Available online at www.sciencedirect.com

ScienceDirect

Procedia Materials Science 5 (2014) 1801 – 1809

Procedia Materials Science

www.elsevier.com/locate/procedia

International Conference on Advances in Manufacturing and Materials Engineering, AMME 2014

Frictional Tooth Contact Analysis along Line of Action of a Spur Gear using Finite Element Method

Santosh Patil[a]*, Saravanan Karuppanan[a], Ivana Atanasovska[b], Azmi A Wahab[a]

[a]Department of Mechancial Engineering, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, 31750 Tronoh, Perak, Malaysia.
[b]Institute Kirilo Savic, Vojvode Stepe 51, Belgrade, Serbia

Abstract

In the present paper an attempt has been made to study the effect of static coefficient of friction along the line of action of a spur gear. The basic task was to analyze and determine the shape function which defines the change of contact stresses along the line of action of the meshed gear pair. The analysis was conducted on an involute spur gear pair. A 3D finite element method was used for the modeling and analysis of the contact between the gear tooth flanks. The involute gear profile was generated using a macro in ANSYS APDL. The Lagrangian multiplier contact algorithm was chosen to determine the stresses at the contacting gear pair. The results obtained with this model for the frictionless case were compared to the theoretical calculations to confirm the accuracy of the model used. After the validation, the analysis was extended to the frictional cases. The results showed an increase in the contact stress with the increase in coefficient of friction. The contact stresses have been observed to increase about 10% for an increase in coefficient of friction from 0 to 0.3.

Selection and peer-review under responsibility of Organizing Committee of AMME 2014

Keywords: Finite element; contact stress; line of action; gear pair

1. Introduction

The gears have been a research point of interest for many decades and still continue to be the same. The contact stress determination has been studied to understand the pitting resistance and surface hardness requirement. The determination of the maximum contact stress point in meshed gear pairs is possible if the change of active contact

* Corresponding author. Tel.: +601116256914
E-mail address: santosh_g02051@utp.edu.my, santosh045@gmail.com

2211-8128

Selection and peer-review under responsibility of Organizing Committee of AMME 2014
doi:10.1016/j.mspro.2014.07.399

stress is provided. This helps in precise evaluation of pitting resistance as explained in literature (Atanasovaka and Nikolic 2003).

The load distribution is one of the primary parameter to be evaluated before carrying the stress analysis. Ristivojevic *et al.* (2013) studied the load carrying capacity of spur gear tooth flanks and discussed the load distribution factor over the tooth flanks. They have also determined the change of contact stress along the line of contact of the spur gear pair. Nikolic and Atanasovska (2003) have analyzed the contact stress on meshed teeth's flanks along the path of contact using theoretical analysis and finite element method (FEM). The results showed that the FEM results matched well with the theoretical analysis and hence the FE model can be employed efficiently to evaluate the contact stresses in a meshed gear pair. Various other works (Patil *et al.* 2013; Kleemola and Lehtovaara 2009 and Atanasovaka *et al.* 2007) also showed the methods to evaluate the contact stresses along the path of contact. The contact stress evaluation in a meshed gear pair has been largely carried out neglecting the frictional coefficient. Few researchers have attempted the inclusion of friction coefficient in the calculation of contact stresses. Velex *et al.* (2000) carried out an experimental and numerical investigation on the influence of tooth friction on spur and helical gear dynamics. They noticed that the tooth friction contributes more at low-medium speeds and are negligible at high speeds. Vijayarangan and Ganesan (1994) worked on the contact stress of a pair of mating spur gear, under static conditions, by using 2D FEM and Lagrangian multiplier technique. The results of the work showed a 5% increase in the static contact stress when the friction coefficient was increased from 0 to 0.3. A similar approach has been adopted in this paper for a 3D spur gear pair.

In this paper, the main objective is to find the effect of coefficient of friction on contact stresses along a line of action of a gear pair. The theoretical and finite element analysis was conducted on a particular gear pair. The particulars and specifications of the gear pair are given in Table 1 and the material properties of the gear pair are given in Table 2. The results of the theoretical analysis have been used for the validation of FE results. Further, the effects of coefficient of friction on the contact stress variations have been found using the verified FE model.

2. The theoretical analysis

2.1. Load distribution in meshed gears

The main step in the analysis of the gear pair teeth is to determine the load distribution on the gear flanks during the gear mesh. The load distribution on gear tooth flanks during gear mesh is non-uniform. Hence, the pinion and the gear are differently engaged in load transfer of the gear pair. The load distribution along the path of contact on a gear tooth flank is represented through the factors of load distribution (Fig.1) (Ristivojevic *et al.* 2013 and Pedrero *et al.* 2010). If n tooth pairs are simultaneously meshed and F_x is the load transferred by x^{th} tooth pair, then the factor of load distribution can be defined as:

$$K_{\alpha x} = \frac{F_x}{F},\ F = \sum_{x=1}^{n} F_x \tag{1}$$

where $K_{\alpha x}$ is the load distribution factor, F_x is the load transferred by the observed tooth pair and F is the total load on the gear pair. The interval of load distribution factor values is given by $0 \leq K_{\alpha x} \leq 1$.

Load factors take into consideration the real working conditions and translate the nominal load value into the maximum load value. The maximum load is defined as the maximum normal load F_{bn} on the tooth flanks, which is the result of the maximum tangential load (F_{tmax}), in accordance with:

$$F_{bn1} = F_{bn2} = F_{bn} = \frac{2T_1}{d_{b1}} = \frac{2T_2}{d_{b2}} = \frac{F_{tmax}}{\cos\alpha_w} = \frac{K_A K_V K_\alpha K_\beta F_t}{\cos\alpha_w} \tag{2}$$

where K_A is the application factor, K_V is the internal dynamic factor, K_α is the factor of load distribution among meshed gear pairs, K_β is the factor of load distribution over the facewidth (K_β=constant, with the assumption that the force per unit tooth width is constant during the tooth mesh).

Table 1. Specifications of gear sets

No.	Parameter	Pinion	Gear
1	No. of Teeth	20	20
2	Normal Module	4.5	4.5
3	Normal Pressure angle (deg.)	20	20
4	Helix angle (deg.)	0	0
5	Pitch Diameter (mm)	91.5	91.5
6	Face Width (mm)	20	20
7	Centre Distance (mm)	91.5	
8	Contact Ratio	1.556	
9	Torque (Nm)	302Nm	
10	Speed (rpm)	1000rpm	

Table 2. Material properties of the gear pair

No.	Parameter	Pinion	Gear
1	Material	Structural Steel	Structural Steel
2	Modulus of Elasticity, GPa	210	210
3	Poisson's ratio	0.3	0.3
4	Density, kg/m^3	7830	7830

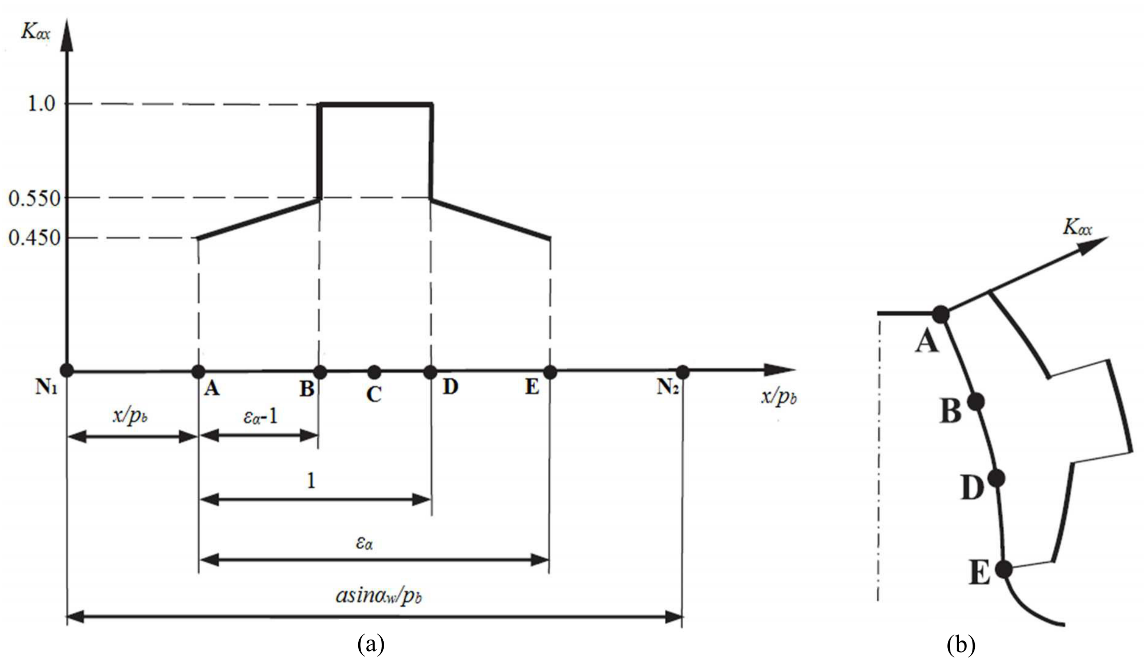

Fig. 1. Load distribution factor (a) along the path of contact; (b) represented on the gear tooth profile

2.2. Radius of curvature of meshed teeth

In the case of involute spur gears, the curvatures' radii of the meshed teeth are continuously varying during the meshing period of a tooth pair, in accordance with the following expression (Equation 3):

$$\rho_x = r_x . sin\, \alpha_x \tag{3}$$

The expressions that correspond to the tooth flanks' contact are shown in Equation 4

$$q = \frac{F_{bn}}{b}; \rho = \frac{1}{2\lambda} = \frac{\rho_1 . \rho_2}{\rho_1 + \rho_2} \text{ and } k_i = \frac{1-\vartheta_i^2}{E_i}, i = 1, 2 \tag{4}$$

2.3. Hertzian contact stress

According to the Hertz contact theory for two cylinders in contact (Hertz 1881 and Hassan 2009), the maximum pressure (ρ_{max}) is represented in the form shown in Equation 5.

$$\rho_{max} = \frac{2q}{\pi . a} = \sqrt{\frac{2}{\pi} . q . \frac{\lambda}{k_1 + k_2}} \tag{5}$$

When the relations of Equations 2, 3, 4 are introduced in Equation 5, then it takes a form applicable for the calculation of maximum Hertzian contact stresses at the region of meshing without slipping for single tooth pair meshing:

$$\sigma_H = \sqrt{\frac{F_{bn}}{b.\rho} . \frac{1}{\pi . \left(\frac{1-\vartheta_1^2}{E_1} + \frac{1-\vartheta_2^2}{E_2}\right)}} \tag{6}$$

where ρ is the equivalent curvature's radius in a contact point.

With the application of Equations 1 and 2 in Equation 6 and also by considering the active stress calculation while the contact is at the pitch point *C*, the contact stress σ_{HC} can be found and is represented as in Equation 7.

$$\sigma_{HC} = \sqrt{\frac{1}{\pi . \left(\frac{1-\vartheta_1^2}{E_1} + \frac{1-\vartheta_2^2}{E_2}\right)}} . \sqrt{\frac{F_{tmax}}{b.d_1} . \frac{u+1}{u} . \frac{2}{cos^2 \alpha . tg \alpha_w}} \tag{7}$$

The ratio of active stress in any contact point σ_{Hx} and active stress for contact at the pitch point σ_{HC} can be used to monitor the change of active stress on the tooth flanks. After appropriate transformations, the ratio can be written in the following form:

$$\frac{\sigma_{Hx}}{\sigma_{HC}} = Z_{\rho x} . \sqrt{\frac{K_{\alpha x}}{K_{\alpha C}}} \tag{8}$$

where $K_{\alpha x}$ and $K_{\alpha C}$ are the load distribution factors for meshed gears at any point and at the pitch point, respectively and $Z_{\rho x}$ is the curve radius factor at any point on the line of contact.

The theoretical calculation of the active contact stress on the tooth flanks along the line of contact has been plotted in Fig. 2. These results will later be used for the validation of the FEA results.

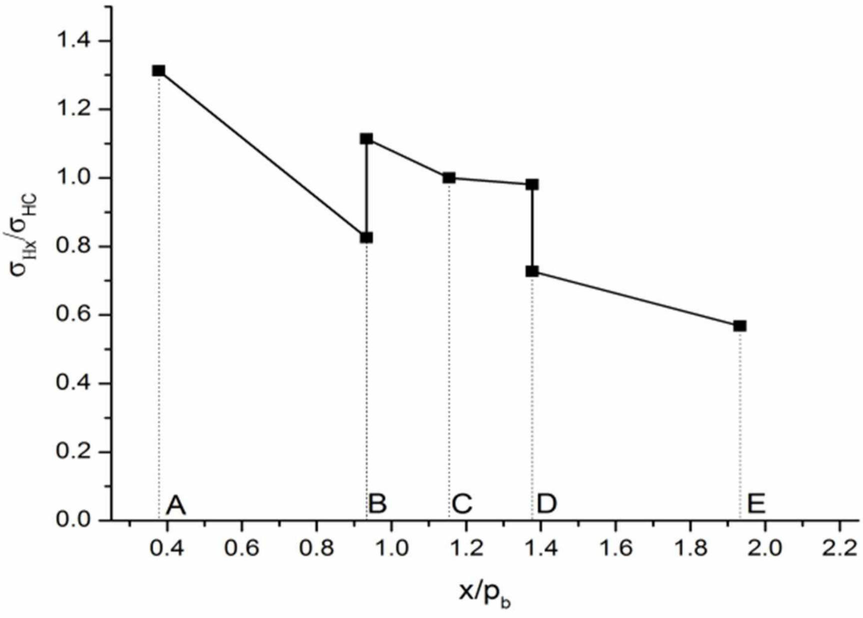

Fig. 2. Active contact stress on tooth flanks along the line of contact

3. Finite element model of gear pairs

Precise 3D contact finite element model was prepared for the selected gear pair. The detailed specification of the gear pair is mentioned in Table 1. The material used for the gear pair was structural steel and its properties are shown in Table 2. The involute gear profile was generated using an ANSYS APDL macro. The involute profile was further used to complete the tooth full profile and then copied three times to create a three-tooth segment. The segment was then extruded to create a three dimensional pinion. Similar steps were followed to create the gear, which completed the gear pair. The discretization of the gear model was done using SOLID 185 (ANSYS 2009), an eight-noded brick finite element without midside nodes, suitable for determination of stress, strain and deformation with optimum utilization of computing time and resources.

The meshed model of the gear pair along with their boundary conditions for all characteristic contact points are shown in Fig.3. The first contact point is *A,* point of change from period of two tooth pairs in contact to single tooth pair period is *B,* pitch point contact is point *C.* The point of change from single tooth pair contact period to two tooth pair in contact is *D* and the last contact point is *E.* The contact boundary condition between the gear pair has been modeled using CONTA 174 and TARGE 170 element types (ANSYS 2009). The coefficient of friction between the contacts was varied in the interest range of 0 to 0.3. Lagrangian multiplier algorithm was chosen for solving the nonlinear contact problem.

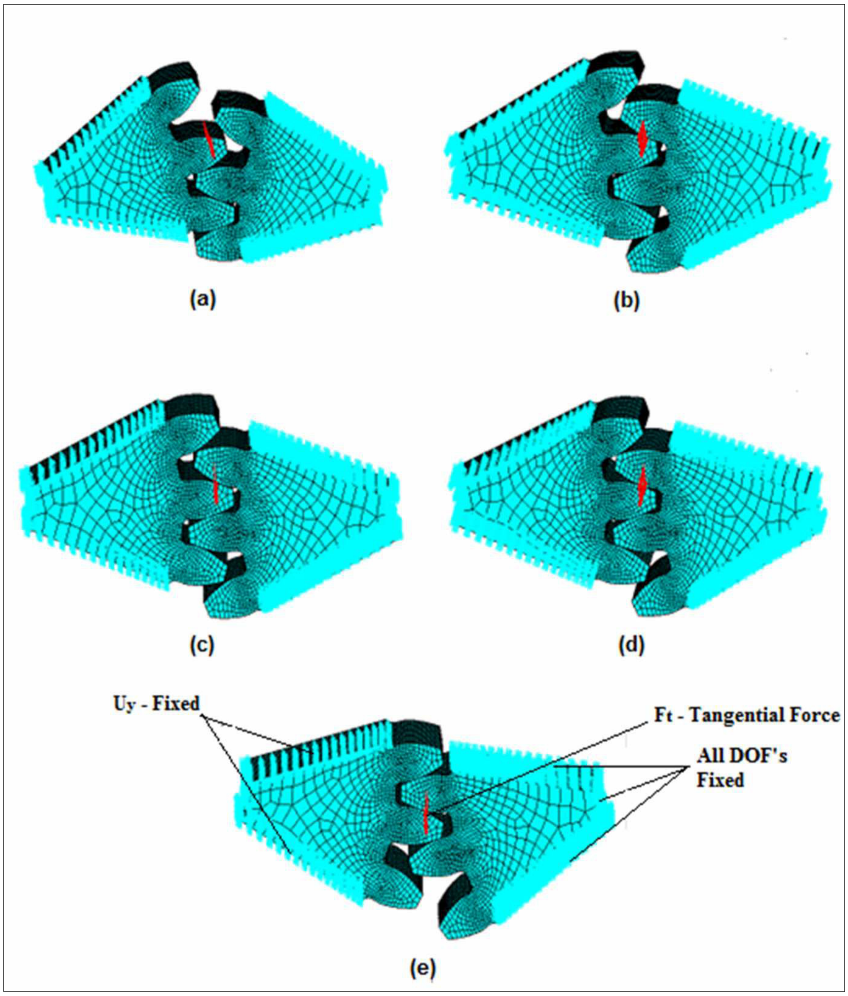

Fig. 3. Meshed models along with boundary conditions at (a) point *A*; (b) point *B*; (c) point *C*; (d) point *D*; (e) point *E*

4. Results and discussion

4.1. Validation of the model

The presented FE model used to calculate the contact stress along the line of contact was validated by the theoretical analysis which was based on Hertzian theory of two cylinder contacts. The theoretical results in Fig. 2 were used for the validation. The FE calculation was done on the tooth pair contact models as shown in Fig. 3 and the von Mises equivalent stress fields are given in Fig. 4. The equivalent stress distribution at the characteristic points of the meshed gear pair can be observed. The maximum stress points in each diagram are represented by the red color and its values are easily readable from the scale shown below each diagram. The model described in this paper corresponds to the ideal gear, so these FE results can be compared with the theoretical results. Fig. 5 shows a comparison of relative contact stress along the line of contact between FE model results and the theoretical results. The characteristic point contacts were selected and their corresponding values were plotted, because calculation of values corresponding to every point of contact is tedious and would be time consuming. Fig. 5 shows that a good level of coincidence exists between the FE model results and the results obtained from theoretical analysis.

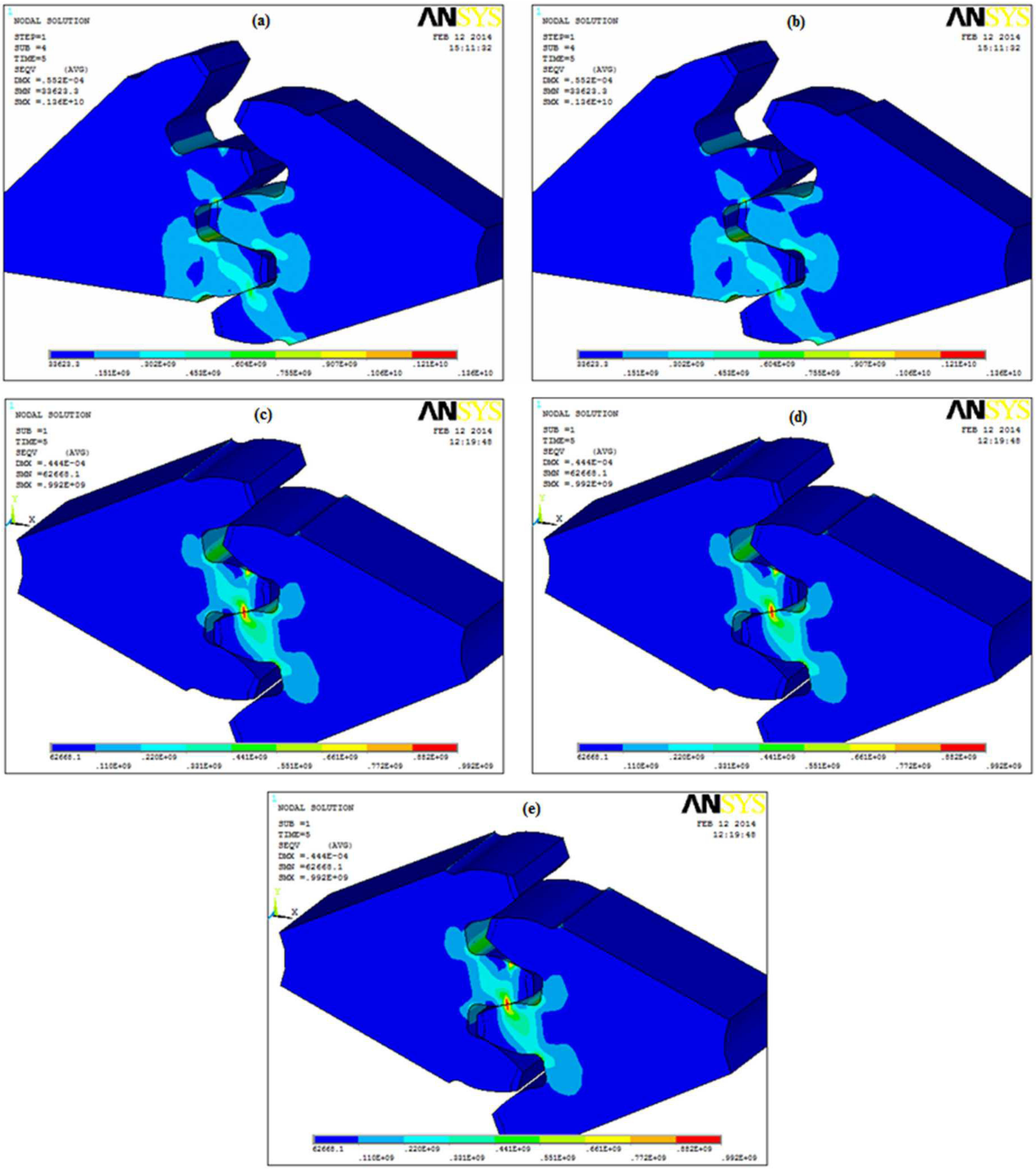

Fig. 4. von Mises' equivalent contact stress fields at: (a) point *A*; (b) point *B*; (c) point *C*; (d) point *D*; (e) point *E*

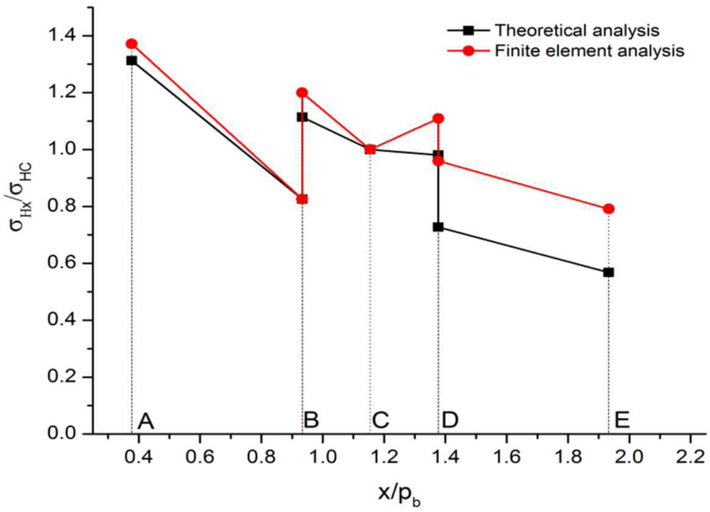

Fig. 5. The change of relative contact stress along the line of contact

4.2. Effect of coefficient of friction on contact stress (gears of same size)

The validated FE model is further utilized to analyze the contact stresses for different coefficient of friction. The evaluation of the active contact stress on the tooth flanks of a meshed gear were conducted by varying the coefficient of friction from 0 to 0.3. The results of the analysis for different coefficient of frictions are as shown in Fig. 6. It was observed that the contact stress value increased as the coefficient of friction value increased from 0 to 0.3. Due to friction, the stresses at the point of contact increased and hence this provides with a possibility of a friction factor (K_f) to be included while calculating the contact stresses in meshed gears. The contact stresses rise 1.4%, 5% and 10% for an equivalent coefficient of friction value of 0.1, 0.2 and 0.3, respectively. The parametric study on different gear sets would yield an appropriate factorial value to be incorporated while calculating the active contact stresses.

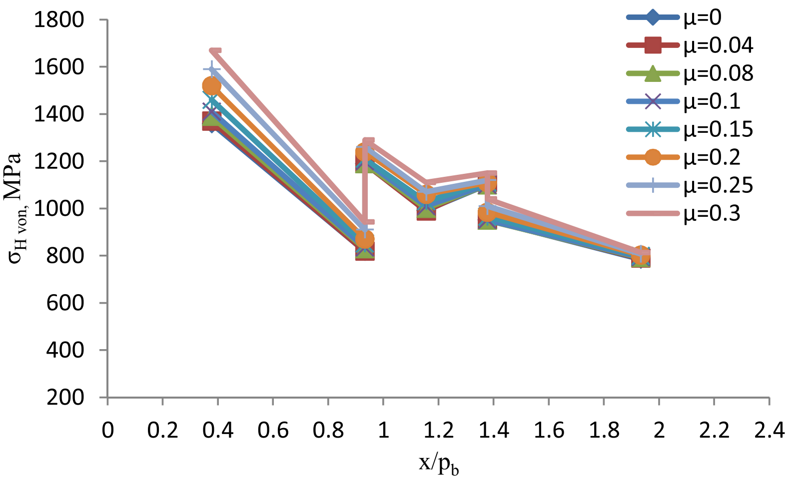

Fig. 6. The maximum contact stress along the line of contact for different coefficient of friction

5. Conclusion

Based on the theoretical and FE analysis conducted in the present paper, following conclusions can be obtained.

- The presented FE model results have a high degree of coincidence with the theoretical results; hence the model proved to be suitable in the analysis of active contact stresses on the tooth flanks of a gear pair.
- The contact stress calculated for different coefficient of friction showed that the contact stresses increased with the increasing value of static coefficient of friction. A 10% rise has been observed when the friction coefficient value was increased from 0 to 0.3.
- The increase in the contact stresses with the increase in friction coefficient provides with an idea of friction factor to be incorporated while calculating the contact stress in meshed gears.

Acknowledgements

The authors would like to thank Universiti Teknologi PETRONAS for the facilities, system support and financial aid (URIF fund with cost center - 015 3AA B10).

References

Atanasovska, I., and Nikolic, V., 2003. The analysis of contact stress on meshed teeth's flanks along the path of contact for a tooth pair. Facta Universitatis- Mechanics, Automatic Control and Robotics 3(15), 1055-1066.

Ristivojevic, M., Lazovic, T., and Vencl, A., 2013. Studying the load carrying capacity of spur gear tooth flanks. Mechanism and Machine Theory 59, 125-137.

Patil Santosh, Karuppanan, S., and Wahab, A. A., 2013. Contact pressure evaluation of a gear pair along the line of action using finite element analysis. Applied Mechanics and Materials 393, 403-408.

Kleemola, J., and Lehtovaara, 2009. Experimental simulation of gear contact along the line of action. Tribology International 42(10), 1453-1459.

Atanasovska, I., Nikolic, V., Momcilovic, D., and Vulovic, S., 2007. Developing of gear FEM model for nonlinear contact analysis. 1st International Congress of Serbian Society of Mechanics, 695-703.

Velex, P., and Cahouet, V., 2000. Experimental and numerical investigation on the influence of tooth friction in spur and helical gear dynamics. Journal of Mechanical Design 122, 515-522.

Vijayarangan, S., and Ganesan, N., 1994. Static contact stress analysis of a spur gear tooth using the finite element method, including frictional effects. Computers and Structures 51(6), 765-770.

Pedrero, J. I., Pleguezuelos, M., Artés, M., and Antona, J. A. 2010. Load distribution model along the line of contact for involute external gears. Mechanism and Machine Theory 45(5), 780-794.

Hertz, H., 1881. Fixed on the contact of elastic bodies, *Gesammelte Werke* (P. Lenard,ed.), Bd. 1, (J.A. Barth, Leipzig, 1895) pp. 155-173. Originally published in Journal of Pure and Applied Mathematics 92,156-171.

Hassan, A. R., 2009. Contact stress analysis of spur gear teeth pair. *World Academy of Science, Engineering and Technology, 58*, 611-616

ANSYS Release 12.0, 2009. Structural analysis guide, ANSYS Inc., Canonsburg, PA – 15317, USA.