

## Accepted Manuscript

A theoretical model of bank lending: Does ownership matter in times of crises?

Michael Brei, Alfredo Schclarek

PII: S0378-4266(14)00121-6

DOI: <http://dx.doi.org/10.1016/j.jbankfin.2014.03.038>

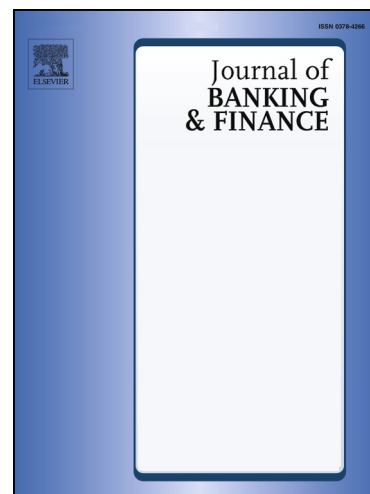
Reference: JBF 4412

To appear in: *Journal of Banking & Finance*

Received Date: 16 September 2013

Revised Date: 15 February 2014

Accepted Date: 27 March 2014



Please cite this article as: Brei, M., Schclarek, A., A theoretical model of bank lending: Does ownership matter in times of crises?, *Journal of Banking & Finance* (2014), doi: <http://dx.doi.org/10.1016/j.jbankfin.2014.03.038>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

# A theoretical model of bank lending: does ownership matter in times of crises?\*

Michael Brei<sup>†</sup>      Alfredo Schclarek<sup>‡</sup>

February 2014

## Abstract

The present study investigates theoretically the lending responses of government-owned and private banks in the event of unexpected financial shocks. Our model predicts that public banks provide more loans to the real sector during times of crises, compared to private banks which cut down on lending and increase liquidity holdings. We put forth three reasons for this heterogeneous behavior. First, the objective of public banks, in contrast to their private peers, is not only to maximize profits given risks, but also to stabilize and promote the recovery of the economy. Second, public banks may suffer less deposit withdrawals or avoid a bank run in a severe crisis, because the state has better access to additional funds making a recapitalization more likely. And finally, public banks may suffer less deposit withdrawals due to their higher credibility in promising a future recapitalization in the case of a severe crisis.

*JEL classification:* G01; G21; G28; H81; E51; E44

*Keywords:* financial crises; bank lending; public banks; bank runs; monetary policy.

---

\*We would like to thank Daniel Heymann, Diego Bastourre, Alejandro Gay, Jorge Oviedo, Kurt von Mettenheim, Olivier Butzbach, Lars Norden, Ata Can Bertay, Luis Giorgio and Mauro Alessandro for comments and discussions; and seminar participants at Central Bank of Brazil VIII Annual Seminar on Risk, Financial Stability and Banking (Sao Paulo, Brazil), SASE 24th Annual Conference (Boston, USA), 16th Annual LACEA Meeting 2011 (Santiago de Chile, Chile), XLV Reunion Anual Asociación Argentina de Economía Política (Buenos Aires, Argentina), 12th Annual GDN Conference (Bogota, Colombia), Workshop "The pro-development role of banking and finance in the economic periphery" (Bellagio, Italy), and the Lunch Seminar at University Paris Ouest. All errors and omissions remain those of the authors.

<sup>†</sup>EconomiX, University Paris Ouest, France; e-mail: michael.brei@uni-bonn.de.

<sup>‡</sup>Department of Economics, Universidad Nacional de Córdoba, Argentina and CON-ICET; e-mail: alfredo@eco.unc.edu.ar; www.cbaeconomia.com.

## Contents

<b>1</b>	<b>Introduction</b>	<b>3</b>
<b>2</b>	<b>The theoretical model</b>	<b>6</b>
2.1	General framework . . . . .	6
2.2	Entrepreneurs . . . . .	7
2.3	Private and public banks . . . . .	8
2.4	Depositors . . . . .	10
2.5	Solution by backward induction . . . . .	11
2.5.1	Solution of period 1 . . . . .	11
2.5.2	Solution of period 0 . . . . .	13
2.6	Optimal behavior of agents in period 1 . . . . .	14
2.7	Recapitalization of public banks in period 1 . . . . .	20
<b>3</b>	<b>Concluding remarks</b>	<b>23</b>
<b>A</b>	<b>Appendix</b>	<b>25</b>
A.1	Proof $E_0(\delta_{PR}) = 1$ . . . . .	25

## 1 Introduction

Since the onset of the global financial crisis, the balance sheets of banks worldwide have continuously come under stress. The freezing of money markets, the significant asset write downs and the associated fall in bank capital have led to liquidity and solvency problems in many financial systems, especially in the United States and Europe. Accordingly, central banks have acted as lenders of last resort, and they have also intervened in money markets as dealers of last resort by directly buying up toxic assets (Mehrling, 2011). In addition, the fiscal authorities have implemented rescue programs involving individually targeted capital injections or debt guarantees, and system-wide interventions such as increases in deposit insurance. In some extreme but not isolated cases, we have even seen nationalizations of private banks, such as in the case of Iceland, England or Ireland. The justification for the state intervention has not only been to prevent the bankruptcy of systemically important institutions but also that the injections of capital and liquidity allow banks to supply more credit to the productive sector.

One of the major risks of a cut back in lending is that the problems in the financial sector end up becoming a problem in the real sector due to the difficulties of firms to obtain bank credit to finance profitable investment projects. Through this channel, a strictly financial crisis spreads to the real sector, worsening the general economic situation and potentially creating a backlash on the financial sector. It appears that the different types of capital and liquidity provisions for banks have prevented the collapse of the financial sector, but it is not clear whether they have been successful in increasing productive credit to the real sector, or whether they have made the financial system safer and more stable. There is evidence that bank balance sheet strength plays an important role in determining banks' responses to a financial crisis, in the sense that banks with higher capitalization and/or lower dependence on wholesale funding may counteract a potential credit crunch that spills over to the real sector, see amongst others (Ivashina and Scharfstein, 2010; Allen and Paligorova, 2011; Puri et al., 2011; Jimenez et al., 2012; Holton et al., 2012; Brei et al., 2013).

Concurrently, the role that government-owned banks may play in the financial system and in the economy in general has come to attract more attention, following a prolonged period of financial liberalization. Indeed, there has been a continuous move towards financial privatization since the 1970s, both in advanced and emerging economies alike, based on the view that liberalized banking sectors are associated with a more efficient, competitive, and sounder financial system (see, amongst others, (Krueger, 1974; Shleifer and Vishny, 1994; LaPorta et al., 2002)). The main argument of this line of thought is that government control of banks tends to be associated with distortions in the allocation of savings, because banks' decisions are biased by political objectives resulting in politically connected lending prob-

lems. The recent experience with the global financial crisis, however, has put this view into question, since a number of highly privatized banking systems, such as the United States and the United Kingdom, have collapsed. Indeed, there exists increasing evidence that public banks have played an important counter-cyclical role in the banking system, helping the economies to recover from financial turmoil (see, amongst others, Allen et al. (2013), Bertay et al. (2012), Brei and Schclarek (2013), and Haas et al. (2012)). Thus, without denying that public banks may be more inefficient than private banks and that advances in institutional quality are needed, it is necessary to reassess the costs and benefits of state-owned banks.

The recent empirical literature on public banks focuses on the cyclical properties of bank lending using information on the financial statements of large samples of banks. The work of Micco and Panizza (2006) suggests that lending by public banks is less pro-cyclical than that of private banks. Similar results are reached by Bertay et al. (2012) who find evidence that public bank lending is less adversely affected during economic downturns than private bank lending, while during booms private banks' lending tends to outpace that of public banks. A related strand of literature focuses on the differential crisis responses of private and public banks. Several cross-country studies suggest that public banks may play a stabilizing role during financial crises, by providing more lending to the economy than their private competitors relative to normal times (see, amongst others, Allen et al. (2013), Brei and Schclarek (2013), Haas et al. (2012), and Cull and Peria (2013)). In addition, the evidence for the pro-active role of banks during crises is supported by a number of country-specific studies (see, Coleman and Feler (2012) for Brazil, Foos (2009) for Germany, Lin et al. (2012) for Japan, Davydov (2013) for Russia, Leony and Romeu (2011) for South Korea, and Onder and Ozyldirim (2013) for Turkey).

The theoretical literature is much less abundant with some notable exceptions. Andrianova et al. (2008) develop a locational model of banking that distinguishes between public and private banks. They show that public banks can play an important role in the banking system but this depends on the institutional quality of a given country. More specifically, in the presence of opportunistic private banks and poor institutional quality, the nonexistence of state banks may lead to financial disintermediation. Andries and Billon (2010) build a theoretical model in which banks face a risk of failure in bad states of the economy, i.e. when productive firms suffer a low productivity state. They put forth that public banks have a more stable deposit base, because depositors perceive that their funds are better protected in times of crisis in the case of public banks. This mechanism helps government-owned banks to insulate their slowdown of lending from downturns when the economy is hit by a financial shock.

Against these backdrops, our study investigates the differential lending responses of public and private banks from a theoretical perspective. In

particular, we develop an overlapping generation model of three periods in which depositors, firms, and private and public banks interact, based on the consumer liquidity demand model of Allen and Gale (1998) and the firm liquidity demand model of Holmstrom and Tirole (1998). However, instead of focusing on the consumption preferences of depositors and borrowers' net worth, the focus of our analysis is set on the portfolio allocation decisions of banks in response to the riskiness of the borrowing firms' investment project. Depending on the size of the riskiness of the investment project, banks decide to grant a certain amount of productive lending (illiquid asset) and hold a proportion of liquid funds. In a crisis, when faced by a mild negative shock, which imply a moderate increase in the riskiness of the investment project, banks partially liquidate the investment projects and increase their liquid asset holdings. However, in a severe crisis, when the increase in the riskiness is large, depositors run on banks and the entire investment project has to be liquidated. In other words, a bank's role of a stable liquidity provider during crises, owing to inflows of funds from investors, which seek a safe haven during market stress (Kashyap et al., 2002; Gatev and Strahan, 2006) may break down during a severe crisis (Acharya and Mora, 2012). In addition, we investigate how an actual and/or promised future bank recapitalization may avoid a bank run. Note also that we model a crisis period through an exogenous increases in the variance of the return of the investment project.<sup>1</sup> Although we do not deny that a crisis episode usually brings about a reduction in expected asset returns, we also view crises episodes as a regime in which the system suffers high aggregate uncertainty and thus high volatility. Further, by focusing on the variance, we are better able to model banks' portfolio allocation changes between liquid and illiquid assets and, as will become clearer below, distinguish between public and private banks.

Using the above stated framework we model the differential crisis responses of private and public banks as a function of different levels of risks in the economy. We model three possible causes by which the portfolio allocation and lending responses might differ. First, public banks might be less risk averse than private banks and more willing to accept riskier lending in an economic downturn, because their objective is not only to maximize profits given risk, but also to sustain growth by the supply of lending to the economy. This implies that, in response to an increase in risk, public banks prefer an asset portfolio with a higher proportion of loans to entrepreneurs and less liquid asset holdings compared to private banks. Second, public banks may suffer less deposit withdrawals, or even avoid a bank run, in a severe crisis, because their owners have more financial resources for a recap-

---

<sup>1</sup>Other papers that incorporate time-varying variance include (Brunnermeier and Pedersen, 2009), (Brunnermeier and Sannikov, 2013), (He and Xiong, 2012) and (Morris and Shin, 2009)

italization, or are more willing to recapitalize their banks, compared to the shareholders of private banks. And finally, depositors perceive that public banks have a higher probability of being recapitalized in the future in the case of a severe crisis, and, thus, are less willing to withdraw funds or run on public banks.

The paper is organized as follows. In section 2 we present the theoretical model and the final section 3 concludes.

## 2 The theoretical model

This section presents a theoretical model that offers a framework to model the differential behavior across private and public banks in times of crises. The model offers a potential description of the conditions and mechanisms giving rise to the empirical evidence presented in the introduction. Concretely, the model allows us to replicate the observed behavior: in times of crisis, private banks reduce their productive lending to a higher extent than public banks.

The model presents three possible explanations for the observed behavior. First, the objective of public banks, in contrast to private banks, is not only to maximize profits given risk, but also to stabilize and promote the recovery of the economy in a downturn. That is, they take into account externalities and social welfare opposed to private banks. Second, public banks might suffer less deposit withdrawals than private banks during crisis periods, or their presence might even avoid a bank run in a severe crisis, because public banks are more likely to be recapitalized, since the fiscal authorities tend to have more financial resources than private shareholders. And finally, public banks may suffer less deposit withdrawals, because private depositors perceive them as safer and find a future recapitalization in times of distress more likely. We start by analyzing the first argument in the following subsections and address the final two explanations in subsection 2.7.

### 2.1 General framework

The economy is characterized by a simple overlapping generation model of three-period-lived agents. It is populated by four types of agents: entrepreneurs (firms), private and public banks, and depositors (consumers). Banks act as financial intermediaries collecting deposits from consumers, and in turn they lend out the funds to entrepreneurs. There is a continuum of entrepreneurs with unit mass, where each entrepreneur has access to an investment project that requires an initial variable investment in period 0 and generates a payoff in period 2. In addition, there are continuums of private and public banks with unit mass, where each bank is endowed with an initial amount of liquid funds in period 0 and no endowments in periods 1

and 2. Banks maximize their profits by receiving funds from depositors and choosing their asset portfolio which is composed of credit to entrepreneurs and liquid funds. Finally, there is a continuum of depositors with unit mass, where each depositor is endowed with an initial amount of funds in period 0 and no funds in periods 1 and 2.

## 2.2 Entrepreneurs

We assume that entrepreneurs have access to an investment project but they require borrowing from banks in order to finance the project. For simplicity reasons, and without affecting our results, we assume that entrepreneurs do not earn the return of the project in period 2, but rather the banks receive all the proceeds. The investment project requires an initial variable investment  $I$  in period 0 and it returns a stochastic amount  $RI$  in period 2, where  $R \geq 0$  is the gross rate of return of the project. Although  $R$  is realized in period 2, its real expected value,  $E(R)$ , is known in period 0 with accuracy, with  $E(R) \geq 1$ . However, the true variance of  $R$  is not known with certainty until period 2, i.e. the exact distribution function is uncertain in period 0.

In period 0, agents have beliefs about the real variance of the project,  $V_0(R)$ . In the intermediate period 1, agents observe a signal which can be thought of as a leading economic indicator. This signal predicts with perfect accuracy the real variance of  $R$ . Thus, after the signal is observed, agents revise their beliefs about the variance of the investment project in accordance to the true variance  $V_1(R)$ . Both the deposit contract between consumers and banks and the credit contract between banks and firms is contingent on the leading economic indicator, and hence on the real distribution function of  $R$ . We define normal times when the real value of the variance is equal or smaller to the beliefs of period 0 ( $V_1(R) \leq V_0(R)$ ). When the variance is larger than this threshold we have two regions, the crisis region and the severe crisis region. We discuss these thresholds in more detail in section 2.6.

In period 1, the investment project may be partially liquidated by banks depending on the realized risk level. Let  $0 \leq \delta \leq 1$  denote the continuation scale in period 1. In the case of a partial liquidation, a proportion of  $(1 - \delta)I$  of the investment project is liquidated and converted into risk-free liquid funds, and  $\delta I$  represents the downsized investment project. We assume that partial liquidation gives less than one unit of liquidity for each unit liquidated, i.e. liquid funds from partial liquidation is  $\beta(1 - \delta)I$ , where  $\beta < 1$ . Note that the partial liquidation and conversion into liquid funds may be interpreted as if the bank had promised the entrepreneur a credit line in period 0 to be used in period 1 but, once the signal is observed, it reduces the actual credit line.<sup>2</sup> Finally, it is only the portion of the project that

<sup>2</sup>Alternatively, it can be interpreted as if the entrepreneur saves some liquid funds to be used in period 1, but that the bank demands him to pay back part of the credit that



is continued that produces a return in period 2 implying that the realized total return is  $\delta RI$ .

### 2.3 Private and public banks

We assume that both private and public banks are risk averse and that they have the same initial level of capital  $A0$  in period 0, i.e.  $A0_{PR} = A0_G = A0$ , where the subscript  $PR$  indicates that the variable corresponds to a private bank and the subscript  $G$  indicates that the variable corresponds to a public or government-owned bank. Banks invest their initial endowment and the received deposits in a portfolio of credits to the entrepreneurs and liquid assets. While the liquid assets are free of any risk, the loans granted to the entrepreneurs are risky in the sense that the realized returns of the investment projects are uncertain. We assume for simplicity that the investment projects that are supported by private and public banks have the same expected return and variance. Furthermore, we assume that banks keep the entire proceeds of the investment projects, i.e. the interest rate paid out to depositors and entrepreneurs is zero.

The expected utility of banks depends on the mean and the variance of the portfolio returns given by  $E(U) = E(R_P) - \frac{\gamma}{2}V(R_P)$ , where  $R_P$  is the return of the portfolio and  $\gamma$  is a positive risk aversion parameter.<sup>3</sup> The expected utility of the private bank in period 0 can be represented as follows:

$$E(U_{PR}) = E_0(\delta_{PR})E(R)I_{PR} + \beta(1 - E_0(\delta_{PR}))I_{PR} + S0_{PR} - \frac{\gamma}{2}E_0(\delta_{PR})^2 I_{PR}^2 V_0(R), \quad (1)$$

where  $E(R)$  is the expected value of the return  $R$ ,  $V_0(R)$  its variance conditional on information in period 0,  $I_{PR}$  the initial investment,  $S0_{PR}$  are liquid asset holdings between period 0 and period 1, and  $E_0(\delta_{PR})$  is the expected fraction of the investment project that will be continued in period 1 conditional on information in period 0. The term  $E_0(\delta_{PR})E(R)I_{PR}$  represents the expected return of the investment project,  $\beta(1 - E_0(\delta_{PR}))I_{PR}$  liquid funds obtained in period 1 after partial liquidation, and  $-\frac{\gamma}{2}E_0(\delta_{PR})^2 I_{PR}^2 V_0(R)$  is the disutility caused by the risk of the investment project.

In the case of public banks, the utility function is similar to that of private banks with the difference that public banks have a disutility from the

---

it lent out in period 0.

<sup>3</sup>These mean-variance preferences are used in models where the environment is uncertain, such as in Mondria (2010), Peress (2010) and Nieuwerburgh and Veldkamp (2010). These preferences lead to the same mean-variance portfolio that obtains under an exponential expected utility function exhibiting constant absolute risk aversion (CARA), so that  $E(R_P) - \frac{\gamma}{2}V(R_P)$  may be rewritten as  $-\frac{1}{\gamma}\ln E(\exp(-R_P\gamma))$ , where  $\gamma = 0$  implies that banks are risk neutral and  $\gamma > 0$  that banks are risk averse. For a more detailed discussion of these types of preferences see Epstein and Zin (1989), Kreps and Porteus (1978), Kreps and Porteus (1979), and Weil (1990), among others.

partial liquidation of investment projects in period 1. The disutility could be justified by the externalities that are caused by partial liquidation such as increases in unemployment benefits, decreases in tax revenues, voters' dissatisfaction or simply reflect the aversion of governments to find themselves in the midst of a crisis. Accordingly, public banks maximize their expected utility generated by the asset portfolio and minimize the partial liquidation in period 1. We assume that the expected utility of public banks in period 0 can be represented by:

$$E(U_G) = E_0(\delta_G)E(R)I_G + \beta(1 - E_0(\delta_G))I_G + S0_G - \theta(1 - E_0(\delta_G))I_G - \frac{\gamma}{2}E_0(\delta_G)^2I_G^2V_0(R) \quad (2)$$

where  $-\theta(1 - E_0(\delta_G))I_G$  is the disutility generated by partial liquidation and  $\theta > 0$ . Note also that for simplicity we assume that both private and public banks have the same risk aversion parameter  $\gamma$ .

Given the assumptions, private banks' maximization problem in period 0 is given by:

$$\begin{aligned} \max_{I_{PR}} & E_0(\delta_{PR})E(R)I_{PR} + \beta(1 - E_0(\delta_{PR}))I_{PR} + S0_{PR} \\ & - \frac{\gamma}{2}E_0(\delta_{PR})^2I_{PR}^2V_0(R) \end{aligned} \quad (3)$$

s.t.  $I_{PR} + S0_{PR} \leq D0_{PR} + A0,$

where the balance sheet constraint implies that the sum of deposits and banks' own capital are invested into loans to entrepreneurs and liquid assets.

In period 1, the maximization problem of private banks is given by:

$$\begin{aligned} \max_{0 \leq \delta_{PR} \leq 1} & \delta_{PR}E(R)I_{PR} + \beta(1 - \delta_{PR})I_{PR} - \frac{\gamma}{2}\delta_{PR}^2I_{PR}^2V_1(R) \\ \text{s.t.} & D0_{PR} - D1_{PR} \leq S0_{PR} + (1 - \delta_{PR})I_{PR}, \end{aligned} \quad (4)$$

where  $D0_{PR} - D1_{PR}$  represent deposit withdrawals of the consumers from the private bank in period 1, and  $S0_{PR} + \beta(1 - \delta_{PR})I_{PR}$  represent liquid assets held by private banks in period 1 defined, by the sum of liquid asset holdings of period 0, and the amount banks receive from entrepreneurs in response to the partial liquidation in period 1. Note that the funds, which the bank receives from partial liquidation, can be used for the repayment of deposits when the stock of liquid funds from period 0,  $S0_{PR}$ , is not high enough to cover the repayment of deposits (discussed bellow).

The maximization problem of public banks in period 0 is given by:

$$\begin{aligned} \max_{I_G} & E_0(\delta_G)E(R)I_G + \beta(1 - E_0(\delta_G))I_G + S0_G \\ & - \theta(1 - E_0(\delta_G))I_G - \frac{\gamma}{2}E_0(\delta_G)^2I_G^2V_0(R) \end{aligned} \quad (5)$$

s.t.  $I_G + S0_G \leq D0_G + A0,$

and that of period 1 is:

$$\begin{aligned} & \max_{0 \leq \delta_G \leq 1} \delta_G E(R)I_G + \beta(1 - \delta_G)I_G - \theta(1 - \delta_G)I_G & (6) \\ & \quad - \frac{\gamma}{2} \delta_G^2 I_G^2 V_1(R) \\ \text{s.t.} \quad & D0_G - D1_G \leq S0_G + (1 - \delta_G)I_G. \end{aligned}$$

## 2.4 Depositors

It is assumed that consumers/depositors value consumption only in period 2 and that they are risk neutral. Thus, their expected utility can be summarized by  $E(U) = E(C_2)$ , where  $C_2$  denotes the consumption in period 2. Note that this setup assumes away intertemporal consumption decisions and risk aversion by consumers, as our model does not analyze the consequences of uncertainty in the intertemporal preferences of consumers on the banking sector, as in Diamond and Dybvig (1983). Consumers have an initial amount of liquid funds,  $LF$ , in period 0 and they do not receive any additional funds in periods 1 and 2.

Regarding consumers' portfolio decision between deposits and cash, we assume that consumers prefer depositing in banks over holding funds in cash, when the expected payoff per unit of deposit is at least unity. In other words, they are ready to deposit all their funds in the bank in period 0 as long as they expect to receive the same amount in period 2. However, if this condition is not met by the banks, consumers will raw al of their funds. Moreover, we assume that the interest rate on deposits is 0. Note that our results do not change if we assume that they receive an infinitesimal small positive interest rate. The maximization problem of consumers in period 0 can thus be summarized by:

$$\begin{aligned} & \max_{C_2} E(C_2) & (7) \\ \text{s.t.} \quad & C_2 \leq D0_{PR} + D0_G + LF0, \quad D0_G + D0_{PR} + LF0 = LF, \\ & D0_{PR} \leq E_0(\delta_{PR})E(R)I_{PR} + \beta(1 - E_0(\delta_{PR}))I_{PR} + S0_{PR}, \\ & D0_G \leq E_0(\delta_G)E(R)I_G + \beta(1 - E_0(\delta_G))I_G + S0_G, \end{aligned}$$

where  $D0_{PR}$  denotes deposits at private banks,  $D0_G$  deposits at public banks,  $LF0$  cash holdings, and  $LF$  the initial endowment. The first constraint is the consumers' budget constraint, while the second constraint is the deposit-cash allocation constraint. The last two conditions represent a deposit constraint that implies that consumers only deposits in banks, if the expected resources available in period 2 are enough for paying back the amount deposited. If the expected resources available in period 2 are not enough, depositors will not deposit any funds in banks.

The maximization problem of consumers in period 1 is given by:

$$\max_{C_2} E(C_2) \quad (8)$$

s.t.

$$C_2 \leq D1_{PR} + D1_G + LF1$$

$$D1_{PR} + D1_G + LF1 = D0_{PR} + D0_G + LF0$$

$$D1_{PR} \leq \delta_{PR}E(R)I_{PR} + \beta(1 - \delta_{PR})I_{PR} + S0_{PR}, \quad (9)$$

$$D1_G \leq \delta_G E(R)I_G + \beta(1 - \delta_G)I_G + S0_G, \quad (10)$$

The first constraint represents the consumers' budget constraint, while the second constraint ensures that the sum of deposits and cash is equal in period 0 and period 1. The final two conditions represent the deposit constraint. Note that if the realized value of  $\delta$  is high enough, so that the deposit constraint is not complied, i.e.  $\delta E(R)I + \beta(1 - \delta)I + S0 < D1$ , there will be a bank run, and depositors withdraw all their funds from banks, and banks are forced to liquidate the investment project in order to meet the demand for liquidity in period 1.<sup>4</sup> In this case, we have that  $\delta = 0$  and the liquid resources left to pay depositors are  $\beta(1 - \delta)I + S0$ . This result is similar to the bank run in Allen and Gale (1998).

## 2.5 Solution by backward induction

In order to solve for the optimal behavior of agents in period 0, we first need to solve the optimization problems in period 1. Subsequently, we can then solve the optimization problems of banks and depositors in period 0.

### 2.5.1 Solution of period 1

Several cases have to be distinguished, when solving the maximization problem 8 of consumers in period 1, since they might or might not withdraw their deposit holdings in private and public banks. In the first case, the deposit constraints for private and public banks 9 and 10 are not violated and we have:

$$\{D1'_{PR}, D1'_G, LF1'\} = \{D0_{PR}, D0_G, LF0\}, \quad (11)$$

where  $D1'_i, i = PR, G$  denotes deposit holdings at private and public banks in period 1, and  $LF1'$  denotes cash holdings. In this case, no deposit outflows occur. In the second case, the deposit constraint for private banks (condition

<sup>4</sup>We follow the literature on banks run, such as Diamond and Dybvig (1983), where a sequential service constraint implies that depositors withdraw their deposits one after the other until the bank is unable to meet any further demand. The sequential service constraint gives depositors an incentive to be the first to run in order to get the full value of their deposits back.

9) is violated, while the deposit constraint for public banks (condition 10) is not violated and it follows that:

$$\{D1'_{PR}, D1'_G\} = \{0, D0_G\} \quad (12)$$

and

$$LF1' = LF0 + D0_{PR}. \quad (13)$$

Finally, when the two deposit constraints for private and public banks (conditions 9 and 10) are both violated, it follows that

$$\{D1'_{PR}, D1'_G\} = \{0, 0\} \quad (14)$$

and

$$LF1' = LF0 + D0_{PR} + D0_G. \quad (15)$$

Next we have to solve the maximization problems of private banks (equation 4) and public banks (equation 6) distinguishing between the three cases outlined above. First, if the two deposit constraints for private and public banks are not violated, it follows that:

$$\delta'_{PR} = \frac{E(R) - 1}{I_{PR}\gamma V_1(R)} \quad (16)$$

and

$$\delta'_G = \frac{E(R) - 1 + \theta}{I_G\gamma V_1(R)}. \quad (17)$$

Second, if only the deposit constraint of private banks is violated and that of public banks is not violated we obtain that:

$$\delta'_{PR} = 0 \quad (18)$$

and

$$\delta'_G = \frac{E(R) - 1 + \theta}{I_G\gamma V_1(R)}. \quad (19)$$

Finally, if the two deposit constraints for private and public banks are binding, it follows that:

$$\delta'_{PR} = 0 \quad (20)$$

and

$$\delta'_G = 0. \quad (21)$$

### 2.5.2 Solution of period 0

Given the optimality conditions for period 1, we now characterize the optimal behavior of consumers and banks in period 0. Starting with the maximization problem 7 of consumers, it follows that the optimal amount of funds deposited in private and public banks  $D0_{PR}^*, D0_G^*$  and the optimal cash holdings  $LF0^*$  verify the following conditions:

$$\{D0_{PR}^*, D0_G^*\} = \begin{cases} \{LF/2, LF/2\} & \text{if } E(R) \geq 1 \\ 0, 0 & \text{if } E(R) < 1 \end{cases} \quad (22)$$

and

$$LF0^* = \begin{cases} 0 & \text{if } E(R) \geq 1 \\ LF & \text{if } E(R) < 1 \end{cases} \quad (23)$$

which imply that consumers will only deposit in banks, if the expected gross return of the investment project per unit of investment is high enough to pay back each unit of deposit. Note also that the optimal proportion of deposits in private and public banks is undetermined and we assume that consumers equally distribute their deposits across private and public banks, i.e.  $D0_{PR}^* = D0_G^* = D0^*$ . However, the conclusions of the model do not change if we assume a different proportion of deposits across private and public banks.

Private banks maximize their expected utility given in equation 3 by choosing between the optimal level of lending to entrepreneurs,  $I_{PR}^*$ , and liquid asset holdings,  $S0_{PR}^*$ . In the optimum, private banks choose the following asset portfolio composition:

$$\{I_{PR}^*, S0_{PR}^*\} = \left\{ \frac{E(R) - 1}{E_0(\delta_{PR})\gamma V_0(R)}, D0^* + A0 - \frac{E(R) - 1}{E_0(\delta_{PR})\gamma V_0(R)} \right\}. \quad (24)$$

In addition, taking into account solution 16 for partial liquidation of private banks in period 1, and solution 24 for period 0, it is demonstrated in appendix A.1 that the expected fraction of the investment project that is continued in period 1 conditional on information in period 0 is equal to  $E_0(\delta_{PR}) = 1$ , implying that solution 24 can be rewritten as:

$$\{I_{PR}^*, S0_{PR}^*\} = \left\{ \frac{E(R) - 1}{\gamma V_0(R)}, D0^* + A0 - \frac{E(R) - 1}{\gamma V_0(R)} \right\}. \quad (25)$$

Note also that we have assumed that  $D0^* + A0 \geq \frac{E(R)-1}{\gamma V_0(R)}$ . A similar optimality condition is obtained for public banks which maximize their expected utility given in equation 5:

$$\{I_G^*, S0_G^*\} = \left\{ \frac{E(R) - 1}{\gamma V_0(R)}, D0^* + A0 - \frac{E(R) - 1}{\gamma V_0(R)} \right\}. \quad (26)$$

It follows that private and public banks act initially similarly, since their optimal lending to entrepreneurs  $I_{PR}^*$  and  $I_G^*$  and their optimal liquid asset holdings  $S0_{PR}^*$  and  $S0_G^*$  are equal, i.e.  $I_{PR}^* = I_G^* = I^*$  and  $S0_{PR}^* = S0_G^* = S0^*$ .

## 2.6 Optimal behavior of agents in period 1

Given the optimal behavior of agents in period 0, we can now turn to analyze the optimal behavior of agents in period 1, after they have observed the leading economic indicator represented by the realization of the variance of the investment project  $V_1(R)$ . Regarding the behavior of depositors, it follows from solutions 9, 10, 11, 12, 13, 14, 15, 16, 17, 25, and 26 that:

$$D1_{PR}^* = \begin{cases} D0^* & \text{if } V_1(R) \leq \frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0} \\ 0 & \text{if } V_1(R) > \frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0} \end{cases} \quad (27)$$

$$D1_G^* = \begin{cases} D0^* & \text{if } V_1(R) \leq \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A0)(E(R)-1)} \\ 0 & \text{if } V_1(R) > \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A0)(E(R)-1)} \end{cases} \quad (28)$$

$$LF1^* = \begin{cases} LF0^* & \text{if } V_1(R) \leq \frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0} \\ LF0^* + D0^* & \text{if } \frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0} < V_1(R) \leq \\ & \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A0)(E(R)-1)} \\ LF0^* + 2D0^* & \text{if } V_1(R) > \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A0)(E(R)-1)} \end{cases} \quad (29)$$

These results imply that when the true variance  $V_1(R)$  is smaller or equal to the threshold  $\frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0}$ , depositors do not withdraw funds from neither private nor public banks. Thus, deposits in period 1 do not change relative to period 0, i.e.  $D1_{PR}^* = D1_G^* = D0^*$ . Below this threshold value for the true variance  $V_1(R)$ , we have two regions, namely normal times and the crisis period. Note also that this threshold is an increasing function of the initial level of bank capital,  $A0$ , implying that a better capitalized bank is able to withstand a higher negative shock without suffering a bank run. Above this threshold value, we have a severe crisis period. If  $\frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0} < V_1(R) \leq \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A0)(E(R)-1)}$ , depositors run on private banks but do not withdraw any funds from public banks. When  $V_1(R) > \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A0)(E(R)-1)}$ , depositors run on both private and public banks. These results are presented in figure 1.

Regarding private and public banks' optimal choice of partial continuation in period 1, given the solutions 16, 17, 19, 20, 21, 25, 26, 27, 28, and

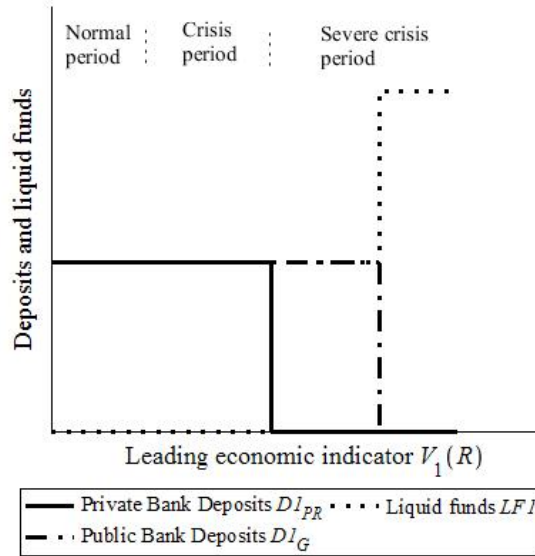


Figure 1: Deposits and liquid funds hold by depositors in period 1

29, it follows that:

$$\delta_{PR}^* = \begin{cases} 1 & \text{if } V_1(R) \leq V_0(R) \\ \frac{V_0(R)}{V_1(R)} & \text{if } V_0(R) < V_1(R) \leq \frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0} \\ 0 & \text{if } V_1(R) > \frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0} \end{cases} \quad (30)$$

$$\delta_G^* = \begin{cases} 1 & \text{if } V_1(R) \leq \frac{(E(R)-1+\theta)V_0(R)}{E(R)-1} \\ \frac{(E(R)-1+\theta)V_0(R)}{(E(R)-1)V_1(R)} & \text{if } \frac{(E(R)-1+\theta)V_0(R)}{E(R)-1} < V_1(R) \leq \\ \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A0)(E(R)-1)} & \\ 0 & \text{if } V_1(R) > \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A0)(E(R)-1)}. \end{cases} \quad (31)$$

This implies that if the realized risk in period 1 is less or equal to the risk perceived in period 0,  $V_1(R) \leq V_0(R)$ , then no partial liquidation occurs. This state is called normal times. Further, if the true variance  $V_1(R)$  lays between the threshold values  $V_0(R)$  and  $\frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0}$ , we have the milder crisis period, because banks partially liquidate the investment project in period 1, but there is no withdrawal of deposits. The severe crisis period occurs



when the true variance  $V_1(R)$  is larger than  $\frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0}$ , and depositors withdraw their deposits from banks which lead to the complete liquidation of the investment project. Note that in the crisis region, it is the optimal behavior of banks that determine partial liquidation, but in the severe crisis region, it is the optimal behavior of depositors that determine the level of partial liquidation. Note also that public banks start partially liquidating the investment project at a higher threshold value of  $V_1(R)$  than private banks. Further, the threshold value for which public banks completely liquidate the investment project is also higher than that of private banks. The results of solutions 30 and 31 are presented graphically in figure 2.

The total lending by banks to entrepreneurs in period 1 is given by  $\delta^* I^*$ . Thus, given the solutions 30 and 31, it follows that the optimal total lending by private banks ( $L1_{PR}^*$ ) and public banks ( $L1_G^*$ ) is:

$$L1_{PR}^* = \begin{cases} I^* & \text{if } V_1(R) \leq V_0(R) \\ \frac{V_0(R)}{V_1(R)} I^* & \text{if } V_0(R) < V_1(R) \leq \frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0} \\ 0 & \text{if } V_1(R) > \frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0} \end{cases} \quad (32)$$

$$L1_G^* = \begin{cases} I^* & \text{if } V_1(R) \leq \frac{(E(R)-1+\theta)V_0(R)}{E(R)-1} \\ \frac{(E(R)-1+\theta)V_0(R)}{(E(R)-1)V_1(R)} I^* & \text{if } \frac{(E(R)-1+\theta)V_0(R)}{E(R)-1} < V_1(R) \leq \\ \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A0)(E(R)-1)} & \\ 0 & \text{if } V_1(R) > \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A0)(E(R)-1)}. \end{cases} \quad (33)$$

These results are presented in figure 3.

We analyze now the liquid asset holdings by private and public banks in period 1. We need to consider the liquid holdings by banks in period 0,  $S0^*$ , given by solutions 25 and 26, the withdrawal of funds by depositors in period 1 given by 27 and 28, and the partial liquidation by banks in period 1 given by solutions 30 and 31. For private banks, it follows that:

$$S1_{PR}^* = \begin{cases} S0^* & \text{if } V_1(R) \leq V_0(R) \\ S0^* + \beta(1 - \delta_{PR}^*) I^* & \text{if } V_0(R) < V_1(R) \leq \frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0} \\ 0 & \text{if } V_1(R) > \frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0} \end{cases} \quad (34)$$

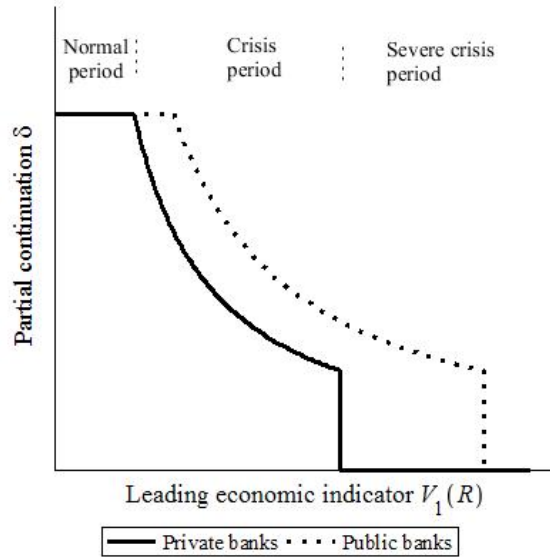


Figure 2: Partial continuation in period 1

where  $(1 - \delta_{PR}^*)I^*$  are the additional liquid asset holdings of private banks implied by the partial liquidation of the investment project. Note also that the threshold  $\frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A_0}$  marks the starting point of the severe crisis period, in which private banks suffer a bank run and leave them without any liquid funds.

For public banks' optimal liquid asset holdings, we have that:

$$S_1^* = \begin{cases} S_0^* & \text{if } V_1(R) \leq \frac{(E(R)-1+\theta)V_0(R)}{E(R)-1} \\ S_0^* + \beta(1 - \delta_G^*)I^* & \text{if } \frac{(E(R)-1+\theta)V_0(R)}{E(R)-1} < V_1(R) \leq \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A_0)(E(R)-1)} \\ 0 & \text{if } V_1(R) > \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A_0)(E(R)-1)} \end{cases} \quad (35)$$

where  $\beta(1 - \delta_G^*)I^*$  are additional liquid funds of public banks obtained by the partial liquidation of the investment project. Note also that when  $V_1(R) \geq \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A_0)(E(R)-1)}$ , the withdrawal of deposits from public banks in period 1 is complete, leaving public banks without any liquid funds. These results are presented graphically in figure 4.

From solutions 30, 31, 32, 33, 34 and 35, it is clear that there are three regions to analyze. The first region corresponds to normal times in which the

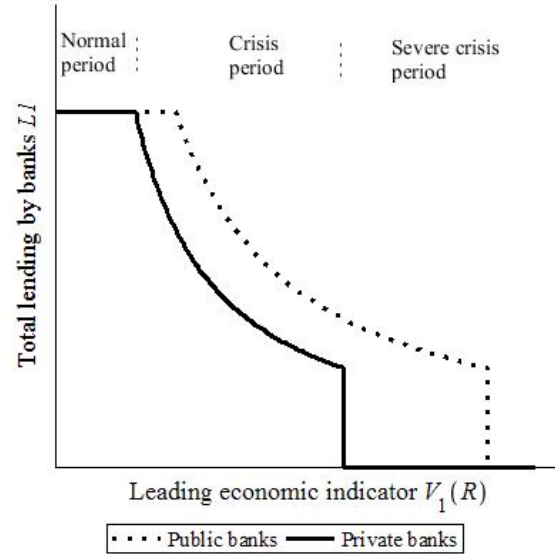


Figure 3: Total lending by banks

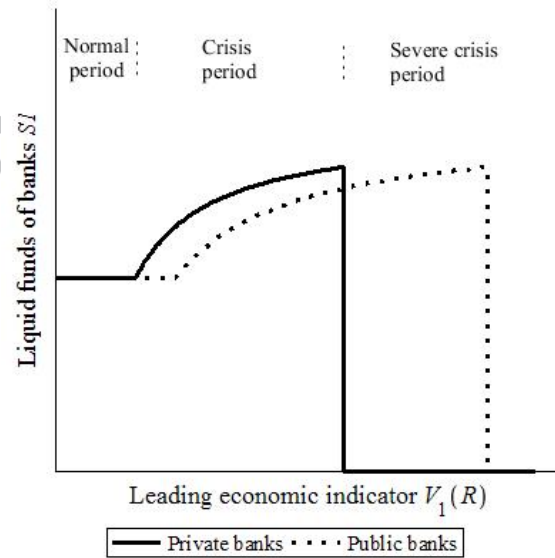


Figure 4: Liquid funds holdings by banks

revised variance is smaller than the prior beliefs, i.e.  $V_1(R) \leq V_0(R)$ . In this region, there is no partial liquidation of the investment project by private and public banks, and depositors do not withdraw any funds from banks. In normal times, private and public banks lend the full amount required by the investment projects and they do not hold any extra liquid assets in period 1 with respect to period 0.

The second region corresponds to a crisis, which occurs when the revised variance is larger than the prior in period 0, but smaller than the threshold at which the withdrawal of deposits is set in motion, i.e.  $V_0(R) < V_1(R) \leq \frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A_0}$ . Within this region we have two subregions given by  $V_0(R) < V_1(R) \leq \frac{(E(R)-1+\theta)V_0(R)}{(E(R)-1)V_1(R)}$  and  $\frac{(E(R)-1+\theta)V_0(R)}{(E(R)-1)V_1(R)} < V_1(R) \leq \frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A_0}$ , respectively. In the first subregion partial liquidation is only carried out by private banks and it is only these that cut back lending to entrepreneurs and start accumulating additional liquid funds. Public banks continue lending to all projects, do not partially liquidate any projects and do not accumulate any additional liquid funds. The reason is that the objective of public banks, in contrast to private banks, is not only to maximize profits, but also to stabilize the economy in response to an adverse shock, due to the disutility that public banks suffer from partial liquidation given by  $\theta$ . In other words, private banks exacerbate the crisis by affecting investment and production, while public banks stabilize the economy. This result can be summarized in the following proposition:

*Proposition 1: During a mild crisis, private banks cut back on lending and hold more liquid assets, while public banks continue lending at the same pace as in normal times and do not accumulate additional liquid funds.*

In the second situation, which could be called a crisis, in contrast to a mild crisis, both private and public banks partially liquidate projects and cut down on lending, while they accumulate additional liquid assets. However, private banks cut down their lending and increase their liquid funds holding by more than public banks. The reason is that public banks dislike partial liquidation of investment projects, which is given by  $\theta$  coefficient. Thus, the asset portfolio of public banks has a higher proportion of loans to firms and less liquid funds than private banks. This result can be summarized in the following proposition:

*Proposition 2: During a crisis, both private and public banks cut down on lending and hold more liquid assets. The reduction in lending of public banks, however, is less pronounced because their objective is also to stabilize the economy.*

Note that the total lending decreases in the value of the revised variance,

i.e. the larger the risk shock, the higher is the slowdown in lending. Note also that within the crisis region, partial liquidation is not affected by the behavior of depositors because they are not withdrawing any funds from banks.

The third region corresponds to a severe crisis, which occurs when the realized variance in period 1 is sufficiently large so that  $V_1(R) > \frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0}$ . Again there are two different subregions. In the first subregion, given by  $\frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A0} < V_1(R) \leq \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A0)(E(R)-1)}$ , depositors withdraw funds from private banks but not from public banks. The reason for the differential response of depositors across private and public banks is that the optimal behavior of public banks is to liquidate a smaller proportion of the investment project than private banks. Thus, the expected resources available in period 2 are larger for public banks than for private banks. This is evident if we consider equations 9 and 10 from where we see that for each unit of investment that is liquidated, the expected resources available in period 2 increases by  $\beta$ , but are reduced by  $E(R)$ , where  $E(R) > 1 > \beta$ . In the second subregion, which occurs when  $V_1(R) > \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A0)(E(R)-1)}$ , both private and public banks experience withdrawals of deposits and they are forced to completely liquidate the investment project to face the demand of liquid funds due to the run of depositors, i.e. all the liquid funds obtained from partial liquidation are handed over to depositors. Note that in this region, it is the withdrawal of deposits that determines partial liquidation and lending, and not the optimal behavior of banks. This result can be summarized in the following proposition:

*Proposition 3: During a severe crisis, both private and public banks cut down on lending because they have no liquid funds left due to the withdrawal of funds by depositors. However, the deposit run affects first private banks, and at a later stage, if the crisis worsens, it also affects public banks.*

## 2.7 Recapitalization of public banks in period 1

The model is now analyzed in the presence of recapitalizations in periods 1 and 2. By modeling explicitly the possibility of a recapitalization in periods 1 and 2, banks will have more funds at their disposal to pay back depositors in period 2 in case of a crisis. In addition, although both types of banks may be recapitalized, we investigate how the results change when only public banks receive a capital injection by the government. Clearly, this assumption implies that depositors are less inclined to withdraw deposits from public banks in period 1. Note that the reason for doing this strong assumption is not to deny that a private recapitalization may be possible, but to acknowledge that the state is usually in a better position or more willing to recapitalize banks than the private sector and, thus, it could be that public

banks have a higher probability of being recapitalized during a crisis. This argument is justified by the observation that the owner of public banks, the fiscal authorities, tend to have more financial resources, or be more willing to recapitalize their banks, than the shareholders of private banks, because governments are also interested in avoiding the negative externalities that a bank run generates. A possible explanation for this observation, that has been argued by Holmstrom and Tirole (1998), is that the state has the capacity to tax agents, while private agents cannot.<sup>5</sup> Another argument is that the shareholders of private banks might shy away from raising equity from stock markets, when stock prices are low. In terms of our model, the funds for the recapitalization may come from a tax levied on consumers that hold liquid funds in periods 1 and/or 2.

We assume that in period 1, the recapitalization of public banks is  $A1_G > 0$  and the expected recapitalization of public banks in period 2 is  $E(A2_G) > 0$ . Note that the recapitalization of public banks only modifies the behavior of consumers, when the expected resources in period 2 are not high enough, i.e. if  $\delta_G E(R)I^* + \beta(1 - \delta_G)I^* + S0 < D1$ . If they are enough without recapitalization, the behavior of depositors is not modified by the recapitalization. Thus, the maximization problem of consumers in period 1 is now given by:

$$\max_{C_2} E(C_2) \quad (36)$$

s.t.

$$C_2 \leq D1_{PR} + D1_G + LF1$$

$$D1_{PR} + D1_G + LF1 = 2D0^* + LF0$$

$$D1_{PR} \leq \delta_{PR}E(R)I^* + \beta(1 - \delta_{PR})I^* + S0^*, \quad (37)$$

$$D1_G \leq \delta_G E(R)I^* + \beta(1 - \delta_G)I^* + S0^* + A1_G + E(A2_G), \quad (38)$$

The recapitalization implies that the expected resources available in period 2 for depositors of public banks is  $\delta_G E(R)I^* + \beta(1 - \delta_G)I^* + S0^* + A1_G + E(A2_G)$ . Thus, the optimal behavior of depositors of public banks, previously given by equation 28, is now

$$D1_G^* = \begin{cases} D0^* & \text{if } V_1(R) \leq \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A0-A1_G-E(A2_G))(E(R)-1)} \\ 0 & \text{if } V_1(R) > \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A0-A1_G-E(A2_G))(E(R)-1)} \end{cases} \quad (39)$$

<sup>5</sup>Actually this taxation capability could even be used by the state to recapitalize private banks in times of distress. This possibility is not analyzed as our objective is to explain potential reasons for which private and public banks might react differently to a crisis. In addition, if private banks are recapitalized by the state, they would become partly public banks.

where the threshold value for which public banks suffer a bank run,  $V_1(R) > \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A_0-A1_G-E(A2_G))(E(R)-1)}$ , is larger than the previous threshold without the recapitalization in period 1,  $A1_G$ , and the expected recapitalization of period 2,  $E(A2_G)$ .

With respect to the optimal lending response of public banks, previously given by equation 33, we now get:

$$L1_G^* = \begin{cases} I^* & \text{if } V_1(R) \leq \frac{(E(R)-1+\theta)V_0(R)}{E(R)-1} \\ \frac{(E(R)-1+\theta)V_0(R)}{(E(R)-1)V_1(R)} I^* & \text{if } \frac{(E(R)-1+\theta)V_0(R)}{E(R)-1} < V_1(R) \leq \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A_0-A1_G-E(A2_G))(E(R)-1)} \\ 0 & \text{if } V_1(R) > \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A_0-A1_G-E(A2_G))(E(R)-1)}. \end{cases} \quad (40)$$

As a result, within the first subregion of the severe crisis case, now given by  $\frac{(E(R)-\beta)V_0(R)}{(1-\beta)I^*-A_0} < V_1(R) \leq \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A_0-A1_G-E(A2_G))(E(R)-1)}$ , public banks do not suffer any deposit withdrawal, while private banks suffer a bank run, not only because public banks partially liquidate less than private banks, but also because public banks are recapitalized in period 1 and consumers expect them to be recapitalized in period 2. These results are presented in figure 5.

And finally, the optimal liquid asset holdings, previously given by equation 35, are now given by:

$$S1_G^* = \begin{cases} S0^* & \text{if } V_1(R) \leq \frac{(E(R)-1+\theta)V_0(R)}{E(R)-1} \\ S0^* + \beta(1 - \delta_G^*)I^* + A1_G & \text{if } \frac{(E(R)-1+\theta)V_0(R)}{E(R)-1} < V_1(R) \leq \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A_0-A1_G-E(A2_G))(E(R)-1)} \\ 0 & \text{if } V_1(R) > \frac{(E(R)-\beta)(E(R)-1+\theta)V_0(R)}{((1-\beta)I^*-A_0-A1_G-E(A2_G))(E(R)-1)}. \end{cases} \quad (41)$$

In our model, the actual recapitalization, or even an expected future recapitalization, is effective in counteracting a bank run in the case of a high adverse risk shock that puts the economy in a severe crisis situation. The avoidance of the bank run implies that the severe crisis situation does not translate into a complete breakdown of the real sector and that, at

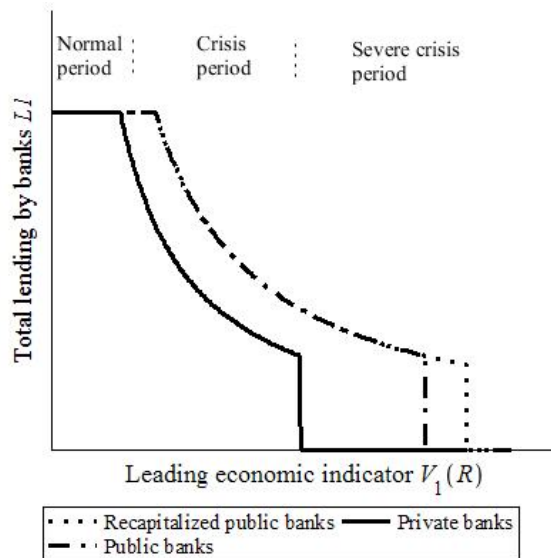


Figure 5: Total lending by recapitalized public banks

least, a fraction of the productive investment projects are realized. Note the important result that the mere possibility of a future recapitalization avoids a bank run with severe real consequences. Accordingly, it might happen that, if the realized gross return  $R$  in period 2 is larger than the expected value of the gross return  $E(R)$ , then there is not even a need for an actual recapitalization in period 2. Furthermore, the promise of a future recapitalization may avoid a bank run in period 1 even if the actual recapitalization in period 2 is not carried out. It is this potential breach that led us to assume that depositors only expect that public banks will be recapitalized in period 2, since we assume that the state is perceived as more credible than private bankers in a severe crisis situation. Of course, if the government is not credible or has a track record of breaking its promises, a more profound credibility analysis should be made, but this is out of the scope of this paper and is left for future research. Further, the possibility of a future recapitalization raises also moral hazard considerations that are not analyzed in this paper.

### 3 Concluding remarks

The present paper examines from a theoretical perspective the lending behavior of private and public banks in response to adverse economic shocks. We develop a theoretical framework that models the interactions of depositors, firms, and private and public banks. The results indicate that lending



during normal times is similar across private and public banks. During a financial crisis, however, private banks' lending decreases to a larger extent than that of public banks. These results indicate that public banks play a counter-cyclical role in their banking systems, while private banks play a more pro-cyclical role.

We propose three possible explanations for the differential lending responses of private and public banks during financial and economic crises. First, their objective function is different. Public banks not only maximize expected profits, but they also experience a disutility when investment projects in the real sector have to be scaled down in response to an unexpected adverse financial shock. Second, public banks may suffer less deposit withdrawals or avoid a bank run in a severe crisis, because the state has better access to additional funds during a severe crisis which makes a recapitalization more likely. And finally, public banks may be subject to less deposit withdrawals due to their higher credibility in promising a future recapitalization in the case of a severe crisis. There is empirical evidence in Brei and Schclarek (2013) indicating that the differential behavior of public and private banks during recent crises is best explained by the difference in their objective function. However, more empirical research is needed to test the importance of these potential explanations before we can be conclusive.

The policy implications of the paper are that, if public banks operate within certain margins of efficiency, then there is a role for an active credit policy through public banks to stabilize the economy during recessions and financial crises. Moreover, public bank lending may be used as a complement to standard monetary and fiscal policy in recessions and crisis times, as they do not only maximize private profits, but also take into account the externalities implied by their lending decisions. In addition, the presence of public banks might decrease the probability of a systemic bank run, as depositors might perceive public banks as safer. Finally, the results have implications for the optimal composition of a banking sector. Clearly, a certain proportion of public banks in the banking sector allows a more effective countercyclical policy.

## A Appendix

### A.1 Proof $E_0(\delta_{PR}) = 1$

Considering solutions 16 and 24 we have that

$$\delta'_{PR} = \begin{cases} 1 & \text{if } V_1(R) \leq \frac{E(R)-1}{\gamma I_{PR}} \\ \frac{E(R)-1}{\gamma I_{PR} V_1(R)} & \text{if } V_1(R) > \frac{E(R)-1}{\gamma I_{PR}} \end{cases} \quad (42)$$

and

$$I_{PR} = \frac{E(R) - 1}{E_0(\delta_{PR})\gamma V_0(R)}. \quad (43)$$

Considering

$$V_1(R) \leq \frac{E(R) - 1}{\gamma I_{PR}}$$

from the threshold of solution 42 and using equation 43, we have that

$$V_1(R) \leq \frac{E(R) - 1}{\gamma \frac{E(R)-1}{E_0(\delta_{PR})\gamma V_0(R)}},$$

which is equal to

$$V_1(R) \leq E_0(\delta_{PR})V_0(R).$$

Rearranging and taking into account that in period 0 it must be true that  $V_1(R) = V_0(R)$ , we get that

$$E_0(\delta_{PR}) \geq 1,$$

which imply that

$$E_0(\delta_{PR}) = 1.$$

QED

## References

- Acharya, V. V. and Mora, N. (2012). Are banks passive liquidity backstops? deposit rates and flows during the 2007-2009 crisis. NBER Working Papers 17838.
- Allen, F. and Gale, D. (1998). Optimal financial crises. *The Journal of Finance*, 53(4):1245–1284.
- Allen, F., Jackowicz, K., and Kowalewski, O. (2013). The effects of foreign and government ownership on bank lending behavior during a crisis in central and eastern europe. Wharton Financial Institutions Center Working Paper No. 13-25.
- Allen, J. and Paligorova, T. (2011). Bank loans for private and public firms in a credit crunch. Bank of Canada Working Papers 2011-13.
- Andrianova, S., Demetriades, P., and Shortland, A. (2008). Government ownership of banks, institutions, and financial development. *Journal of Development Economics*, 85(1-2):218–252.
- Andries, N. and Billon, S. (2010). The effect of bank ownership and deposit insurance on monetary policy transmission. *Journal of Banking & Finance*, 34(12):3050–3054.
- Bertay, A., Demirguc-Kunt, A., and Huizinga, H. (2012). Bank ownership and credit over the business cycle: Is lending by state banks less procyclical? Policy Research Working Paper 6110, World Bank, Washington, DC.
- Brei, M., Gambacorta, L., and von Peter, G. (2013). Rescue packages and bank lending. *Journal of Banking and Finance*, 37.
- Brei, M. and Schclarek, A. (2013). Public bank lending in times of crisis. *Journal of Financial Stability*, forthcoming.
- Brunnermeier, M. K. and Pedersen, L. H. (2009). Market liquidity and funding liquidity. *Review of Financial Studies*, 22(6):2201–2238.
- Brunnermeier, M. K. and Sannikov, Y. (2013). A macroeconomic model with a financial sector. Working Paper, Princeton University.
- Coleman, N. and Feler, L. (2012). Bank ownership, lending, and local economic performance during the 2008 financial crisis. Brown University, Department of Economics, manuscript.
- Cull, R. and Peria, M. S. M. (2013). Bank ownership and lending patterns during the 2008-2009 financial crisis: Evidence from latin america and eastern europe. *Journal of Banking and Finance*, 37(12):4861 – 4878.

- Davydov, D. (2013). Should public banks be privatized? evidence from the financial crisis. University of Vaasa, Department of Accounting and Finance, manuscript.
- Diamond, D. W. and Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91:401–419.
- Epstein, L. G. and Zin, S. E. (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, 57(4):937–69.
- Foos, D. (2009). Lending conditions, macroeconomic fluctuations, and the impact of bank ownership. University of Mannheim, Department of Banking and Finance, manuscript.
- Gatev, E. and Strahan, P. E. (2006). Banks' advantage in hedging liquidity risk: Theory and evidence from the commercial paper market. *The Journal of Finance*, 61:867892.
- Haas, R. D., Korniyenko, Y., Loukoianova, E., and Pivovarsky, A. (2012). Foreign banks and the vienna initiative: Turning sinners into saints? IMF Working Paper 12/117.
- He, Z. and Xiong, W. (2012). Dynamic bank runs. *The Review of Financial Studies*, 25(6):1799–1843.
- Holmstrom, B. and Tirole, J. (1998). Private and public supply of liquidity. *Journal of Political Economy*, 106(1):1–40.
- Holton, S., Lawless, M., and McCann, F. (2012). Firm credit in europe: A tale of three crises. Research Technical Paper 4/RT/12, Central Bank of Ireland.
- Ivashina, V. and Scharfstein, D. (2010). Bank lending during the financial crisis of 2008. *Journal of Financial Economics*, 97(3):319–338.
- Jimenez, G., Ongena, S., Peydro, J. L., and Saurina, J. (2012). Credit supply versus demand: Bank and firm balance-sheet channels in good and crisis times. European Banking Center Discussion Paper No. 2012-003.
- Kashyap, A. K., Rajan, R., and Stein, J. C. (2002). Banks as liquidity providers: An explanation for the coexistence of lending and deposit-taking. *The Journal of Finance*, 57:33–73.
- Kreps, D. M. and Porteus, E. L. (1979). Temporal von neumann-morgenstern and induced preferences. *Journal of Economic Theory*, 20(1):81–109.

- Kreps, D. M. and Proteus, E. L. (1978). Temporal resolution of uncertainty and dynamic choice theory. *Econometrica*, 46(1):185–200.
- Krueger, A. O. (1974). The political economy of the rent-seeking society. *American Economic Review*, 64(3):291–303.
- LaPorta, R., Lopez-De-Silanes, F., and Shleifer, A. (2002). Government ownership of banks. *Journal of Finance*, 57(1):265–301.
- Leony, L. and Romeu, R. (2011). A model of bank lending in the global financial crisis and the case of korea. *Journal of Asian Economics*, 22(4):322 – 334.
- Lin, Y., Srinivasan, A., and Yamada, T. (2012). The bright side of lending by state owned banks: Evidence from japan. National University of Singapore, Department of Finance, manuscript.
- Mehrling, P. (2011). *The New Lombard Street*. Princeton University Press, Princeton, NJ.
- Micco, A. and Panizza, U. (2006). Bank ownership and lending behavior. *Economics Letters*, 93:248–254.
- Mondria, J. (2010). Portfolio choice, attention allocation, and price comovement. *Journal of Economic Theory*, 145(5):1837–1864.
- Morris, S. and Shin, H. S. (2009). Illiquidity component of credit risk. Working Paper.
- Nieuwerburgh, S. V. and Veldkamp, L. (2010). Information acquisition and under-diversification. *Review of Economic Studies*, 77(2):779–805.
- Onder, Z. and Ozyldirim, S. (2013). Role of bank credit on local growth: Do politics and crisis matter? *Journal of Financial Stability*, 9(1):13 – 25.
- Peress, J. (2010). The tradeoff between risk sharing and information production in financial markets. *Journal of Economic Theory*, 145(1):124–155.
- Puri, M., Rocholl, J., and Steffen, S. (2011). Global retail lending in the aftermath of the us financial crisis: Distinguishing between supply and demand effects. *Journal of Financial Economics*, 100(3):556–578.
- Shleifer, A. and Vishny, R. W. (1994). Politicians and firms. *The Quarterly Journal of Economics*, 109(4):995–1025.
- Weil, P. (1990). Nonexpected utility in macroeconomics. *The Quarterly Journal of Economics*, 105(1):29–42.

Highlights “A theoretical model of bank lending: does ownership matter in times of crises?”

- During a crisis, private and public bank lending is reduced in comparison to normal times.
- During a crisis, public banks lend more than private banks.
- One reason is the objective of public banks, in contrast to their private peers, is not only to maximize profits given risks, but also to stabilize and promote the recovery of the economy.
- Also, public banks may suffer less deposit withdrawals or avoid a bank run in a severe crisis, because the state has better access to additional funds making a recapitalization more likely.
- Finally, public banks may suffer less deposit withdrawals due to their higher credibility in promising a future recapitalization in the case of a severe crisis.
- Governments can play an active counter-cyclical role in their banking systems directly through government-owned banks.