{r+\delta}\right) \\ = \; & d\left(r + \frac{r(\lambda_{AA}^3+\lambda_{AB}^3)}{r+\delta}\right) + c\left(r + \frac{r(\lambda_{AA}^2+\lambda_{AB}^2)}{r+\delta}\right) \\ = \; & \frac{r}{r+\delta}[(c+d)(r+\delta) + c(\lambda_{AA}^2+\lambda_{AB}^2) + d(\lambda_{AA}^3+\lambda_{AB}^3)] \end{aligned}$$

$$U_A^3 = z\frac{c+d}{(ad-bc)} + \frac{w}{r+\delta}\frac{[c\lambda_{AA}^2 + d\lambda_{AA}^3]}{(ad-bc)} + \frac{w_0}{r+\delta}\frac{[c\lambda_{AB}^2 + d\lambda_{AB}^3]}{(ad-bc)}$$

$$rU_A^3 = z(1 - f_{AA}^3 - f_{AB}^3) + wf_{AA}^3 + w_0 f_{AB}^3, \quad f_{Aj}^3 = \frac{c\lambda_{Aj}^2 + d\lambda_{Aj}^3}{(r+\delta)(c+d) + c\lambda_{AA}^2 + d\lambda_{AA}^3 + c\lambda_{AB}^2 + d\lambda_{AB}^3}$$

where $j = A, B$. Alternatively, $ad - bc$ can be rewritten as:

$$ad - bc = \frac{r}{r+\delta}[(a+b)(r+\delta) + a(\lambda_{AA}^2+\lambda_{AB}^2) + b(\lambda_{AA}^3+\lambda_{AB}^3)]$$

which allows me to show that ($j = A, B$):

$$rU_A^2 = z(1 - f_{AA}^2 - f_{AB}^2) + wf_{AA}^2 + w_0 f_{AB}^2, \quad f_{Aj}^2 = \frac{a\lambda_{Aj}^2 + b\lambda_{Aj}^3}{(r+\delta)(a+b) + a\lambda_{AA}^2 + b\lambda_{AA}^3 + a\lambda_{AB}^2 + b\lambda_{AB}^3}$$

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