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A fuzzy solution approach for multi objective supplier selection ☆

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ABSTRACT

In today's severe competitive environment the selection of appropriate suppliers is a significantly important decision for effective supply chain management. Appropriate suppliers reduce purchasing costs, decrease production lead time, increase customer satisfaction and strengthen corporate competitiveness. In this study a multiple sourcing supplier selection problem is considered as a multi objective linear programming problem. Three objective functions are minimization of costs, maximization of quality and maximization of on-time delivery respectively. In order to solve the problem, a fuzzy mathematical model and a novel solution approach are proposed to satisfy the decision maker's aspirations for fuzzy goals. The proposed approach can be efficiently used to obtain non-dominated solutions. A numerical example is given to illustrate how the approach is utilized.

1. Introduction

One of the most significant business decisions faced by purchasing managers in a supply chain is the selection of appropriate suppliers while trying to satisfy multi criteria based on price, quality, customer service and delivery. Hence supplier selection is a multi-criteria decision making problem which includes both qualitative and quantitative factors some of which may conflict. Dickson (1966) identified 23 criteria that have been considered by purchasing managers in various supplier selection problems. Reviews of supplier selection criteria and methods can be found in studies belong to Weber, Current, and Benton (1991), Degraeve, Labro, and Roodhooft (2000), De Boer, Labro, and Morlacchi (2001), Aissaoui, Haouari, and Hassini (2007), and Ho, Xu, and Dey (2010).

Moreover, just like in most real-world decision making problems, uncertainty is another important property of supplier selection problems. Informational vagueness because of the tangible and intangible factors of supplier selection problems must be taken into account to reach effective configuration and coordination of supply chains. A detailed classification and review of qualitative techniques for supply chain planning under uncertainty can be found in Peidro, Mula, Poler, and Lario (2009) review paper.

Basically there are two kinds of supplier selection problem (Xia & Wu, 2007): (1) *Single sourcing*: Constraints are not considered in the supplier selection process. The buyer only needs to make one decision, which supplier is the best. (2) *Multiple sourcing*: Some limitations such as supplier's capacity, quality, and delivery are considered in the supplier selection process. In other words not only one supplier can satisfy the buyer's total requirements and the buyer needs to purchase some part of demand from one supplier and the other part from another supplier to compensate for the shortage of capacity or low quality of the first supplier. In these circumstances buyers need to make two decisions: which suppliers are the best, and how much should be purchased from each selected supplier?

In this study multiple sourcing supplier selection is considered as a multi criteria decision making problem with informational vagueness. Fuzziness stems from the aspiration levels of the monetary cost, quality requirements, delivery targets and the demand level. In this study a fuzzy mathematical model based on the augmented max–min operator (Lai & Hwang, 1993) is proposed. The model is integrated a novel fuzzy solution approach which gives an opportunity to the DM to obtain her/his own preferred achievement levels.

The remainder of the paper is organized as follows. In Section 2, literature is reviewed in detail and existing studies are classified according to their solution approaches. Mentioned criticisms with the related approaches highlight the contribution of the presented fuzzy model and approach to the literature. In Section 3, the considered multi objective supplier selection model is explained in detail. In Section 4, after preliminary definitions of fuzzy mathematical programming, fuzzy additive model, augmented max–min model and the proposed fuzzy model for the supplier selection problem are defined. In Section 5, the proposed fuzzy solution approach is presented. In Section 6, the approach is illustrated by a sample problem. Conclusions and future directions appear in the final section.

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2. Literature review

Supplier selection problem has been a focus area of research since 1960. The quantitative techniques for supplier evaluation and selection can be categorized into three classes (Wang & Yang, 2009): (1) *multi-attribute decision making* includes the linear weighting method and the analytical hierarchy process (AHP), (2) *mathematical programming models* include the linear programming models, mixed integer programming, multi-objective programming and data envelopment analysis, (3) *intelligent approaches* as for the last class include neural network based methods, expert systems, fuzzy decision making, hybrid approaches such as integrated AHP and linear programming, combined AHP, data envelopment analysis (DEA) and neural network has been applied for the supplier selection.

Among the quantitative techniques, mathematical programming models have been extensively used for supplier selection problem. This study focus on fuzzy multi-objective mathematical programming models for supplier selection. Since Weber and Current (1993) introduced a multi-objective mixed integer programming model for supplier selection and order allocation among the selected suppliers, several authors (Dahel, 2003; Pokharel, 2008; Rezaei & Davoodi, 2011; Tsai & Wang, 2010; Weber, Current, & Desai, 2000; Xia & Wu, 2007) proposed multi objective programming models to this problem.

In literature a few of the studies have been addressed both multi objectives and fuzziness in the problem. In this study the literature is reviewed for the studies which handle supplier selection by fuzzy multi objective mathematical programming (Table 1). It is observed that majority considers fuzziness of the aspiration levels attained to the objectives (Faez, Ghodsypour, & O'Brien, 2009; Ku, Chang, & Ho, 2010; Kumar, Vrat, & Shankar, 2004; Lee, Kang, & Chang, 2009; Wang, Chen, Wang, & Su, 2010; Wang & Yang, 2009; Özgen, Önüt, Gülsün, Tuzkaya, & Tuzkaya, 2008) and/or right hand side constants (Amid, Ghodsypour, & O'Brien, 2006, 2009; Arikan, 2011; Kumar, Vrat, & Shankar, 2006; Madronero, Peidro, & Vassant, 2010; Yücel & Güneri, 2011). Wu, Zhang, Wu, and Olson (2010) consider objectives' coefficients as fuzzy numbers.

Objective functions are constructed based on four basic supplier evaluation criteria as price, quality, customer service and delivery. According to the literature review, the most preferred objective functions related the *price* criterion are minimizing the net cost, total monetary cost, total purchasing price, the net price of the product after discounts, adding transport cost and ordering cost, total order cost, and purchase amount. *Quality* criterion related objective functions are minimizing the net rejections, total rejection rate of product, total amount of defective units. *Delivery* criterion related objective functions are minimizing the net late deliveries, number of delivery lateness, number of late items, maximizing the total amount of on-time deliveries. Some other objective functions considered in the literature are minimizing the negative effect of vendor service rating, minimizing the negative effect of the economic environment (Wu et al., 2010), maximizing service and minimization of risk (Ku et al., 2010). Objectives' aspiration levels are determined often by using ideal solutions. Other fuzzy parameters are buyer demand, vendors' quotas, and budget amount allocated to vendors, maximum capacity of vendors as right hand side constants.

The fuzzy multi objective supplier selection studies based on mathematical modeling in the literature have been employed six solution approaches (Table 1) which are Zimmermann's max–min approach, Tiwari et al.'s additive model, fuzzy goal programming with weights, fuzzy programming with modified fuzzy or operator, sequential quadratic programming and augmented max–min model, respectively.

A detailed review and classification of solution approaches of fuzzy mathematical programming can be found in Lai and Hwang's (1992) study. A detailed review and classification of solution approaches of fuzzy multi objective decision making problems can be found in Lai and Hwang's (1996) study. Arikan and Güngör (2007) are also classified the approaches according to the fuzzy parameter in a multi objective programming model. When the model has fuzzy aspiration levels attained to the objective functions and/or right hand side constants then fuzzy programming models can be generated by using fuzzy operators (Süer, Arikan, & Babayigit, 2009). Fuzzy model parameters defined mathematically by using membership functions. The relationship between each membership function is defined by using fuzzy operators (see e.g. in Luhandjula, 1982; Pedrycz, 1983; Werners, 1988; Yager, 1980, 1988; Zimmermann & Zysno, 1980). Zimmermann's max–min approach (Zimmermann, 1978) uses min operator which corresponds to the set-theoretic intersection in fuzzy mathematical modeling. In the literature, due to the ease of computation, the most frequently used aggregate operator is min-operator. Tiwari, Dharmar, and Rao (1987) proposed an additive model in which membership functions are combined using the add operator. The

Table 1
Fuzzy multi objective supplier selection studies.

	Approach	Source of fuzziness	Membership functions	Reference(s)
1.	Zimmermann's max–min approach	Fuzzy aspiration levels for objective functions and (or) vendors' quotas as RHS	–Isosceles triangular –Triangular –Right triangular	Kumar et al. (2004), Kumar et al. (2006), Özgen et al. (2008) and Wang and Yang (2009)
2.	Tiwari et al.'s weighted additive model	Fuzzy aspiration levels for objective functions and or fuzzy demand as a RHS constant and or vendors' quotas as RHS	–Isosceles triangular –Triangular –Right triangular	Amid, Ghodsypour, and O'Brien (2006), Amid, Ghodsypour, and O'Brien (2009); Faez, Ghodsypour, and O'Brien (2009), Wang and Yang (2009), Wang et al. (2010), Ku et al. (2010), Yücel and Güneri (2011)
3.	Fuzzy goal programming with weights (traditional representation)	Fuzzy aspiration levels for objective functions	–Triangular	Lee et al. (2009)
4.	Fuzzy programming with modified Werner's fuzzy or operator	Fuzzy aspiration levels for objective functions, maximum capacity of the vendors as RHS, budget amount allocated to vendors as RHS	–S curve	Madronero et al. (2010)
5.	Sequential quadratic programming (possibilistic approach)	Fuzzy model parameters as objective function coefficients and RHS constants	–Trapezoidal	Wu et al. (2010)
6.	Lai and Hwang's augmented max–min model	Fuzzy aspiration levels for objective functions and demand level as RHS	–Triangular –Right triangular	Arikan (2011)

model maximizes achievement levels in total and the solution may include zero level achievement(s). Then it is obtained an unbalanced fuzzy optimal solution (Lee & Li, 1993). In such a case, Lai and Hwang's augmented max–min operator (Lai & Hwang, 1993, 1996) will be appropriate for the solution (Arikan, 2011). Additive and augmented max–min models guarantee non-dominated solution whereas Zimmermann's max–min does not (Lee & Li, 1993).

Rest of the approaches mentioned in Table 1 which are sequential quadratic programming, fuzzy goal programming and fuzzy programming with modified Werner's fuzzy or operator, have some disadvantages. Wu et al. (2010) utilized sequential quadratic programming which does not consider objectives simultaneously. Lee et al. (2009) used traditional representation for fuzzy goal programming where total weighted deviation from each fuzzy aspiration is minimized. Although the solution is a non-dominated one, the model does not prohibit the unbalanced solution case. Furthermore, the traditional representation may restrict the types of the membership functions which are defined for the fuzzy aspirations. Madronero et al. (2010) used Werner's (1988) fuzzy or operator to define the fuzzy decision. In their model, demand is considered as a crisp value. Fuzzy model with Werner's fuzzy or operator has $\gamma \in [0,1]$ parameter which represent the compensation level. When $\gamma = 0$, the model becomes equivalent to the additive model; when $\gamma = 1$, then the model becomes equivalent to Zimmermann's max–min model. Determination of gamma parameter makes the model implementation harder.

3. Multi objective supplier selection model

A typical linear model for supplier selection problems is (Amid, Ghodsypour, & O'Brien, 2006; Weber & Current, 1993) as follows:

Index set

i İndex for suppliers, for all *i* = *1, 2, . . . , n*

Decision variable

x_i The number of units purchased from the *i*th supplier

Parameters

D Aggregate demand of the item over a fixed planning period.

n Number of suppliers competing for selection

P_i Per unit net purchase cost from supplier *i*

F_i Percentage of the accepted units delivered by the supplier *i*

S_i Percentage of the on-time deliveries by the supplier *i*

C_i Capacity of *i*th supplier

The multi objective programming problem formulation for supplier selection is as follows:

$$\text{Minimize} \quad z_1 = \sum_{i=1}^{n} P_i(x_i) \tag{1}$$

$$\text{Maximize} \quad z_2 = \sum_{i=1}^{n} F_i(x_i) \tag{2}$$

$$\text{Maximize} \quad z_3 = \sum_{i=1}^{n} S_i(x_i) \tag{3}$$

s.t.

$$\sum_{i=1}^{n} x_i = D \tag{4}$$

$$x_i \leqslant C_i \quad \text{for all } i = 1, \ldots, n \tag{5}$$

$$x_i \geqslant 0 \quad \text{and integer} \tag{6}$$

Objective function (1) minimizes total monetary cost. Objective function (2) maximizes total quality of purchased items. Objective function (3) maximizes service level of purchased items. Constraint (4) ensures that the overall demand satisfied. Constraint (5) means that order quantity of each supplier should be equal or less than its capacity. Constraint (6) prohibits negative orders.

In this study aspiration levels of objectives and demand assumed as fuzzy.

4. Fuzzy models for supplier selection

4.1. Preliminary definitions

Consider the fuzzy multi objective programming problem (7) with *l* fuzzy objective functions and *s* fuzzy constraints:

$$\begin{aligned} &\text{Find } x \\ &\text{s.t.} \\ &c_k x \tilde{\geqslant} z_k, \quad k \in I_1 \\ &c_k x \tilde{\leqslant} z_k, \quad k \in I_2 \\ &a_r x \cong b_r, \quad r \in T \\ &x \in X \end{aligned} \tag{7}$$

where

$I_1 \cup I_2 = \{1, ...l\}$, $I_1 \cap I_2 = \emptyset$, and X is a set of deterministic linear constraints and sign restrictions.

$c_k x = \sum_{i=1}^{n} c_{ki} x_i \quad k = 1, \ldots, l$
$a_r x = \sum_{i=1}^{n} a_{ri} x_i r = 1, 2, \ldots, s$

For $k \epsilon I_{1,2}$, z_k is the imprecise aspiration level for the *k*th objective function.

$z_k \in [z_k^L, z_k^U]$ denote the imprecise lower and upper bounds respectively for the *k*th fuzzy objective function.

$b_r \in [b_r^L, b_r^U]$ denote the imprecise lower and upper bounds respectively for the *r*th fuzzy constraints.

According to fuzzy mathematical programming, each fuzzy objective and constraint are defined in terms of fuzzy subsets with the appropriate membership functions denoted by $\mu_k(c_k x)$ for $k \in I_{1,2}$(8,9) and $\mu_r(a_r x)$ *for* $r \in T$ (10), respectively. Assuming that membership functions are linear, mathematical definitions (Lai & Hwang, 1992) are as follows:

$$\mu_k(c_k x) = \begin{cases} 1 & \text{if } c_k x \geqslant z_k^U \\ \frac{(c_k x) - z_k^L}{z_k^U - z_k^L} & \text{if } z_k^L \leqslant c_k x \leqslant z_k^U, \quad \forall k \in I_1 \\ 0 & \text{if } c_k x \leqslant z_k^L \end{cases} \tag{8}$$

$$\mu_k(c_k x) = \begin{cases} 1 & \text{if } c_k x \leqslant z_k^L \\ \frac{z_k^U - (c_k x)}{z_k^U - z_k^L} & \text{if } z_k^L \leqslant c_k x \leqslant z_k^U, \quad \forall k \in I_2 \\ 0 & \text{if } c_k x \geqslant z_k^U \end{cases} \tag{9}$$

$$\mu_r(a_r x) = \begin{cases} 0 & \text{if } a_r x \leqslant b_r^L \\ \frac{a_r x - b_r^L}{b_r - b_r^L} & \text{if } b_r^L \leqslant a_r x \leqslant b_r \\ \frac{b_r^U - (a_r x)}{b_r^U - b_r} & \text{if } b_r \leqslant a_r x \leqslant b_r^U \\ 0 & \text{if } a_r x \geqslant b_r^U \end{cases}, \quad \forall r \in T \tag{10}$$

Eq. (8) represents linear monotone increasing membership function $\mu_k(c_k x)$ for maximization type objectives with fuzzy aspiration levels. Eq. (9) represents linear monotone decreasing membership function $\mu_k(c_k x)$ for minimization type objectives with fuzzy aspiration levels and Eq. (10) is a triangular membership function $\mu_r(a_r x)$ for constraints.

4.2. Fuzzy additive model

Fuzzy additive model based on Tiwari et al. (1987) study for the multi objective programming model (7) is given in (11).Variables denoted by λ_k and λ_r represents achievement levels of fuzzy objective functions and fuzzy constraints respectively.

$$\begin{aligned} &\max \left(\sum_{k=1}^{l}\lambda_k + \sum_{r=1}^{s}\lambda_r\right)\Big/(l+s) \\ &\text{s.t.} \\ &\lambda_k \leqslant \mu_k(c_k x), \quad k \in I_1 \cup I_2 \\ &\lambda_r \leqslant \mu_r(a_r x), \quad r \in T \\ &\lambda_k, \lambda_r \in [0,1], \quad k = 1,\ldots,l; \;\; r = 1,\ldots,s \\ &x \in X \end{aligned} \tag{11}$$

4.3. Augmented max–min model

Augmented max–min model (12) based on Lai and Hwang's (1993, 1996) approach for model (7) is as follows:

$$\begin{aligned} &\max \lambda + \left\{\sum_{k=1}^{l}\mu_k(c_k x) + \sum_{r=1}^{s}\mu_r(a_r x)\right\}/(l+s) \\ &\text{s.t.} \\ &\lambda \leqslant \mu_k(c_k x), \quad k = 1,2,\ldots,l \\ &\lambda \leqslant \mu_r(a_r x), \quad r = 1,2,\ldots,s \\ &x \in X \\ &\lambda \in [0,1] \end{aligned} \tag{12}$$

λ is the minimum satisfaction degree and defined as follows:

$$\min_{k,r}\{\mu_k(c_k x), \mu_r(a_r x)\}, \quad \text{for } k = 1,2,\ldots l; r = 1,2,\ldots,s. \tag{13}$$

4.4. The proposed fuzzy model

λ is the minimum satisfaction degree defined in (13)

$$\begin{aligned} &\max \lambda + \left\{\sum_{k=1}^{l}\mu_k(c_k x) + \sum_{r=1}^{s}\mu_r(a_r x)\right\}/(l+s) \\ &\text{s.t.} \\ &\lambda \leqslant \mu_k(c_k x), \quad k = 1,\ldots,l \\ &\lambda \leqslant \mu_r(a_r x), \quad r = 1,\ldots,s \\ &\mu_k(c_k x) \geqslant \alpha_k, \quad k = 1,\ldots,l \\ &\mu_r(a_r x) \geqslant \alpha_r, \quad r = 1,\ldots,s \\ &x \in X \\ &\alpha_k, \alpha_r, \lambda \in [0,1] \end{aligned} \tag{14}$$

Parameters α_k and α_r represent the minimum acceptable achievement levels for the kth objective and rth constraint respectively determined by the DM.

5. The proposed fuzzy solution approach

The algorithmic steps (Fig. 1) of the approach are given as follows:

(1) Construct the multi objective model (7).
(2) Determine fuzzy parameters as aspiration levels of objectives and/or right hand side constants. (For model (7), fuzzy right hand side constant is defined as demand level.)

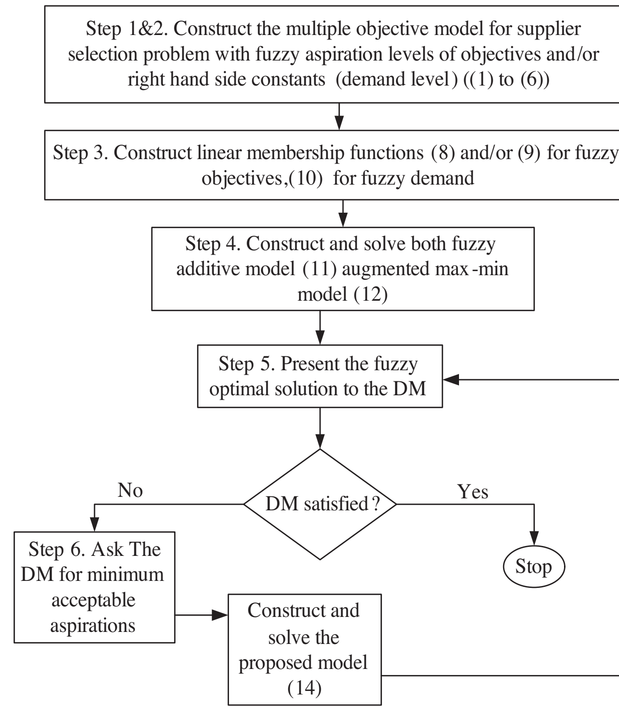

Fig. 1. Flowchart of the proposed fuzzy solution approach.

Table 2
Suppliers' source data for the illustrative example.

Supplier number i	P_i Net price	F_i Quality (%)	S_i Delivery (%)	C_i Capacity
1	5	80	90	400
2	7	90	80	450
3	4	85	85	450
Bounds	[3550, 4900]	[660, 702.5]	[657.5, 700]	–

(3) Construct the linear membership functions for each fuzzy goal (8) and (9) and fuzzy demand (10).
(4) Construct and solve both fuzzy additive model (11) and augmented max–min model (12).
(5) Present the fuzzy optimal solution(s) to the DM. If the DM is satisfied then stop. Otherwise, go to step 6.
(6) Ask the DM for the minimum acceptable aspiration level for each fuzzy objective and/or fuzzy constraint. Construct and solve the proposed model (14). Go to step 5.

In step 5 the model (7), fuzzy parameters and membership functions are assumed as valid. Otherwise, the analyst may turn back to step 1 or step 2 or step 3 for validation check.

6. Illustrative example

The following example problem is taken from Yücel and Güneri's (2011) study. For supplying a new product, a textile company desires to select appropriate suppliers based on three purchasing criteria which are net price, quality and on-time delivery. Yücel and Güneri (2011) proposed a *weighted* additive model in which relative weights for each respective objectives are 0.277, 0.253 and 0.239, and relative weights for fuzzy demand constraint is 0.231. However in this study, it is assumed that each fuzzy objectives and the fuzzy constraint related with the fuzzy demand level have the same relative importance to reach the achievement levels.

Table 3
Solution summaries to the illustrative example.

	Fuzzy additive model	Augmented max–min model
x_1, x_2, x_3	175,200,450	240,130,450
Min z_1 [3550, 4900]	4075.0	3910.0
Max z_2 [660, 702.5]	702.5	691.5
Max z_3 [657.5, 700]	700.0	702.5
Demand [750, 800, 875]	825.0	820.0
$\mu_1(c_1x)$	0.611	0.733
$\mu_2(c_2x)$	1.000	0.741
$\mu_3(c_3x)$	1.000	1.000
$\mu_1(a_1x)$	0.666	0.733

Table 4
Solution summary for the proposed fuzzy model.

x_1, x_2, x_3	245,123,450
Min z_1 [3550, 4900]	3886.0
Max z_2 [660, 702.5]	689.2
Max z_3 [657.5, 700]	701.4
Demand [750, 800, 875]	818.0
$\mu_1(c_1x)$	0.751
$\mu_2(c_2x)$	0.687
$\mu_3(c_3x)$	1.000
$\mu_1(a_1x)$	0.760

During the application of the step 4 of the proposed algorithm it is observed that Fuzzy Additive Model solution (with equal weights) has the same solution as Yücel and Güneri's weighted model solution. The suppliers' quantitative information is given in Table 2.

Step 1: Fuzzy multi objective supplier selection model is as follows:

$$\begin{aligned}
&\text{Find} \quad x_1, x_2, x_3 \\
&\text{s.t.} \\
&5x_1 + 7x_2 + 4x_3 \tilde{\leqslant} z_1 \\
&0.80x_1 + 0.90x_2 + 0.85x_3 \tilde{\geqslant} z_2 \\
&0.90x_1 + 0.80x_2 + 0.85x_3 \tilde{\geqslant} z_3 \\
&x_1 + x_2 + x_3 \cong 800 \\
&x_1 \leqslant 400 \\
&x_2 \leqslant 450 \\
&x_3 \leqslant 450 \\
&x_1, x_2, x_3 \geqslant 0
\end{aligned} \tag{15}$$

Step 2: Bounds for fuzzy objectives are mentioned in Table 2. Fuzzy triangular number for fuzzy demand is defined with [750, 800, 875].

Step 3: Related membership functions are constructed by using (8)–(10) equations with the mentioned bounds of aspirations mentioned in Table 2 and Step 2.

Step 4: Model (11) and (12) are utilized for the illustrative example and fuzzy optimal solutions are obtained by GAMS computer programming package. Their summaries are given in Table 3. Both of the solutions are non-dominated. When fuzzy additive model solution is investigated, it is observed that the first objective function has the least achievement level. On the other hand augmented max–min model solution maximizes the minimum achievement level and it gives better achievement levels for the net price criteria and for the overall demand level. Meanwhile quality achievement level 25.6% decrease. Augmented max–min model gives more balanced solution.

Step 5: Let us assume that the DM does not satisfy with both solutions. S(he) wants at least 0.75 achievement for the first fuzzy objective and for the fuzzy demand level.

Step 6: Model (14) is utilized for the illustrative example with 0.75 as the minimum acceptable achievement level for both $\alpha_{i=1}$ and $\alpha_{r=1}$. The corresponding fuzzy mathematical model for the illustrative example is given in (16). The solution is summarized in Table 4.

$$\begin{aligned}
&\text{Maximize} \quad \lambda + [\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4]/4 \\
&\text{s.t.} \\
&\lambda \leqslant \lambda_1 \\
&\lambda \leqslant \lambda_2 \\
&\lambda \leqslant \lambda_3 \\
&\lambda \leqslant \lambda_4 \\
&\lambda_1 \leqslant \tfrac{4900-(5x_1+7x_2+4x_3)}{1350} \\
&\lambda_2 \leqslant \tfrac{0.80x_1+0.90x_2+0.85x_3-660}{42.5} \\
&\lambda_3 \leqslant \tfrac{0.90x_1+0.80x_2+0.85x_3-657.5}{42.5} \\
&\lambda_4 \leqslant \tfrac{875-(x_1+x_2+x_3)}{75} \\
&\lambda_4 \leqslant \tfrac{x_1+x_2+x_3-750}{50} \\
&\lambda_1 \geqslant 0.75 \\
&\lambda_4 \geqslant 0.75 \\
&x_1 \leqslant 400 \\
&x_2 \leqslant 450 \\
&x_3 \leqslant 450 \\
&x_1, x_2, x_3 \geqslant 0 \\
&\lambda, \lambda_{2,3} \in [0, 1] \\
&\lambda_{1,4} \in [0.75, 1]
\end{aligned} \tag{16}$$

The algorithm is terminated with the assumption that the DM is satisfied with the current fuzzy optimal solution. This solution is also a non dominated one. Minimum acceptable achievements for the first objective and for the demand level are greater than 0.75 as the DM wishes. This solution is a balanced solution with the minimum achievement level 0.687. It is also greater than the minimum achievement level of fuzzy additive model solution (0.611).

7. Conclusions and future directions

In this study a multi sourcing supplier selection problem is considered as a multi objective linear programming problem. The literature reviewed for the studies which handle supplier selection by fuzzy multi objective mathematical programming and they are investigated according to their solution approaches. A typical multi objective supplier selection model which considers three objective functions as minimization of costs, maximization of quality and maximization on-time delivery with fuzzy aspiration levels respectively and fuzzy demand is employed to construct fuzzy mathematical models. Each fuzzy parameter is represented mathematically by using an appropriate linear membership function. Both fuzzy additive and augmented max–min models give non-dominated solutions. Augmented max–min model solution is balanced additionally. The proposed model is exactly same as the augmented max–min model except the additional constraints related with the DM's preferred achievement levels. Hence, both the proposed model and the solution approach give an opportunity to the DM to obtain her/his own preferred achievement levels for the objectives and for the demand level in a non-dominated solution case.

In this study a typical and a very well-known multi objective supplier selection model is transformed into convex fuzzy programming models with a single objective function. This transformation reduces the dimension of the system and results less computational complexity.

The proposed fuzzy model and approach can be employed for any kind of multi objective programming problems. They can be utilized even the multi objective model is defined crisply, by using the ideal solution as for the bounds of linear memberships. The model and the approaches with the same mathematical definitions comes handy when additionally each supplier's capacity is assumed as fuzzy with a linear membership function. Because, fuzzy capacity and minimization objective with a fuzzy aspiration level have the same mathematical definitions from the fuzzy programming point of view. As a future direction, trapezoidal memberships for demand, nonlinear memberships for price, quality and delivery and fuzziness in each supplier's capacity may be considered.

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