Effect of intermediate principal stresses on compressive strength of Phra Wihan sandstone

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ABSTRACT: A polyaxial load frame has been used to determine the compressive strength of Phra Wihan sandstone under true triaxial stresses. The rock specimens are cubic shape with a nominal dimension of $5 \times 5 \times 5$ cm^3. Under a low σ_3, σ_1 at failure initially increases with σ_2 and then decreases with σ_2. Under a high σ_3, σ_1 increases with σ_2. The Coulomb and modified Wiebols & Cook failure criteria derived from the characterization test results predict the sandstone strengths in terms of $J_2^{1/2}$ as a function of J_1 under true triaxial stresses. The modified Wiebols & Cook criterion describes the failure stresses better than does the Coulomb criterion.

1   INTRODUCTION

The effects of confining pressures at great depths on the mechanical properties of rocks are commonly simulated in a laboratory by performing triaxial compression testing of cylindrical rock core specimens. A significant limitation of these conventional methods is that the intermediate and minimum principal stresses are equal during the test while the actual in-situ rock is normally subjected to an anisotropic stress state where the maximum, intermediate and minimum principal stresses are different ($\sigma_1 \neq \sigma_2 \neq \sigma_3$). It has been commonly found that compressive strengths obtained from conventional triaxial testing can not represent the actual in-situ strength where the rock is subjected to an anisotropic stress state (Yang et al. (2007), Haimson (2006), Tiwari & Rao (2006), Tiwari & Rao (2004) and Haimson & Chang (1999)).

From the experimental results on brittle rocks obtained from Colmenares & Zoback (2002) and Haimson (2006) it can be generally concluded that in a $\sigma_1 - \sigma_2$ diagram, for a given σ_3, σ_1 at failure initially increases with σ_2 to a certain magnitude, and then it gradually decreases as σ_2 increases. The effect of σ_2 is more pronounced under higher σ_3. The effect of σ_2 is related to the stress-induced anisotropic properties and behavior of the rock and to the end effect at the interface between the rock surface and loading platen in the direction of σ_2 application. The effect is smaller in homogeneous and fine-grained rocks than in coarse-grained rocks where pre-existing micro-cracks are not uniformly distributed.

Several failure criteria have been developed to describe the rock strength under true triaxial stress states. Comprehensive reviews of these criteria have been given recently by

Colmenares & Zoback (2002), Al-Ajmi & Zimmerman (2005) and Haimson (2006). Among several other criteria, the Mogi and modified Wiebols & Cook criteria are perhaps the most widely used to describe the rock compressive strengths under true triaxial stresses. Obtaining rock strengths under an anisotropic stress state is not only difficult but also expensive. A special loading device (e.g. polyaxial loading machine or true triaxial load cell) is required. As a result test data under true triaxial stress conditions have been relatively limited. Most researchers above have used the same sets of test data (some obtained over a decade ago) to compare with their new numerical simulations or field observations (notably on breakout of deep boreholes) and to verify their new strength criteria and concepts. Due to the cost and equipment availability for obtaining true triaxial strengths, in common engineering practices application of a failure criterion that can incorporate the three-dimensional stresses has been very rare used.

The objective of this study is to determine the effects of the minimum principal stress on the compressive strength of Phra Wihan sandstones. The Coulomb and modified Wiebols & Cook failure criteria derived from the results of conventional tests are used to describe the compressive strengths of the rocks under true triaxial stress states. Assessment of the predictive capability of the two criteria is also made.

2   ROCK SAMPLES

The tested sandstone is from Phra Wihan formation (PW sandstone). It is fine-grained quartz sandstone. It is selected primarily because of its highly uniform texture, density and strength. Its average grain size is 0.1-1.0 mm. The rock is commonly found in the north and northeast of Thailand. Its mechanical properties and responses play a significant role in the stability of tunnels, slope embankments and dam foundations in the region. For the polyaxial compression testing cubical block specimens are cut and ground to have a nominal dimension of $5 \times 5 \times 5$ cm^3.

3   POLYAXIAL COMPRESSION TESTS

A polyaxial load frame equipped with a cantilever beam system (Walsri et al., 2009) has been used to apply constant and uniform lateral stresses (confining pressures) to the cubical specimen while the axial stress is increased at a constant rate until failure occurs (Figure 1). The compression tests are performed to determine the compressive strengths and deformations of the PW sandstone under true triaxial stresses. The intermediate (σ_2) and minimum (σ_3) principal stresses are maintained constant while σ_1 is increased until failure occurs. In this study, σ_2 is varied from 6.6 to 60 MPa, and σ_3 from 2.3 to 15 MPa. Neoprene sheets are used to minimize the friction at all interfaces between the loading platen and the rock surface.

Figure 2 plots the stress-strain curve from the start of loading to failure for some specimens. The elastic modulus and Poisson's ratio are calculated. The three-dimensional principal stress-strain relations given by Jaeger & Cook (1979) can be simplified to obtain a set of governing equations for a isotropic material as follows:

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\nu}{E}(\sigma_2 + \sigma_3) \tag{1}$$
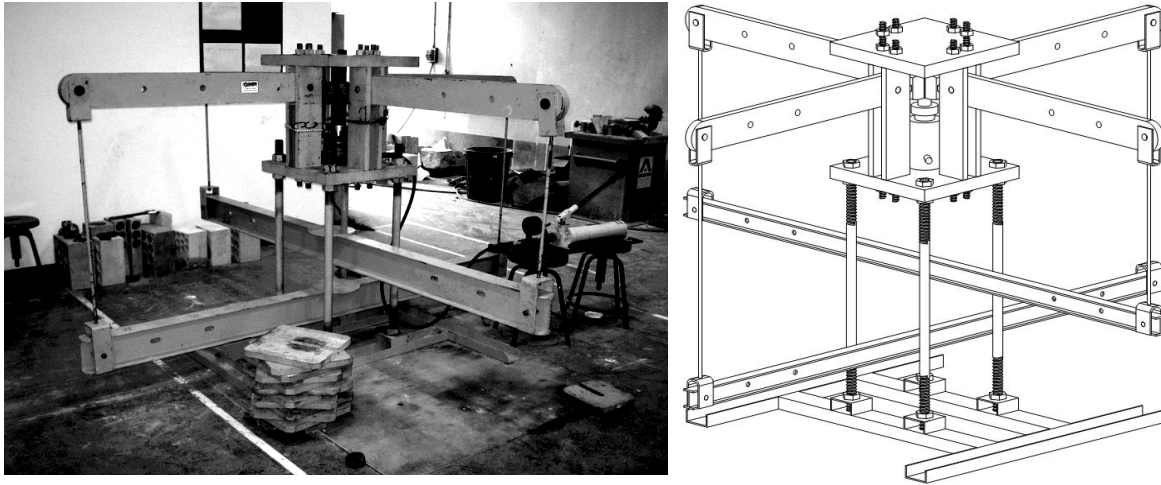
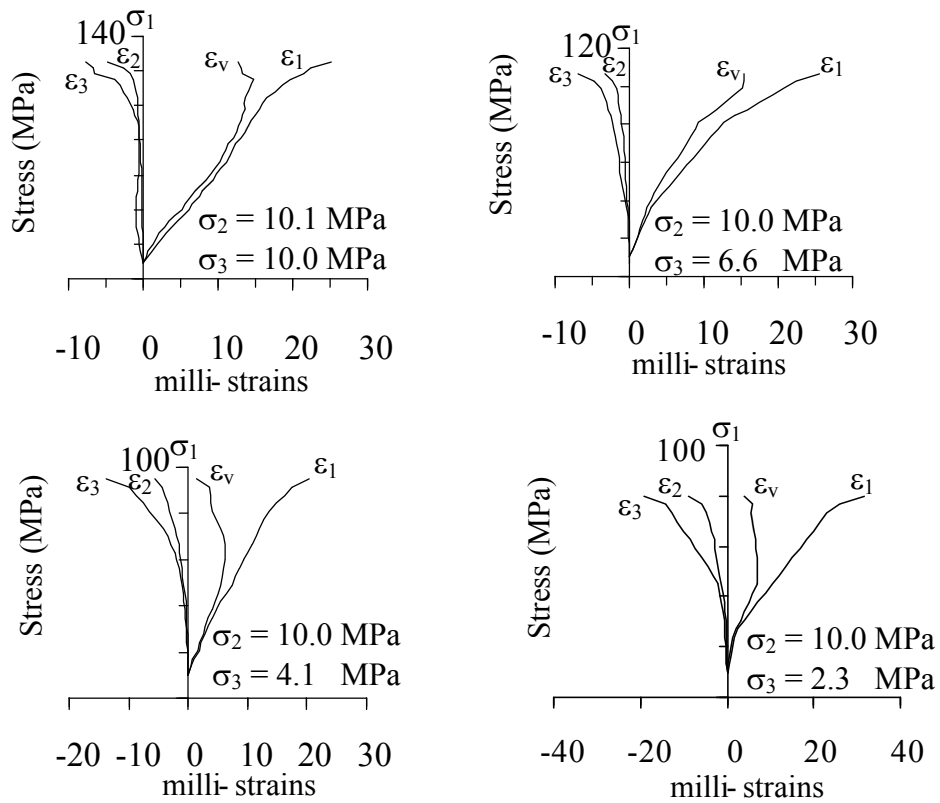Figure 1. Polyaxial load frame developed for compressive strength testing under true triaxial stresses.

Figure 2. Examples of stress-strain curves obtained from polyaxial compressive strength test.

$$\varepsilon_2 = \frac{\sigma_2}{E} - \frac{\nu}{E}(\sigma_1 + \sigma_3) \tag{2}$$

$$\varepsilon_3 = \frac{\sigma_3}{E} - \frac{\nu}{E}(\sigma_1 + \sigma_2) \tag{3}$$

where σ_1, σ_2 and σ_3 are principal stresses, ε_1, ε_2 and ε_3 are principal strains, E is elastic modulus, and v is Poisson's ratio.

The calculations of the Poisson's ratios and tangent elastic moduli are made at 50% of the maximum principal stress. Table 1 summarizes the results for each sample.

Figure 3 plots σ_1 at failure as a function of σ_2 tested under various σ_3's for PW sandstone. The results show the effects of the intermediate principal stress, σ_2, on the maximum stresses at failure by the failure envelopes being offset from the condition where $\sigma_2=\sigma_3$. For all minimum principal stress levels, σ_1 at failure increases with σ_2. The effect of σ_2 tends to be more pronounced under a greater σ_3. These observations agree with those obtained elsewhere (e.g. Haimson & Chang (1999), Colmenares & Zoback (2002) and Haimson (2006)). Post-failure observations suggest that compressive shear failures are predominant in the specimens tested under low σ_2 while splitting tensile fractures parallel to σ_1 and σ_2 directions dominate under higher σ_2 (Figure 4). The observed splitting tensile fractures under relatively high σ_2 suggest that the fracture initiation has no influence from the friction at the loading interface in the σ_2 direction. As a result the increase of σ_1 with σ_2 should not be due to the interface friction.

Table 1. Summary of the polyaxial compressive strength test results on PW sandstone.

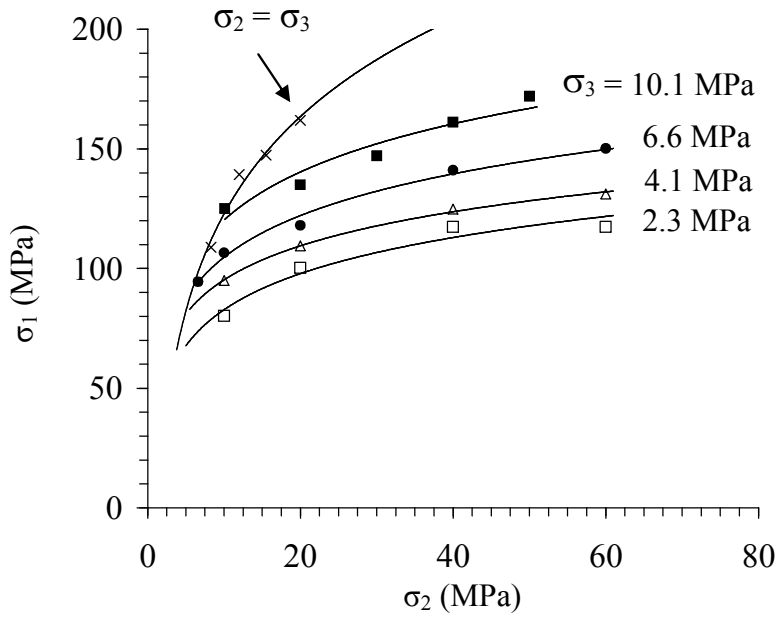| Specimen No.	σ_3 (MPa)	σ_2 (MPa)	σ_1 (MPa)	E (GPa)	v
PWSS-PX-38		10.0	80.3	9.87	0.28
PWSS-PX-03	2.3	20.0	100.3	10.75	0.25
PWSS-PX-58		40.0	117.4	10.89	0.25
PWSS-PX-55		60.0	117.4	10.74	0.28
PWSS-PX-44		10.0	95.0	10.62	0.23
PWSS-PX-08	4.1	20.0	109.5	9.87	0.28
PWSS-PX-27		40.0	124.8	11.88	0.25
PWSS-PX-01		60.0	131.2	10.74	0.27
PWSS-PX-42	6.5	6.6	94.5	9.19	0.27
PWSS-PX-30		10.0	106.6	10.78	0.25
PWSS-PX-33	6.6	20.0	118.0	11.14	0.28
PWSS-PX-45		40.0	141.1	10.92	0.28
PWSS-PX-02		60.0	150.2	10.72	0.27
PWSS-PX-21	8.3	8.3	108.9	9.31	0.26
PWSS-PX-23	10.0	10.1	125.1	9.67	0.28
PWSS-PX-34		20.0	135.0	10.49	0.24
PWSS-PX-35	10.1	30.0	147.1	9.60	0.24
PWSS-PX-32		40.0	161.2	9.24	0.27
PWSS-PX-49		50.0	172.0	10.72	0.27
PWSS-PX-29	12.0	12.0	139.2	9.83	0.28
PWSS-PX-37	15.5	15.5	147.4	10.33	0.26
PWSS-PX-46	20.0	20.0	161.9	9.55	0.28
Mean ± SD.				10.31 ± 0.71	0.26 ± 0.02

Figure 3. Maximum principal stress (σ_1) at failure as a function of σ_2 for various σ_3 values.

[80.3, 10.0, 2.3] [100.3, 20.0, 2.3] [117.4, 40.0, 2.3] [117.4, 60.0, 2.3]

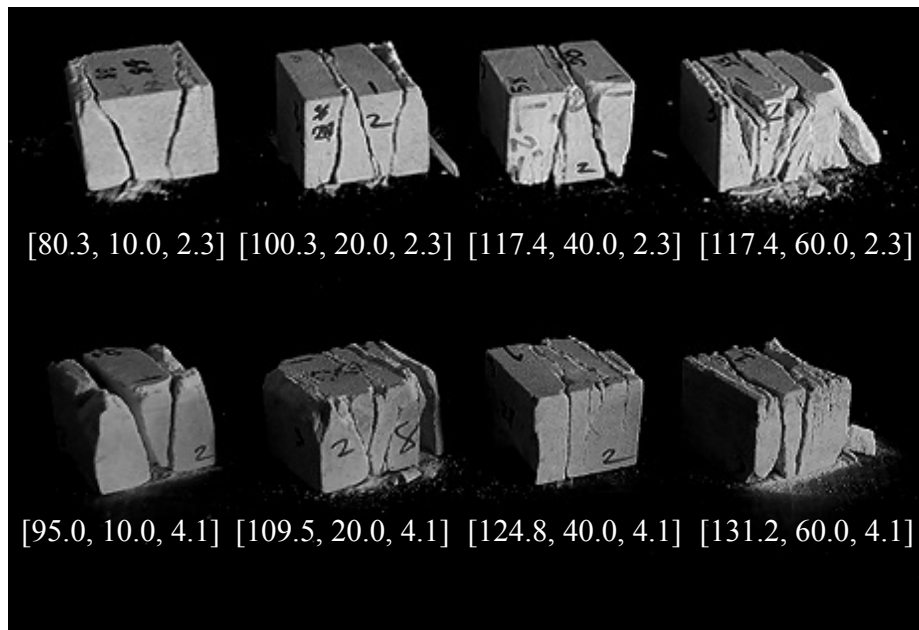[95.0, 10.0, 4.1] [109.5, 20.0, 4.1] [124.8, 40.0, 4.1] [131.2, 60.0, 4.1]

Figure 4. Some post-test specimens of PW sandstone. Number in blankets indicate [$\sigma_1,\sigma_2,\sigma_3$] at failure

4   STRENGTH CRITERIA

The Coulomb and modified Wiebols & Cook failure criteria are used to describe the polyaxial strengths of the PW sandstone. They are selected because the Coulomb criterion has been widely used in actual field applications while the modified Wiebols & Cook criterion has been claimed by many researchers to be one of the best representations of rock strengths under polyaxial compression. To represent the rock strengths under true triaxial stresses the second order stress invariant ($J_2^{1/2}$) and the first order stress invariant or the mean stress (J_1) are calculated from the test results by the following relations (Jaeger & Cook, 1979):

$$J_2^{1/2} = \sqrt{(1/6)\{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2\}} \tag{4}$$

$$J_1 = (\sigma_1 + \sigma_2 + \sigma_3)/3 \tag{5}$$

In this study, the Coulomb criterion is derived from the triaxial compressive strengths of the rocks where σ_2 and σ_3 are equal. The friction angle is 48.8°, and the cohesion is 8.8 MPa. Figure 5 compares the polyaxial test results with those predicted by the Coulomb criterion for PW sandstones. The predictions are made for σ_3=2.3, 4.1, 6.6, 8.3, 10.1, 12.0, 15.5 and 20.0 MPa. (as used in the tests) and under stress conditions from $\sigma_2 = \sigma_3$. In the $J_2^{1/2}$ - J_1 diagram, $J_2^{1/2}$ increases with σ_3 but it is independent of J_1 because the Coulomb criterion ignores σ_2 in the strength calculation. Under a low σ_2 and σ_3 the Coulomb prediction tends to agree with the test results obtained from the PW sandstone. Except for this case, no correlation between the Coulomb predictions and the polyaxial strengths can be found. The inadequacy of the predictability of Coulomb criterion under polyaxial stress states obtained here agrees with a conclusion drawn by Colmenares & Zoback (2002).

The modified Wiebols & Cook criterion given by Colmenares & Zoback (2002) defines $J_2^{1/2}$ at failure in terms of J_1 as:

$$J_2^{1/2} = A + BJ_1 + CJ_1^2 \tag{6}$$

The constants A, B and C depend on rock materials and the minimum principal stresses (σ_3). They can be determined under the conditions where $\sigma_2 = \sigma_3$, as follows (Colmenares & Zoback, 2002):
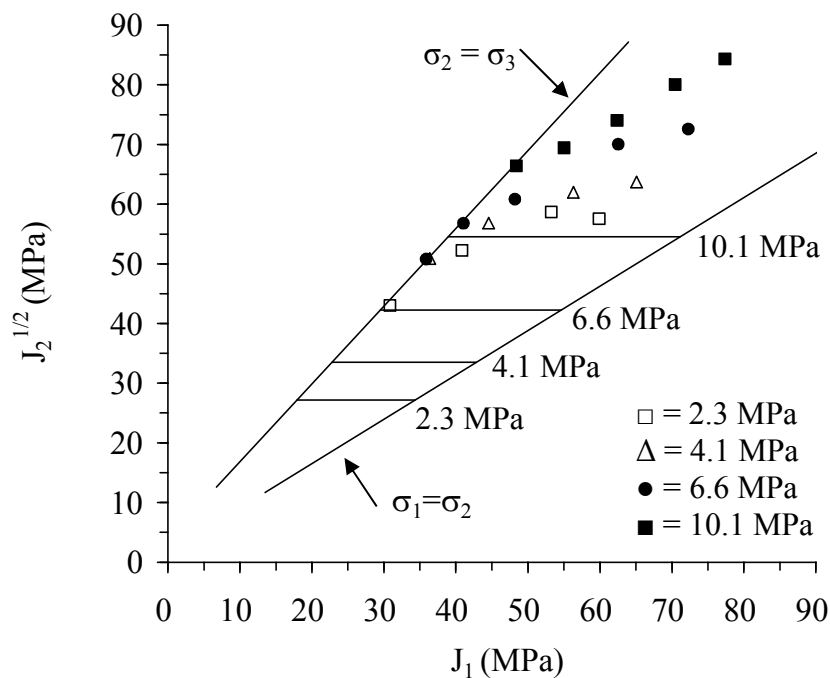

Figure 5. $J_2^{1/2}$ as a function of J_1 from testing PW sandstone compared with the Coulomb criterion predictions (lines).

$$C = \frac{\sqrt{27}}{2C_1 + (q-1)\sigma_3 - C_0} \times \left(\frac{C_1 + (q-1)\sigma_3 - C_0}{2C_1 + (2q+1)\sigma_3 - C_0} - \frac{q-1}{q+2} \right) \tag{7}$$

where: $C_1 = (1 + 0.6\mu_i)C_0$

C_0 = uniaxial compressive strength of the rock.

$\mu_i = \tan\phi$

$q = \{(\mu_i^2 + 1)^{1/2} + \mu_i\}^2 = \tan^2(\pi/4 + \phi/2)$

$$B = \frac{\sqrt{3}(q-1)}{q+2} - \frac{C}{3}(2C_0 + (q+2)\sigma_3 \tag{8}$$

$$A = \frac{C_0}{\sqrt{3}} - \frac{C_0}{3}B - \frac{C_0^2}{9}C \tag{9}$$

The numerical values of A, B and C for PW sandstones are given in Table 2 for each σ_3 tested. Substituting these constants into equation (6), the upper and lower limits of $J_2^{1/2}$ for each rock type can be defined under conditions of $\sigma_2 = \sigma_3$ and $\sigma_1 = \sigma_2$. The predictions are made for σ_3=2.3, 4.1, 6.6, 8.3, 10.1, 12.0, 15.5 and 20.0 MPa. Figure 6 compares the test results with those predicted by the modified Wiebols & Cook criterion. The predictions agree well with the test results. This conforms to the results obtained by Colmenares & Zoback (2002). The predictive capability of the modified Wiebols & Cook criterion can be improved as the minimum principal stress increases.

5 CONCLUSIONS

True triaxial compressive strengths of PW sandstone have been determined in this study. Cubical specimens with a nominal dimension of 5×5×5 cm^3 are prepared. A polyaxial load frame equipped with cantilever beam is used to apply constant σ_2 and σ_3 while σ_1 (along the long axis) is increased until failure is induced. The strength results clearly show that σ_2 affects the maximum stress, σ_1 at failure for PW sandstone. Under true triaxial compressive stresses the modified Wiebols and Cook criterion can predict the compressive strengths of the tested sandstones reasonably well. Due to the effect of σ_2 the Coulomb criterion can not represent the rock strengths under true triaxial compressions, particularly under high σ_2 to σ_3 ratios. The mean misfit is 51.65 for the Coulomb criterion and is 2.93 for the modified Wiebols & Cook criterion.

Table 2. Modified Wiebols and Cook parameters for PW sandstone.

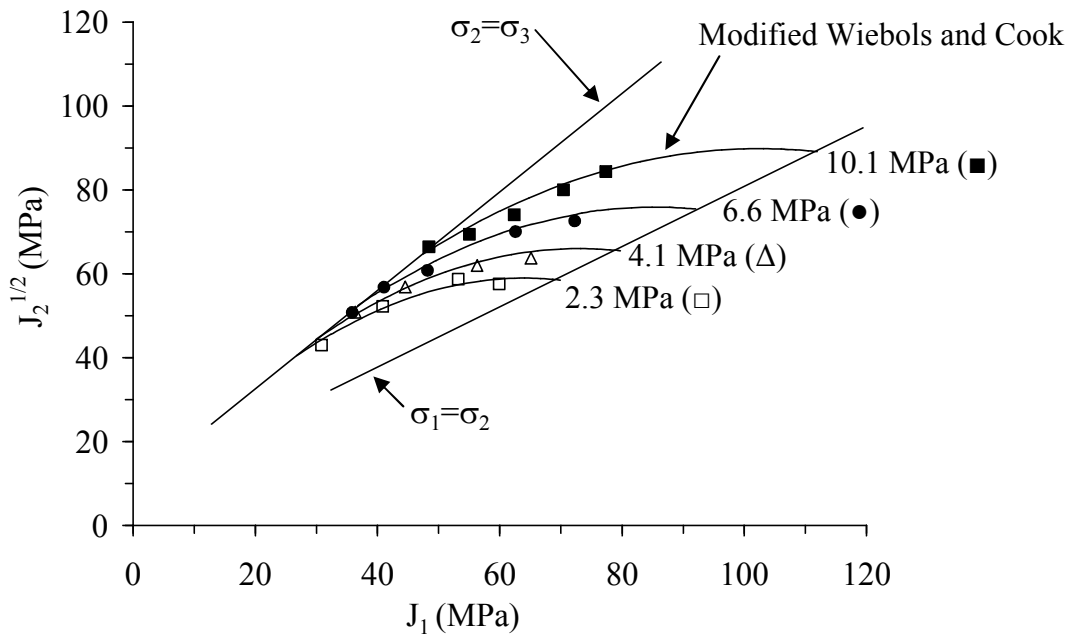| σ_3 (MPa)	Properties		
	A (MPa)	B	C (MPa^{-1})
2.3	4.311	1.706	-0.013
4.1	3.910	1.701	-0.012
6.6	3.486	1.697	-0.010
10.1	3.053	1.694	-0.008

Figure 6. $J_2^{1/2}$ as a function of J_1 from testing PW sandstone compared with the modified Wiebols & Cook criterion predictions (lines).

ACKNOWLEDGTMENT

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