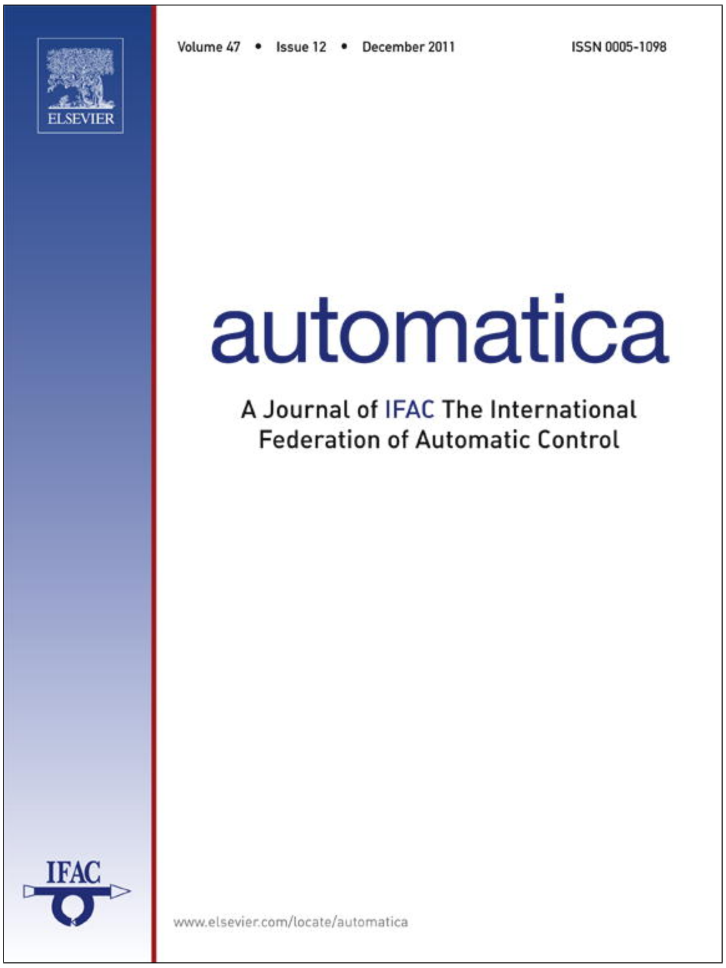
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Brief paper

Adaptive boundary control of a flexible marine installation system[☆]

Wei He [a,b], Shuzhi Sam Ge [a,b,c,1], Shuang Zhang [a,b]

[a] Department of Electrical & Computer Engineering, National University of Singapore, Singapore 117576, Singapore

[b] Centre for Offshore Research & Engineering, National University of Singapore, Singapore 117576, Singapore

[c] Robotics Institute, School of Computer Science and Engineering, University of Electronic Science and Technology of China, Chengdu 611813, China

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ABSTRACT

In this paper, boundary control of a marine installation system is developed to position the subsea payload to the desired set-point and suppress the cable's vibration. Using Hamilton's principle, the flexible cable coupled with vessel and payload dynamics is described as a distributed parameter system with one partial differential equation (PDE) and two ordinary differential equations (ODEs). Adaptive boundary control is proposed at the top and bottom boundaries of the cable, based on Lyapunov's direct method. Considering the system parametric uncertainty, the boundary control schemes developed achieve uniform boundedness of the steady state error between the boundary payload and the desired position. The control performance of the closed-loop system is guaranteed by suitably choosing the design parameters. Simulations are provided to illustrate the applicability and effectiveness of the proposed control.

1. Introduction

In recent years, with the increasing trend towards oil and gas exploitation in deep water, accurate position control for marine installation operations has attracted increasing attention. Due to the requirements for high accuracy and efficiency arising from the modern ocean industry, improving reliability and efficiency of installation operations during oil and gas production in the ocean environment has become a challenging research topic in offshore engineering. Traditional marine installation systems consist of vessel dynamic positioning and crane manipulation to obtain the desired position and heading for the payload (Engineer, 2005; Rowe, Mackenzie, & Snell, 2001). Such methods become difficult in deeper waters due to the long cable between the surface vessel and the payload. One solution for alleviating the precision installation problem is the addition of thrusters attached the payload for the installation operation (How, Ge, & Choo, 2010).

Such a marine installation system consists of an ocean surface vessel, a flexible string-type cable and a subsea payload to be positioned for installation on the ocean floor, as depicted in Fig. 1. The surface vessel, to which the top boundary of the cable is connected, is equipped with a dynamic positioning system with an active thruster. The bottom boundary of the cable is a payload with an end-point thruster attached. This thruster is used for dynamic positioning of the payload. The total marine installation system is subjected to environmental disturbances including ocean currents, waves, and wind. A cable that spans a long distance can produce large vibrations under relatively small disturbances, which will degrade the performance of the system and result in a larger offset from the target installation site. The control for the dynamic positioning of the payload is challenging due to the unpredictable exogenous disturbances such as fluctuating currents and transmission of motions from the surface vessel through the lift cable. Taking into account the unknown time-varying ocean disturbances of the cable leads to the appearance of oscillations, which make the problem of control of the marine installation system relatively difficult.

The dynamics of a flexible mechanical system modeled by a PDE is difficult to control due to the infinite dimensionality of the system. Approaches to control infinite dimensional PDE systems such as the finite element method, Galerkin's method and the assumed modes method (Armaou & Christofides, 2000; Balas, 1978b; Christofides & Armaou, 2000; Sakawa, Matsuno, & Fukushima, 1985; Vandegrift, Lewis, & Zhu, 1994) are based on truncated finite dimensional models of the system. The truncated models are obtained via model analysis or spatial discretization, in which the flexibility is represented by a finite number of modes. The problems arising from the truncation procedure in the modeling need to be carefully treated in practical applications. A

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E-mail addresses: hewei.ac@gmail.com (W. He), samge@nus.edu.sg (S.S. Ge), zhangshuang@nus.edu.sg (S. Zhang).

[1] Tel.: +65 6516 6821; fax: +65 6779 1103.

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potential drawback in the above control design approaches is that the control can cause the actual system to become unstable due to excitation of the unmodeled, high frequency vibration modes (i.e. spillover effects) (Ge, Lee, & Zhu, 1998). Spillover effects which result in instability of the system have been investigated in Balas (1978a) and Meirovitch and Baruh (1983) when the control of the truncated system is restricted to a few critical modes. The control order needs to be increased with the number of flexible modes considered to achieve high accuracy of performance and the control may also be difficult to implement from the engineering point of view since full state measurements or observers are often required. In an attempt to overcome the above shortcomings of the truncated model based control, boundary control combining with other control methodologies such as sliding model control (Zhu & Ge, 1998), energy based robust control (Lee, Ge, & Wang, 2001), the averaging method (Hong & Bentsman, 1994), the backstepping method (Krstic & Smyshlyaev, 2008a,b), and robust adaptive control (He, Ge, How, Choo, & Hong, 2011; Qu, 2001; Yang, Hong, & Matsuno, 2004) have been developed.

On the basis of Lyapunov's direct method, the authors in Fung and Tseng (1999), How, Ge, and Choo (2009), Li, Hou, and Li (2008), Nguyen and Hong (2010), Qu (2001), Rahn, Zhang, Joshi, and Dawson (1999), Shahruz and Krishna (1996) and Yang, Hong, and Matsuno (2005) presented results for the boundary control of flexible systems. In all these works, boundary control is designed for vibration suppression without consideration of the dynamic position control. Recently, by combining the backstepping method with adaptive control design, a novel boundary controller and observer are designed for stabilizing the string and beam model and tracking the target system (Krstic, Guo, Balogh, & Smyshlyaev, 2008; Krstic & Smyshlyaev, 2008c; Smyshlyaev, Guo, & Krstic, 2009). However, this boundary control method is hard to apply to the marine installation system due to difficulties in finding a proper gain kernel. For a marine installation system, the dynamic position control of the payload is as vital as the vibration suppression of the cable. It is therefore necessary to consider both vibration suppression and dynamic positioning in the control design.

2. Problem formulation and preliminaries

For the marine installation system shown in Fig. 1, frame X–Y is the fixed inertia frame, and frame x–y is the local reference frame fixed along the vertical direction of the surface vessel. The top boundary of the cable is at the vessel and the bottom boundary of the cable is at the underwater payload. Forces from thrusters on the vessel and payload are the control inputs of the system, and the boundary position and slope of the cable are used as the feedback signals in the control design. p_d is the desired target position, $p(t)$ is the position of the vessel, $w(x, t)$ is the elastic transverse reflection with respect to frame x–y at the position x for time t, and $y(x, t) := p(t) + w(x, t)$ is the position of the cable with respect to frame X–Y at the position x for time t. Note that $w(L, t) = 0$ is due to the connection between the vessel and the top boundary of the cable.

In this paper, we consider the transverse degree of freedom only. We assume that the original position of the vessel is directly above the subsea payload with no horizontal offset, and that the payload is filled with seawater.

Remark 1. We use the notation $(*)'$, $(*)''$, $(*)'''$ and $(*)''''$ representing the first-, second-, third-, and fourth-order derivatives of $(*)$ with respect to x respectively, and $(\dot{*})$ and $(\ddot{*})$ denoting the first- and second-order derivatives of $(*)$ with respect to time t, respectively, for clarity.

2.1. Dynamic analysis

The kinetic energy of the installation system E_k can be represented as

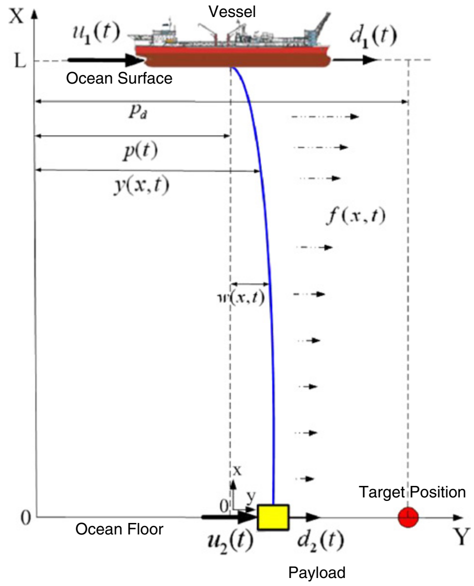

Fig. 1. A typical flexible marine installation system.

$$E_k = \frac{1}{2}M[\dot{y}(L,t)]^2 + \frac{1}{2}\rho\int_0^L [\dot{y}(x,t)]^2 dx + \frac{1}{2}m[\dot{y}(0,t)]^2, \tag{1}$$

where x and t represent the independent spatial and time variables respectively, M denotes the mass of the surface vessel, m denotes the mass of bottom payload, $y(L, t) = p(t)$, $\dot{y}(L, t) = \dot{p}(t)$ and $\ddot{y}(L, t) = \ddot{p}(t)$ are the position, velocity and acceleration of the vessel respectively, $\rho > 0$ is the uniform mass per unit length of the cable, and L is the length of the cable.

The potential energy E_p due to the strain energy of the cable can be obtained from

$$E_p = \frac{1}{2}T\int_0^L \left[w'(x,t)\right]^2 dx, \tag{2}$$

where T is the tension of the cable. The definition of $y(x, t)$ yields $y'(x, t) = w'(x, t)$. Then we have

$$E_p = \frac{1}{2}T\int_0^L \left[y'(x,t)\right]^2 dx. \tag{3}$$

The virtual work done by the ocean current disturbance on the vessel, the cable and the payload is given by

$$\delta W_f = \int_0^L f(x,t)\delta y(x,t)dx + d_1(t)\delta y(L,t) + d_2(t)\delta y(0,t), \tag{4}$$

where $f(x, t)$ is the distributed transverse load on the cable due to the hydrodynamic effects of the ocean current, waves and wind, $d_1(t)$ denotes the environmental disturbance on the vessel, and $d_2(t)$ denotes the environmental disturbance on the payload. The virtual work done by damping on the vessel, the cable and the payload is represented by

$$\delta W_d = -\int_0^L c\dot{y}(x,t)\delta y(x,t)dx - c_1\dot{y}(L,t)\delta y(L,t) - c_2\dot{y}(0,t)\delta y(0,t), \tag{5}$$

where c is the damping coefficient of the cable, c_1 denotes the damping coefficient of the vessel, and c_2 denotes the damping coefficient of the payload. We introduce the control u_1 applied to the top boundary of the cable from the thruster attached to the vessel, and the control u_2 applied to the bottom boundary of the cable from the thruster attached to the payload. The virtual work done by the boundary control is written as

$$\delta W_m = u_1(t)\delta w(L,t) + u_2(t)\delta w(0,t). \tag{6}$$

Then, we have the total virtual work done on the system as

$$\delta W = \delta W_f + \delta W_d + \delta W_m. \tag{7}$$

Using Hamilton's principle (Goldstein, 1951), we further obtain the governing equation of the system as

$$\rho\ddot{y}(x,t) - Ty''(x,t) + c\dot{y}(x,t) = f(x,t), \tag{8}$$

$\forall(x,t) \in (0,L) \times [0,\infty)$, and the boundary conditions of the system as

$$u_1(t) + d_1(t) - c_1\dot{y}(L,t) - M\ddot{y}(L,t) - Ty'(L,t) = 0, \tag{9}$$

$$u_2(t) + d_2(t) - c_2\dot{y}(0,t) - m\ddot{y}(0,t) + Ty'(0,t) = 0, \quad \forall t \in [0,\infty). \tag{10}$$

2.2. Ocean current disturbance

The effect of a time-varying ocean current $U(x,t)$ on a cable is modeled as a distributed load (Blevins, 1977; Faltinsen, 1990). The distributed load on the flexible cable can be expressed as a combination of a mean drag and an oscillating drag modeled as

$$f(x,t) = \frac{1}{2}\rho_s C_D(x,t)U(x,t)^2 D + A_D\cos(4\pi f_v t + \theta), \tag{11}$$

where ρ_s is the sea water density, $C_D(x,t)$ is the drag coefficient, D is the pipe outer diameter, f_v is the shedding frequency, θ is the phase angle, and A_D is the amplitude of the oscillatory part of the drag force, typically 20% of the first term in $f(x,t)$ (Faltinsen, 1990). The non-dimensional vortex shedding frequency can be expressed as

$$f_v = \frac{S_t U(x,t)}{D}, \tag{12}$$

where S_t is the Strouhal number.

Assumption 1. For the distributed load $f(x,t)$ on the cable, the disturbance $d_1(t)$ on the vessel, and the disturbance $d_2(t)$ on the payload, we assume that there exist constants $\bar{f} \in R^+$, $\bar{d}_1 \in R^+$ and $\bar{d}_2 \in R^+$ such that $|f(x,t)| \le \bar{f}$, $\forall(x,t) \in [0,L] \times [0,\infty)$, $|d_1(t)| \le \bar{d}_1$, $\forall t \in [0,\infty)$ and $|d_2(t)| \le \bar{d}_2$, $\forall t \in [0,\infty)$. This is a reasonable assumption as the time-varying disturbances $f(x,t)$, $d_1(t)$ and $d_2(t)$ have finite energy and hence are bounded, i.e. $f(x,t) \in \mathcal{L}_\infty([0,L])$, $d_1(t) \in \mathcal{L}_\infty$ and $d_2(t) \in \mathcal{L}_\infty$.

2.3. Preliminaries

For the convenience of stability analysis, we present the following lemmas and properties for the subsequent development.

Lemma 1 (*Ge, He, How, & Choo, 2010, Rahn, 2001*). *Let $\phi_1(x,t)$, $\phi_2(x,t) \in R$ with $x \in [0,L]$ and $t \in [0,\infty)$; the following inequalities hold:*

$$\phi_1\phi_2 \le |\phi_1\phi_2| \le \phi_1^2 + \phi_2^2, \quad \forall\phi_1,\phi_2 \in R. \tag{13}$$

$$|\phi_1\phi_2| = \left|\left(\frac{1}{\sqrt{\delta}}\phi_1\right)\left(\sqrt{\delta}\phi_2\right)\right| \le \frac{1}{\delta}\phi_1^2 + \delta\phi_2^2, \quad \forall\phi_1,\phi_2 \in R \text{ and } \delta > 0. \tag{14}$$

Lemma 2 (*Horn & Johnson, 1990*). *The Rayleigh–Ritz theorem: let $A \in R^{n\times n}$ be a real, symmetric, positive-definite matrix; therefore, all the eigenvalues of A are real and positive. Let $\lambda_{\min}$ and $\lambda_{\max}$ denote the minimum and maximum eigenvalues of A, respectively; then for $\forall x \in R^n$, we have*

$$\lambda_{\min}\|x\|^2 \le x^T A x \le \lambda_{\max}\|x\|^2, \tag{15}$$

where $\|\cdot\|$ denotes the standard Euclidean norm.

Lemma 3 (*Ge et al., 2010, Hardy, Littlewood, & Polya, 1959*). *Let $\phi(x,t) \in R$ be a function defined on $x \in [0,L]$ and $t \in [0,\infty)$ that satisfies the boundary condition*

$$\phi(0,t) = 0, \quad \forall t \in [0,\infty). \tag{16}$$

then the following inequalities hold:

$$\phi^2 \le L\int_0^L [\phi']^2 dx, \quad \forall x \in [0,L]. \tag{17}$$

Property 1 (*Queiroz, Dawson, Nagarkatti, & Zhang, 2000*). *If the kinetic energy of the system (8)–(10), given by Eq. (1), is bounded $\forall t \in [0,\infty)$, then $\dot{y}(x,t)$, $\dot{y}'(x,t)$ and $\dot{y}''(x,t)$ are bounded $\forall(x,t) \in [0,L] \times [0,\infty)$.*

Property 2 (*Queiroz et al., 2000*). *If the potential energy of the system (8)–(10), given by Eq. (3), is bounded $\forall t \in [0,\infty)$, then $y'(x,t)$ and $y''(x,t)$ are bounded $\forall(x,t) \in [0,L] \times [0,\infty)$.*

3. Control design

The control objective is to design the boundary control to position the subsea payload to the desired set-point p_d and simultaneously suppress the vibrations of the cable in the presence of the time-varying ocean disturbance. The control forces $u_1(t)$ and $u_2(t)$ are from the thruster in the vessel and the thruster attached to the subsea payload respectively. In this section, Lyapunov's direct method is used to construct the boundary control $u_1(t)$ and $u_2(t)$ at the top and bottom boundaries of the cable and to analyze the stability of the closed-loop system. When T, m and c_2 are unknown, the boundary control is designed to compensate the system parametric uncertainty.

To stabilize the system given by governing equation (8) and boundary condition Eqs. (9) and (10), we propose the following boundary control:

$$u_1(t) = -k_v\dot{y}(L,t) - \mathrm{sgn}[\dot{y}(L,t)]\bar{d}_1, \tag{18}$$

$$u_2(t) = -P\hat{\Phi} - k_s u_a - \mathrm{sgn}(u_a)\bar{d}_2 - k_p(y(0,t) - p_d), \tag{19}$$

where $\mathrm{sgn}(\cdot)$ denotes the signum function, k_v, k_p and k_s are the positive control gains, and the vectors P, $\hat{\Phi}$, and the auxiliary signal u_a are defined as

$$P = [y'(0,t) \quad -\dot{y}'(0,t) \quad -\dot{y}(0,t)], \tag{20}$$

$$\hat{\Phi} = [\hat{T} \quad \hat{m} \quad \hat{c}_2]^T. \tag{21}$$

$$u_a = \dot{y}(0,t) - y'(0,t). \tag{22}$$

The parameter vector Φ is defined as

$$\Phi = [T \quad m \quad c_2]^T. \tag{23}$$

The adaptation law is designed as

$$\dot{\hat{\Phi}} = \Gamma P^T u_a - r\Gamma\hat{\Phi}, \tag{24}$$

where $\Gamma \in R^{3\times 3}$ is a diagonal positive-definite matrix and r is a positive constant. We define all the eigenvalues of Γ as real and positive, and the maximum and minimum eigenvalues of matrix Γ as $\lambda_{\max}$ and $\lambda_{\min}$ respectively. The parameter estimate error vector $\tilde{\Phi} \in R^3$ is defined as

$$\tilde{\Phi} = \Phi - \hat{\Phi}. \tag{25}$$

After differentiating the auxiliary signal Eq. (22), multiplying the resulting equation by m, and substituting Eq. (10), we obtain

$$m\dot{u}_a(t) = Ty'(0,t) + d_2 - m\dot{y}'(0,t) - c_2\dot{y}(0,t) + u_2 = P\Phi + d_2 + u_2. \quad (26)$$

Substituting Eq. (19) into Eq. (26) and substituting Eq. (25) into Eq. (24), we have

$$m\dot{u}_a = P\tilde{\Phi} - k_s u_a + d_2 - \mathrm{sgn}(u_a)\bar{d}_2 - k_p(y(0,t) - p_d), \quad (27)$$

$$\dot{\hat{\Phi}} = -\Gamma P^T u_a + r\Gamma\hat{\Phi}. \quad (28)$$

Remark 2. The proposed boundary control does not require distributed sensing and all the signals in the boundary control can be measured by sensors or obtained by a backward difference algorithm. $y(L,t)$ and $y(0,t)$ can be sensed by two global positioning systems (GPS) located in the vessel and the end-point thruster respectively. $y'(0,t)$ can be measured by an inclinometer at the bottom boundary of the cable. In practice, the effect of measurement noise from sensors is unavoidable, which will affect the controller implementation, especially when high order differentiation terms with respect to time exist. In our proposed controller Eqs. (18) and (19), $\dot{y}(L,t)$, $\dot{y}(0,t)$ and $\dot{y}'(0,t)$ with only one time differentiation with respect to time can be calculated with a backward difference algorithm.

Consider the Lyapunov function candidate

$$V_a = V_1 + V_2 + \Delta + \frac{1}{2}\tilde{\Phi}^T\Gamma^{-1}\tilde{\Phi}, \quad (29)$$

where the energy term V_1, an auxiliary term V_2 and a small crossing term Δ are defined as

$$V_1 = \frac{\beta}{2}\rho\int_0^L [\dot{y}]^2 dx + \frac{\beta}{2}T\int_0^L [y']^2 dx + \frac{\beta}{2}M[\dot{y}(L,t)]^2 + \frac{\beta k_p}{2}[y(0,t) - p_d]^2, \quad (30)$$

$$V_2 = \frac{1}{2}mu_a^2, \quad (31)$$

$$\Delta = \alpha\rho\int_0^L (x-L)\dot{y}y' dx, \quad (32)$$

where α and β are two positive weighting constants.

Lemma 4. *The Lyapunov function candidate given by (29) can be upper and lower bounded as*

$$\lambda_{1a}(V_1 + V_2 + \|\tilde{\Phi}\|^2) \le V_a \le \lambda_{2a}(V_1 + V_2 + \|\tilde{\Phi}\|^2), \quad (33)$$

where λ_{1a} and λ_{2a} are two positive constants defined as

$$\lambda_{1a} = \min\left(1 - \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)}, \frac{1}{2\lambda_{\max}}\right), \quad (34)$$

$$\lambda_{2a} = \max\left(1 + \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)}, \frac{1}{2\lambda_{\min}}\right). \quad (35)$$

Proof. Substitution of Ineq. (13) into Eq. (32) yields

$$|\Delta| \le \alpha\rho L\int_0^L ([y']^2 + [\dot{y}]^2)dx \le \alpha_1 V_1, \quad (36)$$

where

$$\alpha_1 = \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)}. \quad (37)$$

Then, we obtain

$$-\alpha_1 V_1 \le \Delta \le \alpha_1 V_1. \quad (38)$$

Considering α as a small positive weighting constant satisfying $0 < \alpha < \frac{\min(\beta\rho,\beta T)}{2\rho L}$, we can obtain

$$\alpha_2 = 1 - \alpha_1 = 1 - \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)} > 0, \quad (39)$$

$$\alpha_3 = 1 + \alpha_1 = 1 + \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)} > 1. \quad (40)$$

Then, we further have

$$0 \le \alpha_2 V_1 \le V_1 + \Delta \le \alpha_3 V_1. \quad (41)$$

Given the Lyapunov function candidate in Eq. (29), we obtain

$$0 \le \lambda_1(V_1 + V_2) \le V_1 + V_2 + \Delta \le \lambda_2(V_1 + V_2), \quad (42)$$

where $\lambda_1 = \min(\alpha_2, 1) = \alpha_2$ and $\lambda_2 = \max(\alpha_3, 1) = \alpha_3$ are positive constants. Utilizing the properties of matrix Γ and Lemma 2, we have

$$\frac{1}{2\lambda_{\max}}\|\tilde{\Phi}\|^2 \le \frac{1}{2}\tilde{\Phi}^T\Gamma^{-1}\tilde{\Phi} \le \frac{1}{2\lambda_{\min}}\|\tilde{\Phi}\|^2. \quad (43)$$

Combining Ineqs. (41) and (43), we have Eq. (33). □

Lemma 5. *The time derivative of the Lyapunov function in (29) can be upper bounded with*

$$\dot{V}_a \le -\lambda_a V_a + \varepsilon, \quad (44)$$

where λ_a and ε are two positive constants.

Proof. Differentiating Eq. (29) with respect to time leads to

$$\dot{V}_a = \dot{V}_1 + \dot{V}_2 + \dot{\Delta} + \tilde{\Phi}^T\Gamma^{-1}\dot{\tilde{\Phi}}. \quad (45)$$

Substituting the governing equation Eq. (8), using the boundary conditions and Lemmas 1–3, we obtain

$$\begin{aligned}\dot{V}_a \le & -\left(\beta c + \frac{\alpha\rho}{2} - \beta\delta_2 - \frac{\alpha cL}{\delta_4}\right)\int_0^L [\dot{y}]^2 dx \\ & - \left(\frac{\alpha T}{2} - 4k_pL - \alpha L\delta_3 - \alpha cL\delta_4\right)\int_0^L [y']^2 dx \\ & - \beta(k_v + c_1)[\dot{y}(L,t)]^2 - \left(k_s - k_p - \frac{\beta T}{2}\right)u_a^2 \\ & - \left(\frac{\beta T}{2} - \frac{\alpha\rho L}{2} - \frac{\beta k_p\delta_1}{2}\right)[\dot{y}(0,t)]^2 \\ & - \left(\frac{\beta T}{2} - \frac{\alpha TL}{2}\right)[y'(0,t)]^2 - k_p\left(1 - \frac{\beta}{2\delta_1}\right)[y(0,t) - p_d]^2 \\ & - \frac{r}{2}\|\tilde{\Phi}\|^2 + \frac{r}{2}\|\Phi\|^2 + \left(\frac{\beta}{\delta_2} + \frac{\alpha L}{\delta_3}\right)\int_0^L \bar{f}^2 dx + 4k_p p_d^2 \\ \le & -\lambda_{3a}(V_1 + V_2) + \varepsilon,\end{aligned} \quad (46)$$

where the constants k_v, k_p, k_s, α, β, δ_1, δ_2, δ_3 and δ_4 are chosen to satisfy the following conditions:

$$\alpha < \frac{\min(\beta\rho, \beta T)}{2\rho L}, \quad (47)$$

$$\frac{\beta T}{2} - \frac{\alpha\rho L}{2} - \frac{\beta k_p\delta_1}{2} \ge 0, \quad (48)$$

$$\frac{\beta T}{2} - \frac{\alpha TL}{2} \ge 0, \quad (49)$$

$$\sigma_1 = \beta c + \frac{\alpha\rho}{2} - \beta\delta_2 - \frac{\alpha c L}{\delta_4} > 0, \tag{50}$$

$$\sigma_2 = \frac{\alpha T}{2} - 8k_p L - \alpha L\delta_3 - \alpha c L\delta_4 > 0, \tag{51}$$

$$\sigma_3 = \beta(k_v + c_1), \tag{52}$$

$$\sigma_4 = 1 - \frac{\beta}{2\delta_1} > 0, \tag{53}$$

$$\sigma_5 = k_s - k_p - \frac{\beta T}{2} > 0, \tag{54}$$

$$\lambda_{3a} = \min\left(\frac{2\sigma_1}{\beta\rho}, \frac{2\sigma_2}{\beta T}, \frac{2\sigma_3}{\beta M}, \frac{2\sigma_4}{\beta}, \frac{2\sigma_5}{m}, \frac{r}{2}\right) > 0, \tag{55}$$

$$\varepsilon = \left(\frac{\beta}{\delta_2} + \frac{\alpha L}{\delta_3}\right)\int_0^L \bar{f}^2 dx + 4k_p p_d^2 + \frac{r}{2}\|\Phi\|^2 > 0. \tag{56}$$

From Ineqs. (42) and (46) we have

$$\dot{V}_a \le -\lambda_a V_a + \varepsilon, \tag{57}$$

where $\lambda_a = \lambda_{3a}/\lambda_{2a}$. □

Theorem 1. *For the system dynamics described by (8) and boundary conditions (9) and (10), under Assumption 1, and the boundary control (18) and (19), given that the initial conditions are bounded, we can conclude that the closed-loop system is uniformly bounded, and the system boundary error signal $e(t) = y(0,t) - p_d$ will remain within the compact set Ω_a defined by*

$$\Omega_a := \{e \in R|\ |e| \le D_a\}, \tag{58}$$

where $D_a = \sqrt{\frac{2}{\beta k_p \lambda_{1a}}\left(V_a(0) + \frac{\varepsilon}{\lambda_a}\right)}$.

Proof. From Eq. (57), we obtain

$$V_a \le \left(V_a(0) - \frac{\varepsilon}{\lambda_a}\right)e^{-\lambda_a t} + \frac{\varepsilon}{\lambda_a} \le V_a(0)e^{-\lambda_a t} + \frac{\varepsilon}{\lambda_a}, \tag{59}$$

which implies that V_a is bounded. Utilizing Ineq. (17) and Eq. (30), we obtain that $w(x,t)$ is uniformly bounded as follows:

$$|w(x,t)| \le \sqrt{\frac{2L}{\beta T\lambda_{1a}}\left(V_a(0)e^{-\lambda_a t} + \frac{\varepsilon}{\lambda_a}\right)},$$
$$\forall(x,t) \in [0,L] \times [0,\infty) \tag{60}$$

and we have

$$\frac{\beta k_p}{2}[y(0,t) - p_d]^2 \le V_1 \le V_1 + V_2 \le \frac{1}{\lambda_{1a}}V_a \in \mathcal{L}_\infty, \tag{61}$$

$$|y(0,t) - p_d| \le \sqrt{\frac{2}{\beta k_p \lambda_{1a}}\left(V_a(0)e^{-\lambda_a t} + \frac{\varepsilon}{\lambda_a}\right)}. \quad \square \tag{62}$$

Remark 3. From Eq. (61), we can state that V_1 and V_2 are bounded $\forall t \in [0,\infty)$. Use of boundedness of V_1 and V_2 produces that $\dot{y}(x,t), y'(x,t)$ are bounded $\forall(x,t) \in [0,L] \times [0,\infty)$ and u_a is bounded $\forall t \in [0,\infty)$. Then, we can obtain that potential energy Eq. (3) is bounded. Using Property 2, we can further obtain that $y''(x,t)$ is bounded. From the boundedness of $\dot{y}(x,t)$, we can state that $\dot{y}(0,t)$ and $\dot{y}(L,t)$ are bounded. Therefore, we can conclude that the kinetic energy of the system Eq. (1) is also bounded. Using Property 1, we can obtain that $\dot{y}(x,t)$ and $\dot{y}'(x,t)$ are also bounded $\forall(x,t) \in [0,L] \times [0,\infty)$. Applying Assumption 1, Eq. (8) and the above statements, we can state that $\ddot{y}(x,t)$ is also bounded $\forall(x,t) \in [0,L] \times [0,\infty)$. From Eq. (59), we can obtain that the parameter estimate error $\tilde{\Phi}$ is bounded $\forall t \in [0,\infty)$. Then, we can state the proposed control Eqs. (18) and (19) ensuring that all internal system signals including $y(x,t), y'(x,t), \dot{y}(x,t), \dot{y}'(x,t)$ and $\ddot{y}(x,t)$ are uniformly bounded. Since $\hat{\Phi}$, $y'(x,t)$ and $\dot{y}(x,t)$ are all bounded $\forall(x,t) \in [0,L] \times [0,\infty)$, we can conclude that the boundary control Eqs. (18) and (19) are also bounded $\forall t \in [0,\infty)$.

Table 1
Parameters of the marine installation system.

Parameter	Description	Value
L	Length of the cable	1000.00 m
D	Diameter of the cable	0.05 m
M	Mass of the vessel	9.60×10^7 kg
m	Mass of the payload	4×10^5 kg
c_1	Damping of the vessel	9.20×10^3 N s/m
c_2	Damping of the payload	9.20×10^2 N s/m
T	Tension	4.00×10^6 N
ρ	Mass per unit length	8.02 kg/m
ρ_s	Sea water density	1024.00 kg/m^3
c	Distributed damping	1.00 N s/m^2
p_d	Desired set-point	50.00 m

Remark 4. In the above analysis, the deflection of the cable $w(x,t)$ can be made arbitrarily small provided that the design control parameters are appropriately selected. By choosing the proper values of α and β, it is shown that the increase in the control gains k_v and k_s will result in a larger σ_3 and σ_5, which will lead to a greater λ_3. Then the value of λ_a will increase, which will reduce the size of Ω_a and yield a better vibration suppression performance.

Remark 5. Even though $y(0,t)$ may be far from the desired position p_d, it is guaranteed that the steady bottom boundary state error $y(0,\infty) - p_d$ can be made arbitrarily small provided that the design parameters are appropriately selected. It is easily seen that the increase in the control gains k_v and k_s will result in a better tracking performance. However, increasing k_v and k_s will lead to a high gain control scheme. Therefore, in practical applications, the design parameters should be adjusted carefully to achieve suitable transient performance and control action.

4. Numerical simulations

The cable, initially at rest, is excited by a distributed transverse disturbance due to ocean current. The corresponding initial conditions of the marine installation system are given as

$$y(x,0) = 0, \tag{63}$$
$$\dot{y}(x,0) = 0. \tag{64}$$

The system parameters are given in Table 1.

In our simulation experiments, the ocean surface current velocity $U(t)$ is modeled as a mean flow with worst case sinusoidal components to simulate the cable with a mean deflected profile. The sinusoids have frequencies of $\omega_i = \{0.867, 1.827, 2.946, 4.282\}$, for $i = 1$–4, corresponding to the four natural modes of vibration of the cable. The surface current $U(t)$ is expressed as

$$U(t) = \bar{U} + U'\sum_{i=1}^{4}\sin(\omega_i t), \quad i = 1, 2, \ldots, 4, \tag{65}$$

where $\bar{U} = 2$ ms^{-1} is the mean flow current and $U' = 0.2$ is the amplitude of the oscillating flow. In the simulation, we assume that the full current load is applied from $x = 1000$ m to $x = 0$ m and thereafter linearly declines to zero at the ocean floor, $x = 0$, to obtain a depth dependent ocean current profile $U(x,t)$. The distributed load $f(x,t)$ is generated from Eq. (11) with

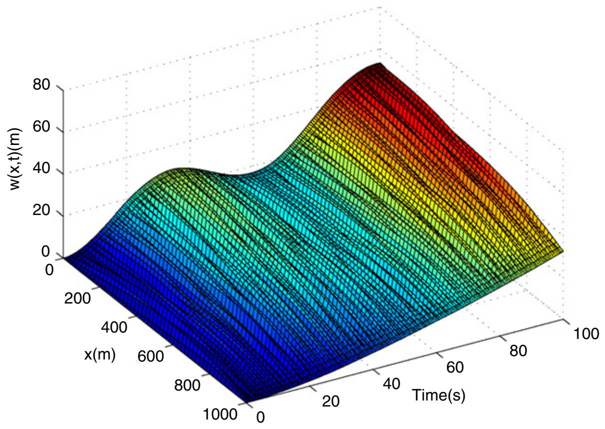

Fig. 2. Position of the cable without control.

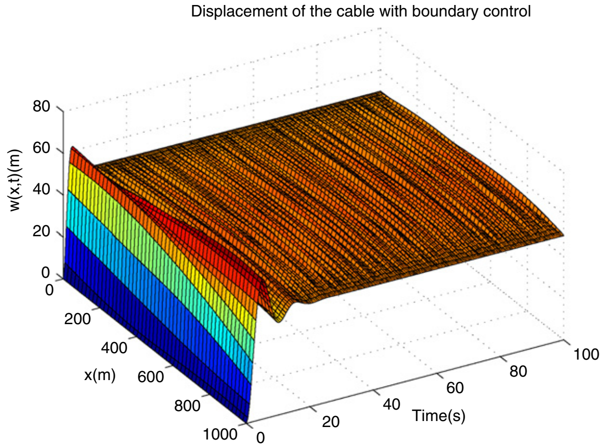

Fig. 3. Position of the cable with robust adaptive boundary control.

$C_D = 1, \theta = 0, S_t = 0.2$ and $f_v = 2.625$. The disturbance $d_1(t)$ on the vessel is generated from

$$d_1(t) = [3 + 0.8\sin(0.7t) + 0.2\sin(0.5t) + 0.2\sin(0.9t)] \times 10^6. \quad (66)$$

The disturbance $d_2(t)$ on the payload is given by

$$d_2(t) = [3 + 0.8\sin(0.7t) + 0.2\sin(0.5t) + 0.2\sin(0.9t)] \times 10^4. \quad (67)$$

The position of the cable for free vibration, i.e., $u_1(t) = u_2(t) = 0$, exposed to ocean disturbance is shown in Fig. 2. It is clear that the system is unstable and the vibration of the cable is quite large. The position of the cable with adaptive control Eqs. (18) and (19), by choosing $k_v = 4\times10^7$, $k_p = 1\times10^5$, $k_s = 1.5\times10^5$, $r = 0.001$ and $\Gamma = \text{diag}\{5\times10^6, 1\times10^4, 5\times10^6\}$, under ocean disturbance is shown in Fig. 3. Fig. 3 illustrates that the proposed boundary control is able to bring the subsea payload to the desired position $p_d = 50$ m and stabilize the cable in a small neighborhood of its equilibrium position.

5. Conclusion

In this paper, both position control and vibration suppression have been investigated for a flexible marine installation system subject to ocean disturbance. To fully compensate for the effect of unknown system parameters, a signum term and an auxiliary signal term have been introduced to develop an adaptive boundary control law. All the signals of the closed-loop system have been proved to be uniformly bounded by using Lyapunov's direct method. The simulation results have illustrated that the proposed control is able to position the payload to the desired set-point and suppress the vibration of the cable with a good performance.

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Wei He received his B.Eng. in automatic control from South China University of Technology (SCUT), Guangzhou, China, in 2006. He is currently working toward a Ph.D. degree in the Department of Electrical and Computer Engineering, National University of Singapore (NUS), Singapore. His current research interests include distributed parameter systems, marine cybernetics and robotics.

Shuzhi Sam Ge is founding Director of Social Robotics Lab, Interactive Digital Media Institute and Full Professor in the Department of Electrical and Computer Engineering, the National University of Singapore and the Robotics Institute, School of Computer Science and Engineering, University of Electronic Science and Technology of China, Chengdu, China. He received his Ph.D. from Imperial College, in 1993, and his B.Sc. degree from Beijing University of Aeronautics & Astronautics (BUAA), in 1986.

He has (co-)authored three books and over 300 international journal and conference papers. He is the Editor-in-Chief of the International Journal of Social Robotics. He has served/is serving as an Associate Editor for a number of flagship journals including IEEE Transactions on Automatic Control, IEEE Transactions on Control Systems Technology, IEEE Transactions on Neural Networks, and Automatica. He also serves as an Editor of the Taylor & Francis Automation and Control Engineering Series. He served as Vice President of Technical Activities, 2009–2010, and is serving as Vice President for Membership Activities, 2011–2012, IEEE Control Systems Society. He is a fellow of IEEE, a fellow of IFAC and a Fellow of IET. His current research interests include social robotics, multimedia fusion, medical robots, and intelligent systems.

Shuang Zhang received her B.Sc. in information and computing science from Shandong University (SDU) in 2007, and her M.Eng. degree in automatic control from South China University of Technology (SCUT) in 2010. Now, she is working toward a Ph.D. degree in the Department of Electrical and Computer Engineering, National University of Singapore (NUS), Singapore. Her current research interests include infinite dimensional systems and flexible systems.